



Welcome to

BYJU'S Classes

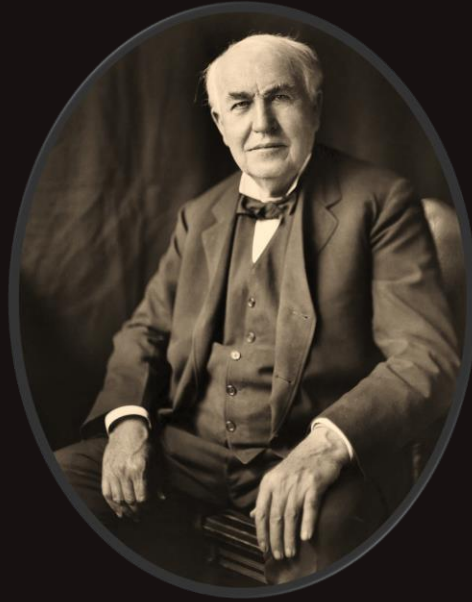


THE CURRENT WAR

B

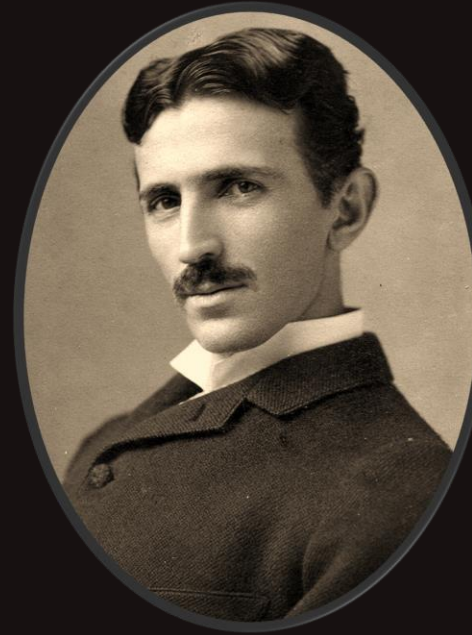


DIRECT CURRENT, 1880s



DC

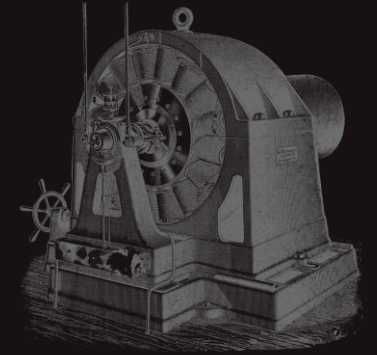
Edison invented a compact electric bulb which could glow for a sustained period of time. Edison was a proponent of DC which was the prevalent form of electric current at that time.



AC

At the same time when Edison discovered the electric bulb, a young student by the name of Nikola Tesla came up with different vision.

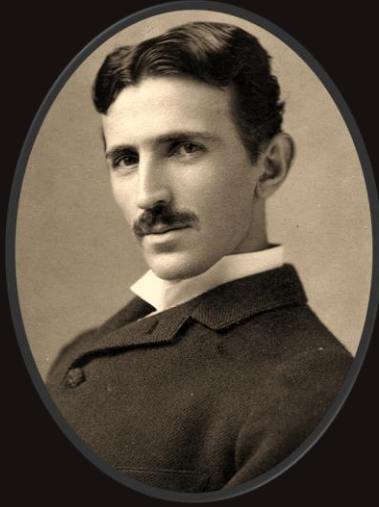
He believed that AC is the more efficient form of current.



THE
ALTERNATING CURRENT SYSTEM,
1888

THE CURRENT WAR

B



NIKOLA TESLA



GEORGE
WESTINGHOUSE

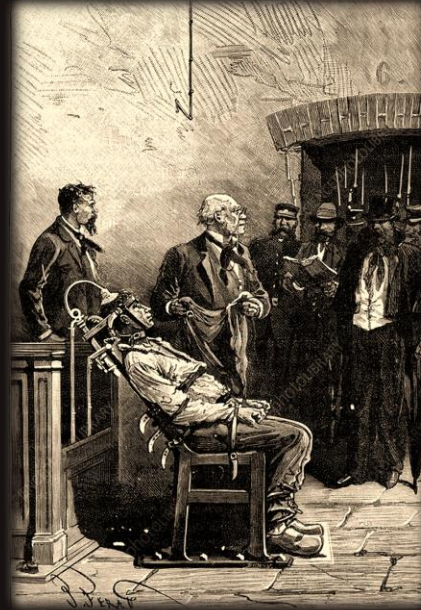
In July 1888

- ~ Tesla pitched his idea about AC motor to George Westinghouse, the richest person of USA at that time, and told him that transmission of electricity would be more efficient using AC.
- ~ George Westinghouse got impressed with Tesla's idea and Tesla started building AC power plants with the help of "Westinghouse Electric co."

THE CURRENT WAR

B

EDISON electrocuted animals and human to show that AC was **too dangerous** to use.



- ~ Since Tesla continuously tried to transmit power at a large distance, either the current or the voltage should be high, since $P = VI$. If current is high, then there will be huge joule heating (I^2R). So, the only option to distribute large power was producing high voltage.
- ~ If voltage becomes high, then that would not be safe and this weakness was exploited by Edison and he started anti AC campaign.
- ~ At the same time, transformers were invented and this helped Tesla a lot. Tesla started using step up and step down transformer when it required to generate or reduce high voltage.

TESLA-WESTINGHOUSE NIAGARA FALLS POWER PLANT(1895)

B

It was Tesla who pitched in, and his AC made it possible and the Niagara project goes to Westinghouse Electric co. Edison had to just watch it and the pivotal battle of the current is won by Tesla's AC.



If we take a video of tube light in slo-mo. We can see the light is not constant. It is constantly fluctuating. This is because the current we get in our houses is AC.

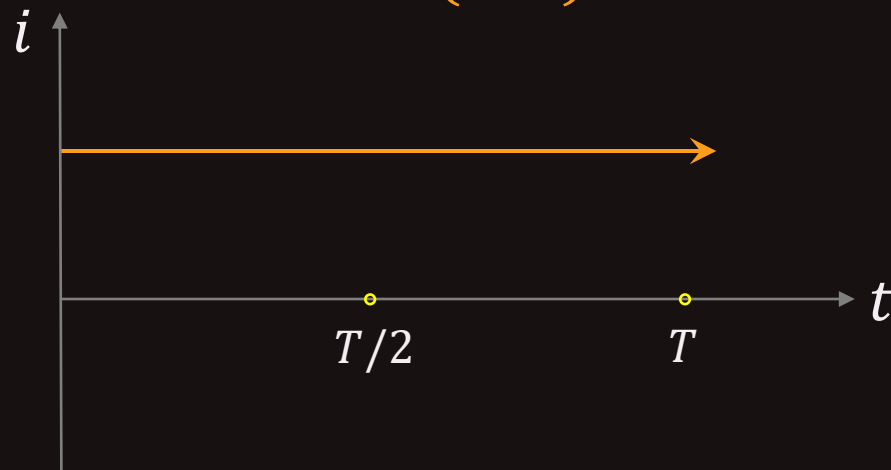


ALTERNATING CURRENT



An electric current which **periodically reverses its direction** in contrast to direct current which flows only in one direction.

Direct Current (DC)



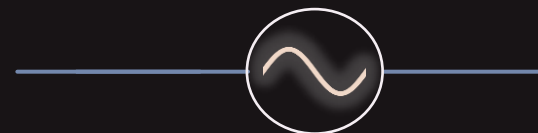
DC Source



Alternating Current (AC)



AC Source

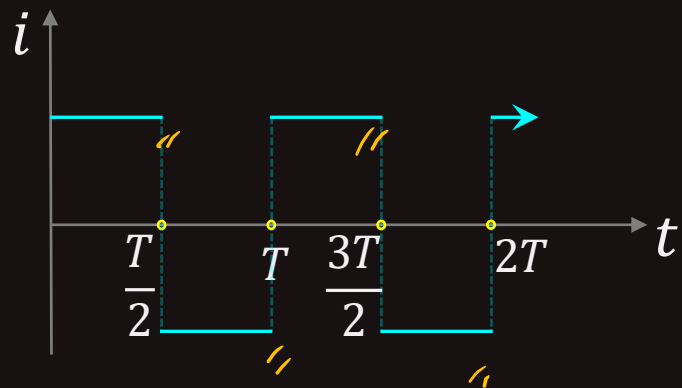




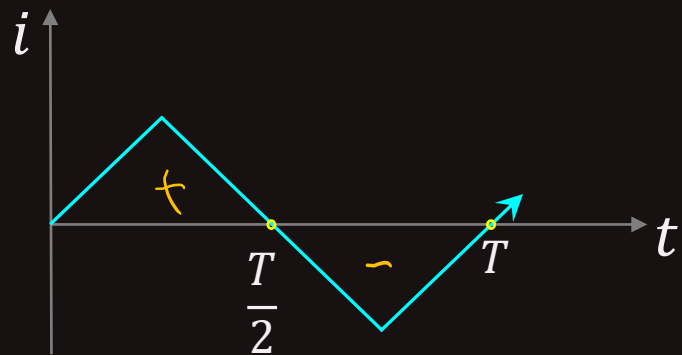
ALTERNATING CURRENT



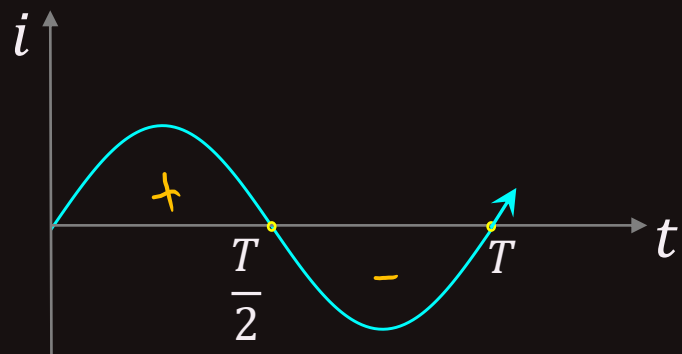
An electric current which **periodically reverses its direction** in contrast to direct current which flows only in one direction.



Square Wave AC



Triangular Wave AC



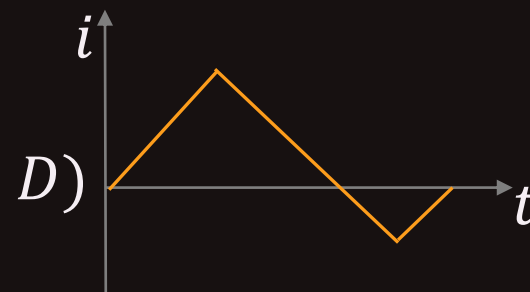
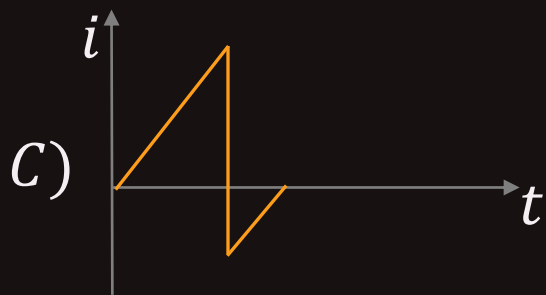
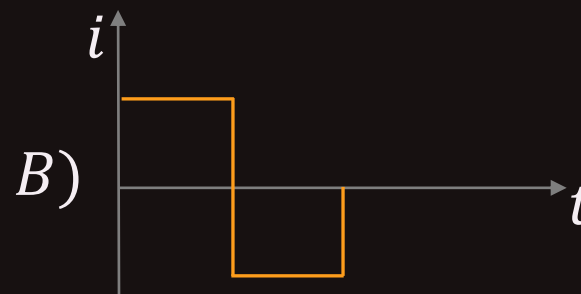
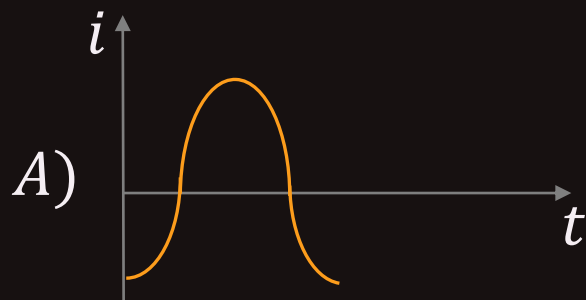
Sinusoidal AC



QUESTION



Variation of current with time for four types of generators are shown in the figures. Which amongst them can be called AC.



only A



A & D



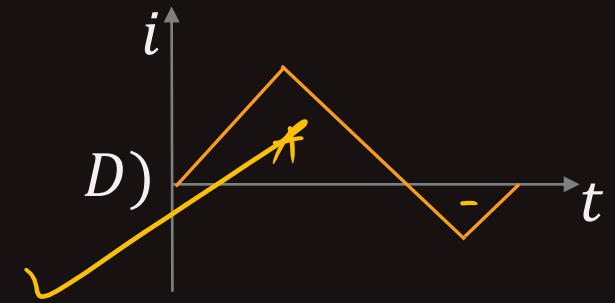
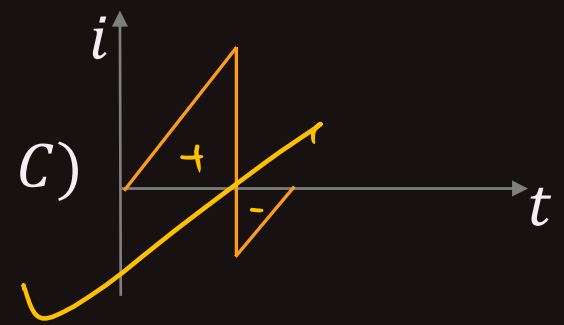
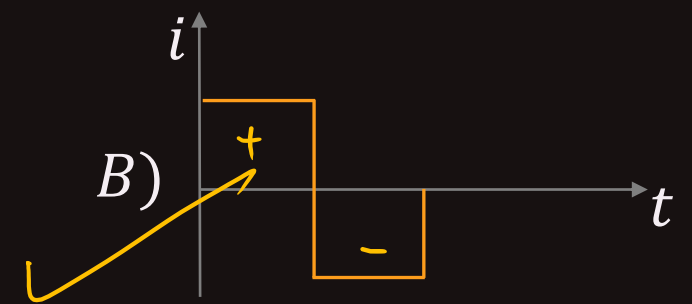
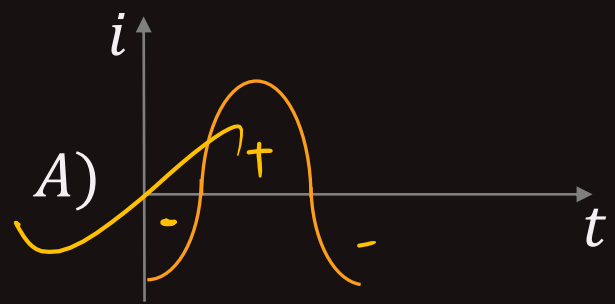
A, B, C, D



A & B



SOLUTION



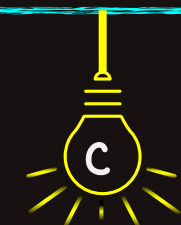
In all the given cases, current is changing direction with time. Hence all these cases are of alternating currents.



only A



A & D



A, B, C, D



A & B

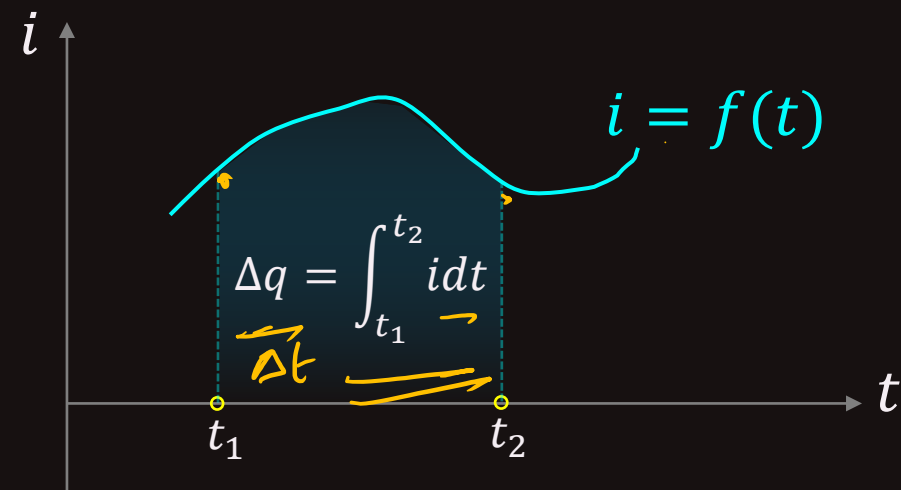


MEAN OR AVERAGE VALUE OF CURRENT



$$\text{Average current } (i_{av}) = \frac{\Delta q}{\Delta t}$$

$$\text{Average current representations: } i_{av} = \langle i \rangle = \overline{(i)}$$



Average current (i_{av}) for time varying current is,

$$i_{av} = \frac{\Delta q}{\Delta t} = \frac{\int_{t_1}^{t_2} i dt}{t_2 - t_1}$$

$$e_{av} = \frac{1}{\Delta t} \int_{t_1}^{t_2} \mathcal{E} \cdot dt$$

★

$$\sim i_{av} = \frac{1}{\Delta t} \int_{t_1}^{t_2} i dt \sim$$

$$\int_{t_1}^{t_2} dt$$



MEAN OR AVERAGE VALUE OF CURRENT

Average value of an AC is equal to that of DC for which the amount of charge that flows in a given amount of time is the same as that of AC.

$$\star \Delta q_{DC} = i_{DC} \Delta t$$

$$\star \Delta q_{AC} = \int_{t_1}^{t_2} i dt$$

$\Delta q_{DC} = \Delta q_{AC}$

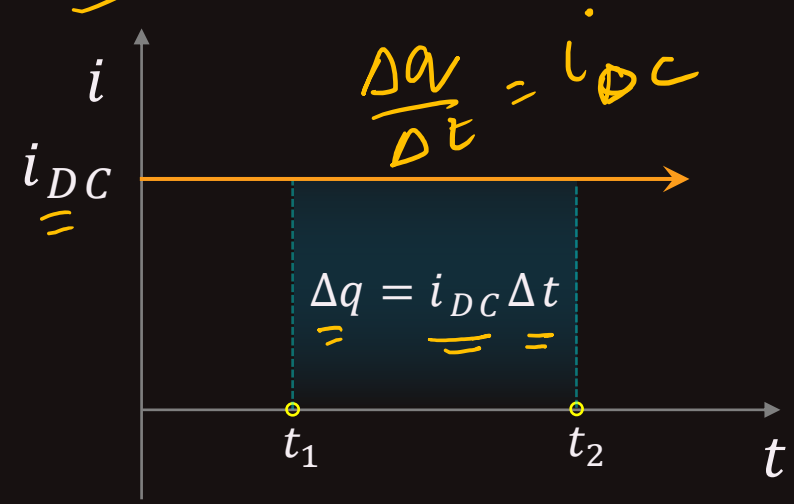
$$i_{DC} \Delta t = \int_{t_1}^{t_2} i dt$$

$$i_{DC} = \frac{1}{\Delta t} \int_{t_1}^{t_2} i dt = (i_{av})$$

If $\Delta q_{DC} = \Delta q_{AC} \Rightarrow i_{av} = i_{DC}$

$$\star \sim i_{av} = \frac{1}{\Delta t} \int_{t_1}^{t_2} i dt \sim$$

B

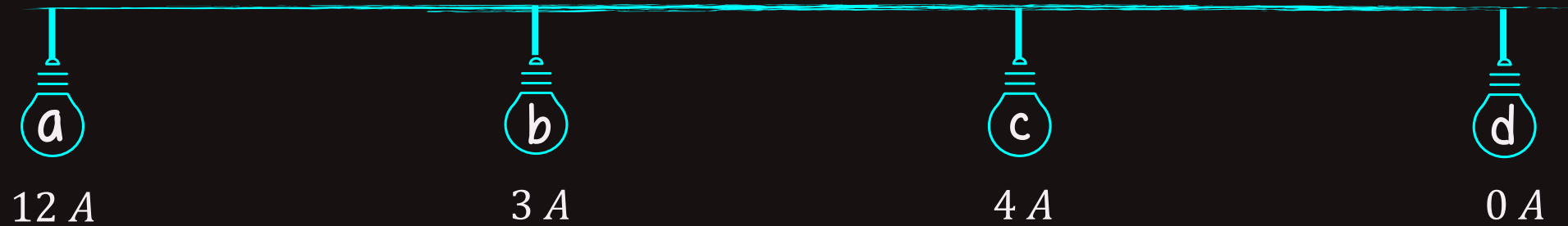




QUESTION



If $i = 3t^2$, find average current in 2 s.





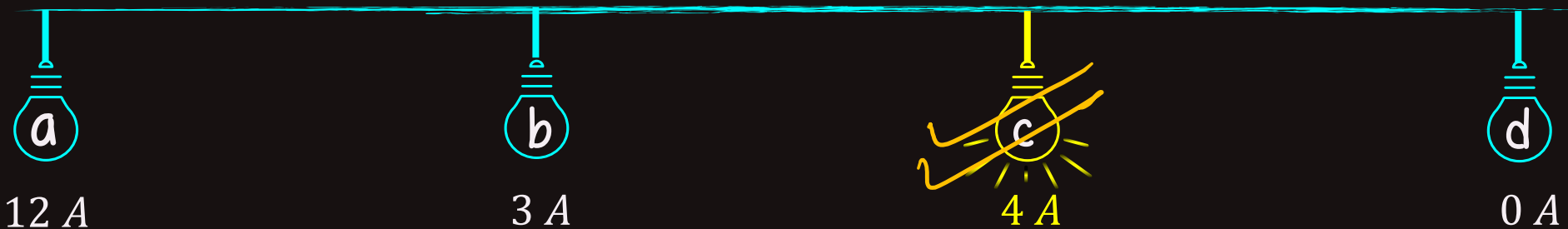
$$i = 3t^2 \rightarrow i_{av} = \frac{1}{\Delta t} \int_{t_1}^{t_2} i dt \leftarrow \begin{matrix} t_1 = 0 \text{ s} \\ t_2 = 2 \text{ s} \end{matrix}$$

$$i_{av} = \frac{1}{2} \int_0^2 3t^2 dt$$

$$i_{av} = \frac{1}{2} \left(\frac{3t^3}{3} \right)_0^2 = \frac{8}{2} \text{ A}$$

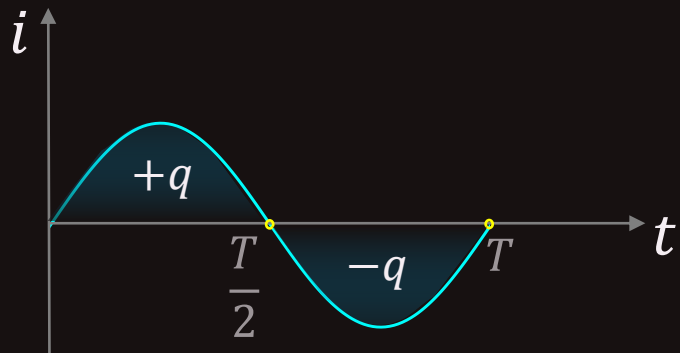
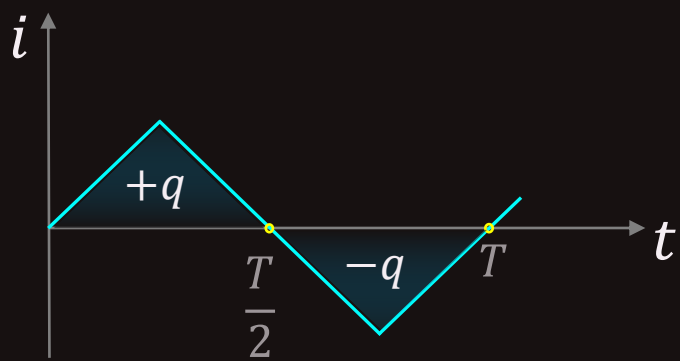
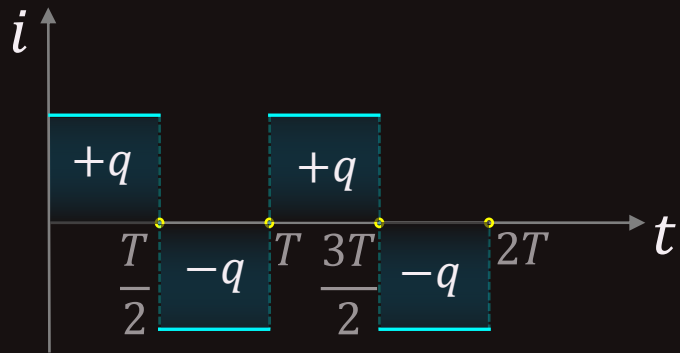
$$i_{av} = 4 \text{ A}$$

$$\int x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$



MEAN OR AVERAGE VALUE OF AC

B



$$i_{av} = \langle i \rangle = \bar{i} = \frac{\int_{t_1}^{t_2} i dt}{\int_{t_1}^{t_2} dt}$$

$$\therefore \int_0^T i dt = q - q = 0$$

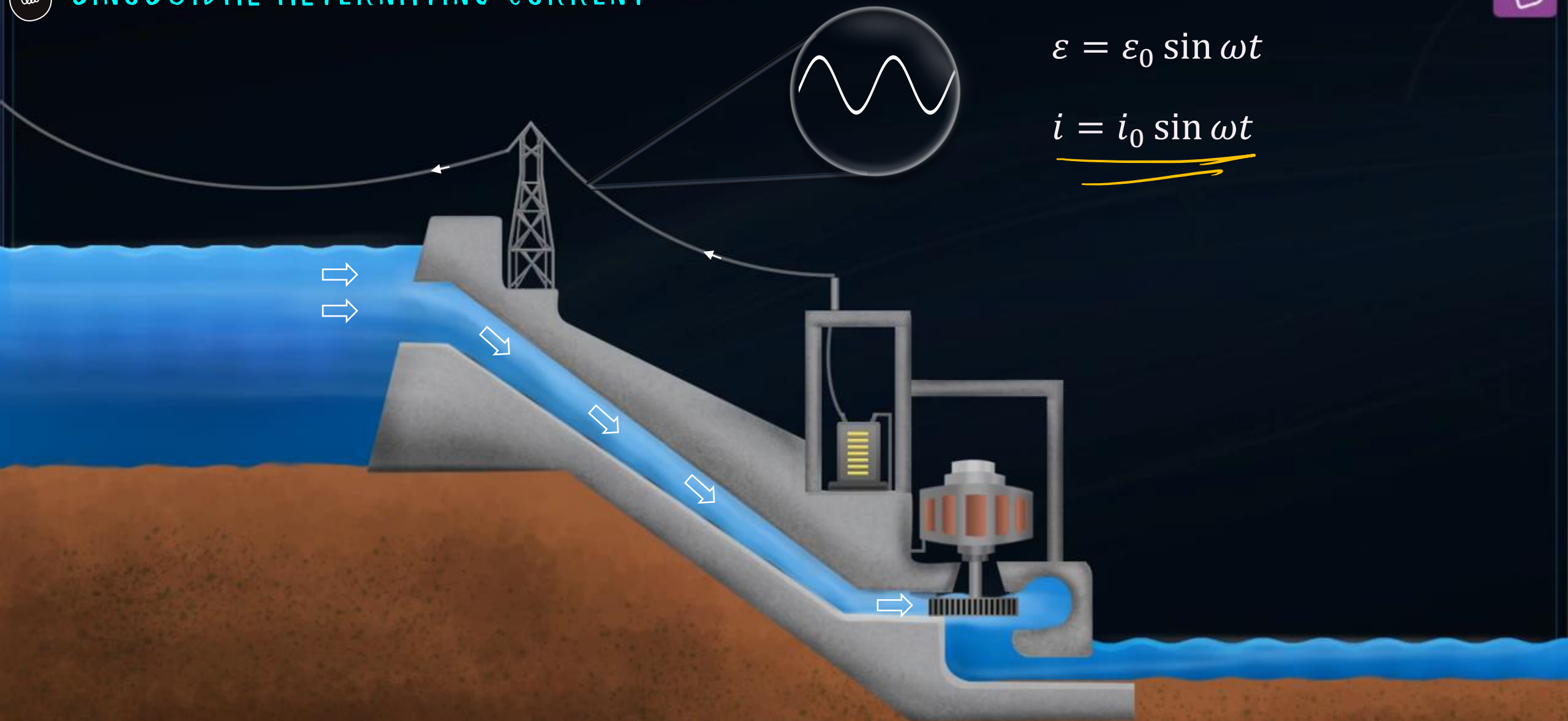
$$i_{av} = \frac{1}{\Delta t} \int_0^T i dt = 0$$

i_{av} for full cycle of AC is **zero**



SINUSOIDAL ALTERNATING CURRENT

B



In this case, current is generated through the hydroelectric power plant, where water waves at high speed hit the turbine that results in generation of AC current.



AVERAGE CURRENT OF A SINUSOIDAL AC



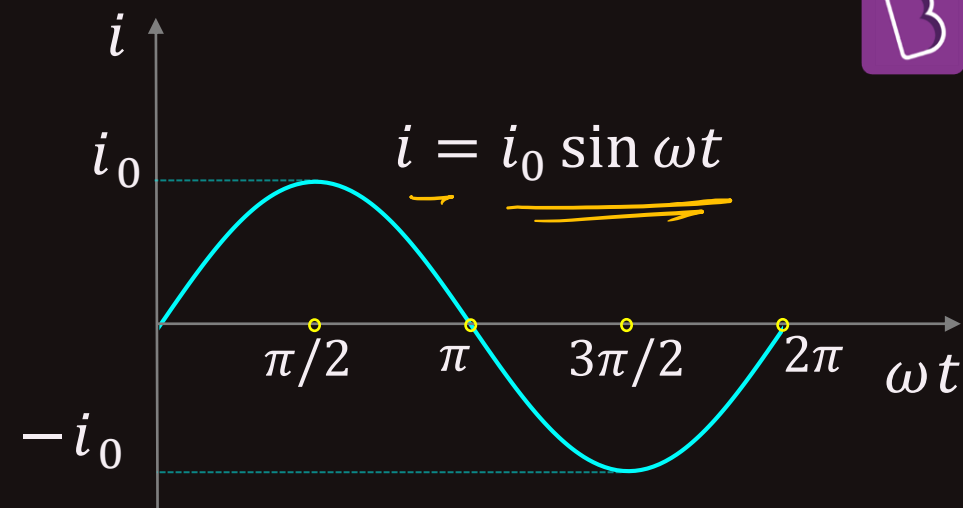
For full cycle

$$i_{av} = \frac{1}{(T - 0)} \int_0^T i_0 \sin \omega t \, dt \quad \left(i_{av} = \frac{1}{\Delta t} \int_{t_1}^{t_2} i \, dt \right)$$

$$i_{av} = \frac{1}{T} i_0 \left(-\frac{\cos \omega t}{\omega} \right)_0^T = \frac{i_0}{T\omega} (\cos 0 - \cos \omega T)$$

$$i_{av} = \frac{i_0}{T\omega} (\cos 0 - \cos 2\pi) = 0 \quad \left(\omega = \frac{2\pi}{T} \right)$$

$$\sim (i_{av})_{full \, cycle} = 0 \sim$$



$i_{avg} =$

$$\frac{1}{\Delta t} \int_{t_1}^{t_2} i \, dt$$

$$\int \sin \theta \cdot d\theta$$

$$\rightarrow -\frac{\cos 2\theta}{2}$$



AVERAGE CURRENT OF A SINUSOIDAL AC



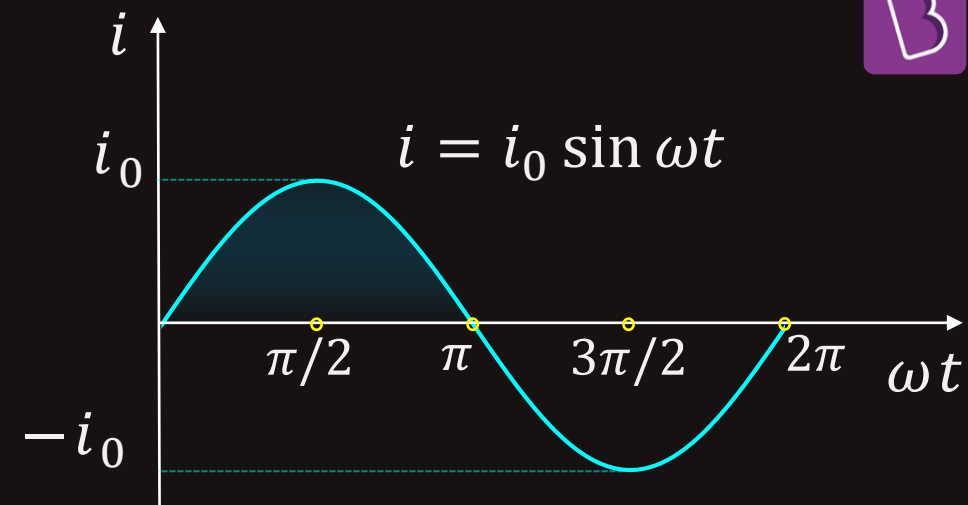
For half cycle

$$i_{av} = \frac{1}{\left(\frac{T}{2} - 0\right)} \int_0^{T/2} i_0 \sin \omega t \, dt$$

$$i_{av} = \frac{2}{T} i_0 \left(-\frac{\cos \omega t}{\omega} \right)_0^{T/2} = \frac{2i_0}{T\omega} \left(\cos 0 - \cos \left(\frac{\omega T}{2} \right) \right)$$

$$\frac{2\pi}{T} \times \frac{T}{2}$$

$$i_{av} = \frac{2i_0}{T\omega} \underbrace{(\cos 0 - \cos \pi)}_{1 - (-1)} = \frac{4i_0}{T \times \frac{2\pi}{T}} = \frac{2i_0}{\pi}$$



$$(i_{av})_{half \, cycle} = \frac{2i_0}{\pi}$$





AVERAGE CURRENT OF A SINUSOIDAL AC



For full cycle

$$(i_{av})_{full\ cycle} = 0$$



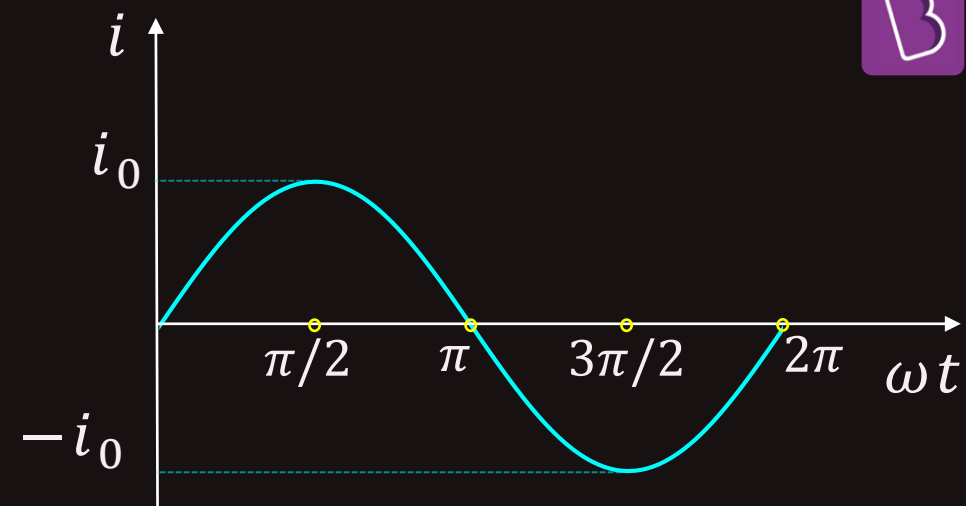
For half cycle

$$(i_{av})_{half\ cycle} = \frac{2i_0}{\pi}$$



For half cycle

$$(\varepsilon_{av})_{half\ cycle} = \frac{2\varepsilon_0}{\pi}$$



These are true if and only if:

$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin \omega t$$



ROOT MEAN SQUARE(RMS) VALUE



B



The procedure of finding the RMS value of any function is just doing the mathematical operation in the reverse order of the name i.e., find square of the function \Rightarrow find its mean \Rightarrow find square root.



$$i_{rms} = \sqrt{\langle i^2 \rangle}$$

$$i_{rms} = \sqrt{\frac{1}{\Delta t} \int_{t_1}^{t_2} i^2 dt}$$



$$\varepsilon_{rms} = \sqrt{\langle \varepsilon^2 \rangle}$$

$$\varepsilon_{rms} = \sqrt{\frac{1}{\Delta t} \int_{t_1}^{t_2} \varepsilon^2 dt}$$

Root mean square (rms)

$$\sqrt{\langle x^2 \rangle} = x_{rms}$$

RMS VALUE OF A SINUSOIDAL AC

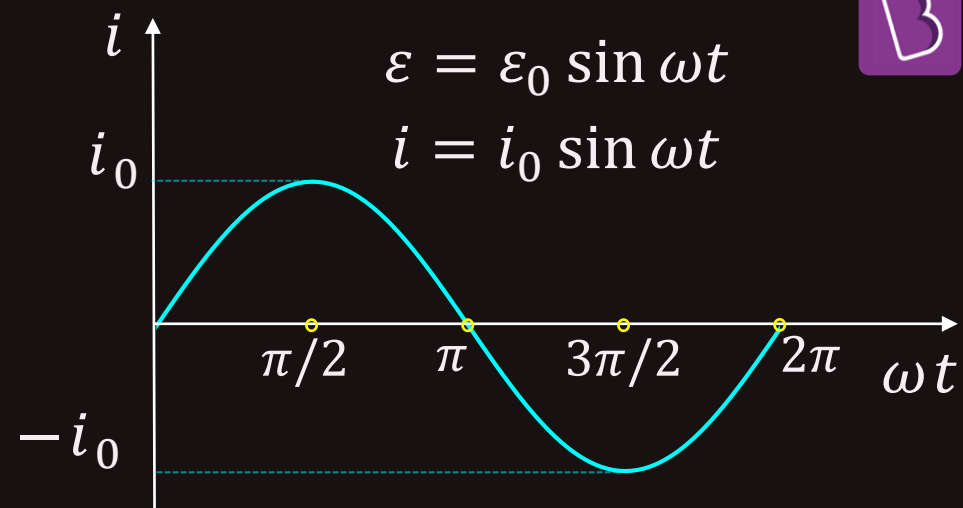
$$i_{rms} = \sqrt{\langle i^2 \rangle}$$

$$i_{rms} = \sqrt{\langle i_0^2 \sin^2 \omega t \rangle}$$

$$i_{rms}^2 = \frac{1}{T} \int_0^T i_0^2 \sin^2 \omega t \, dt = \frac{i_0^2}{T} \int_0^T \sin^2 \omega t \, dt$$

$$i_{rms}^2 = \frac{i_0^2}{T} \int_0^T \frac{1 - \cos 2\omega t}{2} \, dt$$

$$i_{rms}^2 = \frac{i_0^2}{2T} \left(t - \frac{\sin 2\omega t}{2\omega} \right)_0^T$$



$$\left(\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \right)$$

RMS VALUE OF A SINUSOIDAL AC

$$i_{rms} = \sqrt{\langle i^2 \rangle}$$

$$i_{rms}^2 = \frac{i_0^2}{2T} \left(t - \frac{\sin 2\omega t}{2\omega} \right) \Big|_0^T \quad \left(T = \frac{2\pi}{\omega} \right)$$

$$i_{rms}^2 = \frac{i_0^2}{2T} \left(\left(T - \frac{\sin 2\omega \left(\frac{2\pi}{\omega} \right)}{2\omega} \right) - (0 - \sin 0) \right) = \frac{i_0^2}{2T} T$$

$$i_{rms}^2 = \frac{i_0^2}{2}$$

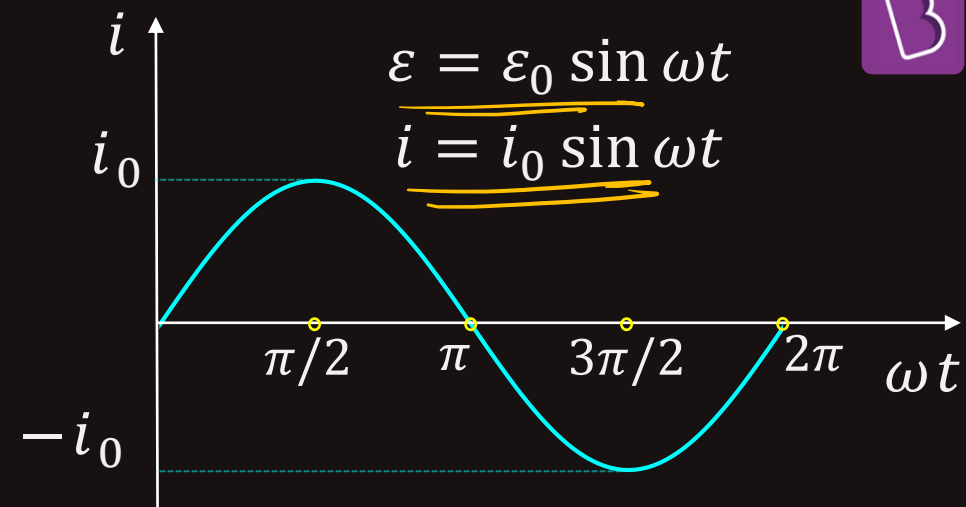
$$\text{AC Source} \quad i_{rms} = \frac{i_0}{\sqrt{2}} \quad \text{AC Load}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}}$$

$$\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}}$$

$$(i_{av})_{half\ cycle} = \frac{2i_0}{\pi}$$

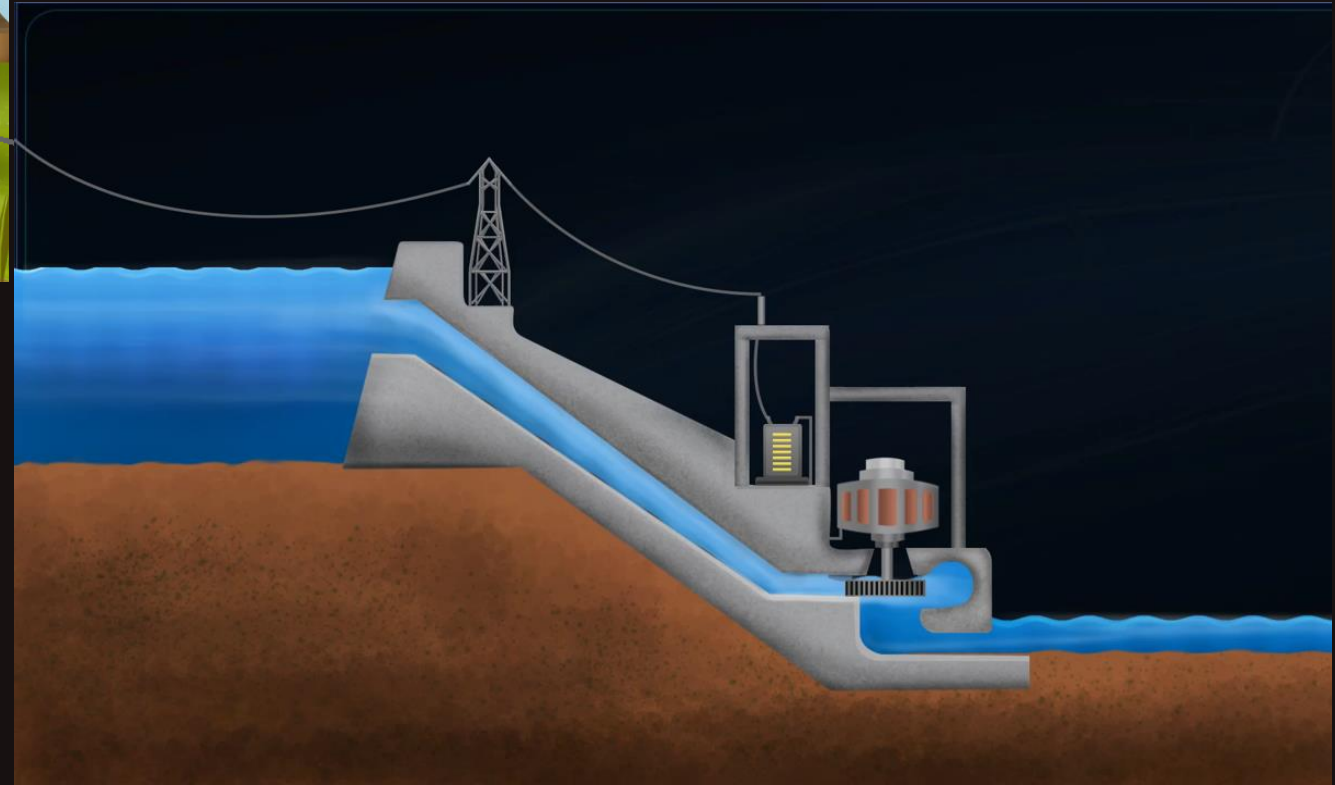
$$(\varepsilon_{av})_{half\ cycle} = \frac{2\varepsilon_0}{\pi}$$





~ This AC voltage is made usable as per the voltage requirement of the users using step up or step down transformers.

~ The AC generated in power plant can be transmitted over large distances using transmission cables at high voltage.




SIGNIFICANCE OF RMS VALUE



Household current \rightarrow sinusoidal AC ($\varepsilon = \varepsilon_0 \sin \omega t$)
 $220\text{ V}, 50\text{ Hz}$

 $\varepsilon_{rms} = 220\text{ V}$

 $\varepsilon_{av} = 0\text{ V}$

 $\varepsilon_0 = \sqrt{2} \varepsilon_{rms}$

$$\varepsilon_0 = \sqrt{2} \times 220 = 311.12\text{ V} \approx 311\text{ V}$$

If problem states only ε (not $\varepsilon_0, \varepsilon_{rms}, \varepsilon_{av}$)
then, consider it as ε_{rms}

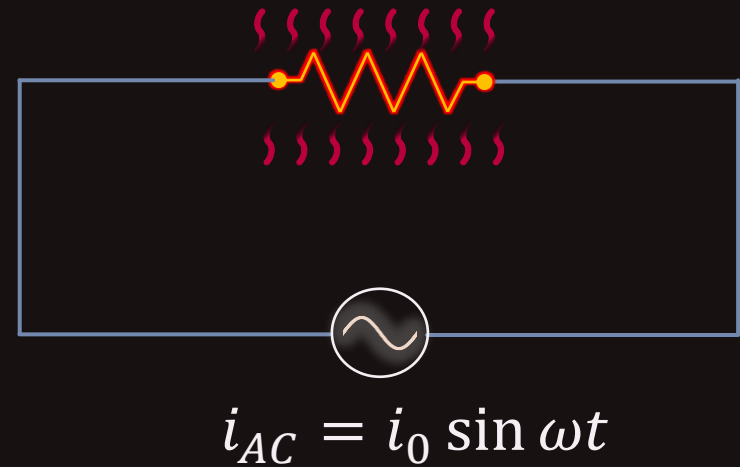


SIGNIFICANCE OF RMS VALUE



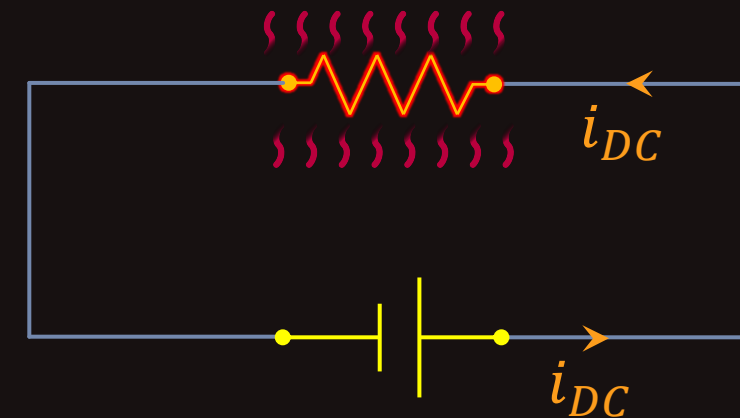
Heat produced in AC circuit through resistor R in time t_1 to t_2

$$H_{AC} = \int_{t_1}^{t_2} i_{AC}^2 R dt \quad i_{rms} = \sqrt{\frac{1}{\Delta t} \int_{t_1}^{t_2} i_{AC}^2 dt}$$



Heat produced in DC circuit through resistor R in time t_1 to t_2

$$H_{DC} = i_{DC}^2 R \Delta t$$



SIGNIFICANCE OF RMS VALUE

$$H_{AC} = \int_{t_1}^{t_2} i_{AC}^2 R dt \quad i_{rms} = \sqrt{\frac{1}{\Delta t} \int_{t_1}^{t_2} i_{AC}^2 dt}$$

$$H_{DC} = i_{DC}^2 R \Delta t$$

If we require to have $\Delta H_{DC} = \Delta H_{AC}$, then,

$$i_{DC}^2 R \Delta t = \int_{t_1}^{t_2} i_{AC}^2 R dt$$

$$i_{DC} = \sqrt{\frac{1}{\Delta t} \int_{t_1}^{t_2} i_{AC}^2 dt} = i_{rms}$$

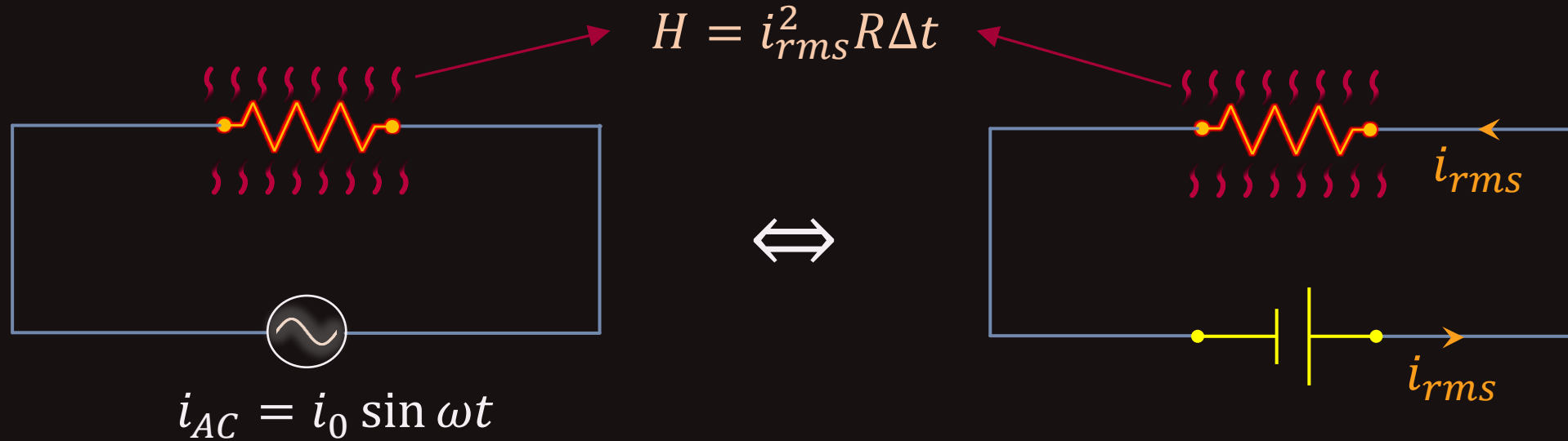
\therefore If $\Delta H_{DC} = \Delta H_{AC} \Rightarrow \textcircled{i_{rms}} = i_{DC} \Rightarrow i_{rms}$ is actually the DC equivalent of an AC



$$i_{AC} = i_0 \sin \omega t$$



SIGNIFICANCE OF RMS VALUE



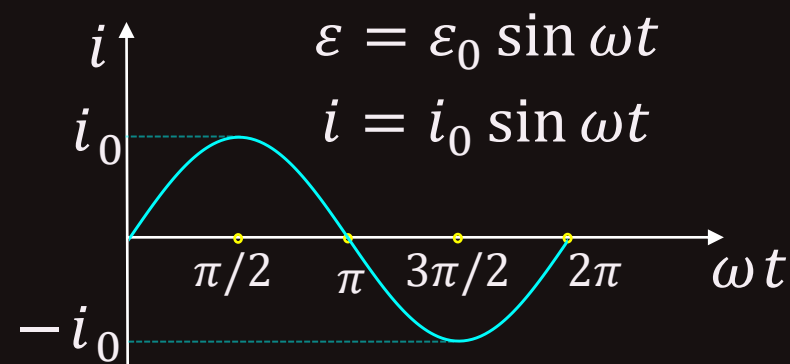
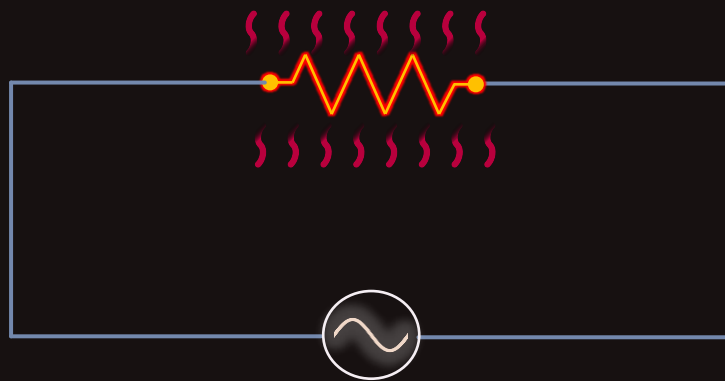
RMS value of a given AC can be defined as that DC value which produces same heat in a resistance which the AC produces in that resistance in same duration.

i_{rms} is the effective DC value of a given AC

SIGNIFICANCE OF RMS VALUE

B

$$H = i_{rms}^2 R \Delta t$$



$$i_{AC} = i_0 \sin \omega t$$

DC devices cannot measure alternating current or emf. Normal Ammeter, Voltmeter will show **only zero for AC**

Hot Wire Ammeter & Hot Wire Voltmeter are used to measure AC. They measure RMS value of i & ε .



Useful Results :

$$\textcircled{\sim} \overline{\sin \omega t} = 0$$

$$\textcircled{\sim} \overline{\cos \omega t} = 0$$

$$\textcircled{\sim} \overline{\sin^2 \omega t} = \frac{1}{2}$$

$$\textcircled{\sim} \overline{\cos^2 \omega t} = \frac{1}{2}$$

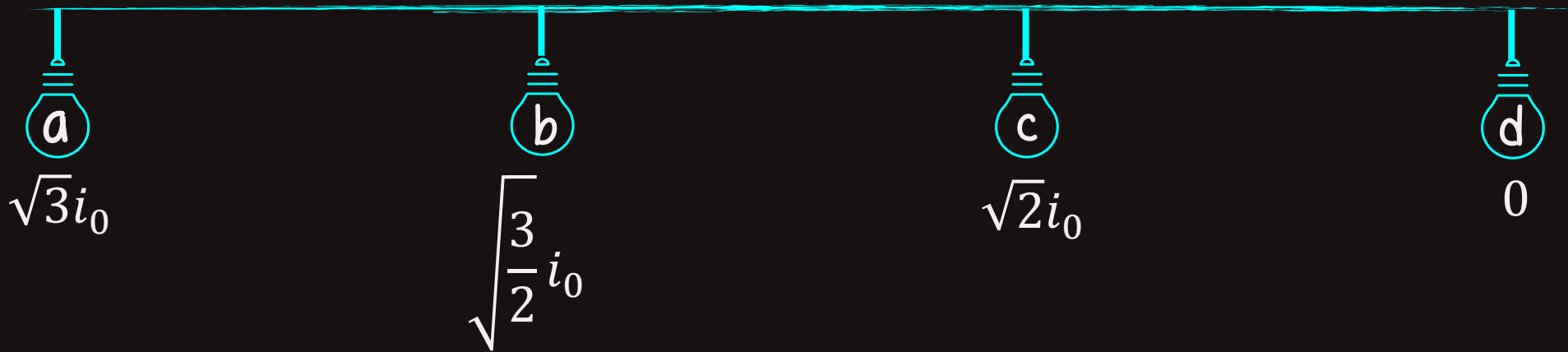
The overhead bar denotes the average value of each of the functions and the average is taken over a complete cycle



QUESTION



Find RMS current for $i = i_0 + i_0 \sin \omega t$.





SOLUTION



$$i = i_0 + i_0 \sin \omega t$$

$$i_{rms} = \sqrt{i^2} \Rightarrow i_{rms} = \sqrt{(i_0 + i_0 \sin \omega t)^2}$$

$$i_{rms} = \sqrt{i_0^2 + i_0^2 \sin^2 \omega t + 2i_0^2 \sin \omega t} \quad [\text{Since } i_0 \text{ is constant, } \overline{i_0^2} = i_0^2]$$

$$i_{rms} = \sqrt{i_0^2 + i_0^2 \times \frac{1}{2} + 2i_0^2 \times 0} = \sqrt{\frac{3 \times i_0^2}{2}} \Rightarrow i_{rms} = i_0 \sqrt{\frac{3}{2}}$$

$$\overline{\sin \omega t} = 0$$

$$\overline{\sin^2 \omega t} = \frac{1}{2}$$

a
 $\sqrt{3}i_0$

b
 $\sqrt{\frac{3}{2}}i_0$

c
 $\sqrt{2}i_0$

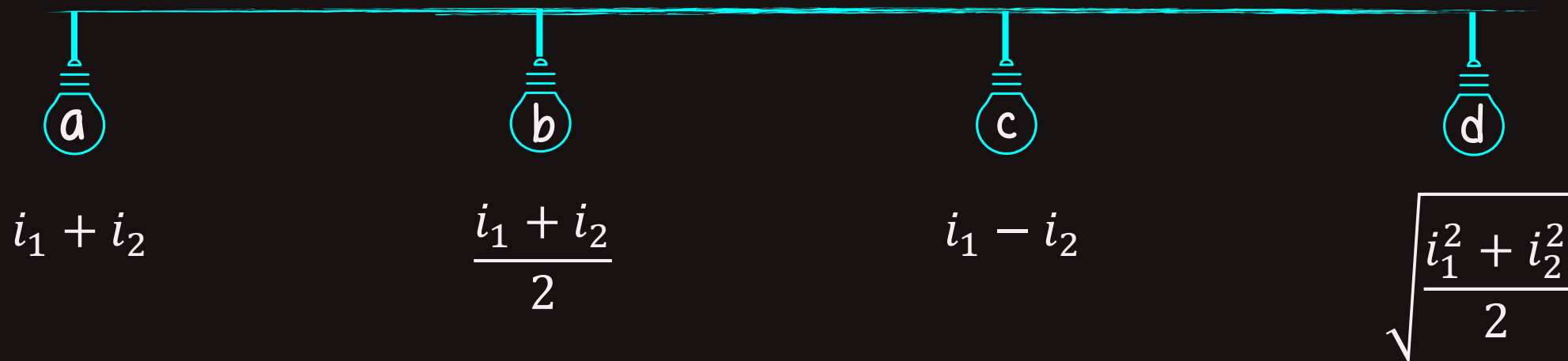
d



QUESTION



Find RMS current for $i = i_1 \sin \omega t + i_2 \cos \omega t$.





$$i = i_1 \sin \omega t + i_2 \cos \omega t$$

$$i_{rms} = \sqrt{i^2}$$

$$i_{rms} = \sqrt{(i_1 \sin \omega t + i_2 \cos \omega t)^2}$$

$$i_{rms} = \sqrt{i_1^2 \sin^2 \omega t + i_2^2 \cos^2 \omega t + 2i_1 i_2 \sin \omega t \cos \omega t}$$

$$i_{rms} = \sqrt{i_1^2 \times \frac{1}{2} + i_2^2 \times \frac{1}{2} + 2i_1 i_2 \times 0}$$

$$i_{rms} = \sqrt{\frac{i_1^2 + i_2^2}{2}}$$



$$i_1 + i_2$$



$$\frac{i_1 + i_2}{2}$$



$$i_1 - i_2$$



$$\sqrt{\frac{i_1^2 + i_2^2}{2}}$$

Short cut for full cycle

$$\odot \quad \overline{\sin \omega t} = 0$$

$$\odot \quad \overline{\cos \omega t} = 0$$

$$\odot \quad \overline{\sin^2 \omega t} = \frac{1}{2}$$

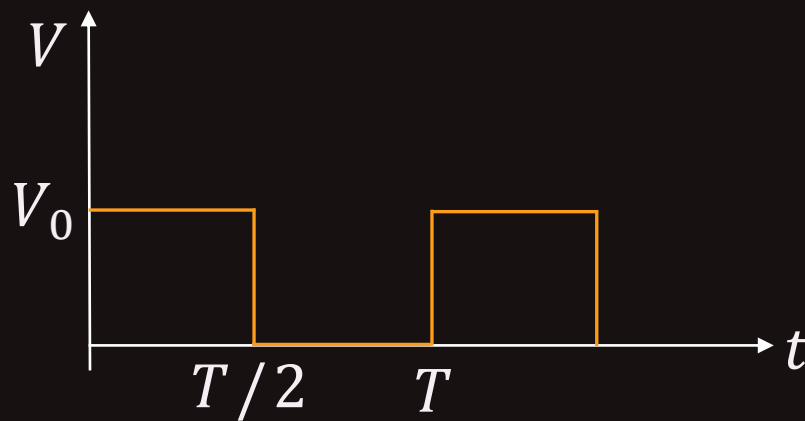
$$\odot \quad \overline{\cos^2 \omega t} = \frac{1}{2}$$



QUESTION



Find RMS value of potential difference V shown in the figure is



$$\frac{V_0}{\sqrt{3}}$$



$$V_0$$



$$\frac{V_0}{\sqrt{2}}$$



$$\frac{V_0}{2}$$



SOLUTION



B

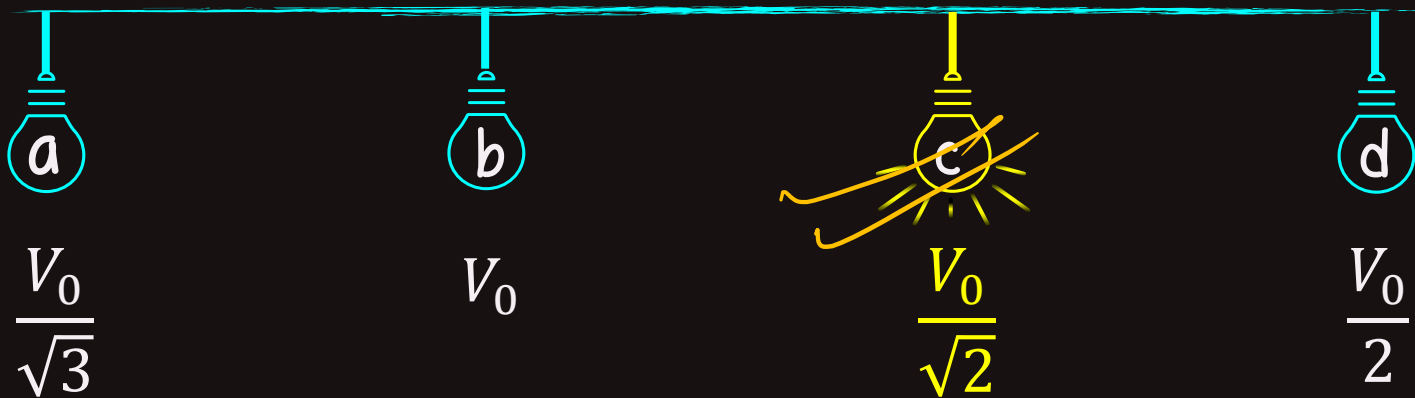
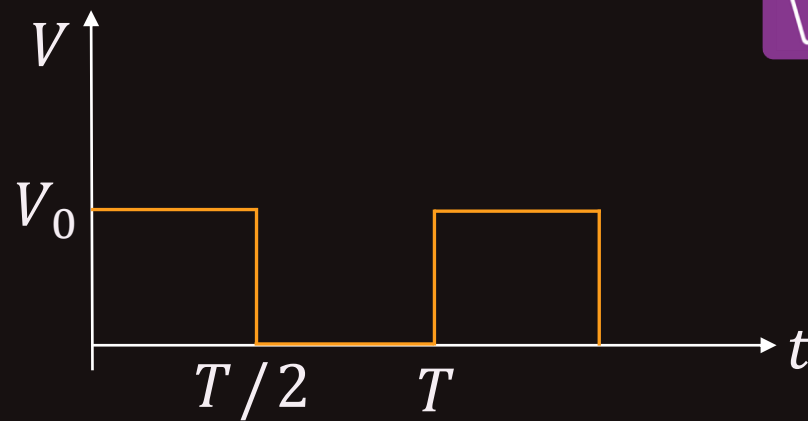
$$V_{rms} = \sqrt{\frac{1}{\Delta t} \int_0^T V^2 dt}$$

$$V_{rms}^2 = \frac{1}{T} \left(\int_0^{T/2} V_0^2 dt + \int_{T/2}^T 0 dt \right)$$

$$V_{rms}^2 = \frac{1}{T} V_0^2 \left(\frac{T}{2} - 0 \right) + 0$$

$$V_{rms}^2 = \frac{V_0^2}{2}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}} //$$



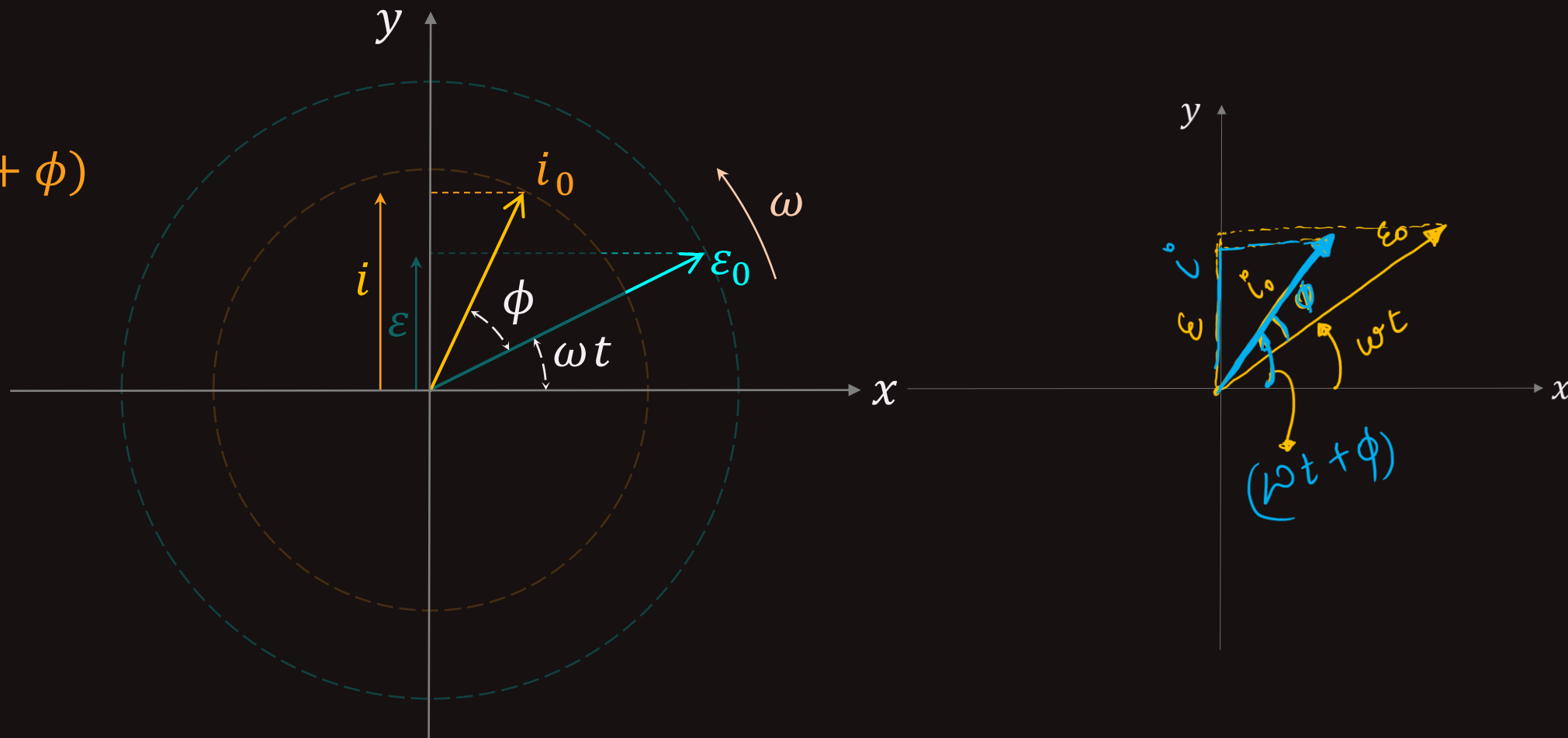


PHASOR DIAGRAM

A diagram that represents AC and voltage of same frequency as rotating vectors (phasors) along with proper phase angle between them.

$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin(\omega t + \phi)$$





PHASOR DIAGRAM

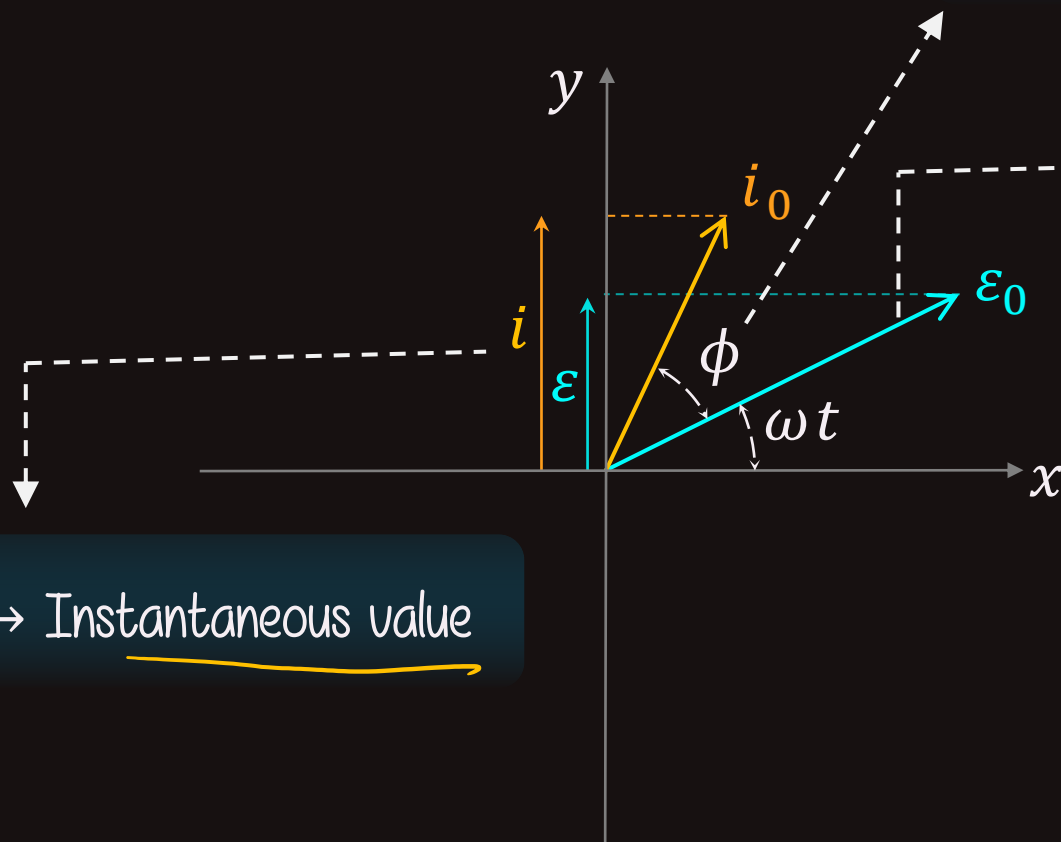
B

$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin(\omega t + \phi)$$

$$\text{Phase difference} = \left| \begin{array}{l} \text{Phase of } i \\ (\omega t + \phi) \end{array} - \begin{array}{l} \text{Phase of } \varepsilon \\ (\omega t) \end{array} \right|$$

Length of arrow → Peak value



Projection on y axis → Instantaneous value



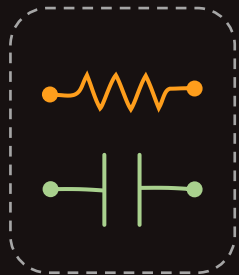
AC CIRCUITS

B

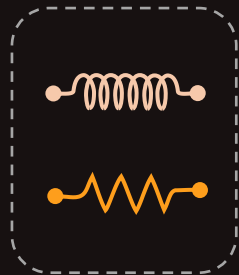
I. An AC source connected only to:



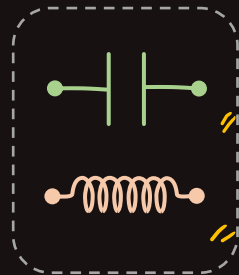
II. An AC source connected to more than one element.



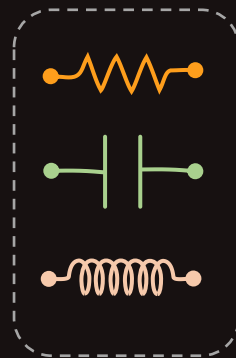
RC
Circuit



LR
Circuit



LC
Circuit



LCR
Circuit

PURE RESISTIVE AC CIRCUIT

Apply KVL in clockwise direction:

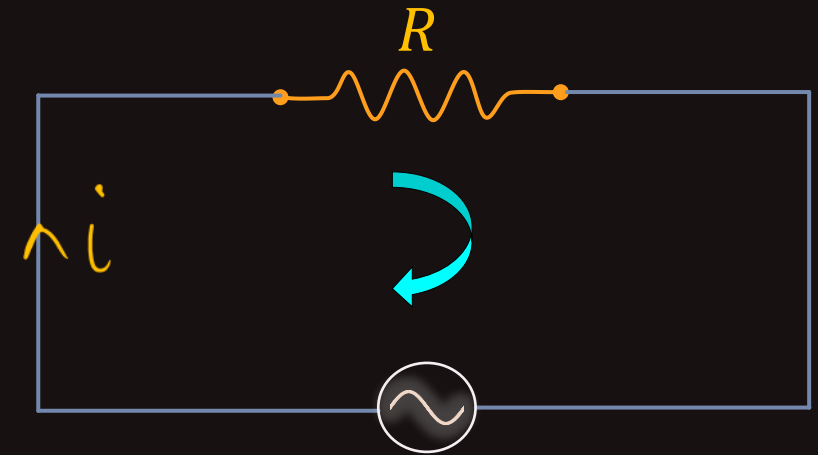
$$\varepsilon - iR = 0 \Rightarrow i = \frac{\varepsilon}{R}$$

$$i = \frac{\varepsilon_0 \sin \omega t}{R}$$

$$i = i_0 \sin \omega t \Rightarrow \underset{\substack{\uparrow \\ \text{Peak current}}}{i_0} = \frac{\varepsilon_0}{R} \quad \text{Peak voltage}$$

$$\text{Phase difference } (\phi) = \omega t - \omega t = 0$$

Current is **in phase** with potential



$$\underline{\varepsilon = \varepsilon_0 \sin \omega t}$$

In AC circuit, hinderance to the current is defined by $\frac{\varepsilon_0}{i_0}$

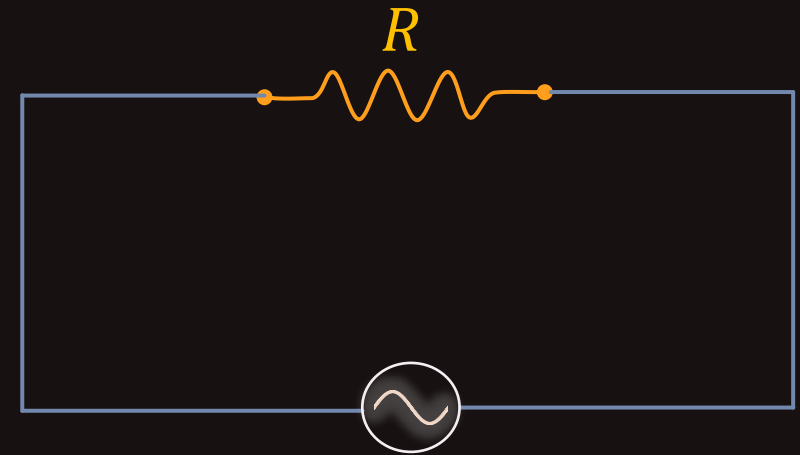
For purely resistive circuit

$$\frac{\varepsilon_0}{i_0} = R$$

PURE RESISTIVE AC CIRCUIT

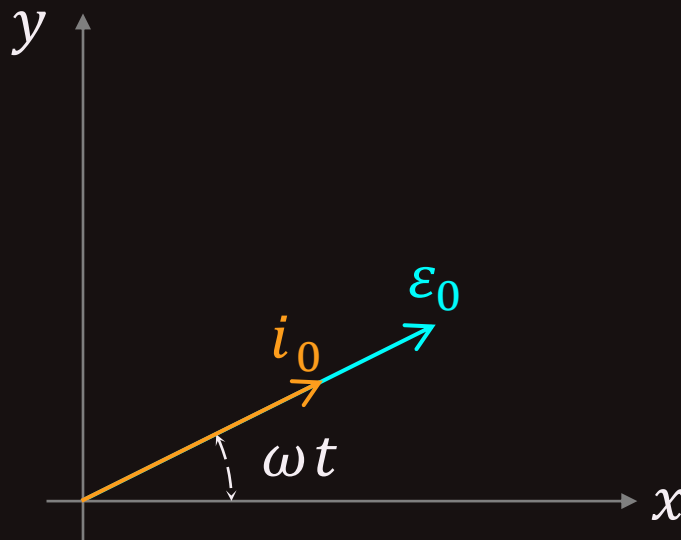
Current is **in phase** with potential

$$i_0 = \frac{\epsilon_0}{R}$$

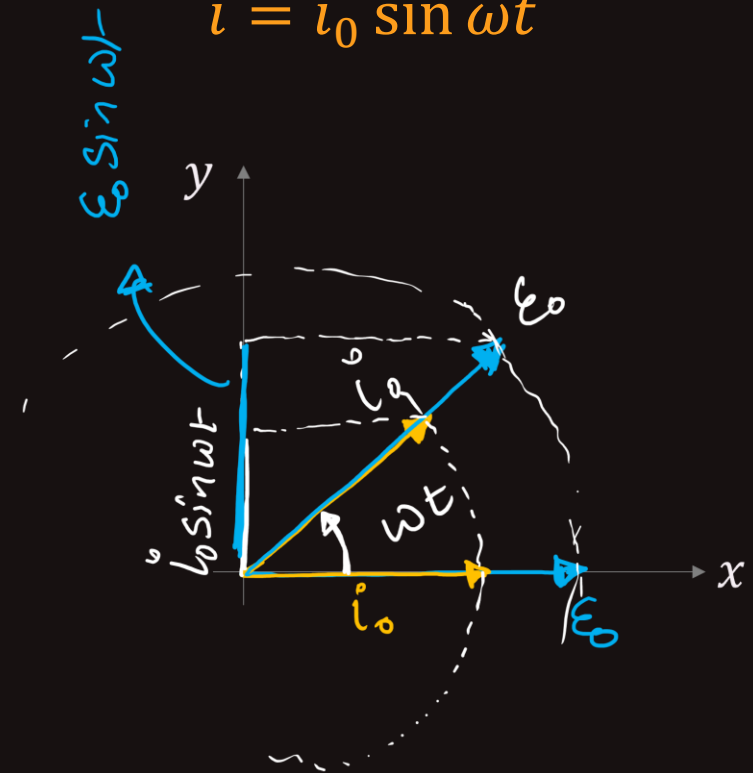


Phasor diagram

- Since the current and potential are in phase, their phasors will overlap with each other.
- Since the magnitude of peak value of current is less than that of the potential, current phasor will be smaller than voltage phasor.



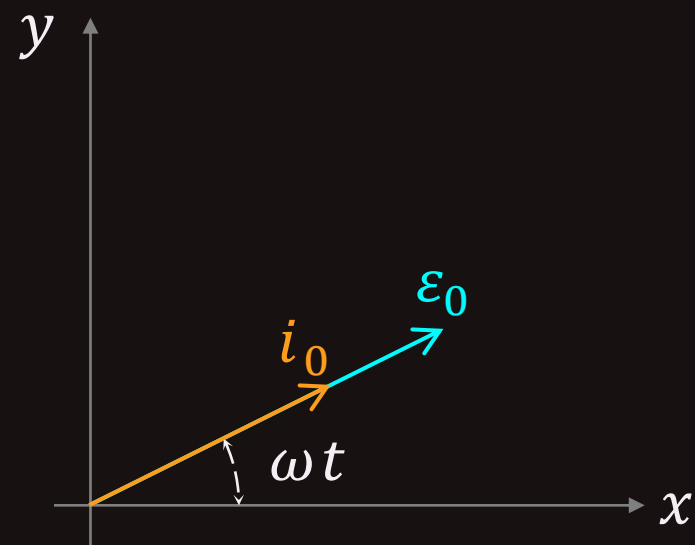
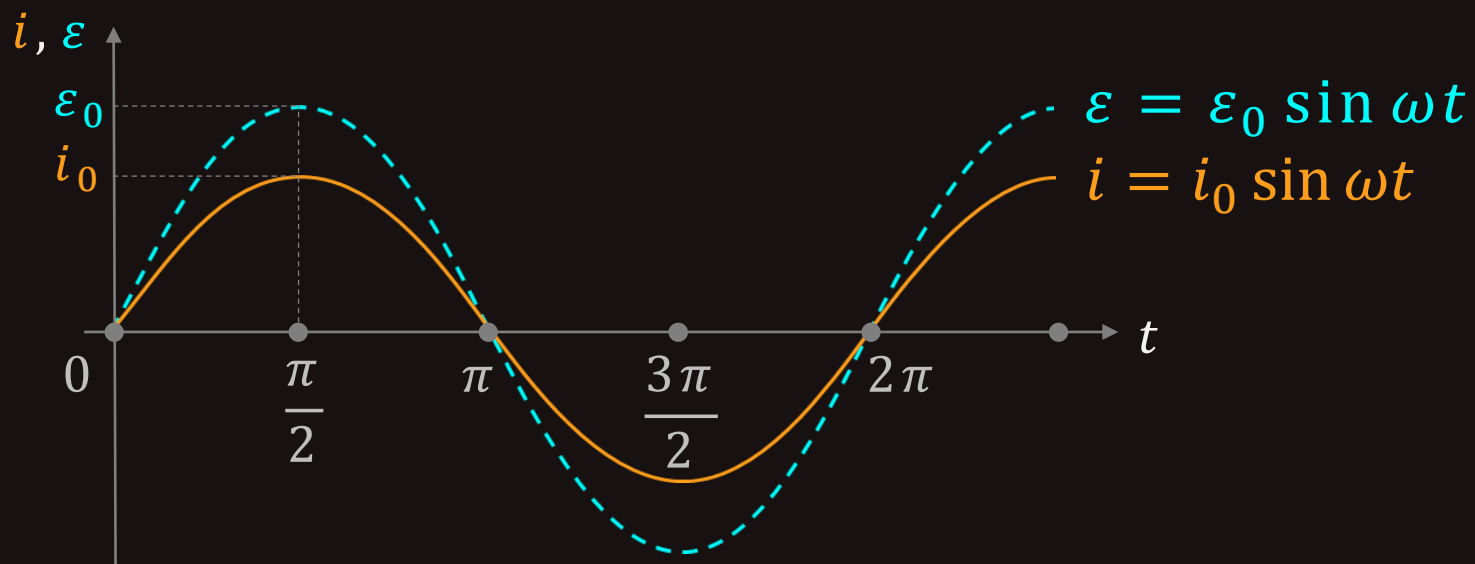
$$\begin{aligned}\epsilon &= \epsilon_0 \sin \omega t \\ i &= i_0 \sin \omega t\end{aligned}$$



PURE RESISTIVE AC CIRCUIT

Wave diagram

- Since the current and potential both are represented by sine function with same angular frequency (ω), their waveform will be same as shown in the figure below.
- Since $i_0 = \frac{\varepsilon_0}{R}$, the magnitude of peak value of current is less than that of the potential.



Phasor diagram

$$\varepsilon = \varepsilon_0 \sin \omega t$$
$$i = i_0 \sin \omega t$$
$$i_0 < \varepsilon_0$$



PURE INDUCTIVE AC CIRCUIT



Potential drop across inductance, $V_L = L \frac{di}{dt}$

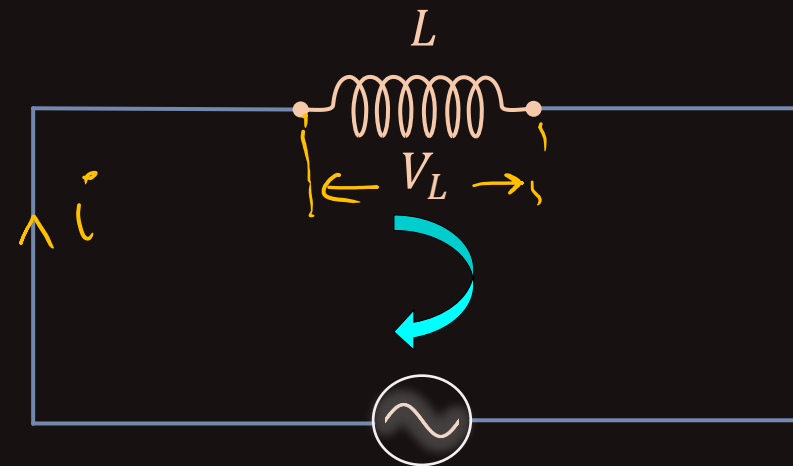
Apply KVL ;

$$\varepsilon - V_L = 0$$

$$L \frac{di}{dt} = \varepsilon \Rightarrow di = \frac{\varepsilon}{L} dt$$

$$i = \int di = \int \frac{\varepsilon_0 \sin \omega t}{L} dt$$

$$i = \frac{\varepsilon_0}{L} \frac{(-\cos \omega t)}{\omega} = -\frac{\varepsilon_0}{L\omega} \cos \omega t$$



$$\varepsilon = \varepsilon_0 \sin \omega t$$



PURE INDUCTIVE AC CIRCUIT

B

$$i = \frac{\varepsilon_0}{L} \frac{(-\cos \omega t)}{\omega} = -\frac{\varepsilon_0}{L\omega} \cos \omega t$$

$$i = -\frac{\varepsilon_0}{L\omega} \sin\left(\frac{\pi}{2} - \omega t\right) = \frac{\varepsilon_0}{L\omega} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

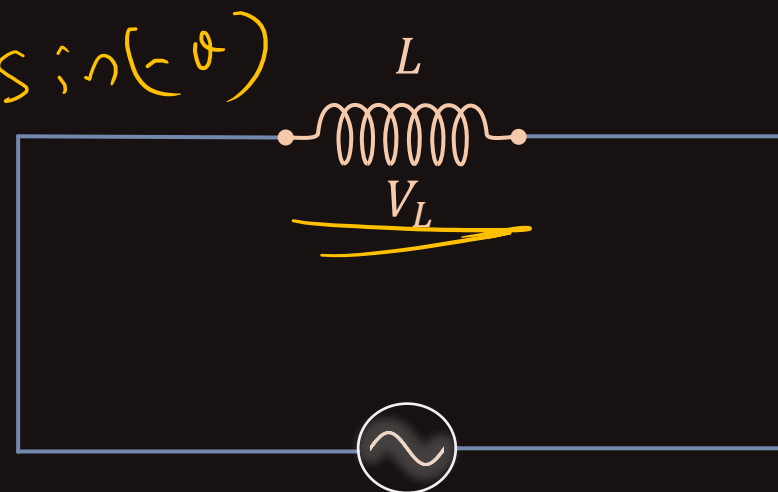
$$i_0 = \frac{\varepsilon_0}{L\omega} = \frac{\varepsilon_0}{X_L}$$

$$X_L = L\omega$$

Inductive reactance

SI Unit : Ohm (Ω)

$$-\sin \theta = \sin(-\theta)$$



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$\frac{\varepsilon_0}{i_0} = X_L = \omega L$$

$$\omega = 2\pi f$$

$$\omega \propto f$$

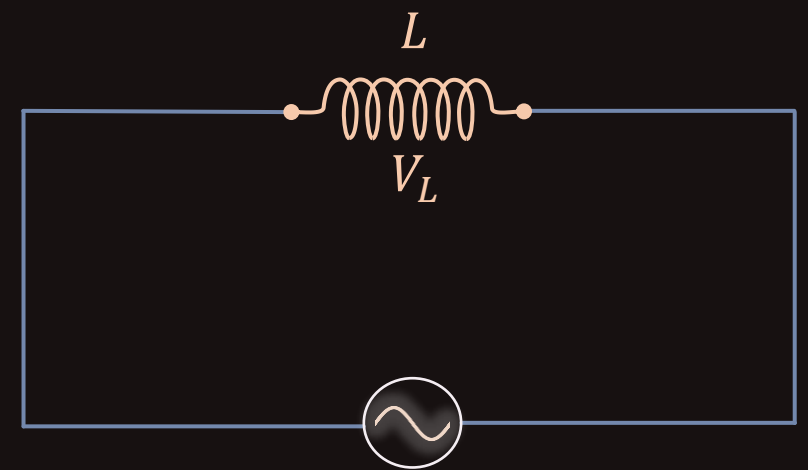
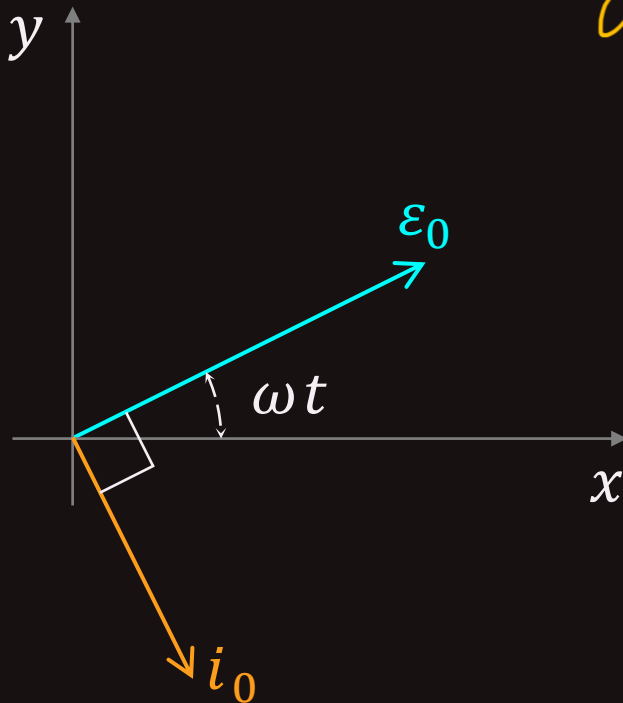
PURE INDUCTIVE AC CIRCUIT

$$\text{Phase difference } (\phi) = \left(\omega t - \frac{\pi}{2} \right) - \omega t = -\frac{\pi}{2}$$

Current **lags** potential by **90°**

At $\omega t \rightarrow +ve$
 $i \rightarrow -ve$

Phasor diagram



$$\varepsilon = \varepsilon_0 \sin \omega t$$
$$\underline{i = i_0 \sin \left(\omega t - \frac{\pi}{2} \right)}$$

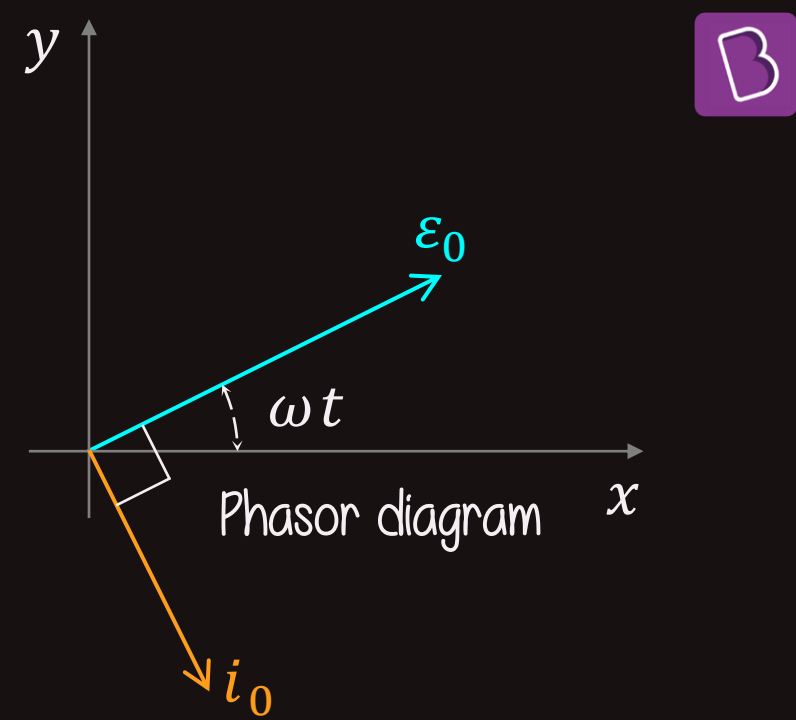
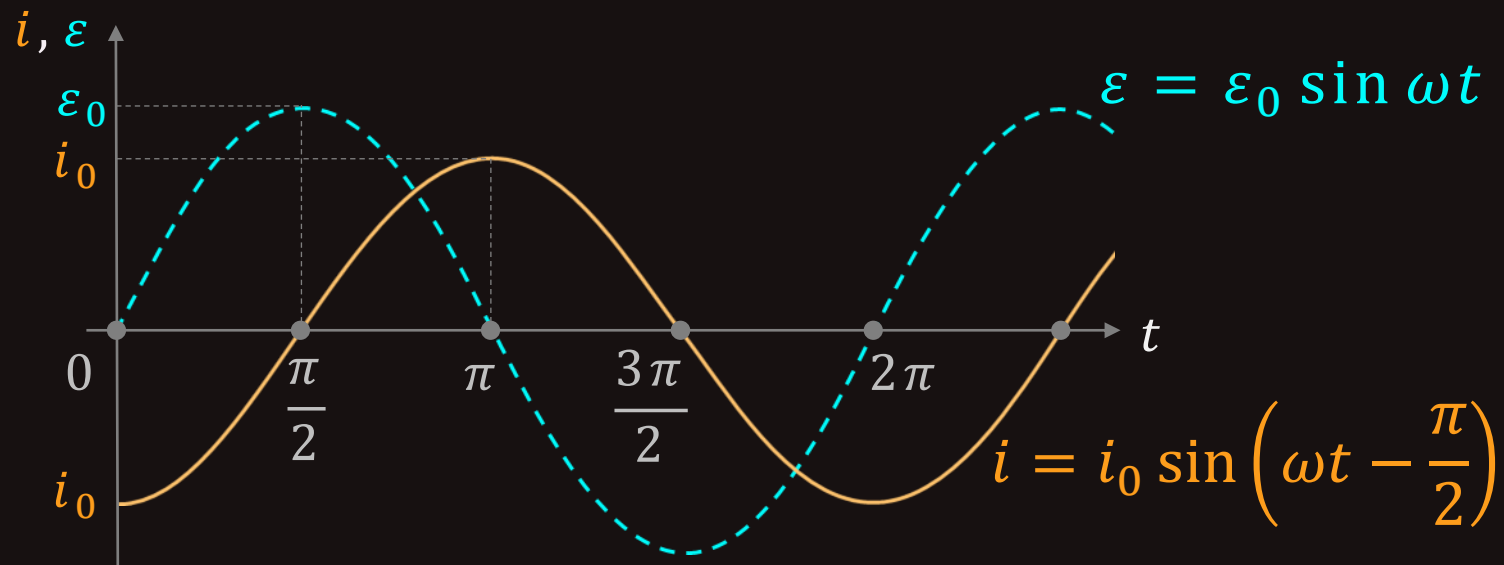


PURE INDUCTIVE AC CIRCUIT

$$\text{Phase difference } (\phi) = \left(\omega t - \frac{\pi}{2} \right) - \omega t = -\frac{\pi}{2}$$

Current **lags** potential by **90°**

 Wave diagram

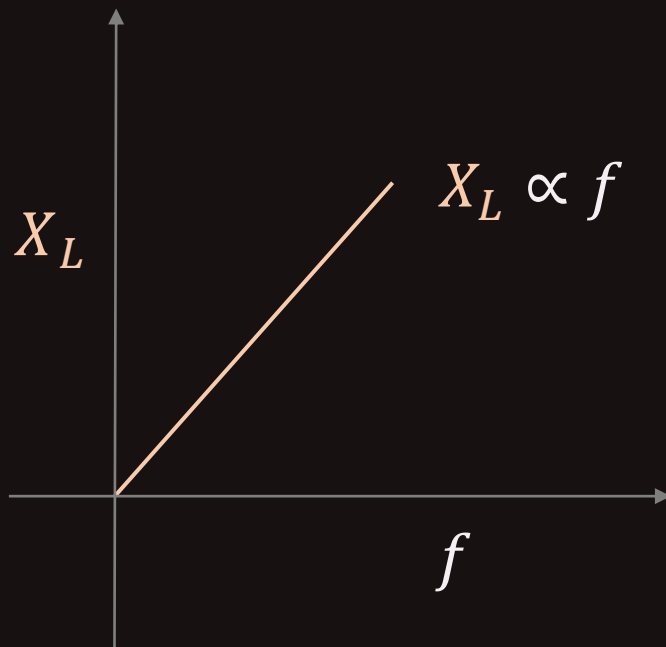


PURE INDUCTIVE AC CIRCUIT

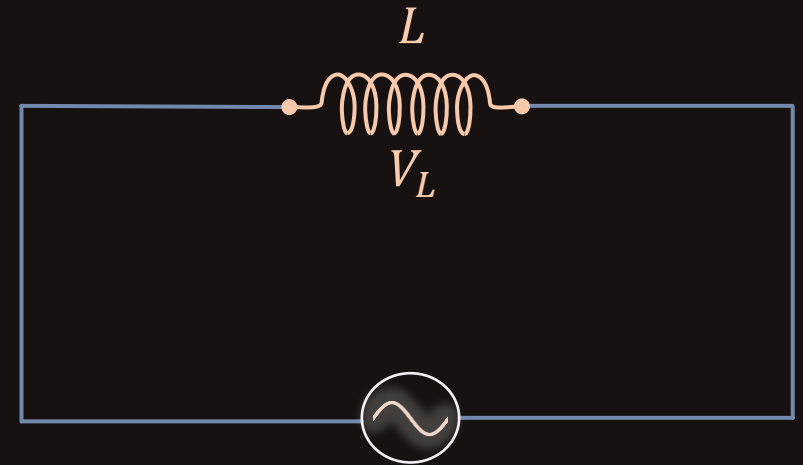
 X_L v/s frequency (f)

$$X_L = L\omega$$

$$X_L = L \times 2\pi f \quad (\because \omega = 2\pi f)$$



$\omega \uparrow$
 $X_L \uparrow$



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin \left(\omega t - \frac{\pi}{2} \right)$$

$$X_L = \omega L$$

$$X_L = 2\pi f \times L$$

$$X_L \propto L$$

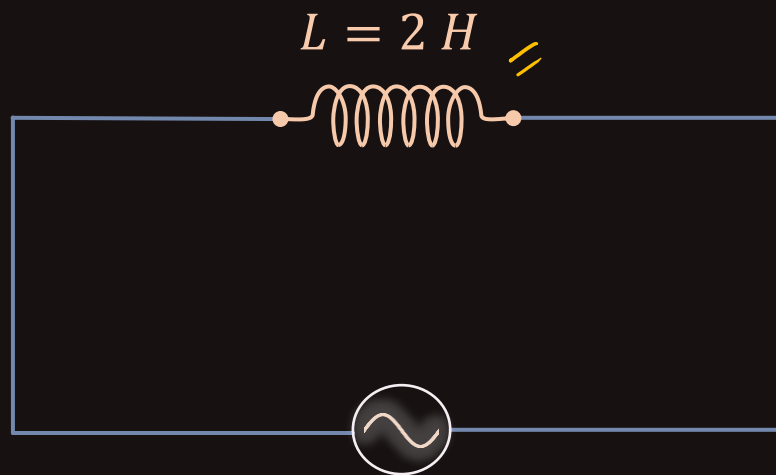
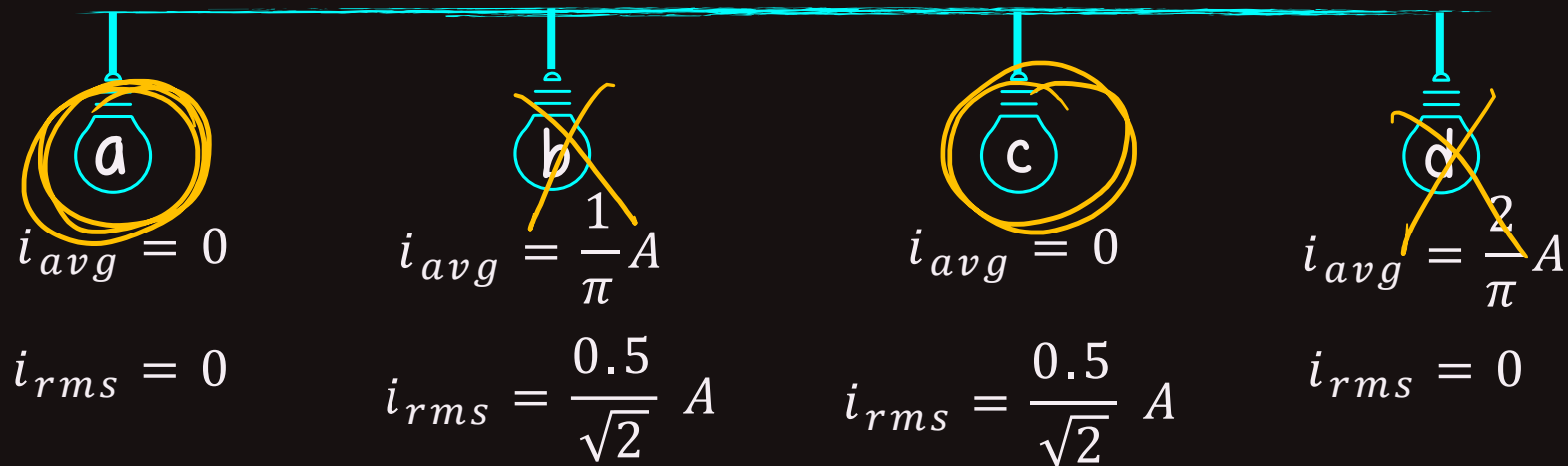


QUESTION

Find i_{avg} and i_{rms} of given circuit.

$$i_{av} = 0$$

B



$$\varepsilon = 10 \sin(10t + 30^\circ)$$



SOLUTION

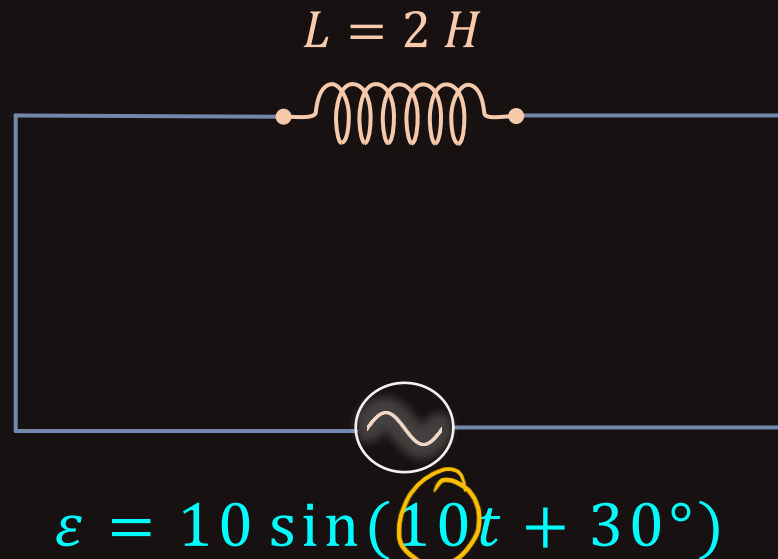


$$\varepsilon = \varepsilon_0 \sin(\omega t + \phi) \quad \varepsilon_0 = 10 \text{ V} \quad \omega = 10 \text{ s}^{-1}$$

$$i_0 = \frac{\varepsilon_0}{X_L} = \frac{\varepsilon_0}{L\omega} = \frac{10}{2 \times 10} = 0.5 \text{ A}$$

$$i_{avg} = 0 \quad (\text{For full cycle of AC } i_{av} = 0)$$

$$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{0.5}{\sqrt{2}} \text{ A}$$



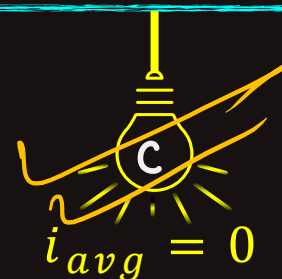
$$i_{avg} = 0$$

$$i_{rms} = 0$$



$$i_{avg} = \frac{1}{\pi} \text{ A}$$

$$i_{rms} = \frac{0.5}{\sqrt{2}} \text{ A}$$



$$i_{avg} = 0$$

$$i_{rms} = \frac{0.5}{\sqrt{2}} \text{ A}$$



$$i_{avg} = \frac{2}{\pi} \text{ A}$$

$$i_{rms} = 0$$



PURE CAPACITIVE AC CIRCUIT

Potential drop across capacitance, $V_C = \frac{q}{C}$

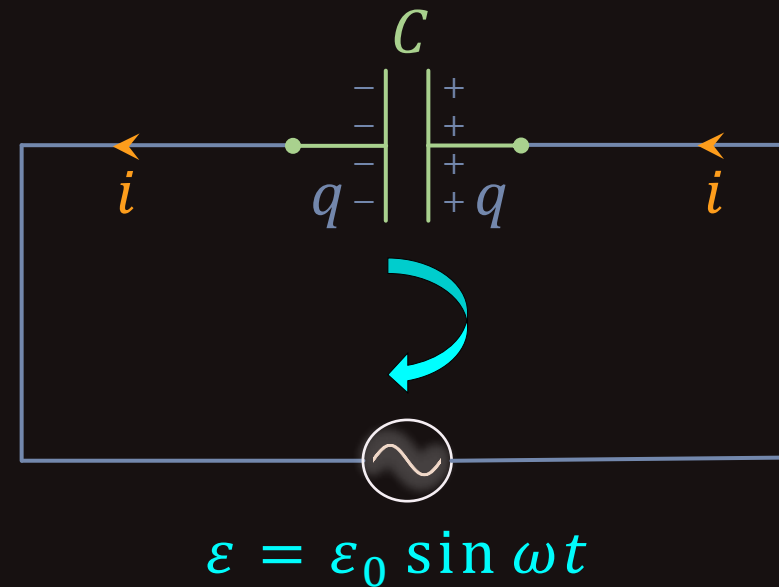
Apply KVL ;

$$\varepsilon - V_C = 0$$

$$\frac{q}{C} = \varepsilon \Rightarrow q = C\varepsilon_0 \sin \omega t$$

$$i = \frac{dq}{dt} = \frac{d(C\varepsilon_0 \sin \omega t)}{dt}$$

$$i = C\omega\varepsilon_0 \cos \omega t$$





PURE CAPACITIVE AC CIRCUIT

$$i = C\omega\varepsilon_0 \cos \omega t$$

$$i = \frac{\varepsilon_0}{\frac{1}{C\omega}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

i_0

$$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i_0 = \frac{\varepsilon_0}{\frac{1}{C\omega}} = \frac{\varepsilon_0}{X_C}$$

$$X_C = \frac{1}{C\omega}$$

Capacitive reactance //

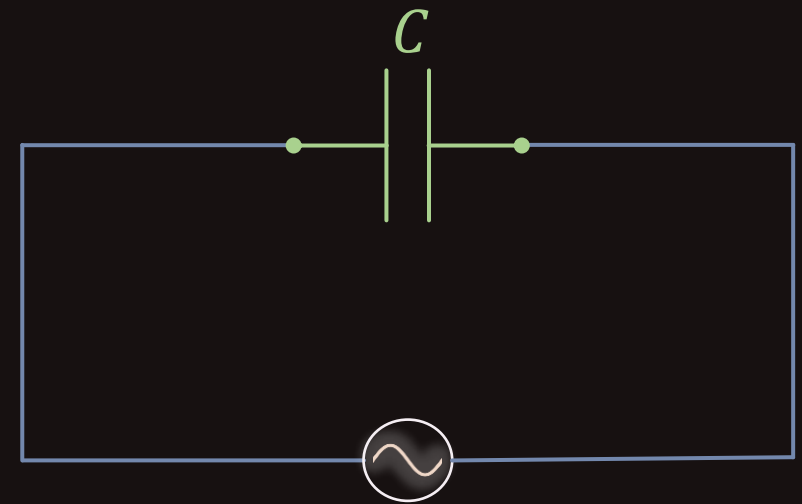
SI Unit : Ohm Ω //

$$\frac{\varepsilon_0}{i_0} = X_C$$

$$X_C = \frac{1}{\omega C}$$

$$X_C \propto \frac{1}{\omega}$$

$\omega \uparrow$
 $X_C \downarrow$



$$\varepsilon = \varepsilon_0 \sin \omega t$$

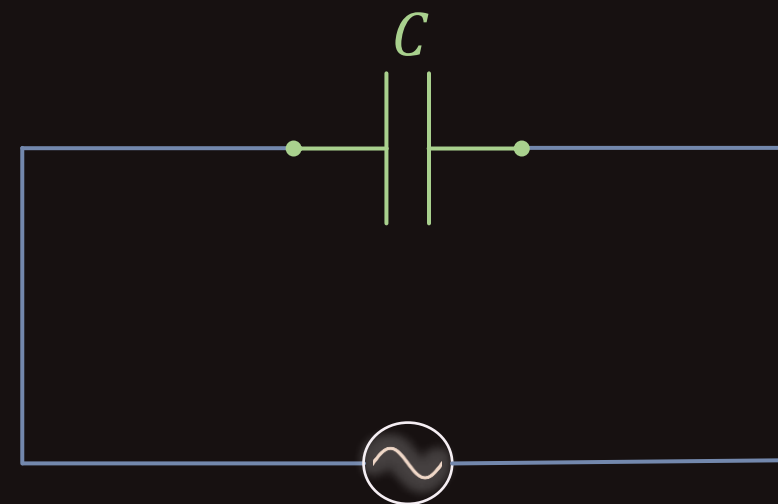
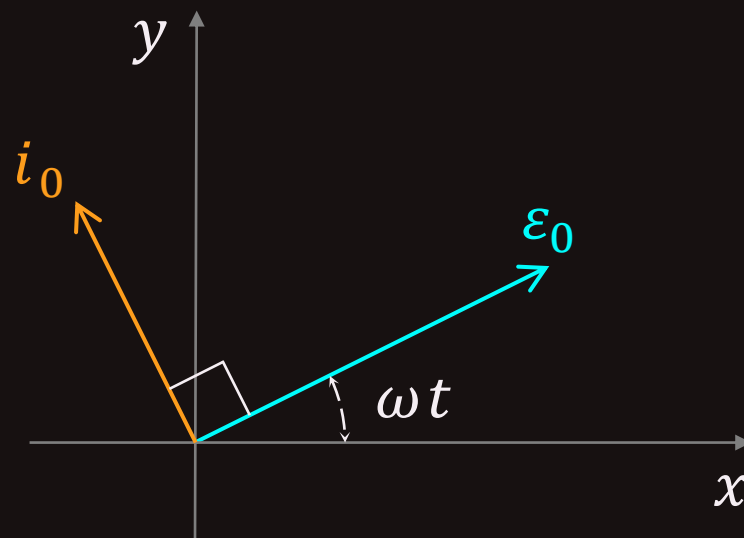
$$\boxed{\frac{\varepsilon_0}{i_0} = \frac{1}{\omega C} = X_C}$$

PURE CAPACITIVE AC CIRCUIT

$$\text{Phase difference } (\phi) = \left(\omega t + \frac{\pi}{2} \right) - \omega t = \frac{\pi}{2}$$

Current **leads** potential by **90°**

 Phasor diagram



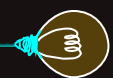
$$\begin{aligned}\varepsilon &= \varepsilon_0 \sin \omega t \\ i &= i_0 \sin \left(\omega t + \frac{\pi}{2} \right)\end{aligned}$$



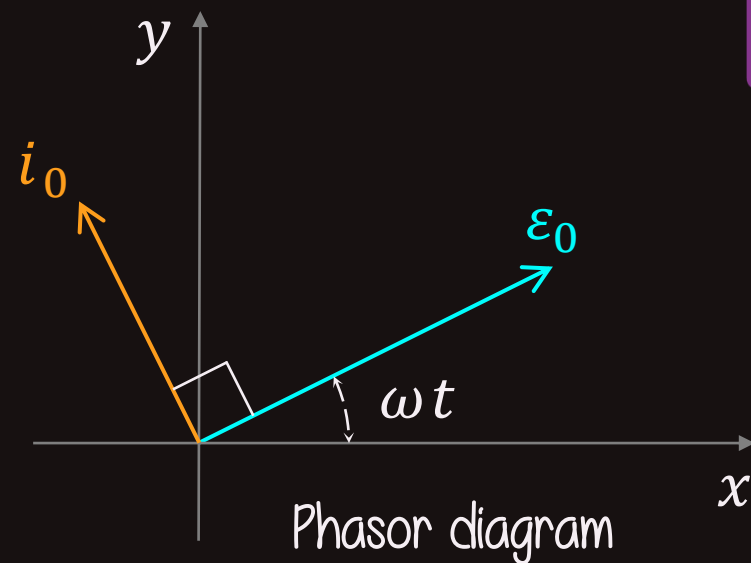
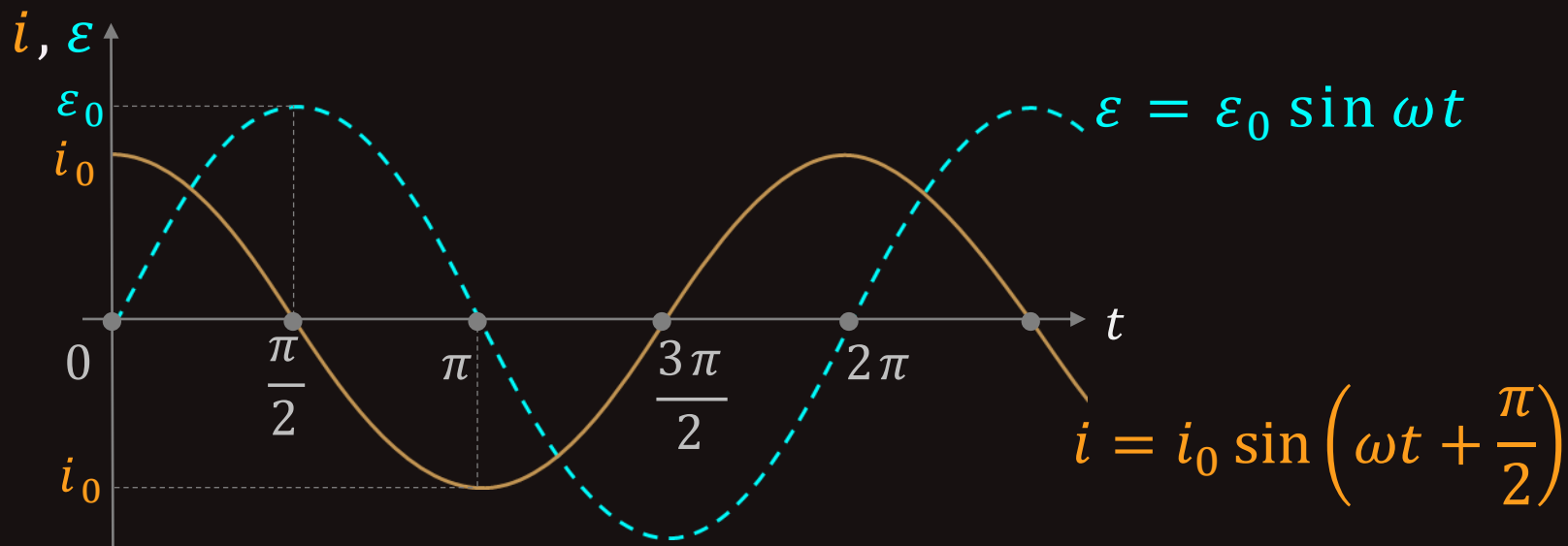
PURE CAPACITIVE AC CIRCUIT

$$\text{Phase difference } (\phi) = \left(\omega t + \frac{\pi}{2} \right) - \omega t = \frac{\pi}{2}$$

Current **leads** potential by **90°**



Wave diagram

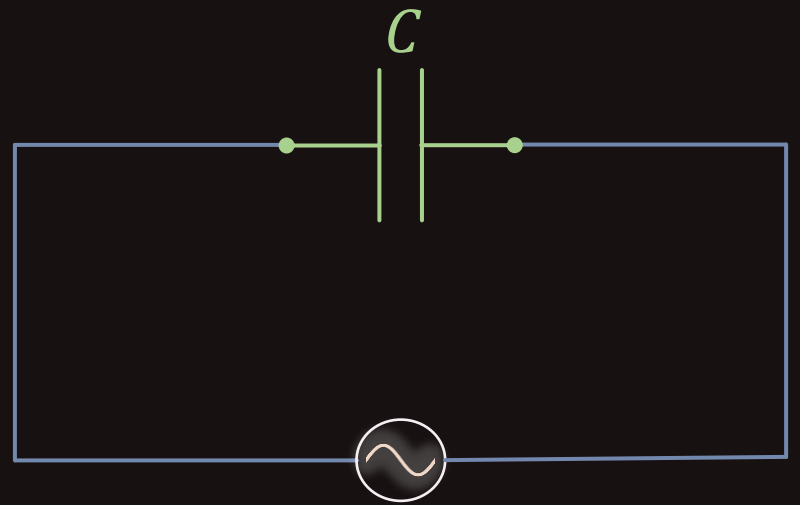
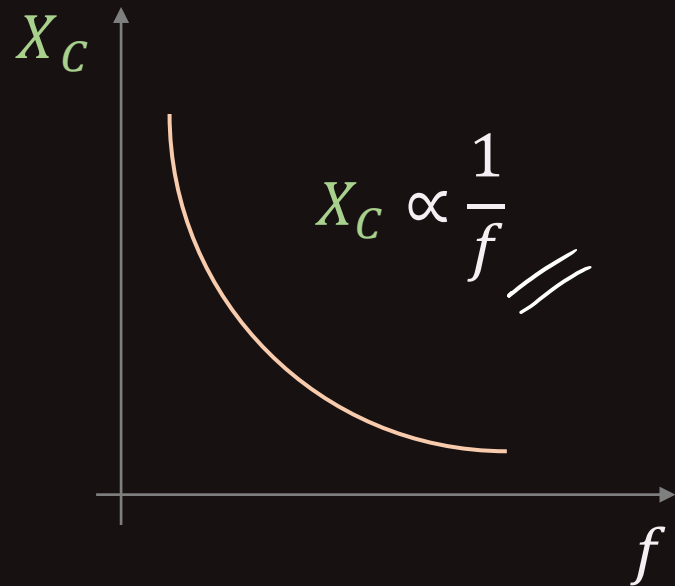


PURE CAPACITIVE AC CIRCUIT

 X_C v/s frequency (f)

$$X_C = \frac{1}{C\omega} \quad (\because \omega = 2\pi f)$$

$$X_C = \frac{1}{C \times 2\pi f}$$



$$\varepsilon = \varepsilon_0 \sin \omega t$$
$$i = i_0 \sin \left(\omega t + \frac{\pi}{2} \right)$$



SIMPLE AC CIRCUITS



Element	Current (emf: $\varepsilon = \varepsilon_0 \sin \omega t$)	ϕ	X(Reactance)
Resistor //	$\frac{\varepsilon_0}{R} \sin \omega t$	0	$R = \frac{\varepsilon_0}{i_0}$
Inductor //	$\frac{\varepsilon_0}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)$	$-\frac{\pi}{2}$	$\omega L = \frac{\varepsilon_0}{i_0}$ $\rightarrow X_L$
Capacitor //	$\frac{\varepsilon_0}{1/\omega C} \sin \left(\omega t + \frac{\pi}{2} \right)$	$\frac{\pi}{2}$	$\frac{1}{\omega C} \rightarrow X_C$



QUESTION



A capacitor of capacity C has reactance X . If capacitance and frequency become double then reactance will be...



$4X$



$X/2$



$X/4$



$2X$

$$X_C = \frac{1}{\omega C}$$



SOLUTION



$$X = \frac{1}{C\omega} = \frac{1}{C2\pi f}$$

$$f' = 2f$$

$$C' = 2C$$

$$X' = \frac{1}{C'2\pi f'} = \frac{1}{(2C) \times 2\pi \times (2f)} = \frac{1}{4(C2\pi f)}$$

$$X' = \frac{X}{4}$$



4X



X/2



X/4



2X



QUESTION



Current in a pure capacitive circuit of $C = 5 \mu F$ is $5 \sin(50t + 30^\circ)$. Find the equation for emf.



$$2 \times 10^6 \sin(50t)$$



$$2 \times 10^4 \sin(50t - 60^\circ)$$



$$2 \times 10^4 \sin(50t + 90^\circ)$$



$$2 \times 10^4 \sin(50t)$$



SOLUTION

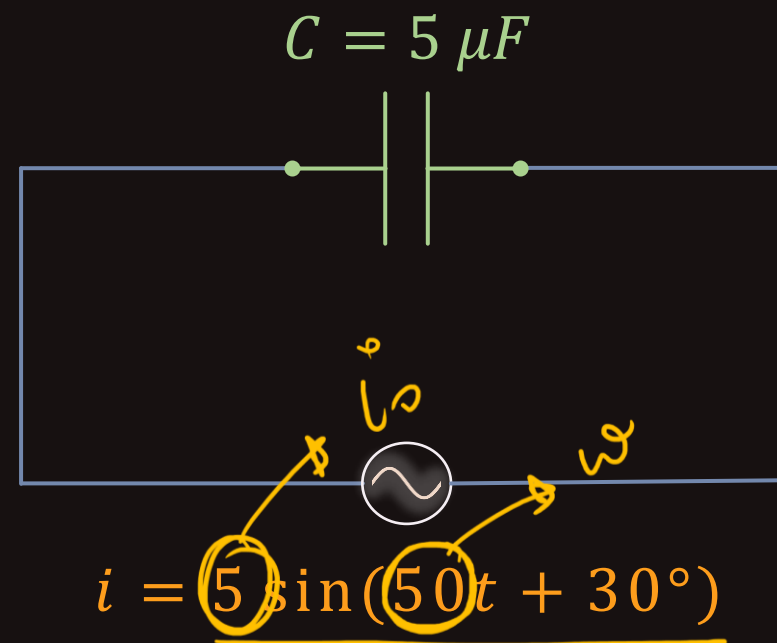


$$i = i_0 \sin(\omega t + \phi) \quad i_0 = 5 \text{ A} \quad \omega = 50 \text{ s}^{-1}$$

$$i_0 = \frac{\varepsilon_0}{X_C} \Rightarrow \varepsilon_0 = i_0 X_C = i_0 \frac{1}{\omega C}$$

$$\varepsilon_0 = 5 \frac{1}{50 \times 5 \times 10^{-6}}$$

$$\varepsilon_0 = 2 \times 10^4 \text{ V}$$



Current leads potential by 90° .

$$\varepsilon = \varepsilon_0 \sin(\omega t + \phi - 90^\circ) \Rightarrow \varepsilon = 2 \times 10^4 \sin(50t + 30 - 90^\circ)$$

$$\varepsilon = 2 \times 10^4 \sin(50t - 60^\circ)$$



$$2 \times 10^6 \sin(50t)$$



$$2 \times 10^4 \sin(50t - 60^\circ)$$

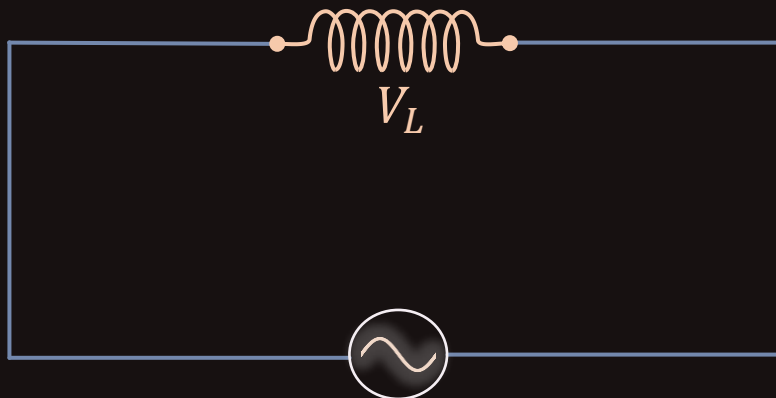


$$2 \times 10^4 \sin(50t + 90^\circ)$$



$$2 \times 10^4 \sin(50t)$$

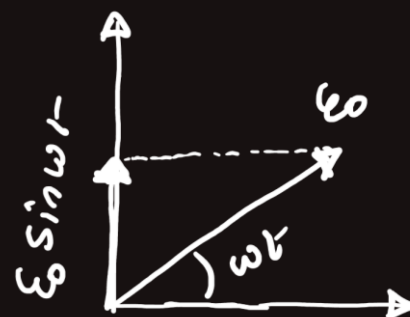
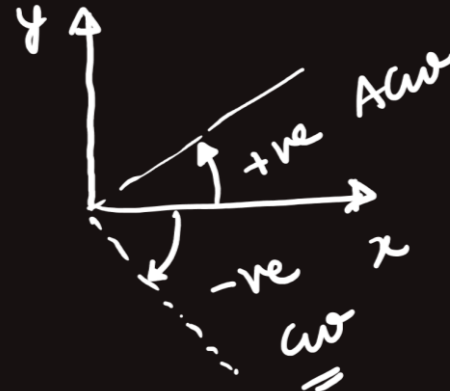
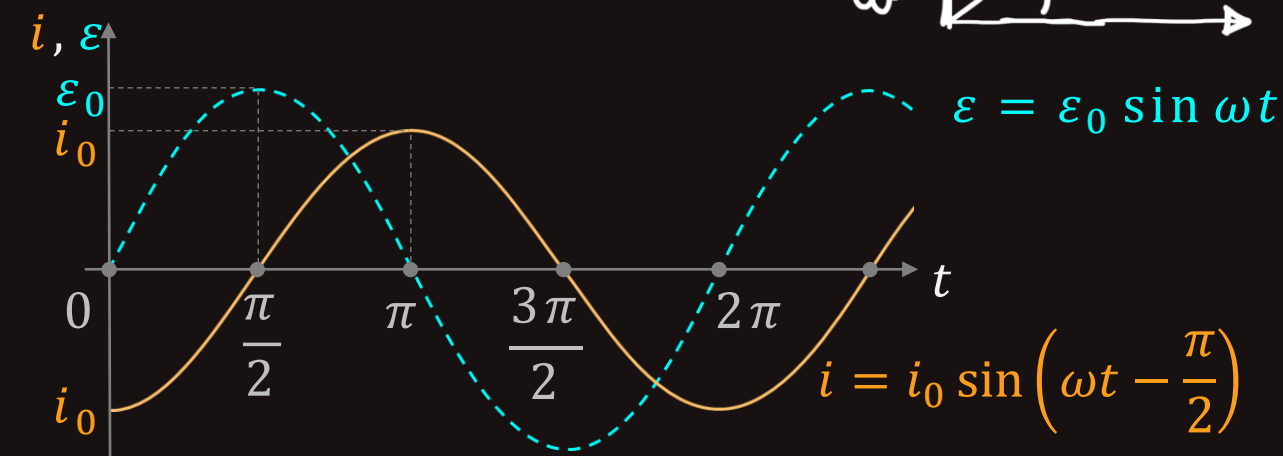
PURE INDUCTIVE CIRCUIT



$$\varepsilon = \varepsilon_0 \sin \omega t$$



Wave diagram



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin \left(\omega t - \frac{\pi}{2} \right)$$

$$0 \rightarrow \frac{\pi}{2}$$

ε increases from zero to maximum, current becomes zero.

$$\frac{\pi}{2} \rightarrow \pi$$

ε decreases from maximum to zero, current grows to maximum.

$$\pi \rightarrow \frac{3\pi}{2}$$

ε increases from zero to negative maximum, current becomes zero.

$$\frac{3\pi}{2} \rightarrow 2\pi$$

ε decreases from negative maximum to zero, current grows to negative maximum.

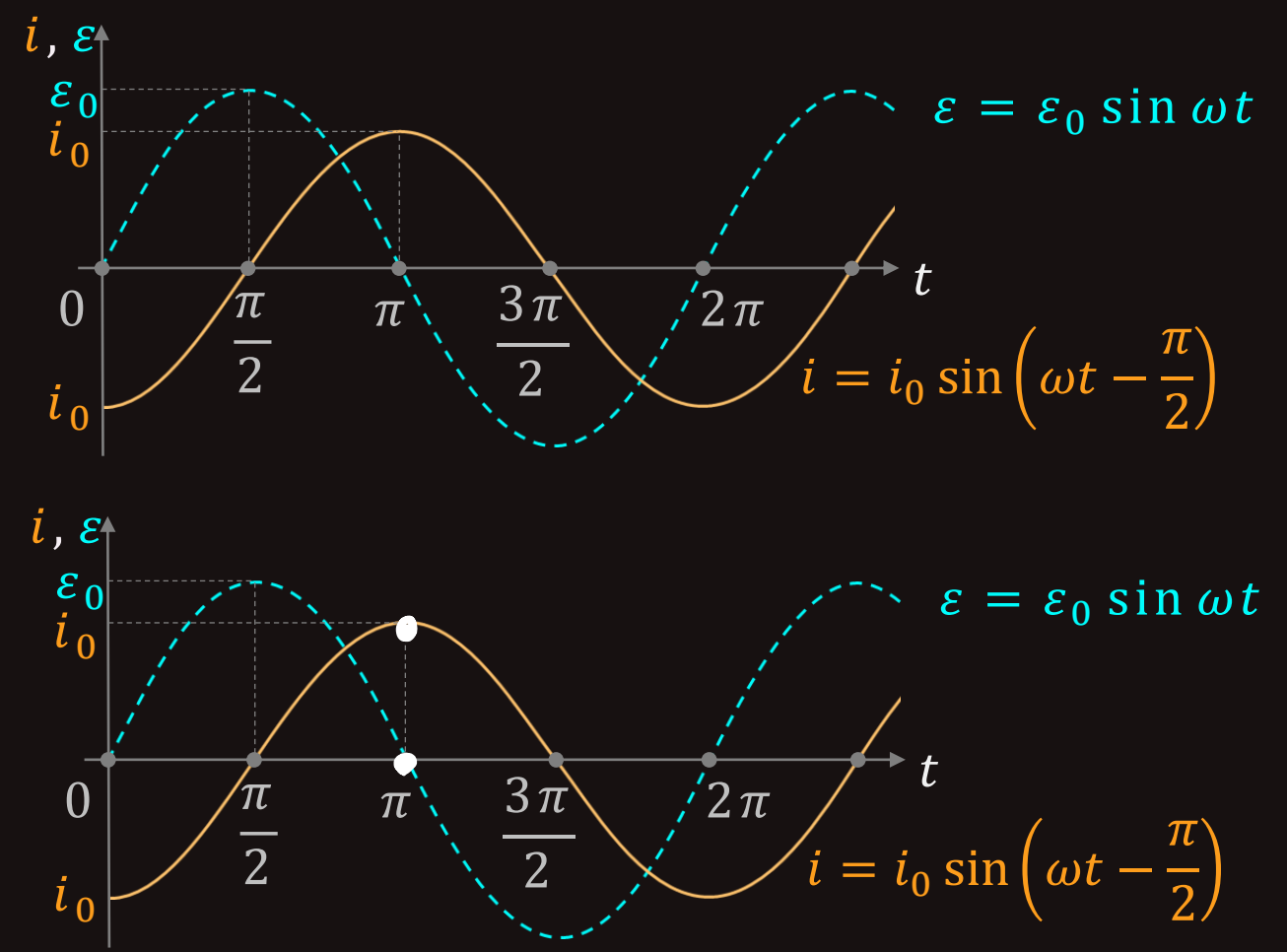
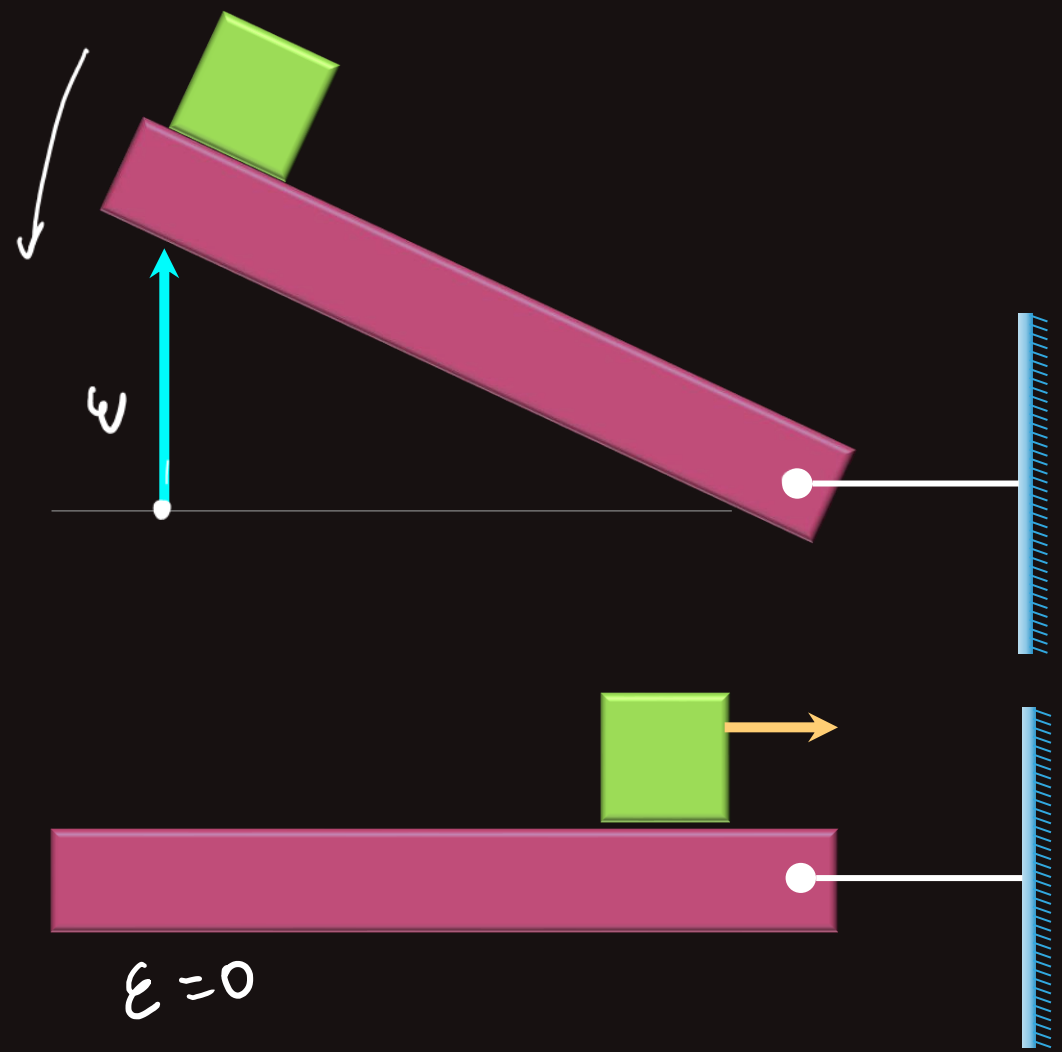
Current tries to follow path of voltage. The phase difference between them always remains to be 90° .



PURE INDUCTIVE CIRCUIT

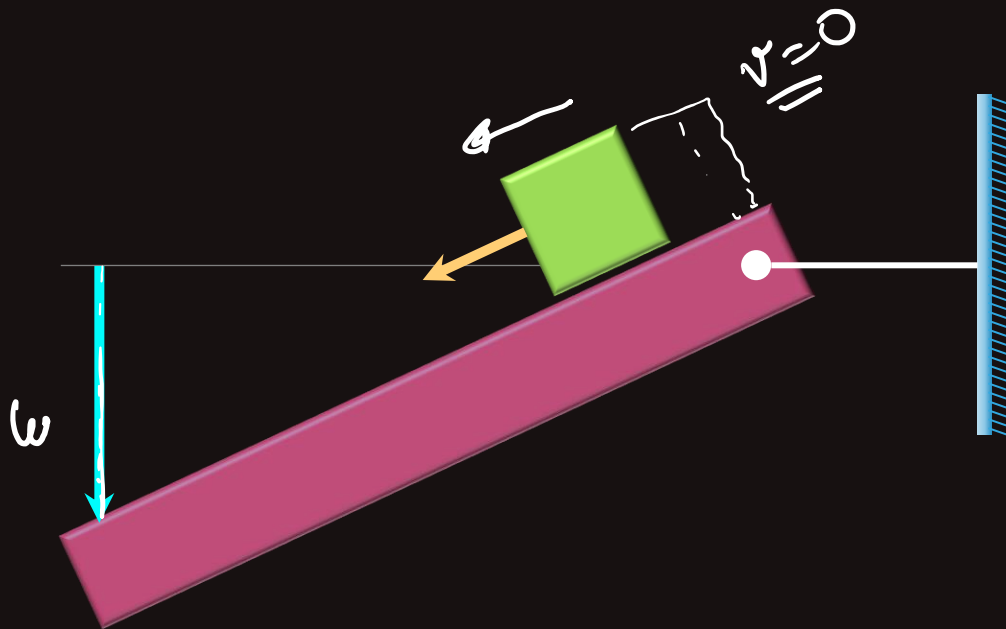
B

The lagging of current can be seen from the mechanical analogy as shown. Height represents the voltage and velocity of block is current. When height is maximum, velocity is zero and vice-versa.

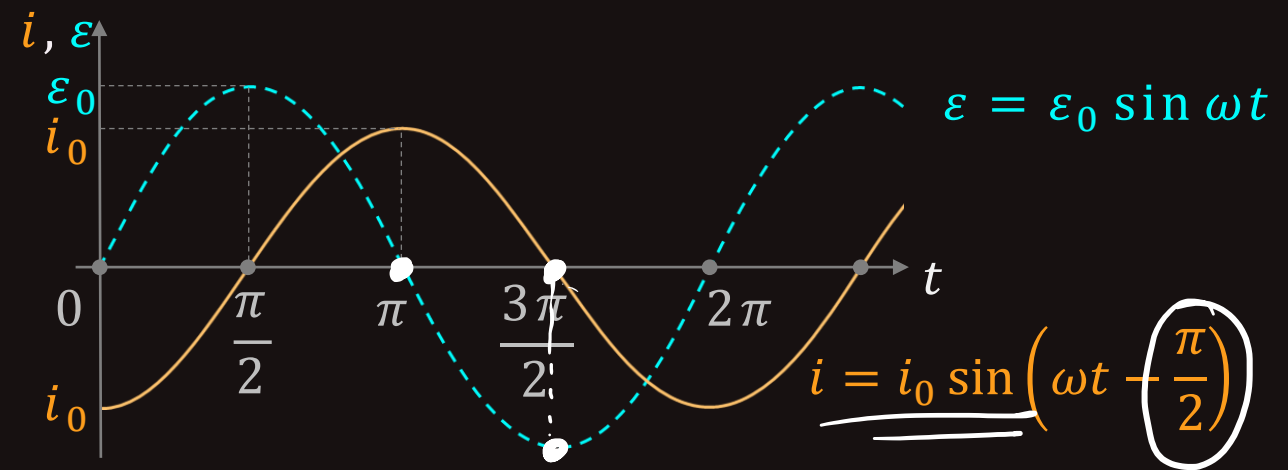


PURE INDUCTIVE CIRCUIT

B



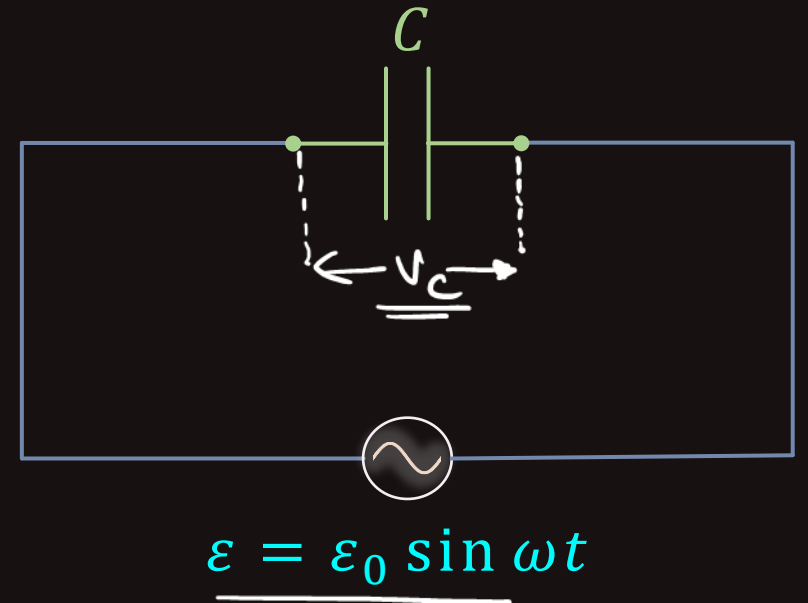
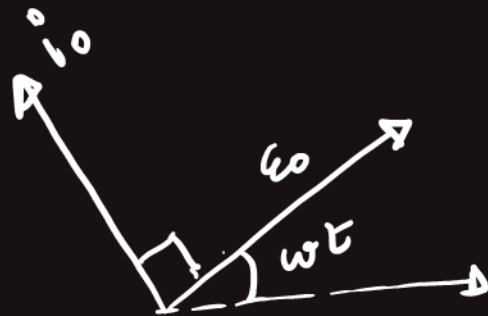
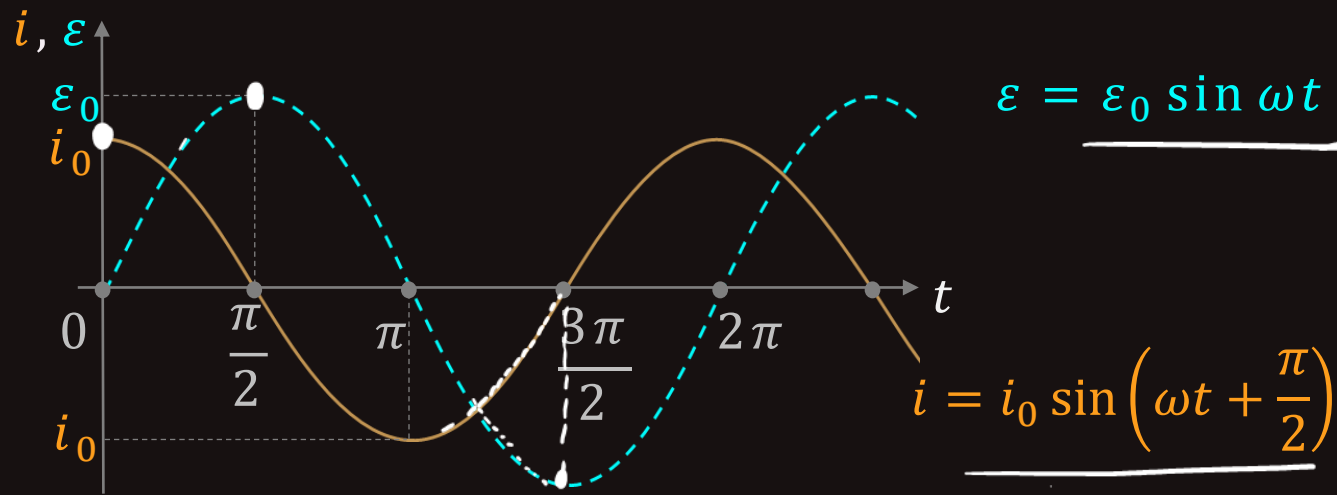
Wave diagram





PURE CAPACITIVE CIRCUIT

Wave diagram





IMPEDANCE (Z)



BOARDS



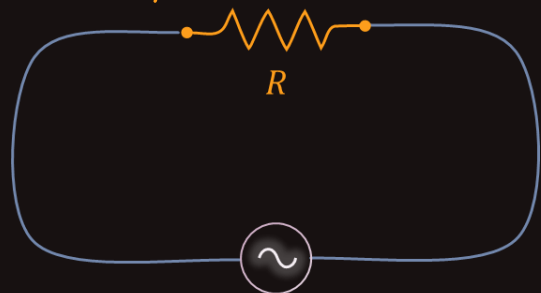
NEET



The relation b/w peak current and peak voltages can be written as

$$\tilde{i}_0 = \frac{\tilde{E}_0}{Z}$$

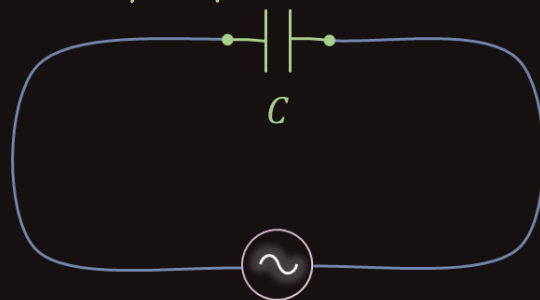
Purely resistive circuit



$$Z = R$$

Ω

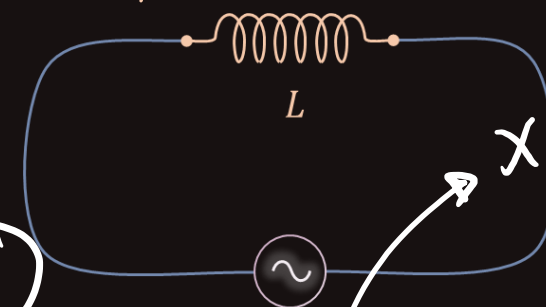
Purely capacitive circuit



$$Z = 1/\omega C$$

$$X_C = \frac{1}{\omega C}$$

Purely inductive circuit



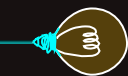
$$Z = \omega L$$

$$X_L = \omega L$$

$$\frac{\tilde{E}_0}{\tilde{i}_0} = Z$$



Z is called impedance.



Impedance is defined as the opposition any circuit shows when voltage is applied to it.



SI unit is Ohm (Ω)



AC CIRCUITS – COMBINATIONS | RC CIRCUIT



$$\mathcal{E}_0^2 = V_R^2 + V_C^2$$

$$\mathcal{E}_0^2 = i_0^2 R^2 + i_0^2 X_C^2$$

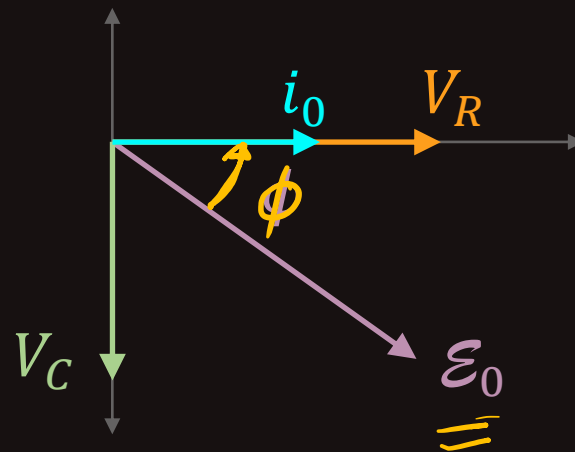
$$i_0^2 Z^2 = i_0^2 R^2 + i_0^2 X_C^2$$

$$Z^2 = R^2 + X_C^2 \Rightarrow Z = \sqrt{R^2 + X_C^2}$$

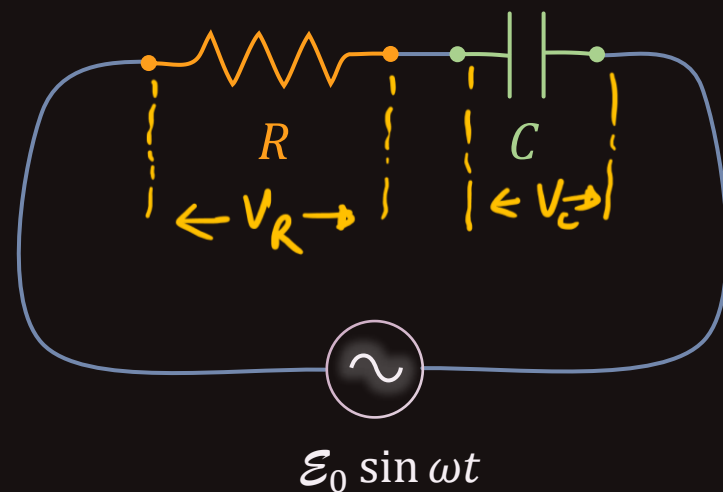
$$X_C = 1/\omega C$$

$$Z = \sqrt{R^2 + (1/\omega C)^2}$$

$$X_C = \frac{1}{\omega C}$$



$$\frac{\mathcal{E}_0}{i_0} = Z$$



$$\mathcal{E}_0 = i_0 Z$$

$$V_R = i_0 R$$

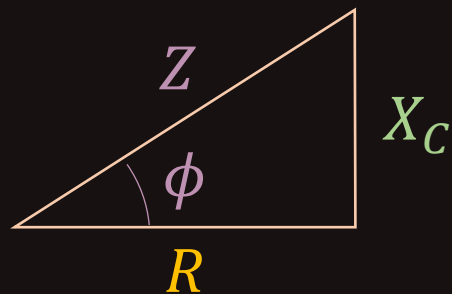
$$V_C = i_0 X_C$$



AC CIRCUITS – COMBINATIONS | RC CIRCUIT

B

$$Z^2 = R^2 + X_C^2$$

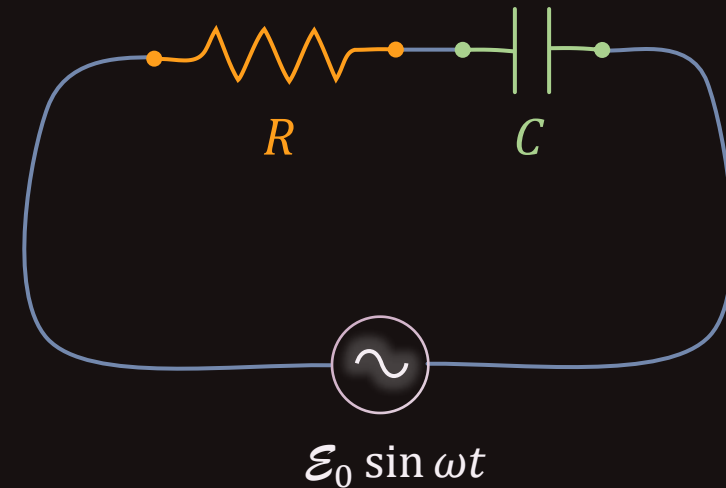
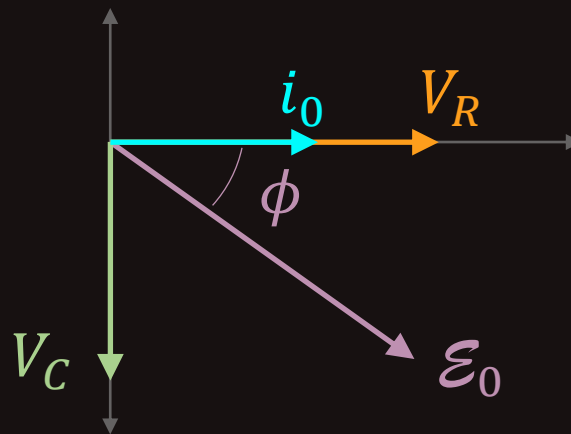


$$\tan \phi = \frac{X_C}{R} \quad \left(X_C = \frac{1}{\omega C} \right)$$

$$\tan \phi = \frac{1}{\omega CR}$$

Peak current (i_0) is given by-

$$i_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (1/\omega C)^2}}$$



$$Z = \sqrt{R^2 + (1/\omega C)^2}$$

$$V_R = i_0 R \quad R = \frac{V_R}{i_0}$$

$$V_C = i_0 X_C \quad X_C = \frac{V_C}{i_0}$$

$$= \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}}$$



AC CIRCUITS – COMBINATIONS | RC CIRCUIT



$$Z = \sqrt{R^2 + (1/\omega C)^2}$$

$$\tan \phi = \frac{1}{\omega CR}$$

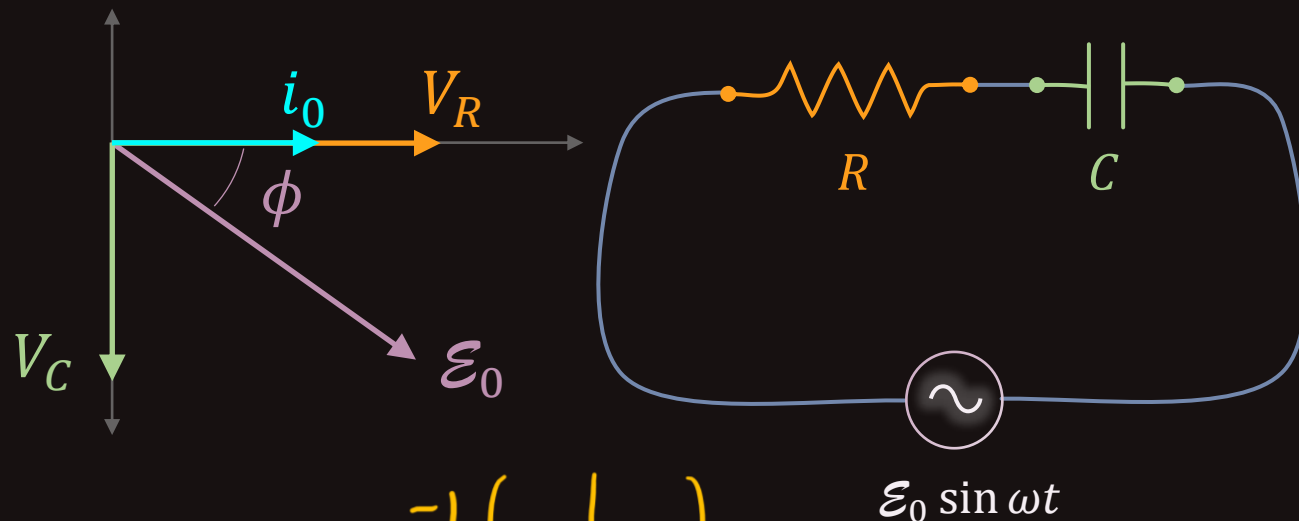
$$i_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (1/\omega C)^2}}$$

Steady state current (i) in the circuit

$$i = \frac{\mathcal{E}_0}{Z} \sin(\omega t + \phi)$$



The current leads the emf by ϕ .



$$\phi = \tan^{-1} \left(\frac{1}{\omega CR} \right)$$

$$i = i_0 \sin(\omega t + \phi)$$

$$i = \frac{\mathcal{E}_0}{Z} \sin(\omega t + \phi)$$



AC CIRCUITS – COMBINATIONS | LR CIRCUIT



$$\underline{\mathcal{E}_0^2 = V_R^2 + V_L^2}$$

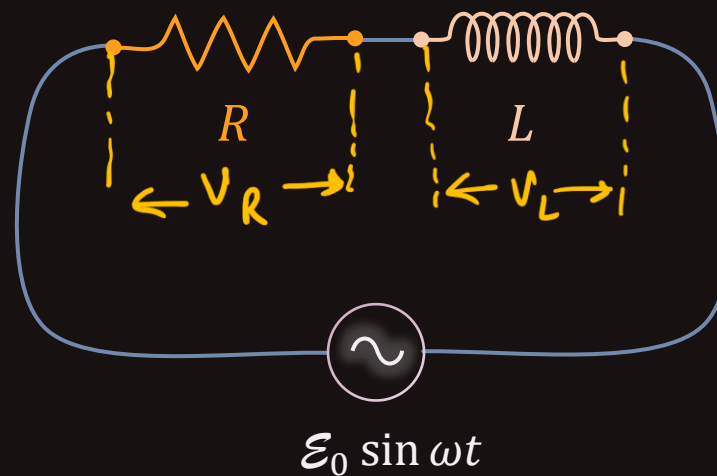
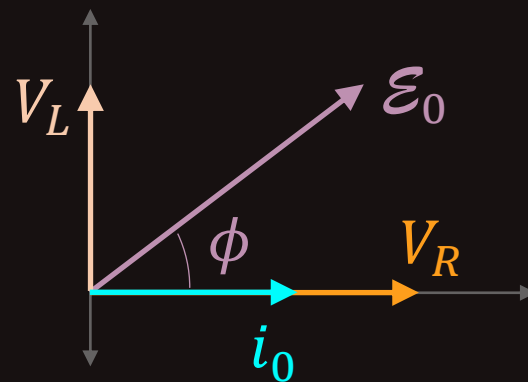
$$\mathcal{E}_0^2 = i_0^2 R^2 + i_0^2 X_L^2$$

$$\cancel{i_0^2} Z^2 = \cancel{i_0^2} R^2 + \cancel{i_0^2} X_L^2$$

$$Z^2 = R^2 + X_L^2 \Rightarrow \underline{Z = \sqrt{R^2 + X_L^2}}$$

$$X_L = \omega L$$

$$\underline{Z = \sqrt{R^2 + (\omega L)^2}}$$

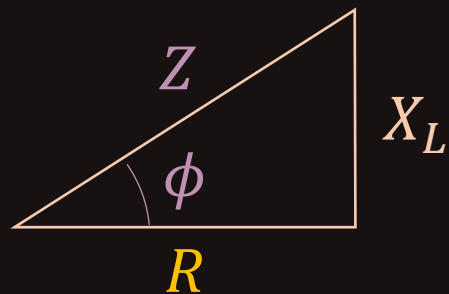




AC CIRCUITS – COMBINATIONS | LR CIRCUIT



$$Z^2 = R^2 + X_L^2$$

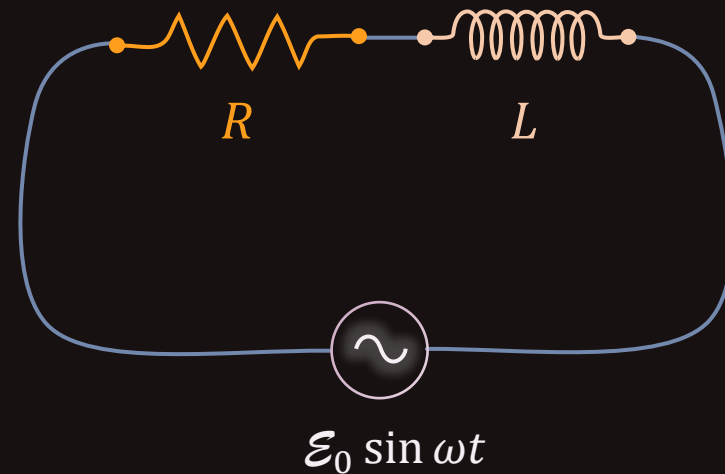
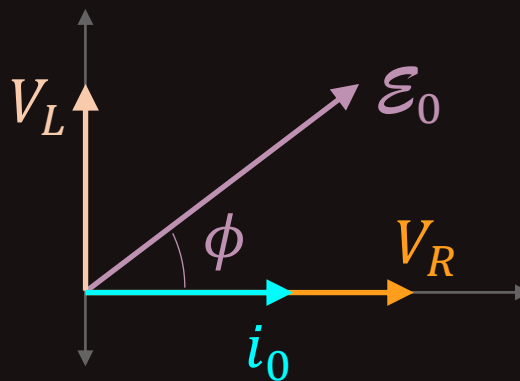


$$\tan \phi = \frac{X_L}{R} \quad (X_L = \omega L)$$

$$\tan \phi = \frac{\omega L}{R}$$

Peak current (i_0) is given by=

$$i_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L)^2}}$$



$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

$$Z = \sqrt{R^2 + (\omega L)^2}$$



AC CIRCUITS – COMBINATIONS | LR CIRCUIT



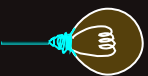
$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\tan \phi = \frac{\omega L}{R}$$

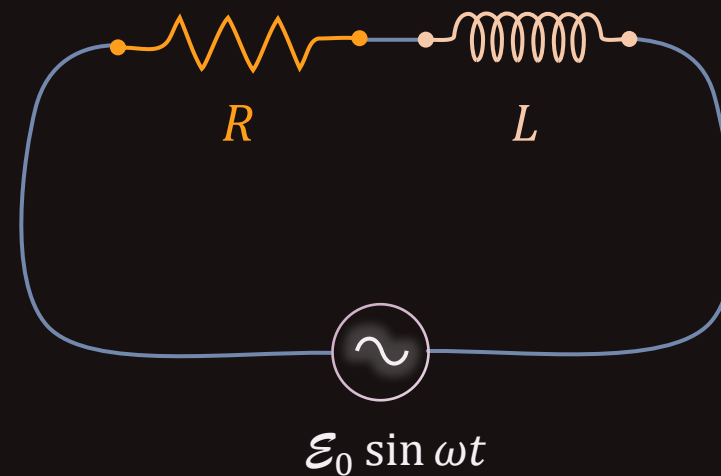
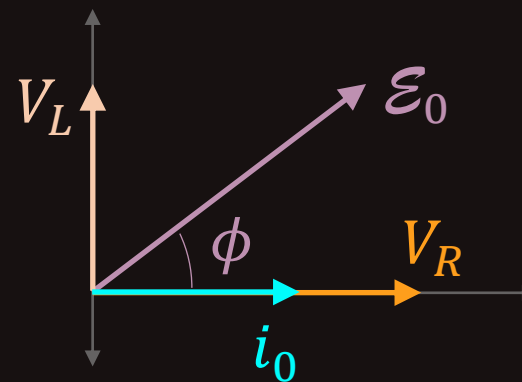
$$i_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L)^2}}$$

Steady state current (i) in the circuit

$$i = \frac{\mathcal{E}_0}{Z} \sin(\omega t - \phi)$$



The current lags the emf by ϕ .



$$i = i_0 \sin(\omega t - \phi)$$

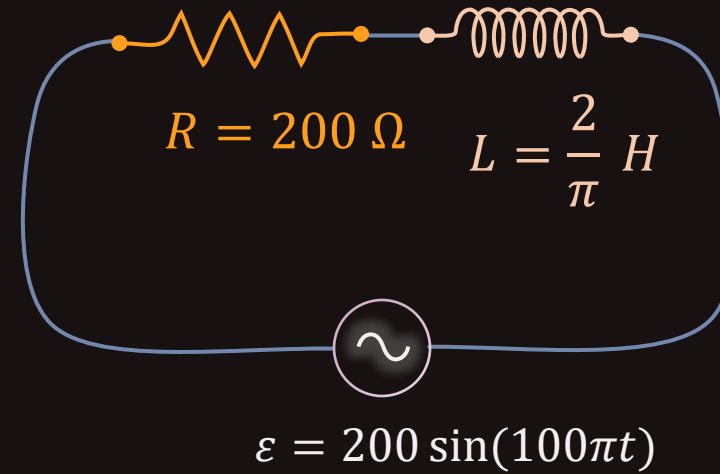


QUESTION



In the given circuit, find

- (a) inductive reactance X_L .
- (b) impedance Z .
- (c) Peak current i_0 .
- (d) $i(t)$





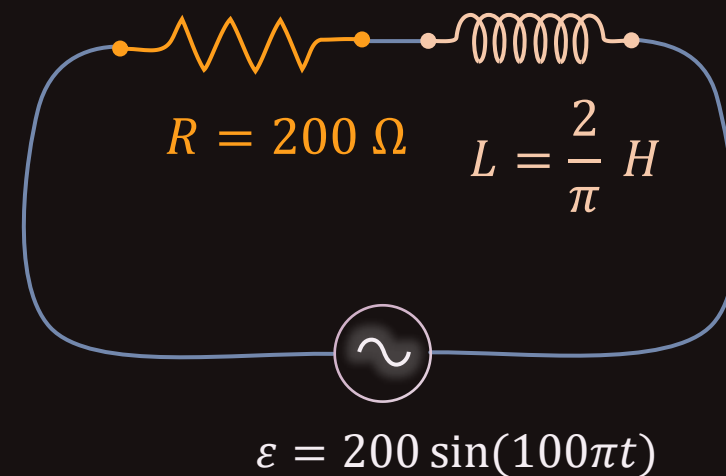
(a) inductive reactance X_L .

$$X_L = \omega L$$

$$X_L = 100\pi \times \frac{2}{\pi} = 200 \, \Omega //$$

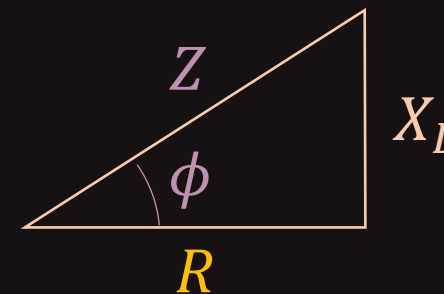
$$\begin{aligned}\mathcal{E} &= \mathcal{E}_0 \sin(\omega t) \\ \mathcal{E} &= 200 \sin(100\pi t)\end{aligned}$$

$$\begin{aligned}\mathcal{E}_0 &= 200 \, \text{V} \\ \omega &= 100\pi \, \text{s}^{-1}\end{aligned}$$



(b) impedance Z .

$$\begin{aligned}Z^2 &= R^2 + X_L^2 \Rightarrow Z = \sqrt{R^2 + X_L^2} \\ Z &= \sqrt{(200)^2 + (200)^2} \\ \therefore Z &= 200\sqrt{2} \, \Omega\end{aligned}$$





SOLUTION



(c) Peak current i_0 .

$$i_0 = \frac{\varepsilon_0}{Z}$$

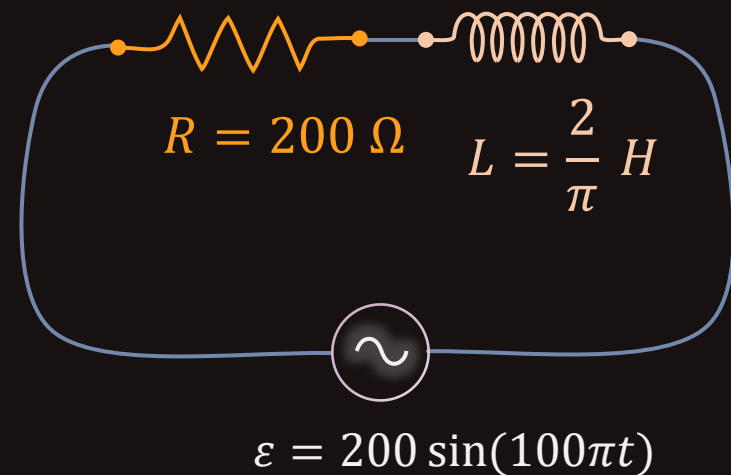
$$\varepsilon_0 = 200 \text{ V}$$

$$i_0 = \frac{200}{200\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ A}$$

$$Z = \frac{\varepsilon_0}{i_0}$$

$$\varepsilon_0 = 200 \text{ V}$$

$$Z = 200\sqrt{2} \Omega$$

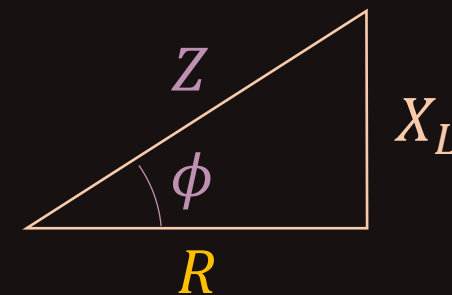
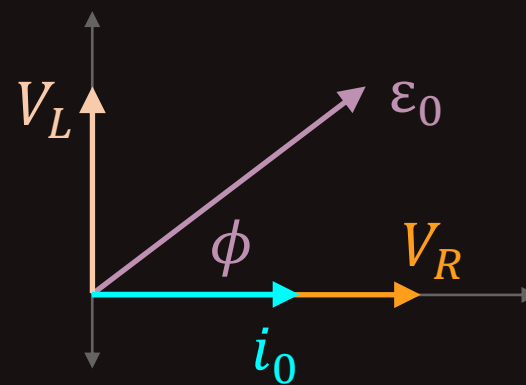


(d) $i(t)$

$$i(t) = i_0 \sin(100\pi t - \phi)$$

$$\tan \phi = \frac{X_L}{R} = \frac{200}{200} = 1 \Rightarrow \phi = \frac{\pi}{4}$$

$$\therefore i(t) = \frac{1}{\sqrt{2}} \sin\left(100\pi t - \frac{\pi}{4}\right)$$





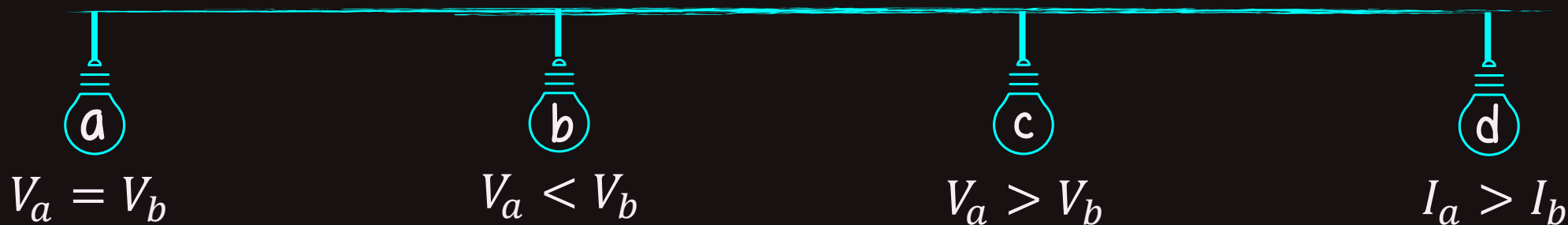
QUESTION



A series R - C circuit is connected to an alternating voltage source. Consider two situations

- (a) When capacitor is air filled.
- (b) When capacitor is mica filled.

Current through resistor is I and voltage across capacitor is V then-

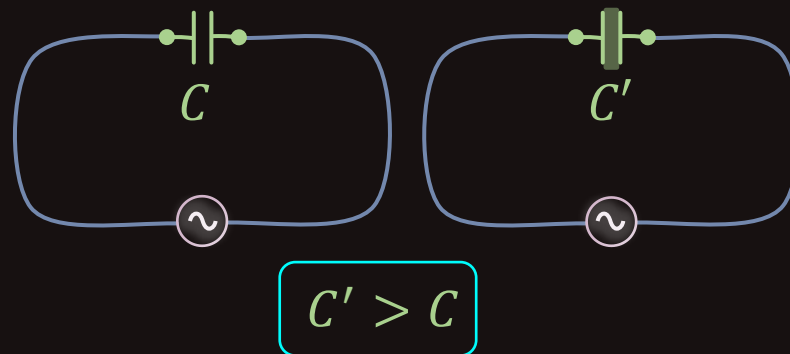




SOLUTION



$$X_C \propto \frac{1}{C}$$



$$X_C = \frac{1}{\omega C}$$

When capacitor is filled with mica, its capacitance increases.

As C increases, X_C decreases

As X_C decreases, voltage across capacitor decreases ($X_C \propto V$).

$$V_a > V_b$$



$$V_a = V_b$$



$$V_a < V_b$$



$$V_a > V_b$$



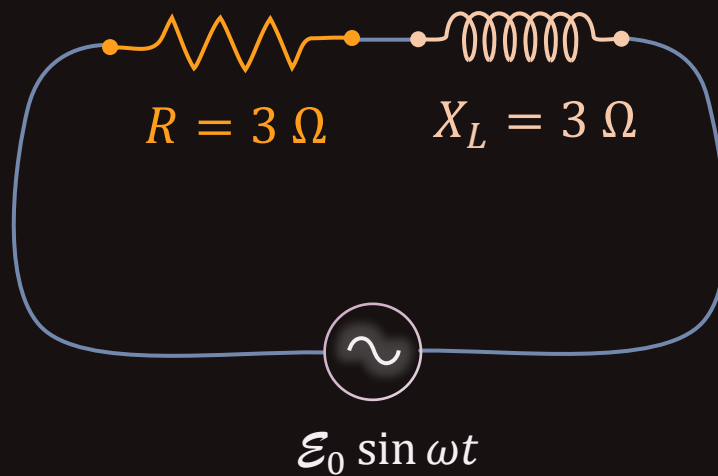
$$I_a > I_b$$



QUESTION



An A.C. voltage is applied to a resistance R and an inductor L in series. If R and the inductive reactance are both equal to $3\ \Omega$, the phase difference between the applied voltage and the current in the circuit is



a
Zero

b
 $\pi/6$

c
 $\pi/4$

d
 $\pi/2$

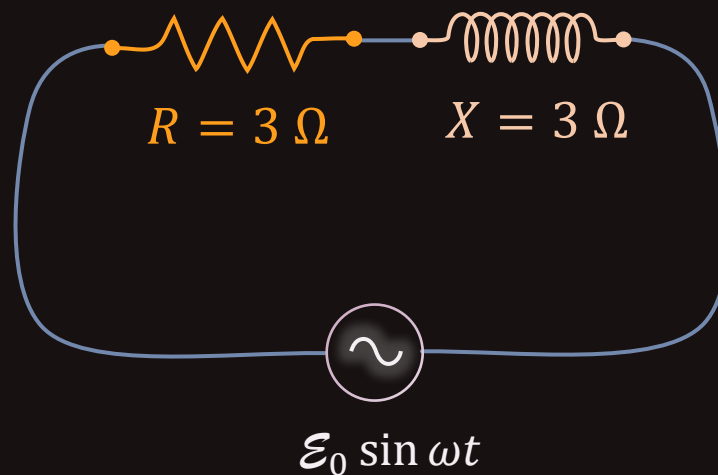


SOLUTION

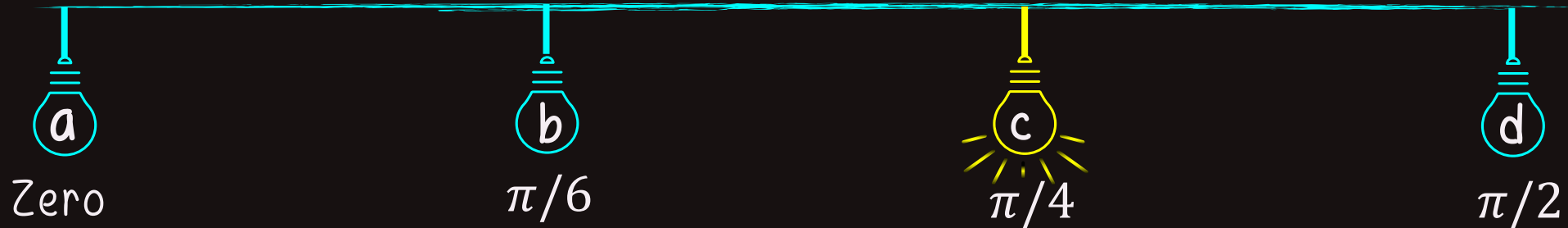
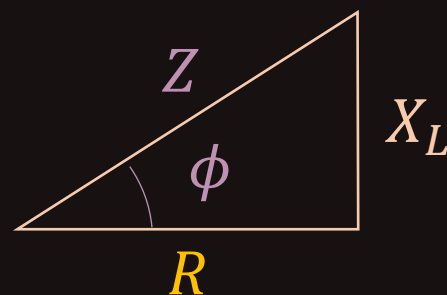


$$\tan \phi = \frac{X_L}{R} = 1$$

$$\phi = 45^\circ \text{ or } \pi/4$$



$$\tan \phi = \frac{X_L}{R}$$

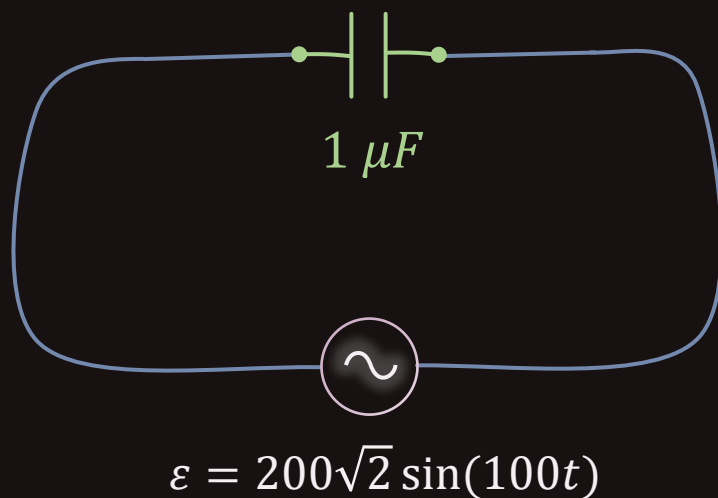




QUESTION



In an A.C. circuit, an alternating voltage, $\varepsilon = 200\sqrt{2} \sin(100t)$ volt is connected to capacitor of capacity $1 \mu F$. The r.m.s. value of current in the circuit is



10 mA



100 mA



200 mA



20 mA



SOLUTION



Alternating voltage, $\varepsilon = 200\sqrt{2} \sin(100t)$

comparing with $\varepsilon = \varepsilon_0 \sin \omega t$

$$\omega = 100 \text{ rad/s} \quad \varepsilon_0 = 200\sqrt{2} \text{ volt}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} \Omega = 10^4 \Omega$$

$$I_0 = \frac{\varepsilon_0}{X_C} = \frac{200\sqrt{2}}{10^4} \text{ A} = 2\sqrt{2} \times 10^{-2} \text{ A} \Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2\sqrt{2} \times 10^{-2}}{\sqrt{2}} = 2 \times 10^{-2} \text{ A} = 20 \text{ mA}$$



$$\varepsilon = 200\sqrt{2} \sin(100t)$$

$$\varepsilon = \varepsilon_0 \sin \omega t$$



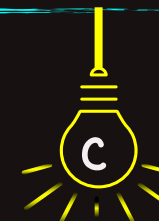
10 mA



100 mA



200 mA



20 mA



POWER IN AC CIRCUITS

The work done by source in time interval dt is,

$$dW = \mathcal{E} i dt$$

$$dW = \mathcal{E}_0 i_0 \sin \omega t \sin(\omega t + \phi) dt$$

$$= \mathcal{E}_0 i_0 (\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi) dt$$

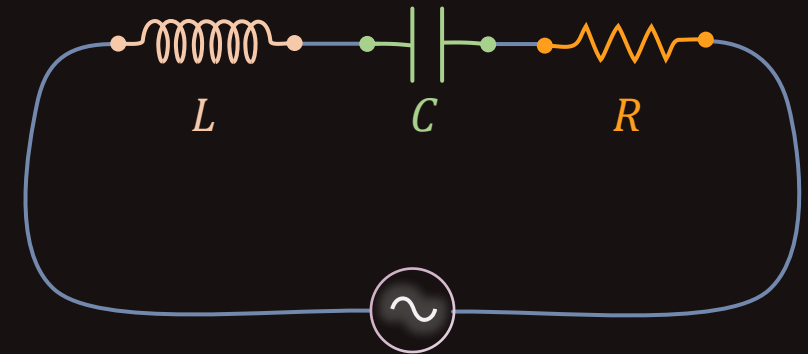
Total work done in a cycle is,

$$W = \mathcal{E}_0 i_0 \cos \phi \int_0^T \sin^2 \omega t dt + \mathcal{E}_0 i_0 \sin \phi \int_0^T \sin \omega t \cos \omega t dt$$

$$= \mathcal{E}_0 i_0 \cos \phi \int_0^T (1 - \cos 2\omega t) dt + \mathcal{E}_0 i_0 \sin \phi \int_0^T \frac{1}{2} \sin 2\omega t dt$$

$$= \frac{\mathcal{E}_0 i_0 \cos \phi}{2} \left[\int_0^T dt - \int_0^T \cos 2\omega t dt \right] + \mathcal{E}_0 i_0 \sin \phi \int_0^T \frac{1}{2} \sin 2\omega t dt$$

$$W = \frac{1}{2} \mathcal{E}_0 i_0 T \cos \phi$$



$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

$$i = i_0 \sin(\omega t + \phi)$$



POWER IN AC CIRCUITS



Total work done in a cycle is,

$$W = \frac{1}{2} \mathcal{E}_0 i_0 T \cos \phi$$



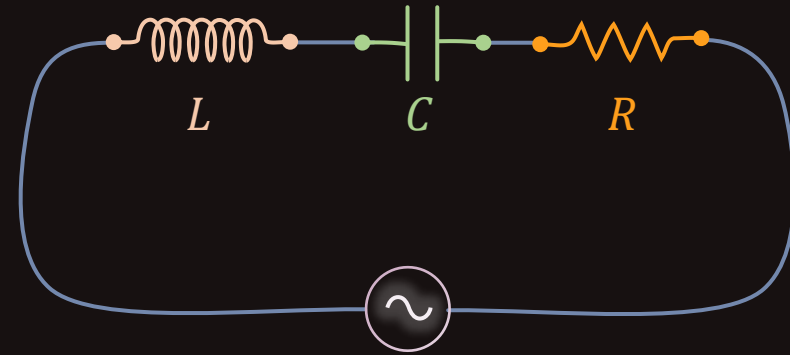
Average Power delivered,

$$P_{avg} = \frac{W}{T}$$

$$P_{avg} = \frac{1}{2} \mathcal{E}_0 i_0 \cos \phi$$

$$= \frac{\mathcal{E}_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos \phi$$

$$\sim P_{avg} = \mathcal{E}_{rms} i_{rms} \cos \phi \sim$$



$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

$$i = i_0 \sin(\omega t + \phi)$$



POWER IN AC CIRCUITS

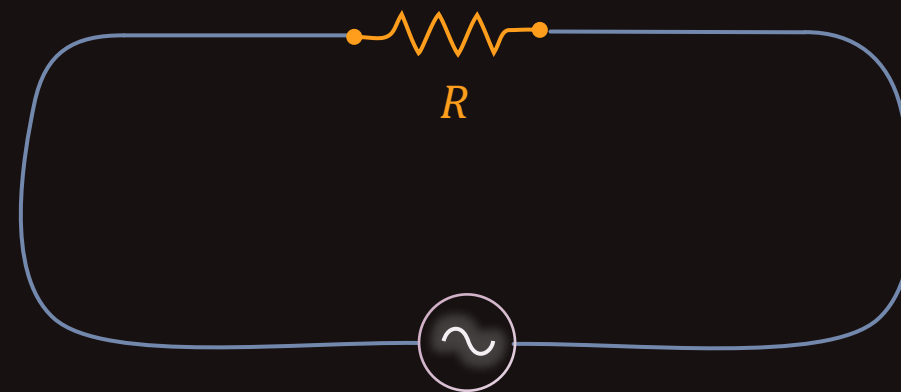
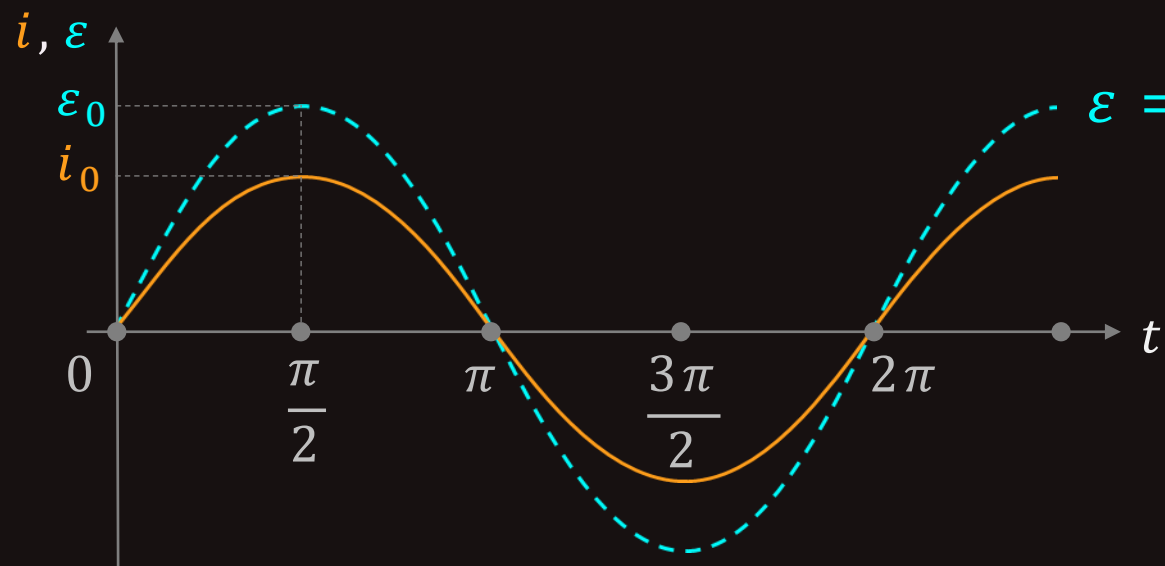
B

$$P_{avg} = \mathcal{E}_{rms} i_{rms} \cos \phi$$

Power Factor



For purely resistive circuit,



$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

$$i = i_0 \sin(\omega t + \phi)$$

$$P_{avg} = \mathcal{E}_{rms} i_{rms} \cos 0^\circ$$

$$P_{avg} = \mathcal{E}_{rms} i_{rms}$$

Power drawn is maximum in a purely resistive circuit



POWER IN AC CIRCUITS

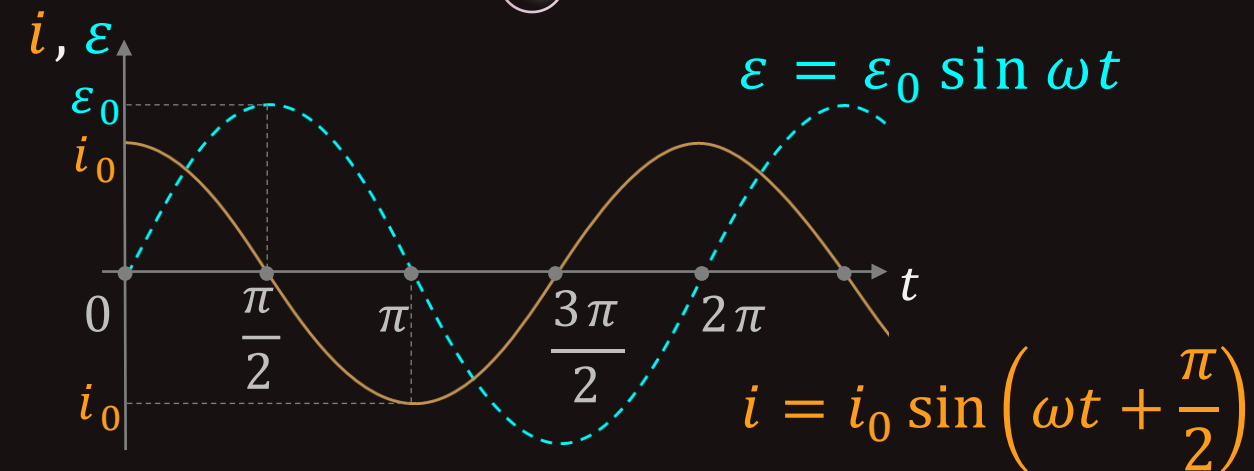
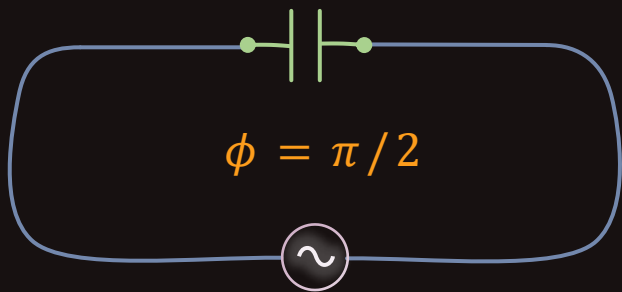


$$\sim P_{avg} = \mathcal{E}_{rms} i_{rms} \cos \phi \sim$$

Power Factor



For purely reactive circuit -

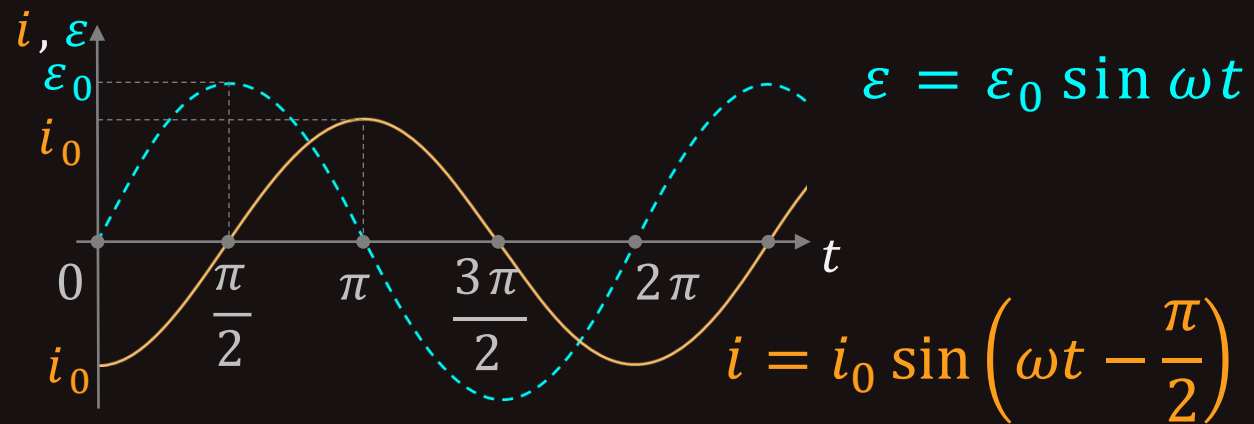
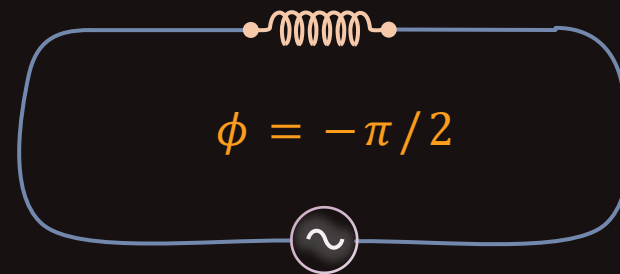


$$\phi = \pi/2 \text{ or } \phi = -\pi/2$$

$$P_{avg} = \mathcal{E}_{rms} i_{rms} \cos \left(\pm \frac{\pi}{2} \right)$$

$$P_{avg} = 0$$

No power is absorbed for a full cycle in purely inductive or purely capacitive circuits





POWER IN AC CIRCUITS | RC CIRCUITS

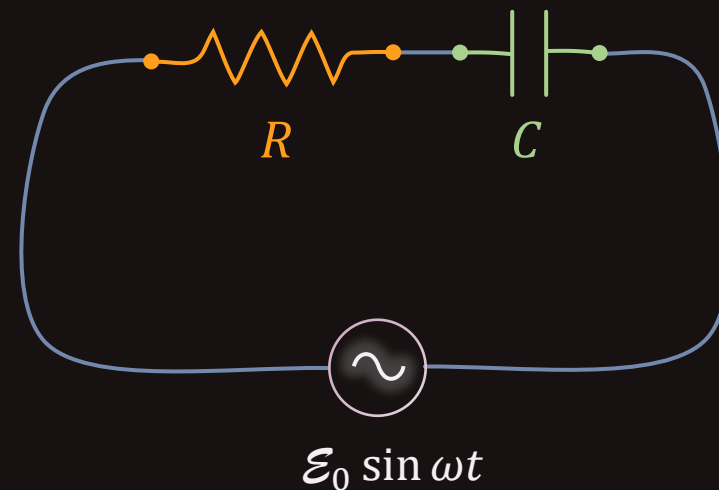
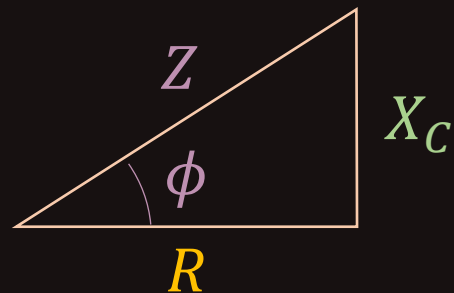
$$P_{avg} = \mathcal{E}_{rms} i_{rms} \cos \phi$$

$$\cos \phi = \frac{R}{Z}$$

$$\cos \phi = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$$

Average Power in RC circuits is,

$$P_{avg} = \mathcal{E}_{rms} i_{rms} \frac{R}{Z}$$







POWER IN AC CIRCUITS | RL CIRCUITS



$$P_{avg} = \mathcal{E}_{rms} i_{rms} \cos \phi$$

$$\cos \phi = \frac{R}{Z}$$


$$\cos \phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$


Average Power in LR circuits is,

$$P_{avg} = \mathcal{E}_{rms} i_{rms} \frac{R}{Z}$$

