Welcome to

BYJU'S Classes

Current Electricity

Current Electricity

0600 BC

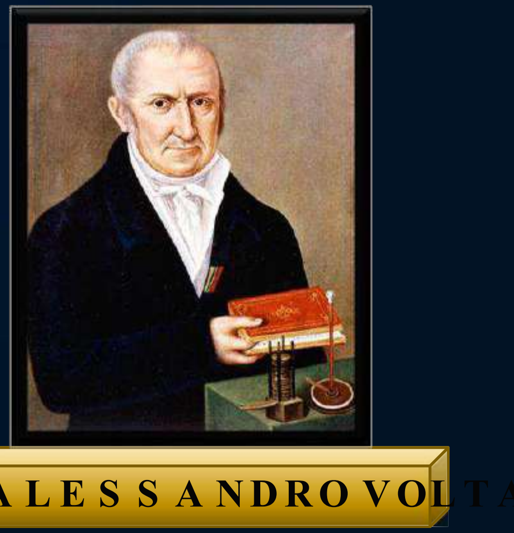
From the knowledge of electrostatics, we know that when two bodies come in contact and get rubbed with each other, the transfer of charges happens. The electricity produced by this method is known as frictional electricity.

But to store this charges we need something by which we can use this charges in recent future.

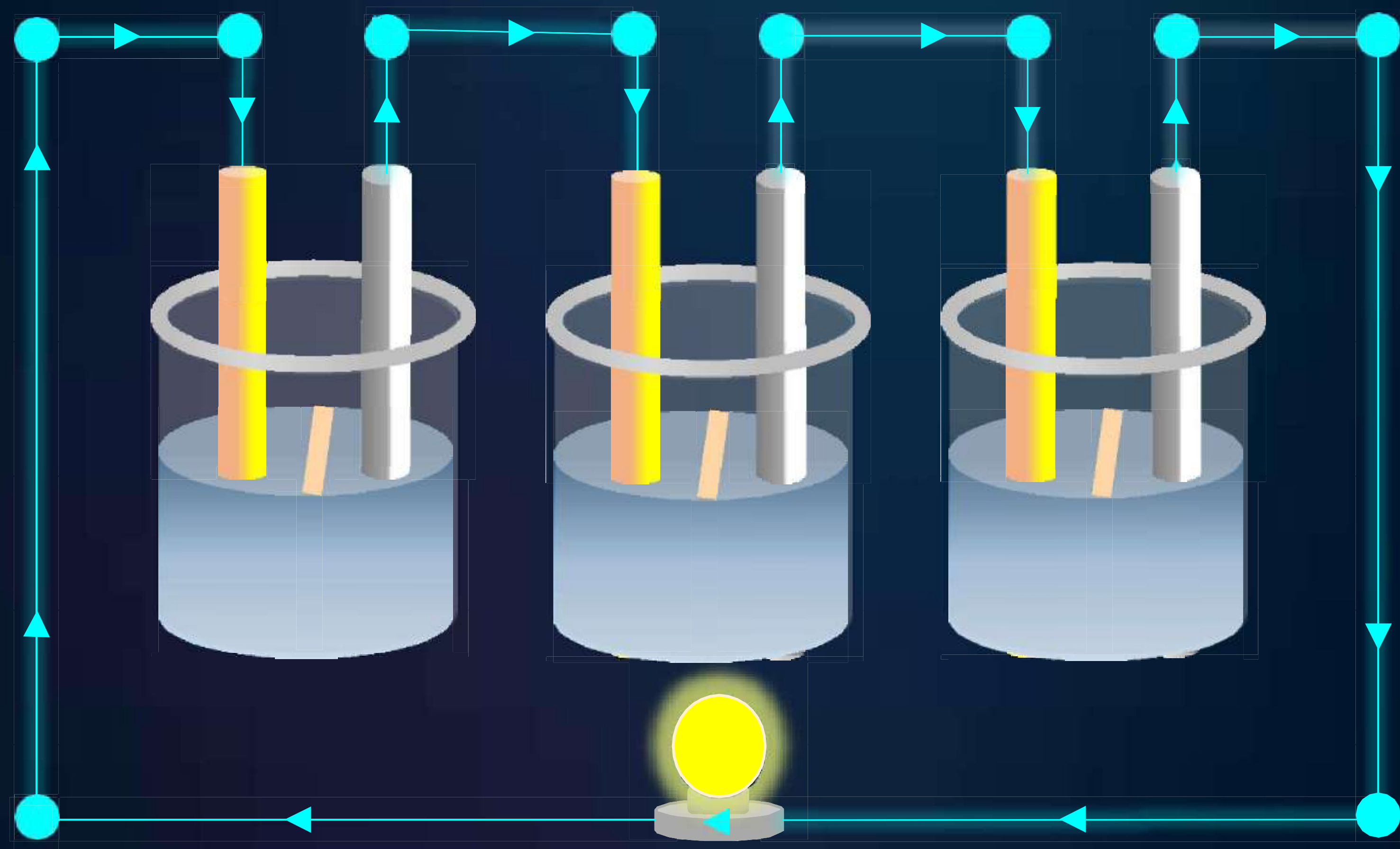
1801 AD

The Italian physicist "Alessandro Voltas" invented first electrical battery called "Voltaic pile" which made up by stacking several pairs of alternating copper (or silver) and zinc discs (electrodes) separated by cloth or cardboard soaked in brine (electrolyte) to increase the electrolyte conductivity.

This battery could continuously provide an electric current.



Current Electricity

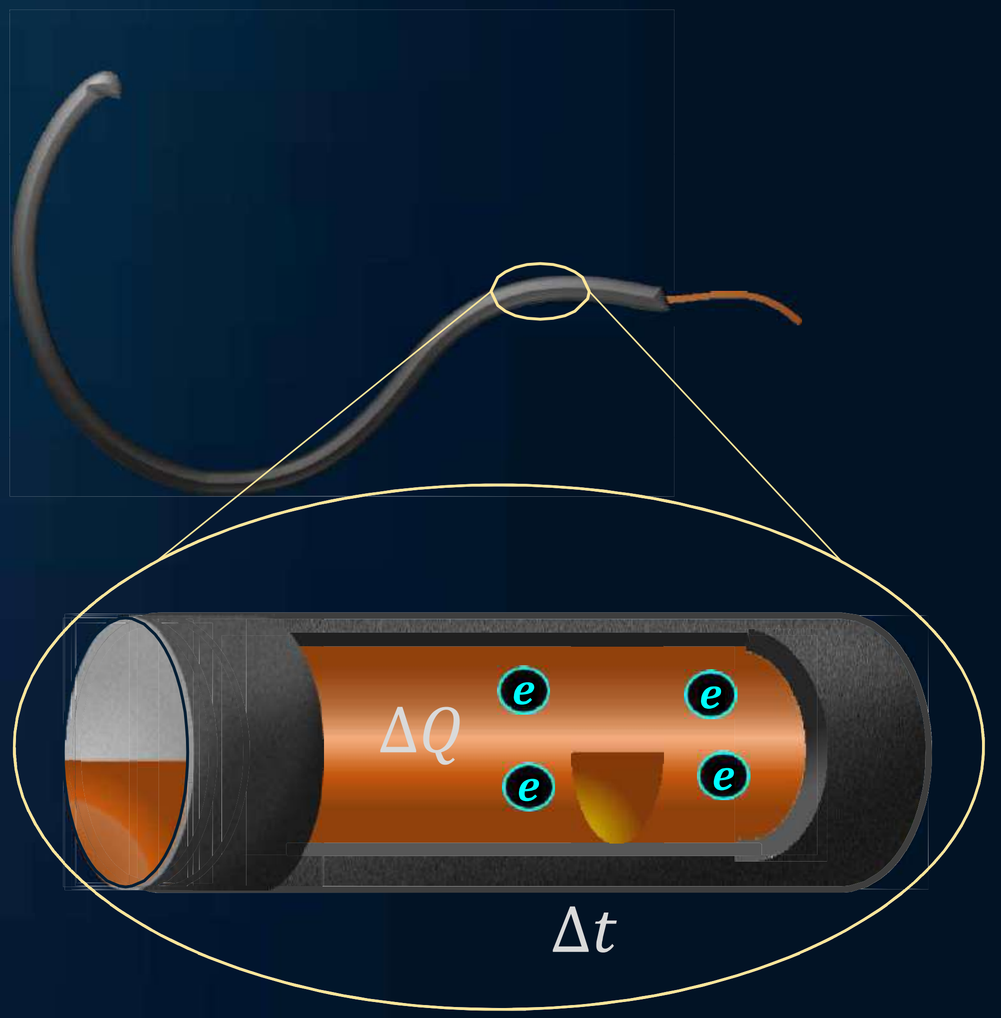


Alessandro Voltas also observed that when the copper and zinc plates were sink in an acid solution more electricity got produced.

Electric Current

The net amount of charge flowing across the area in the time interval Δt, is defined to be the current across the area.

If ΔQ amount of charge (free electrons) is passing through a hypothetical cross-sectional area of a conducting wire in time Δt, then the current is defined as follows:



$$I = \frac{\Delta Q}{\Delta t}$$

If $1\ C$ charge is flowing through a cross-sectional area of the wire in $1\ s$, then that current is called 1 Ampere (A) current.

$$I = \frac{1\ C}{1\ s} = 1\ A$$

Electric Current

❖ Electric current (I) is a fundamental quantity.

➤ Dimensional Formula – $\boxed{[\underbrace{M^0}_{\text{Mass}}\overbrace{L^0}^{\text{Length}}\underbrace{T^0}_{\text{Time}}\overbrace{I^1}^{\text{Current}}]}$

➤ Unit of current – $\boxed{\text{Ampere } (A)}$

Question

A flow of 10^7 electrons per second in a conducting wire constitutes a current of

A) $1.6 \times 10^{-12} A$ B) $1.6 \times 10^{26} A$ C) $1.6 \times 10^{-26} A$ D) $1.6 \times 10^{12} A$

Solution

NEET

If n number of electrons flow through the conducting wire per second, then net charge will be given by,

$\Delta Q = ne$

Therefore, the current is,

$$I = \frac{\Delta Q}{\Delta t} = \frac{ne}{\Delta t}$$

The following data are given:

$$n = 10^7, t = 1\ s, e = 1.6 \times 10^{-19}\ C$$

Hence,

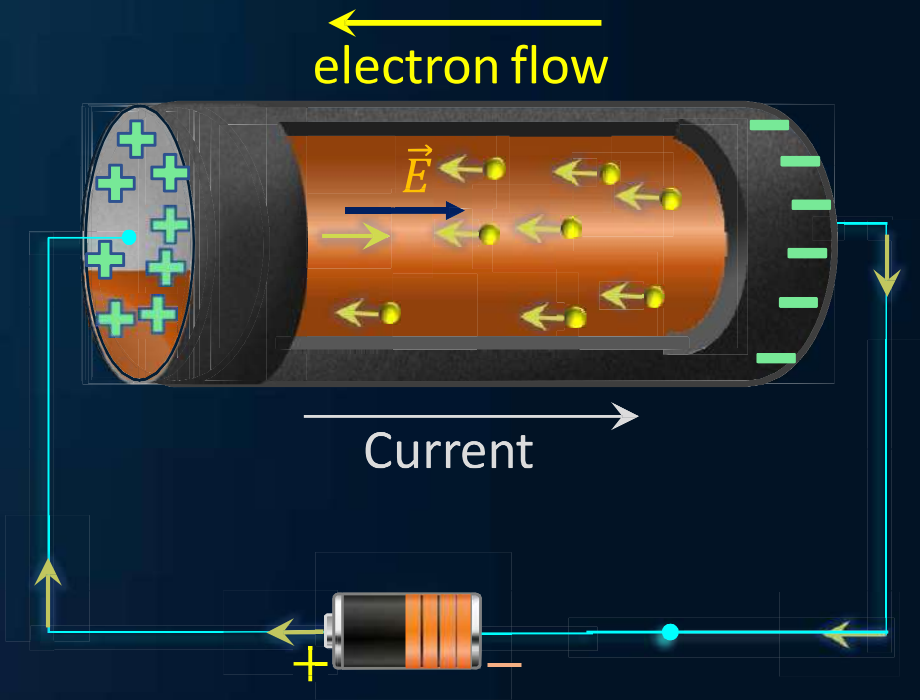
$$I = \frac{10^7 \times 1.6 \times 10^{-19}}{1}$$

$$I = 1.6 \times 10^{-12}\ A$$

Thus, option (A) is the correct answer.

Direction of Conventional current

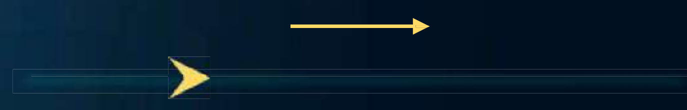
- Current flows from one end of conductor to other end only because of potential difference between two ends.
- Under electrostatic conditions, we know that the electric field inside an isolated conductor is zero and all the free electrons are in random motion so that there is no velocity gradient exists. But when this isolated conductor is connected to the battery, positive and negative charges are induced on that sides of the conductor which are connected to the positive and negative terminal of the battery, respectively, as shown in the figure. Since the conductor is no longer in electrostatic condition, a net electric field $\vec{E}$ is produced inside the conductor.
- The electrons feels an attractive force $\vec{F} = -e\vec{E}$ and hence, the electrons flow from negative terminal to positive terminal of the battery whereas the holes (i.e., positive charges) experiences a repulsive force $\vec{F} = e\vec{E}$ and it flows in opposite direction of the flow of electrons.



- The direction of electric current is taken from positive (high potential) to the negative (low potential) terminal, opposite to the flow of electrons to maintain the methodology that any physical quantity flows from high to low. Example: Gas flows from high pressure region to low pressure region.

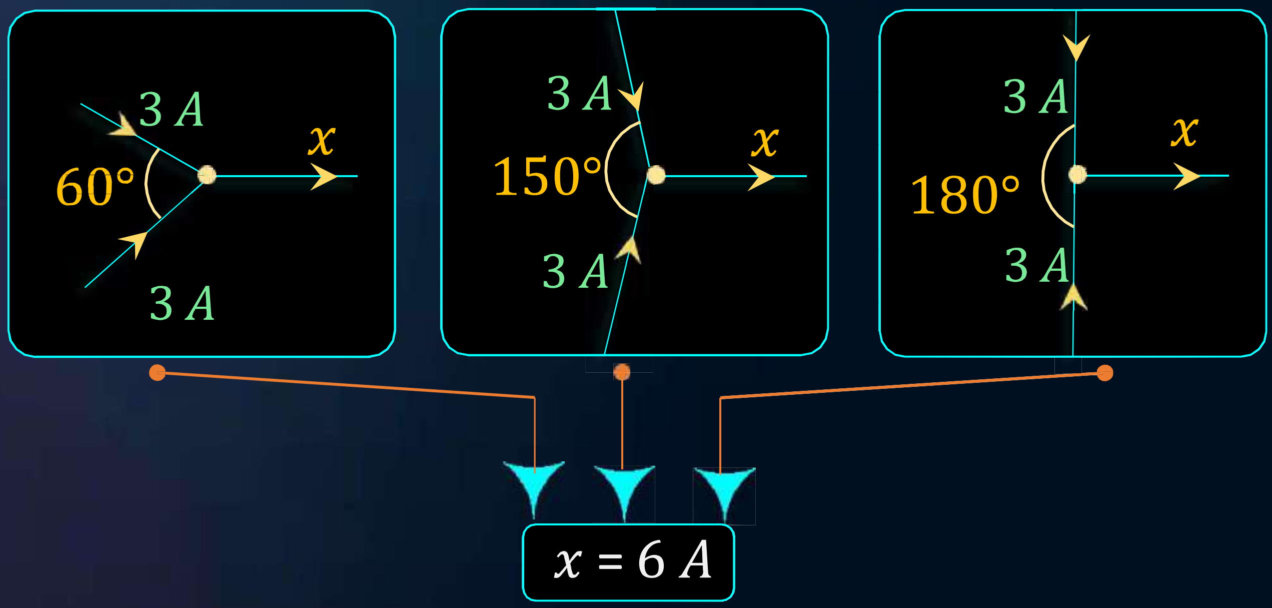
Is current a scalar or a vector quantity?

Since current has both magnitude and direction, it may seem to us that this is a vector quantity. But, if it is a vector quantity is has to obey at least the law of vector addition.



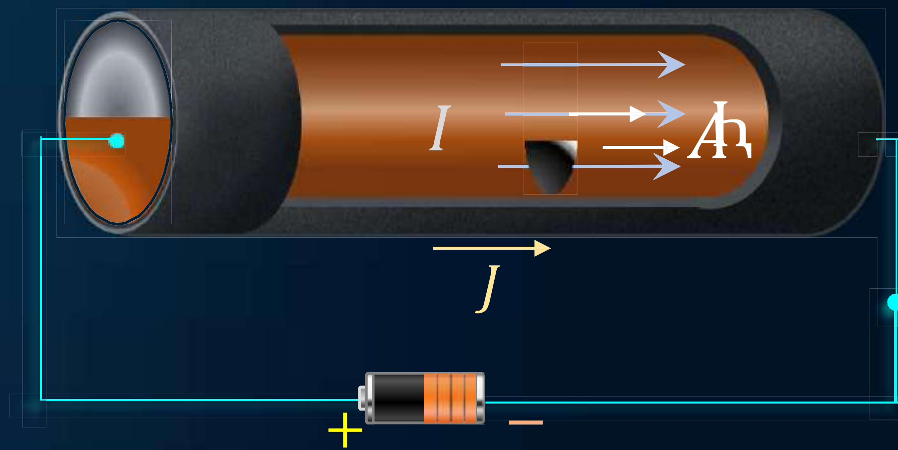
In all three following cases it is seen that although the angle between the direction of flow of current is different, the current $x = 6\ A$. Therefore, it obeys simple algebraic addition. Hence, the conclusion is:

- Electric current is a treated as a Scalar Quantity.
- It does not follow the law of vector addition.



Current density

- The vector quantity that can be associated with the current is current density.
- The amount of electric current traveling per unit cross-section area is known as current density and its direction is always along the direction of current.
- If current I is flowing through an area A perpendicular to the direction current, then mathematically, the current density is defined as follows:

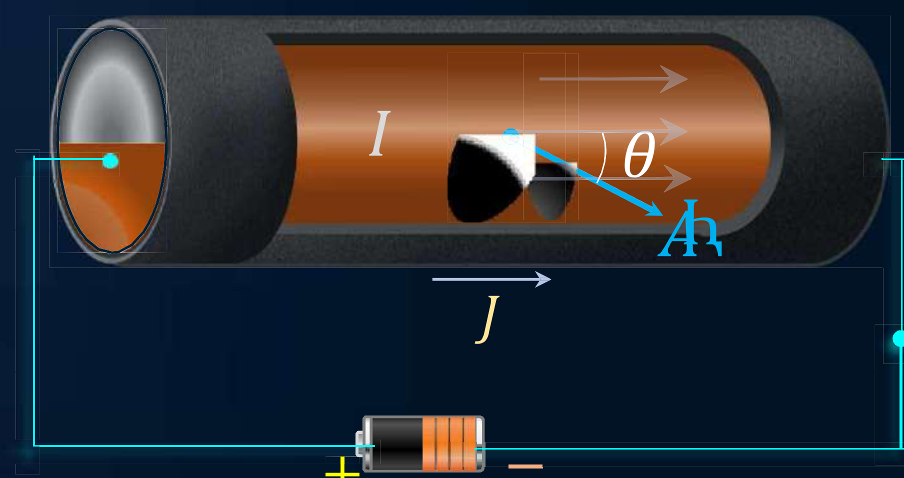


$$J = \frac{I}{A}\left(\frac{Amp}{m^2}\right)$$

$\vec{A} \rightarrow$ direction same as I

$A \rightarrow \perp$ to the current flow

- If current I is flowing through an area A, and the angle between the direction of flow of the current and the direction of the area vector is θ , as shown in the figure, then mathematically, the current density is defined as follows:



$$|\vec{J}\,| = \frac{I}{A\cos\theta}$$

Current density

Therefore, if the current density and the area is known to us, then the current can be found out as follows:

$$I = \vec{J}\cdot\vec{A}$$

For any three dimensional surface, this current can also be represented as follows:

$$I = \int \vec{J}\, d\vec{A}$$

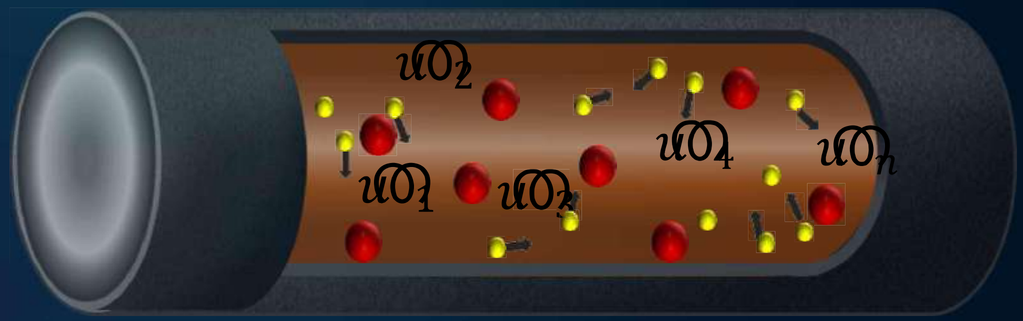
Notes

- Dimensional Formula – $[M^0L^{-2}T^0I^1]$
- More the current in a conductor, the higher will be the current density.
- Current density is a vector quantity having both a direction and a scalar magnitude.
- Current density is important in the designing of electrical and electronic systems.

Average thermal velocity

Consider that the temperature of an isolated iron rod is increased up to a certain level. As the temperature of the rod is increased, the thermal agitation of the free electrons in the rod will be increasing gradually and this in turn will increase the thermal energy of the electrons. Hence, the electrons will be in random motions.

Since the electrons are in random motion, it is possible that the velocity of all electrons due to this thermal agitation points at different directions and hence, the average thermal velocity (not speed) becomes zero.



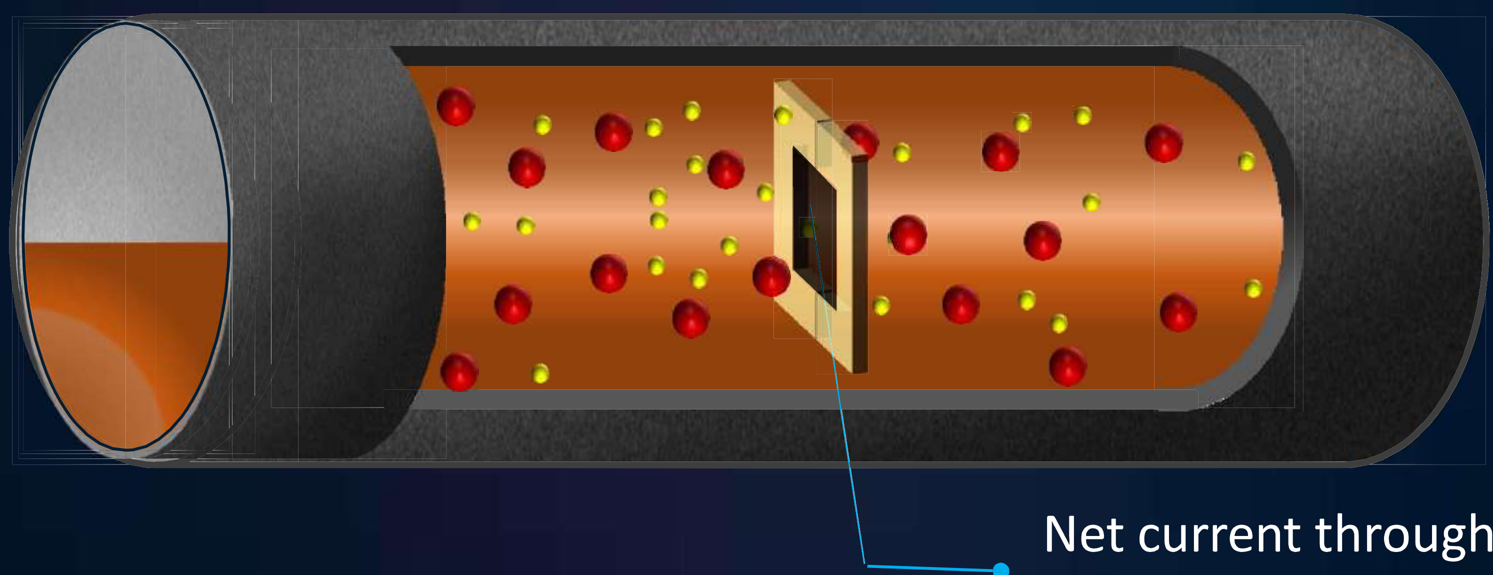
If $\vec{u}_1, \vec{u}_2, \vec{u}_3, \ldots, \vec{u}_n$ are the individual thermal velocities of n number of electrons, then the average thermal velocity is given by,

$$\vec{u}_{avg} = \text{Average thermal velocity} = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \vec{u}_4 + \vec{u}_5 + \cdots + \vec{u}_n}{n} = 0$$

Current Electricity

When the electrons are in random motion due to the thermal agitation if we imagine a hypothetical cross-sectional area as shown in the figure, the net number of electrons crossing this area will be zero during a certain time interval . This is because during that time internal the number of electrons crossing the area from right to left is equal to the number of electrons crossing the area from left to right.

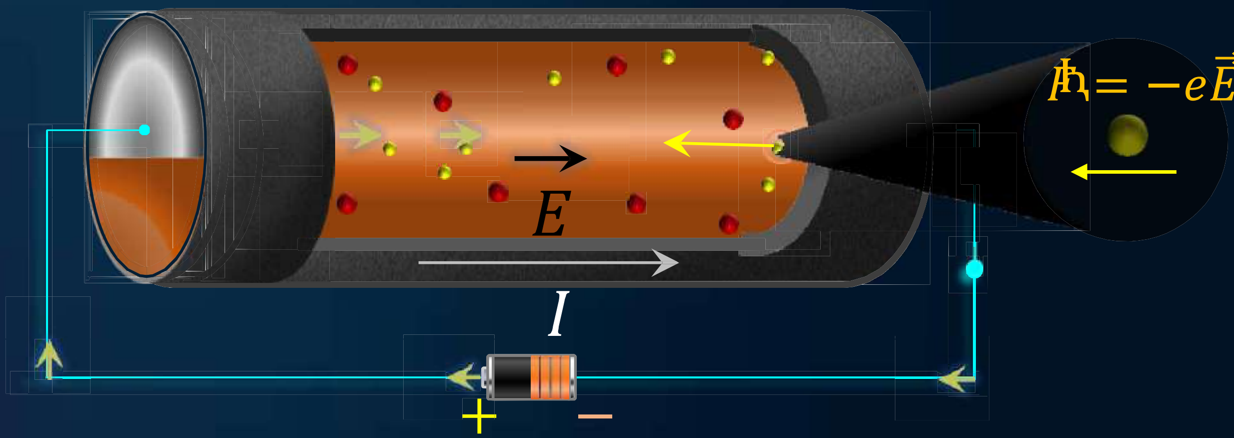
Since no net charge is crossed the hypothetical area, it can be said that the net current through this area will be zero. That’s why an isolated conductor doesn’t give us electric shock.



Net current through this area is Zero

Acceleration of e^-

If the thermally heated iron rod is connected with a battery, then along with the random motion due to thermal agitation the electrons also move from negative terminal to positive terminal of the battery because of the created electric field inside the rod.



Because of the electric field, the force on one electron is given by,

$\vec{F} = -e\vec{E}$

Due to this force on the electron, let the electron starts drifting with acceleration $\vec{a}$. If the mass of the electron is m, then the acceleration of the electron will be given by,

$$\vec{a} = -\frac{e\vec{E}}{m}$$

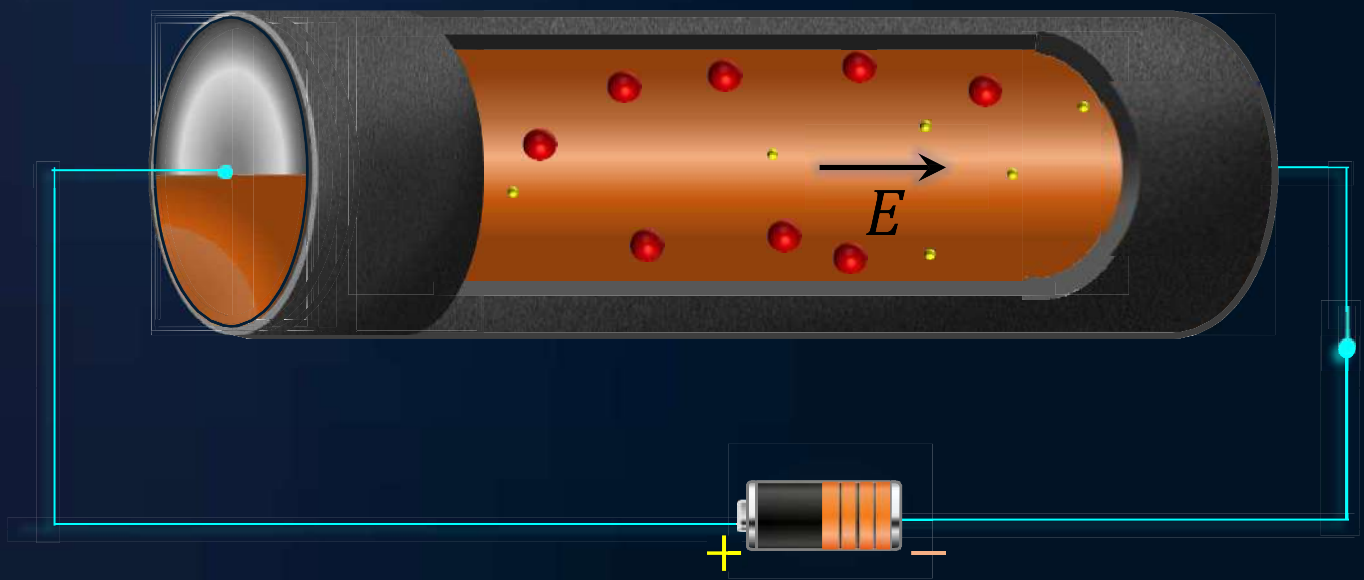
$$|a| = \frac{eE}{m}$$

Drift Velocity

- The average velocity attained by charged particles in a material due to an electric field is called as **drift velocity**.

- If $v_1, v_2, v_3, \ldots\ldots, v_n$ are the individual velocities of n number of electrons due to the electric field, then the average velocity is given by,

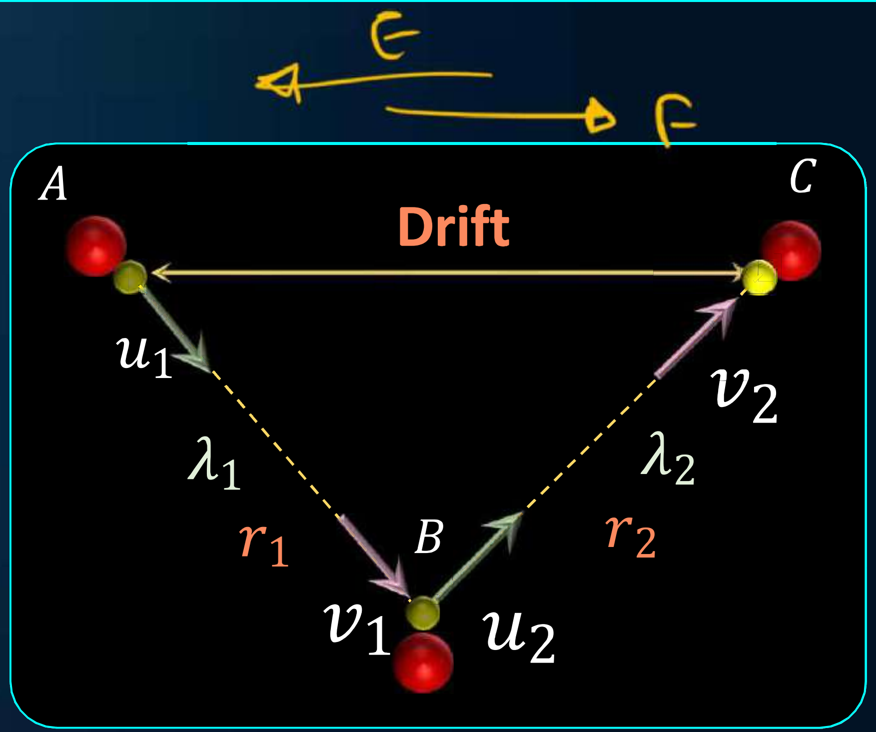
$$v_d = v_{avg} = \frac{v_1 + v_2 + v_3 + \cdots + v_n}{n}$$



Drift Velocity

Let the electric field is acting towards the left. Thus, the force on the electrons will act towards right.

Assume that an electron starts moving with thermal velocity u_1 from point $\boldsymbol{A}$ and gets collided with an atom of the material at point $\boldsymbol{B}$. At point $\boldsymbol{B}$, the velocity of the electron changes to v_1 because of the force due to the electric field. In between point $\boldsymbol{A}$ and $\boldsymbol{B}$, the electron doesn't suffer any collision.



The path between two successive collisions suffered by an electron is known as the free path(λ).

If λ_1, λ_2, λ_3,……., λ_n are the individual free path of n number of electrons, then the mean free path is defined as,

$$\lambda_m = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n}{n}$$

The time between two successive collisions is known as "Relaxation time". It is denoted by "τ".

Drift Velocity

We know that the acceleration of the electron due to the electron electric field is, $|a| = \frac{eE}{m}$

Since this acceleration is constant, the velocity v_1 can be written as,

$$v_1 = u_1 + a\tau_1$$

Now, the average velocity of all electrons is defined as,

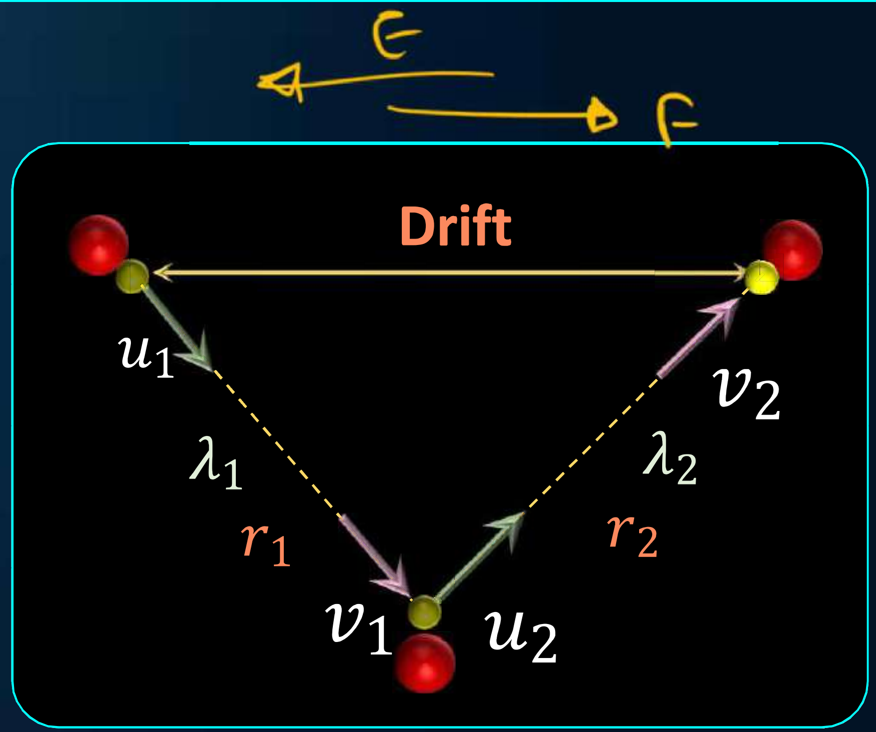
$$\bar{v}_{avg} = \frac{v_1 + v_2 + v_3 \cdots v_n}{n} = \frac{u_1 + u_2 \cdots u_n}{n} + a\frac{(\tau_1 + \tau_2 \cdots \tau_n)}{n}$$

$$\bar{v}_{avg} = \underbrace{\cancel{\bar{u}_{avg}}}_{\text{thermal}}{}^{0} + a\tau_{avg}$$

$$\bar{v}_{avg} = a\tau_{av}$$

Therefore, the drift velocity of the electrons is defined as,

$$v_d = v_{avg} = a r_{avg}$$



$$\Rightarrow \boxed{v_d = v_{avg} = -\frac{eE}{m} r_{avg}}$$

The negative sign refers the fact that the direction of the drift velocity is always opposite to the electric field

Relation between (v_d) & V NEET

We know that the magnitude of drift velocity is,

$$v_d = v_{avg} = \frac{eE}{m} r_{avg}$$

Since a small change in potential difference is defined as $dV = -E.dx$, thus, the magnitude of any potential difference can be written as,

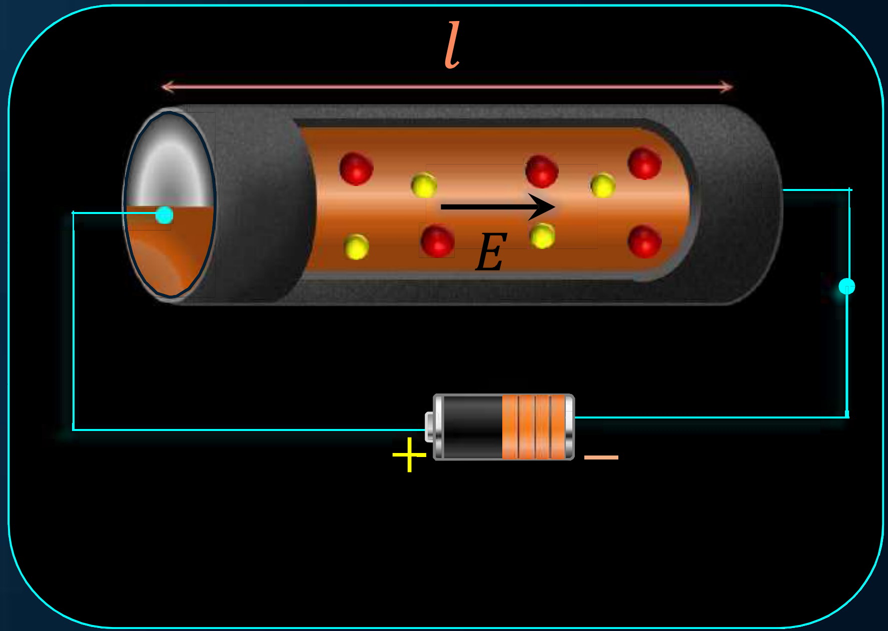
$$\Delta V = E.\Delta x$$

Now, in this case, since the potential difference between two ends of the rod is V and the length of the rod is l, the electric field will be,

$$E = \frac{V}{l}$$

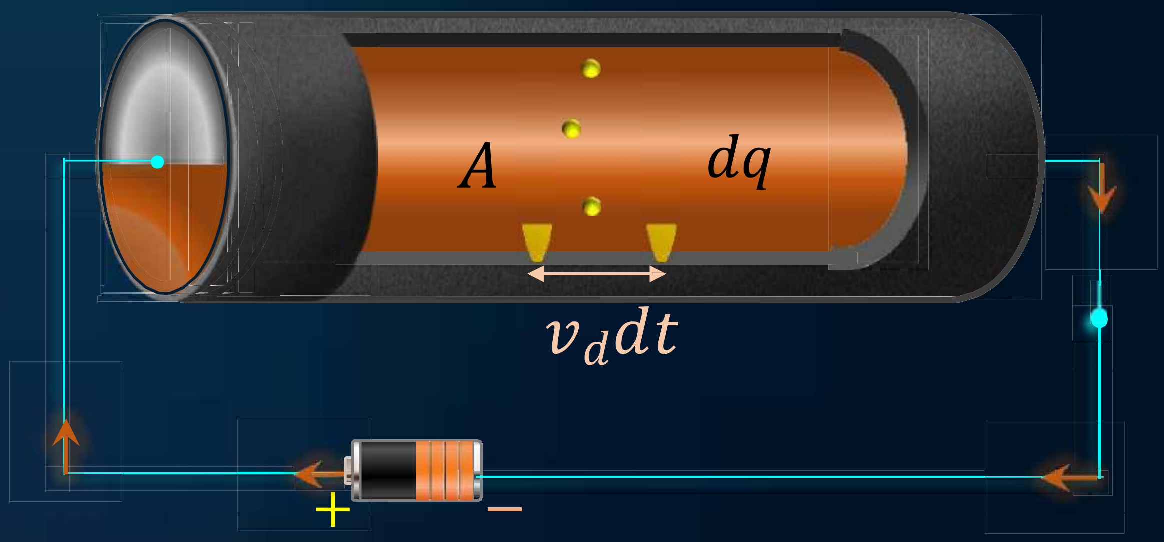
Hence, the expression of the magnitude of drift velocity becomes,

$$\boxed{v_d = \frac{eV}{ml} r_{avg}}$$



Relation between (I) & (v_d)

Let two cross-sections of area A, as shown in the figure.



We know that due to the electric field the average velocity of the electrons is the drift velocity v_d.

If n be the number of electrons per unit volume of the rod and dq amount of charge travels the distance between two cross-sections in time dt, then,

dq = [No. of free e^- in the volume contained by two cross-sections] × [charge of one electron]

$\Rightarrow dq$ = [(Number of electrons per unit volume) × (The volume of the portion contained by those two cross-sections)] × [Charge of one electron]

$$\Rightarrow dq = (n \times v_d dt \times A) \times e$$

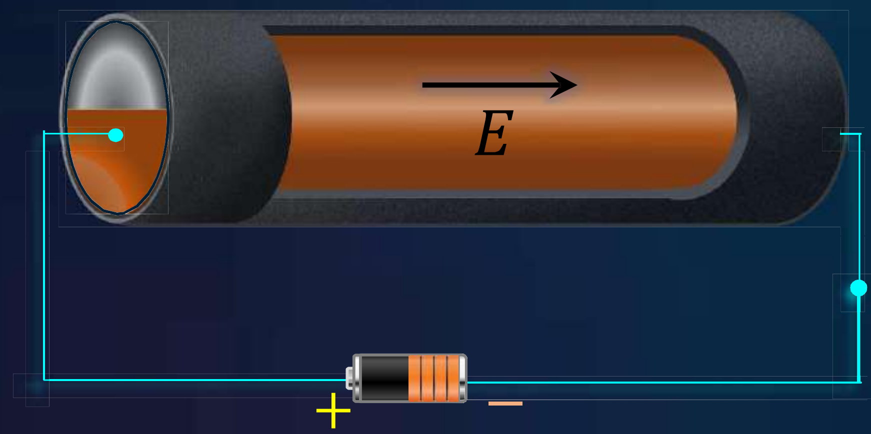
$$\Rightarrow dq = (nev_d A)dt$$

Now since the current is defined as $I = \frac{dq}{dt}$, therefore,

$$\boxed{I = neAv_d}$$

Mobility

The magnitude of the drift velocity per unit electric field :



$$\mu = \frac{v_d}{E}$$

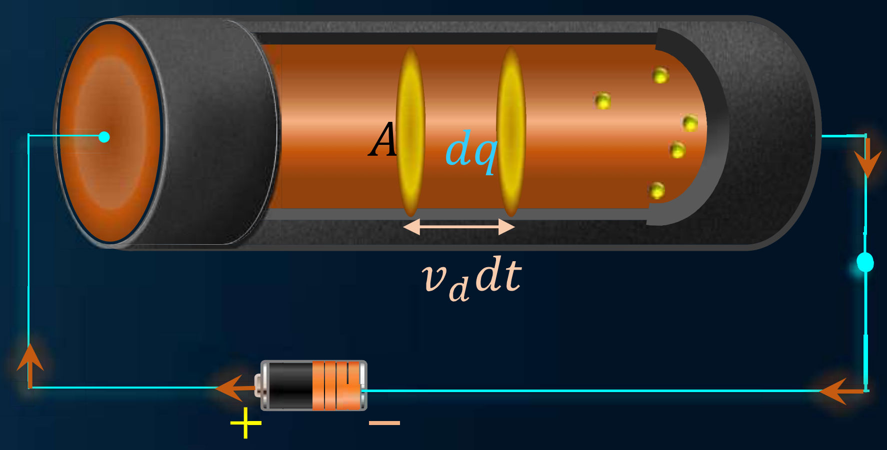
$$v_d = \frac{eE}{m} r_{avg}$$

$$\mu = \frac{e r_{avg}}{m}$$

Relation between (I) & (V)

We know that the expression of drift velocity is given by,

$$\boxed{v_d = \frac{eV}{ml}\tau_{avg}}$$



We also know that the current can be written in terms of drift velocity as,

$$\boxed{I = neAv_d}$$

By combining these two relations, we get,

$$I = \frac{neAeV\tau_{avg}}{ml}$$

$$\Rightarrow V = \left(\frac{ml}{ne^2\tau_{avg}A}\right)I$$

Since all the terms in parenthesis are constants in a given physical condition, thus,

$$\boxed{V \propto I}$$

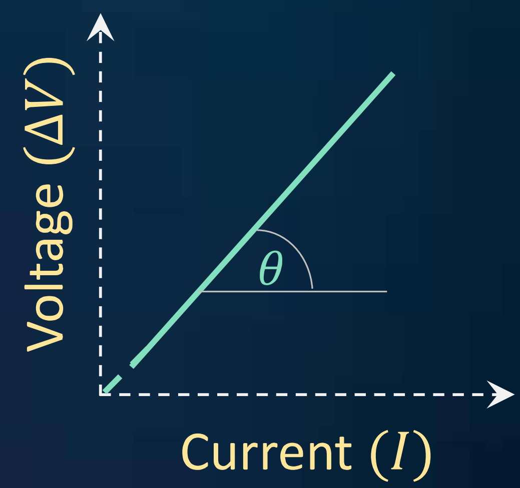
Ohm's Law

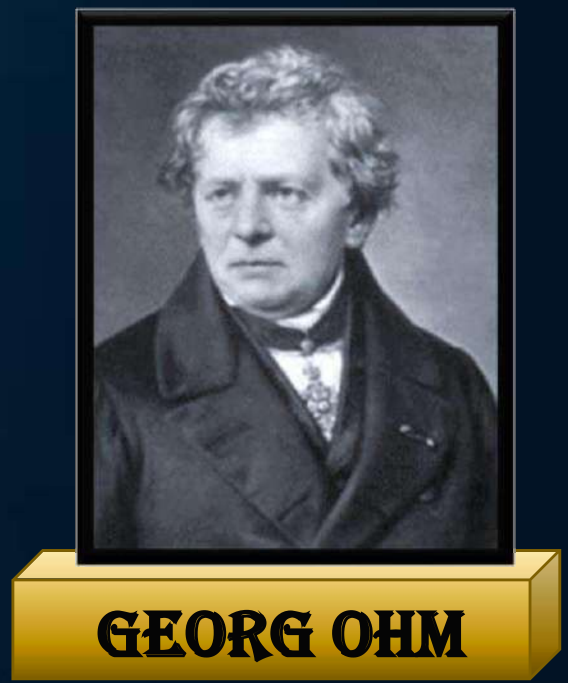
This law states that the voltage across a conductor is directly proportional to the current flowing through it, provided all physical conditions and temperature remain constant.

$$\boxed{\Delta V \propto I} \quad \text{or} \quad \boxed{\Delta V = IR}$$

R = Resistance of the conductor

The above relation is of $y = mx$ type and hence the plot of ΔV vs I will be a straight line passing through origin with slope $m = \tan\theta = \frac{\Delta V}{I} = R\ (Constant)$





Electrical Resistance:

Resistance is a property of conductor due to which it resists the flow of electric current through it.

$$\boxed{R = \frac{V}{I}\ \left(\frac{Volts}{Amp}\right) or\ \Omega\ \text{(Ohm)}}$$

Resistor Symbol:

Question

NEET

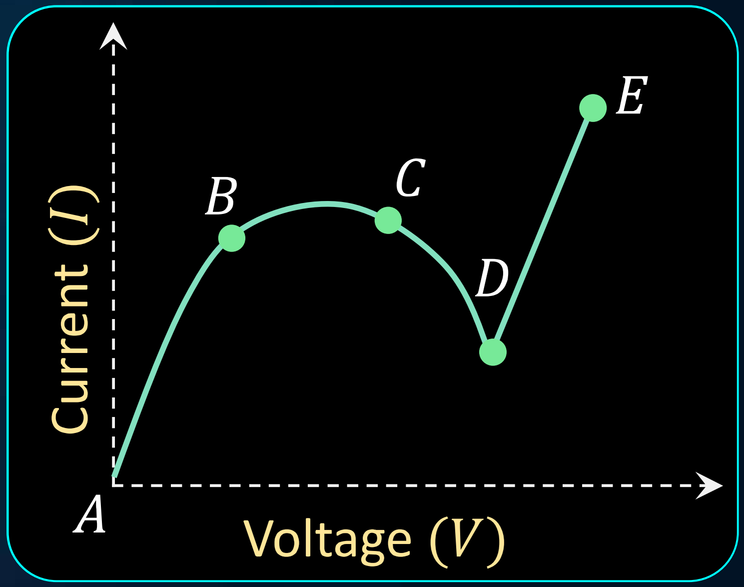
From the graph between current (I) and voltage (V) shown below, identify the portion corresponding to negative resistance

A CD

B DE

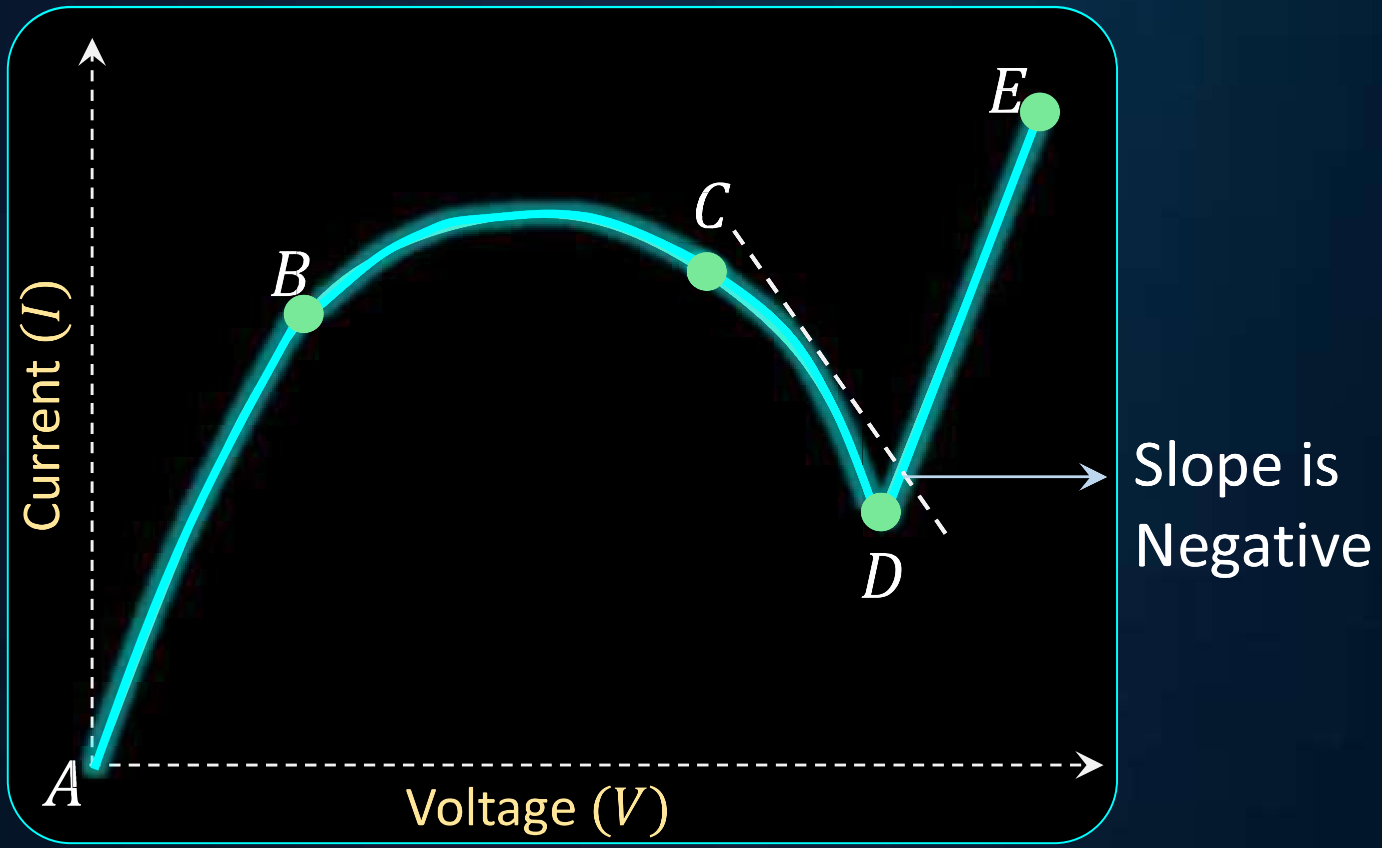
C AB

D BC



Discussion

Negative resistance corresponds to decreasing current with increase in voltage and in the given graph only CD portion represents this. Thus, option (A) is the correct answer.



Vector form of Ohm's Law

We know that the expression of drift velocity is given by,

$$\boxed{v_d = \frac{eV}{ml}\tau_{avg}}$$

We also know that the current can be written in terms of drift velocity as,

$$\boxed{I = neAv_d}$$

By combining these two relations, we get,

$$I = neA\frac{eV}{ml}\tau_{avg}$$

$$\Rightarrow \frac{V}{l} = \frac{mI}{ne^2A\tau_{avg}}$$

Resistivity (ρ)

$$\Rightarrow \frac{V}{l} = \left(\frac{m}{ne^2\tau_{avg}}\right) \times \frac{I}{A}$$

Since the magnitude of electric field and the current density are defined as $E = \frac{V}{l}$ and $J = \frac{I}{A}$, respectively,

we get, $\boxed{\vec{E} = \rho\,\vec{J}}$ [Where $\rho = \frac{m}{ne^2\tau_{avg}}$ = Resistivity]

This relation can also be rearranged as follows:

$$\vec{J} = \vec{E} \times \frac{1}{\rho}$$

Since $\frac{1}{\rho} = \sigma$ (Conductivity)

$$\boxed{\vec{J} = \sigma\vec{E}}$$

S.I unit = Siemens/metre $= S/m$

Electrical Resistance

We know that,

$$V = \left(\frac{ml}{ne^2\tau A}\right) I$$

$$\Rightarrow \frac{V}{I} = \left(\frac{ml}{ne^2\tau A}\right)$$

Now, we also know that,

$$R = \frac{V}{I}$$

$$\boxed{R = \left(\frac{ml}{ne^2\tau A}\right)}$$

By assuming temperature to be constant, all the quantities at the R.H.S. of this expression except l and A are constants. Therefore, $R \propto l$ and $R \propto \frac{1}{A}$.

m = mass of e^-

l = length of the conductor

n = No. of free e^- per unit volume

e = charge on e^-

τ = Avg. Relaxation Time

A = Area of cross-section

Electrical Resistivity

Electrical resistivity is defined as the resistance offered by the material per unit length for unit cross-section.

We have seen that the resistance can be expressed as,

$$R = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A}$$

The term in the parenthesis is mathematically defined as the resistivity. Thus,

$$\rho = \frac{m}{ne^2\tau} = \text{Resistivity} = ohm.m$$

Therefore, the expression of resistance becomes,

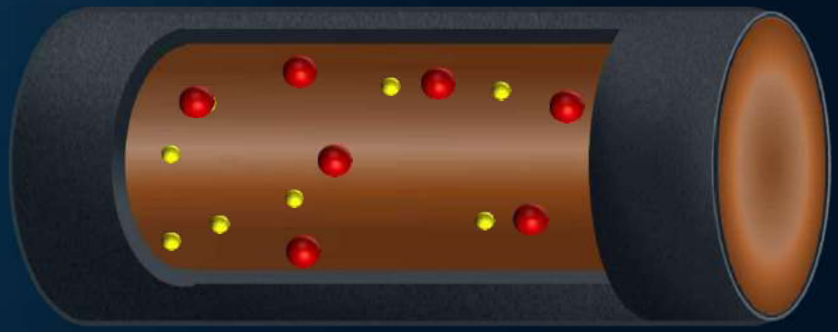
$$\boxed{R = \rho\frac{l}{A}}$$

Note: Resistivity depends on temperature and the material itself

Effect of temperature on Resistance & Resistivity

I) Conductors

When the temperature of a conductor increases, more electrons become free and hence, the number density of the free electrons (i.e., n) increases. Now, due to thermal agitation, the collision of the free electrons with other electrons and with the atoms of the material of the conductor also increases. Therefore, the relaxation time (i.e., τ) decreases.



The resistance is defined as,

$$R = \left(\frac{ml}{ne^2\tau A}\right) \quad \left[\text{where, } \frac{m}{ne^2\tau} = \rho \text{ (resistivity)}\right]$$

As n increases, R will decrease which in turn decreases the resistivity and as τ decreases, R will increase which in turn increases the resistivity.

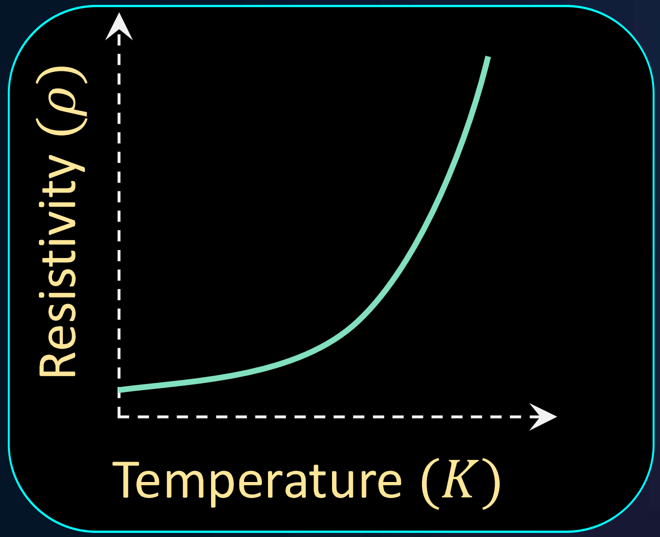
$$T\uparrow \;\begin{cases} n\uparrow \longrightarrow \boxed{R\downarrow, \rho\downarrow} \\ \tau\downarrow \longrightarrow \boxed{R\uparrow, \rho\uparrow} \end{cases}$$

(Since the conductor has plenty of free e^- per unit volume, so the increase in n doesn't have much effect on R & ρ)

Effect of temperature on Resistance & Resistivity

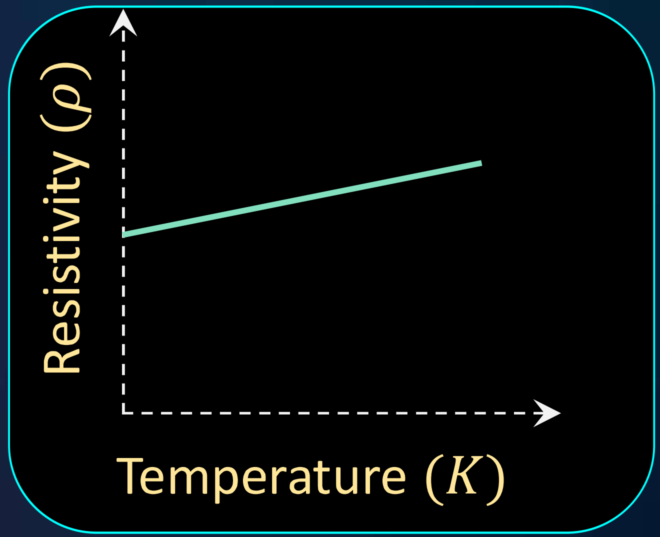
The graphical representation of the resistivity with temperature for conductors and alloys are shown below.

I) Conductors



For copper

II) For alloys



(Manganin, Nichrome, Constantan)

Question

NEET

The resistance of a wire is 'R' ohm. If it is melted and stretched to 'n' times of its original length, its new resistance will be :

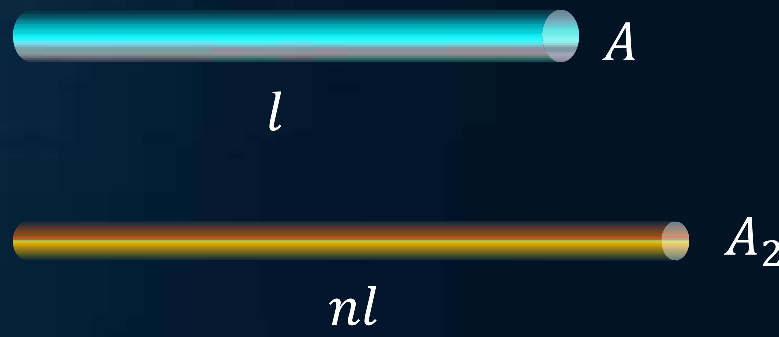
A. $\frac{R}{n}$

B. n^2R

C. $\frac{R}{n^2}$

D. nR

Discussion



Assume that initially the rod has length l and cross-sectional area A, Therefore, The resistance of the rod is,

$$R = \frac{\rho l}{A} \dots (i)$$

If it is melted and stretched to 'n' times of its original length, let its cross-sectional area becomes A_2. Since the volume of the rod remains same, we get the following:

$$V = A \times l = A_2 \times nl$$

$$\therefore A_2 = \frac{A}{n}$$

Hence, the new resistance of the rod becomes,

$$R_2 = \frac{\rho nl}{A_2}$$

$$\Rightarrow R_2 = \frac{\rho nl}{A/n}$$

$$\Rightarrow R_2 = \frac{\rho l}{A} \times n^2$$

$$\Rightarrow R_2 = R \times n^2$$

$$\boxed{R_2 = n^2 R}$$

Thus, option (B) is the correct answer.

Question

NEET

A wire of a certain material is stretched slowly by ten percent. Its new resistance and specific resistance become respectively

A. both remain the same

B. 1.1 times, 1.1 times

C. 1.2 times, 1.1 times

D. 1.21 times, same

Discussion

In the previous problem, we have seen that when the length of the wire increases n times, the cross-sectional area of the rod decreases n times and hence, the resistance increases n^2 times.

Therefore, If old resistance is R, then the new resistance will be,

$R' = n^2 R$

In this problem, the length of the rod is increased by $10\ \%$. Therefore, if the old length of the rod is l, then the new length of the rod will be,

$l' = l' = l + 10\ \%\ l$

$\Rightarrow l' = l + \frac{10}{100} l = l + 0.1l$

$\therefore l = 1.1\ l$

Therefore, in this case, the length of the rod increases by $n\ =\ 1.1$ times.

Hence, the new resistance will be,

$R' = (1.1)^2 R = 1.21\ R$

Since the resistivity (or specific resistance) only depends on the material and the temperature, and in this problem both remains same, the specific resistance remains same.

Thus, option (D) is the correct answer.

Question

NEET

The masses of the wires of copper are in the ratio $1:3:5$ and their lengths are in the ratio $5:3:1$. The ratio of their electrical resistance is:

A. $1:3:5$

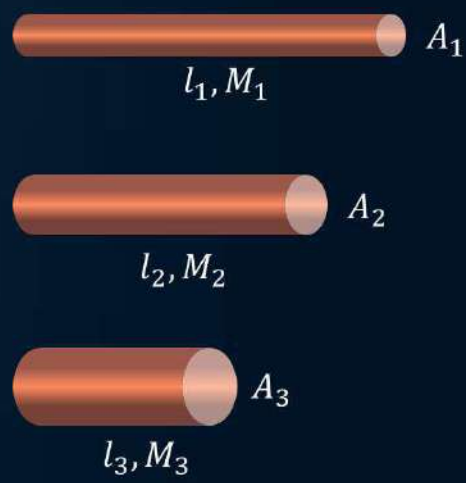
B. $1:25:125$

C. $5:3:1$

D. $125:15:1$

Discussion

It is given that the masses of the wires of copper are in the ratio 1: 3: 5 and their lengths are in the ratio 5: 3: 1.



$M_1 : M_2 : M_3 = 1 : 3 : 5$

$l_1 : l_2 : l_3 = 5 : 3 : 1$

If the length and the cross-section of a rod are l and A, then the resistance of the rod is given by,

$$R = \frac{\rho l}{A}$$

If the density a rod is d, the mass of the rod is given by,

$M = \text{Volume} \times \text{Density} = Al \times d$

$$\therefore A = \frac{M}{ld}$$

Therefore, the expression of resistance is given by,

$$R = \frac{\rho l^2 d}{M} \Rightarrow R \propto \frac{l^2}{M}$$

Therefore, the ratio of resistance of the of the rods is given by,

$$R_1 : R_2 : R_3 = \frac{l_1^2}{M_1} : \frac{l_2^2}{M_2} : \frac{l_3^2}{M_3}$$

$$\Rightarrow R_1 : R_2 : R_3 = \frac{5^2}{1} : \frac{3^2}{3} : \frac{1^2}{5}$$

$$\Rightarrow R_1 : R_2 : R_3 = 25 : 3 : \frac{1}{5}$$

$$\therefore R_1 : R_2 : R_3 = 125 : 15 : 1$$

Thus, option (D) is the correct answer.

Question

The current density in a wire is $10\ A/cm^2$ and the electric field in the wire is $5\ V/cm$. If $\sigma =$ conductivity of the material, then find σ (in S/m).

A. $200\ S/m$

B. $400\ S/m$

C. $100\ S/m$

D. $80\ S/m$

Discussion

From Ohm's law, we get,

$$J = \sigma E$$

$$\Rightarrow \sigma = \frac{J}{E}$$

The current density of the wire is,

$$J = 10\ A/cm^2 = 10\ A/(10^{-4}\ m^2) = 10^5\ A/m^2$$

The electric field in the wire is,

$$E = 5\ V/cm = 5\ V/(10^{-2} m) = 500\ V/m$$

Therefore, the conductivity will be,

$$\sigma = \frac{10^5}{500}$$

$$\boxed{\sigma = 200\ \frac{S}{m}}$$

Temperature coefficient of Resistance

Consider a conducting wire with initial temperature T_0 and initial resistance R_0. Let the temperature of the rod is increased by dT and hence, the resistance of the rod is increased by dR.

This increase in resistance is directly proportional to the increase in temperature and the initial resistance of the rod. Thus, we can write the following:

$$dR \propto dT$$

$$dR \propto R_0$$

By combining these two proportionality relations, we get,

$$\boxed{dR = \alpha R_0 dT}$$

R_0 = Resistance at any reference temperature T_0

α = The proportionality constant known as "Co-efficient of thermal resistance"

Conducting Wire

Temperature coefficient of Resistance

$dR = \alpha R_0 dT$

If the resistance of the rod becomes R_f at any final temperature T_f, then by doing integration of the above equation, we get

$$\int_{R_0}^{R_f} dR = \alpha R_0 \int_{T_0}^{T_f} dT$$

$$\Rightarrow R_f - R_0 = \alpha R_0 (T_f - T_0)$$

$$\Rightarrow R_f = R_0 + \alpha R_0 (T_f - T_0)$$

Therefore, if the temperature difference is defined as $\Delta T = T_f - T_0$, then,

$$\boxed{R_f = R_0(1 + \alpha \Delta T)}$$

Since $R = \frac{\rho l}{A}$, and if the initial and final resistivity are ρ_0 and ρ_f, then,

$$\boxed{\rho_f = \rho_0(1 + \alpha \Delta T)}$$

Temperature coefficient of Resistance

Therefore, the resistance of a material changes with temperature as,

$$R_f = R_0(1 + \alpha\Delta T)$$

Hence, the "Co-efficient of thermal resistance" is given by,

$$\boxed{\alpha = \frac{R_f - R_0}{R_0\Delta T}}$$

I) For Conductors: $\alpha = +ve$

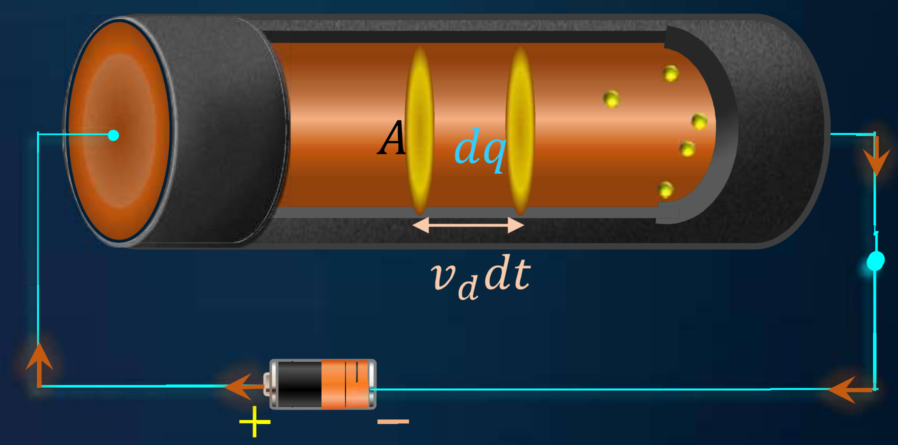
II) For alloys like (Nichrome): $\alpha = 0$

Recap

Relation between I and V :

$$\boxed{I = neAv_d}$$

$$v_d = \frac{eV}{ml}\tau_{avg}$$



$$I = \frac{neAeV\tau_{avg}}{ml}$$

$$V = \left(\frac{ml}{ne^2\tau_{avg}A}\right)I$$

$$\boxed{V \propto I}$$

Recap

Ohm's Law:

This law states that the voltage across a conductor is directly proportional to the current flowing through it, provided all physical conditions and temperature remain constant.

$$\boxed{\Delta V \propto I} \quad \text{or} \quad \boxed{\Delta V = IR}$$

Electrical resistance(R):

Resistance is a property of conductor due to which it resists the flow of electric current through it.

$$\boxed{R = \frac{V}{I} \quad \frac{Volts}{Amp}\, or\ \Omega\ \text{(Ohm)}}$$

$R \propto l$

$R \propto \frac{1}{A}$

$R = \rho \frac{l}{A}$

Recap

Vector form of ohm's Law:

$$\vec{E} = \rho\,\vec{J}$$

$$\vec{J} = \sigma\vec{E}$$

Electrical resistivity:

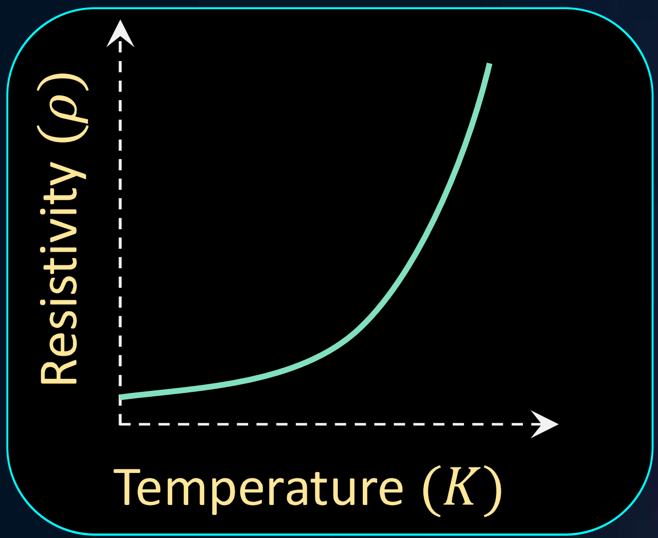
It is defined as the resistance offered by the material per unit length for unit cross-section.

$$\rho = \frac{m}{ne^2\tau} = \text{Resistivity} = ohm.m$$

Recap

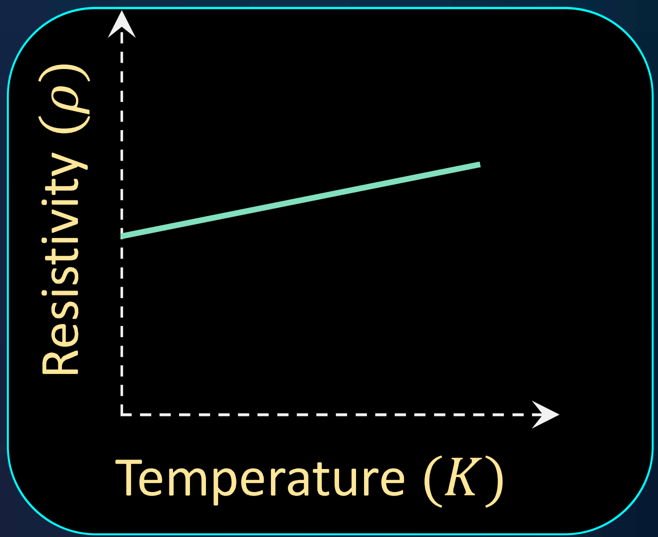
Effect of temperature on resistance:

i) Conductors



For copper

ii) For alloys



(Manganin, Nichrome, Constantan)

$$R_f = R_0(1 + \alpha \Delta T)$$

$$\rho_f = \rho_0(1 + \alpha \Delta T)$$

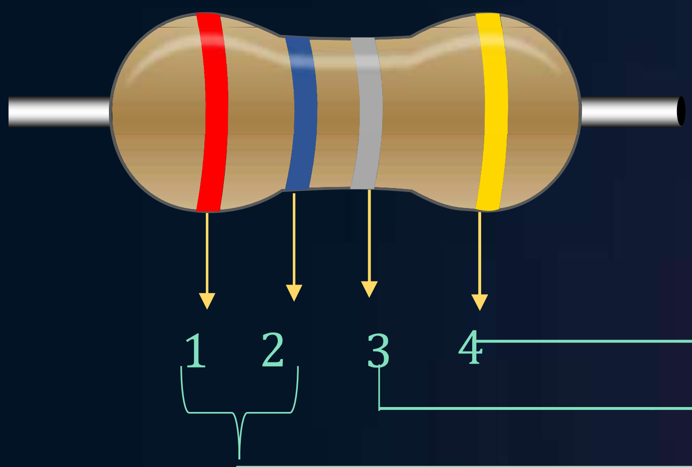
R_0 = Resistance at reference temperature T_0

α = Co-efficient of thermal resistance

Colour coding of carbon resistors

Carbon resistors

4 co-axial colour bands (rings)



4^{th} band → It stands for tolerance or possible variation in percentage about the indicated values.

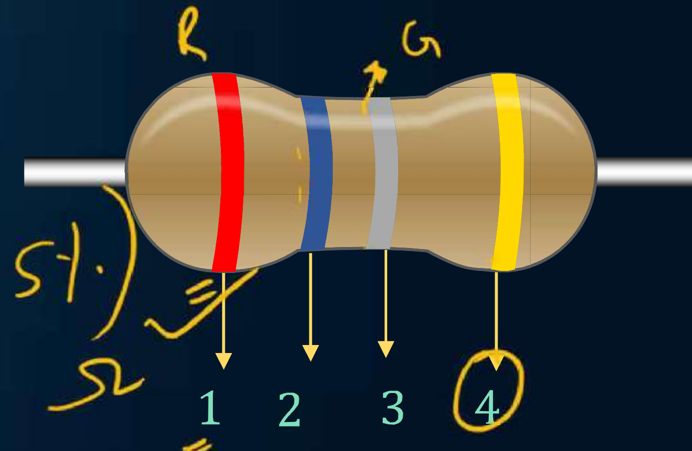
3^{rd} band → Indicates the decimal multiplier

1^{st} and 2^{nd} band → Indicate the first two significant figures of the resistance in ohms.

Colour coding of carbon resistors

Resistor colour codes

Colour	Number	Multiplier	Tolerance
Black	0	$1(10^0)$	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	$\pm 5\%$
Silver		10^{-2}	$\pm 10\%$
No colour			$\pm 20\%$



$(26 \times 10^8 \pm 5\%)\ \Omega$

Trick:

BBROYGBVGW

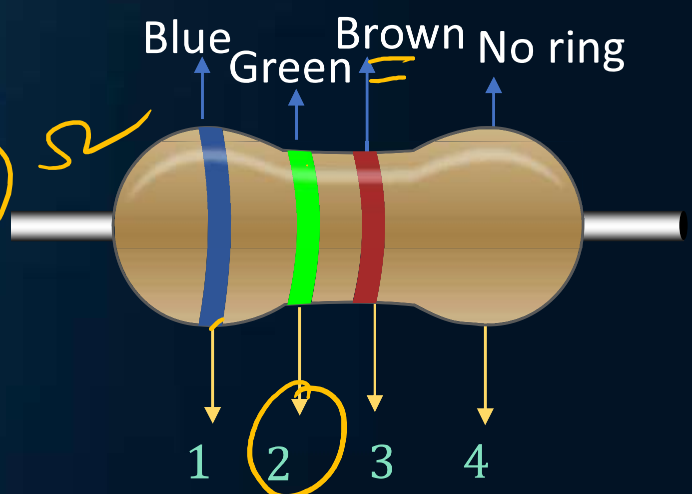
0 1 2 3 4 5 6 7 8 9

BB ROY of Great Britain has a Very Good Wife

Colour coding of carbon resistors

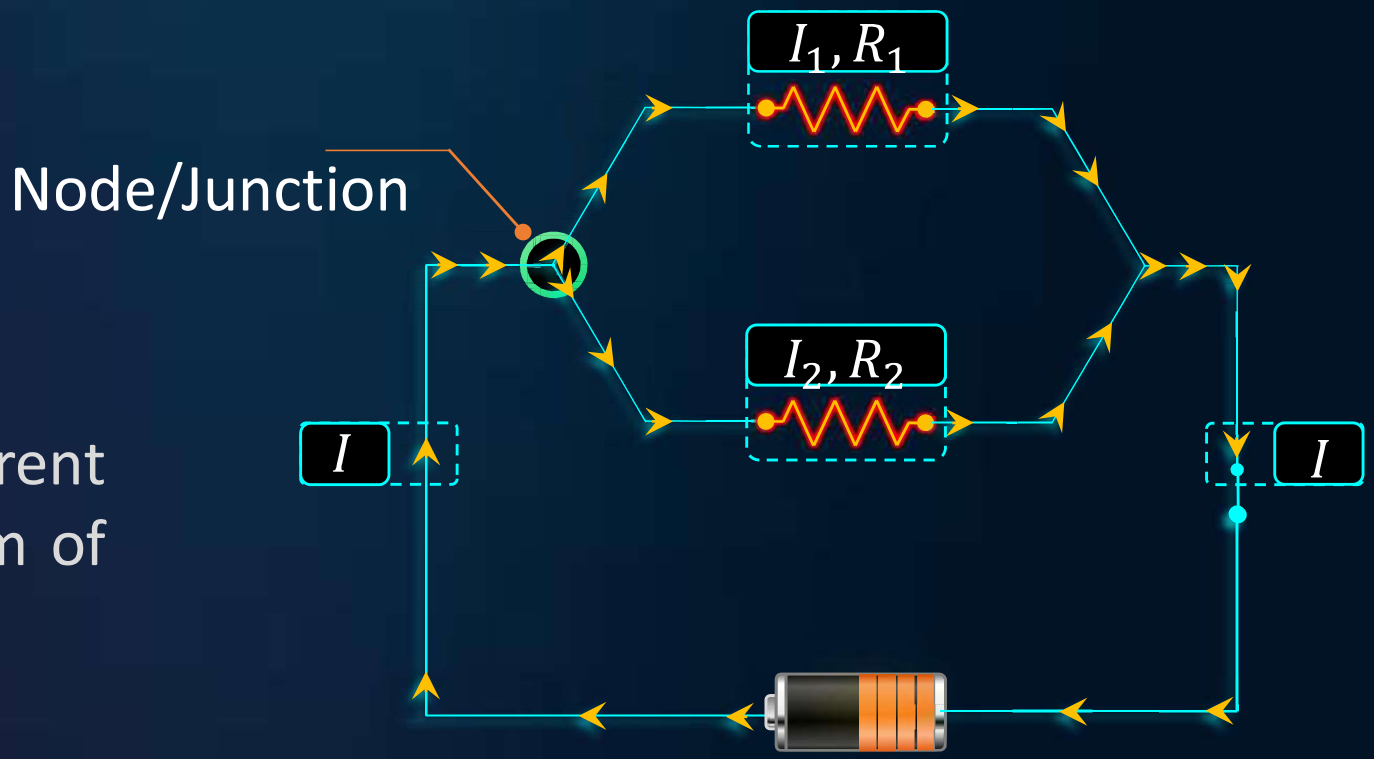
Colour	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	$\pm 5\%$
Silver		10^{-2}	$\pm 10\%$
No colour			$\pm 20\%$

$(65 \times 10 \pm 20\%)\ \Omega$

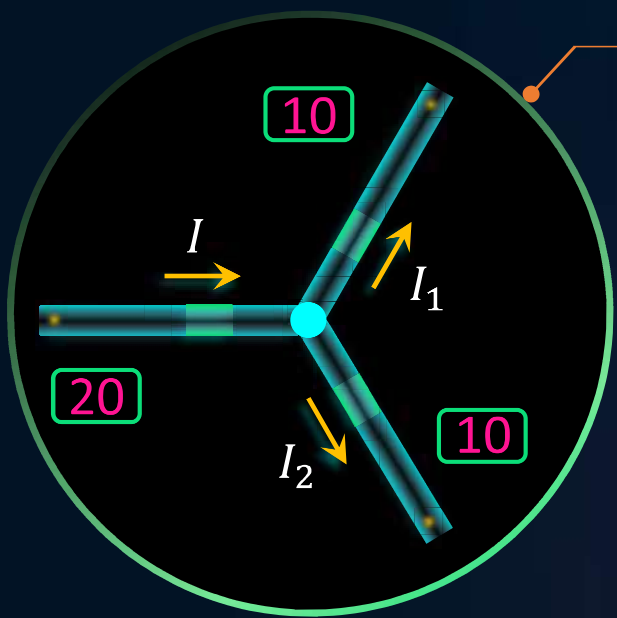


Kirchhoff's Current Law (KCL)

- **Node or Junction :** It is a point in a circuit at which more than two conductors meet.
- At a node or junction net current coming in is equal to net current going out. There's no accumulation of current at the junction.
- The law states that the amount of current flowing into a node is equal to the sum of currents flowing out of it.



Kirchhoff's Current Law (KCL)



Junction

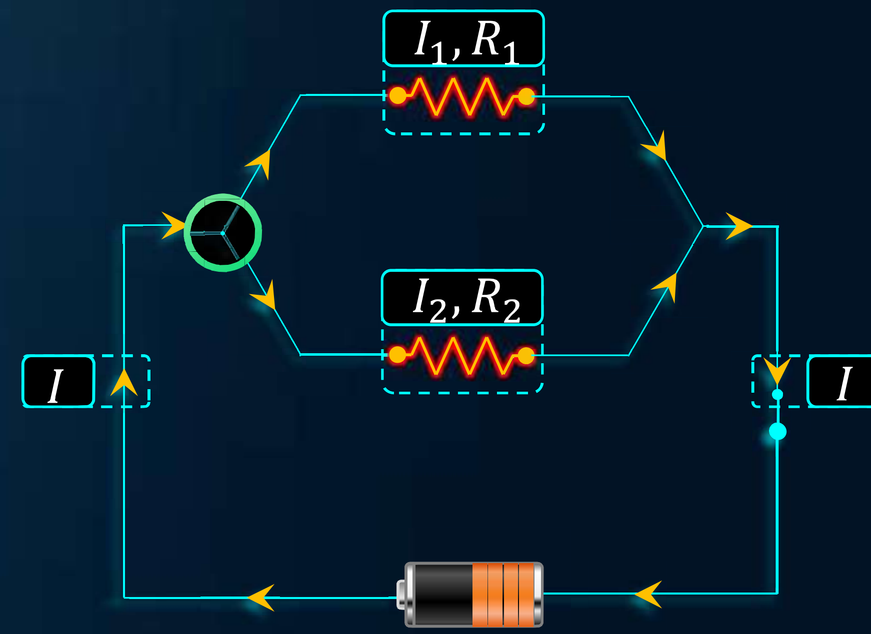
Incoming current= Outgoing current

$$I = I_1 + I_2$$

$$\boxed{I - I_1 - I_2 = 0}$$

$$\boxed{\Sigma I = 0}$$

Net current at a junction is zero.



$$\frac{\Delta Q}{\Delta t} - \frac{\Delta Q_1}{\Delta t} - \frac{\Delta Q_2}{\Delta t} = 0 \Rightarrow \Delta Q = \Delta Q_1 + \Delta Q_2$$

Sum of charges entering the node = Sum of charges leaving the node

Law of conservation of Charge

Question

In given circuit, find the value of I

A. -2 A

B. 8 A

C. 5 A

D. 2 A

Discussion

Method-1:

Incoming current = Outgoing current

$$4\,A + 2\,A = 8\,A + I$$

$$\Rightarrow I = -2\,A$$

Method-2:

Assuming the incoming current towards the junction to be positive and outgoing current from the junction to be negative, we can write,

$$\sum I = 0$$

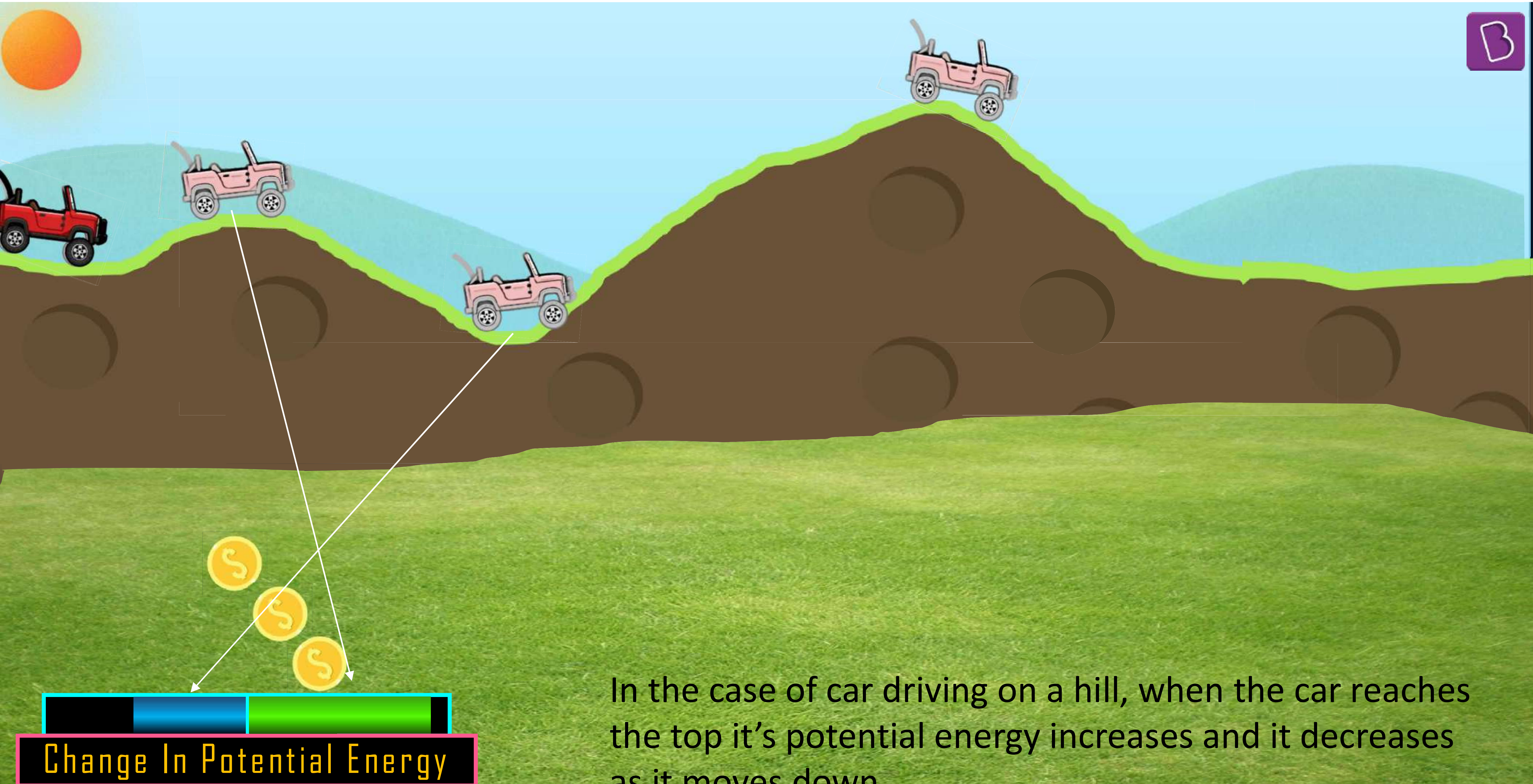
$$+2 + 4 - 8 - I = 0$$

$$I = -2\,A$$

$\longrightarrow$ +ve $\bullet$ $\longrightarrow$ −ve

$-ve$ sign implies actual direction of current is opposite to the assumed direction of current. That means the current I will be towards the junction.

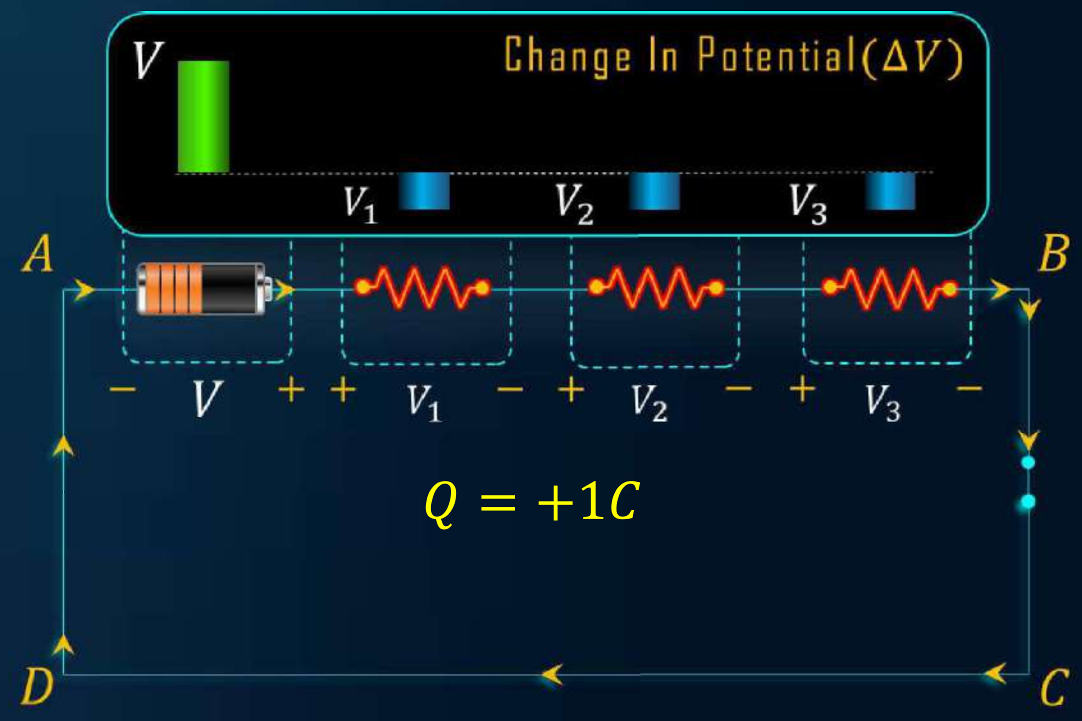
Thus, option (A) is the correct answer.



In the case of car driving on a hill, when the car reaches the top it's potential energy increases and it decreases as it moves down.

Kirchhoff's Voltage Law (KVL)

Suppose charge of $+1C$ is flowing through the circuit at any particular instant. As the charge pass through the resistances, its potential energy decreases.



Work done in carrying a $+1C$ charge once around the circuit $= 1C(V) + 1C(-V_1) + 1C(-V_2) + 1C(-V_3)$

Since the starting point and ending point of the charge is same, the total work done will be zero. Hence, energy will be conserved.

Therefore, KVL is based upon $\boxed{\text{Law of conservation of Energy}}$

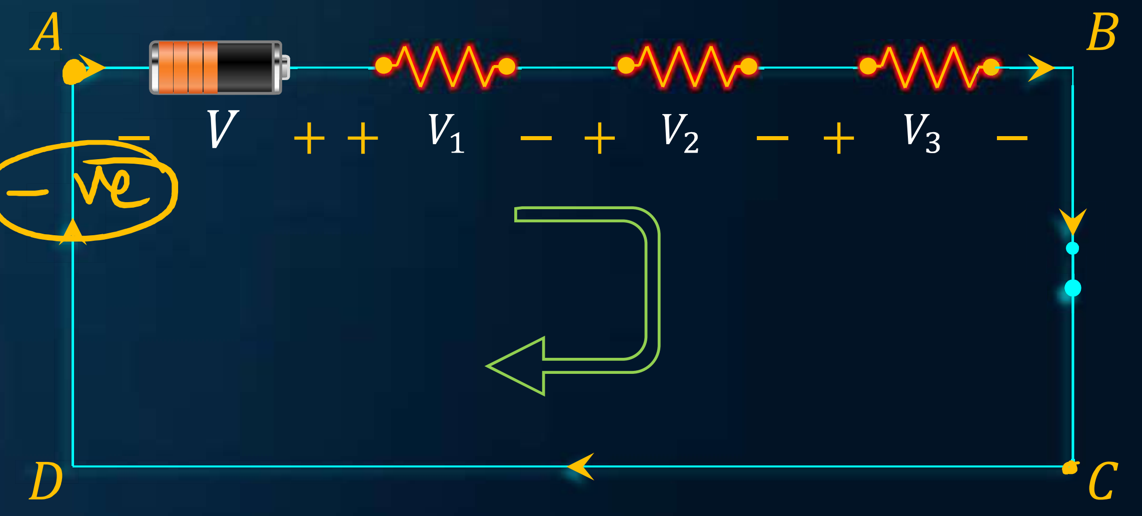
Kirchhoff's Voltage Law (KVL)

- In a closed loop/mesh, the algebraic sum of all the potential differences is zero

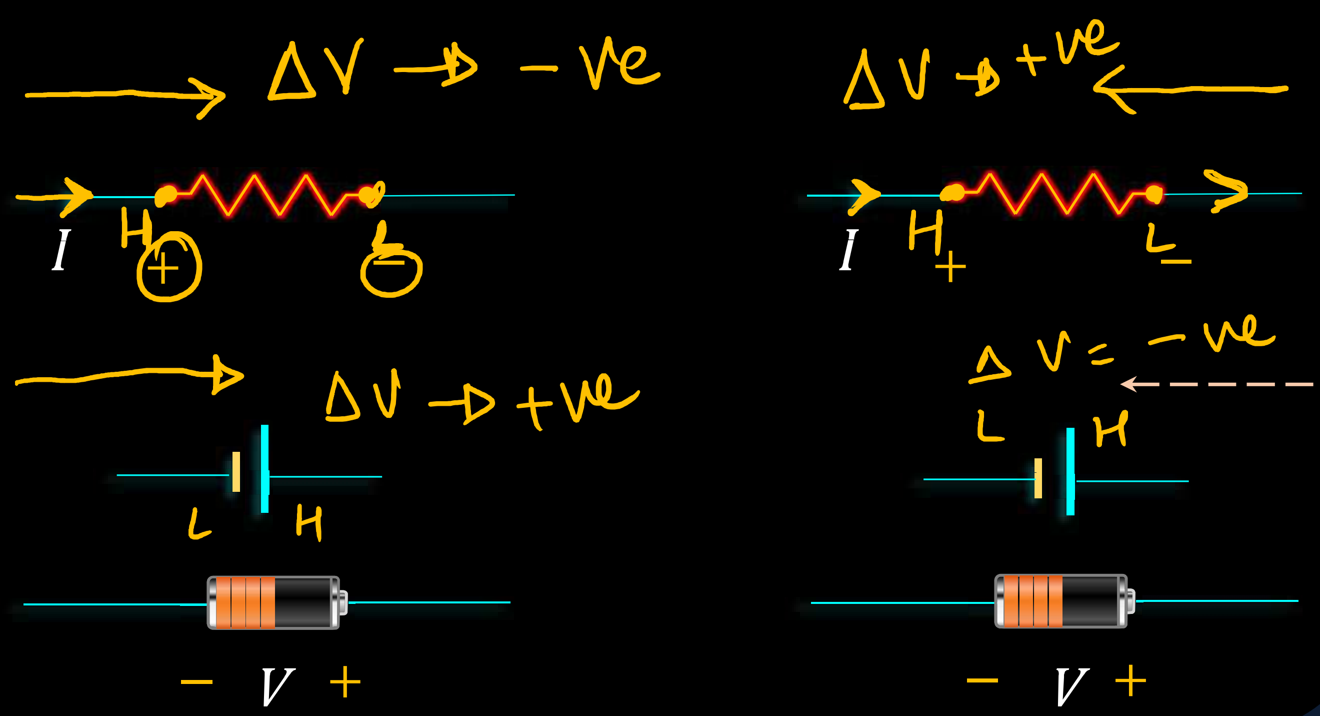
} +ve

i

Same → −ve



Sign Convention



L = Lower Potential

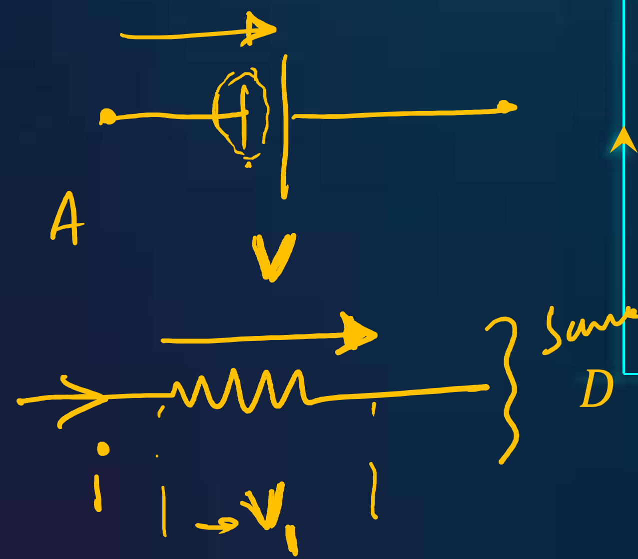
H = Higher Potential

- If the direction in which sum is taken is same as the direction of current then take potential difference as negative.
- If they are in opposite direction then take potential difference as positive.

Kirchhoff's Voltage Law (KVL)

- In a closed loop/mesh, the algebraic sum of all the potential differences is zero

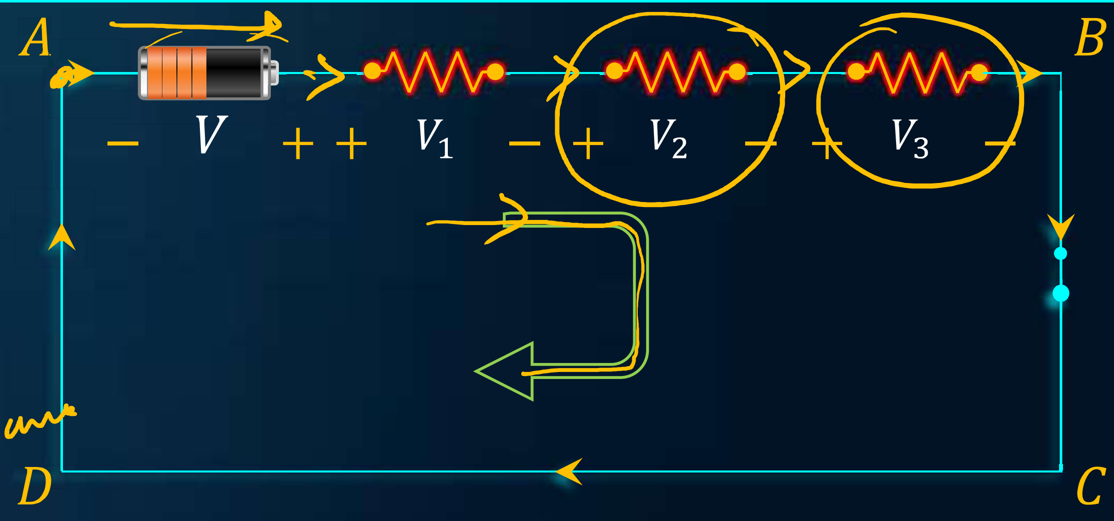
A B C D A



$+V - V_1 - V_2 - V_3 = 0$

$\Sigma \Delta V = 0$

$$\boxed{V = V_1 + V_2 + V_3}$$



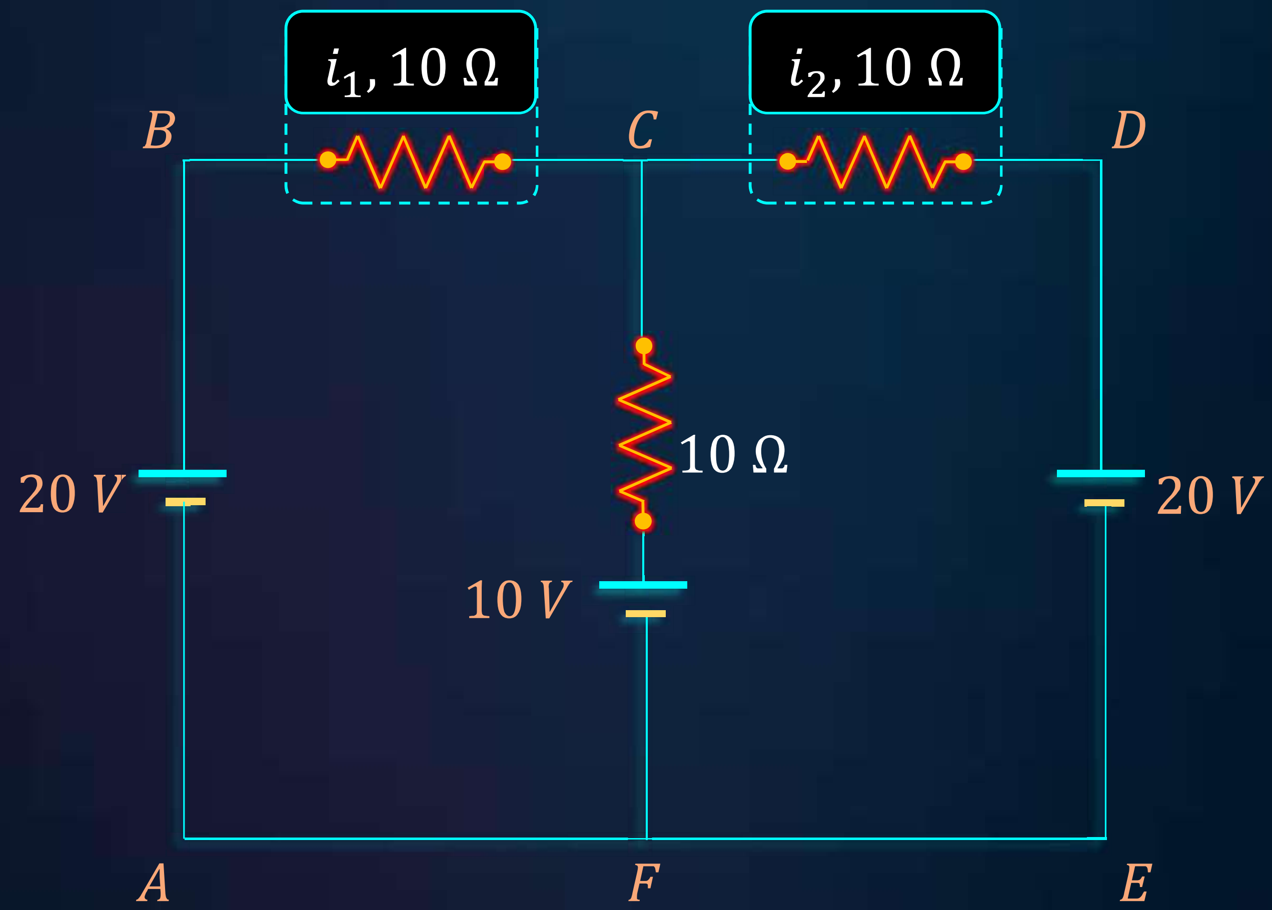
In loop $ABCDA$,

$V - V_1 - V_2 - V_3 = 0$

$\Sigma V = 0$

Question

In given circuit, find i_1 and i_2 respectively



(A) $1\ A, 2\ A$

(B) $\frac{1}{3}\ A, \frac{2}{3}\ A$

(C) $\frac{1}{3}\ A, \frac{1}{3}\ A$

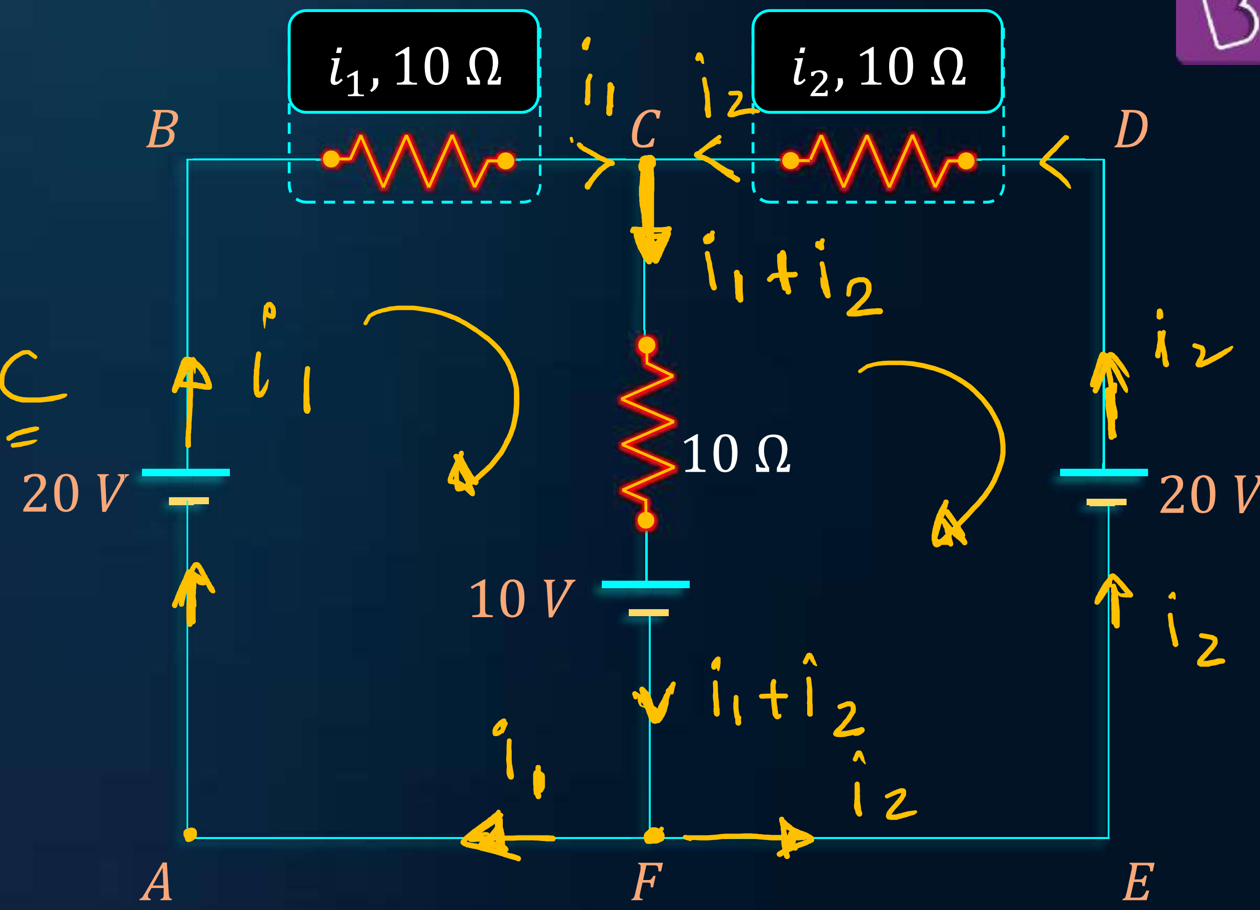
(D) $5\ A, 2\ A$

- Draw current from source
- Distribute current using KCL
- Choose loops with direction

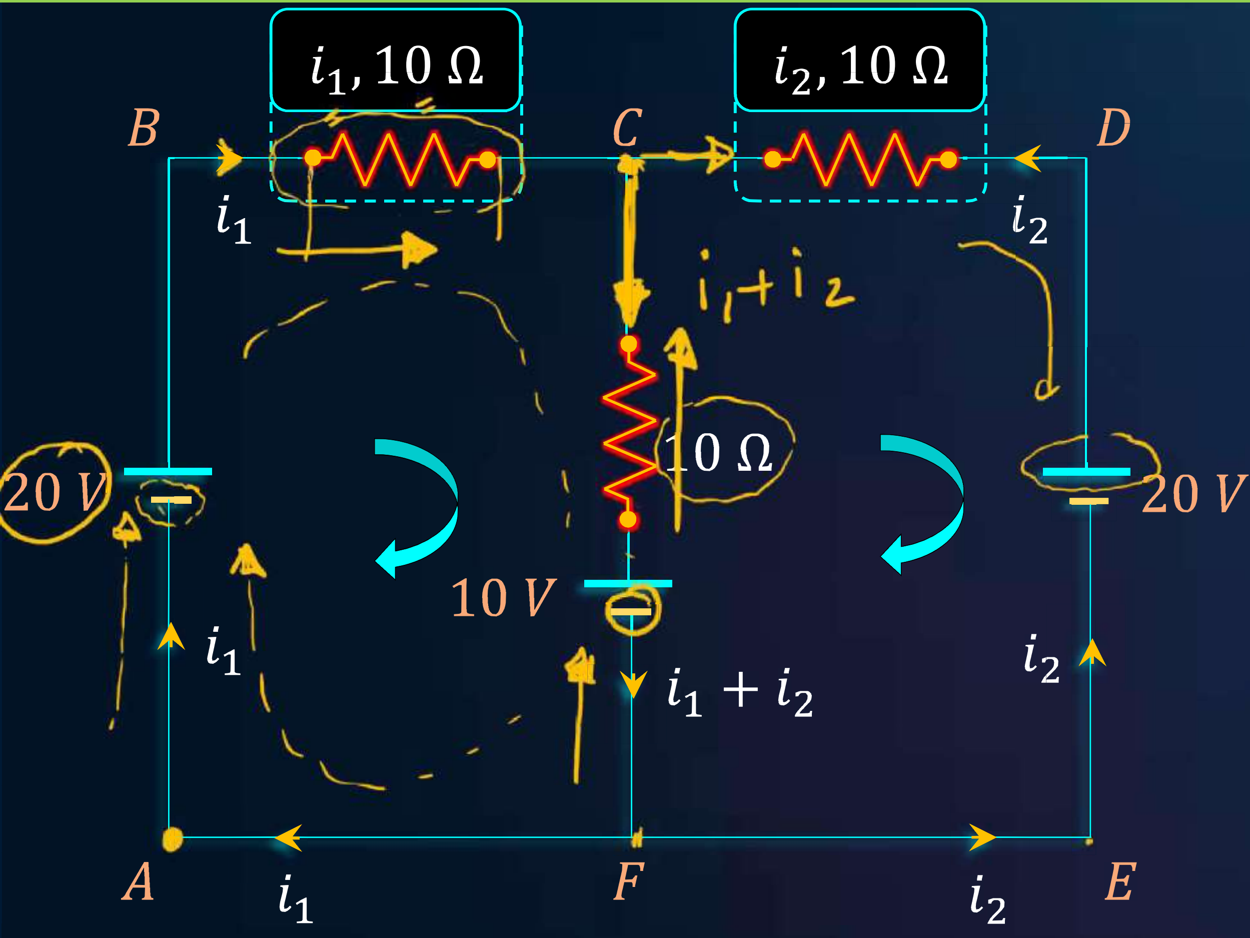
  Clockwise / Anticlockwise
- Apply KVL over selected loop

AB CFA

CD EFC



Discussion



In loop A B C F A

$$+20 - i_1(10) - (i_1 + i_2)(10) - 10 = 0$$

$$2i_1 + i_2 = 1 \quad \text{—} \ (1)$$

In loop C D E F C

$$+i_2(10) - 20 + 10 + (i_1 + i_2)10 = 0$$

$$2i_2 + i_1 = 1 \quad \text{—} \ (2)$$

By solving equation (1) and (2), we get,

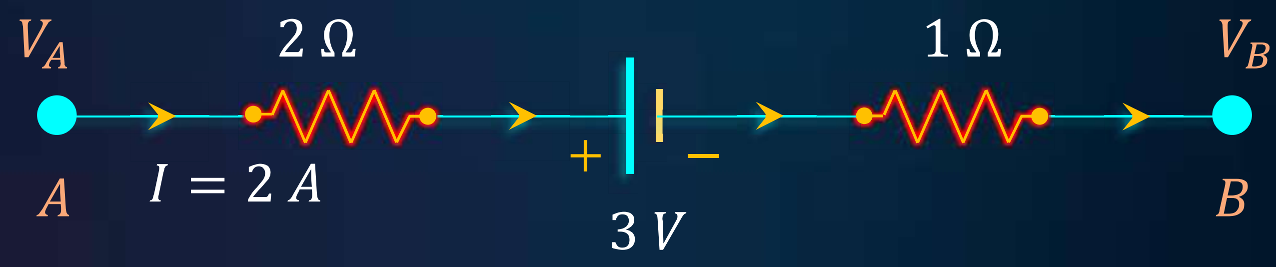
$$i_1 = i_2 = \frac{1}{3}\ A$$

Thus, option (C) is the correct answer.

Question

The potential difference $(V_A - V_B)$ between the points A and B in the given figure is

- A. $+9\ V$
- B. $-3\ V$
- C. $+3\ V$
- D. $+6\ V$



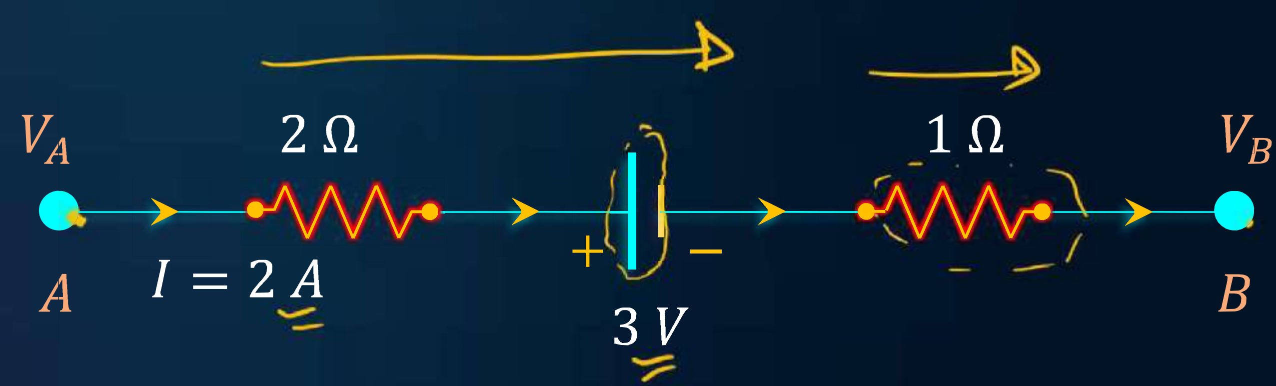
Discussion

Using KVL,

$$V_A - 2 \times I - 3 - 1 \times I = V_B$$

$$\Rightarrow V_A - V_B = 2I + 3 + I$$

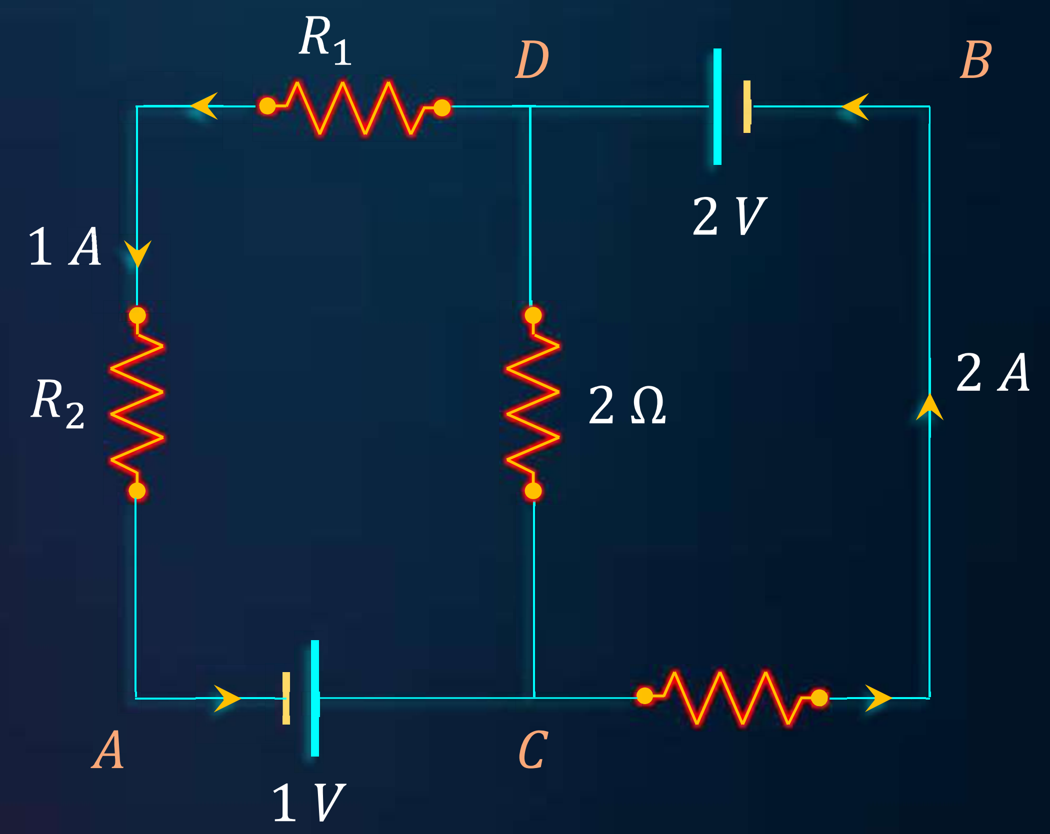
$$\Rightarrow V_A - V_B = 9\ V$$



Thus, option (A) is the correct answer.

Question

In the circuit shown in the figure, if the potential at point A is taken to be zero, the potential at point B is



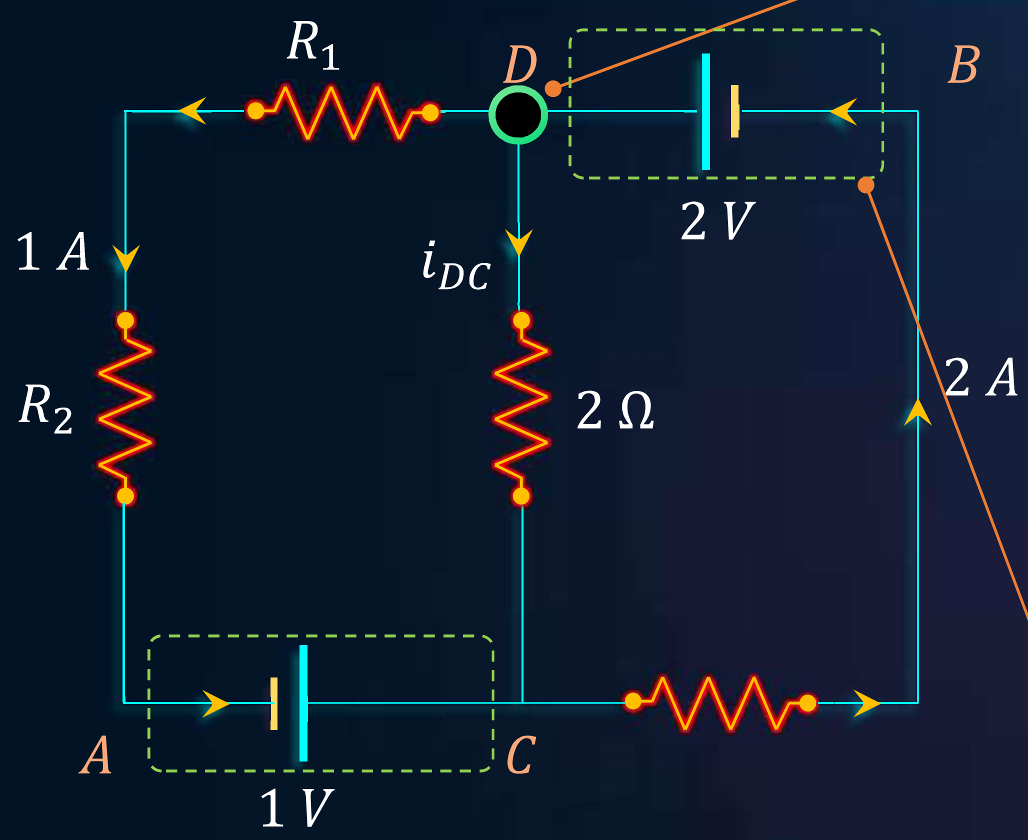
A. $-2\ V$

B. $-1\ V$

C. $+1\ V$

D. $+2\ V$

Discussion



Applying KCL at node D, we get,

$$i_{DC} = 2\,A - 1\,A = 1\,A$$

Therefore, the potential difference between point D and C is given by,

$$V_{DC} = i_{DC} \times 2\,\Omega = 2\,V$$

$$\Rightarrow V_D - V_C = 2\,V$$

$$\Rightarrow V_D = 3\,V \quad [\text{since } V_C = 1\,V]$$

$$V_{DB} = 2\,V$$

$$\Rightarrow V_D - V_B = 2\,V$$

$$\Rightarrow V_B = 1\,V \quad [\text{since } V_D = 3\,V]$$

$$V_{AC} = -1\,V$$

$$V_A - V_C = -1\,V$$

$$V_C = 1\,V \quad [\text{since } V_A = 0\,V]$$

Thus, option (C) is the correct answer.

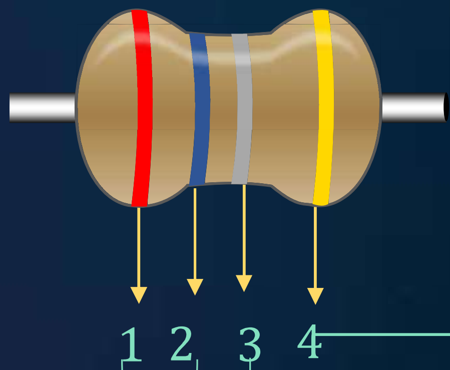
Recap

Colour coding of carbon resistors:

BBROYGBVGW
0 1 2 3 4 5 6 7 8 9

BB ROY of Great Britain has a Very Good Wife

Colour	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	$\pm 5\%$
Silver		10^{-2}	$\pm 10\%$
No colour			$\pm 20\%$



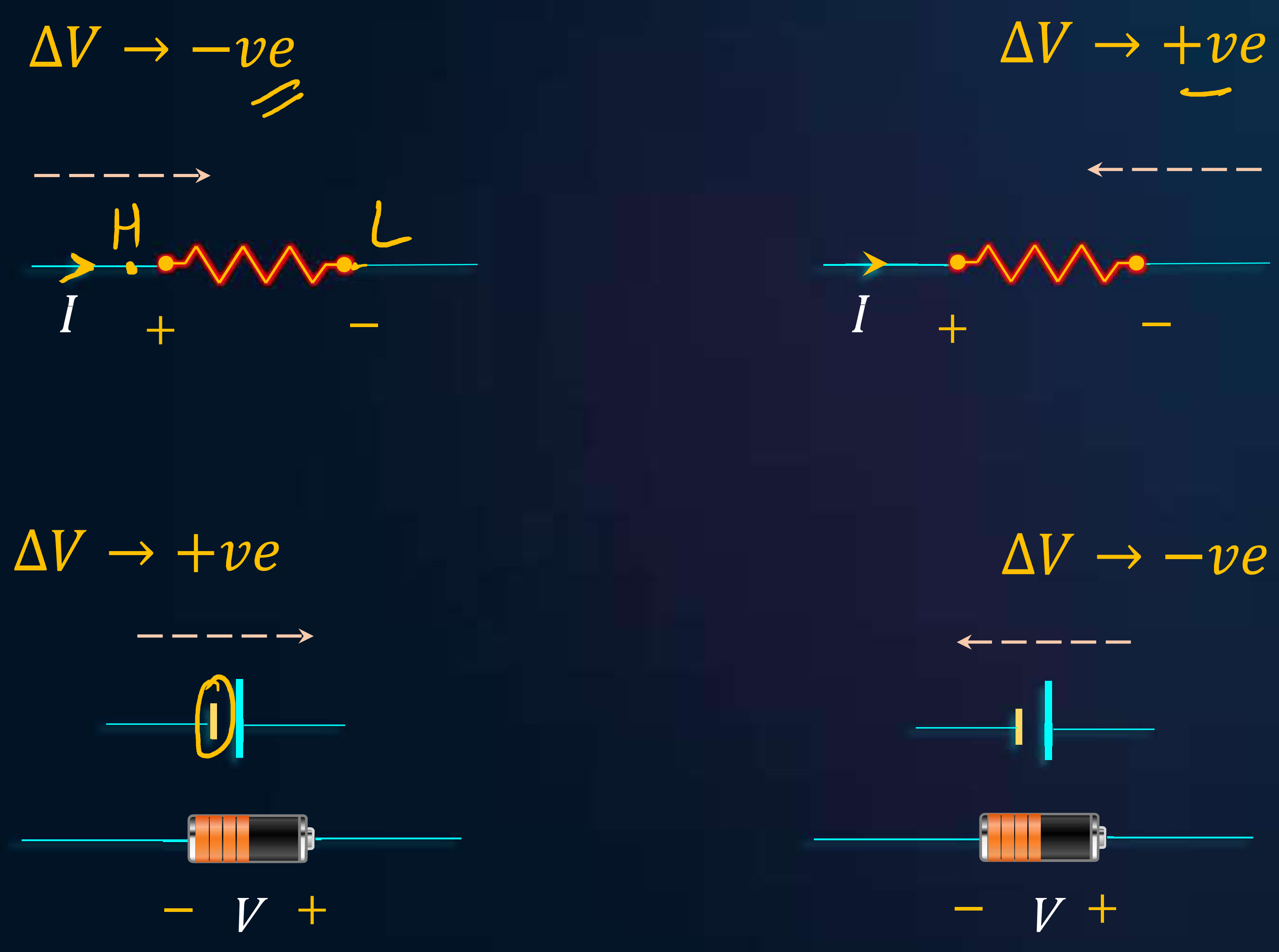
4^{th} band → stands for tolerance or possible variation in percentage about the indicated values.

3^{rd} band→Indicates the decimal multiplier

1,2 band → Indicate the first two significant figures of the resistance in ohms.

Recap

Sign convention:



- If the direction in which sum is taken is same as the direction of current then take the potential difference as negative.
- If they are in opposite direction then take potential difference as positive.

Recap

Kirchhoff's current law:

- Node or Junction : It is a point in a circuit at which more than two conductors meet.

At junction,

- Incoming current = Outgoing current

$$I = I_1 + I_2$$

$$\Sigma I = 0$$

$$\frac{\Delta Q}{\Delta t} - \frac{\Delta Q_1}{\Delta t} - \frac{\Delta Q_2}{\Delta t} = 0$$

Law of conservation of Charge

Recap

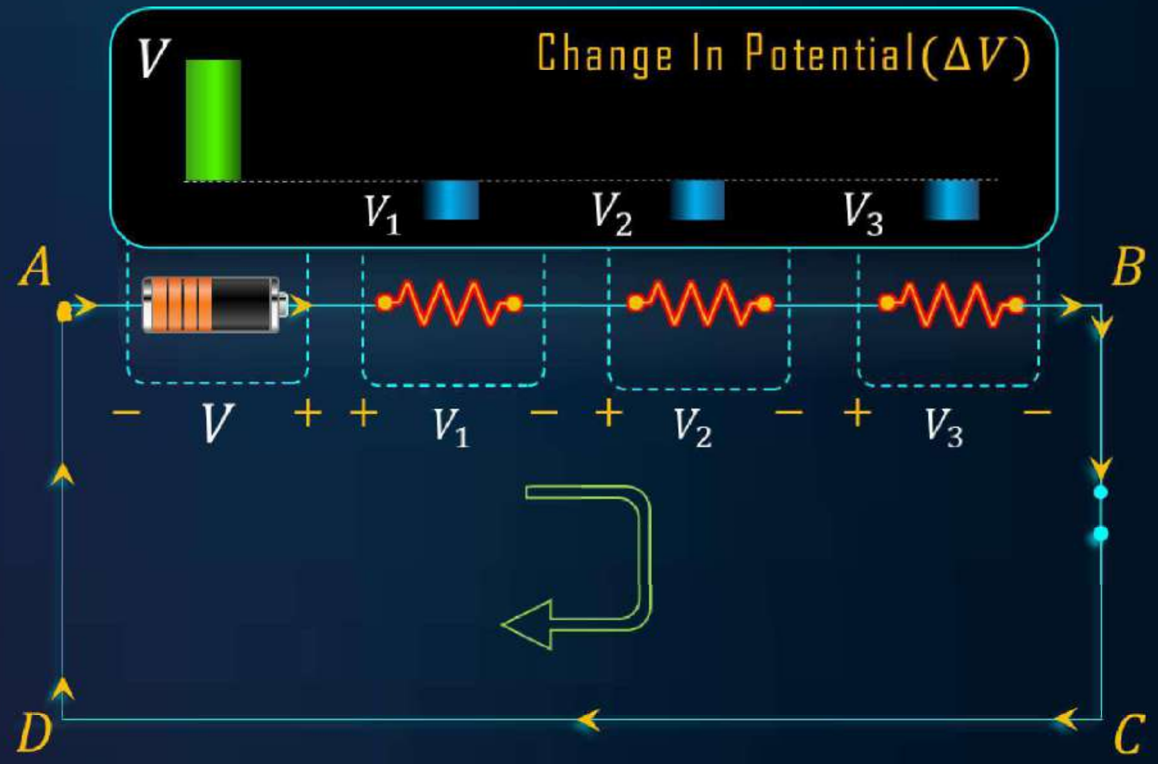
Kirchhoff's voltage law:

In a closed loop/mesh, the algebraic sum of all the potential differences is zero

A B C D A

$$V - V_1 - V_2 - V_3 = 0$$

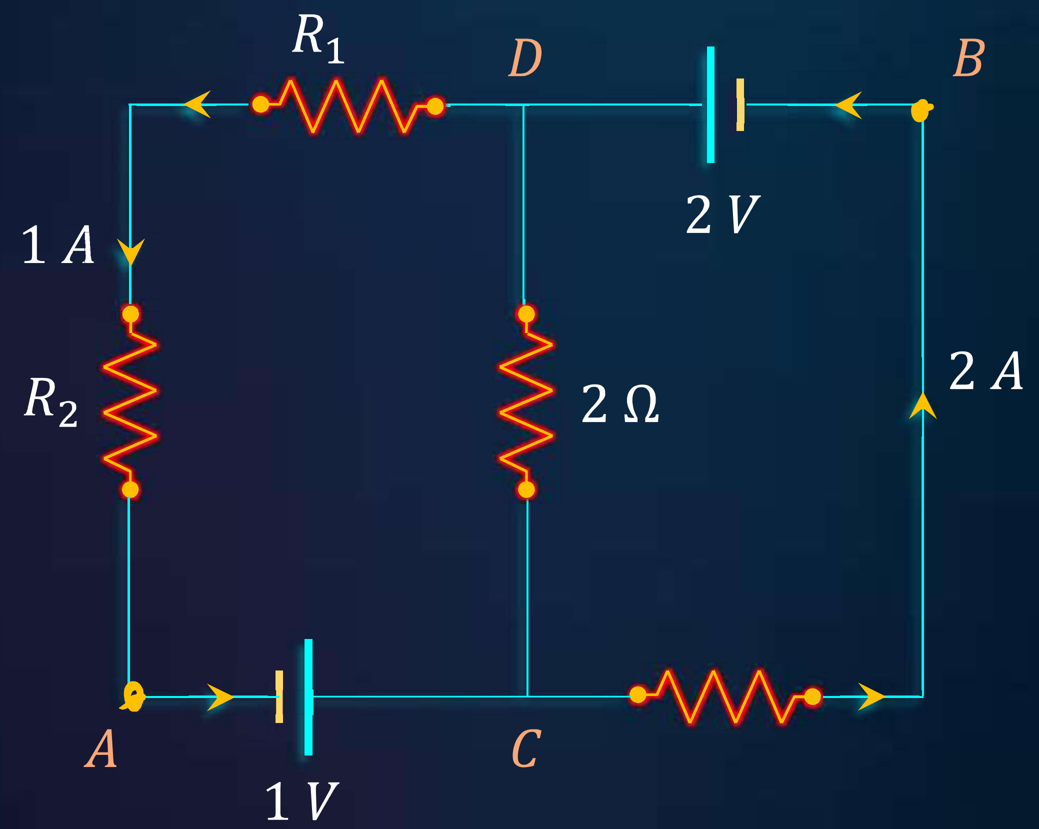
$$\Sigma V = 0$$



Law of conservation of Energy

Question

In the circuit shown in the figure, if the potential at point A is taken to be zero, the potential at point B is



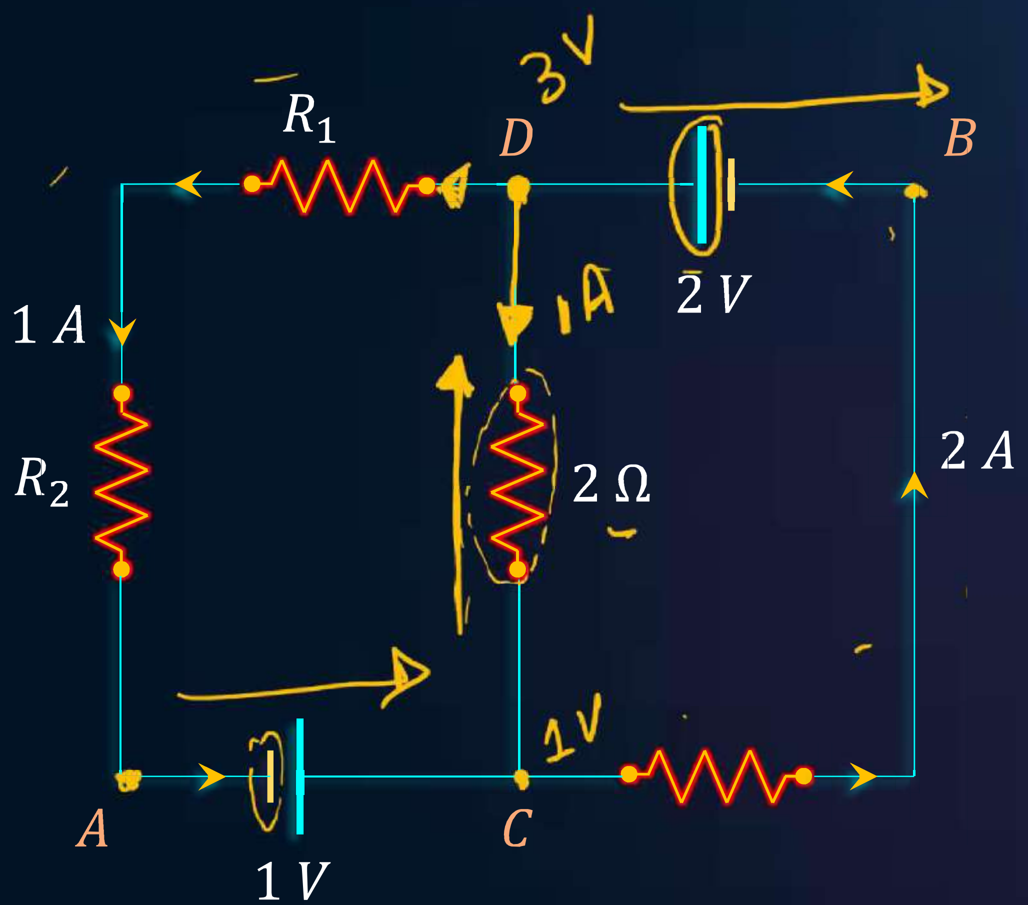
a) $-2\ V$

b) $-1\ V$

c) $+1\ V$

d) $+2\ V$

Discussion



$V_A + 1 = V_C$ ($V_A \to 0$)

$V_C = 1V$

$A \to C \to D \to B$

$V_C + (1)(2) = V_D$

$V_D = 3V$

$D \to B$

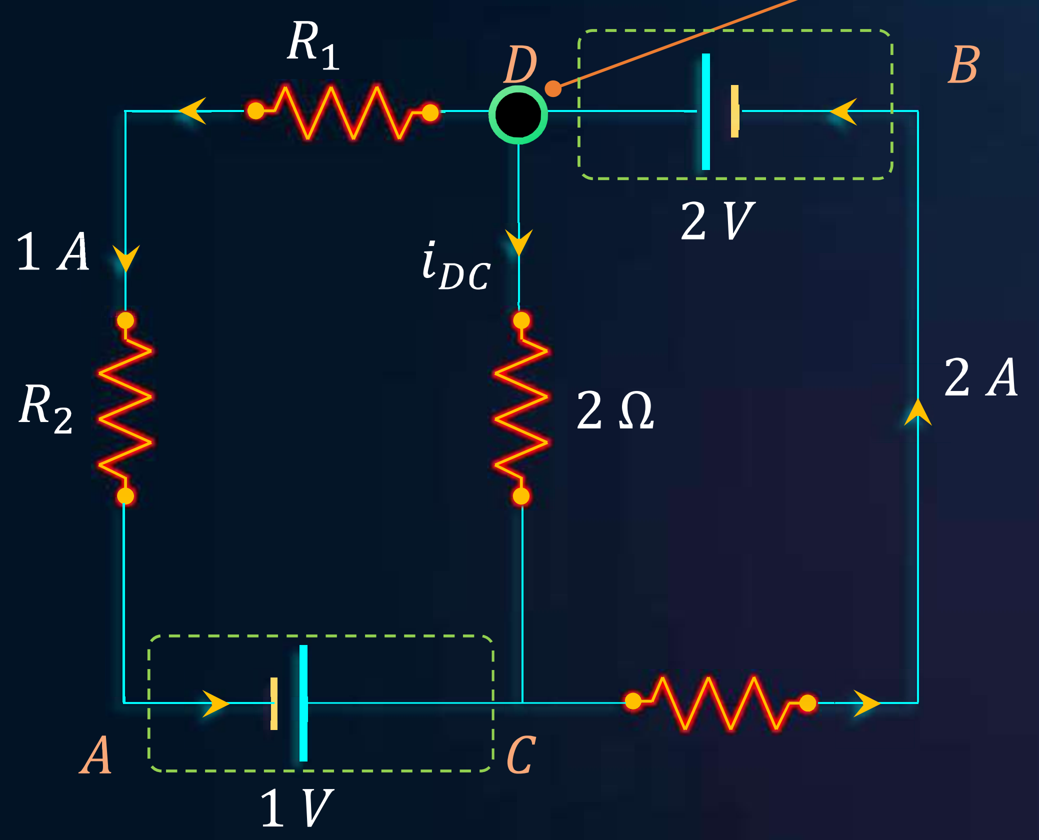
3V

$V_D - 2 = V_B$

$3 - 2 = V_B$

$V_B = 1V$

Summary



Applying KCL ,

$$i_{DC} = 2\ A - 1\ A = 1\ A$$

$$V_{DC} = i_{DC} \times 2\ \Omega = 2\ V$$

$$V_D - V_C = 2\ V$$

$$V_C = 1\ V$$

$$V_D = 3\ V$$

$$V_{DB} = 2\ V$$

$$V_D - V_B = 2\ V$$

$$V_B = 1\ V$$

$$V_{AC} = -1\ V$$

$$V_A - V_C = -1\ V$$

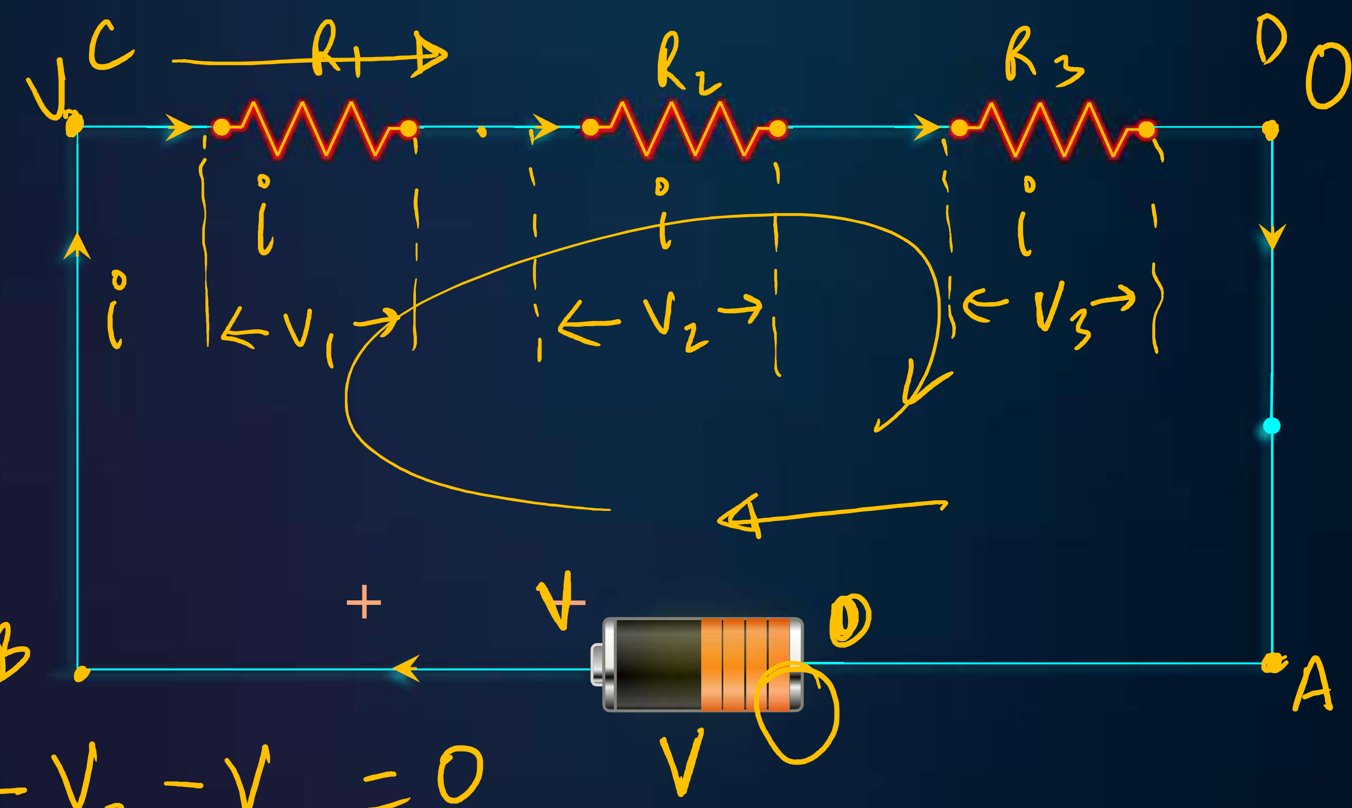
$$V_A = 0\ V$$

$$V_C = 1\ V$$

Thus, option (c) is the correct answer.

Combination of Resistors

Series combination: Voltage



$V_1 = iR_1$

$V_2 = iR_2$

$V_3 = iR_3$

ABCDA

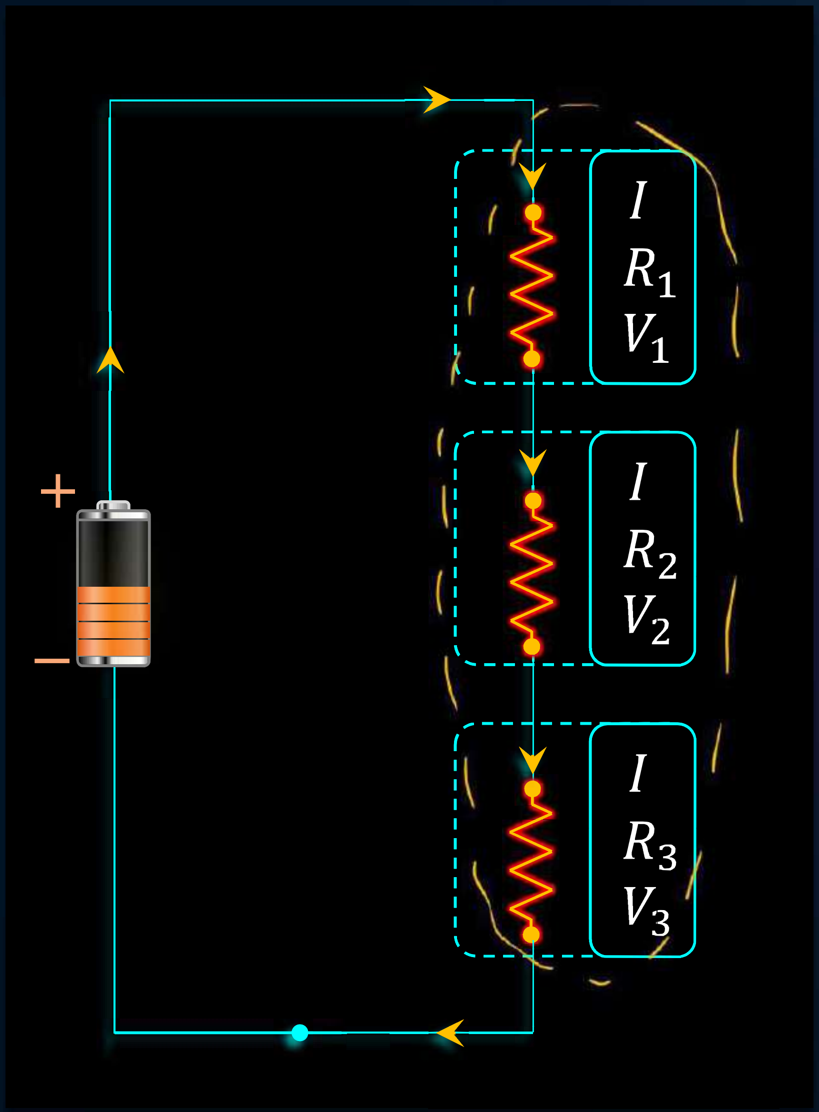
$V - V_1 - V_2 - V_3 = 0$

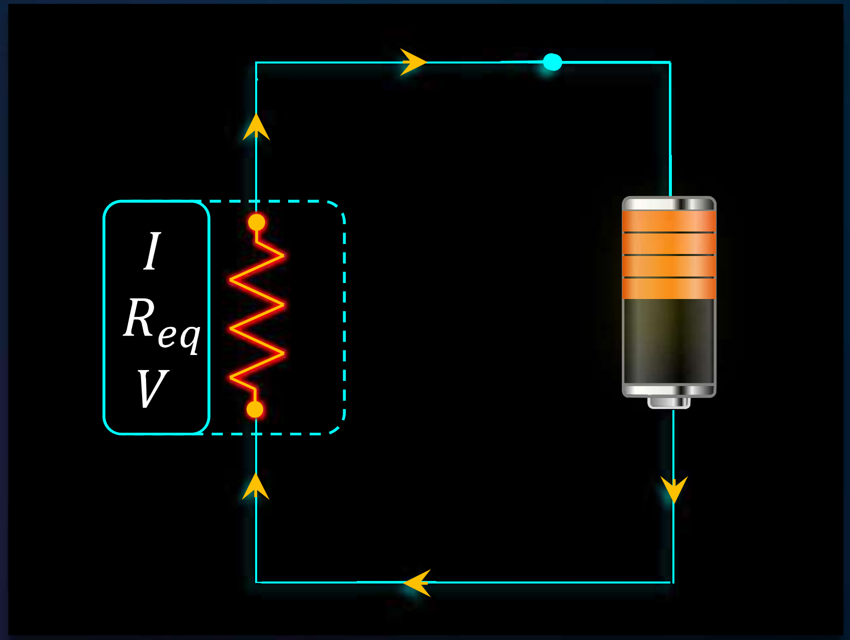
$\Sigma \Delta V = 0$

$V = V_1 + V_2 + V_3$

Combination of Resistors

Series combination: Equivalent resistance





$$V = V_1 + V_2 + V_3$$

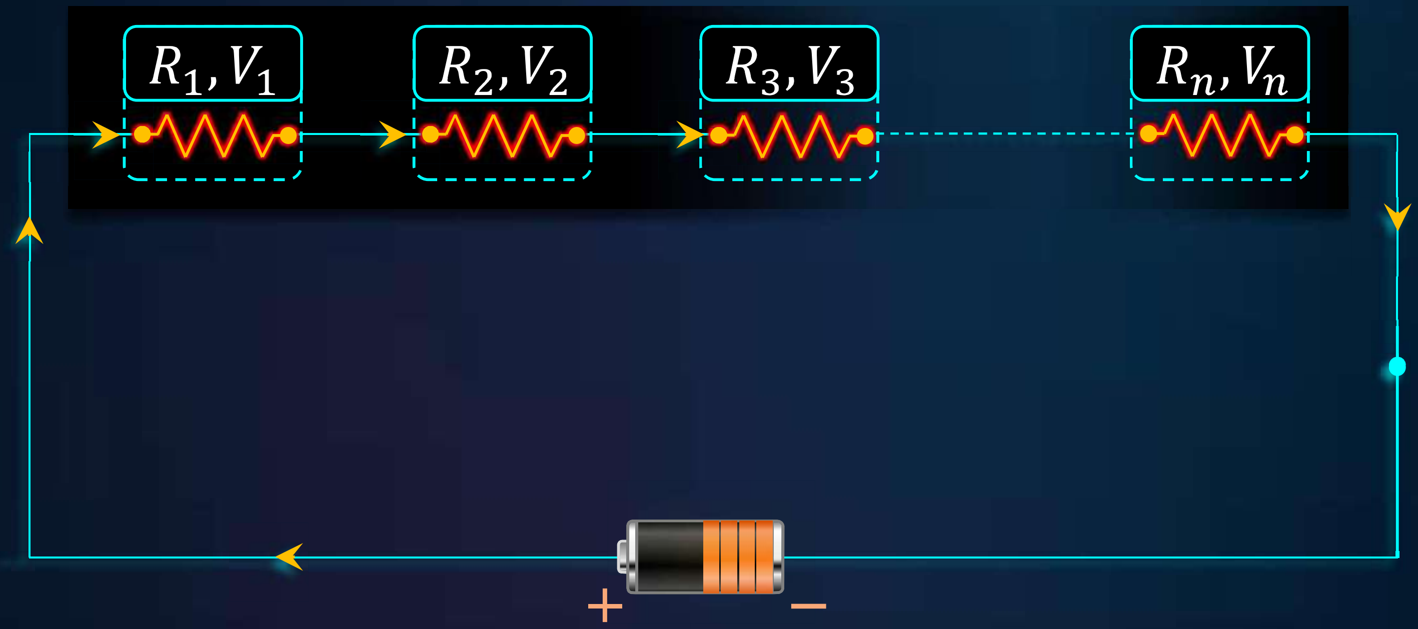
$$\cancel{I} R_{eq} = \cancel{I} R_1 + \cancel{I} R_2 + \cancel{I} R_3$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3 \ldots}$$

Combination of Resistors

Series combination: Many resistors

$$R_{eq} = R_1 + R_2 + R_3$$



$$R_{eq} = R_1 + R_2 + R_3 + \ \dots\dots\dots + R_n$$

$$R_{eq} = nR \quad \text{(Equal resistance)}$$

Combination of Resistors

Parallel combination: Votage and current

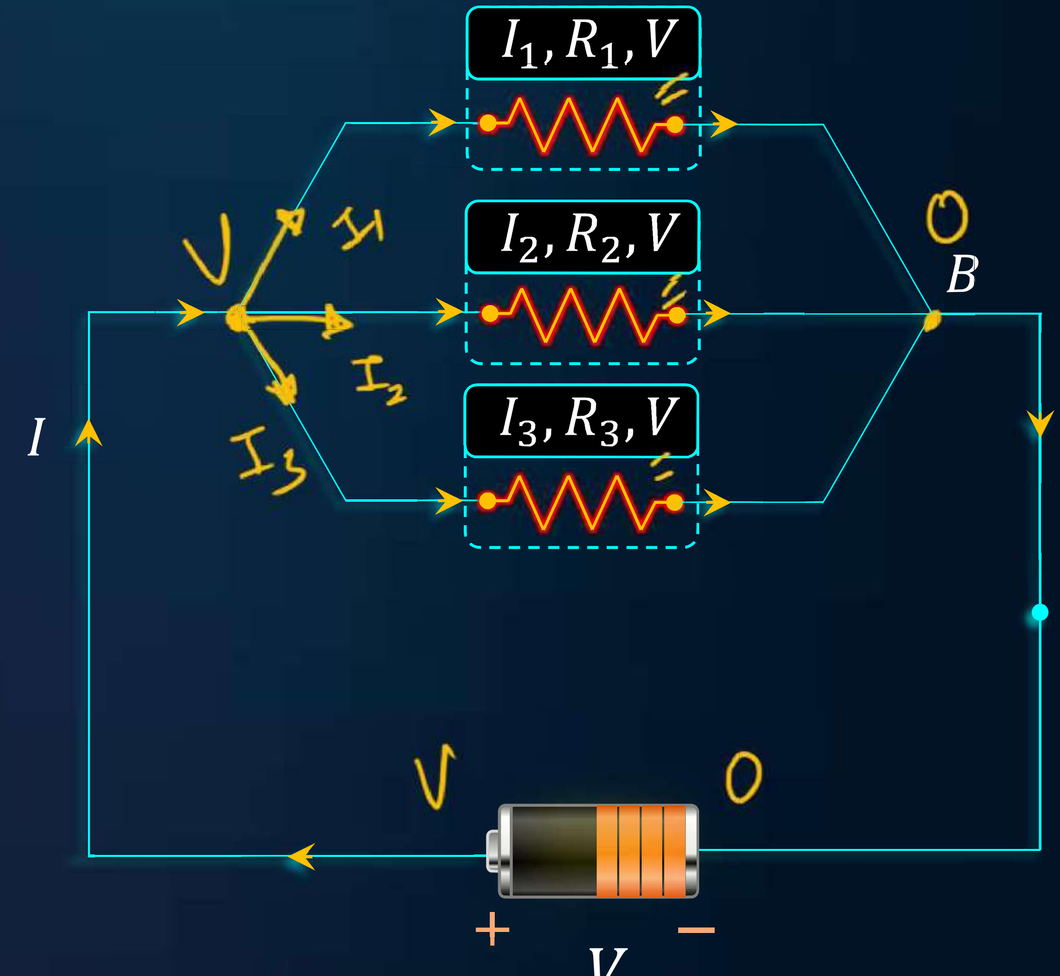
$$I = I_1 + I_2 + I_3$$

$$V = I_1 R_1 \qquad I_1 = \frac{V}{R_1}$$

$$V = I_2 R_2 \qquad I_2 = \frac{V}{R_2}$$

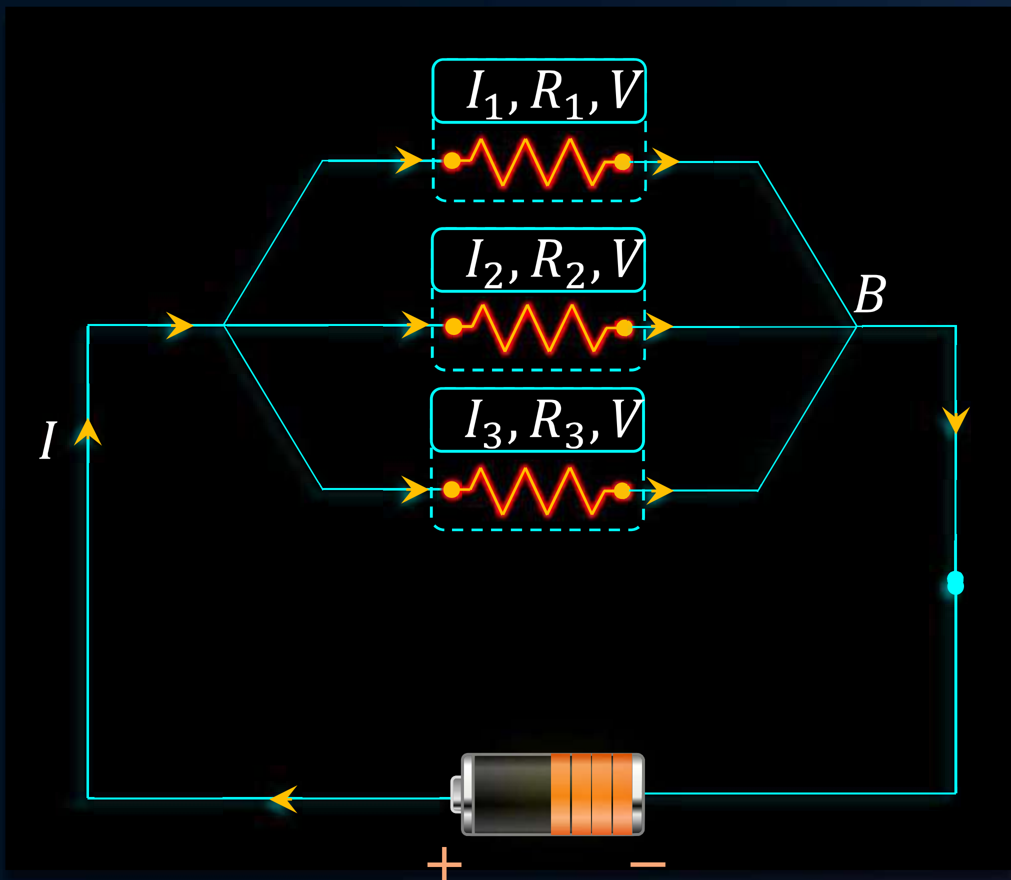
$$V = I_3 R_3 \qquad I_3 = \frac{V}{R_3}$$

$$I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

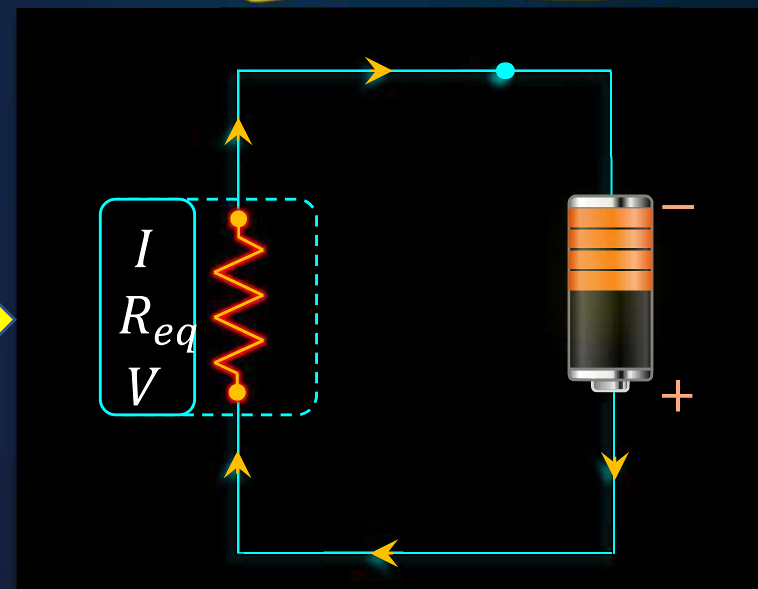


Combination of Resistors

Parallel combination: Equivalent resistance



$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$



- $I = I_1 + I_2 + I_3$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

- $R_{eq} = \frac{R}{n}$ (For n equal resistances)

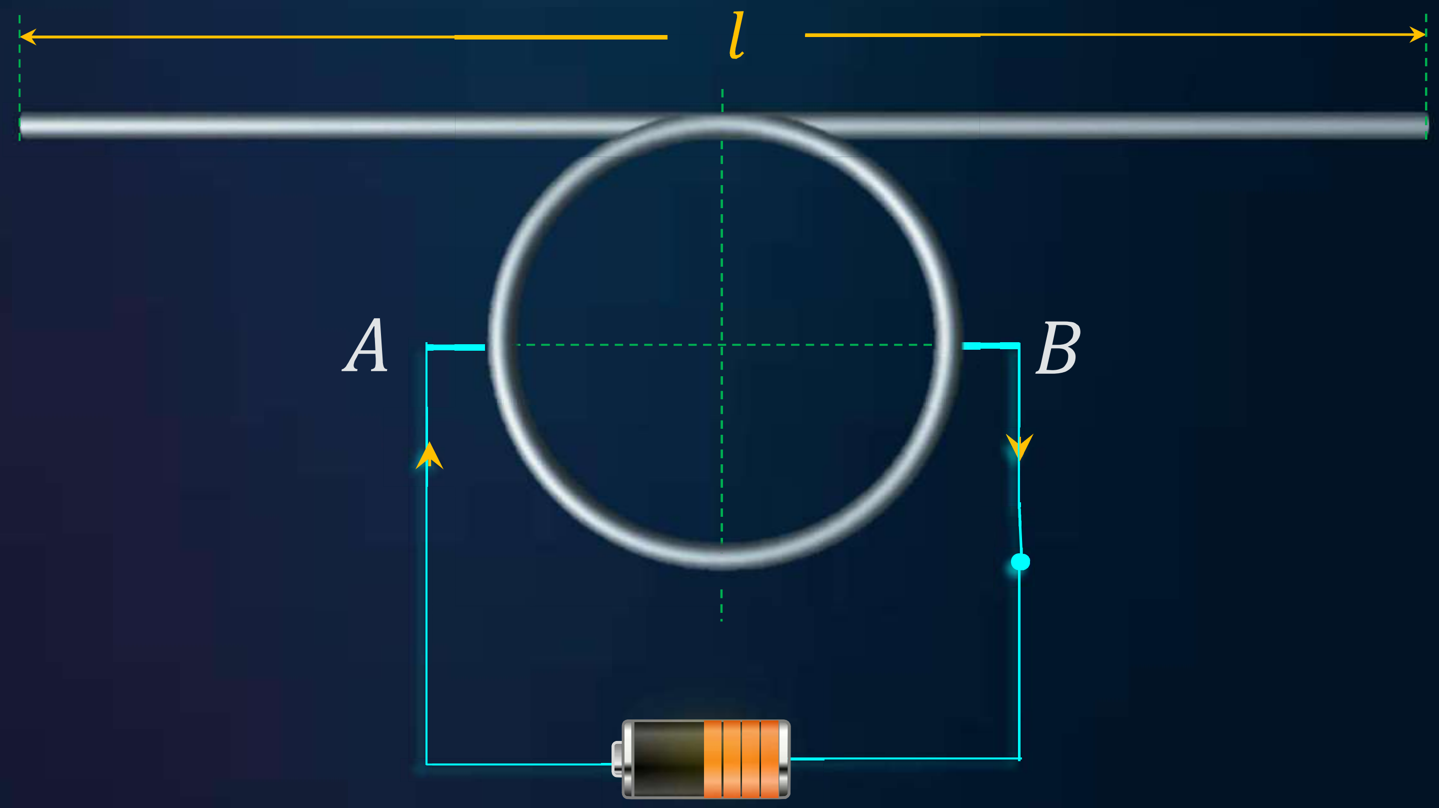
$R = 5\ \Omega$

$R_{eq} = \frac{R}{n} = \frac{5}{3}\ \Omega$

$R_{eq} = \frac{3 \times 6}{3+9} = 2\ \Omega$

Question

When a wire of uniform cross-section A, length l and resistance R is bent into a complete circle, resistance between any two of diametrically opposite points will be



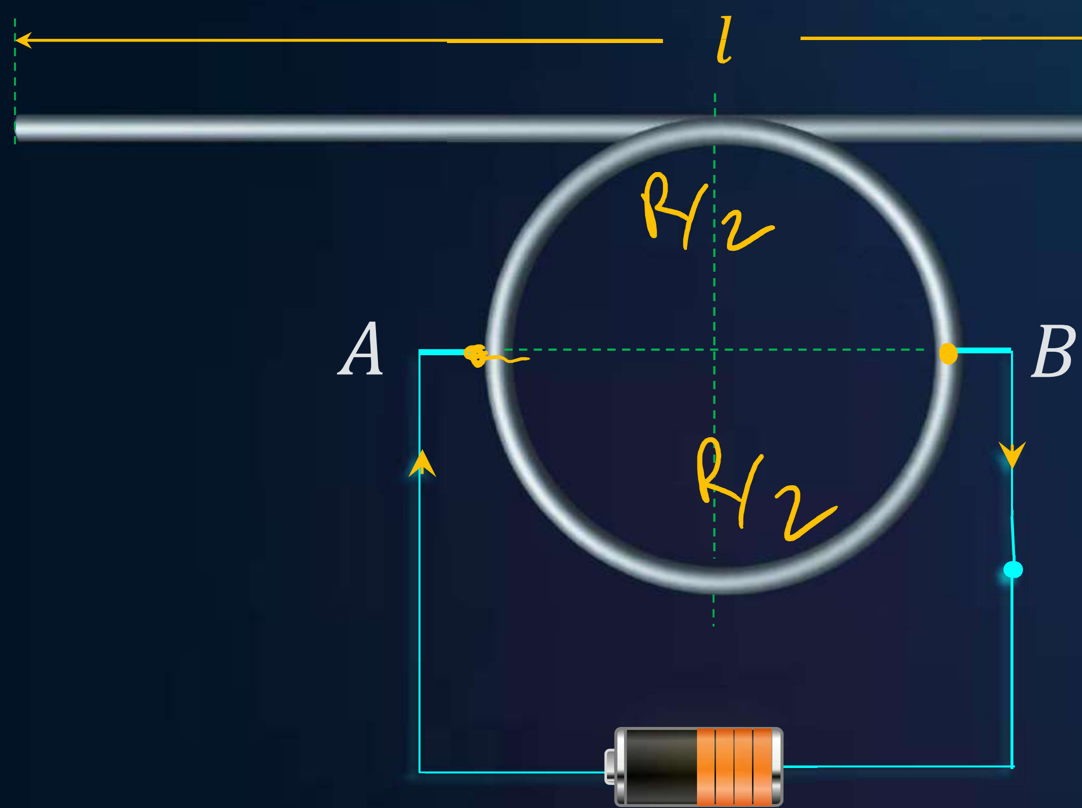
a) $R/4$

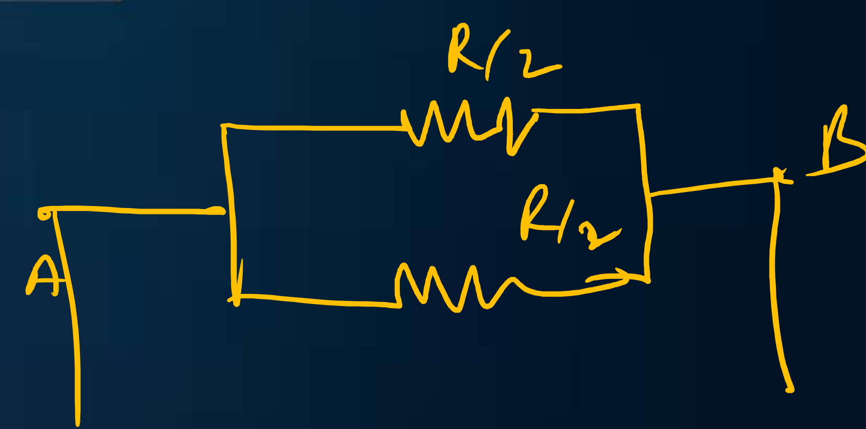
b) $4R$

c) $R/8$

d) $R/2$

Discussion





$$\frac{R}{n}$$

$$R_{eq} = \frac{R}{2 \times 2} = \frac{R}{4}$$

Here, the wire can be considered as parallel combination as shown in the adjacent figure.

Thus, option (a) is the correct answer.

Question

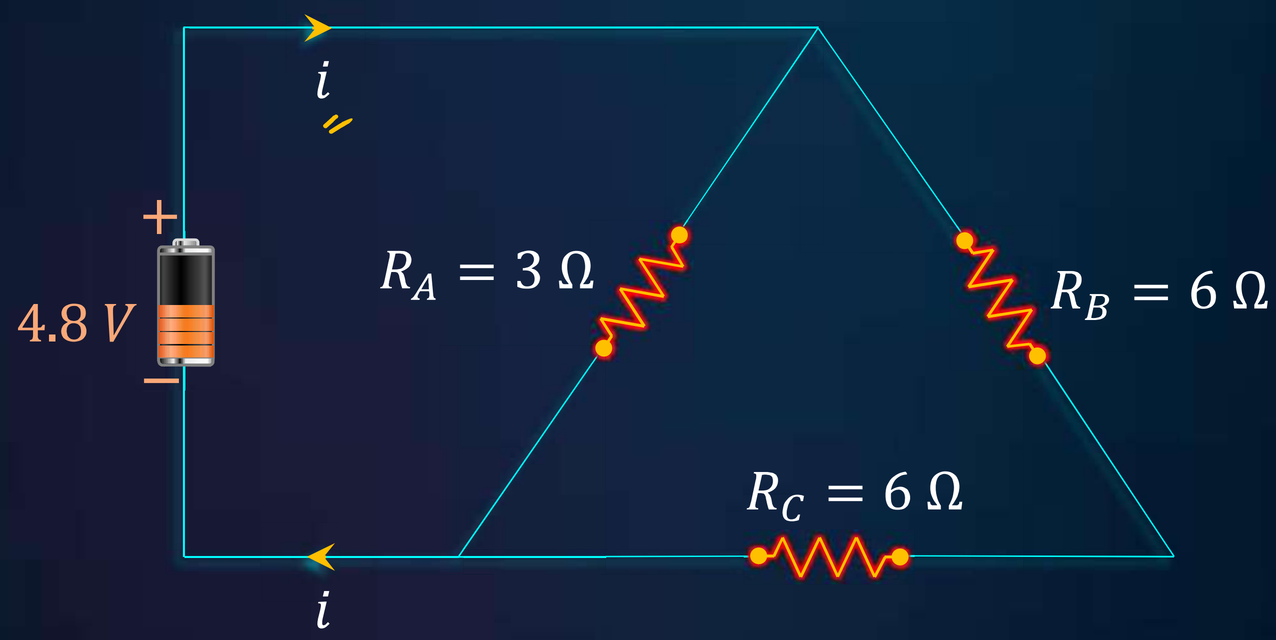
The current (i) in the given circuit is

a) $4.9\ A$

b) $6.8\ A$

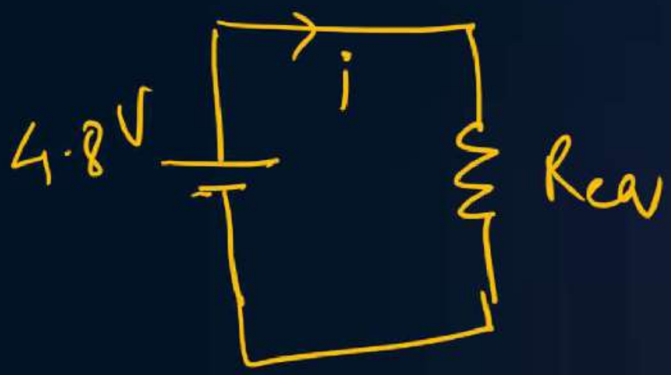
c) $8.3\ A$

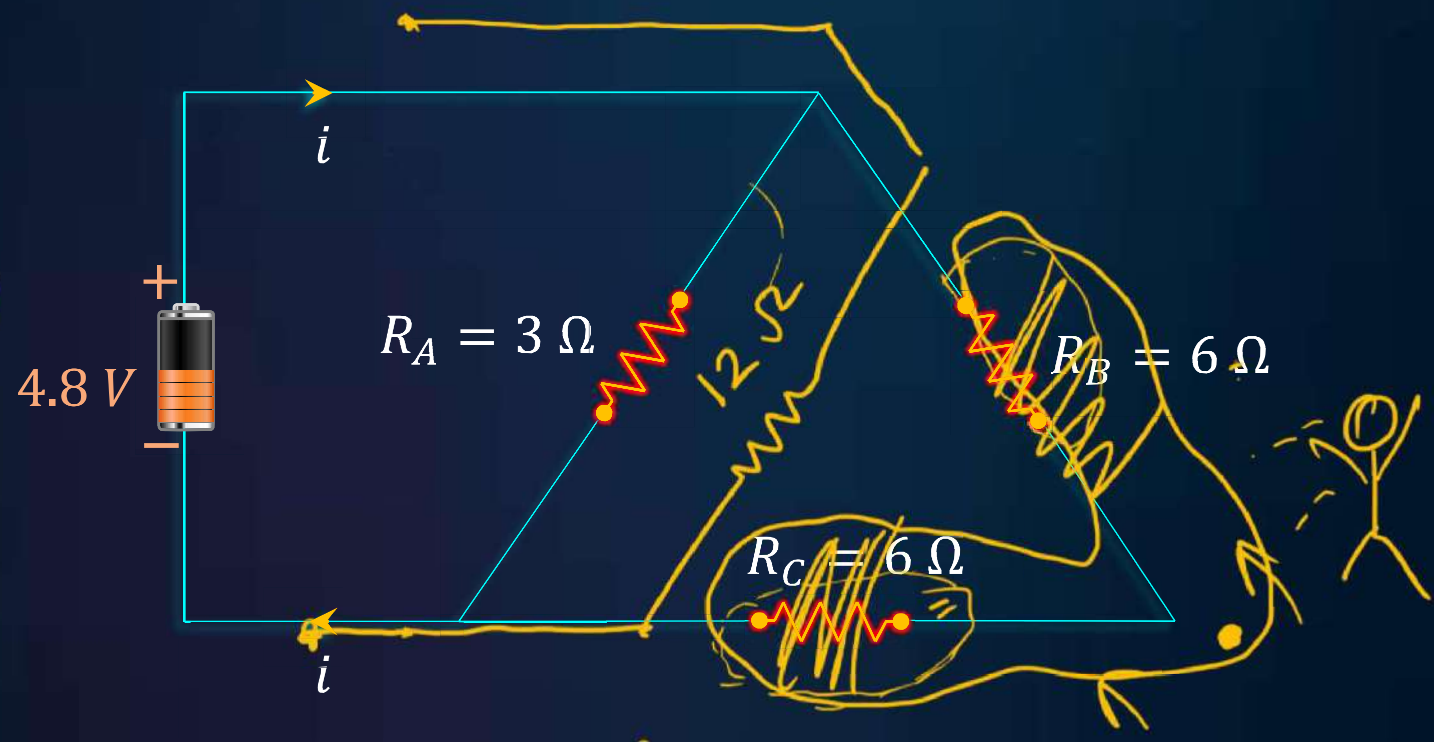
d) $2.0\ A$



Discussion

The given circuit can be replaced in form of single equivalent resistor as shown below.





$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 12}{3 + 12} = \frac{12}{5}\ \Omega$$

$$i = \frac{V}{R_{eq}} = \frac{4.8 \times 5}{12} = 2\ A$$

Thus, option (d) is the correct answer.

Question

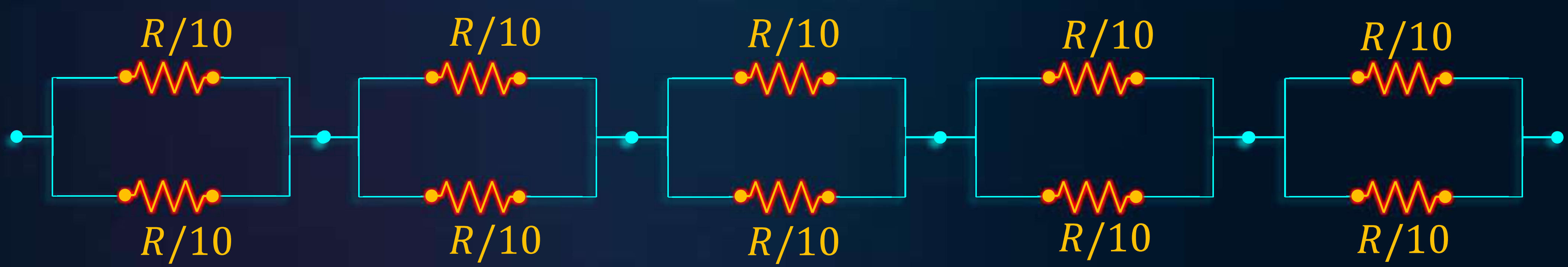
A wire of resistance R is divided in 10 equal parts. Two pieces each are connected in parallel and then such five such combinations are connected in series. The total resistance of the system will be

a) R

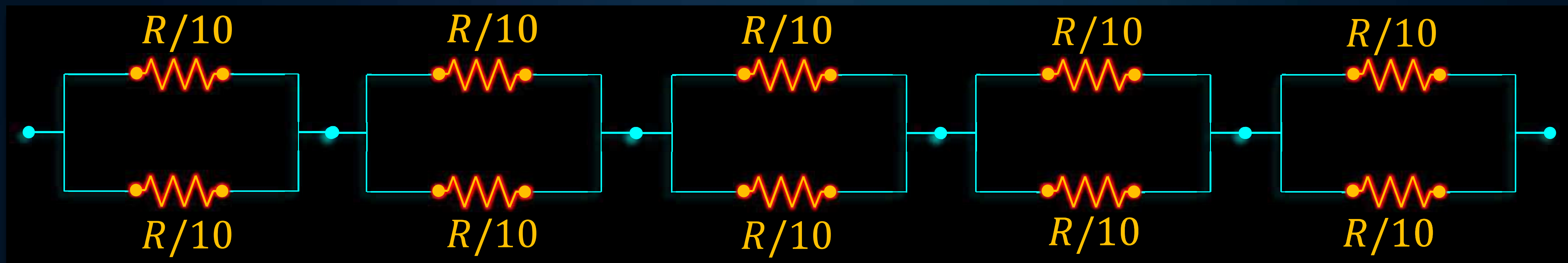
b) $R/4$

c) $R/10$

d) $R/20$



Discussion



Here, equivalent resistance of each parallel loop is:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{(R/10)\times(R/10)}{\left(\frac{R}{10}\right)+)\times\left(\frac{R}{10}\right)}$$

$$= \frac{R}{20}\ \Omega$$

$$R_{eq} = \cancel{5} \times \frac{R}{\cancel{20}\ 4} = \frac{R}{4}\ \Omega$$

Thus, option (b) is the correct answer.

Question

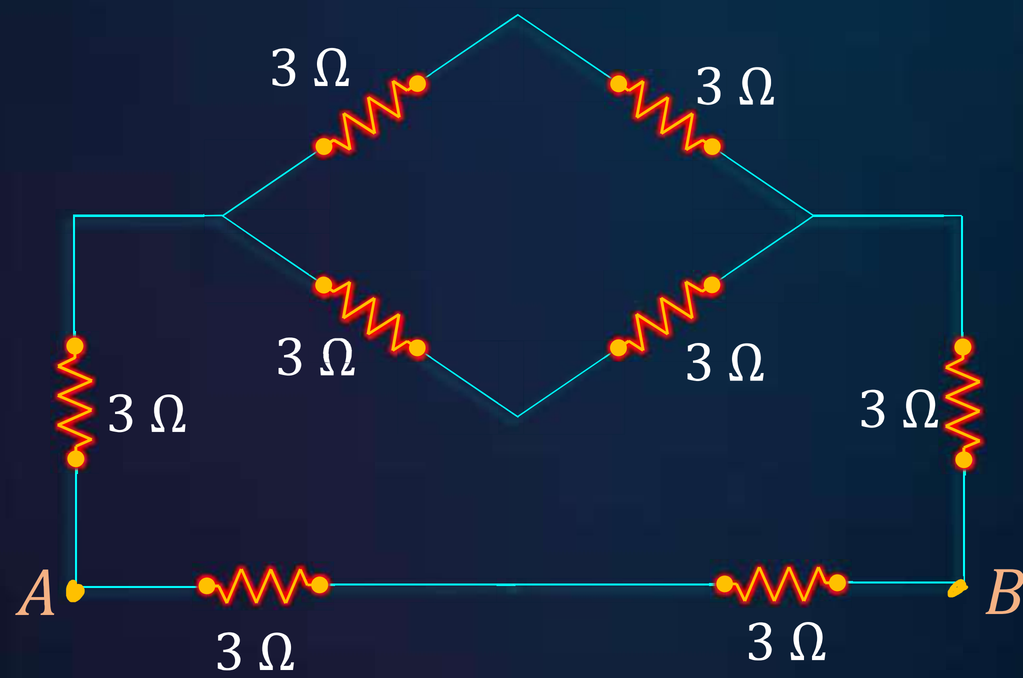
Equivalent resistance between *A* and *B* will be

a) $2\ \Omega$

b) $18\ \Omega$

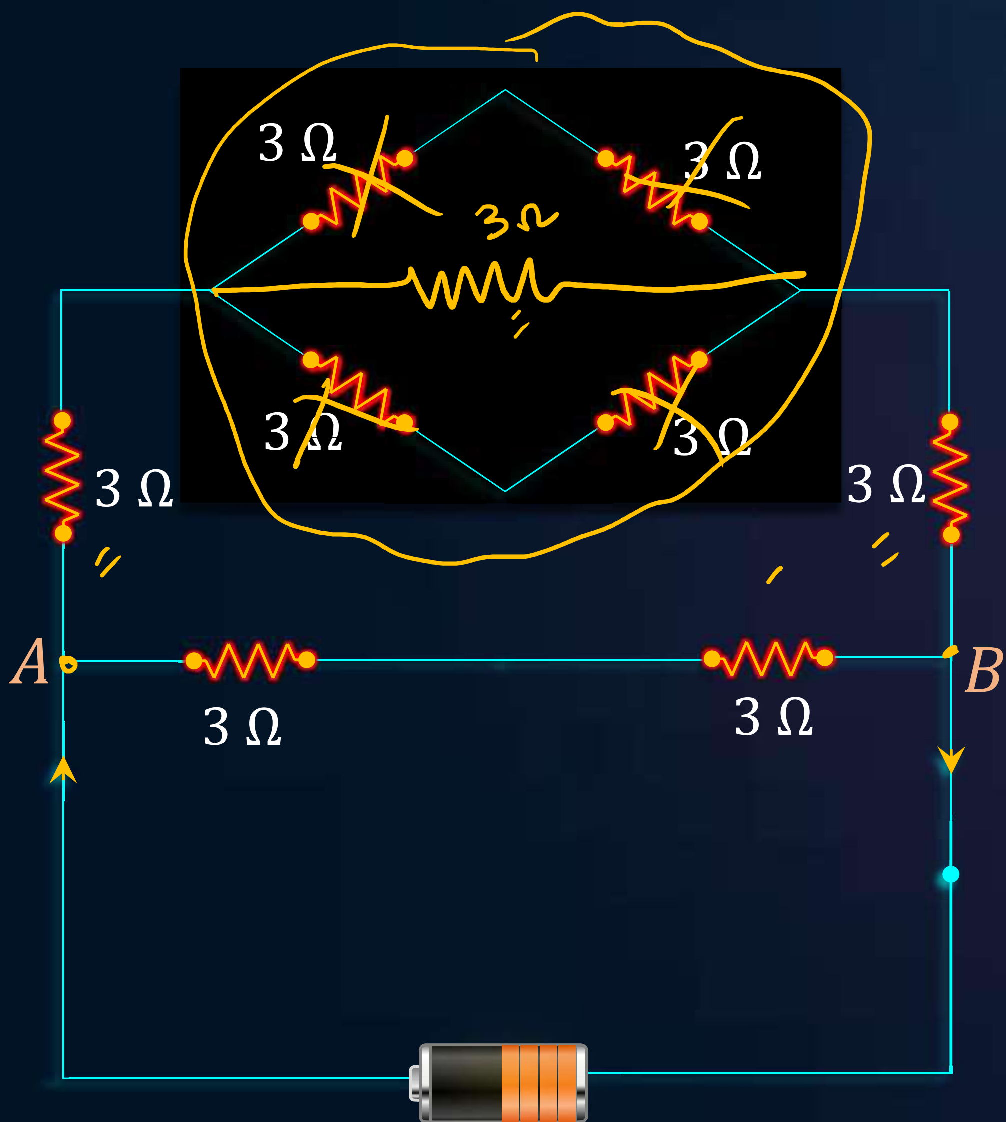
c) $6\ \Omega$

d) $3.6\ \Omega$



Discussion

The given circuit can be replaced in form of a simple loop as shown below.



$$R_{eq} = \frac{6 \times 9}{6+9} = 3.6\ \Omega$$

Thus, option (d) is the correct answer.

Question

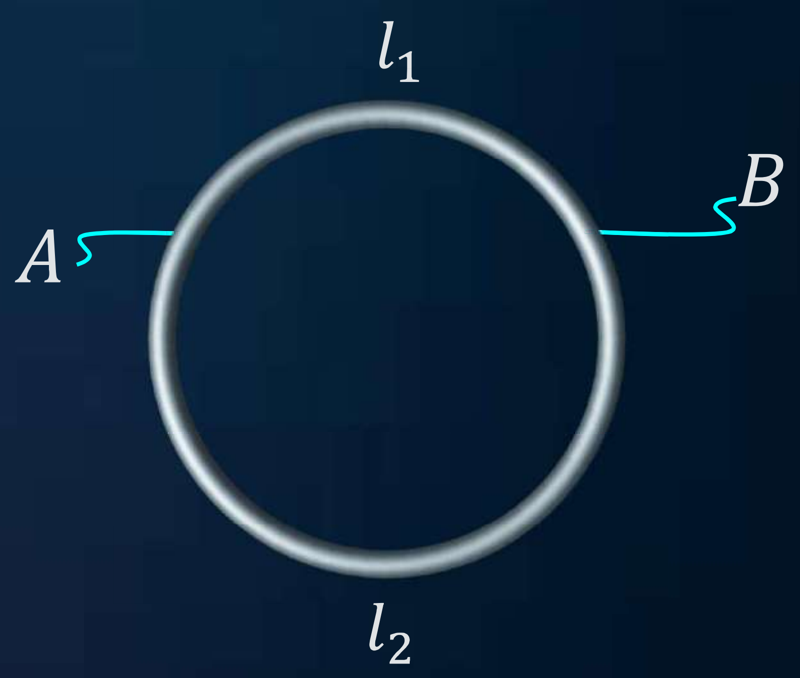
A ring is made of a wire having a resistance $R_0 = 12\ \Omega$. Find the points A and B , as shown in the figure, at which a current carrying conductor should be connected so that the resistance R of the sub circuit between these points is equal to $8/3\ \Omega$.

a) $\frac{l_1}{l_2} = \frac{5}{8}$

b) $\frac{l_1}{l_2} = \frac{1}{3}$

c) $\frac{l_1}{l_3} = \frac{3}{8}$

d) $\frac{l_1}{l_2} = \frac{1}{2}$



Discussion

NEET

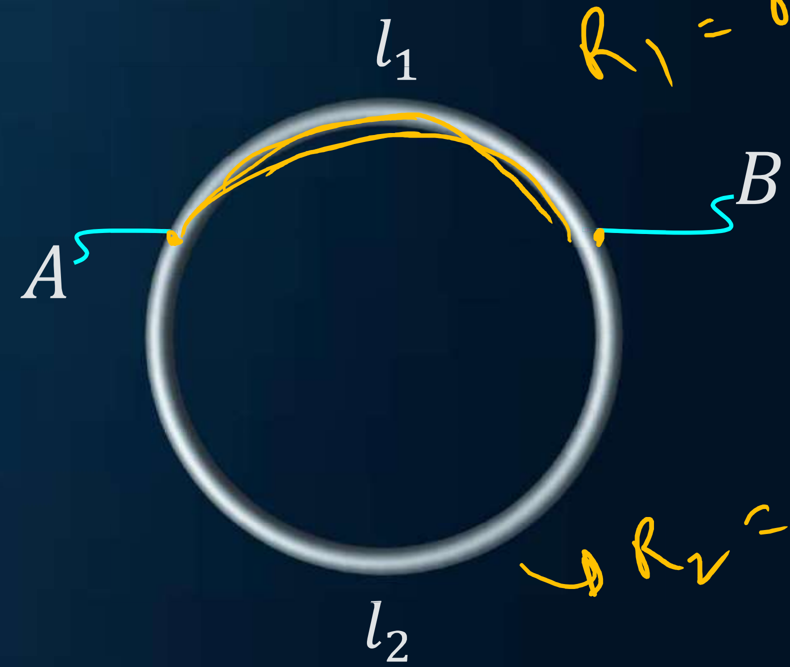
$$R_0 = R_1 + R_2 = 12\ \Omega \quad \text{—— (1)}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{8}{3}\ \Omega$$

$$R_1 R_2 = 32\ \Omega \quad \text{—— (11)}$$

$$R_1 + R_2 = 12\ \Omega \quad \text{—— (1)}$$

$$R_1 = r l_1 \ ; \ R_2 = r l_2$$



$R_1 = r l_1$

$R_2 = r l_2$

$R_2 > R_1$

$l_2 > l_1$

Discussion

$R_1 + R_2 = 12\,\Omega\;....(1)$ $\qquad$ $R_1 R_2 = 32\,\Omega\;....(2)$

4 $\qquad$ 8

$(A-B)^2 = (A+B)^2 - 4AB$

$(R_2 - R_1)^2 = (R_2 + R_1)^2 - 4(R_2 R_1)$

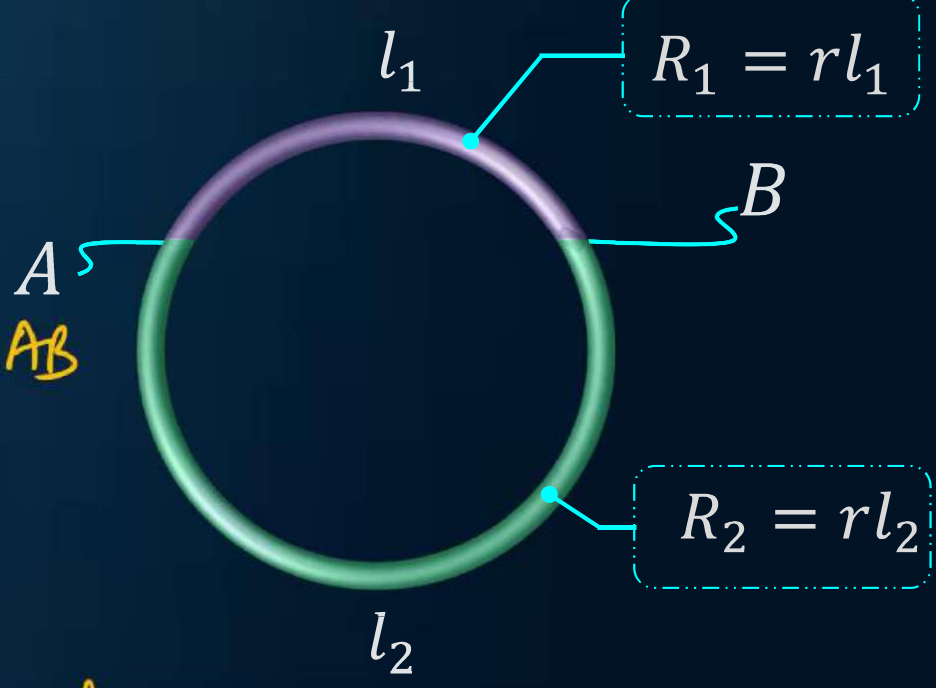
$(R_2 - R_1)^2 = (12)^2 - 4(32)$

$R_2 - \cancel{R_1} = 4 \quad \text{———} \quad (3)$

$\cancel{R_1} + R_2 = 12 \quad \text{———} \quad (1)$

$2R_2 = 16$

$R_2 = 8\,\Omega$



$\frac{R_1}{R_2} = \frac{\cancel{r}l_1}{\cancel{r}l_2} = \frac{\cancel{4}}{\cancel{8}} \quad \frac{1}{2}$

$\frac{l_1}{l_2} = \frac{1}{2}$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{1}{2}$$

Thus, option (d) is the correct answer.

Question

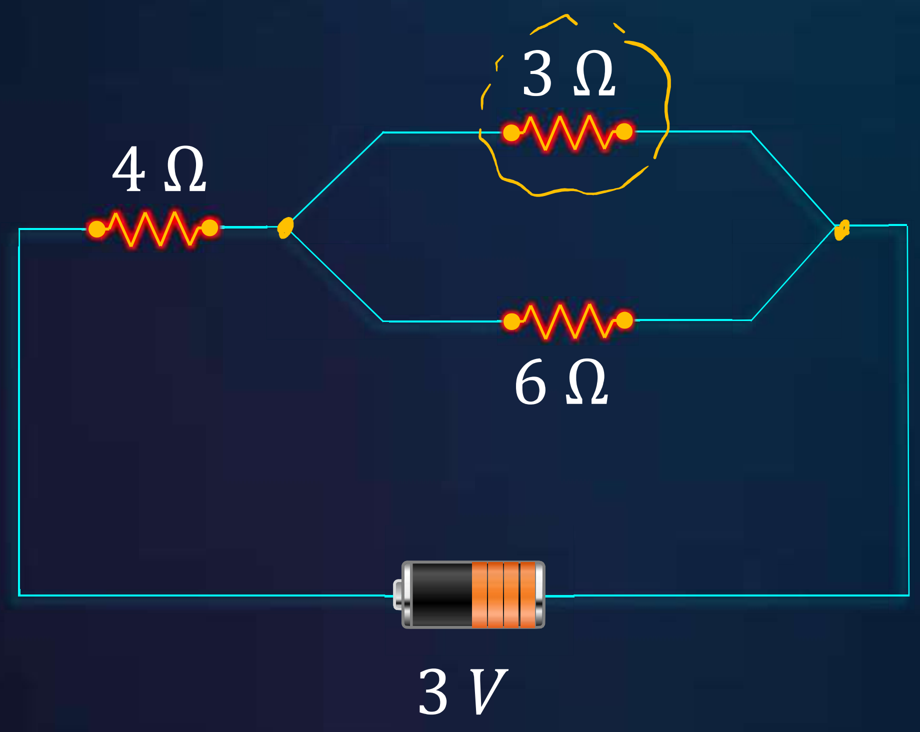
The potential drop across the 3 Ω resistor is

a) $1\ V$

b) $1.5\ V$

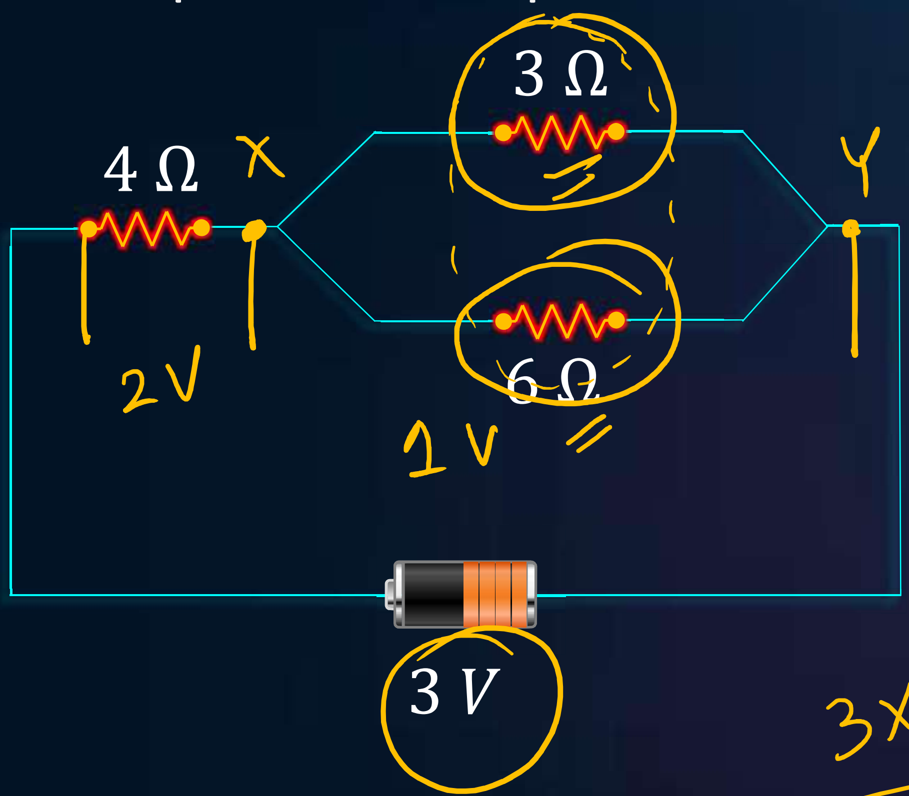
c) $2\ V$

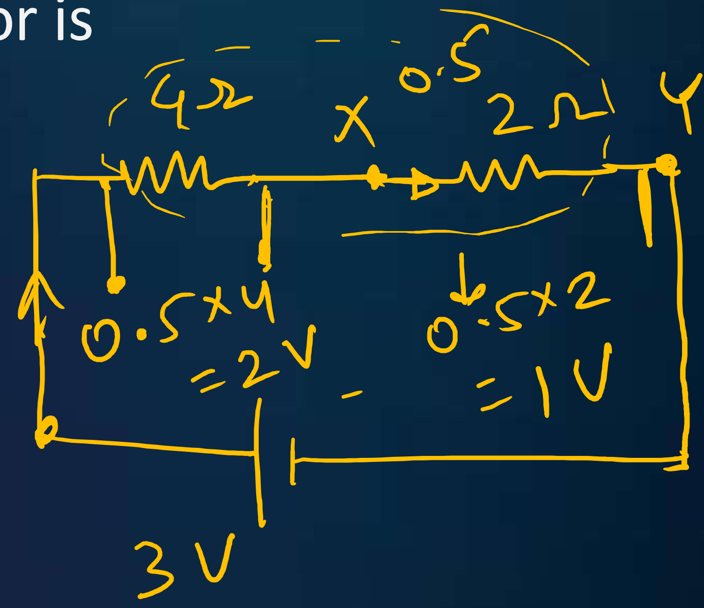
d) $3\ V$



Discussion

The potential drop across the 3 Ω resistor is





$$i = \frac{V}{R} = \frac{3}{6} = 0.5A$$

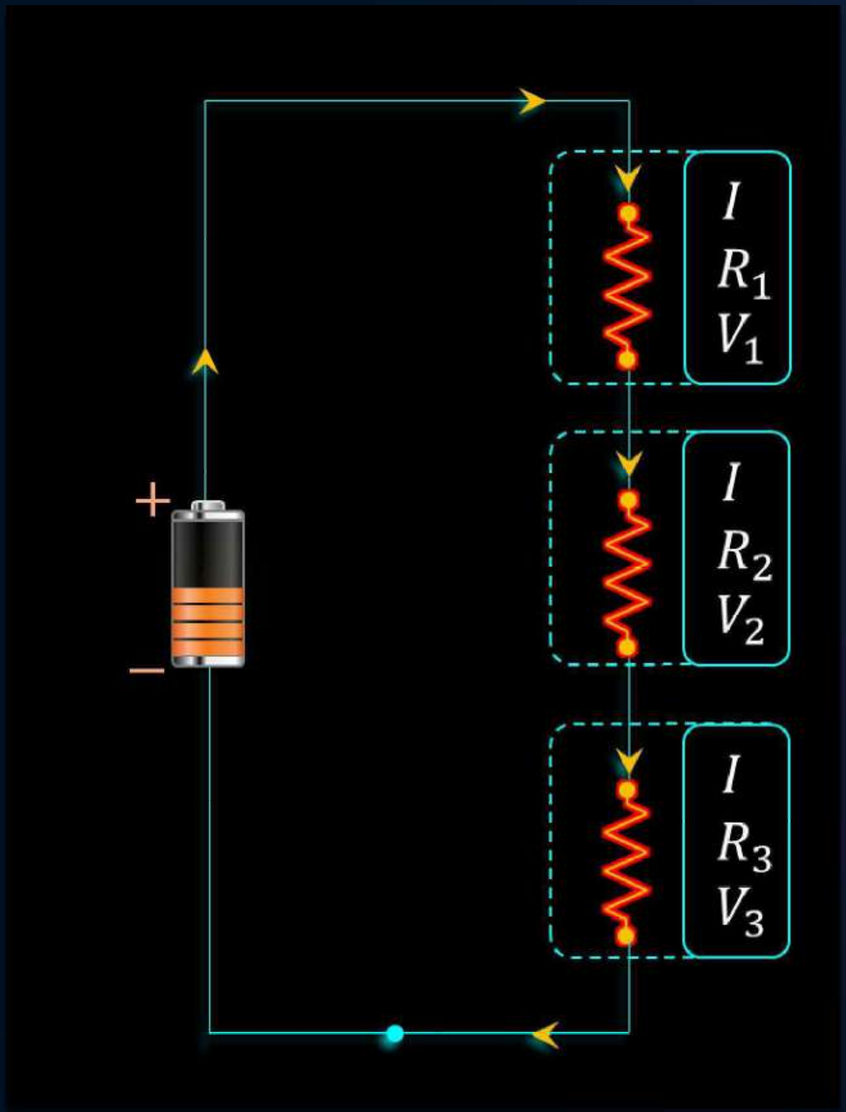
$$\frac{3\times 6}{3+6} = \frac{18}{9} = 2\ \Omega$$

Potential drop , $V_2 = 0.5 \times 2 = 1\ V$

Thus, option (a) is the correct answer.

Recap

Series combination:



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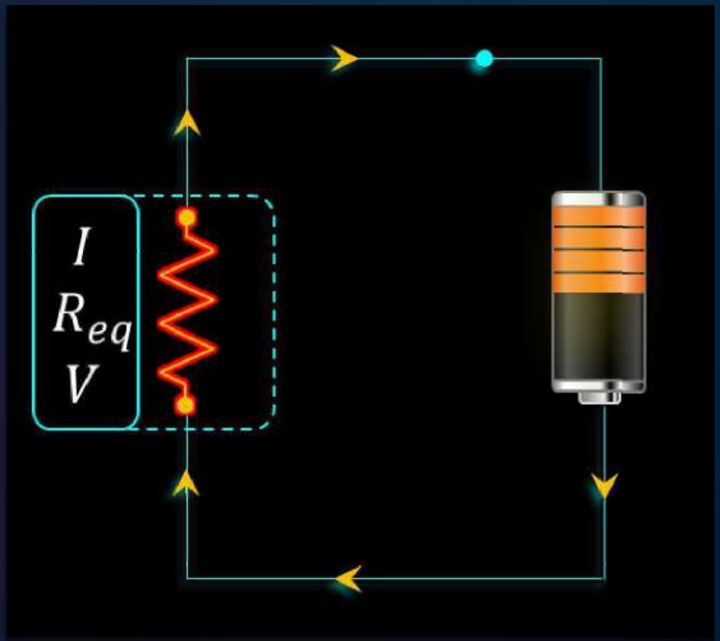


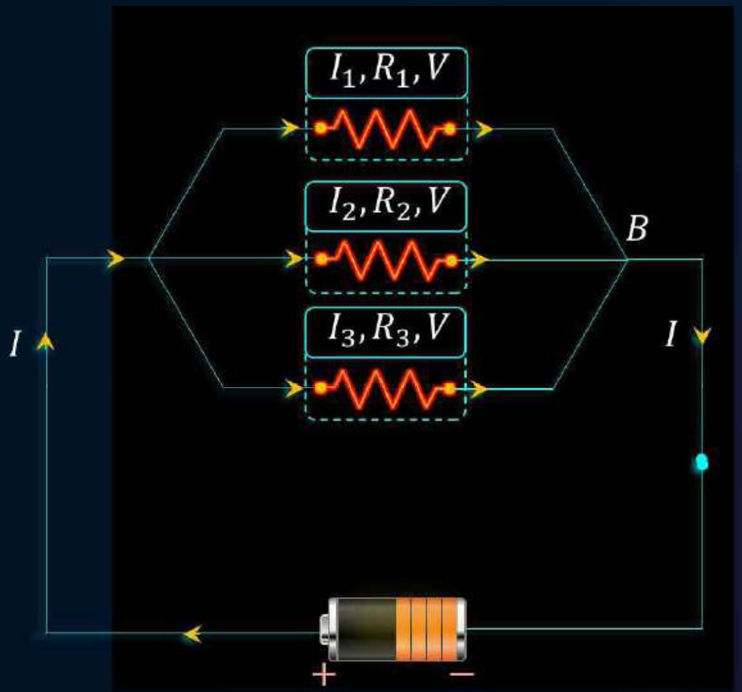
- $V = V_1 + V_2 + \cdots$

$$IR_{eq} = IR_1 + IR_2 + IR_3$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

Recap

Parallel combination:



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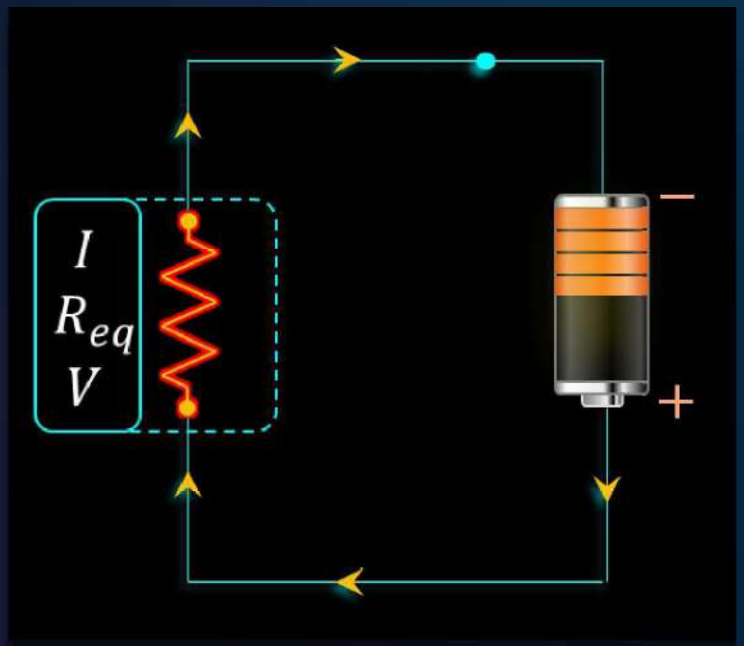


- $I = I_1 + I_2 + I_3$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

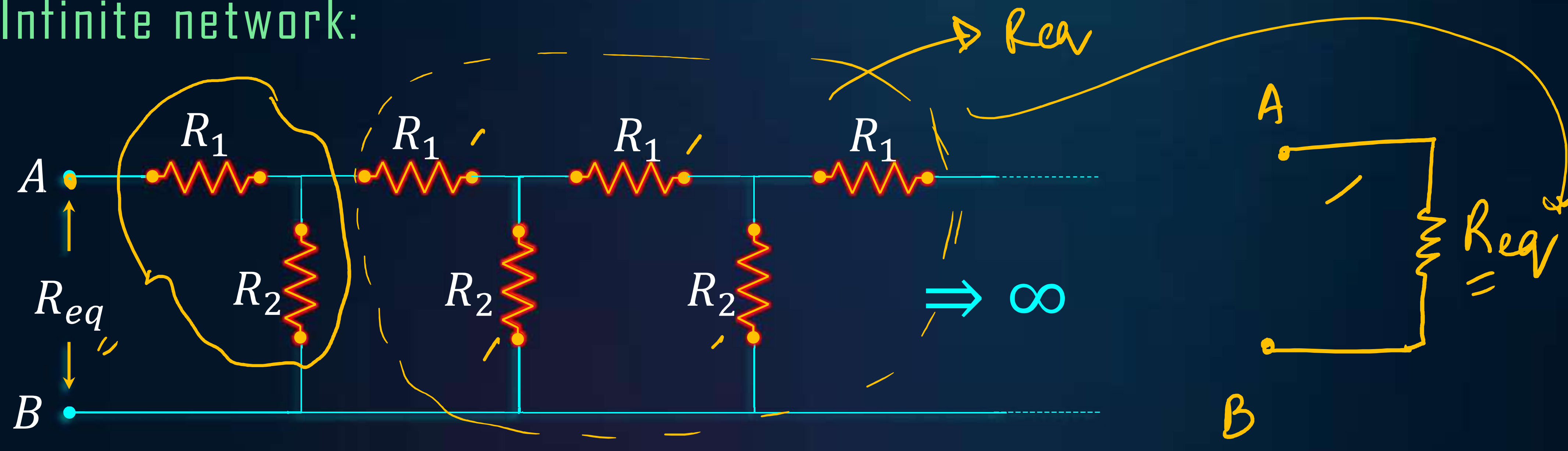
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

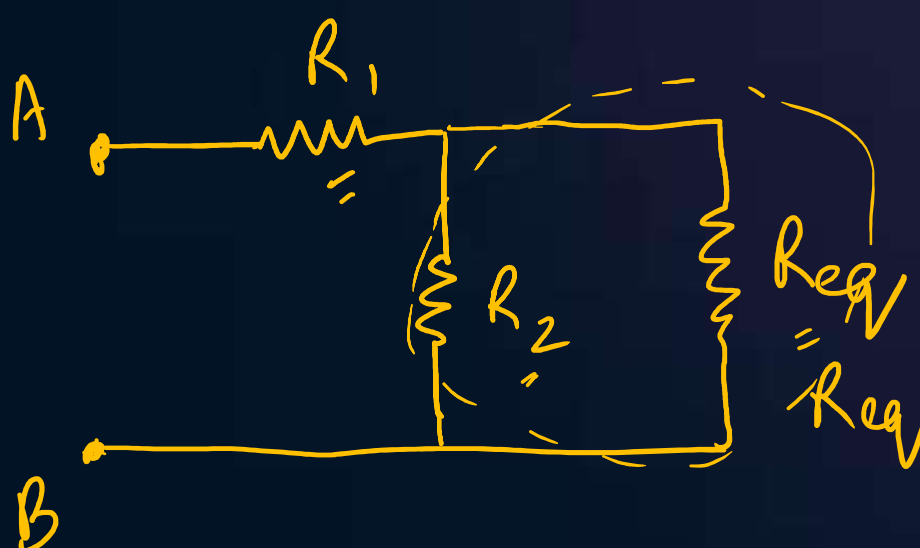
- If only two resistances R_1 and R_2 are in parallel combination, then equivalent resistance will be $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

- If there are n number of resistances of resistance R are in parallel combination, then equivalent resistance will be $R_{eq} = \frac{R}{n}$

Combination of Resistors

Infinite network:

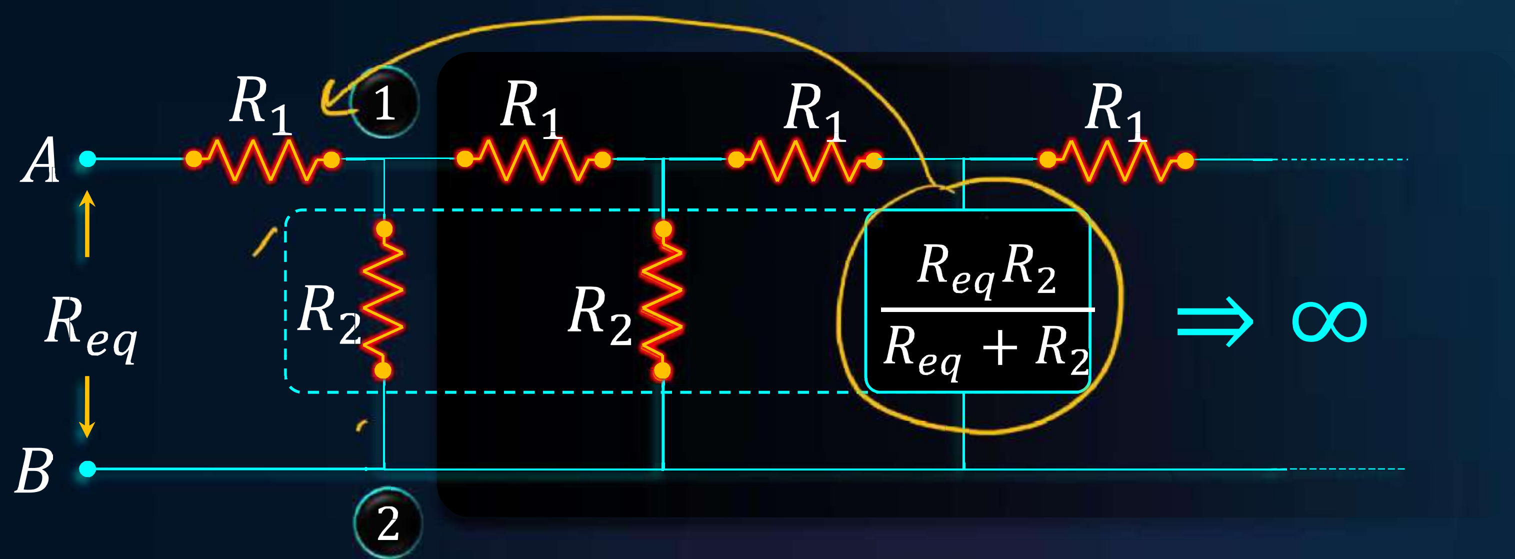


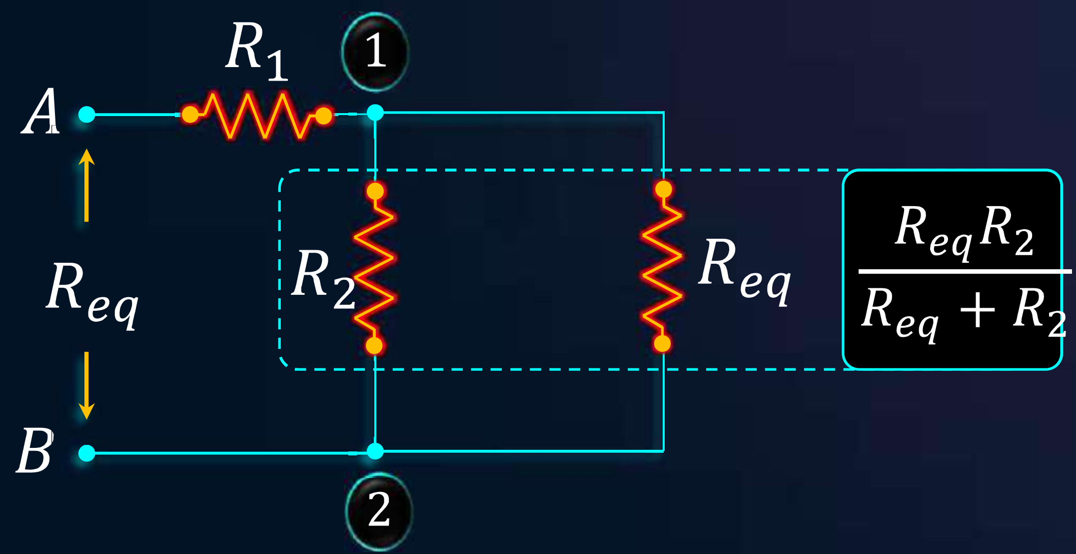


$$R_{eq} = \frac{R_2 R_{eq}}{R_2 + R_{eq}} + R_1$$

- First identify the repeating pattern. Then, by keeping one such pattern, replace the rest of the network by an equivalent resistance R_{eq} as shown. After that, by solving a quadratic equation of R_{eq}, we can find the equivalent resistance.

Infinite network:





$$R_1 + \frac{R_{eq}R_2}{R_{eq}+R_2} = R_{eq}$$

$$R_1(R_{eq}+R_2) + R_{eq}R_2 = R_{eq}(R_{eq}+R_2)$$

$$R_{eq}^2 - R_{eq}R_1 - R_1R_2 = 0$$

$$\Rightarrow R_{eq} = \frac{R_1 \pm \sqrt{R_1^2 + 4R_1R_2}}{2}$$

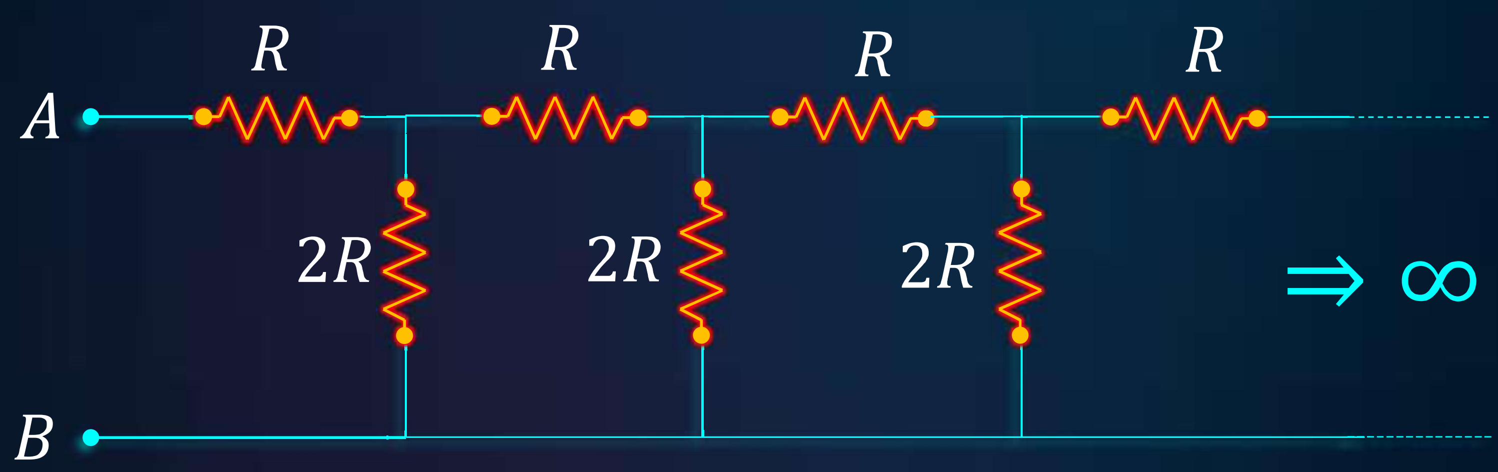
$$\frac{R_1\left(1+\sqrt{1+4R_2/R_1}\right)}{2}$$

$$\frac{R_1\left(1-\sqrt{1+4R_2/R_1}\right)}{2}$$

(Not possible as resistance can't be negative)

Question

An infinite ladder network is arranged with resistances R and $2R$ as shown. The effective resistance between terminals A and B is



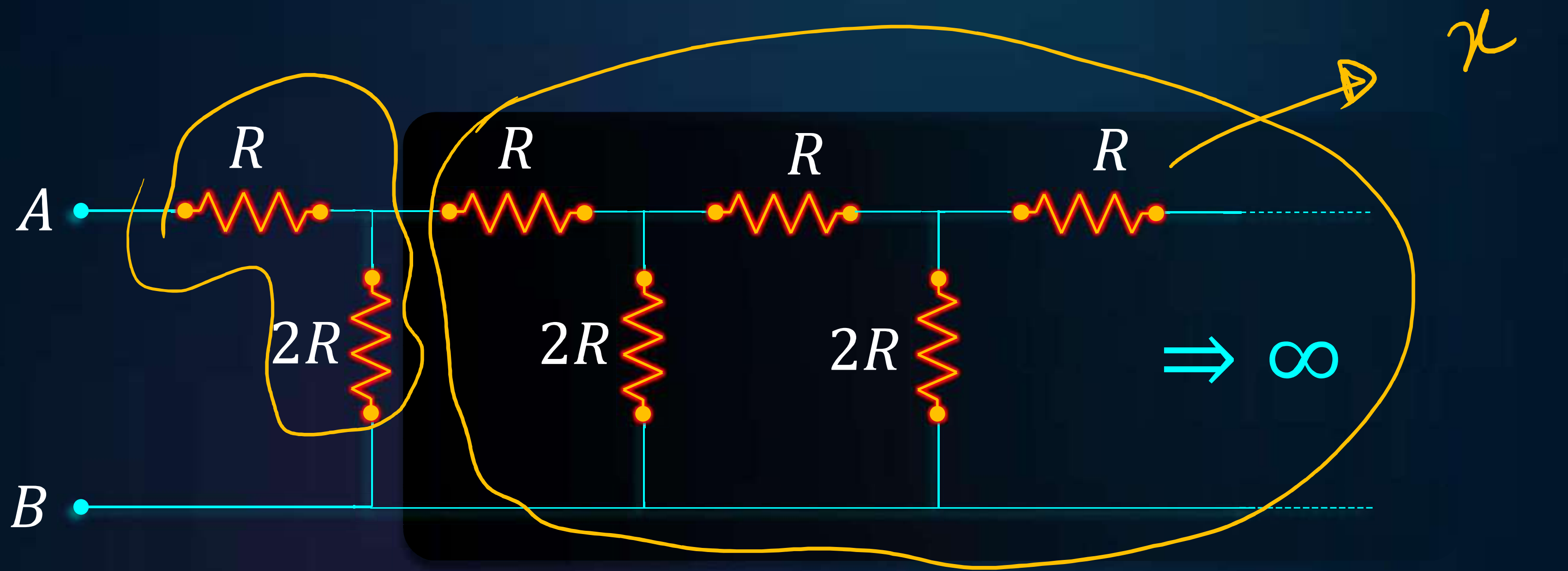
a) ∞

b) R

c) $2R$

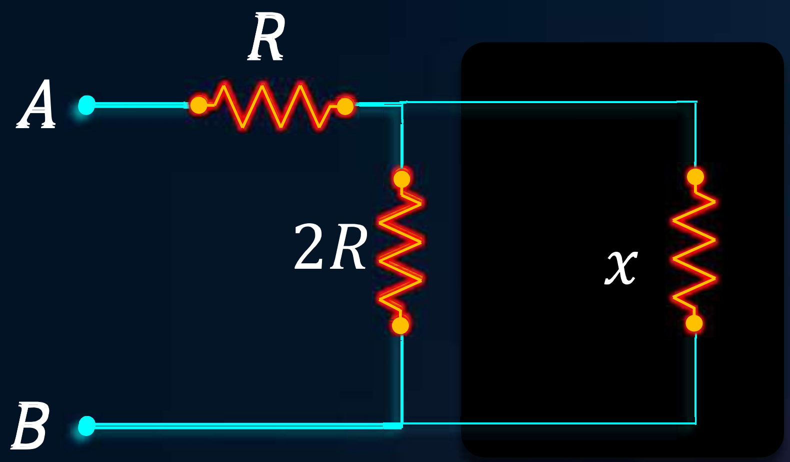
d) $3R$

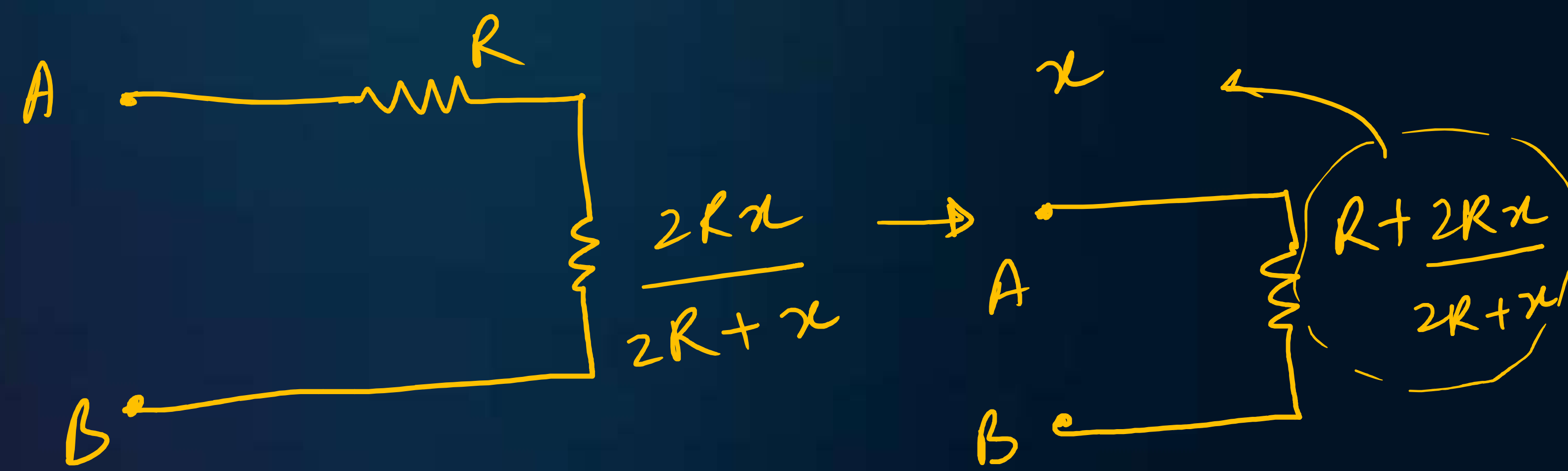
Discussion



$$\frac{2Rx}{2R+x} + R = x$$

Discussion





$$x = R + \frac{2Rx}{2R+x}$$

$$x = \frac{R(2R+x) + 2Rx}{(2R+x)}$$

$$x^2 - xR - 2R^2 = 0$$

$$x = \frac{R \pm \sqrt{R^2 - 8R^2}}{2}$$

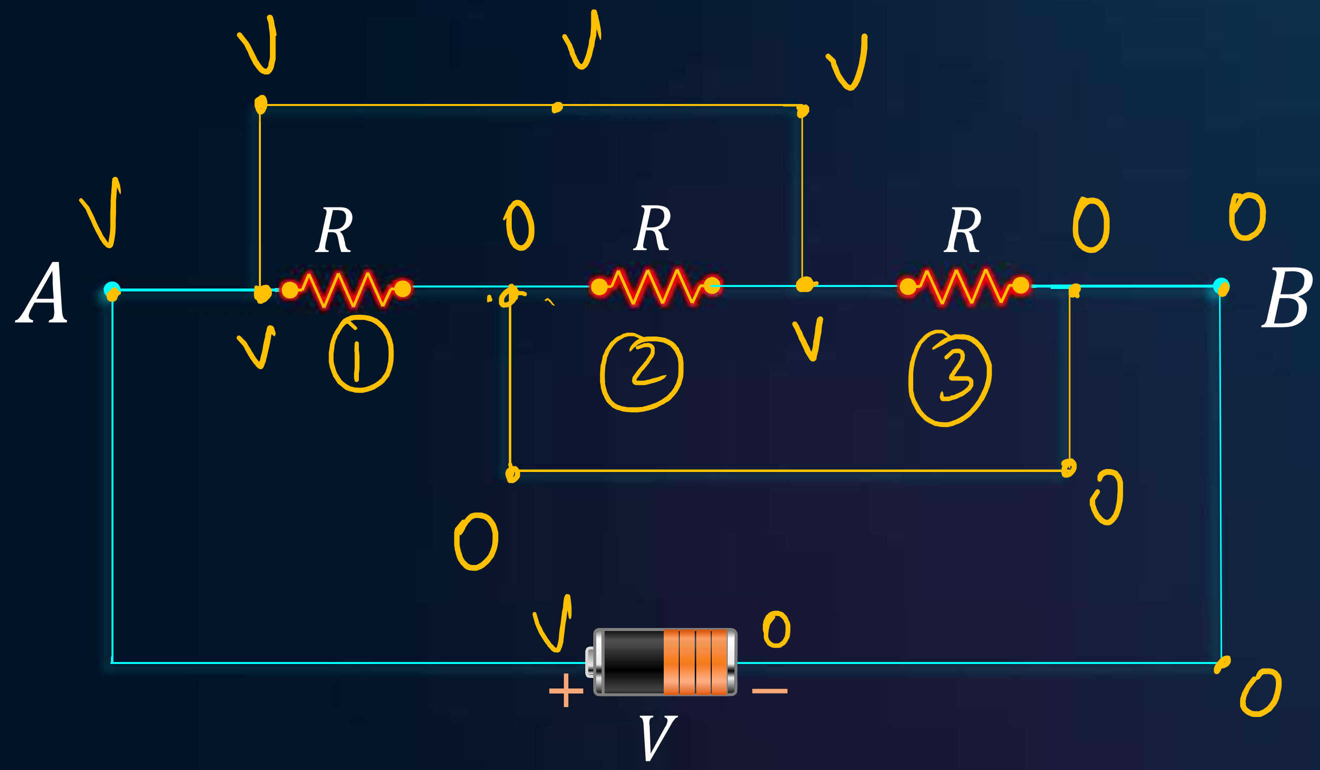
$$x = \frac{R \pm \sqrt{R^2 + 8R^2}}{2} = \frac{4R}{2} \; or \; \cancel{\left(-\frac{2R}{2}\right)}$$

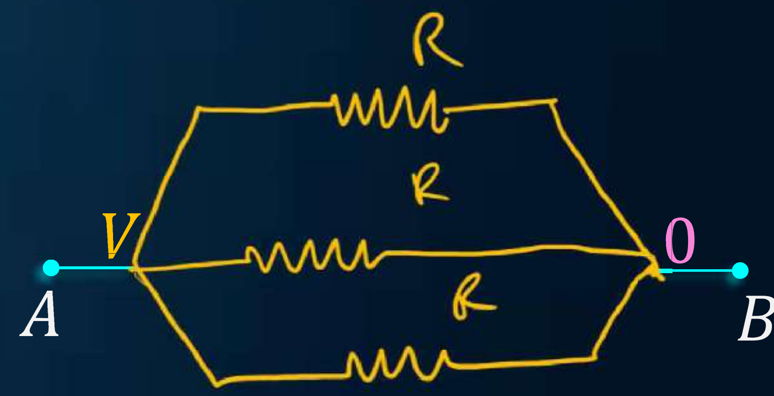
$$x = 2R$$

Thus, option (c) is the correct answer.

Combination of Resistors

Potential method





$$R_{eq} = R/3$$

- Potential across the connecting wires does not change as we assume the wires are of negligible resistance.

Question

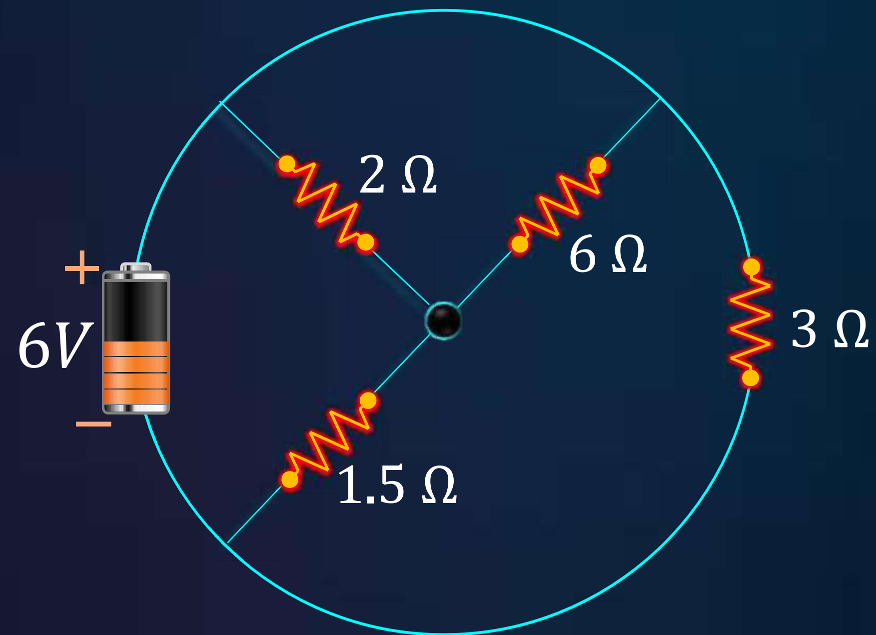
The total current supplied to the circuit by the battery is

a) $1\ A$

b) $2\ A$

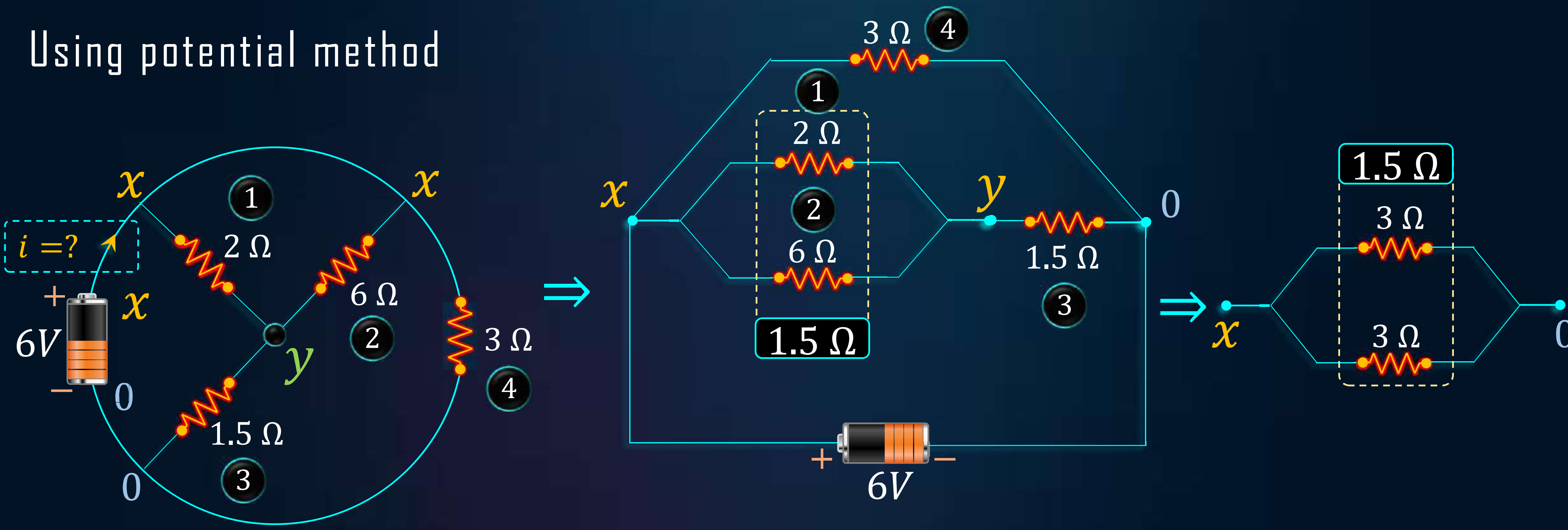
c) $4\ A$

d) $6\ A$



Discussion

Using potential method

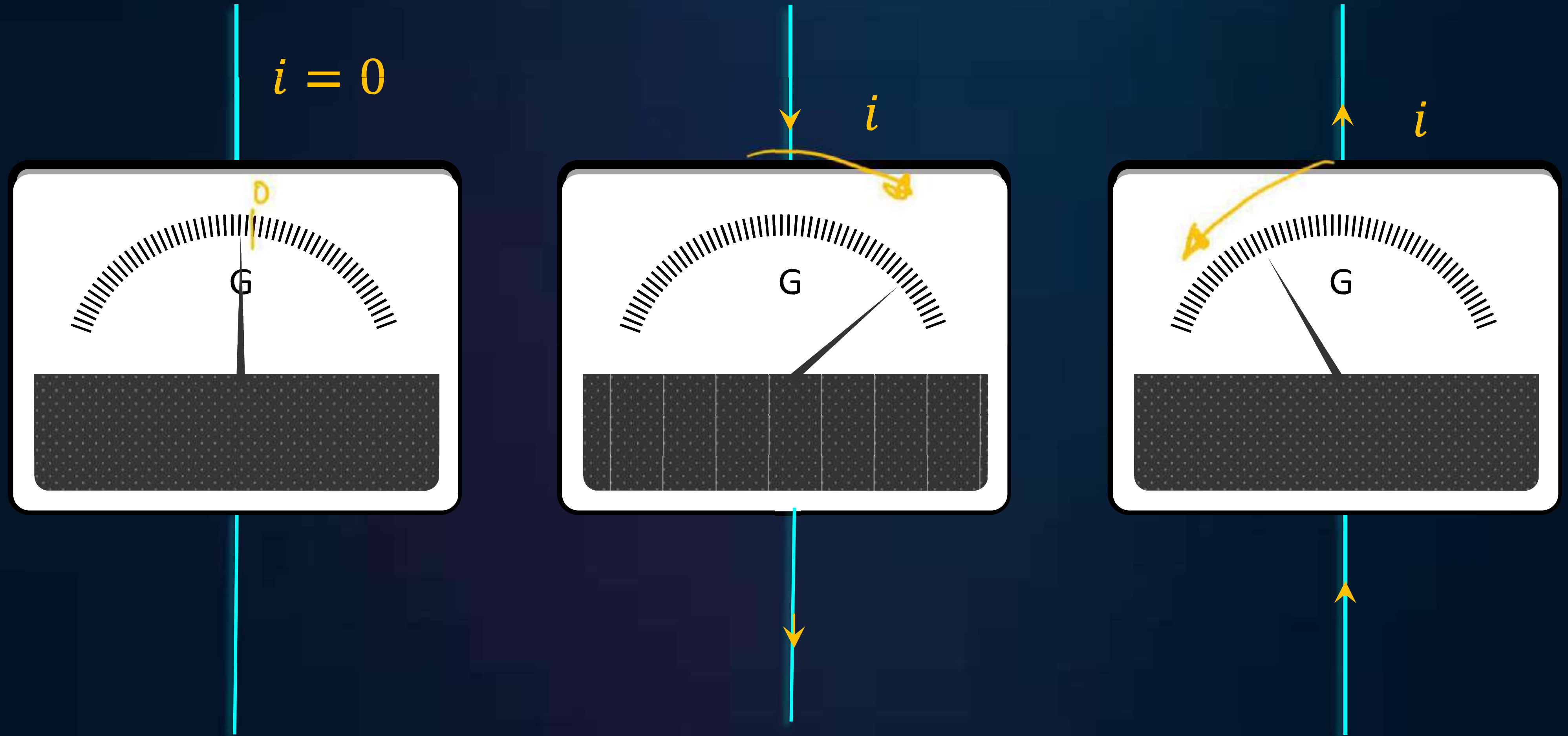


$$i = \frac{V}{R_{eq}} = \frac{6}{1.5} = 4\ A$$

Thus, option (c) is the correct answer.

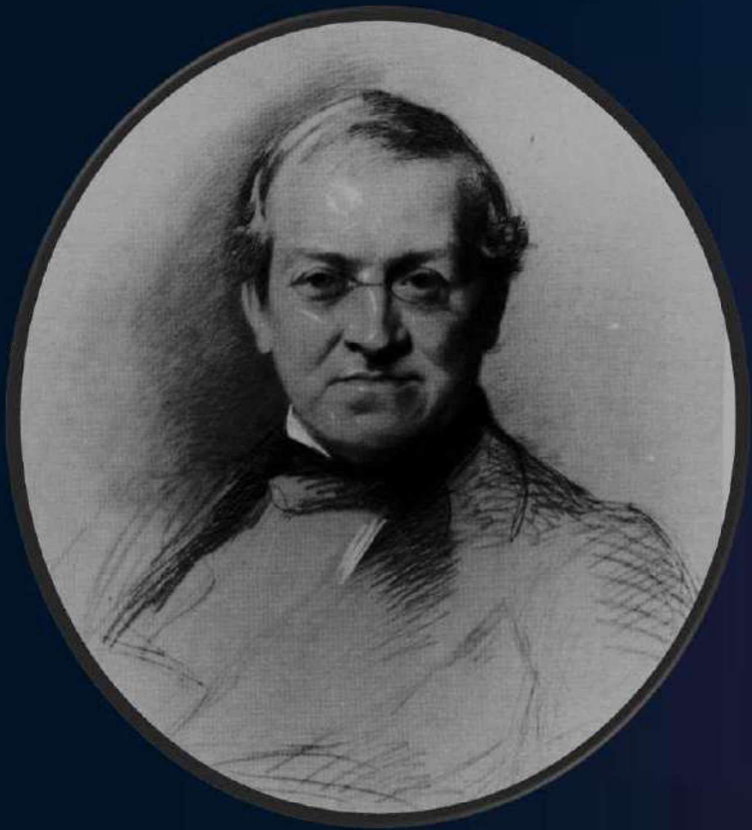
Galvanometer

A device used to detect the presence of current, the direction of flow and compare the magnitudes of two currents.

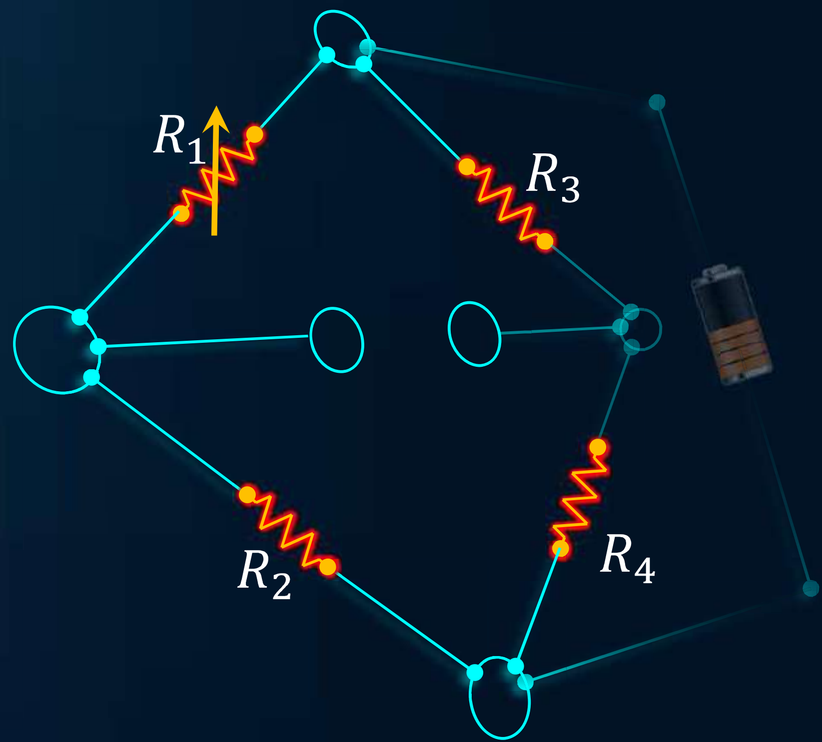


Angle of deflection $(\theta) \propto$ Amount of current (i) flowing through it

Wheatstone Bridge



Charles Wheatstone
(1802-1875)

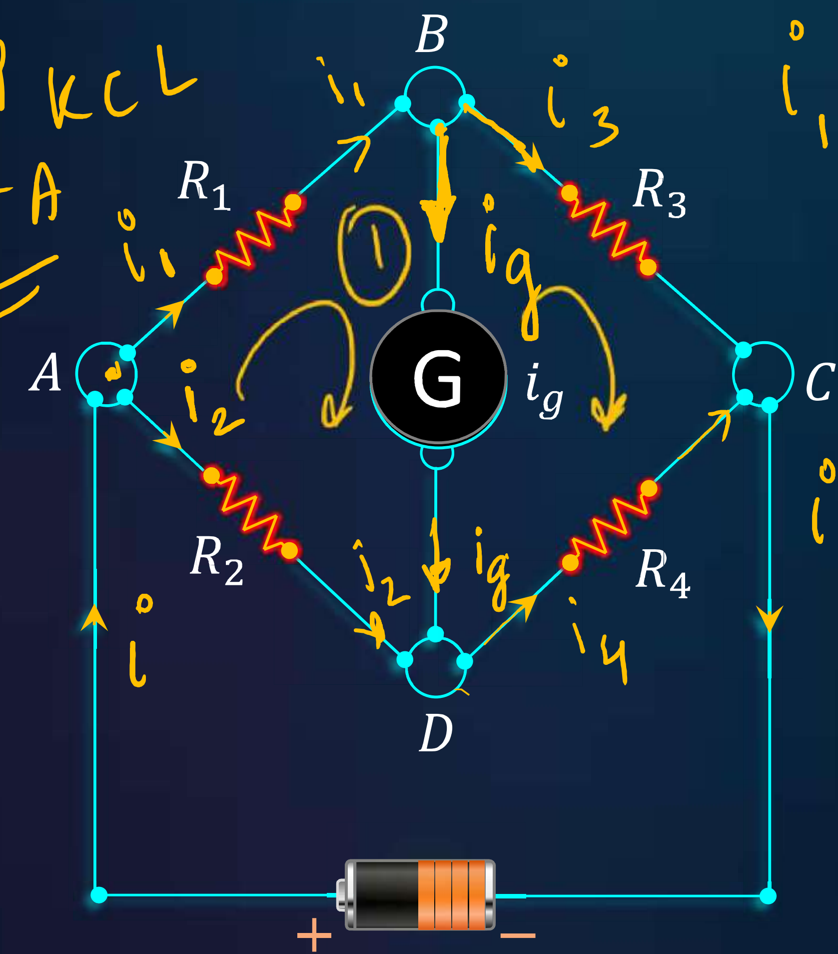


The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance

Wheatstone Bridge

$i = i_1 + i_2$

App KCL
at A

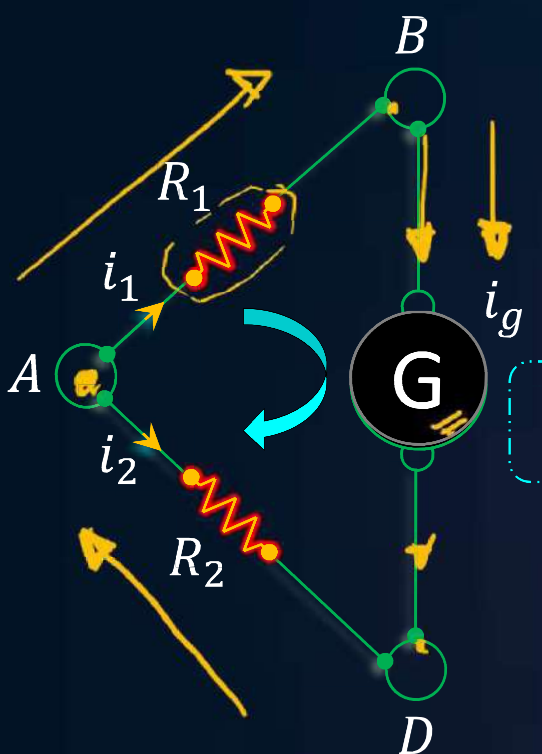


$i_1 = i_g + i_3$

$i_4 = i_2 + i_g$

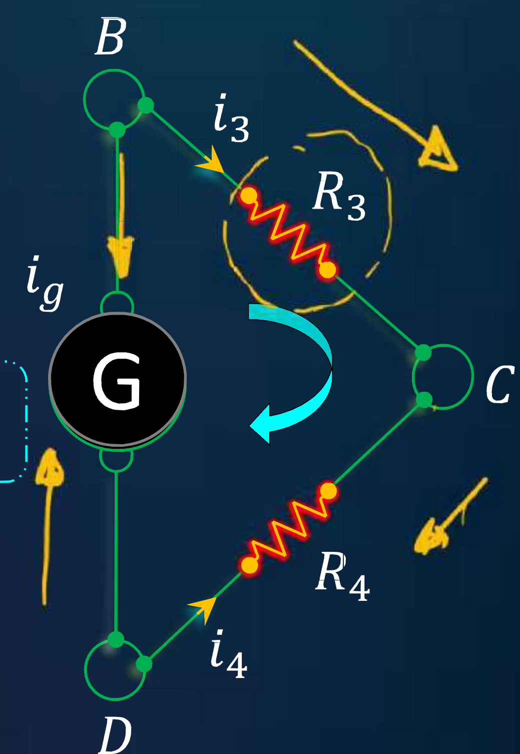
In the case of balanced Wheatstone bridge current through the galvanometer will be zero. Consider two loops *ABDA* and *BCDB* to apply Kirchhoff’s laws.

Wheatstone Bridge



Ref → G

Applying Kirchhoff's Rule



$i = i_1 + i_2$ — (3)

$i_1 = i_g + i_3$ — (4)

ABDA

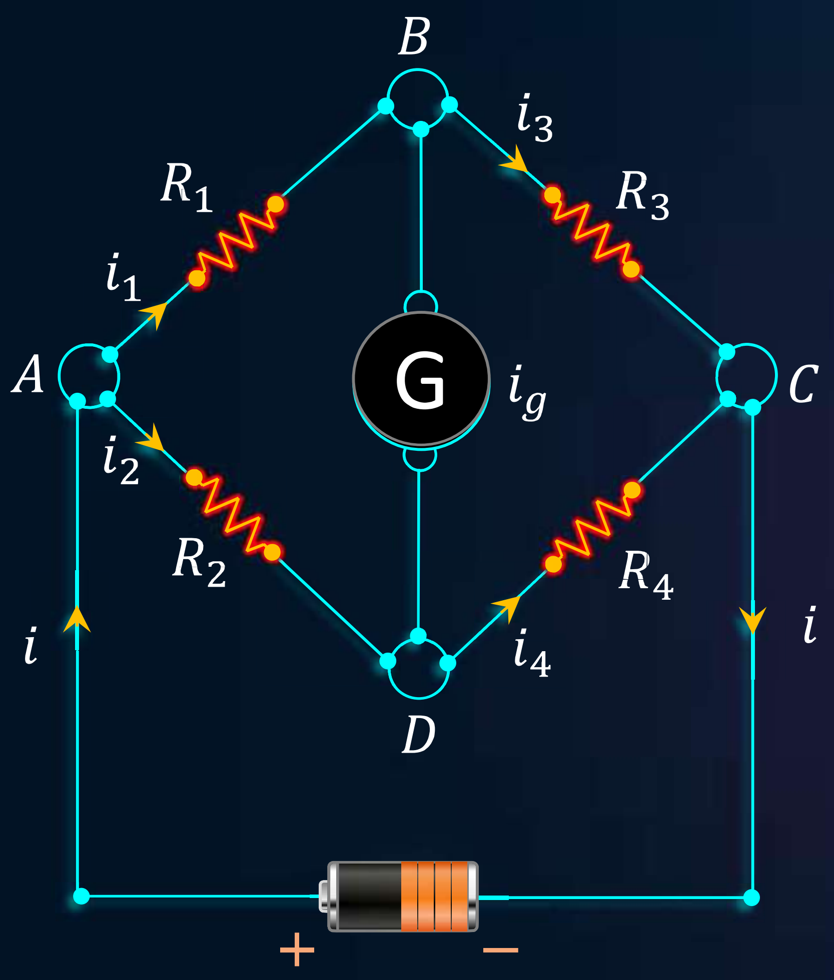
$-i_1 R_1 - i_g G + i_2 R_2 = 0$ — (1)

BCDB

$-i_3 R_3 + i_4 R_4 + i_g G = 0$ — (2)

(G = Resistance of the galvanometer)

Wheatstone Bridge



$G =$ Resistance of the galvanometer

Applying KCL at B :

$$i_1 = i_g + i_3 \ldots\ldots..(1)$$

Applying KCL at D :

$$i_2 + i_g = i_4 \ldots\ldots\ldots(2)$$

Applying KVL to $ABDA$

$$-i_1R_1 - i_gG + i_2R_2 = 0 \ldots\ldots.(3)$$

Applying KVL to $BCDB$

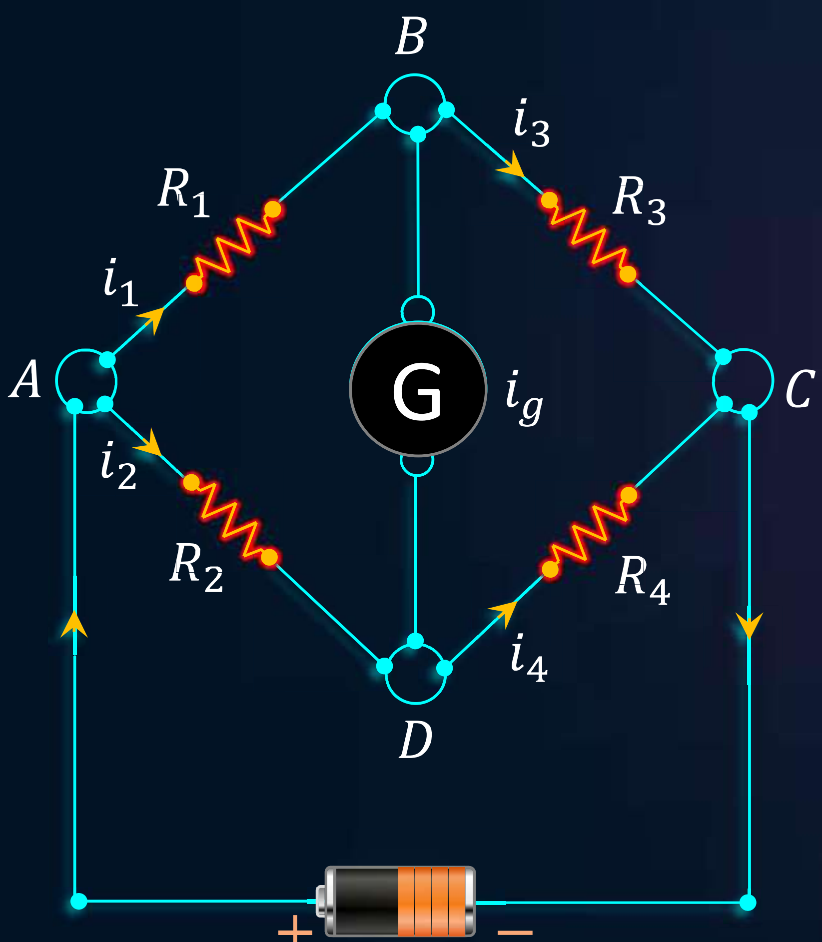
$$-i_3R_3 + i_4R_4 + i_gG = 0 \ldots\ldots\ldots(4)$$

When the Wheatstone bridge is balanced, $i_g = 0$

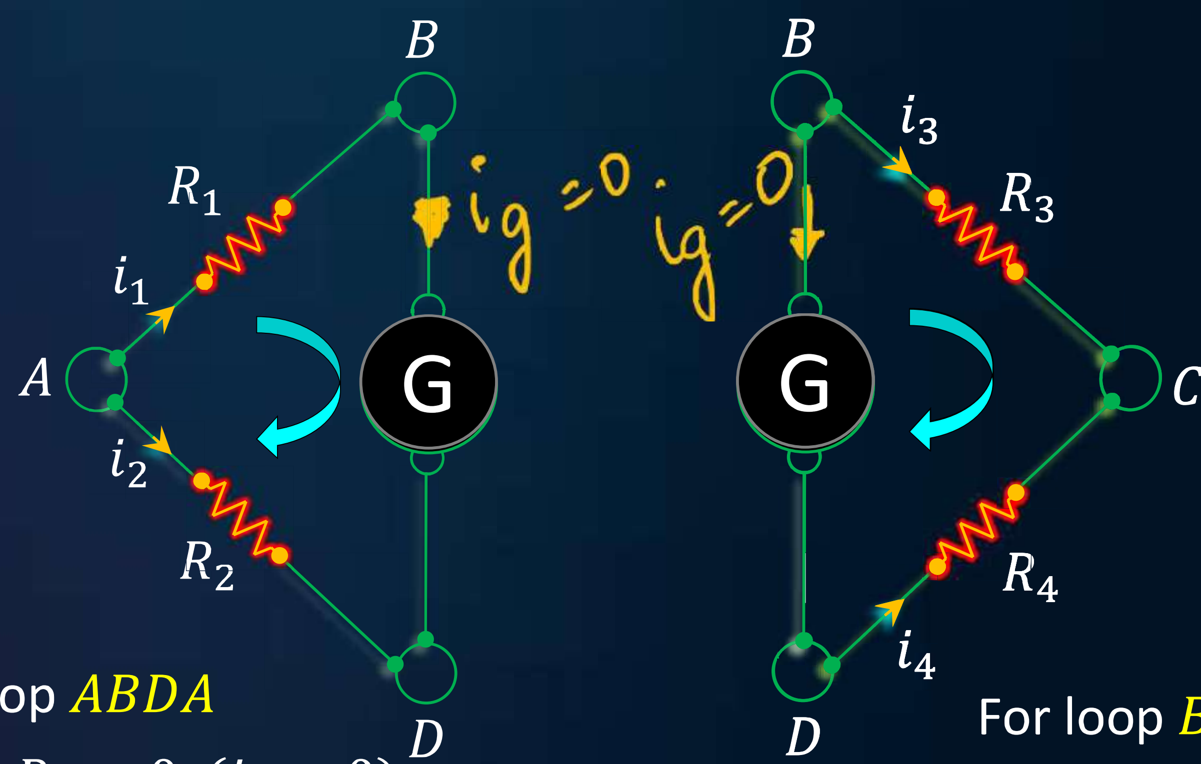
Wheatstone Bridge

Balanced Bridge:

$i_1 = i_3$ $\quad$ $\boxed{i_g = 0}$ $\quad$ $i_2 = i_4$



Applying Kirchhoff's Rule



For loop *ABDA*

$$-i_1R_1 - 0 + i_2R_2 = 0 \quad (i_g = 0)$$

$$\boxed{\frac{i_1}{i_2} = \frac{R_2}{R_1}}$$

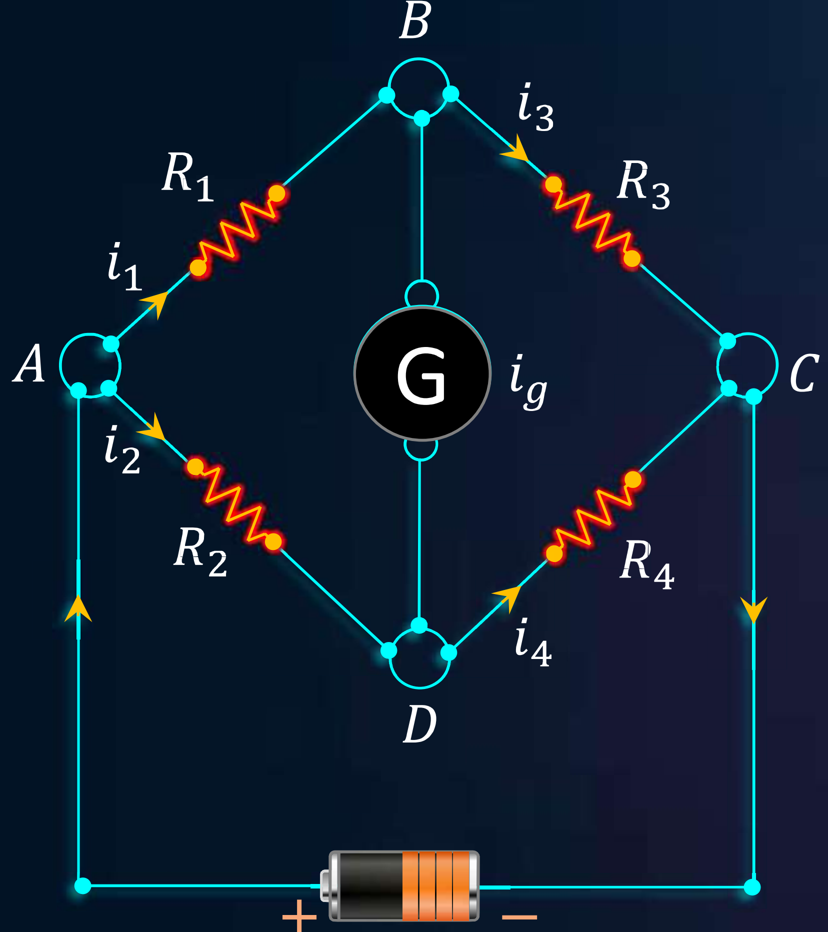
For loop *BCDB*

$$-i_3R_3 + i_4R_4 - 0 = 0$$

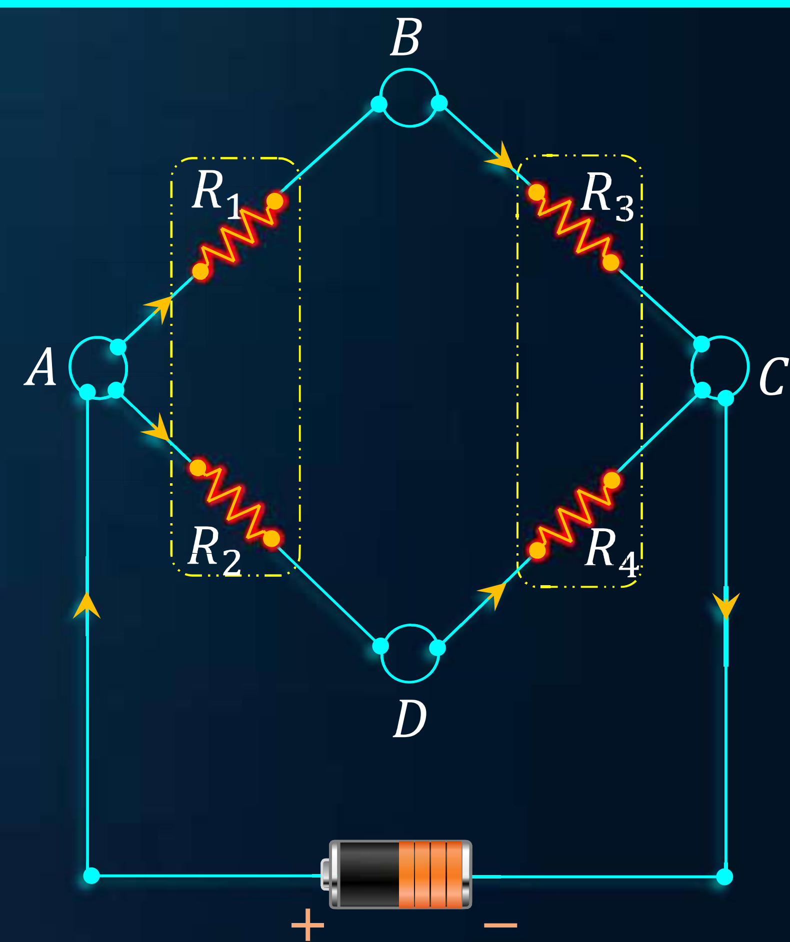
$$\boxed{\frac{i_3}{i_4} = \frac{i_1}{i_2} = \frac{R_4}{R_3}}$$

$$\boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$$

Wheatstone Bridge



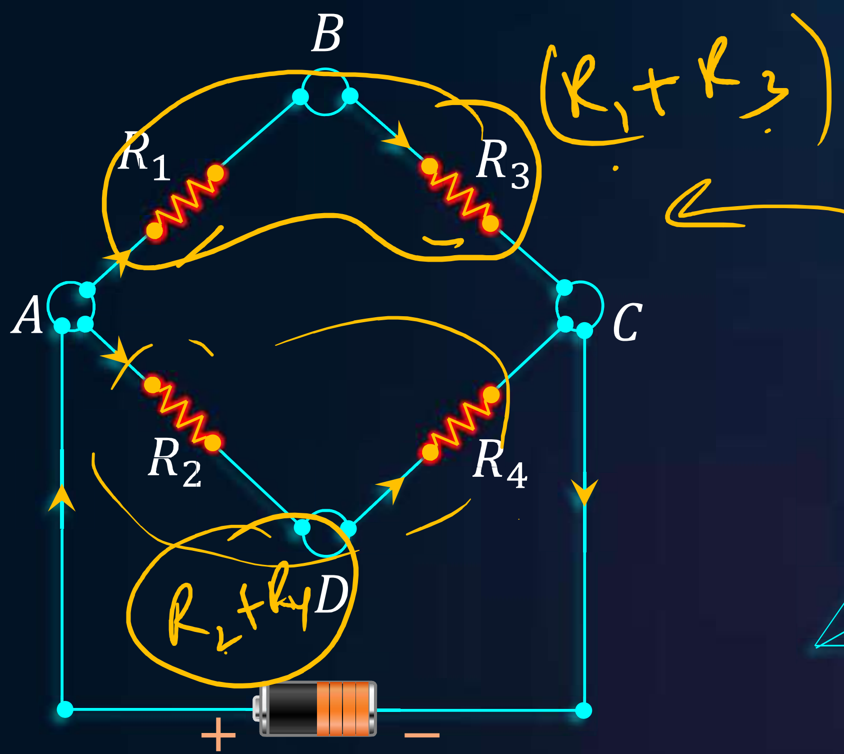
$i_g = 0$

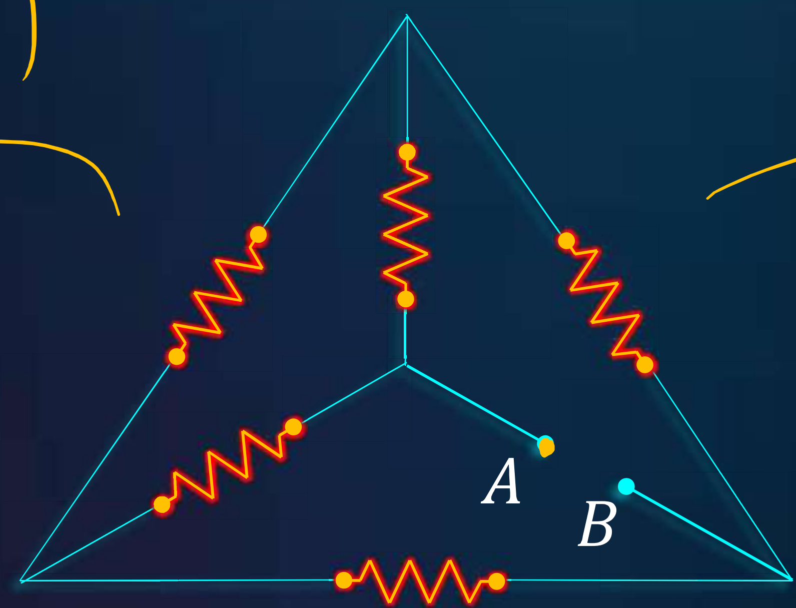


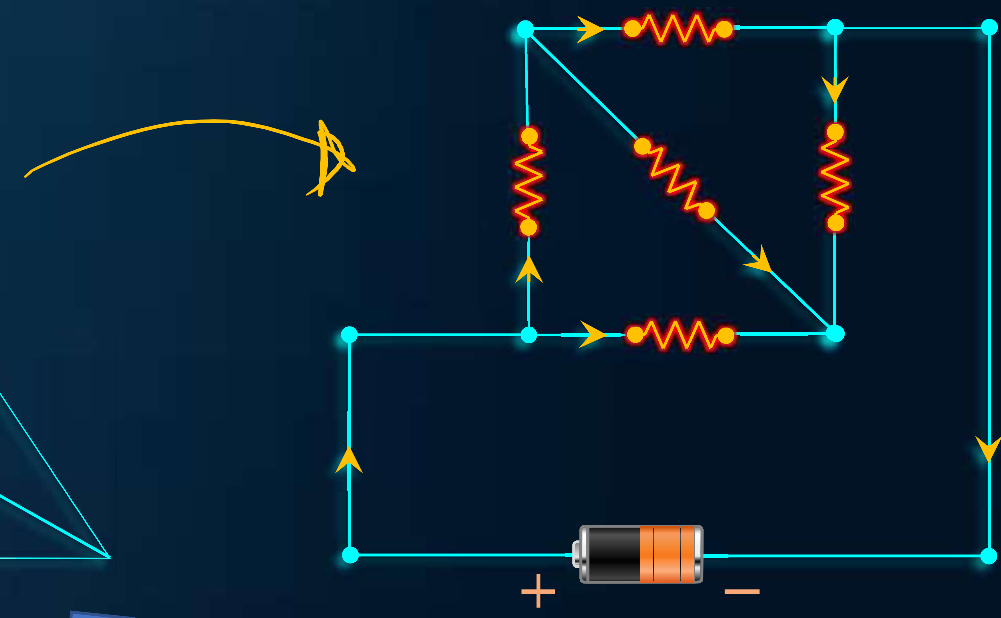
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

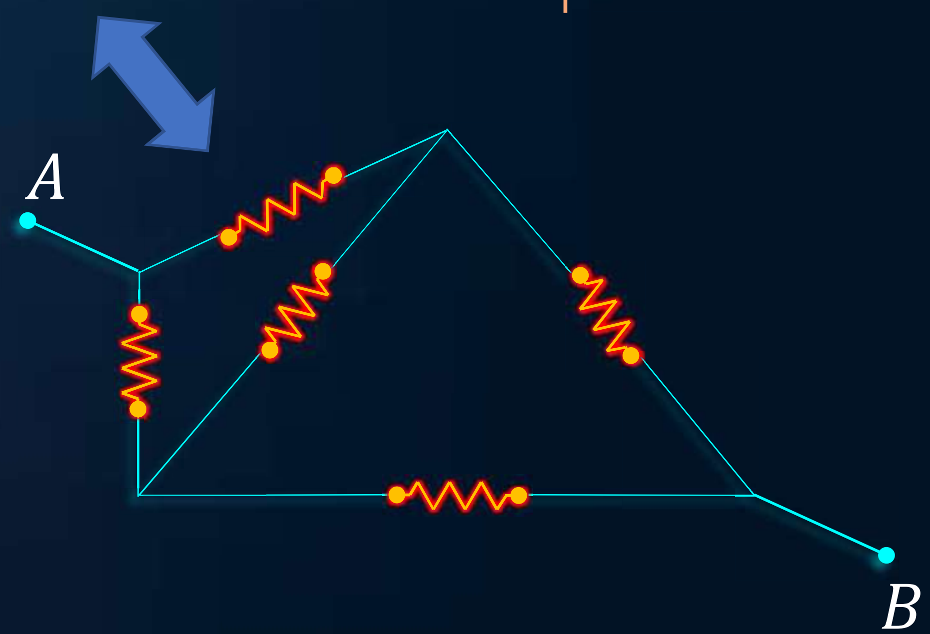
Conditionon of Balanced Wheatstone bridge

Wheatstone Bridge









$$R_{eq} = \frac{(R_1+R_3)(R_2+R_4)}{(R_1+R_3)+(R_2+R_4)}$$

Question

NEET

In a Wheatstone's bridge all the four arms have equal resistance R. If the resistance of the galvanometer arm is also R, the equivalent resistance of the combination as seen by the battery is

a) $R/4$

b) $R/2$

c) R

d) $2R$

Discussion

$$\frac{R}{R} = \frac{R}{R} = 1$$

$$R_{eq} = \frac{\cancel{2}R}{\cancel{2}}$$

$$\frac{R}{R} = \frac{R}{R}$$

Balanced Wheatstone bridge

Equivalent resistance:

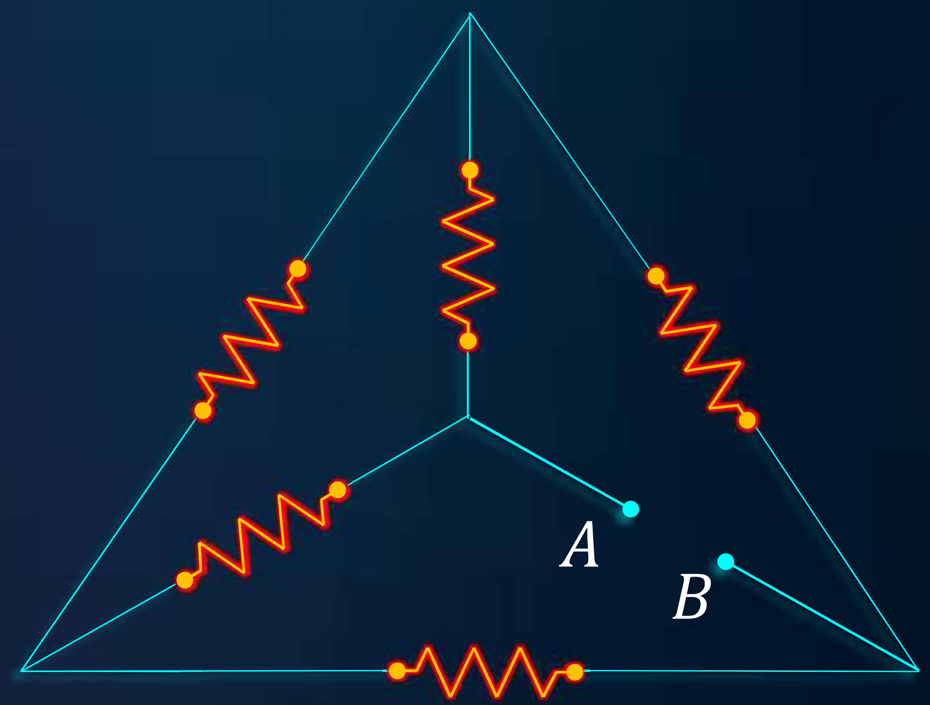
$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} = \frac{2}{2R} = \frac{1}{R} \Rightarrow R_{eq} = R$$

Thus, option (c) is the correct answer.

Question

NEET

In the network shown in the figure, each of the resistance is equal to $2\ \Omega$. The resistance between the points A and B is



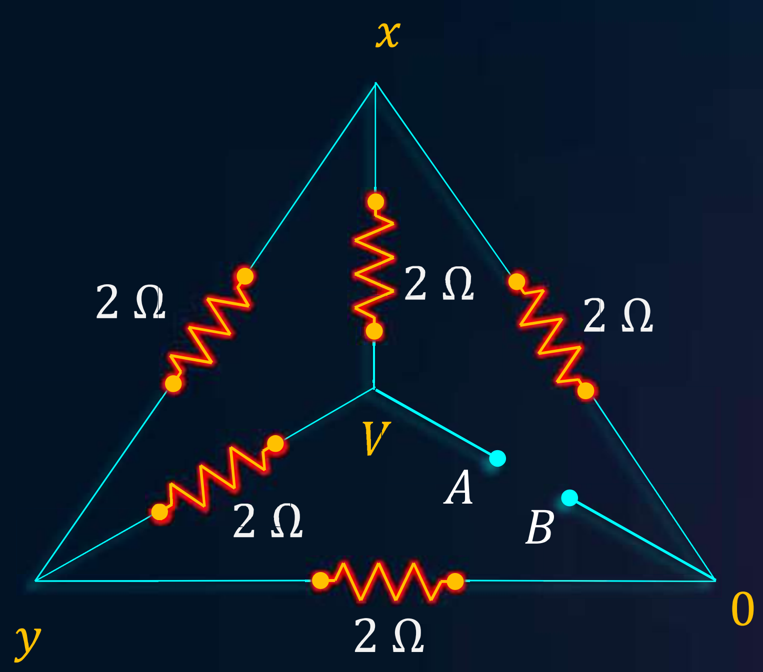
a) $1\ \Omega$

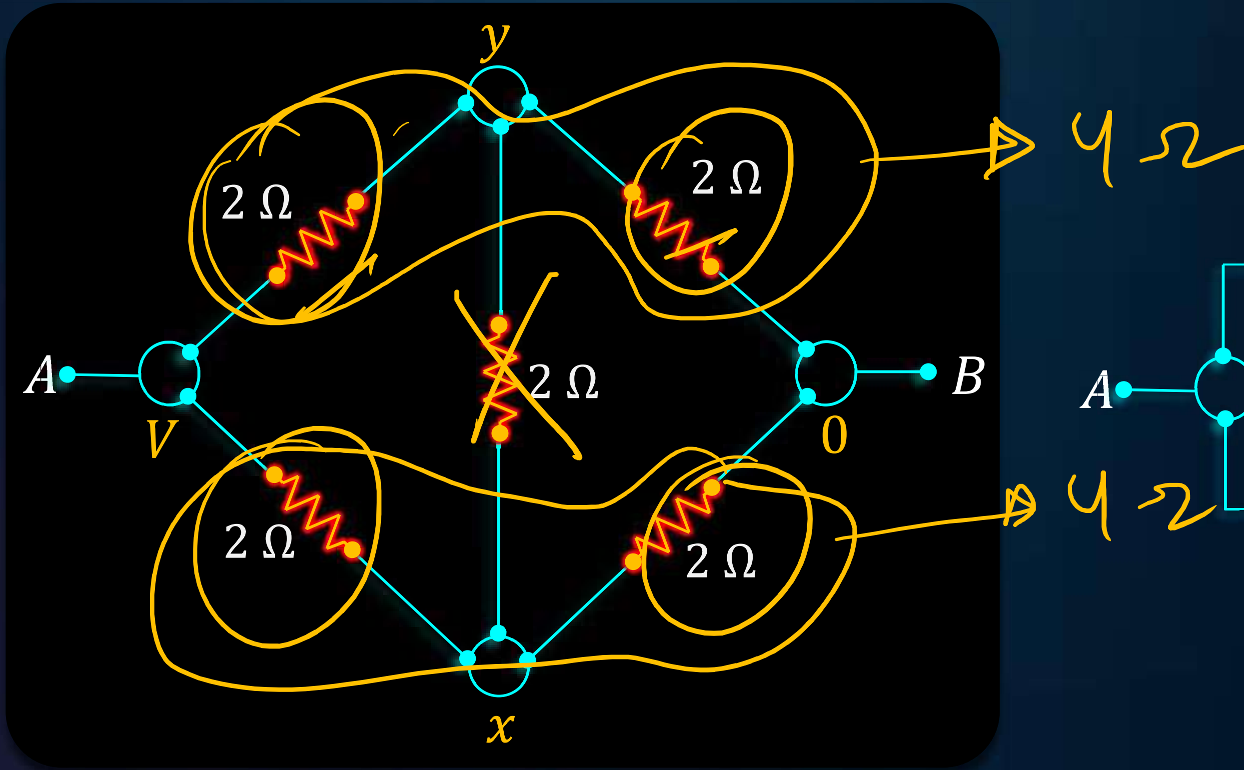
b) $4\ \Omega$

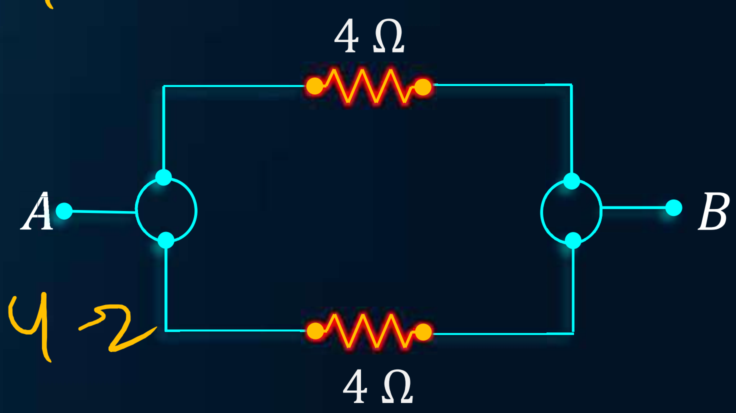
c) $3\ \Omega$

d) $2\ \Omega$

Summary







Circuit is a balanced Wheatstone bridge.

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$R_{eq} = 2\ \Omega$$

Thus, option (d) is the correct answer.

Question

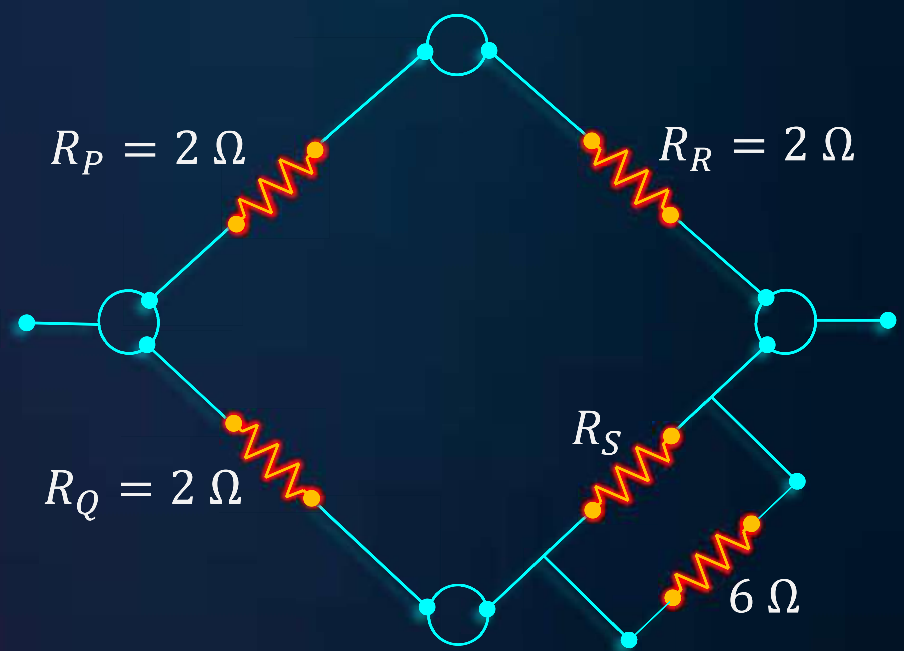
Three resistance P, Q, R each of $2\ \Omega$ and an unknown resistance S form the four arms of a Wheatstone bridge circuit. When a resistance of $6\ \Omega$ is connected in parallel to S the bridge gets balanced. What is the value of S?

a) $3\ \Omega$

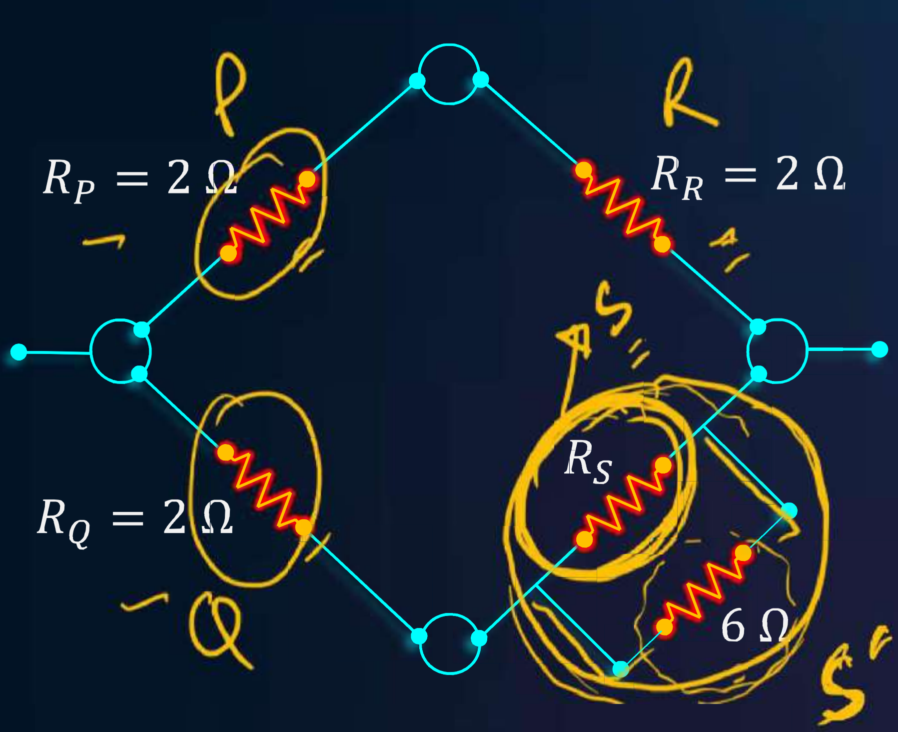
b) $6\ \Omega$

c) $1\ \Omega$

d) $2\ \Omega$



Discussion



$$\frac{R_P}{R_Q} = \frac{R_R}{S'} = 1 \quad \Big| \quad \frac{2}{S'} = 1$$

$$S' = 2$$

Since R_S and $6\ \Omega$ resistances are in parallel combination, thus,

$$S' = \frac{6R_s}{6+R_s} = 2$$

$$6R_s = 12 + 2R_s$$

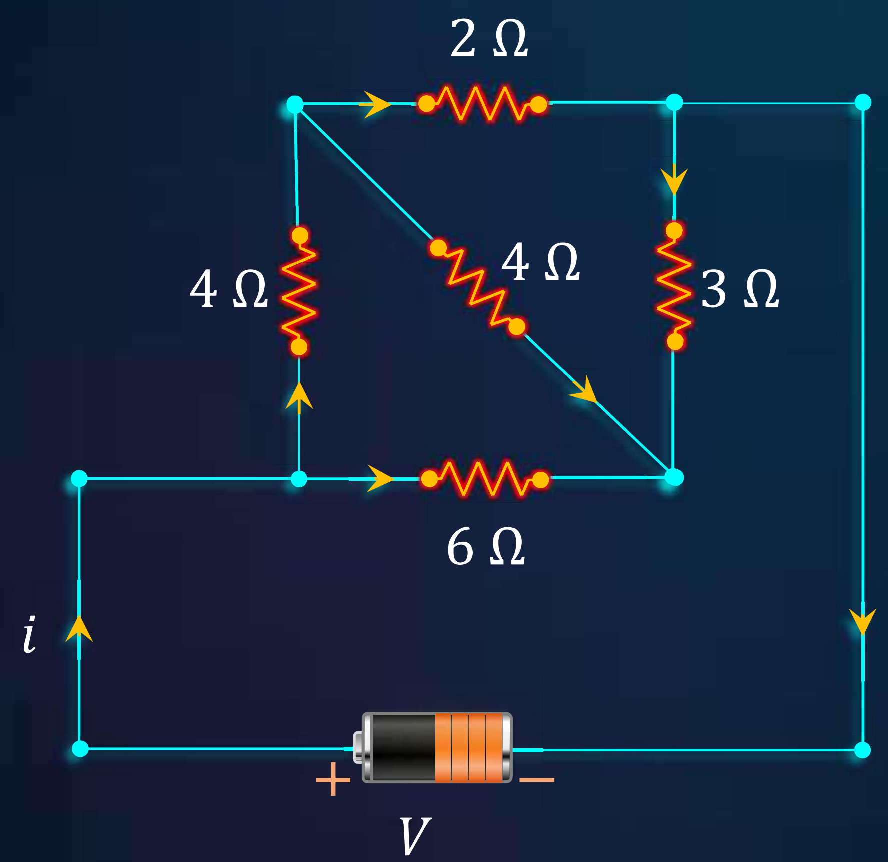
$$R_s = 3\,\Omega$$

Thus, option (a) is the correct answer.

Question

NEET

For the network shown in figure, the value of the current i is



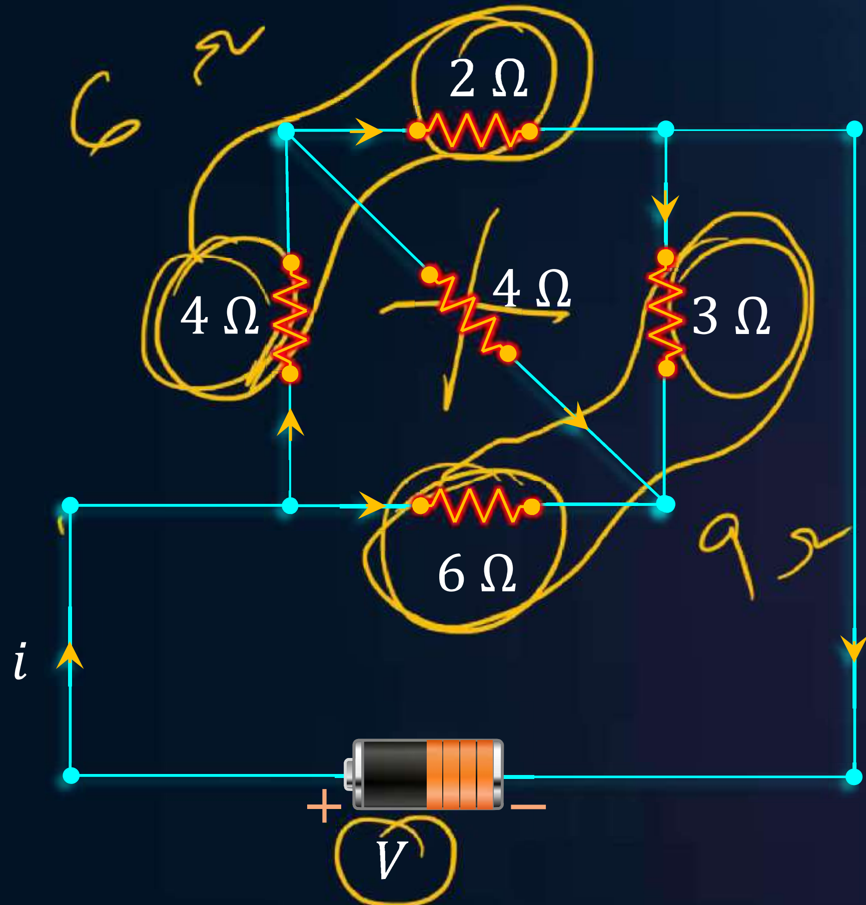
a) $\frac{9V}{35}$

b) $\frac{5V}{18}$

c) $\frac{5V}{9}$

d) $\frac{18V}{5}$

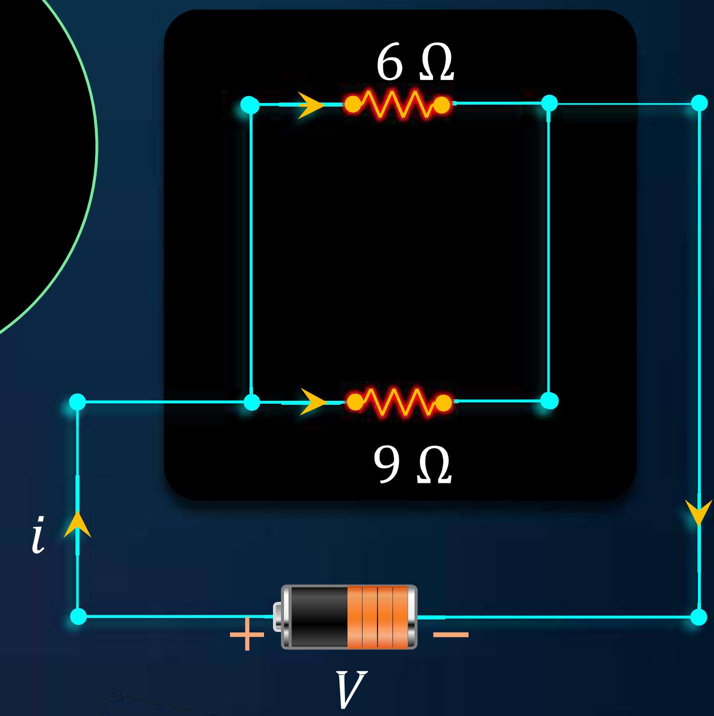
Discussion



$$\frac{4}{6} = \frac{2}{3}$$

Balanced Wheatstone bridge

$\Rightarrow$



$$R_{eq} = \frac{6 \times 9}{6+9}$$

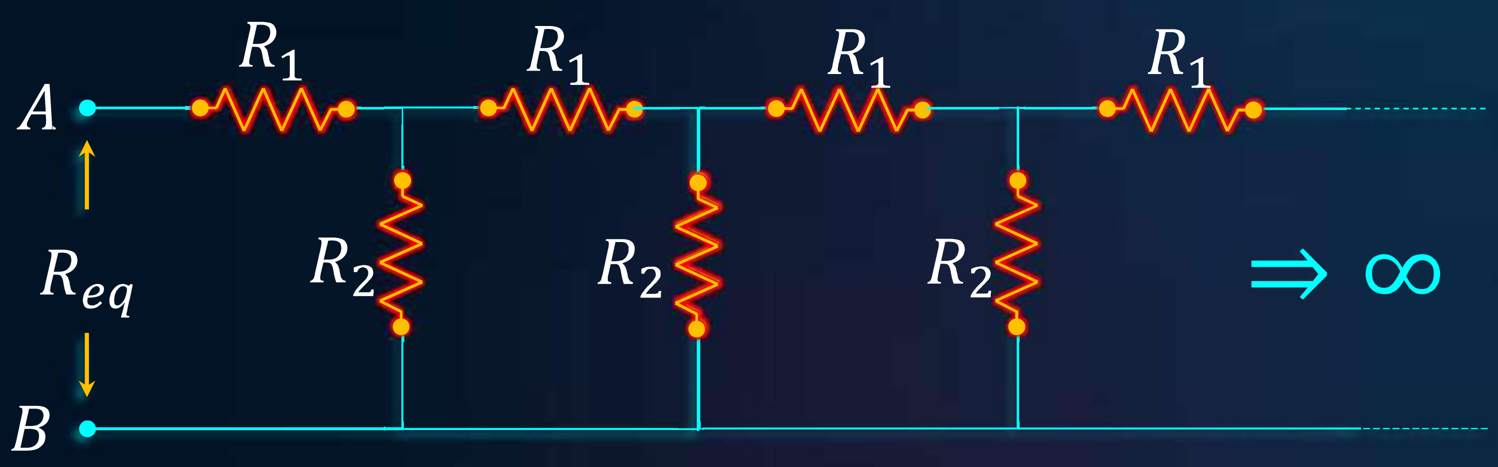
$$R_{eq} = \frac{18}{5}\ \Omega$$

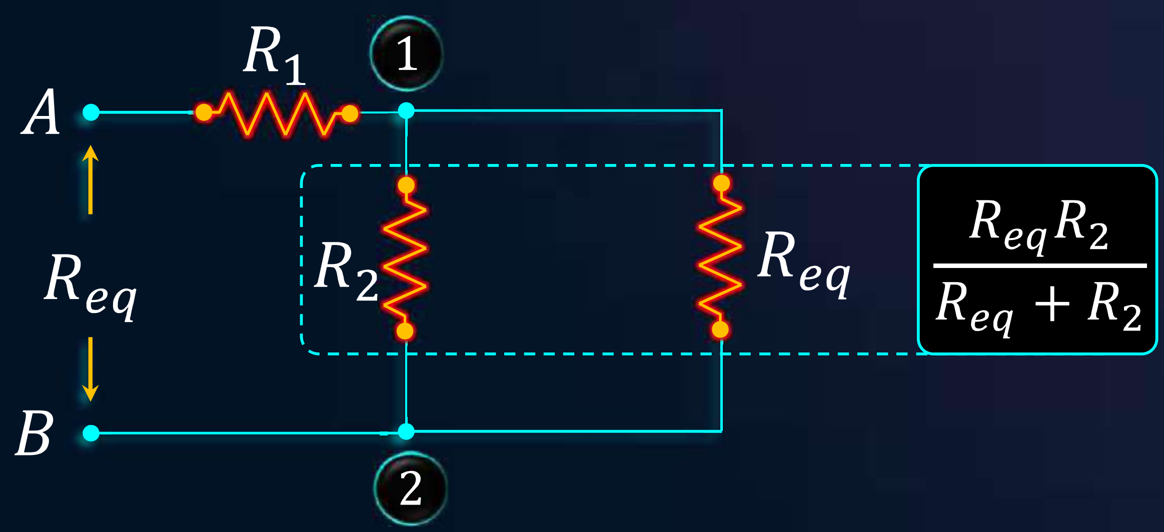
$$i = \frac{V}{R_{eq}} = \frac{5V}{18}$$

Thus, option (b) is the correct answer.

Recap

Infinite network:





- Assume the entire network as R_{eq}.
- Find a pattern with the resistors.

$$R_1 + \frac{R_{eq}R_2}{R_{eq}+R_2} = R_{eq}$$

$$R_1(R_{eq}+R_2) + R_{eq}R_2 = R_{eq}(R_{eq}+R_2)$$

$$R_{eq}^2 - R_{eq}R_1 - R_1R_2 = 0$$

$$\Rightarrow R_{eq} = \frac{R_1 \pm \sqrt{R_1^2 + 4R_1R_2}}{2}$$

$$\boxed{\frac{R_1\left(1+\sqrt{1+4R_2/R_1}\right)}{2}}$$

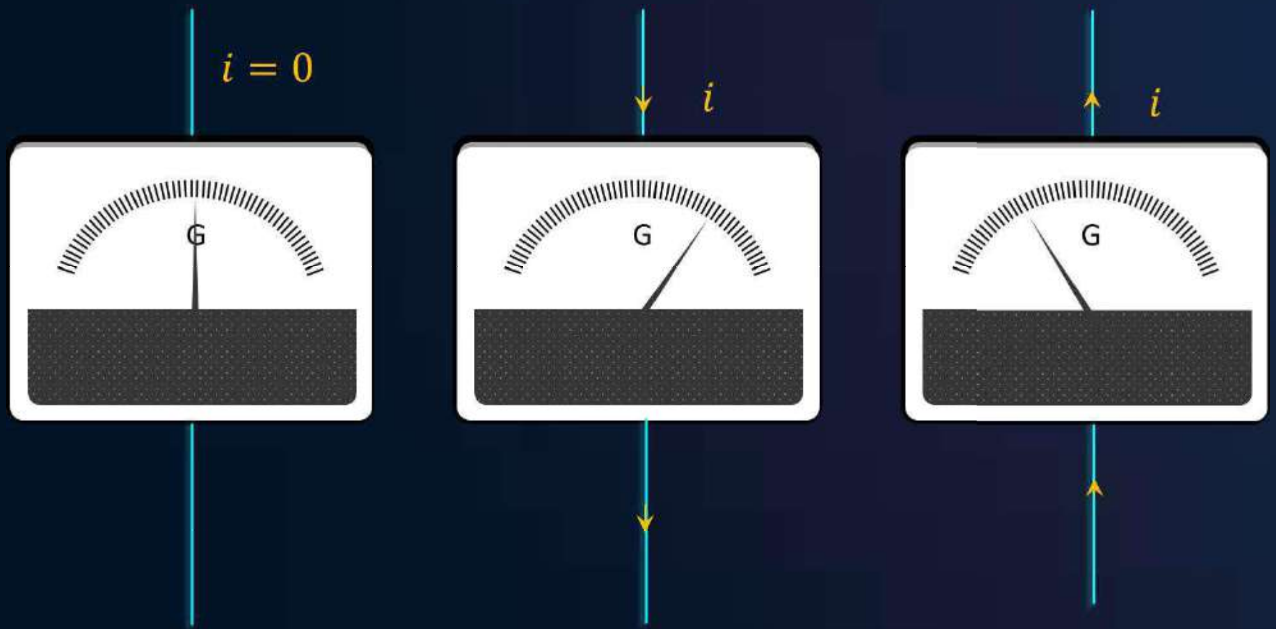
$$\frac{R_1\left(1-\sqrt{1+4R_2/R_1}\right)}{2}$$

(Not possible as resistance can’t be negative)

Recap

Galvanometer:

A device used to detect the presence of current, the direction of flow and compare the magnitudes of two currents.

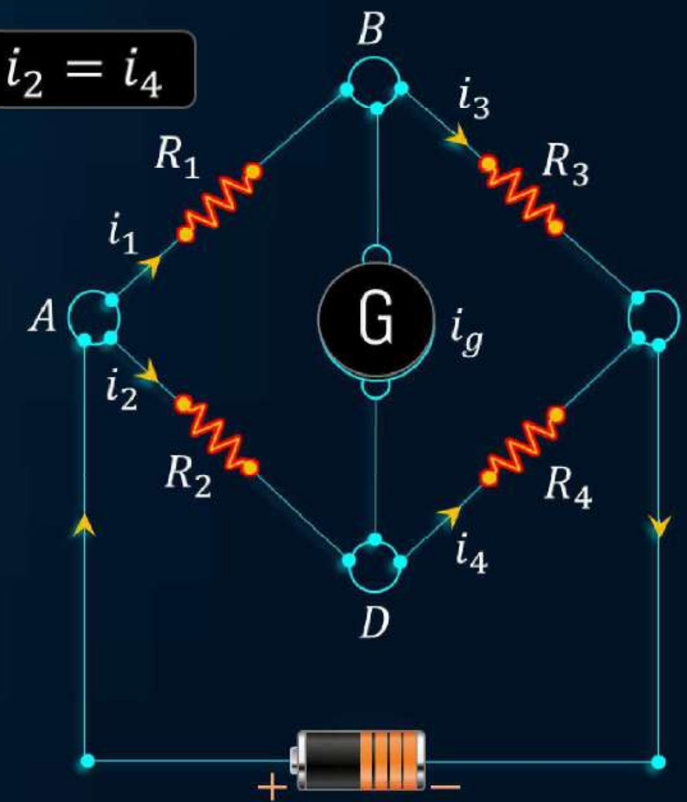


Angle of deflection $(\theta) \propto$ Amount of current (i) flowing through it

Wheatstone bridge:

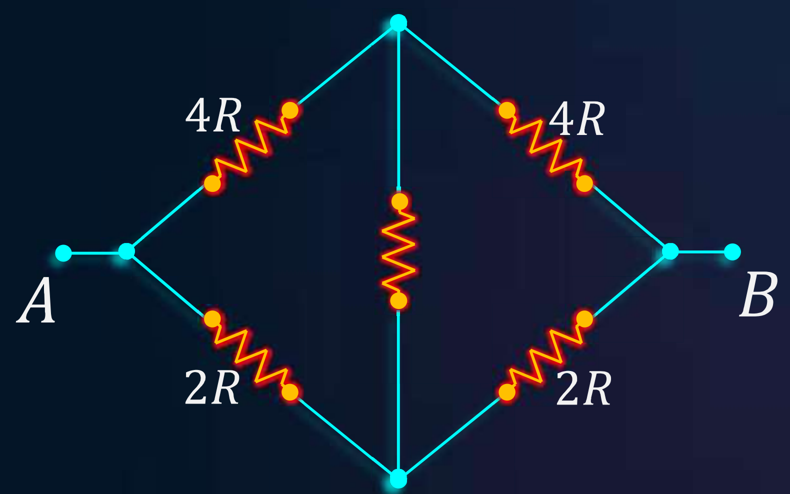
Condition for balanced Wheatstone bridge

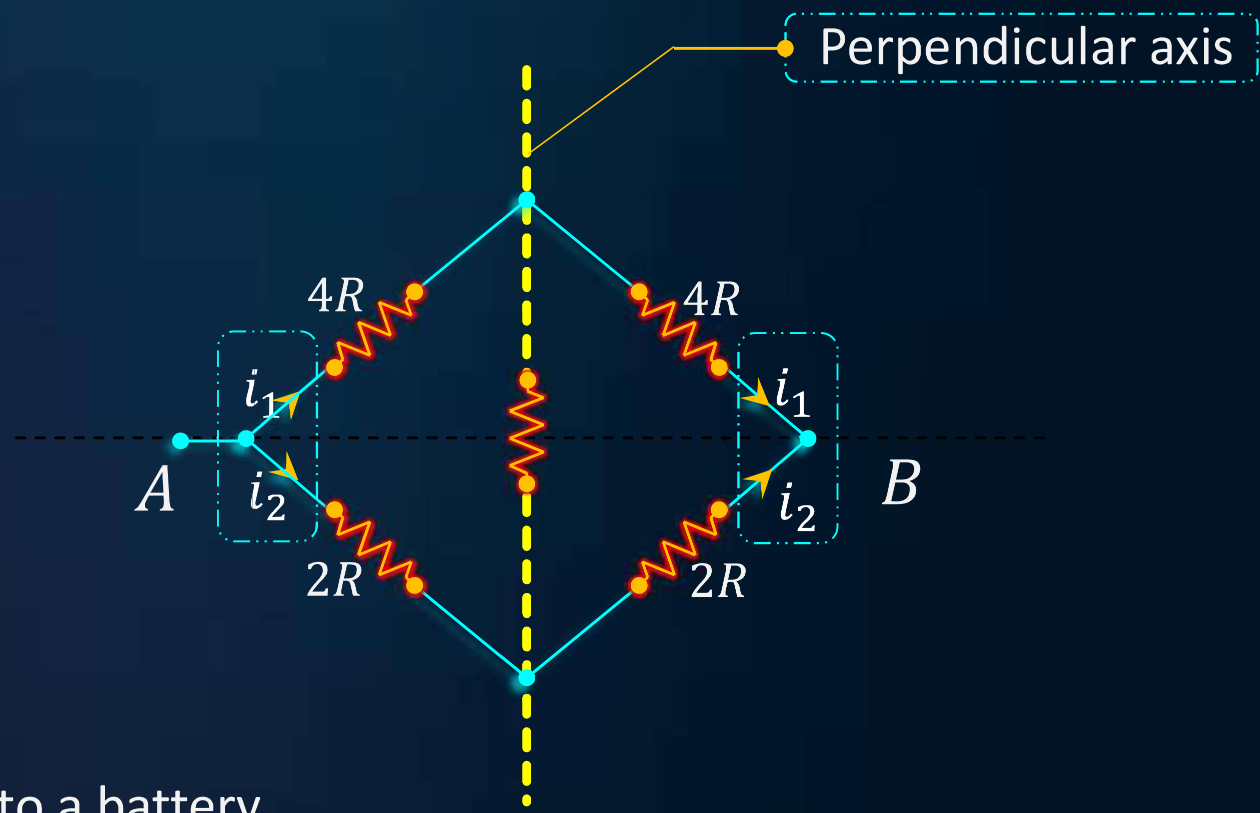
$i_1 = i_3$ $\quad$ $i_g = 0$ $\quad$ $i_2 = i_4$



$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Mirror Symmetry/(Perpendicular axis of symmetry/line symmetry)

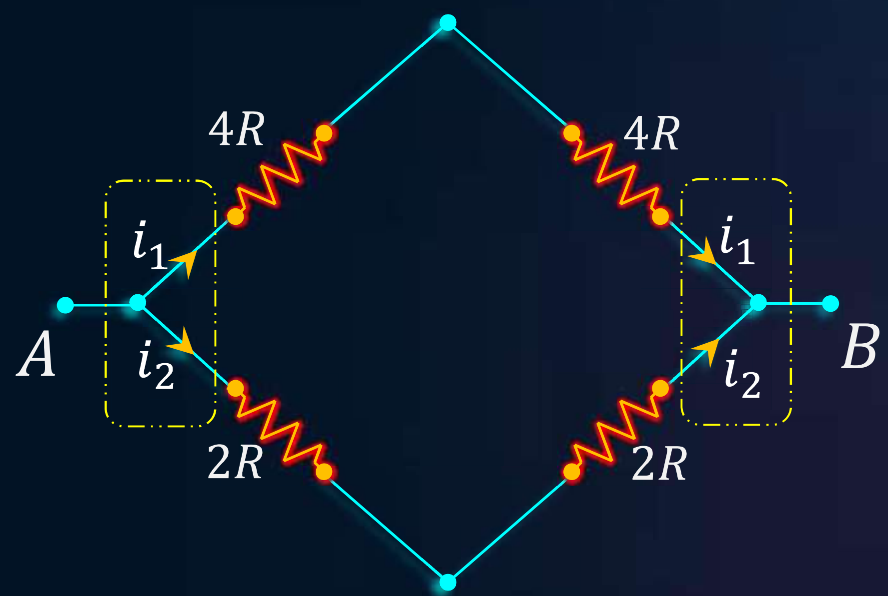




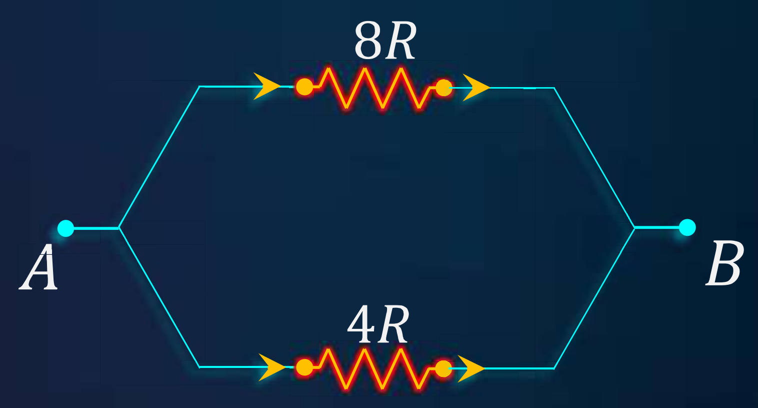
- Assume that point A and point B are connected to a battery.
- Draw a line perpendicular to the line joining A and B such that the resistances are same in both the sides.
- Since the resistances are same in both the sides of the perpendicular axis, current flowing through it will also be same.

Symmetric Circuit

Mirror Symmetry/(Perpendicular axis of symmetry/line symmetry)

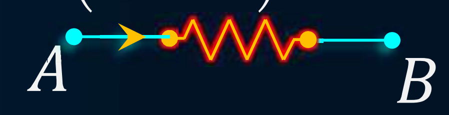


$\Rightarrow$



$\Rightarrow$

$$\left(\frac{8R \times 4R}{8R + 4R}\right) = \frac{8}{3}R$$

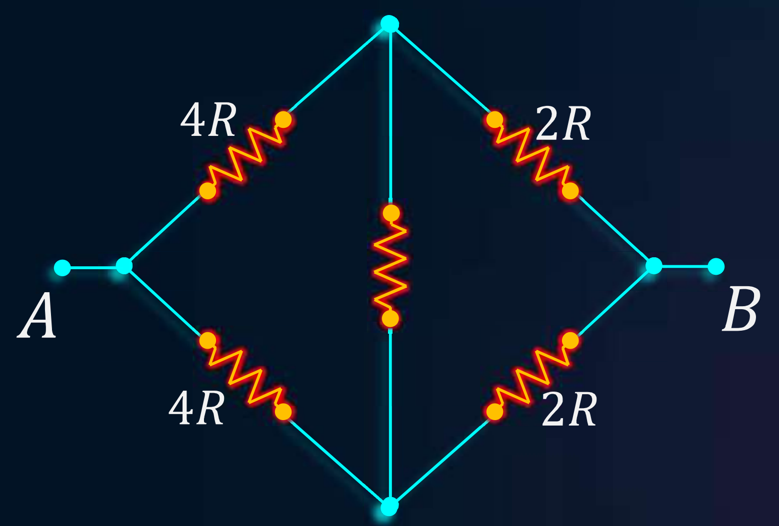


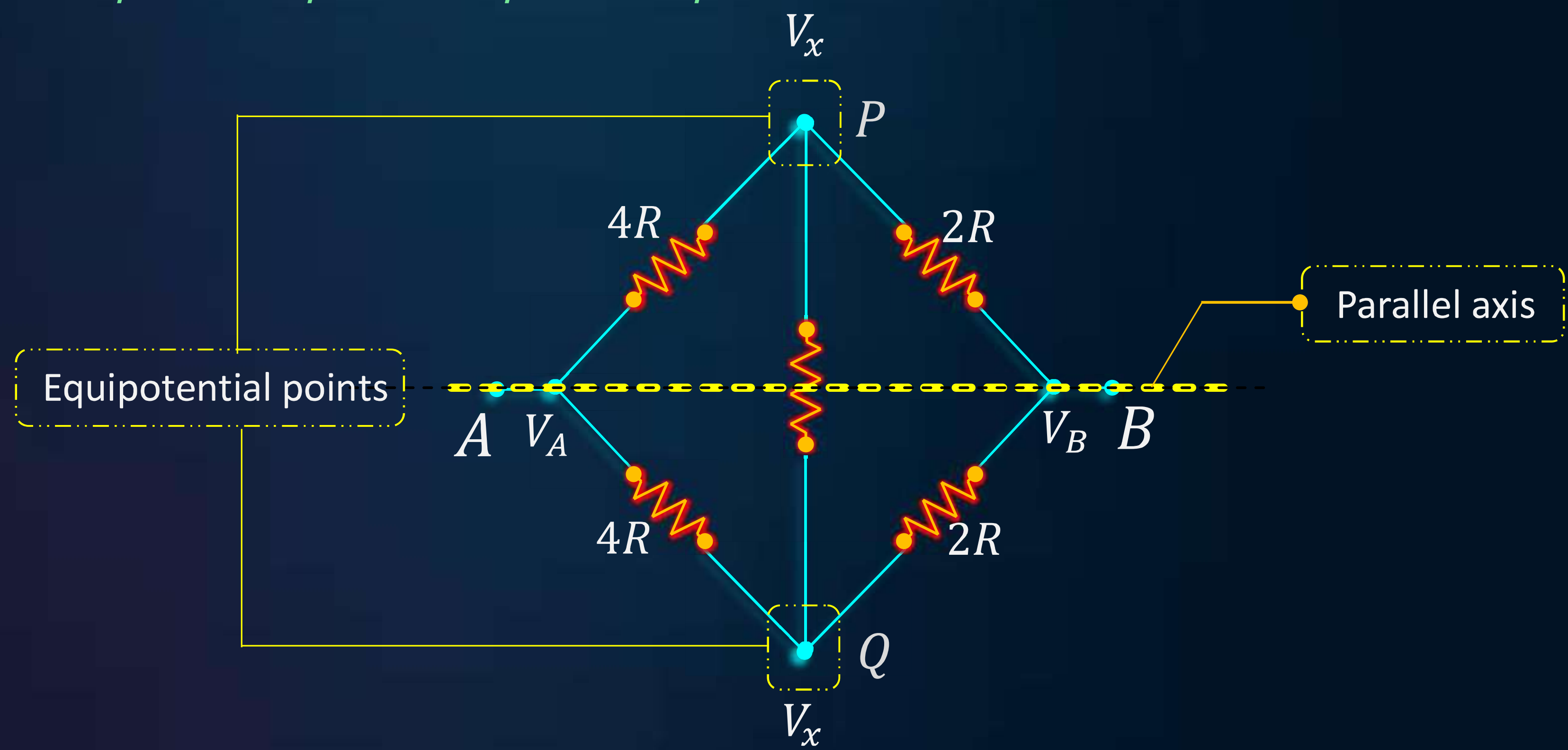
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow \frac{4R}{2R} = \frac{4R}{2R}$$

Balanced Wheatstone Bridge

Symmetric Circuit

Folding Symmetry/(Parallel axis of symmetry/line symmetry)

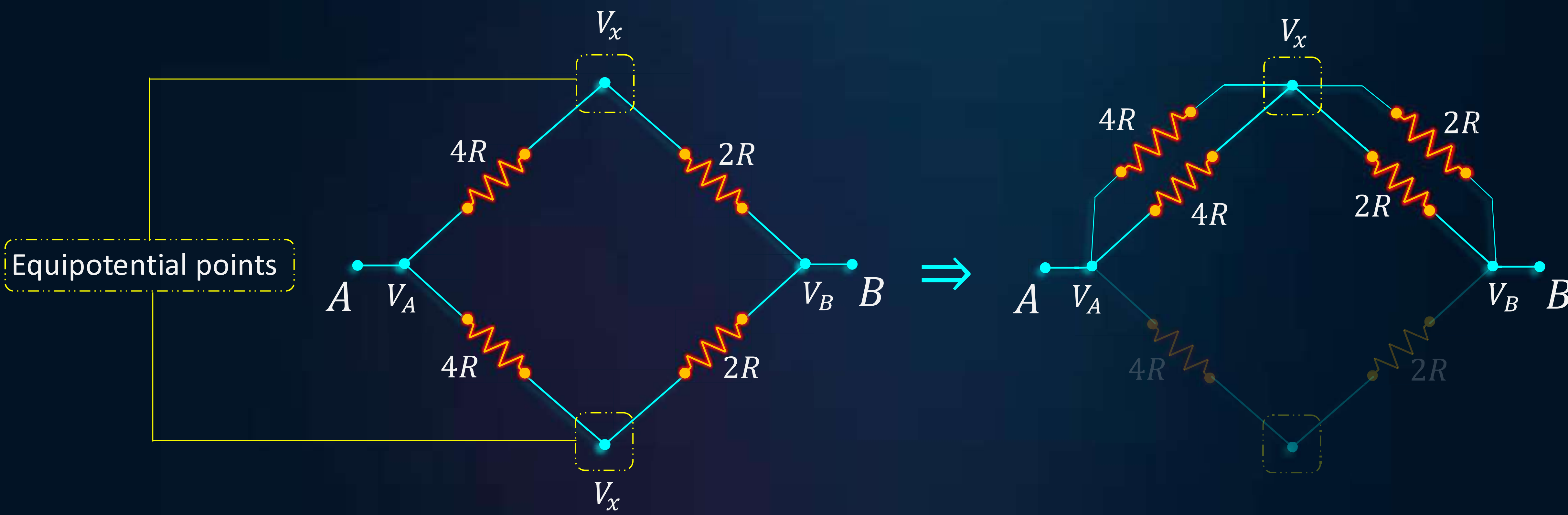




- In this case, the figure is symmetric about the horizontal axis or, parallel axis.
- The points above and below the parallel axis i.e., point P and Q have same potential as the symmetry of the configuration is about the parallel axis. Hence, no current will flow through the resistance connected between the point P and Q .

Symmetric Circuit

Folding Symmetry/(Parallel axis of symmetry/line symmetry)



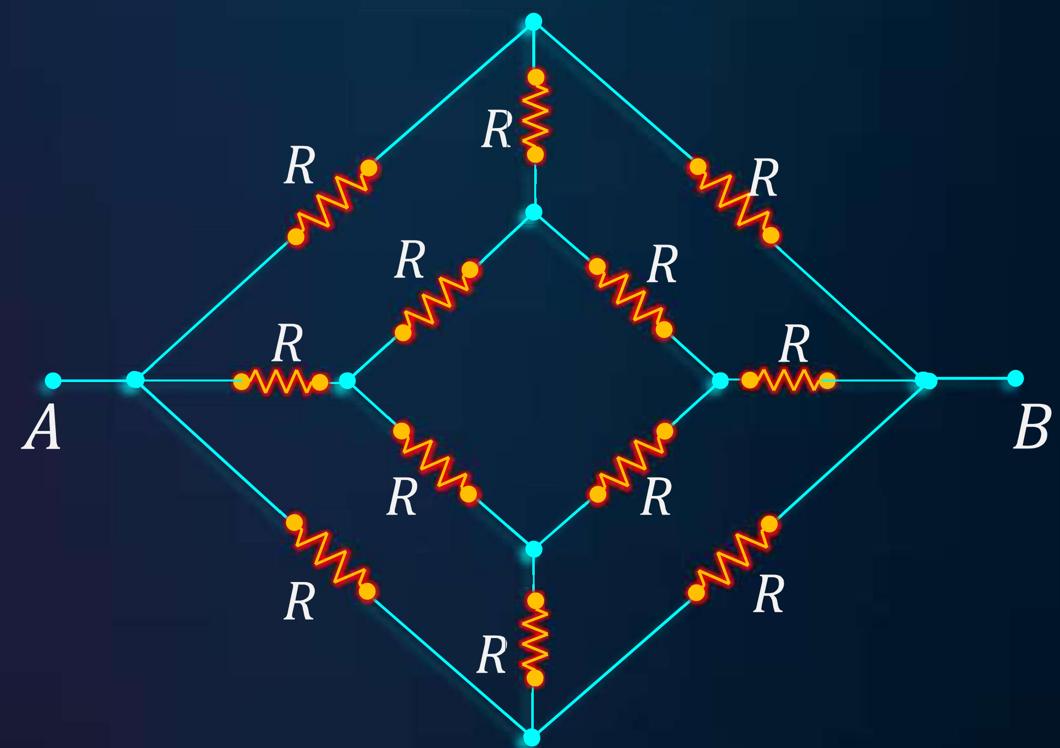
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow \frac{4R}{4R} = \frac{2R}{2R}$$

Balanced Wheatstone Bridge

Question

NEET

Find the equivalent resistance between points A and B in the given circuit.



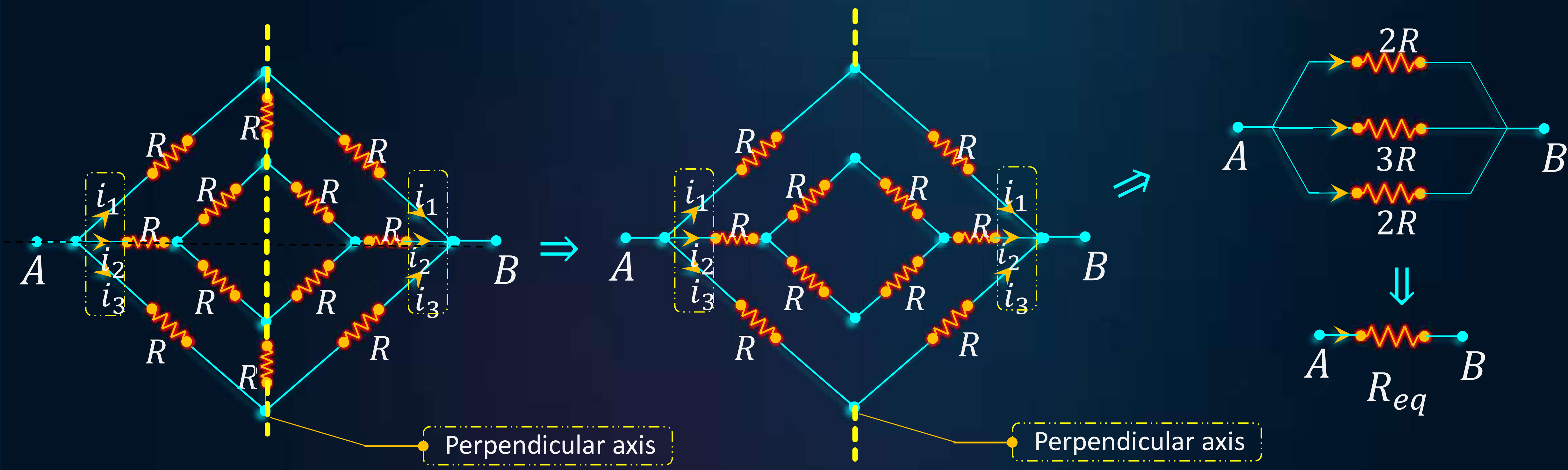
a) $3R/4$

b) $R/4$

c) $R/2$

d) R

Discussion



- In this case we can see that the circuit is symmetrical about the perpendicular axis. So, there will not be any current flowing through the resistors that lie over this axis and we can remove that.

- This circuit is also symmetrical about the parallel axis so if we took the points of common potential and fold the circuit, we will get the same resistance.

The equivalent resistance between points A and B in the given circuit is,

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{3R} + \frac{1}{2R}$$

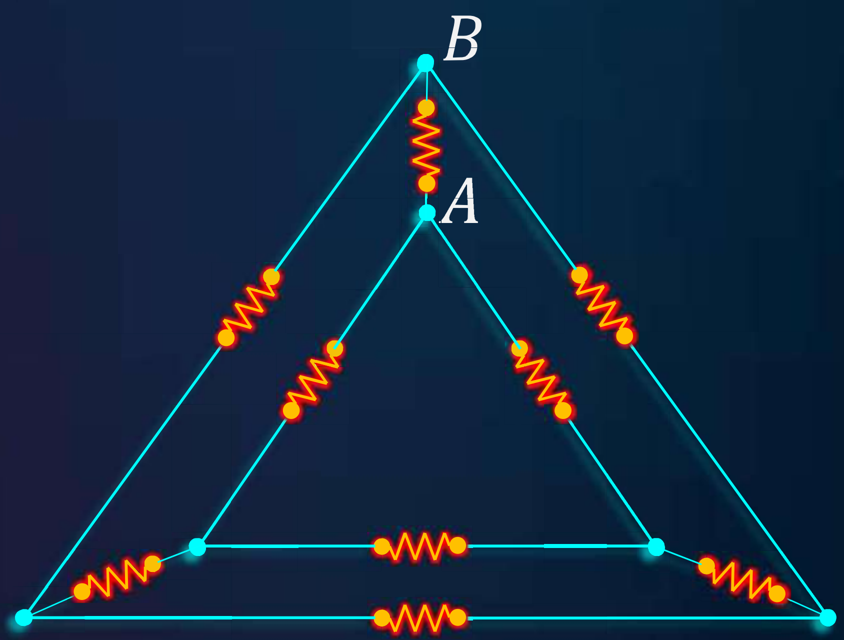
$$\Rightarrow R_{eq} = 3R/4$$

Thus, option (a) is the correct answer.

Question

BOARDS

Find the equivalent resistance between points A and B in the given circuit. Consider all resistances to be R.



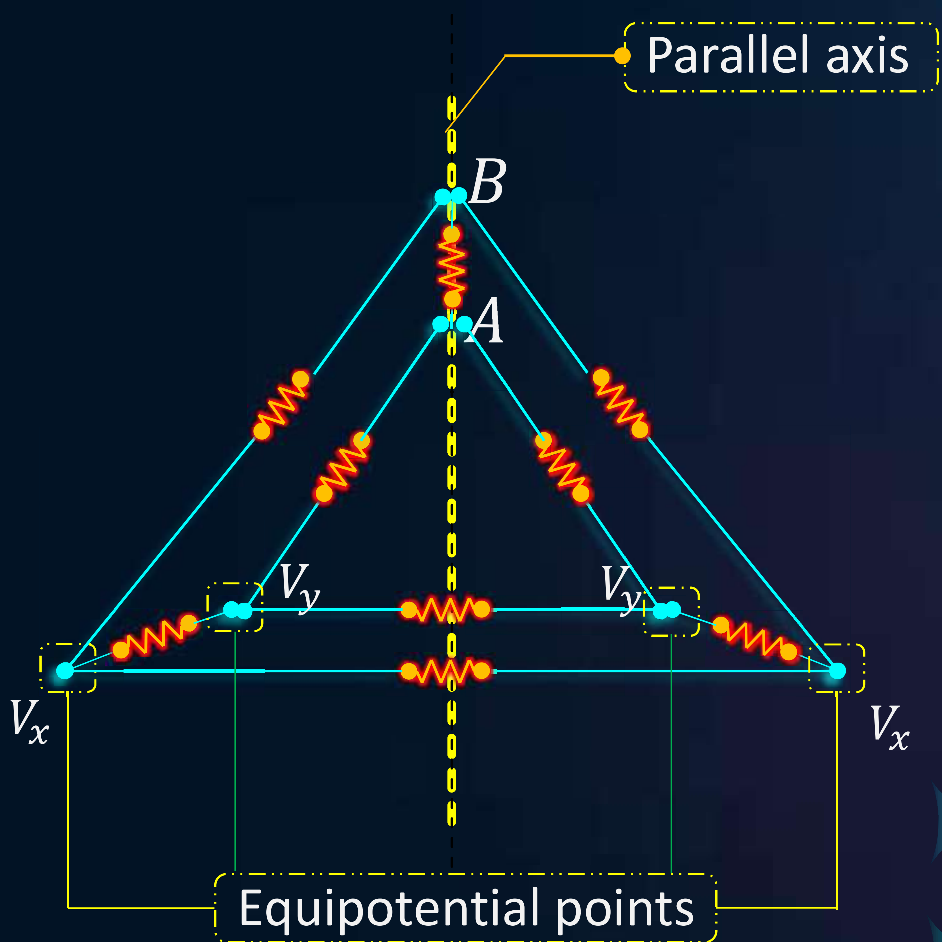
a) $R/4$

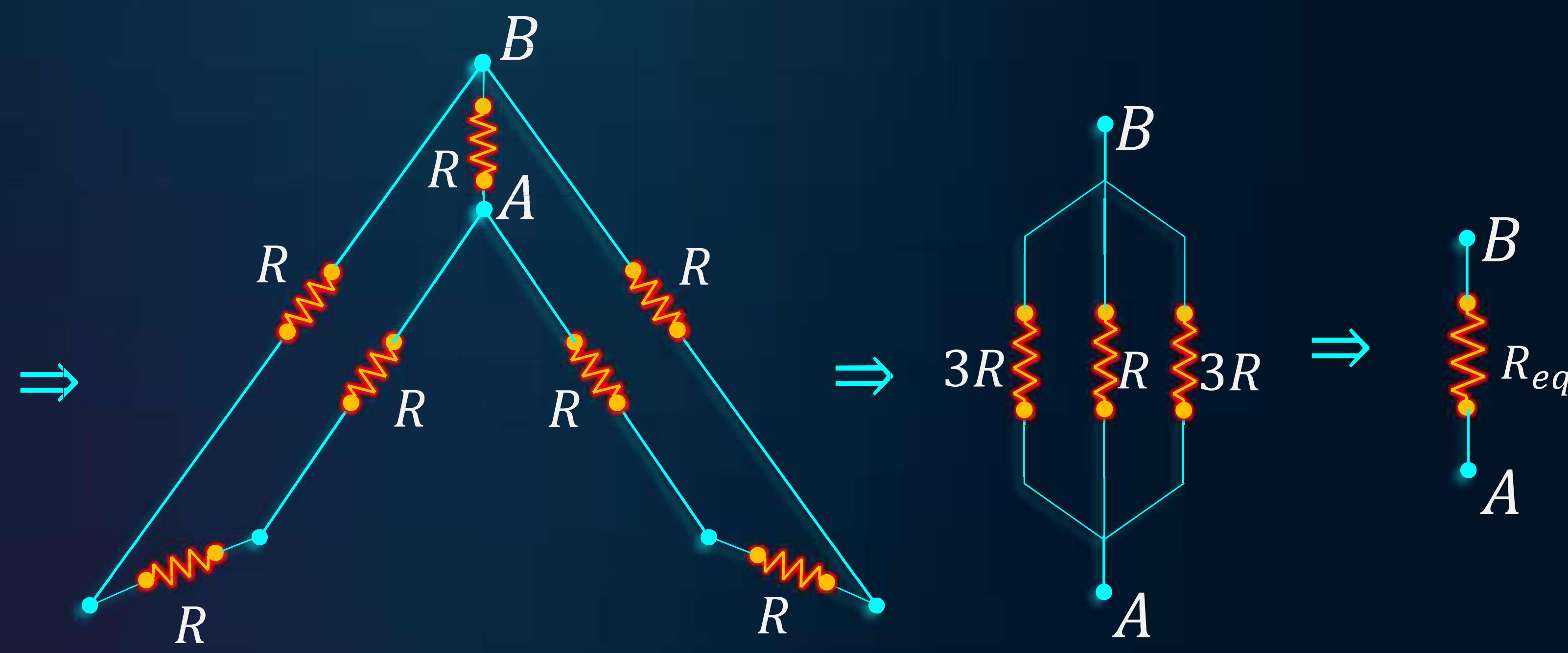
b) $2R/5$

c) $4R/5$

d) $3R/5$

Discussion





- In this case the circuit is only symmetric about the parallel axis i.e., the folding axis.
- Identify the equipotential points and fold the circuit accordingly by removing the resistances between the equipotential points.

$$\frac{1}{R_{eq}} = \frac{1}{3R} + \frac{1}{R} + \frac{1}{3R}$$

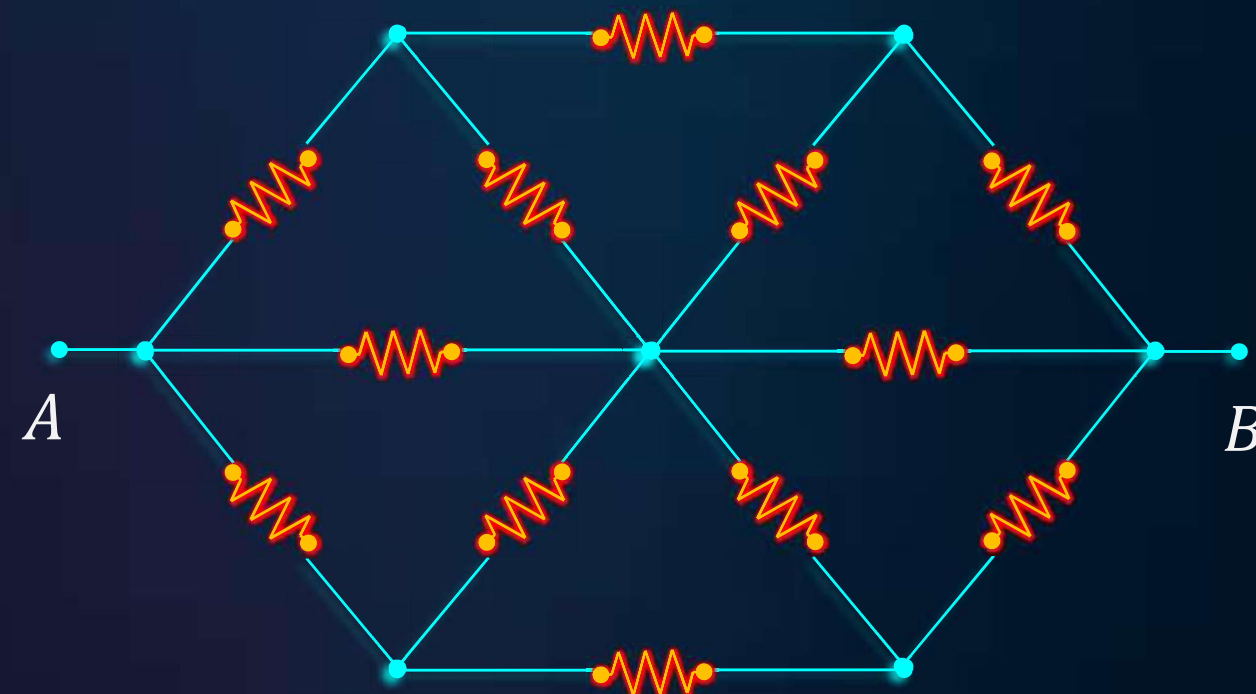
$$R_{eq} = 3R/5$$

Thus, option (d) is the correct answer.

Question

NEET

Find the equivalent resistance between points A and B in the given circuit. Consider all resistances to be R.



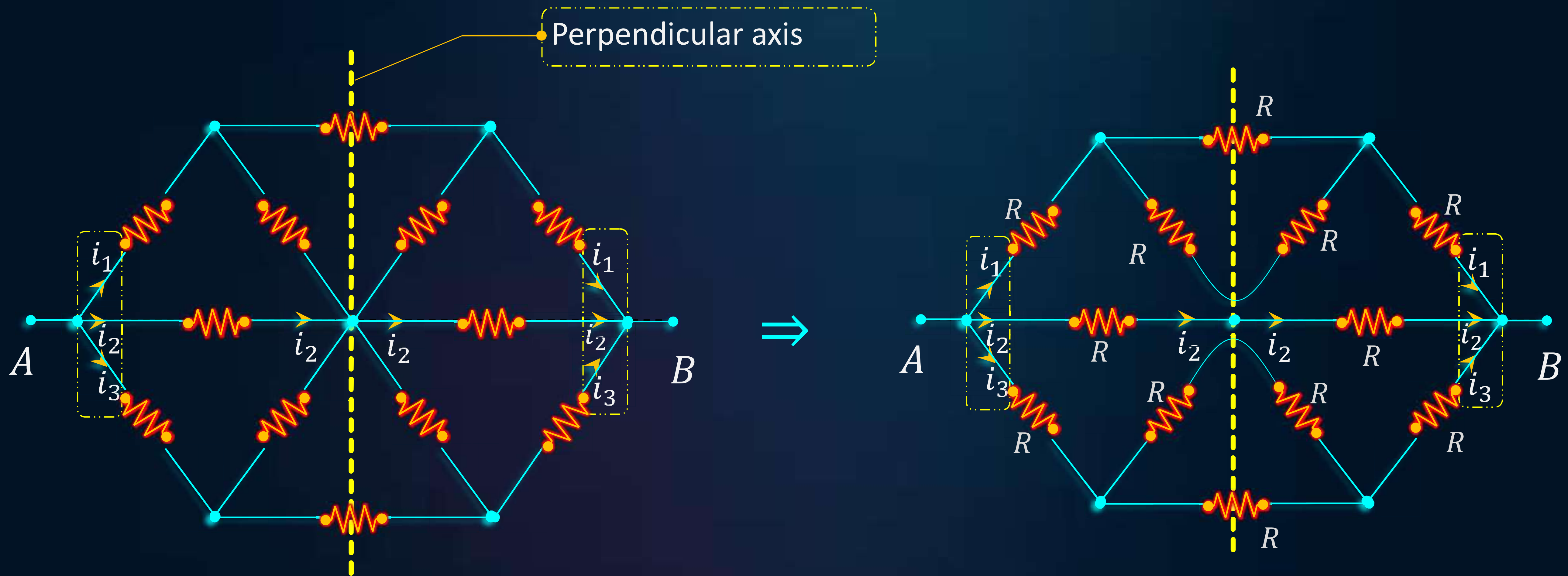
a) $R/4$

b) $2R/5$

c) $4R/5$

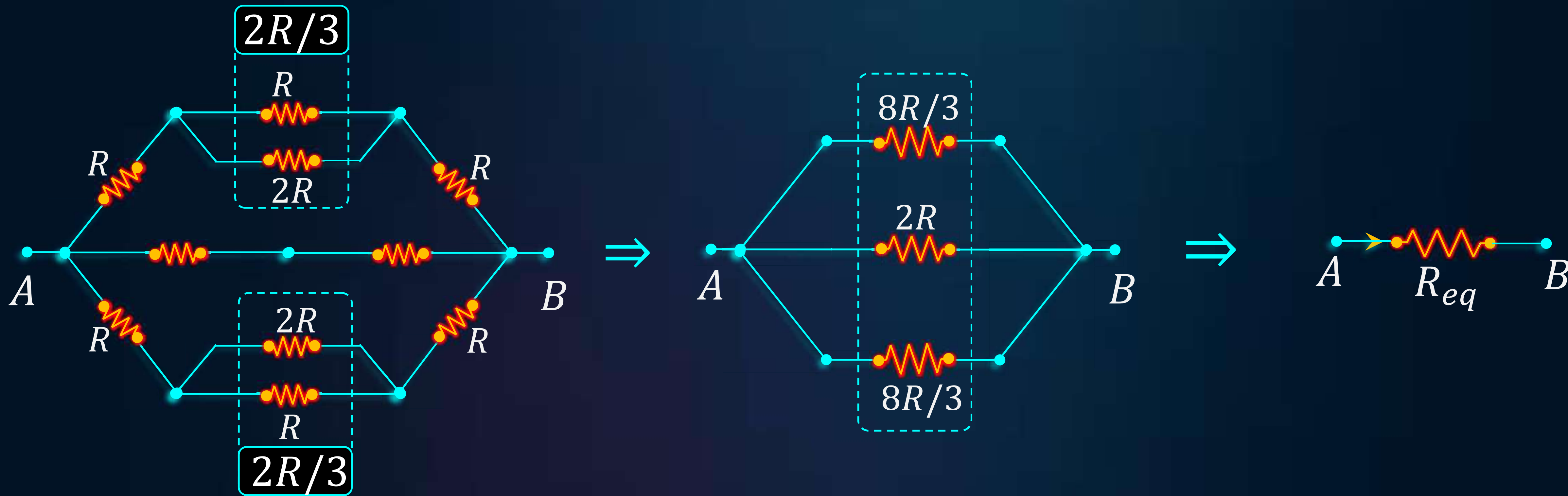
d) $3R/5$

Discussion



- In this case the circuit is symmetric about both parallel axis and perpendicular axis. First, choose the case of perpendicular axis.
- The central junction will be of no use as there will not be any current flowing through it from the other two arms.

Discussion

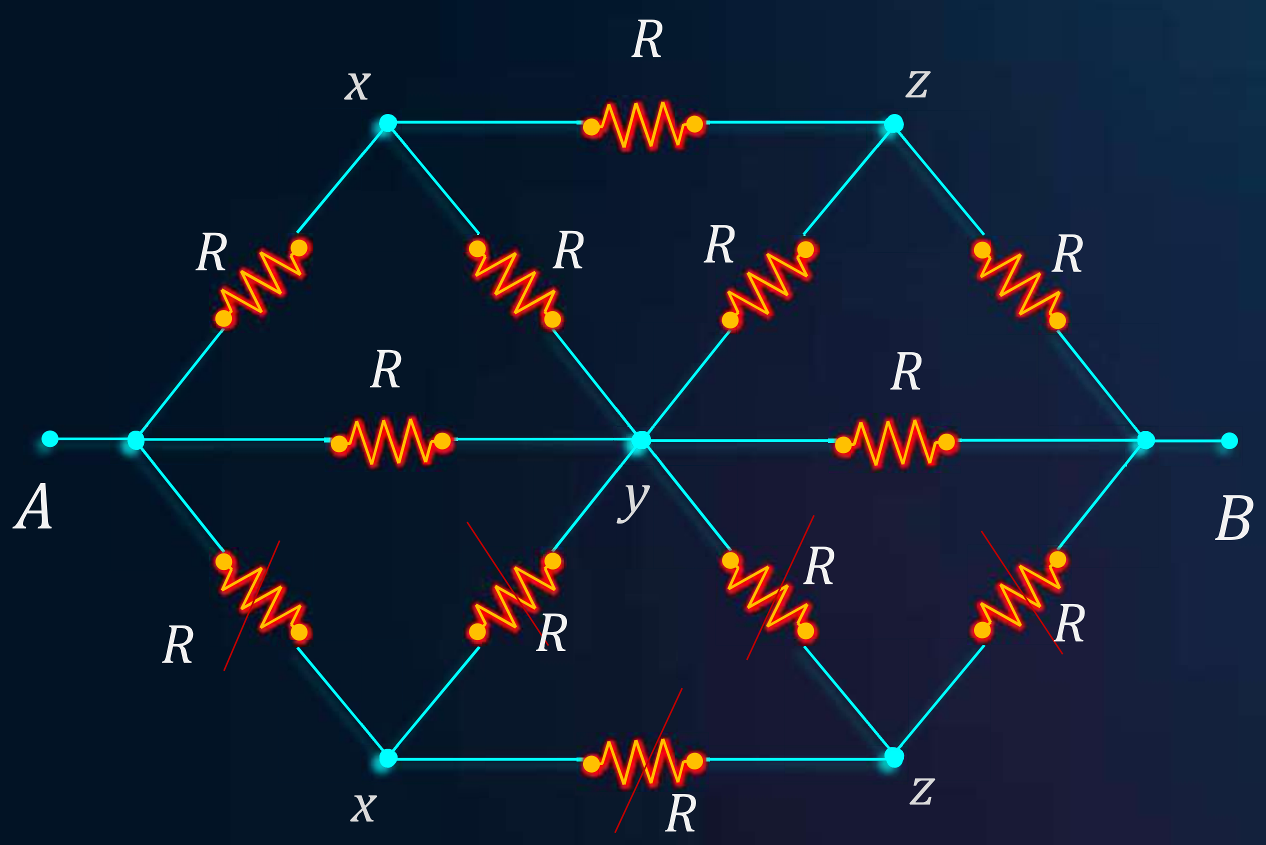


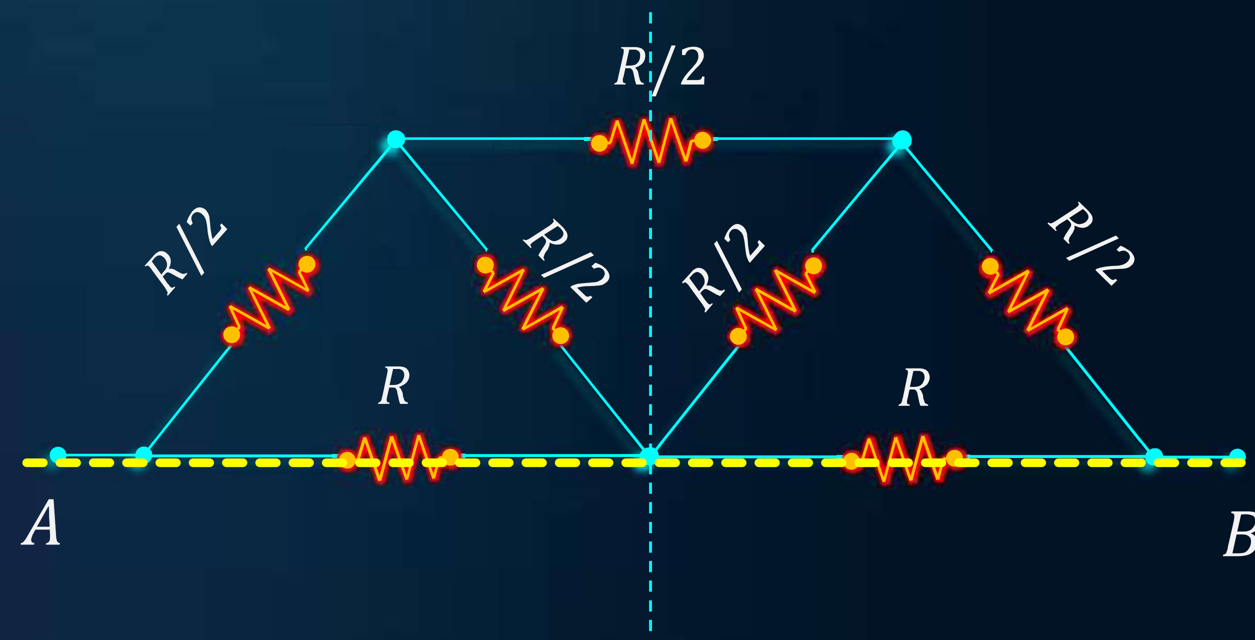
$$\frac{1}{R_{eq}} = \frac{3}{8R} + \frac{1}{2R} + \frac{3}{8R}$$

$$R_{eq} = 4R/5$$

Thus, option (c) is the correct answer.

Discussion

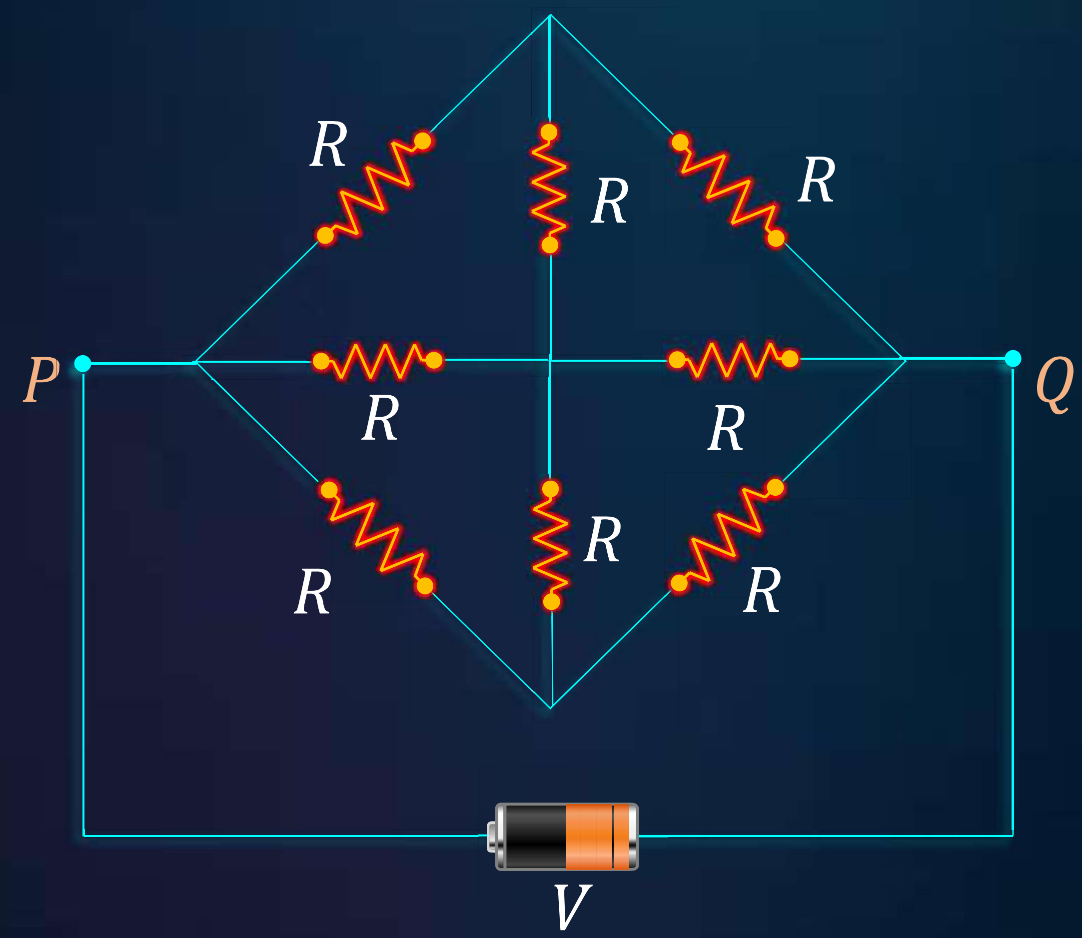




- To solve this circuit using parallel axis, first fold the circuit w.r.t the horizontal axis passing through AB and overlap those equipotential points, and then find equivalent resistance of each parallel combination.

- Then proceed further by using the mirror symmetry.

Symmetric Circuit



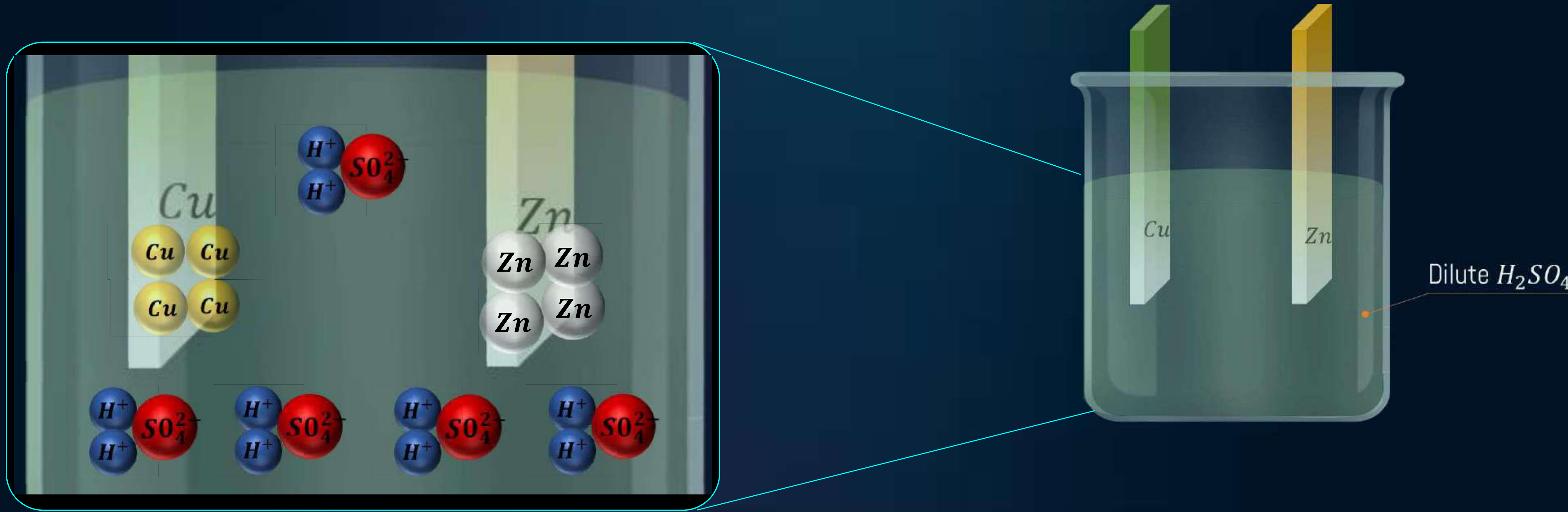
HOME ASSIGNMENT

(This circuit is symmetric about both the perpendicular and parallel axis.)

Cell

- Any device which can continuously maintain the flow of charges (electrons) in a closed circuit is called cell.
- Cells use electrolytic reactions to convert chemical energy into electrical energy.
- A group of cells is called as a battery.

Cell

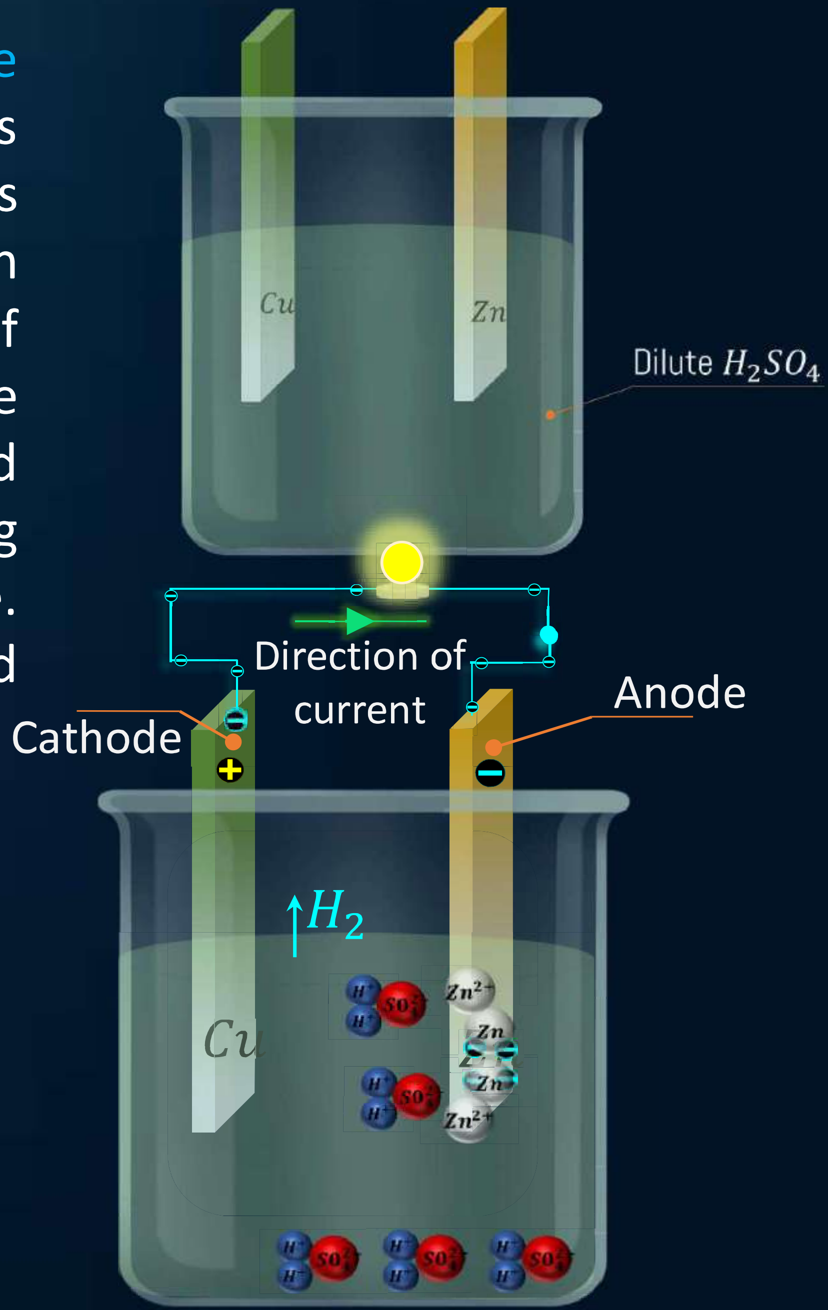


- EMF (electromotive force) is the potential difference between the two terminals of a battery or cell in an open circuit.

Working of a Cell

Whenever two dissimilar metals are immersed inside an electrolyte solution, the more reactive metal will tend to dissolve in the electrolyte as positive metal ions, leaving electrons behind on the metal plate. This phenomenon makes the more reactive metal plate negatively charged. In this case Zn is the most reactive metal. It reacts with negative SO_4^{2-} ion of the sulfuric acid solution (H_2SO_4) and forms zinc sulfate ($ZnSO_4$). As the copper is less reactive metal, the positive hydrogen ions of the sulfuric acid solution tend to get deposited on the copper plate. More zinc ions coming out in the solution means more number of electrons leave in the zinc plate. These electrons then pass through the external conductor connected between zinc and copper plates.

$$Zn + H_2SO_4 \rightarrow ZnSO_4 + H_2$$



On reaching on the copper plate, these electrons then combine with the hydrogen atoms deposited on the plate and form neutral hydrogen atoms. These atoms then combine in pairs to form molecules of hydrogen gas and the gas lastly comes up along the copper plate in form of hydrogen bubbles.

Cell

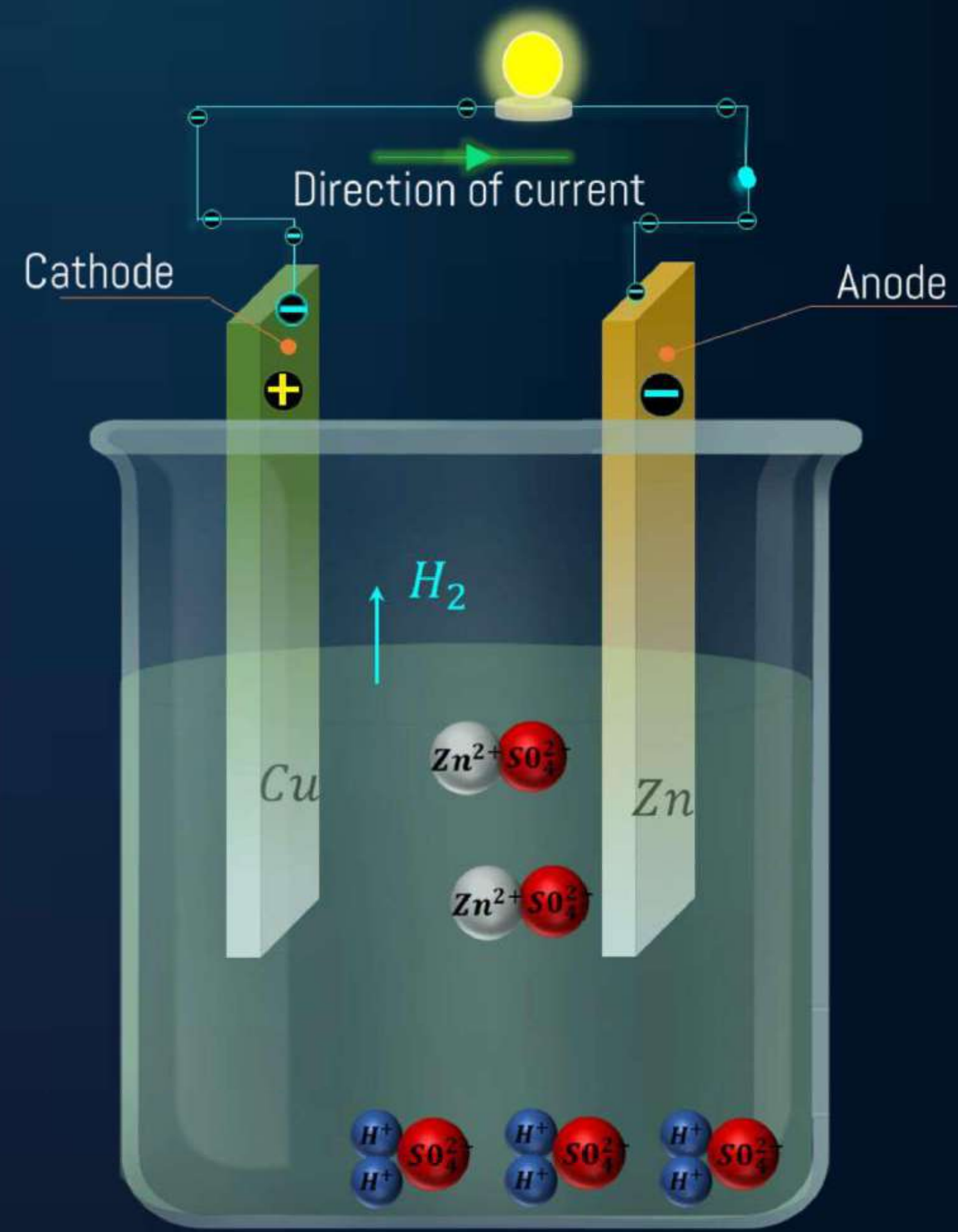
<u>At anode</u> :

$$Zn(s) \rightarrow Zn^{2+} + 2e^-$$

$$Zn^{2+} + H_2SO_4 \rightarrow ZnSO_4 + 2H^+$$

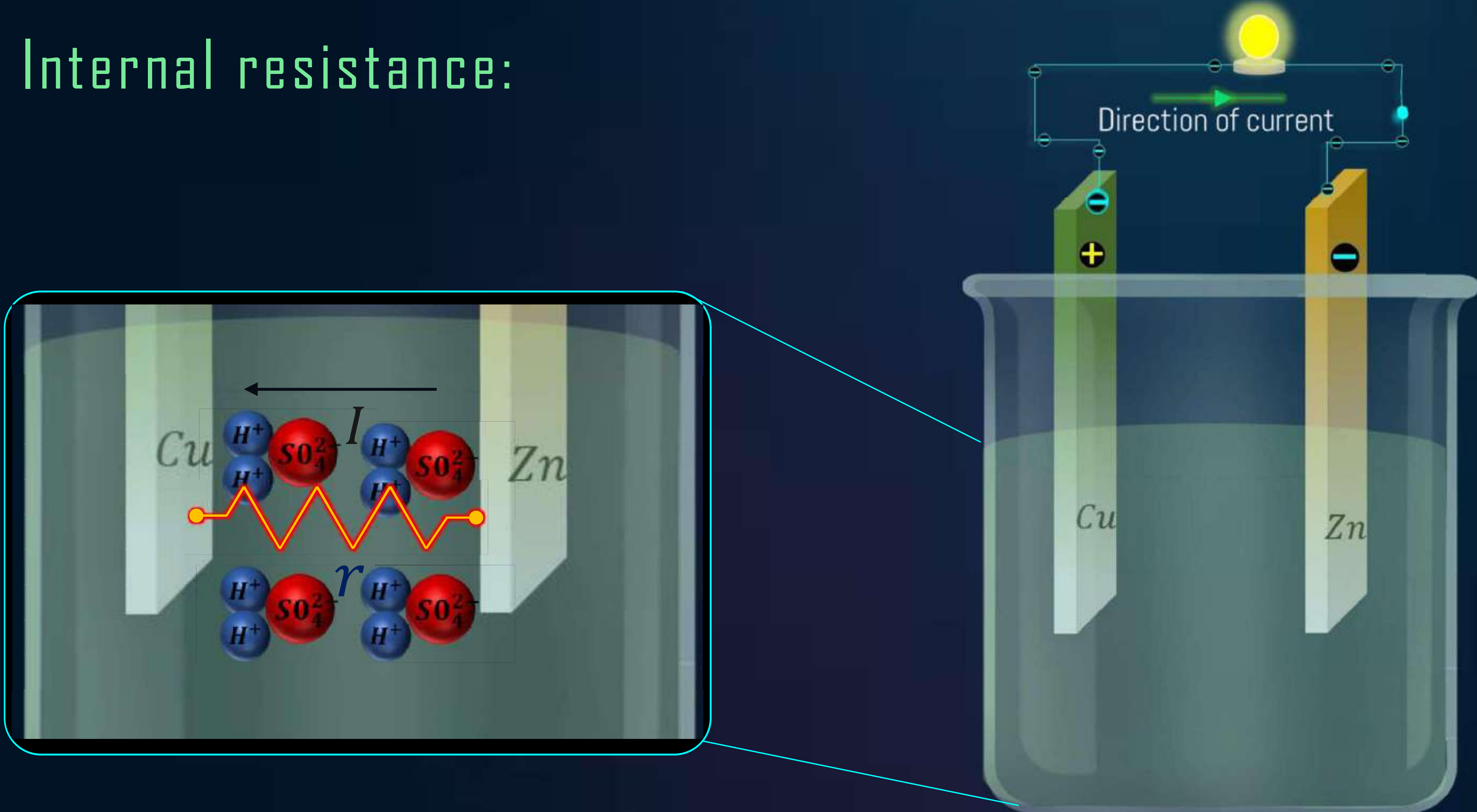
<u>At cathode</u> :

$$2H^+ + 2e^- \rightarrow H_2(g)$$



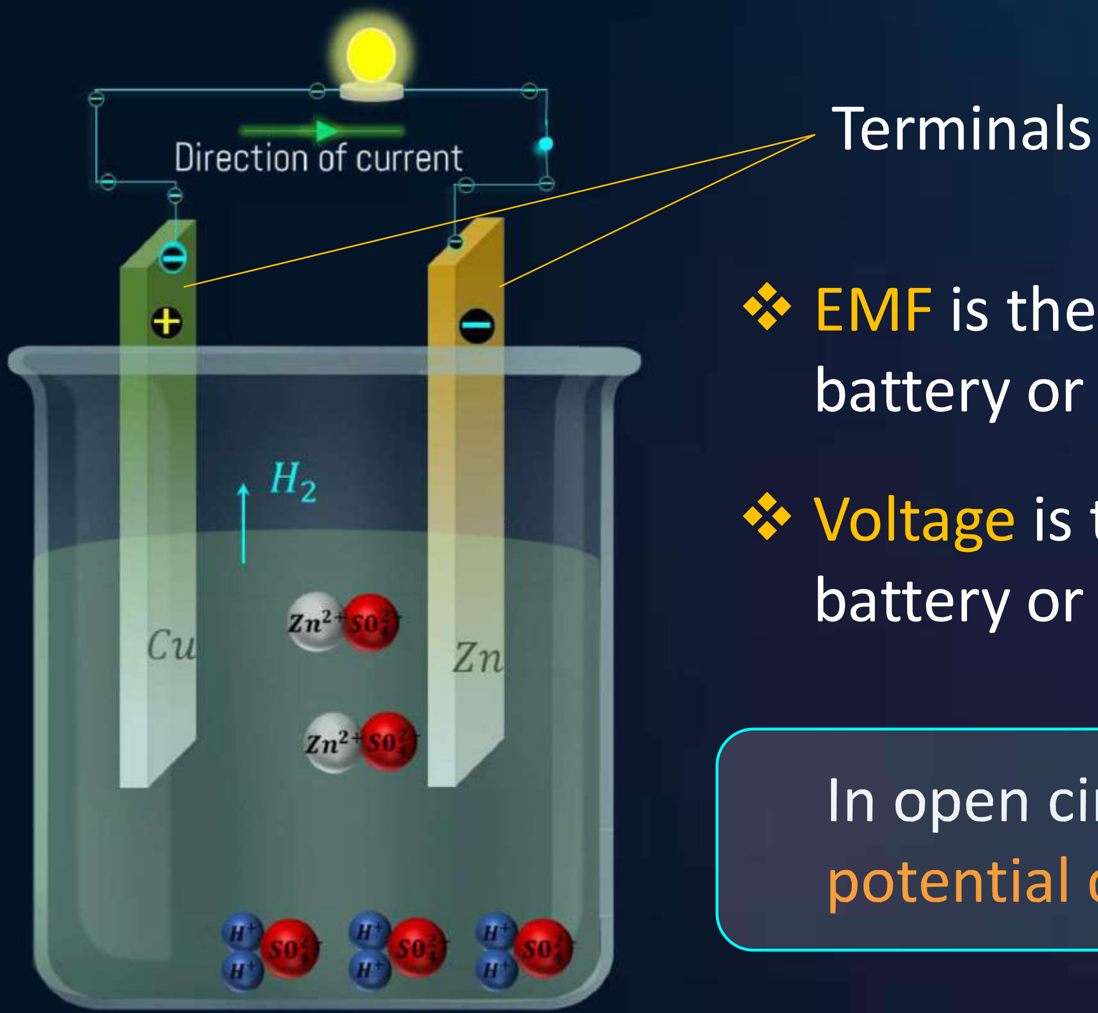
Cell

Internal resistance:



- The electrolyte through which a current flows has a finite resistance r, called internal resistance. This resistance appears due to the hinderance between H_2SO_4 atoms.

Cell

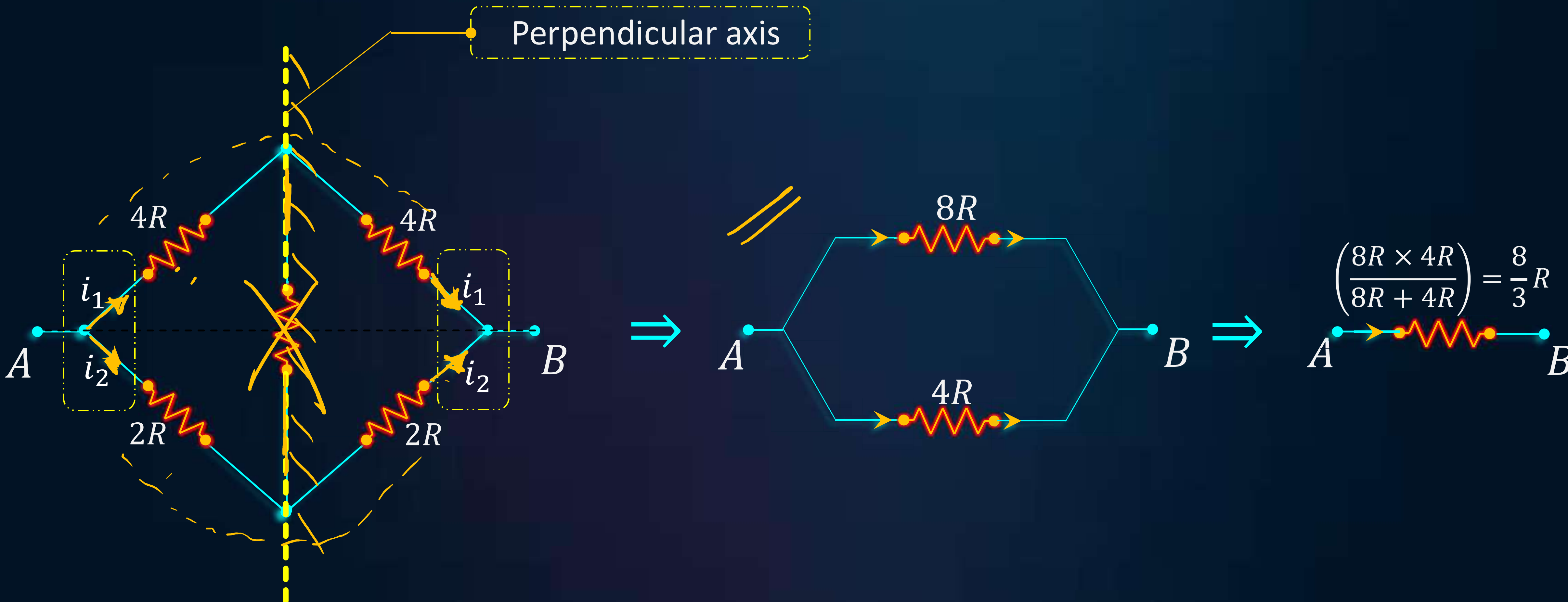


- EMF is the potential difference between the two terminals of a battery or cell in an open circuit.
- Voltage is the potential difference between the two terminals of a battery or cell in an closed circuit.

In open circuit, emf of the battery is equal to the potential difference between the terminals.

Recap

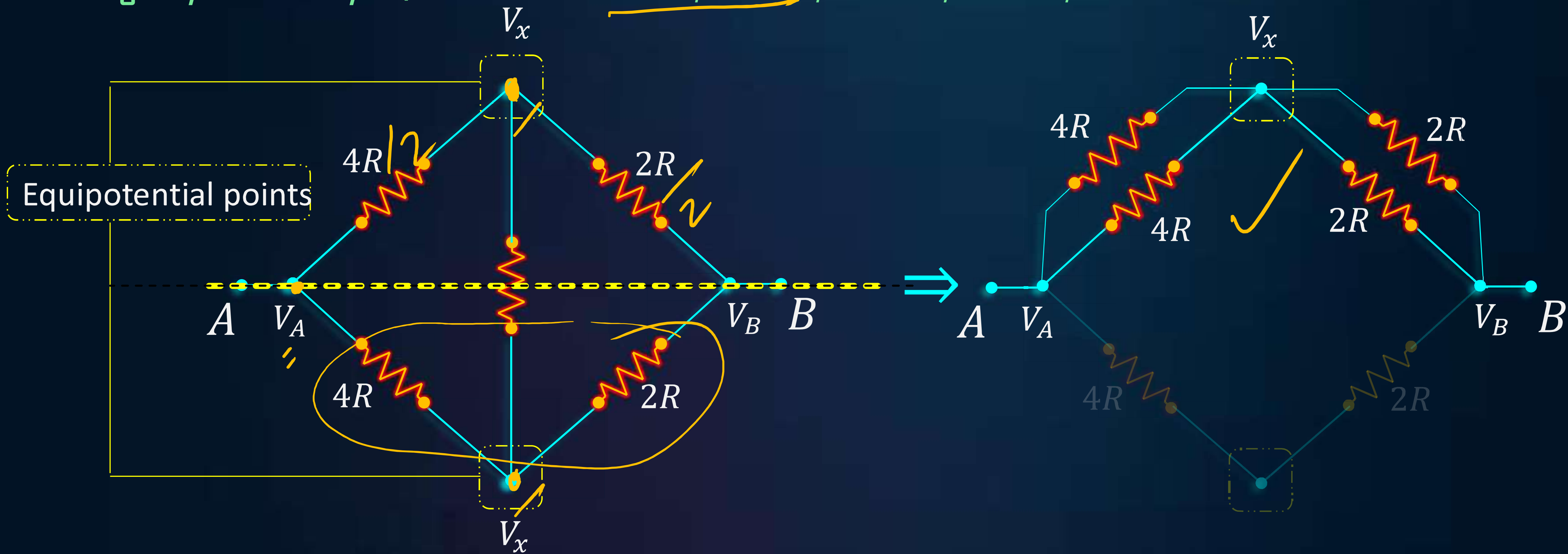
Mirror Symmetry/(Perpendicular axis of symmetry/line symmetry)

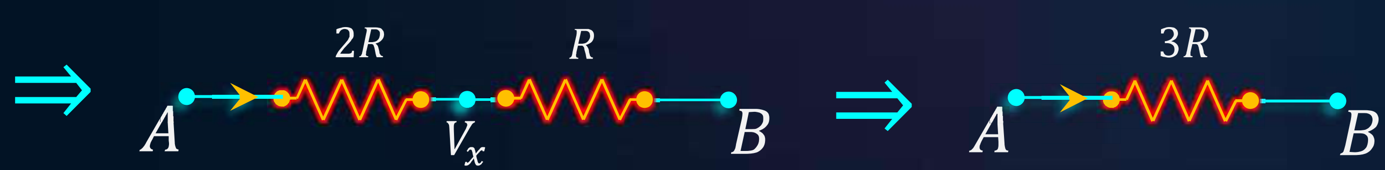


$$\left(\frac{8R \times 4R}{8R + 4R}\right) = \frac{8}{3}R$$

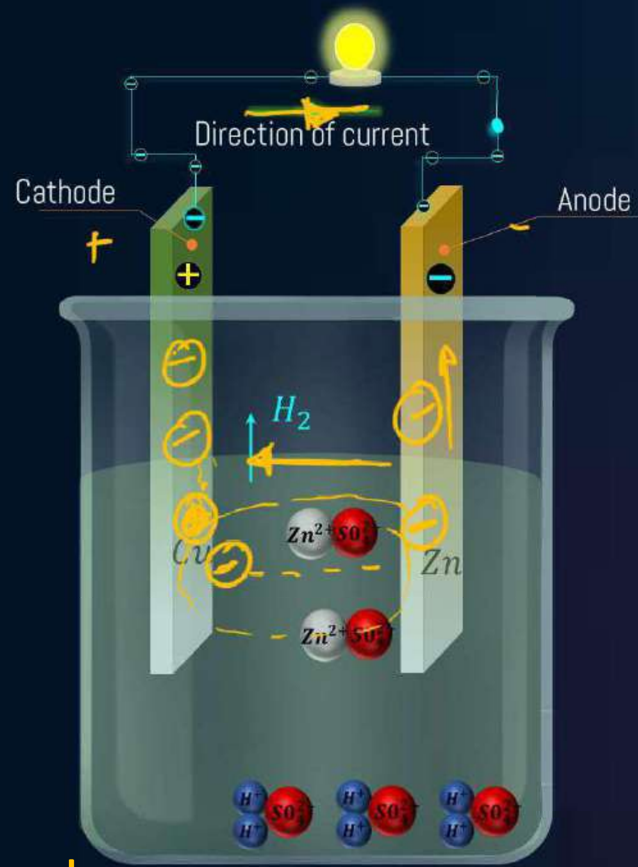
Recap

Folding Symmetry/(Parallel axis of symmetry/line symmetry)





Recap



At anode

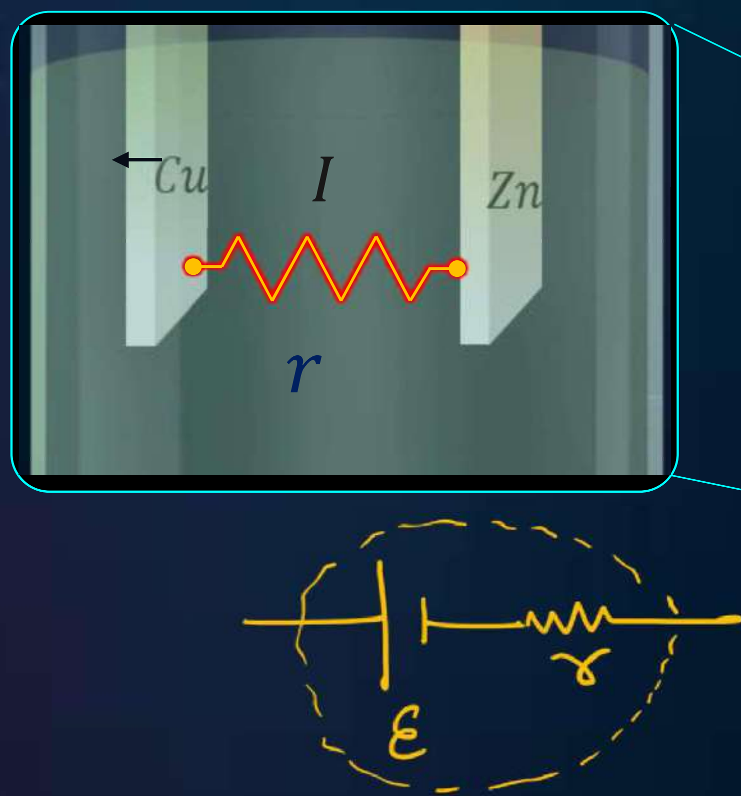
$$Zn(s) \rightarrow Zn^{2+} + 2e^-$$

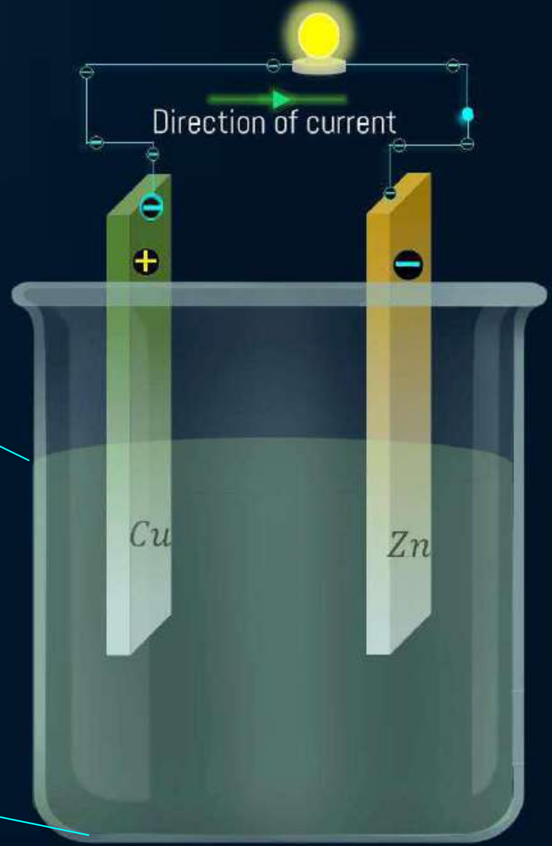
$$Zn^{2+} + H_2SO_4 \rightarrow ZnSO_4 + 2H^+$$

At cathode

$$2H^+ + 2e^- \rightarrow H_2(g)$$

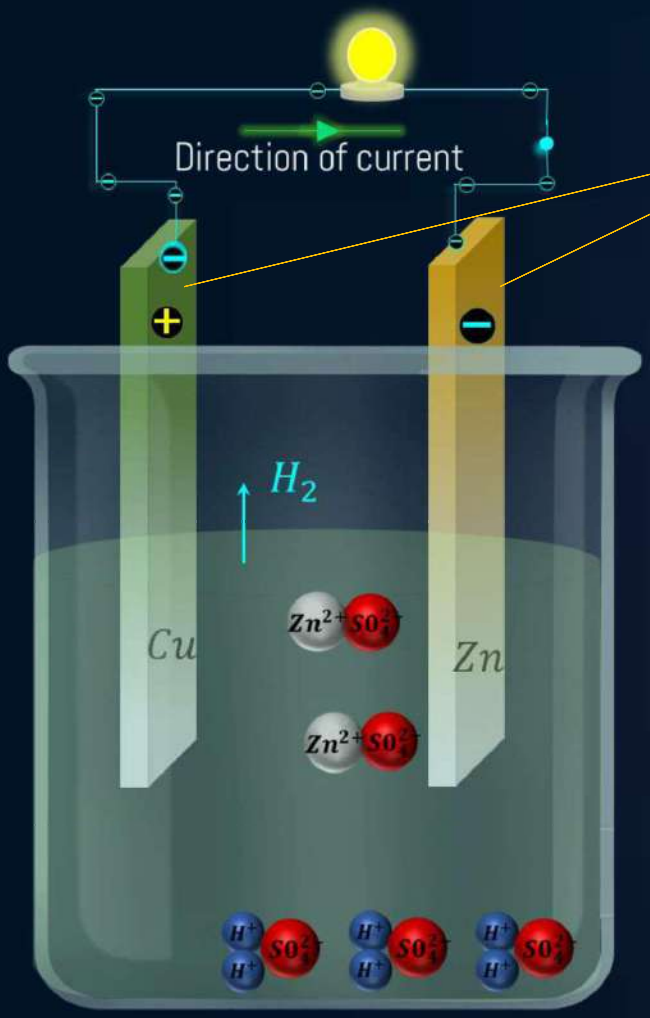
Internal resistance:





The electrolyte through which a current flows has a finite resistance r, called internal resistance.

Cell



Terminals

EMF of a cell is defined as work done by cell in moving unit positive charge in the whole circuit including the cell once.

$$E = \frac{W}{q}$$

S.I Unit $= \dfrac{Joule}{Coulomb}$ $or\ Volt$

EMF is independent of quantity of electrolyte, size of electrodes and distance between the electrodes.

Cell

Relation b/w Emf and Voltage

$$\boxed{V = E - Ir}$$

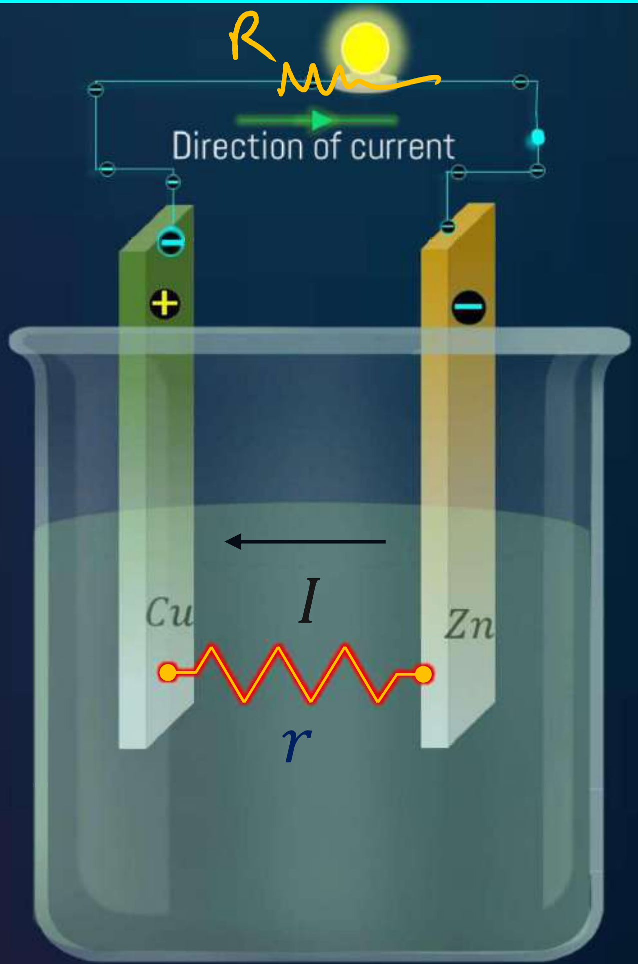
$V = IR$

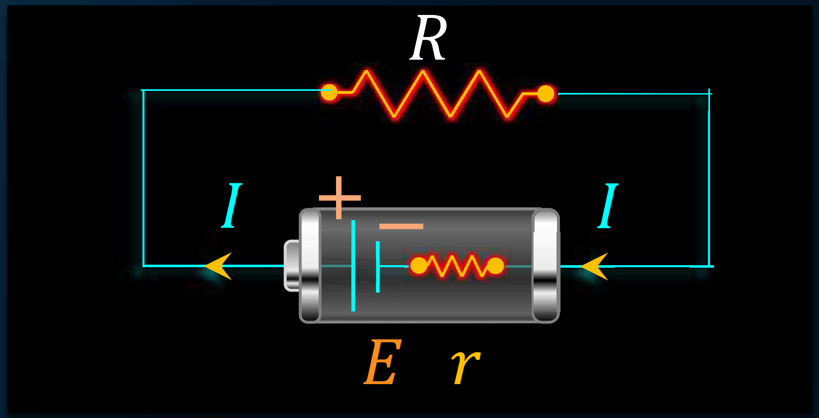
V: Voltage across terminals of a cell

E: Emf of a cell

I: Current flowing in the circuit

r: Internal resistance of a cell



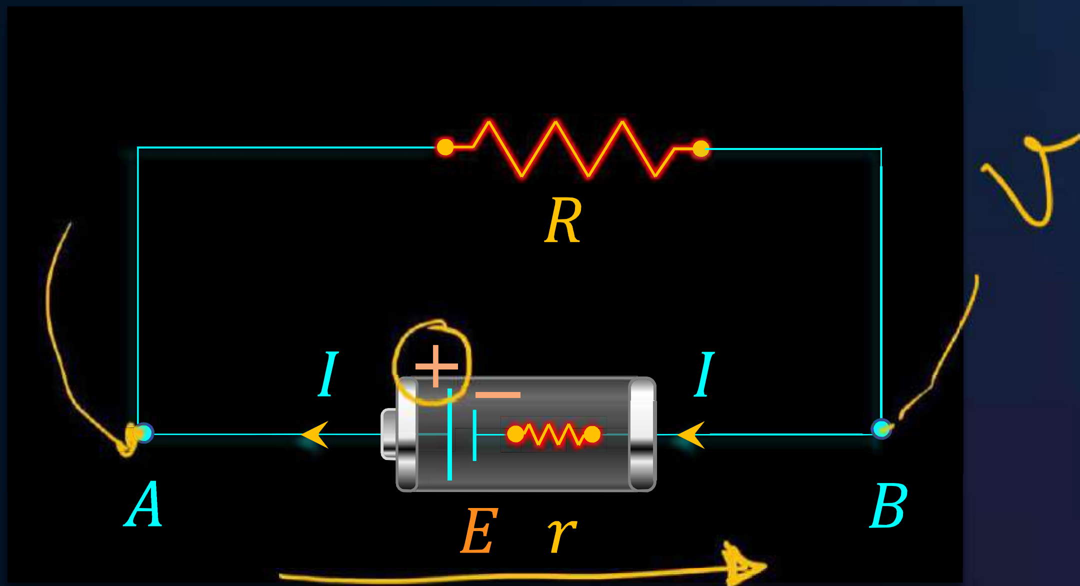


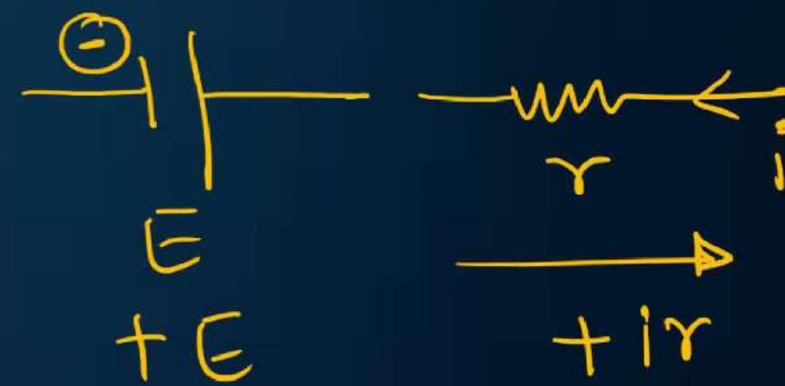
EMF is the potential difference between the two terminals of a battery or cell in an open circuit.

Voltage is the potential difference between the two terminals of a battery or cell in an closed circuit.

Cell

Discharging of a cell :





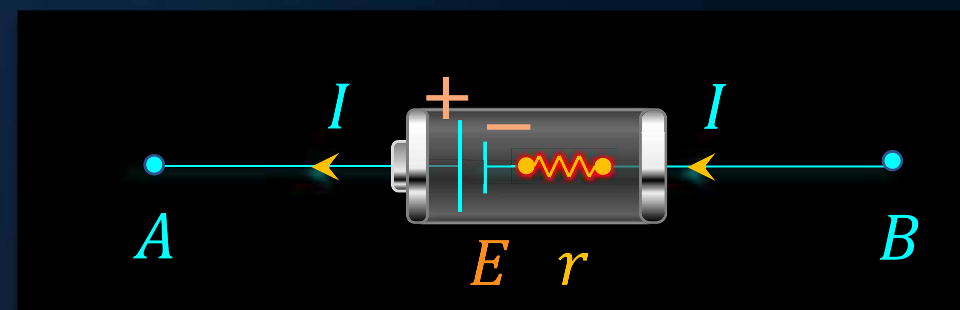
On applying KVL from A to B,

$$V_A - E + Ir = V_B$$

$$V_A - V_B = E - Ir$$

$$\boxed{V = E - Ir}$$

$$I = \frac{E}{R + r} \Rightarrow V < E$$



The cell is getting discharged as it is driving the current.

Cell

Discharging of a cell :

$$I = \frac{E}{R+r} \quad ; \quad V = IR$$

$$V = E - Ir \rightarrow 0$$

open

$$V = \frac{ER}{R+r}$$

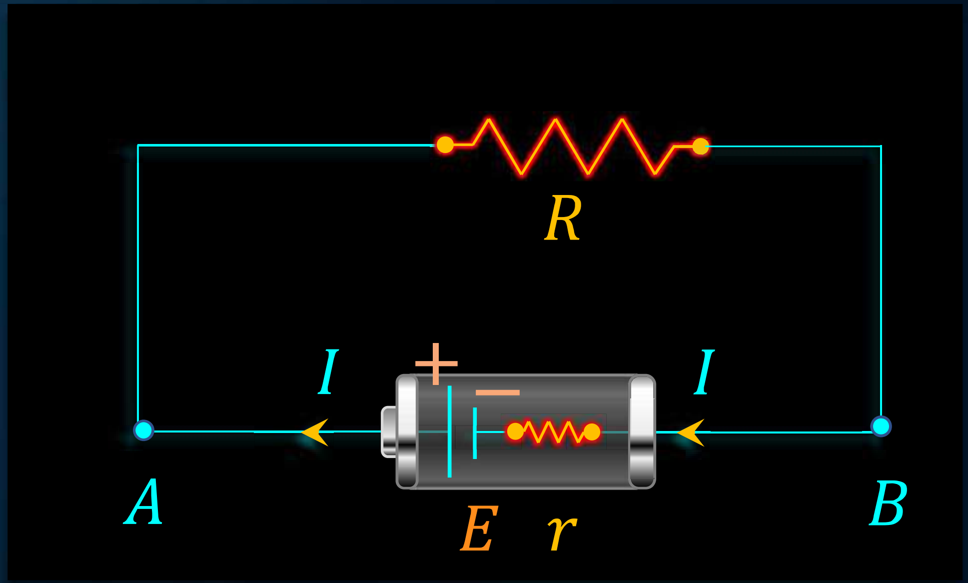
$$V = \frac{E}{\left(1 + \frac{r}{R}\right)}$$

$V = E$

$r = 0$

$I = 0$

$R = \infty$



$$V = \frac{ER}{R+r} = \frac{E}{1 + r/R}$$

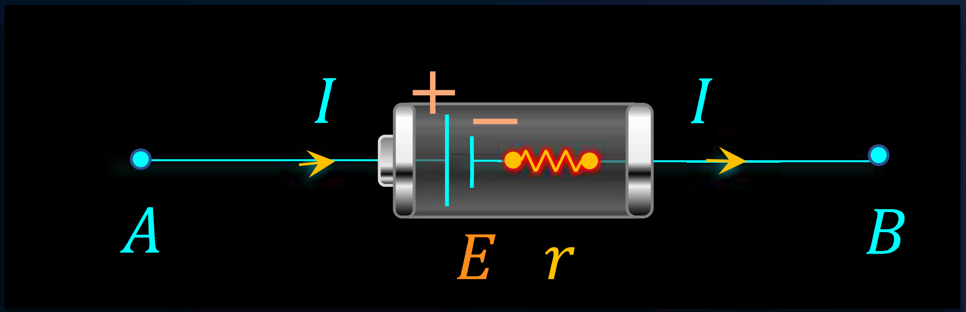
$$V = E$$

$r = 0$ (ideal cell)

$I = 0$ (open circuit)

$R = \infty$ (ideal case)

Charging of a cell :



On applying KVL from A to B,

$$V_A - E - I r = V_B$$

$$\underbrace{V_A - V_B}_{\downarrow} = E + I r$$

$$\boxed{V = E + I r}$$

$$\therefore V > E$$

The cell is getting charged as the current is driven through the cell

Question

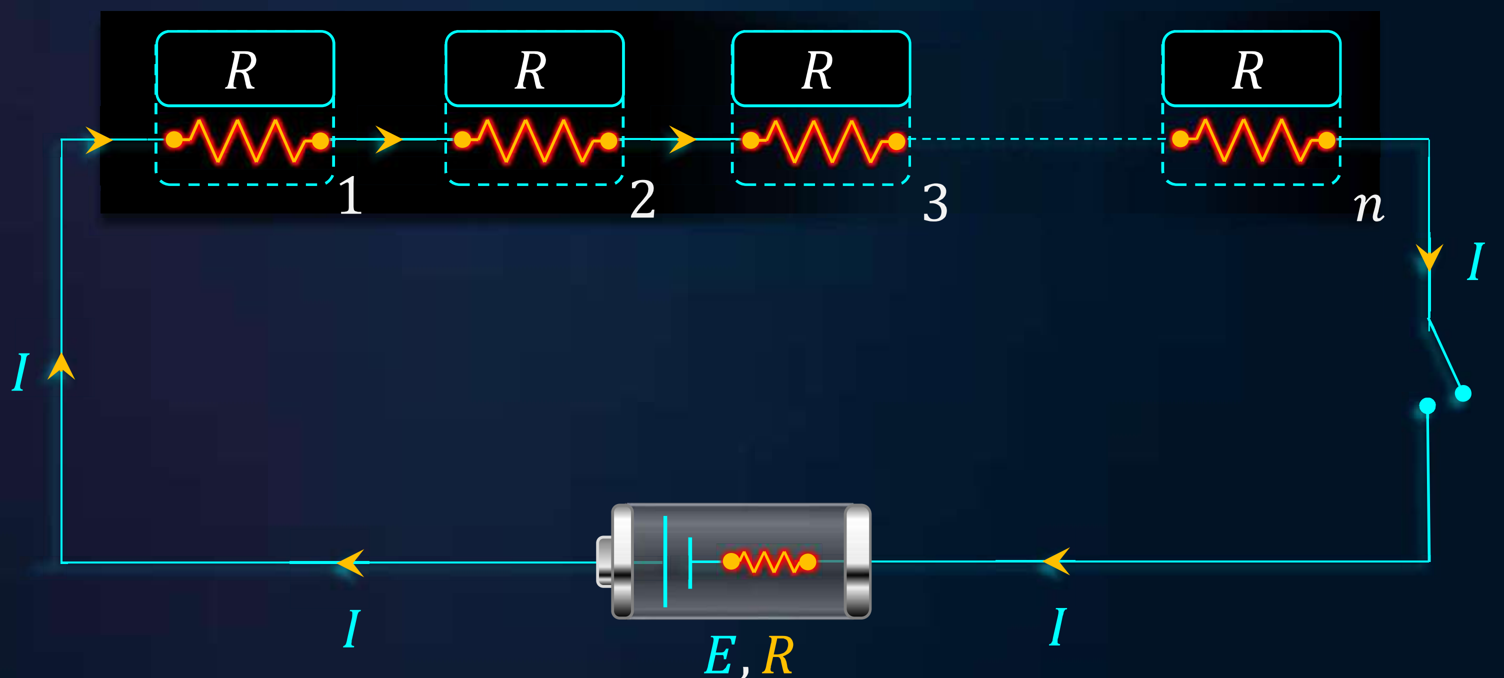
A set of 'n' equal resistors, of value 'R' each, are connected in series to a battery of e.m.f. 'E' and internal resistance 'R'. The current drawn is I. Now the 'n' resistors are connected on parallel to the same battery. Then the current drawn from battery becomes $10I$. The value of 'n' is

a) 10

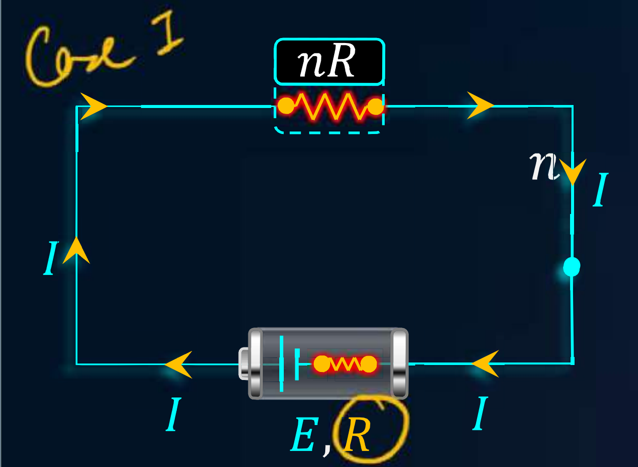
b) 11

c) 20

d) 9



Discussion

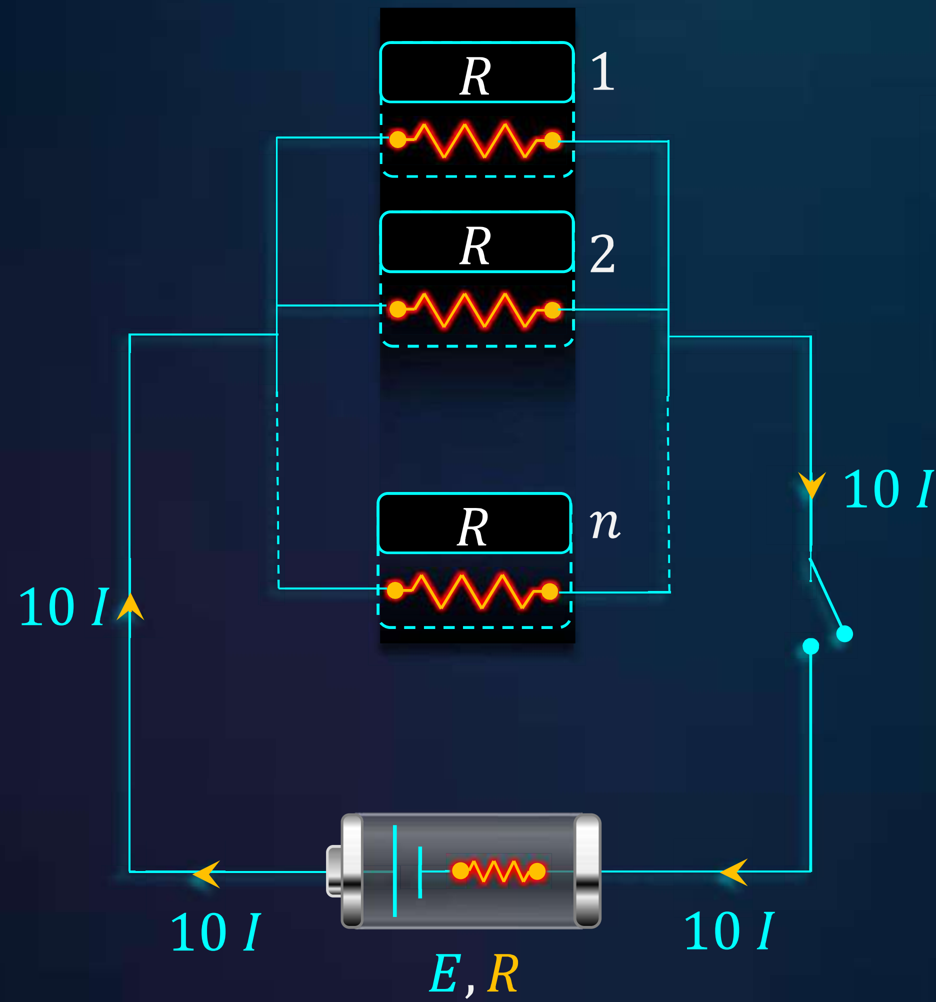


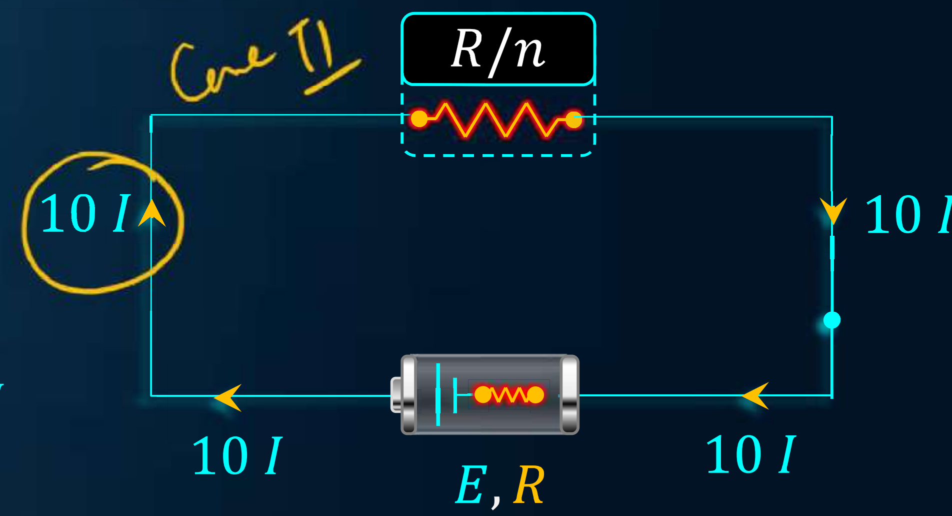
Current drawn when resistors are in series,

Case I

$$I = \frac{E}{nR+R}$$

$$I = \frac{E}{R(n+1)} \quad \text{— (1)}$$





Current drawn when resistors are in parallel,

Case II

$$10\,I = \frac{E}{\frac{R}{n}+R}$$

$$10\,I = \frac{E}{R\left(1+\frac{1}{n}\right)} \quad \text{— (II)}$$

Discussion

$$I = \frac{E}{(n+1)R} \quad \ldots(1)$$

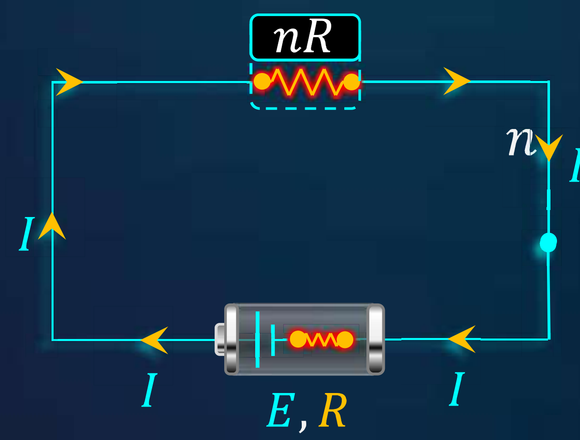
$$10\,I = \frac{E}{\left(\frac{1}{n}+1\right)R} \quad \ldots(2)$$

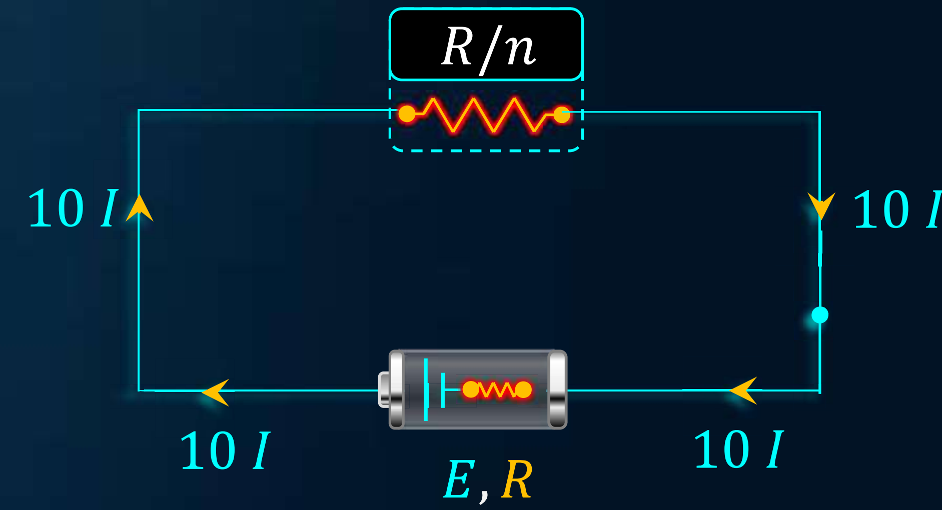
From equation (1) and equation (2),

$$10\,\frac{\cancel{E}}{(n+1)\cancel{R}} = \frac{\cancel{E}}{\left(1+\frac{1}{n}\right)\cancel{R}}$$

$$\Rightarrow n+1 = 10\left(1+\frac{1}{n}\right)$$

$$\Rightarrow n - \frac{10}{n} = 9$$





$$\Rightarrow n^2 - 9n - 10 = 0$$

$$\Rightarrow (n+1)(n-10) = 0$$

$$\Rightarrow n = -1 \quad \boxed{n = 10}$$

Thus, option (a) is the correct answer.

Question

The internal resistance of a $2.1\,V$ cell which gives a current of $0.2\,A$ through a resistance of $10\,\Omega$ is:

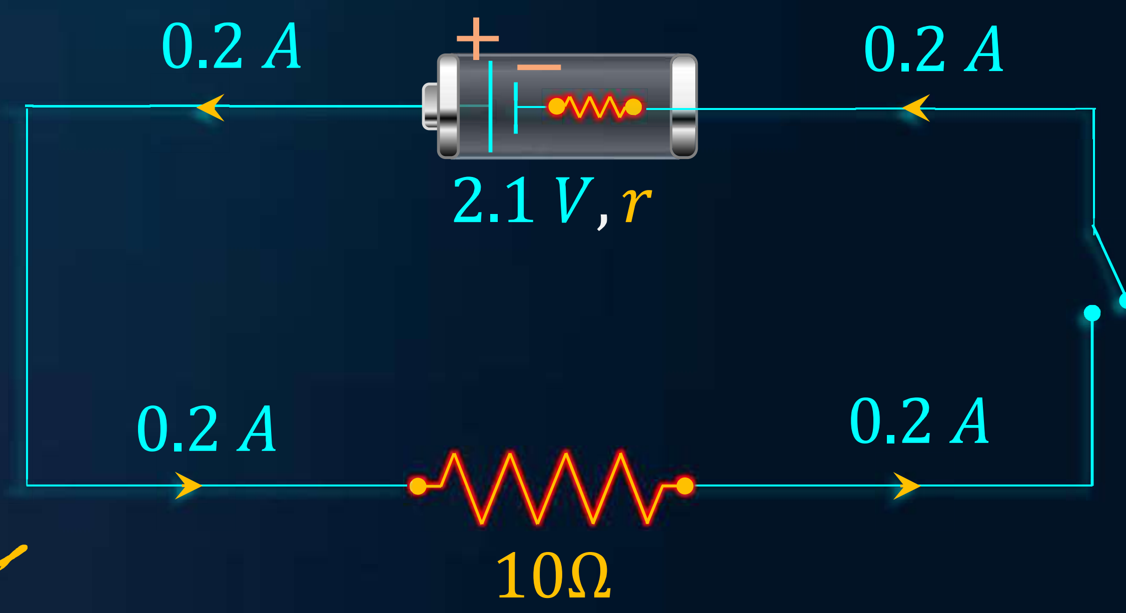
- a) $0.2\,\Omega$
- b) $0.5\,\Omega$
- c) $0.8\,\Omega$
- d) $1.0\,\Omega$

$E = 2.1\,V$

$I = 0.2\,A$

$R = 10\,\Omega$

$$I = \frac{E}{R + r}$$



Discussion

$$I = \frac{E}{R + r}$$

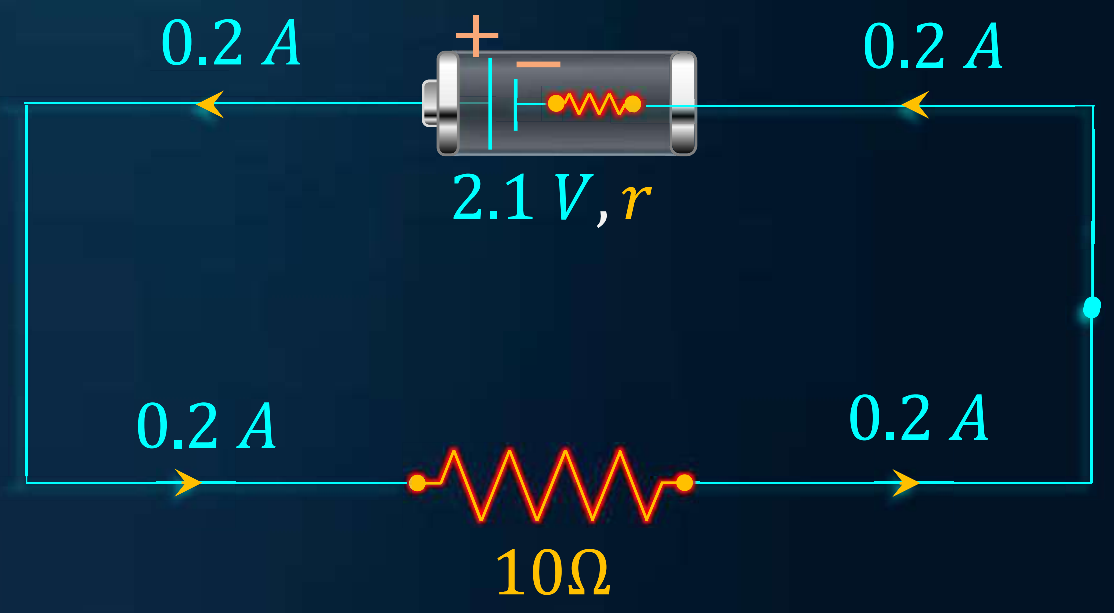
$$0.2 = \frac{2.1}{10 + r}$$

$$2 + 0.2r = 2.1$$

$$r = \frac{2.1 - 2}{0.2}$$

$$r = 0.5\ \Omega$$

Thus, option (b) is the correct answer.



Question

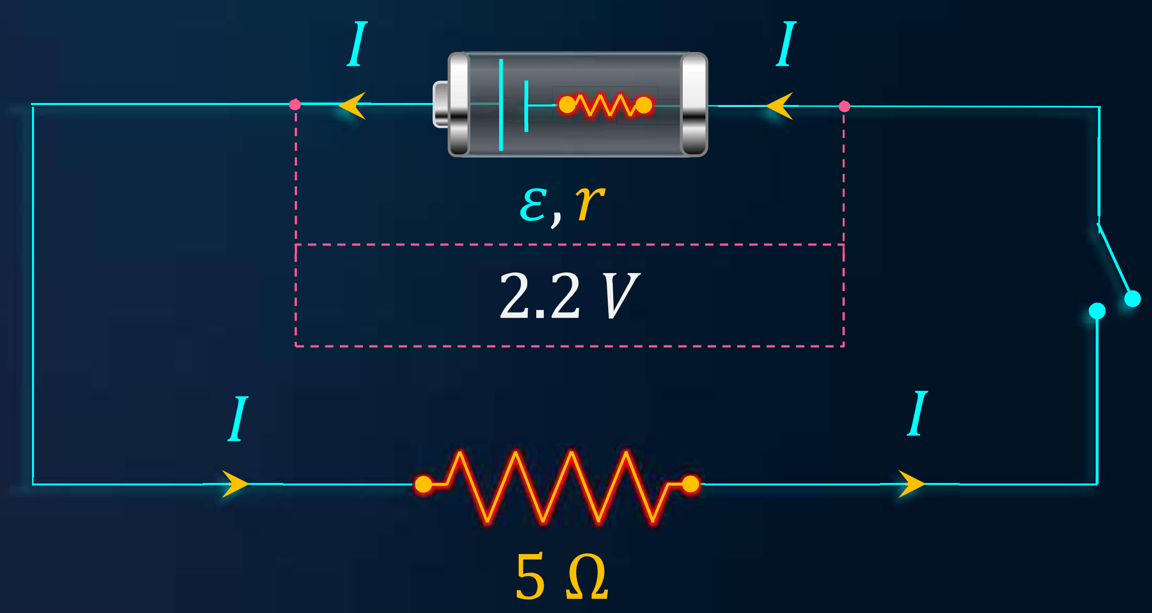
For a cell, terminal potential difference is $2.2\ V$ when circuit is open and reduce to $1.8\ V$ when cell is connected to a resistance of $R = 5\ \Omega$. Determine internal resistance of cell (r).

a) $\frac{10}{9}\ \Omega$

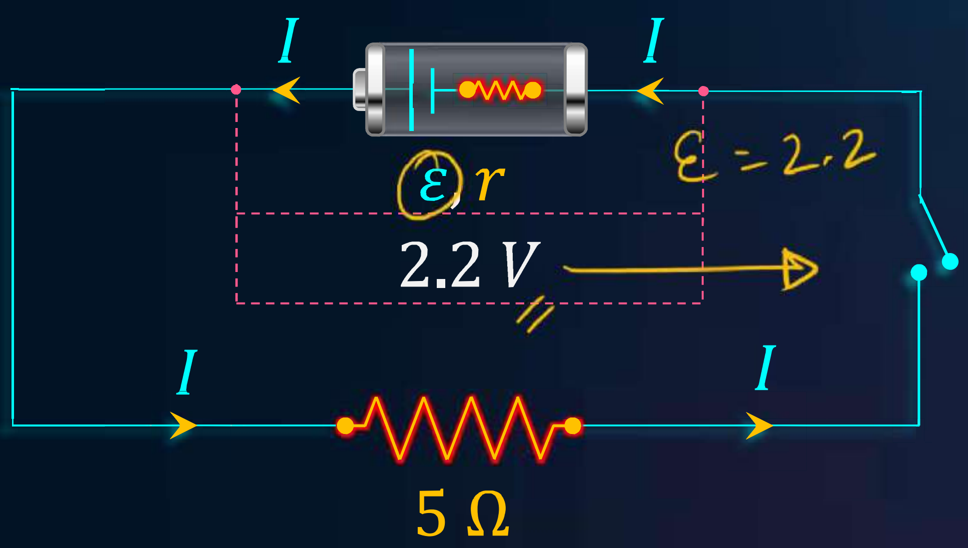
b) $\frac{9}{10}\ \Omega$

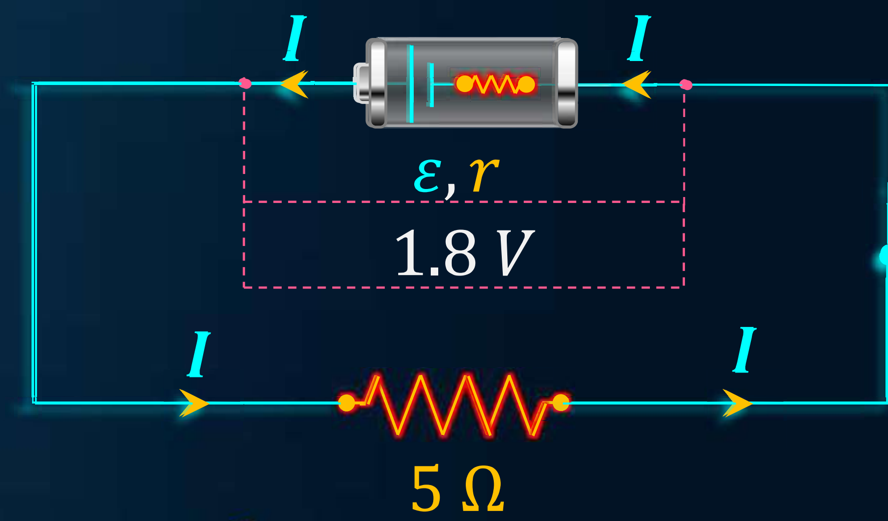
c) $\frac{11}{9}\ \Omega$

d) $\frac{5}{9}\ \Omega$



Discussion





For Closed circuit,

$V = 1.8 \rightarrow V = IR$

$\mathcal{E} = 2.2$

$R = 5\,\Omega \qquad 1.8 = I\,5$

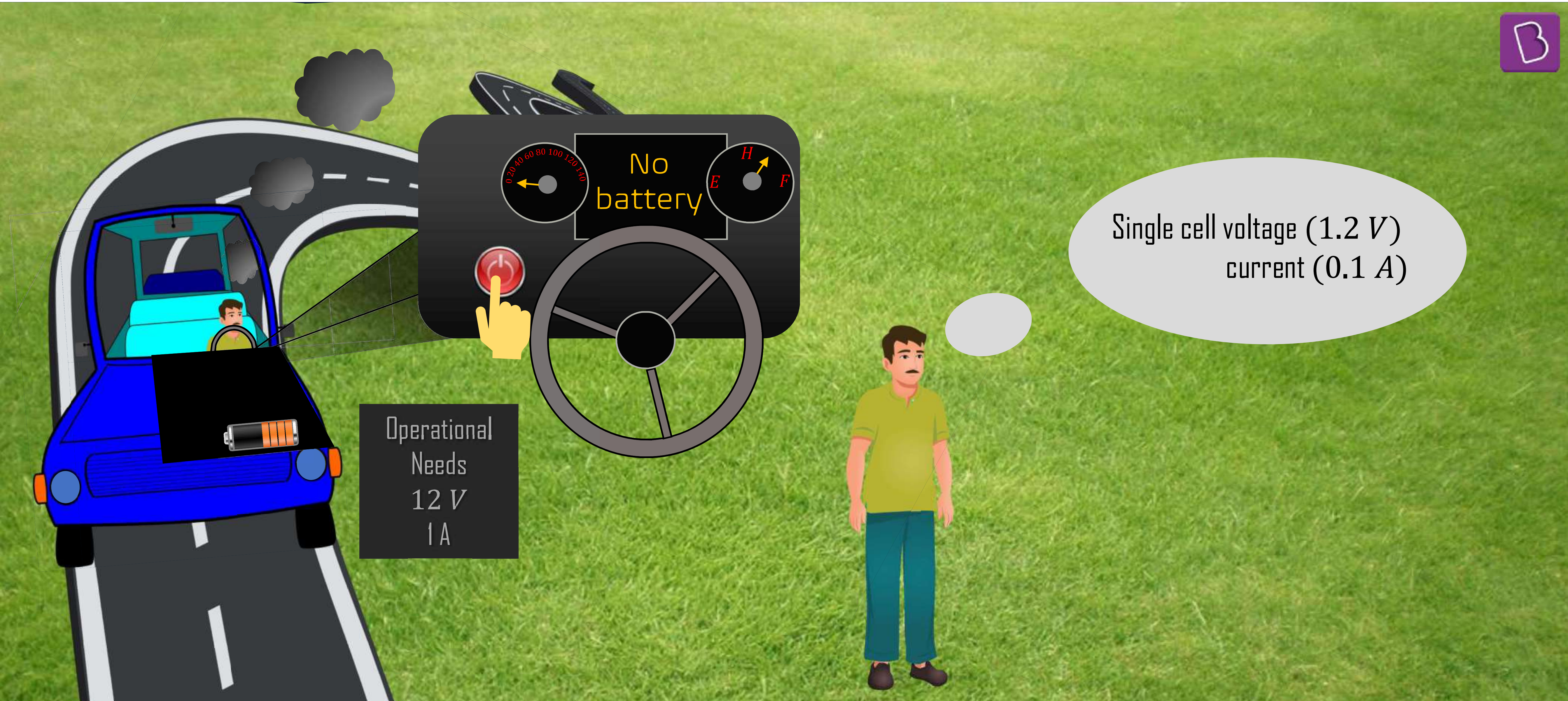
$I = \frac{1.8}{5}\,A$

$V = \mathcal{E} - Ir$

$1.8 = 2.2 - \frac{1.8}{5} \times r$

$$\Rightarrow r = \frac{10}{9}\,\Omega$$

Thus, option (a) is the correct answer.

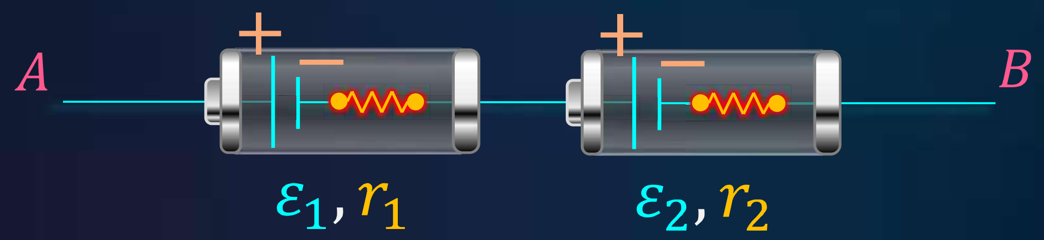


In many daily use cases we use combination of cells to cater our voltage and current requirements.

Combination of Cells

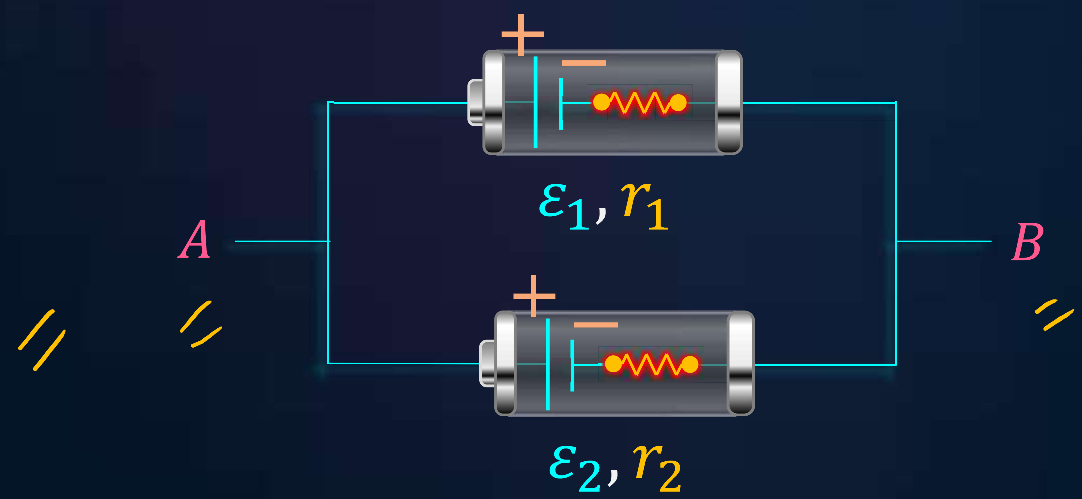
Series combination:

One terminal of a cell joined with only one terminal of other cell.



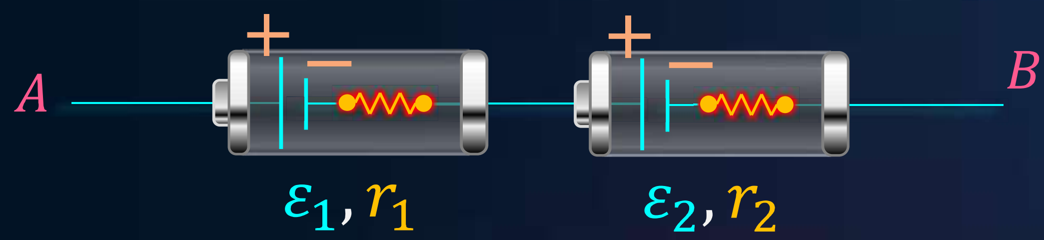
Parallel combination:

One terminal of all cells should join together, Similarly other terminals too should join together.

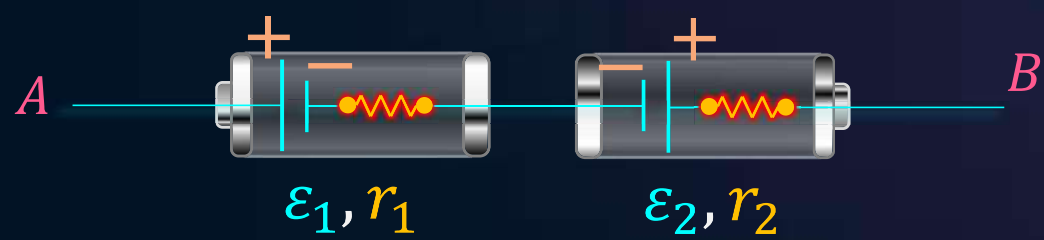


Series Combination of Cells

Polarity

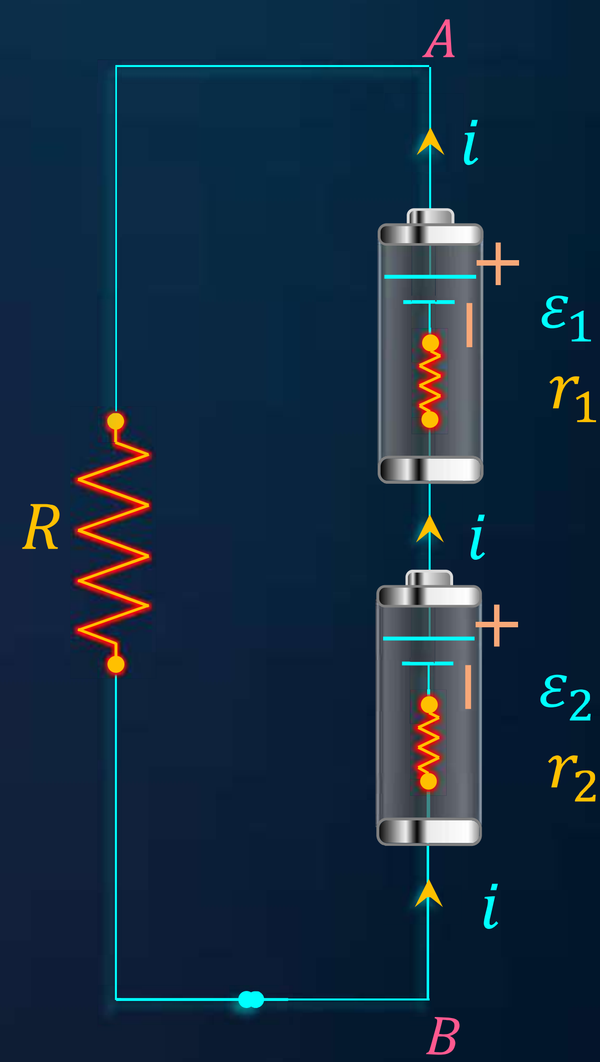


Same polarity



Opposite polarity

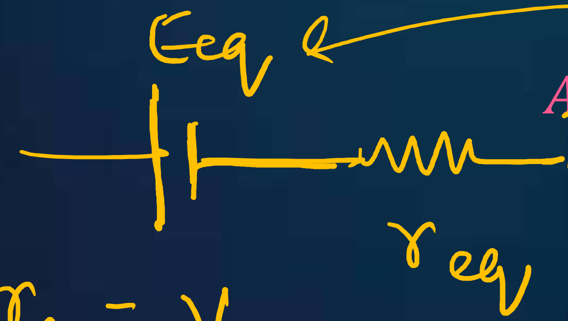
Same polarity

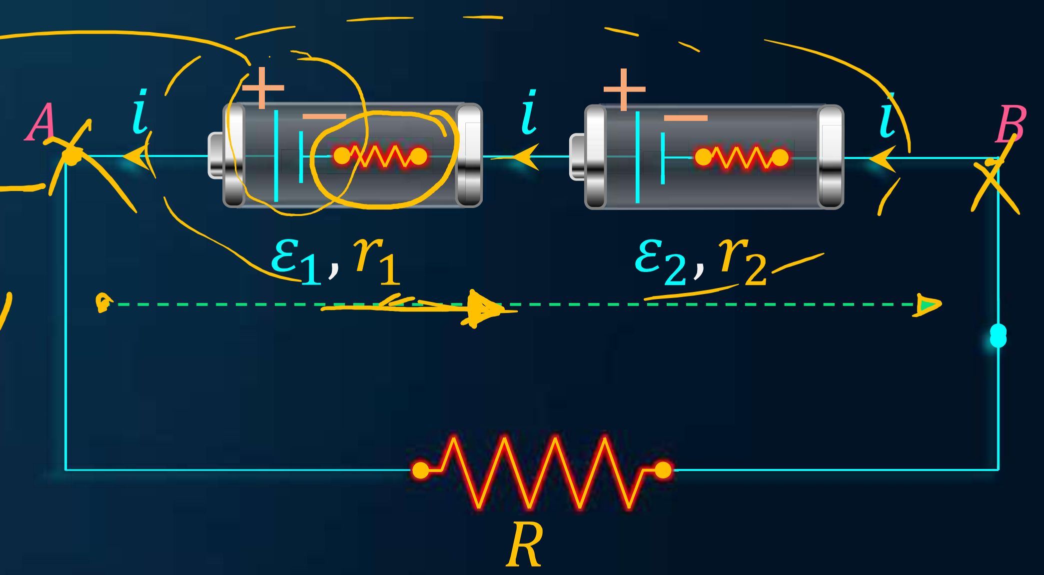


- Same polarity
- Same current flow through all cells

Series Combination of Cells

Apply KVL from A to B



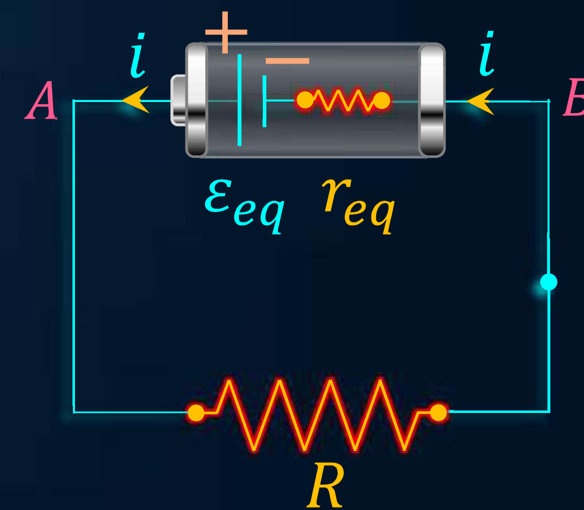


$$V_A - \varepsilon_1 + i r_1 - \varepsilon_2 + i r_2 = V_B$$

$$V_A - V_B = \varepsilon_1 - i r_1 + \varepsilon_2 - i r_2$$

$$V = \varepsilon_1 + \varepsilon_2 - i(r_1 + r_2)$$

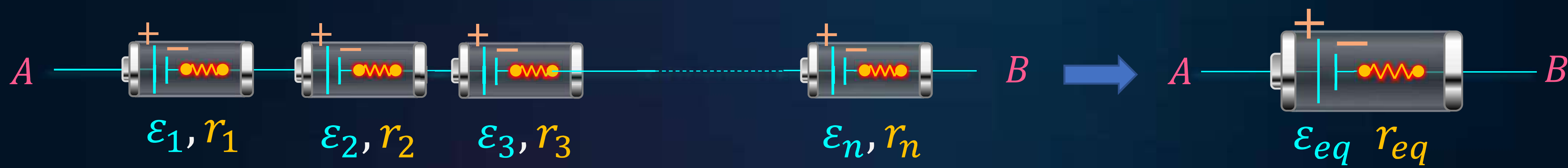
$$V = \varepsilon_{eq} - i\, r_{eq}$$



$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 \; ; \; r_{eq} = r_1 + r_2$$

Series Combination of Cells

- n non-identical cells are connected in series with same polarity



$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \cdots \varepsilon_n$$

$$r_{eq} = r_1 + r_2 \cdots r_n$$

$$\boxed{\varepsilon_{eq} = \sum_{i=1}^{n} \varepsilon_i}$$

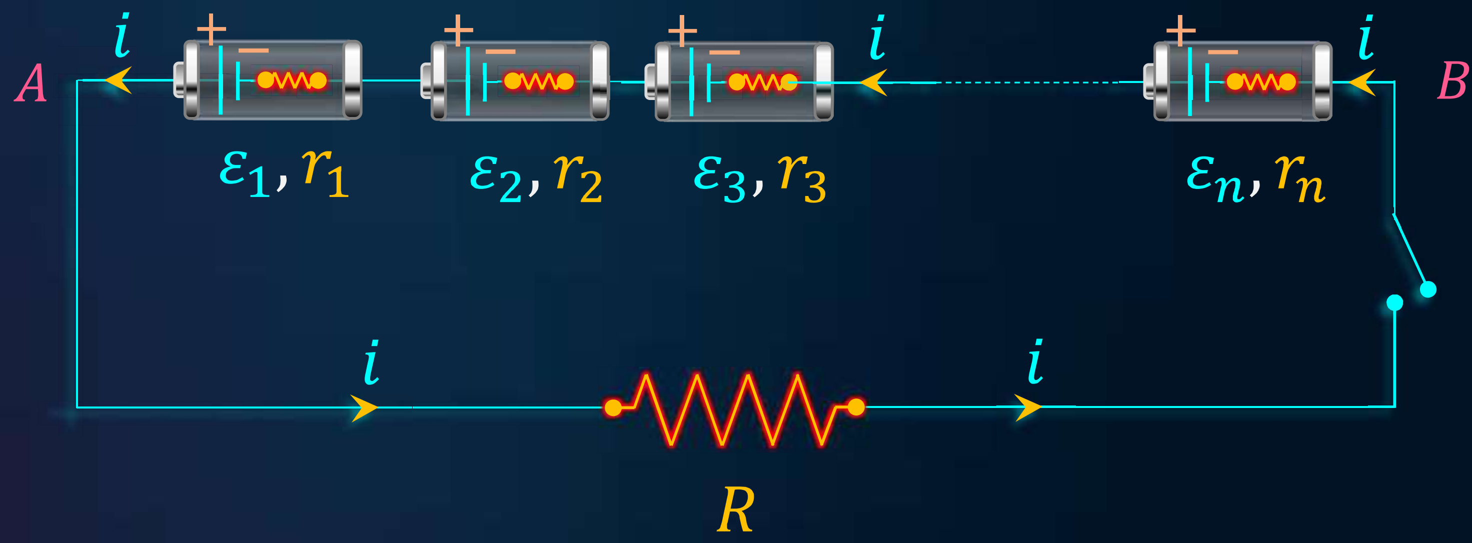
$$\boxed{r_{eq} = \sum_{i=1}^{n} r_i}$$

Series Combination of Cells

- n non-identical cells are connected in series with same polarity

$$\varepsilon_{eq} = \sum_{i=1}^{n} \varepsilon_i$$

$$r_{eq} = \sum_{i=1}^{n} r_i$$



$$i = \frac{E}{R + r_{eq}}$$

$$R_{eq} = R + r_{eq}$$

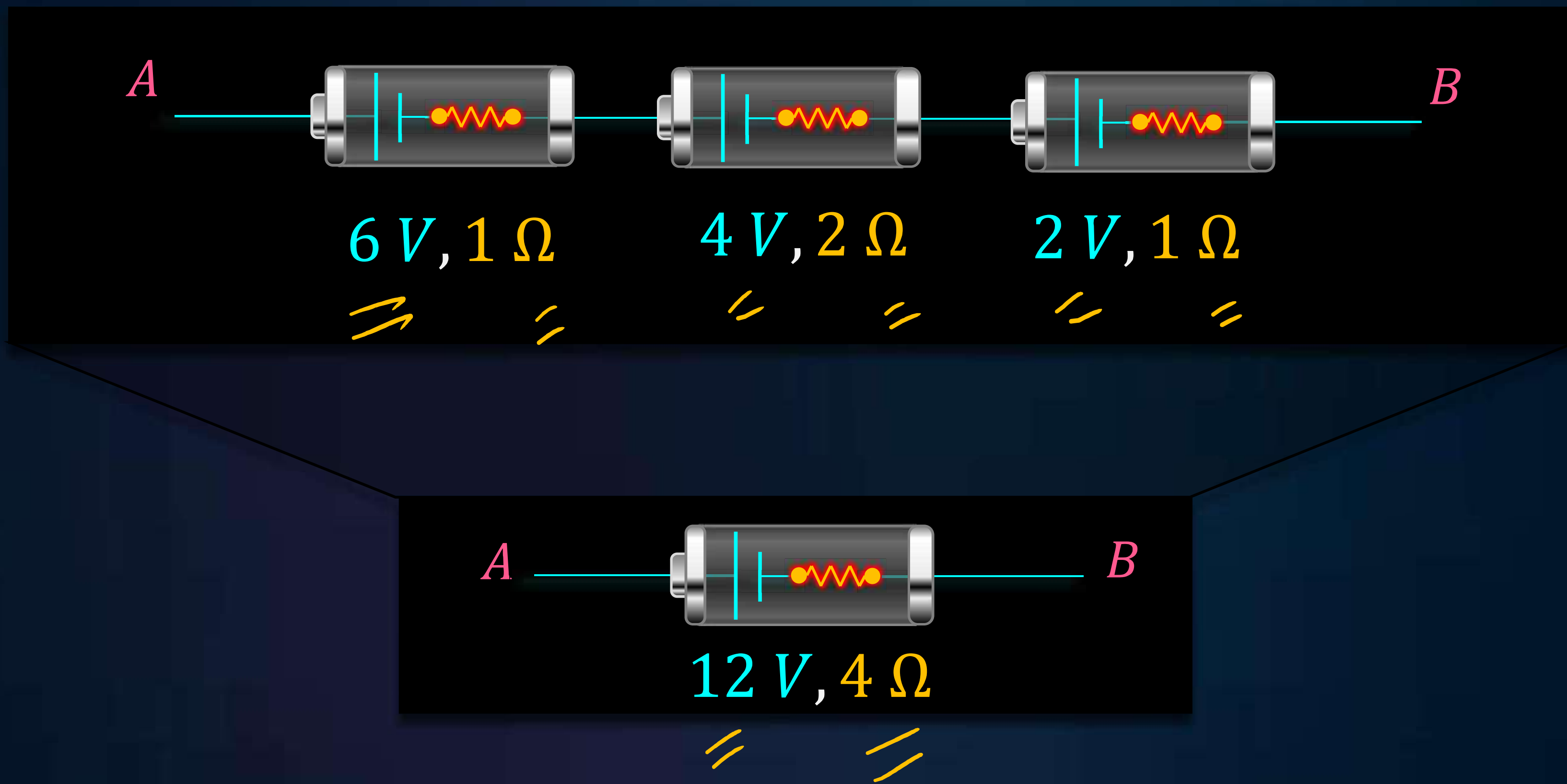
$$i = \frac{\varepsilon_{eq}}{R_{eq}}$$

External resistance and cell resistances are in series connection

$$R_{eq} = R + r_1 + r_2 + r_3 + \cdots r_n$$

$$i = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots \varepsilon_n}{R + r_1 + r_2 + r_3 + \cdots r_n}$$

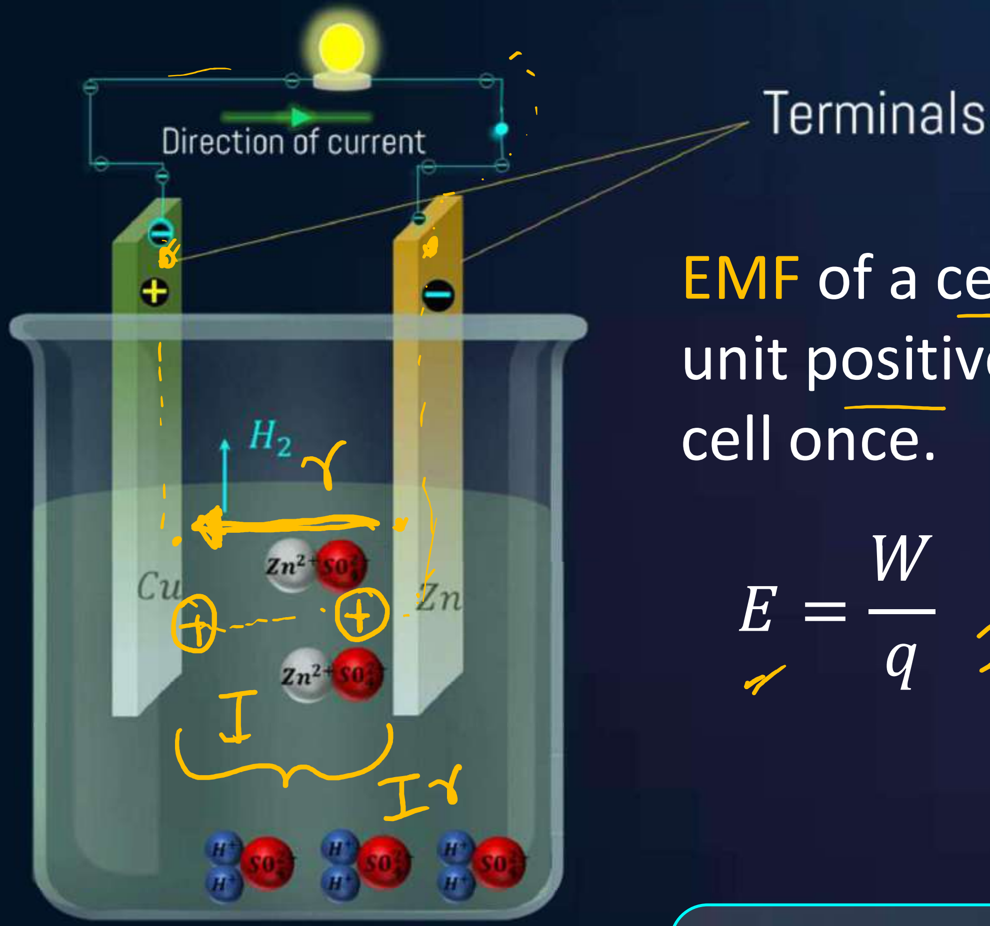
Series Combination of Cells



Recap

Emf of a cell

$$\mathcal{E} = V + Ir$$



EMF of a cell is defined as work done by cell in moving unit positive charge in the whole circuit including the cell once.

$$E = \frac{W}{q}$$ S.I Unit $= \frac{Joule}{Coulomb}$ or $Volt$

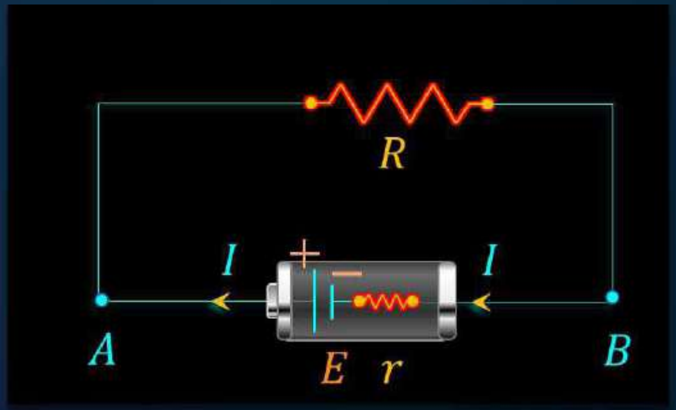
$$\boxed{V = \mathcal{E} - Ir}$$

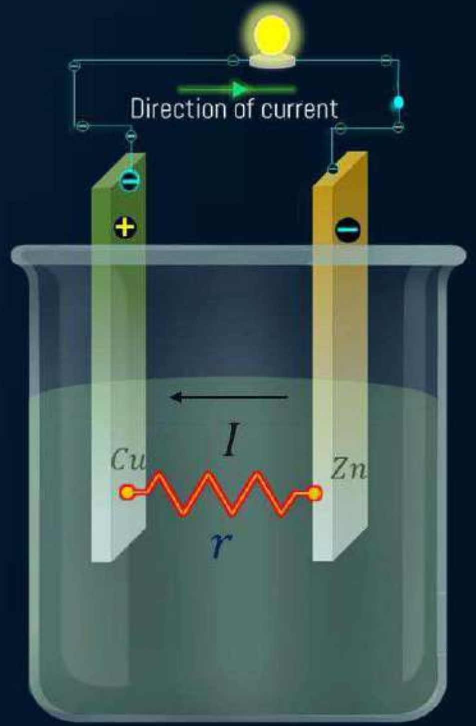
Emf is independent of quantity of electrolyte, size of electrodes and distance between the electrodes.

Recap

Relation b/w Emf and voltage

$$\boxed{V = E - Ir}$$

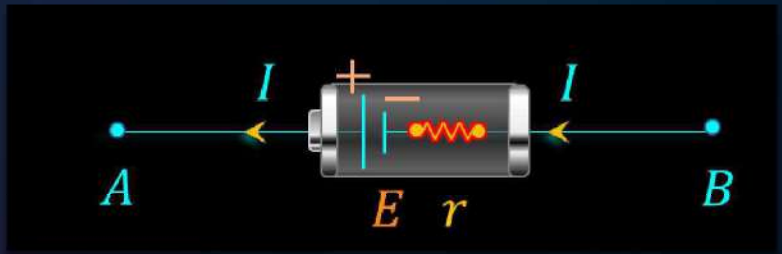




Discharging of a cell

$$I = \frac{E}{R + r}$$

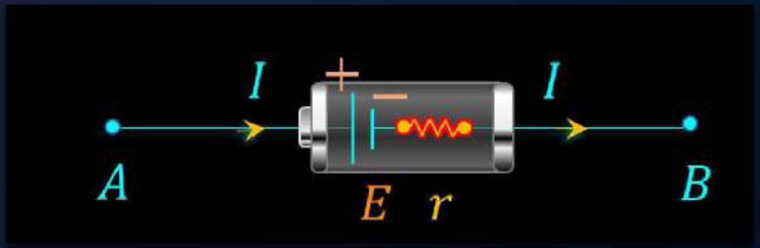
$$V < E$$



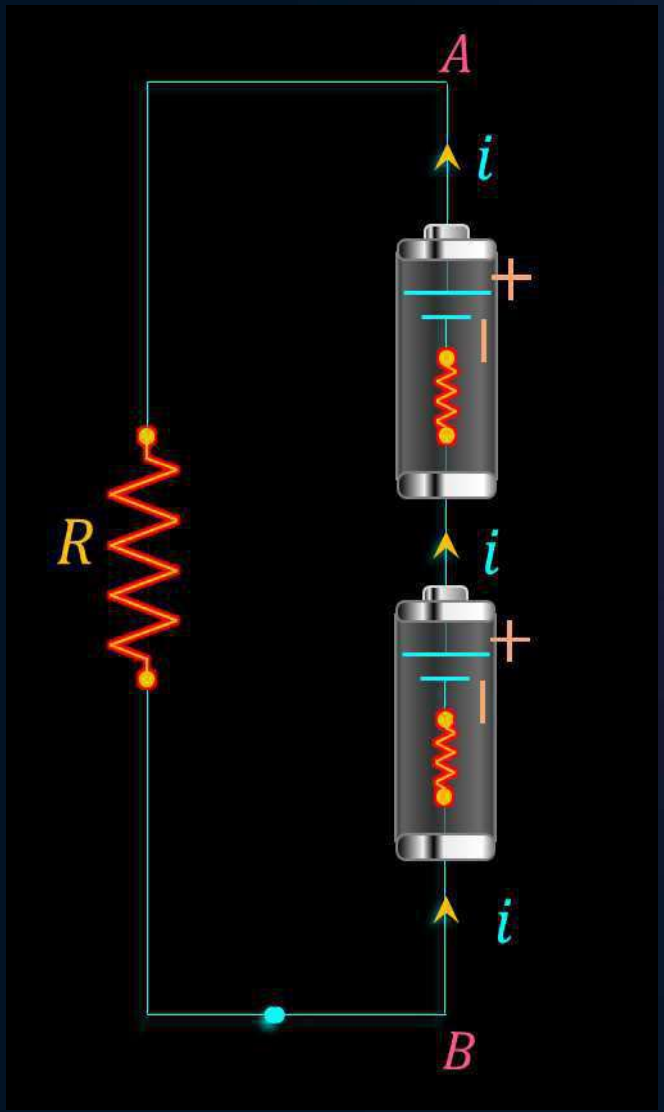
Charging of a cell

$$V > E$$

$$\boxed{V = E + Ir}$$



Series combination of cells



Same polarity:

Same current flow through all cells

$$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2}$$

$$\boxed{r_{eq} = r_1 + r_2}$$

If n non-identical cells are connected in series with same polarity,

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots \varepsilon_n$$

$$r_{eq} = r_1 + r_2 + r_3 + \cdots r_n$$

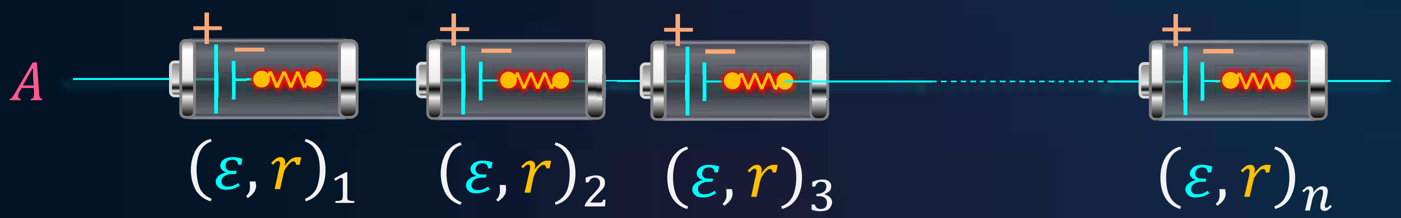
$$\boxed{\varepsilon_{eq} = \sum_{i=1}^{n} \varepsilon_i}$$

$$\boxed{r_{eq} = \sum_{i=1}^{n} r_i}$$

Series Combination of Cells

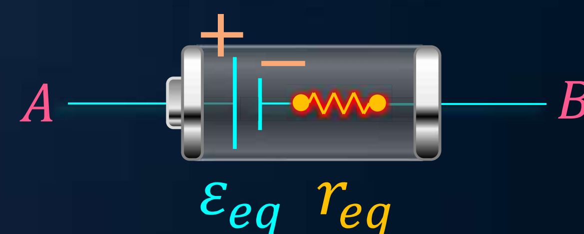
- n identical cells are connected in series with same polarity

Series



$$\varepsilon_{eq} = n\varepsilon$$

$$r_{eq} = nr$$



$$\boxed{\varepsilon_{eq} = n\varepsilon}$$

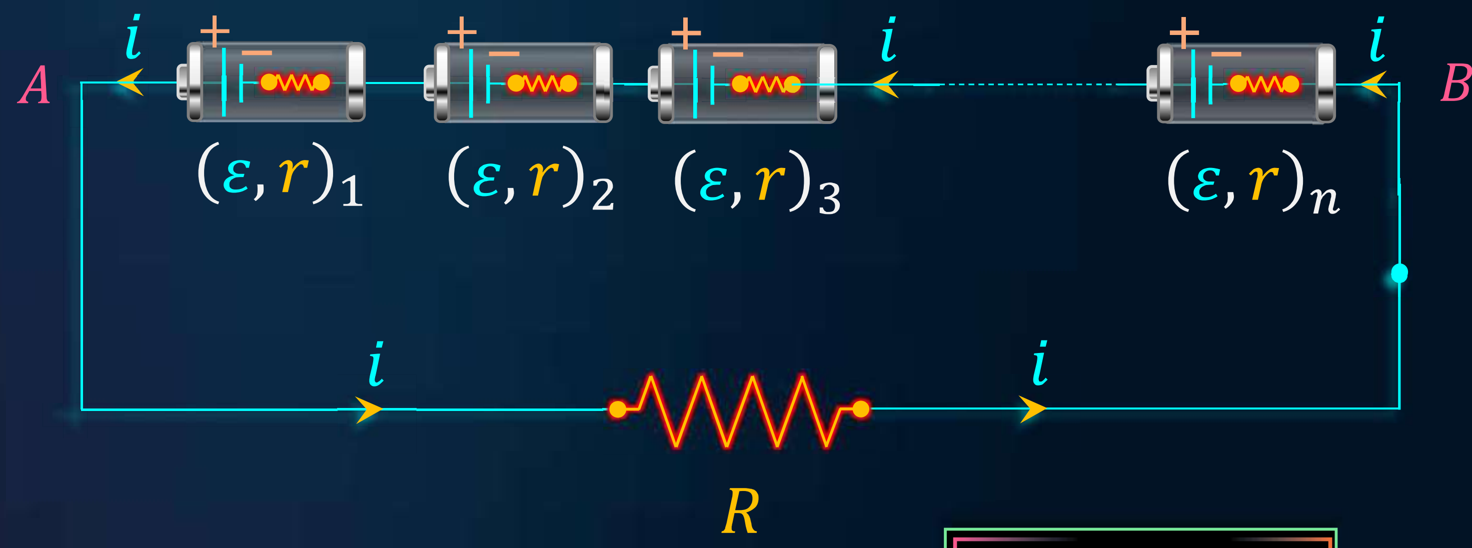
$$\boxed{r_{eq} = nr}$$

Series Combination of Cells

- n identical cells are connected in series with same polarity

$$\varepsilon_{eq} = n\varepsilon$$

$$r_{eq} = nr$$



$$R_{eq} = R + nr$$

$$\boxed{i = \frac{\varepsilon_{eq}}{R_{eq}} = \frac{n\varepsilon}{R + nr}}$$

$$i = \frac{n\varepsilon}{R + nr}$$

$R >> nr \rightarrow i = \frac{n\varepsilon}{R}$

$nr >> R \rightarrow i = \frac{\cancel{n}\varepsilon}{\cancel{n}r} = \frac{\varepsilon}{r}$

$i = \frac{\varepsilon}{r}$

Series Combination of Cells

- n identical cells are connected in series with same polarity

$$\boxed{i = \frac{\varepsilon}{r}}$$

No change in current than current in single cell

$1.2\ V \rightarrow 12\ V$

$1\ A$

Operating voltage

$12\ V$

$nr \gg R$

Example:

$$\varepsilon = 1.2\ V$$

$$n = 10$$

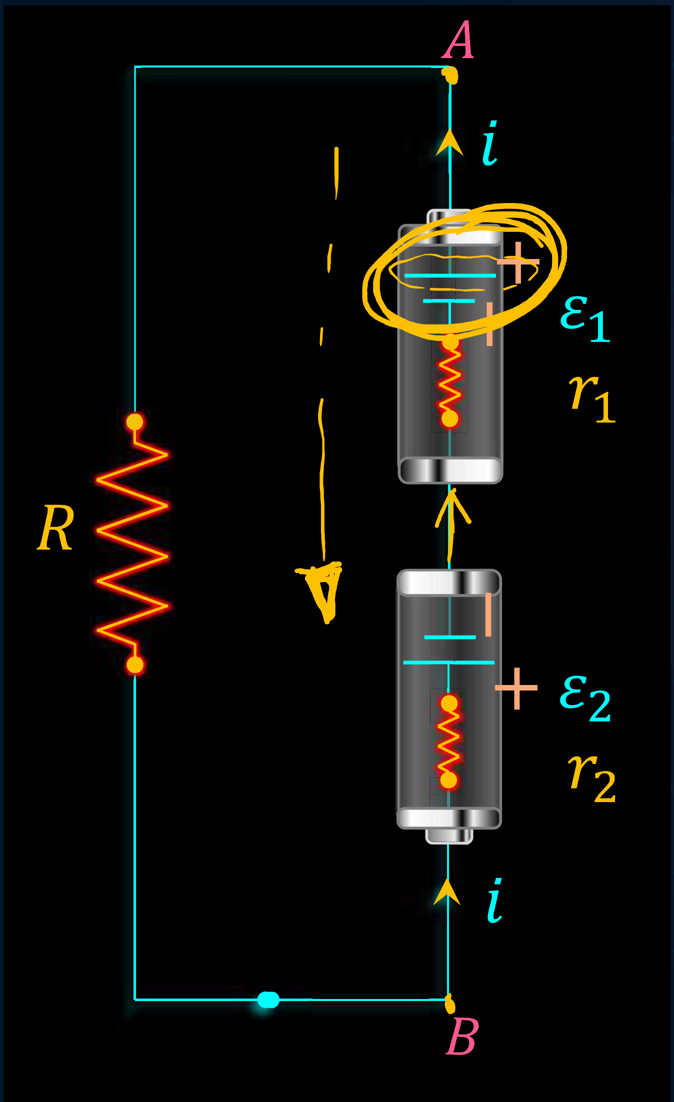
$$\varepsilon_{eq} = 10 \times 1.2 = 12\ V$$

$n\varepsilon = 12$

$n \cdot (1.2) = 12$

$n = 10$

Series Combination of Cells



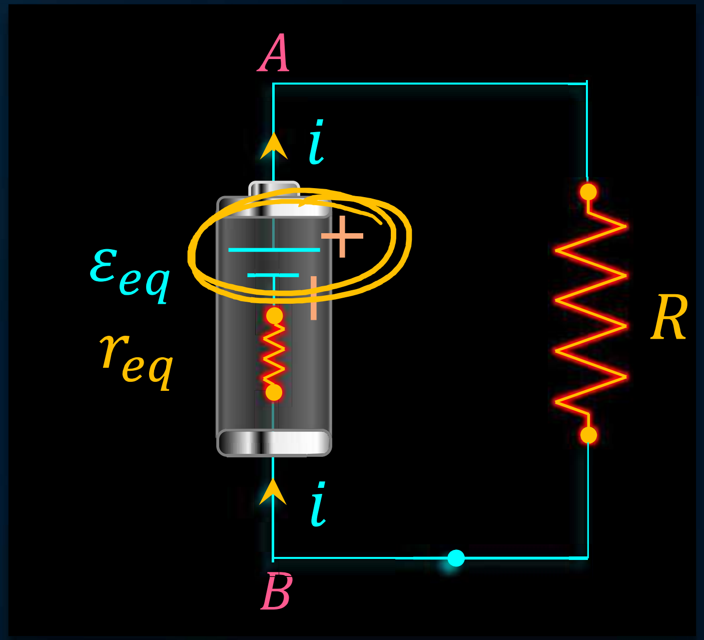
- Opposite polarity
- Same current flow through all cells

$$V_A - \varepsilon_1 + i r_1 + \varepsilon_2 + i r_2 = V_B$$

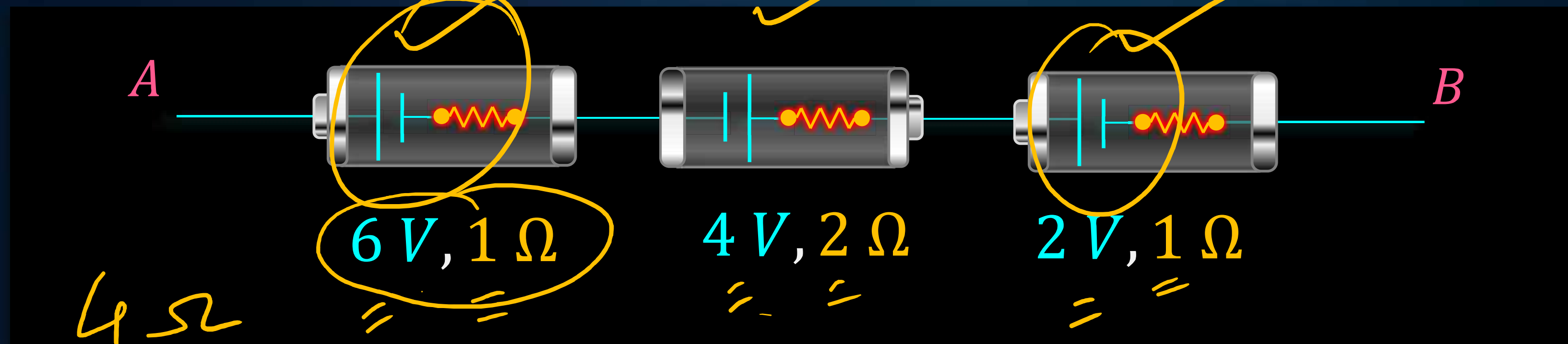
$$V_A - V_B = \varepsilon_1 - \varepsilon_2 - i(r_1 + r_2)$$

- $\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$
- $r_{eq} = r_1 + r_2$

$$V = \varepsilon_{eq} - i r_{eq}$$



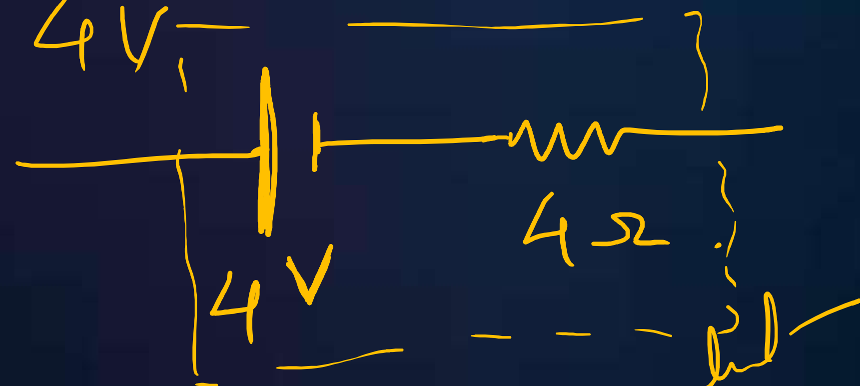
Series Combination of Cells



$r_{eq} = 4\,\Omega$

$\varepsilon_{eq} = +6\,V - 4\,V + 2\,V$

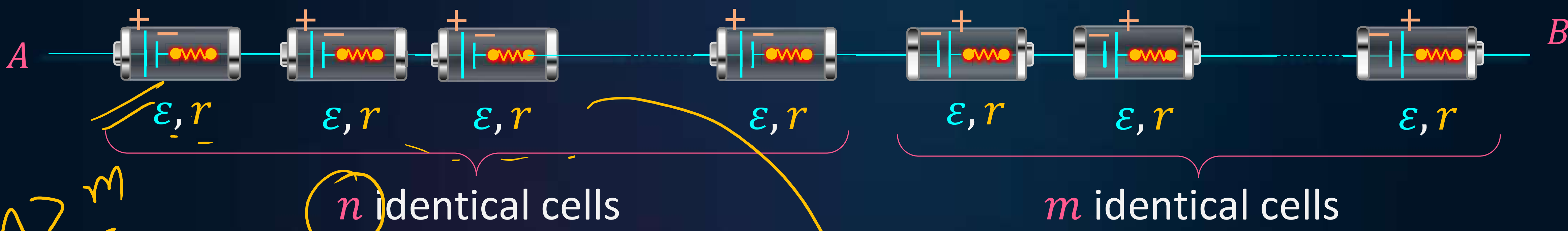
$= 4\,V$



Equivalent

Series Combination of Cells

- n identical cells with same polarity and m same cells with opposite polarity

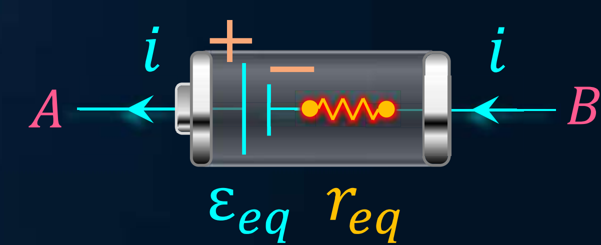


$n > m$

$$\varepsilon_{eq} = n\varepsilon - m\varepsilon$$

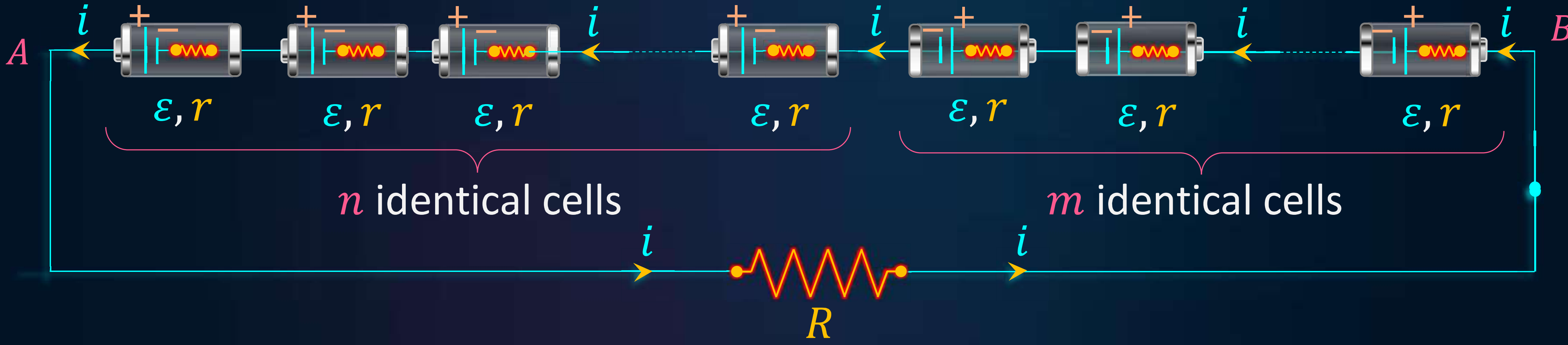
$$= (n - m)\varepsilon$$

$$r_{eq} = (n + m)\, r$$



Series Combination of Cells

- n identical cells with same polarity and m same cells with opposite polarity



$$\boxed{\varepsilon_{eq} = n\varepsilon - m\varepsilon}$$

$$\boxed{r_{eq} = (n+m)r}$$

$$R_{eq} = R + r_{eq}$$

$$\boxed{R_{eq} = R + (n+m)r}$$

$$i = \frac{\varepsilon_{eq}}{R_{eq}} = \frac{(n-m)\varepsilon}{R+(n+m)r}$$

Question

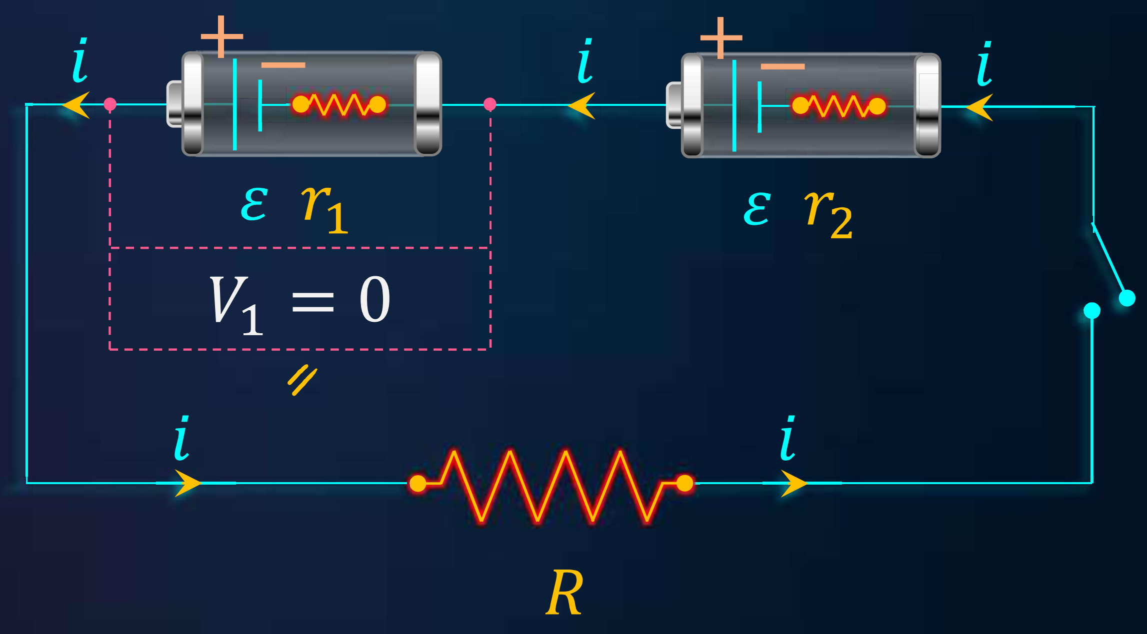
Two cells, having the same e.m.f. are connected in series through an external resistance R. Cells having internal resistances r_1 and r_2 $(r_1 > r_2)$ respectively. When the circuit is closed, the potential difference across the first cell is zero. The value of R is:

a) $r_1 + r_2$

b) $r_1 - r_2$

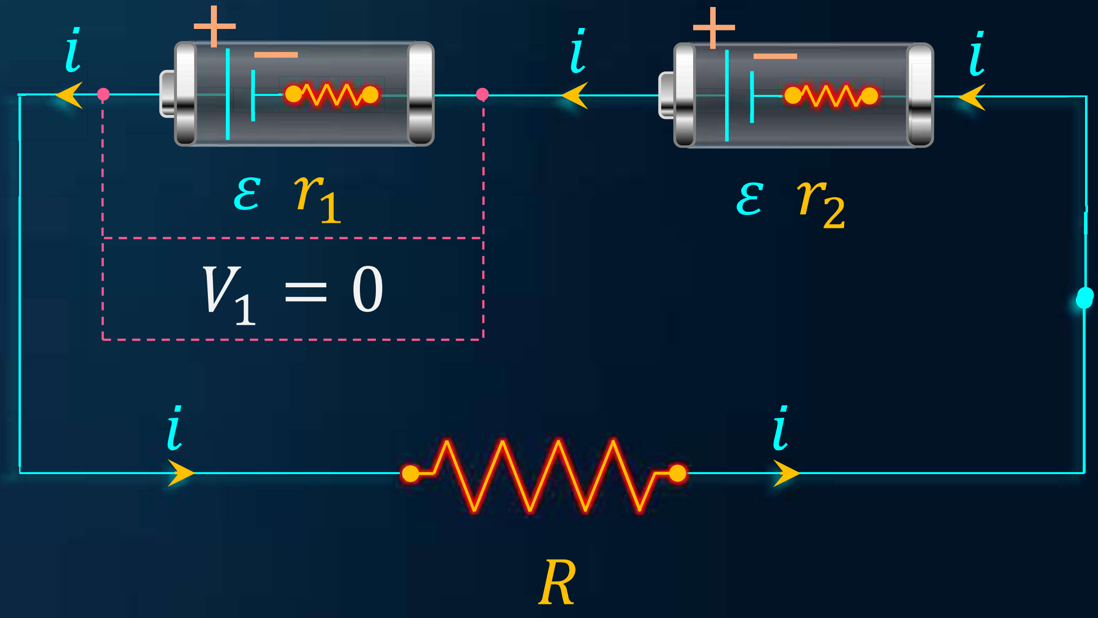
c) $\frac{r_1 + r_2}{2}$

d) $\frac{r_1 - r_2}{2}$



Discussion

External resistance and cell resistances are in series connection



$$R_{eq} = R + r_1 + r_2$$

$$\overset{0}{V_1} = \varepsilon - i r_1$$

$$\overset{0}{i} = \frac{\varepsilon_{eq}}{R_{eq}} = \frac{2\varepsilon}{R + r_1 + r_2}$$

$$\overset{0}{V_1} = \varepsilon - \frac{2\varepsilon r_1}{R + r_1 + r_2}$$

Discussion

$$i = \frac{2\varepsilon}{R + r_1 + r_2}$$

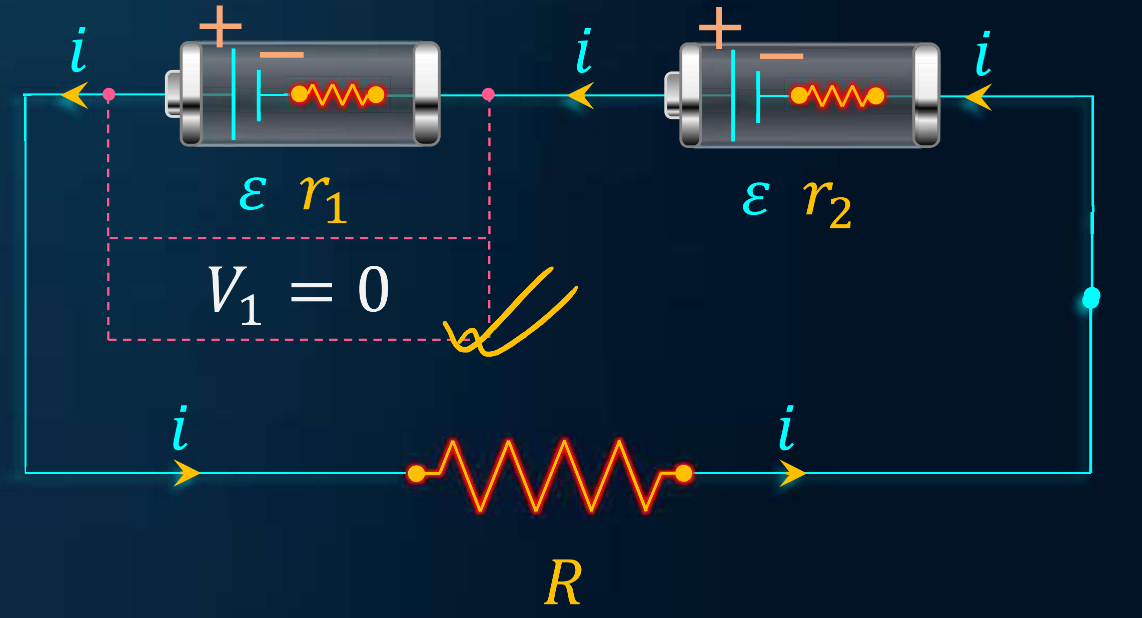
Terminal p.d. across first cell is:

$$\cancel{V_1}^{\,0} = \varepsilon - i r_1$$

$$\cancel{\varepsilon} = \frac{2\cancel{\varepsilon} r_1}{R + r_1 + r_2}$$

$$R + r_1 + r_2 = 2 r_1$$

$$\boxed{R = r_1 - r_2}$$



Thus, option (b) is the correct answer.

Question

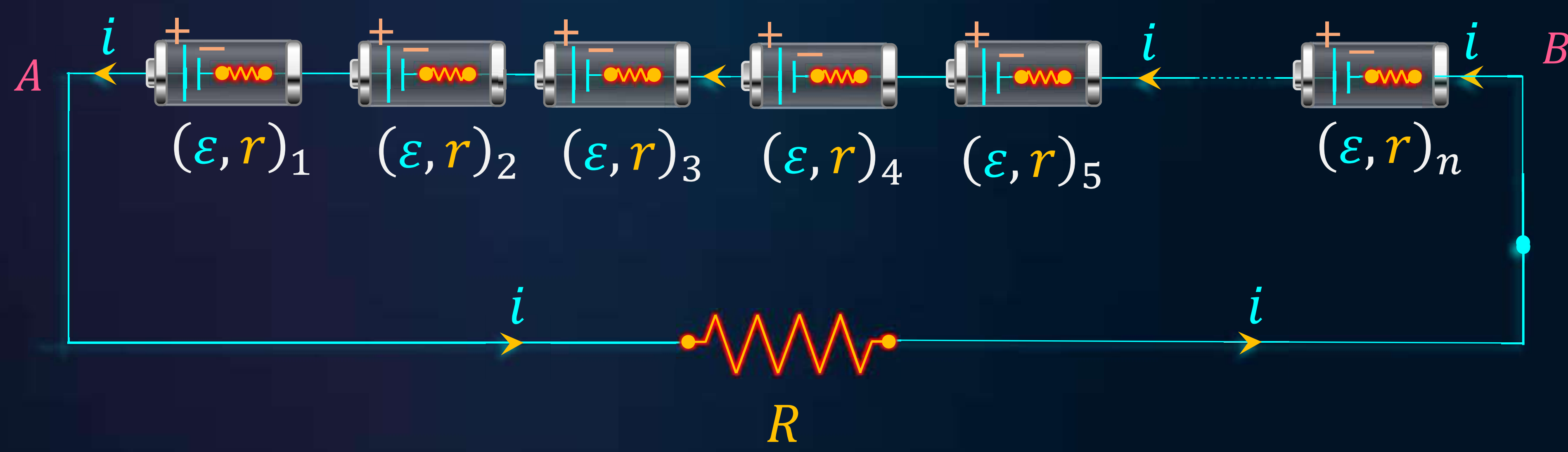
$'n'$ cells each of e.m.f $'\varepsilon'$ and internal resistance $'r'$ are connected in series in same polarity. An external resistance R is connected to the combination. If the polarity of $'m'$ cells are reversed, find current in the external resistor.

a) $\dfrac{(n-m)\varepsilon}{R+(n+m)r}$

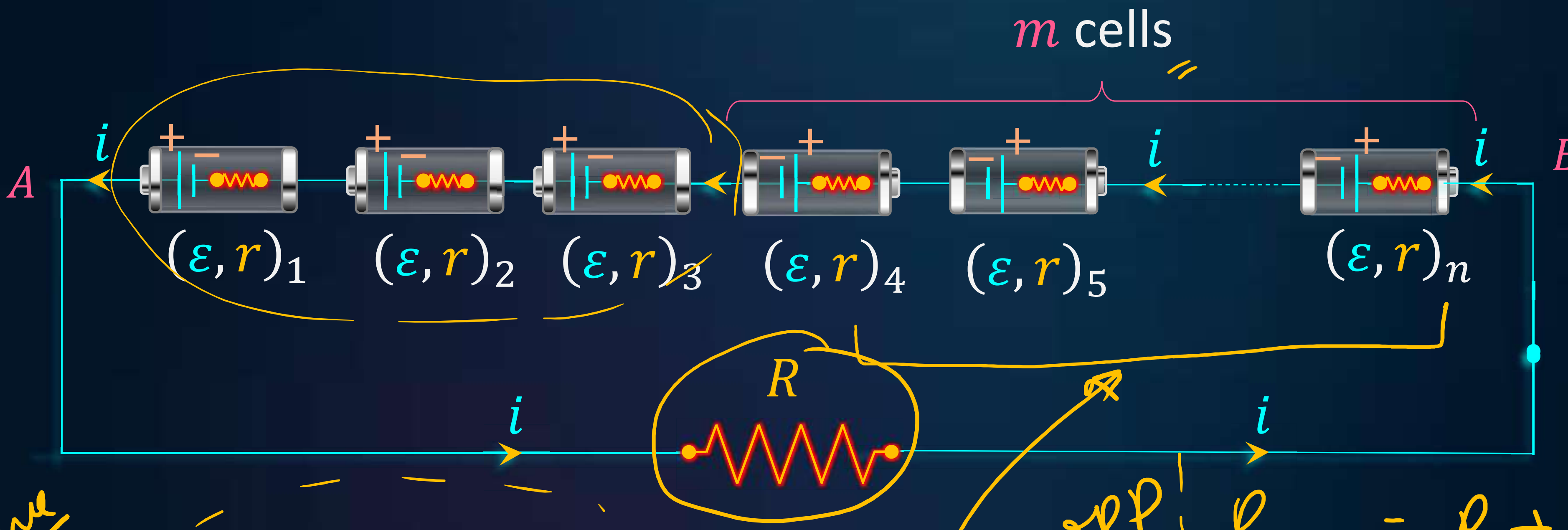
b) $\dfrac{n\,\varepsilon}{R+nr}$

c) $\dfrac{(n-m)\varepsilon}{R+nr}$

d) $\dfrac{(n-2m)\varepsilon}{R+nr}$



Discussion



same

$+(n-m)\varepsilon$

$m\varepsilon$ opp

$R_{eq} = R + nr$

$$\varepsilon_{eq} = (n-m)\varepsilon - m\varepsilon$$
$$= n\varepsilon - m\varepsilon - m\varepsilon$$
$$= (n-2m)\varepsilon$$

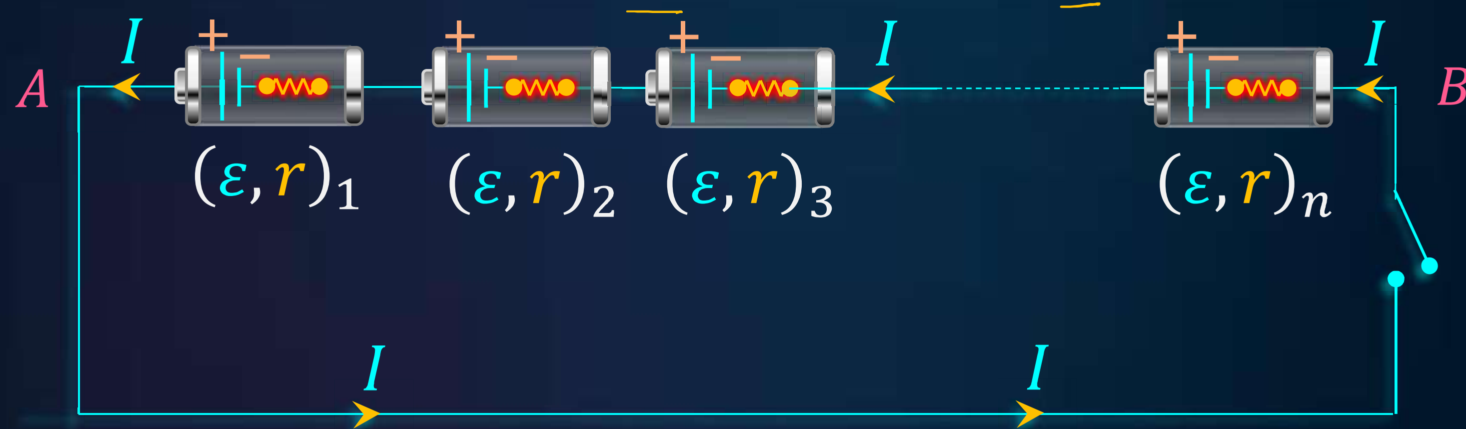
$$i = \frac{\varepsilon_{eq}}{R_{eq}}$$

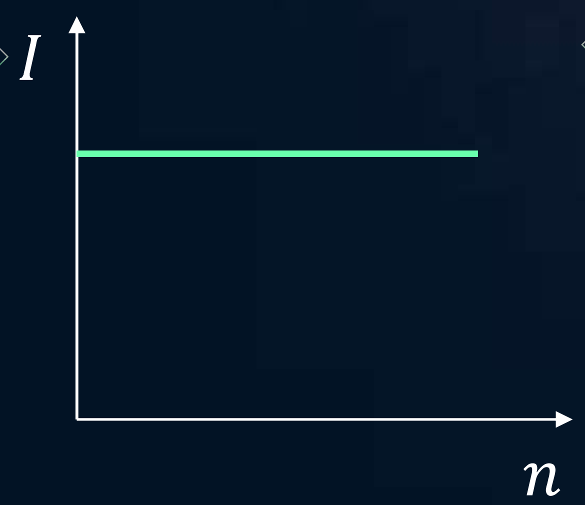
$$i = \frac{(n-2m)\varepsilon}{R+nr}$$

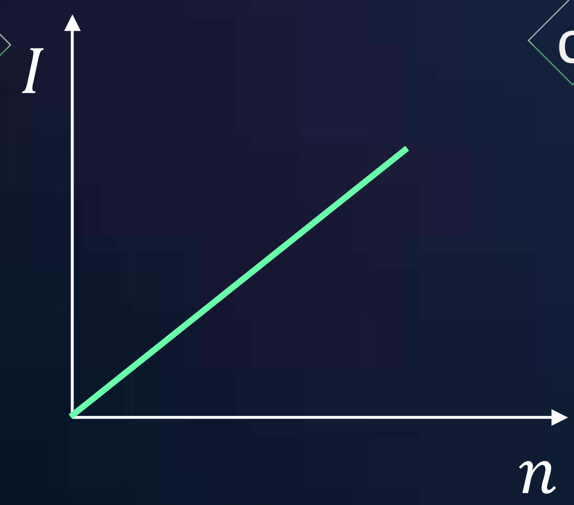
Thus, option (d) is the correct answer.

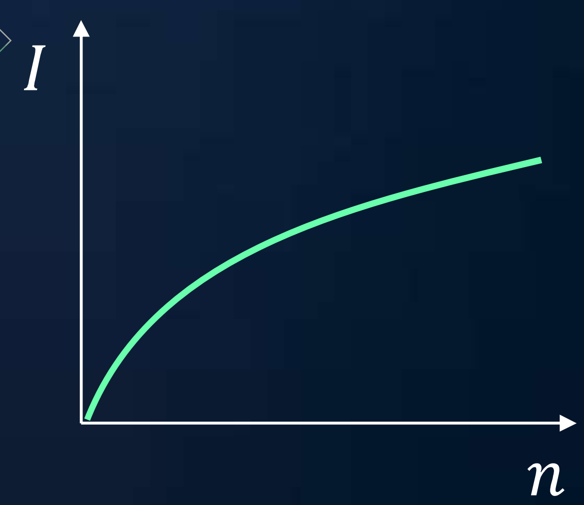
Question

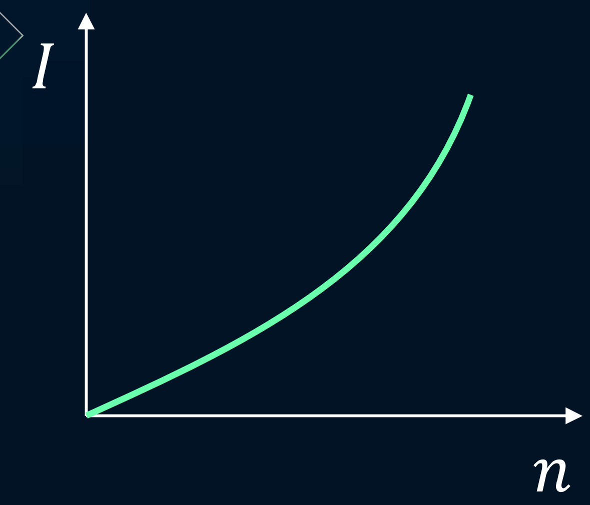
A battery consists of a variable $'n'$ number of identical cells (having internal resistance $'r'$ each) which are connected in series. The terminals of battery are short circuited and the current I is measured. Which of the graphs shows the correct relationship between I and n.



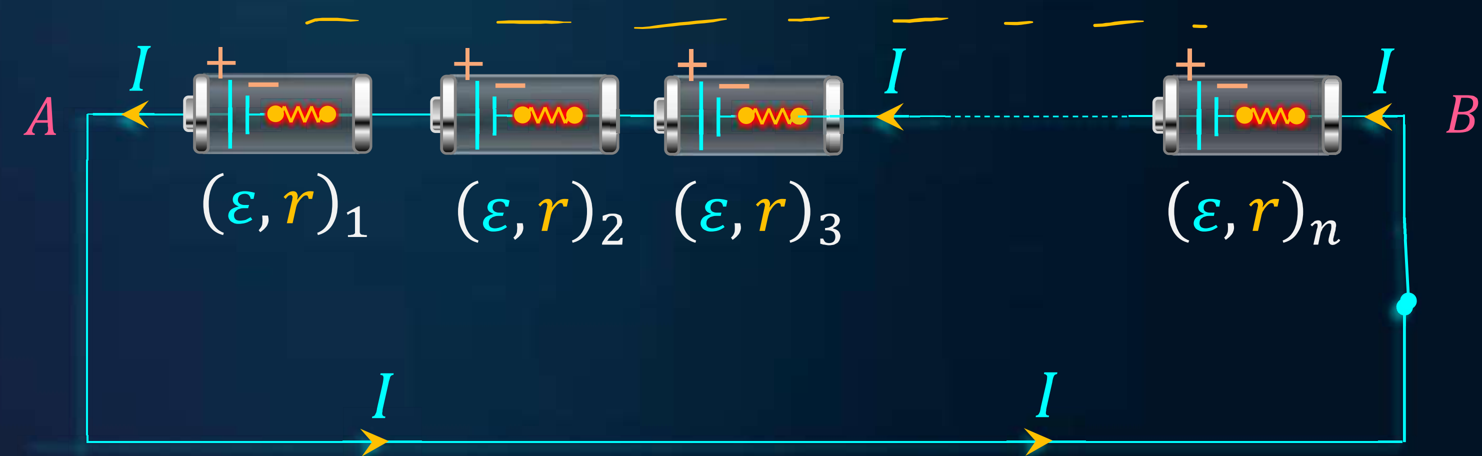
a


b


c


d


Discussion



$$\varepsilon_{eq} = n\varepsilon$$

$$r_{eq} = nr$$

$$R_{eq} = \cancel{R} + r_{eq}$$

0

$$=$$

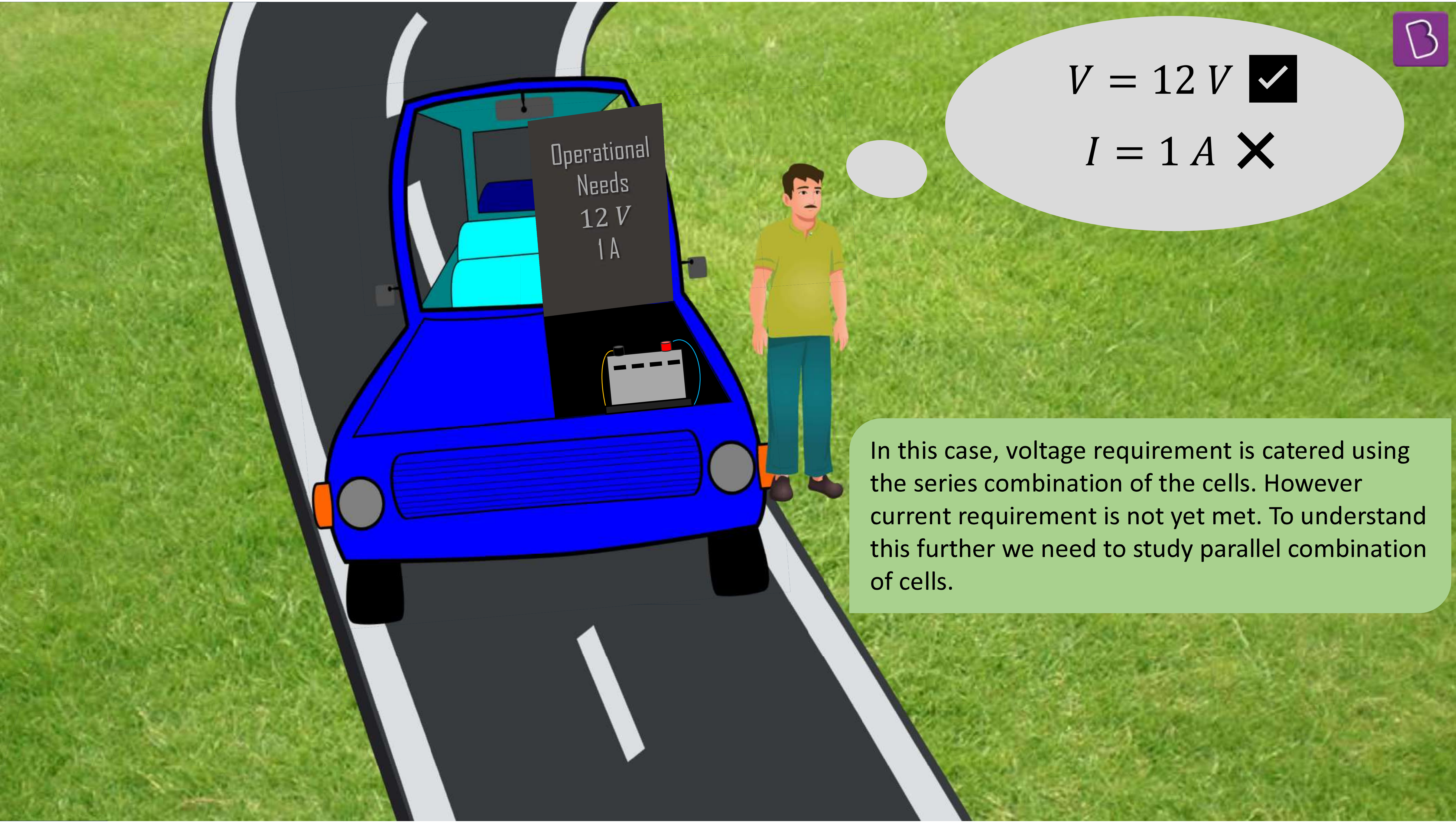
$$\boxed{i = \frac{\varepsilon}{r}}$$

$$i = \frac{\varepsilon}{r}$$

$$i = \frac{\varepsilon_{eq}}{R_{eq}} = \frac{\cancel{n}\varepsilon}{\cancel{n}r}$$

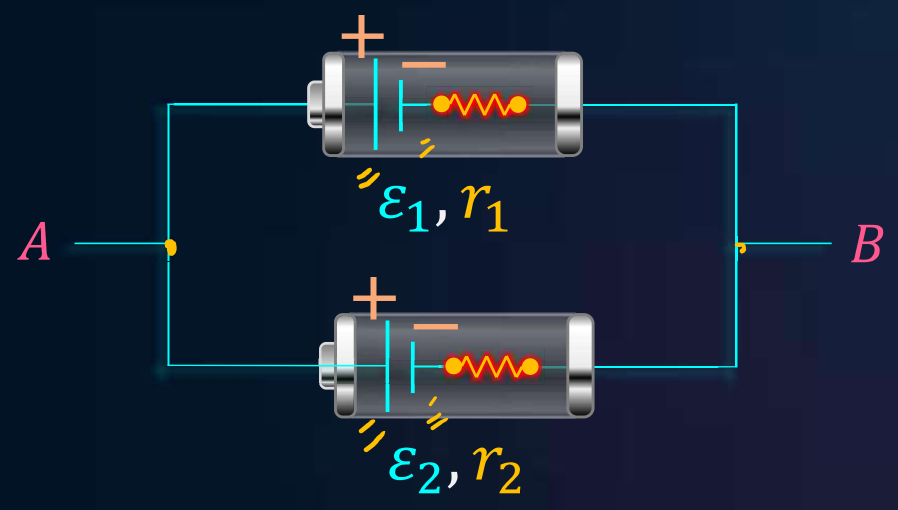
Thus, option (a) is the correct answer.

∴ Current I is constant and independent of n

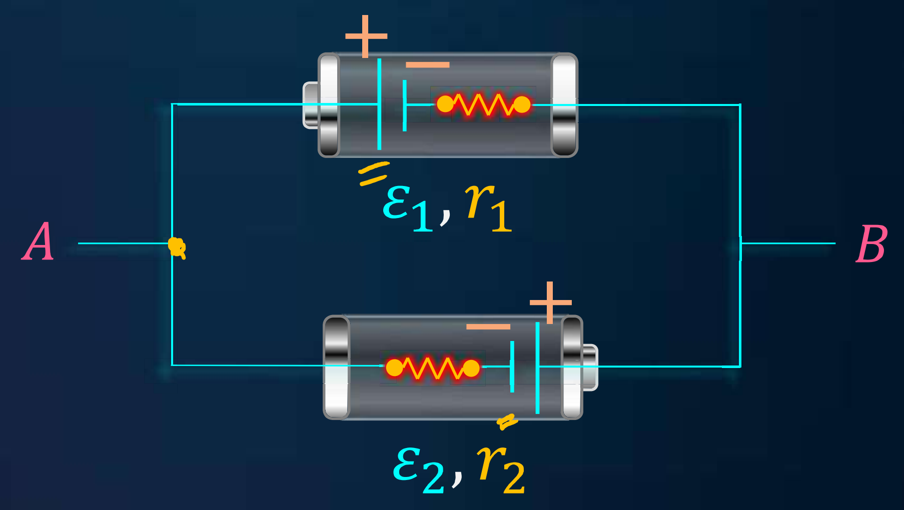

Operational
Needs
$12\ V$
1 A
$V = 12\ V$ ✓
$I = 1\ A$ ✗
In this case, voltage requirement is catered using the series combination of the cells. However current requirement is not yet met. To understand this further we need to study parallel combination of cells.

Parallel Combination of Cells

Polarity



Same polarity



Opposite polarity

Parallel Combination of Cells

$i = i_1 + i_2$ ....(1)

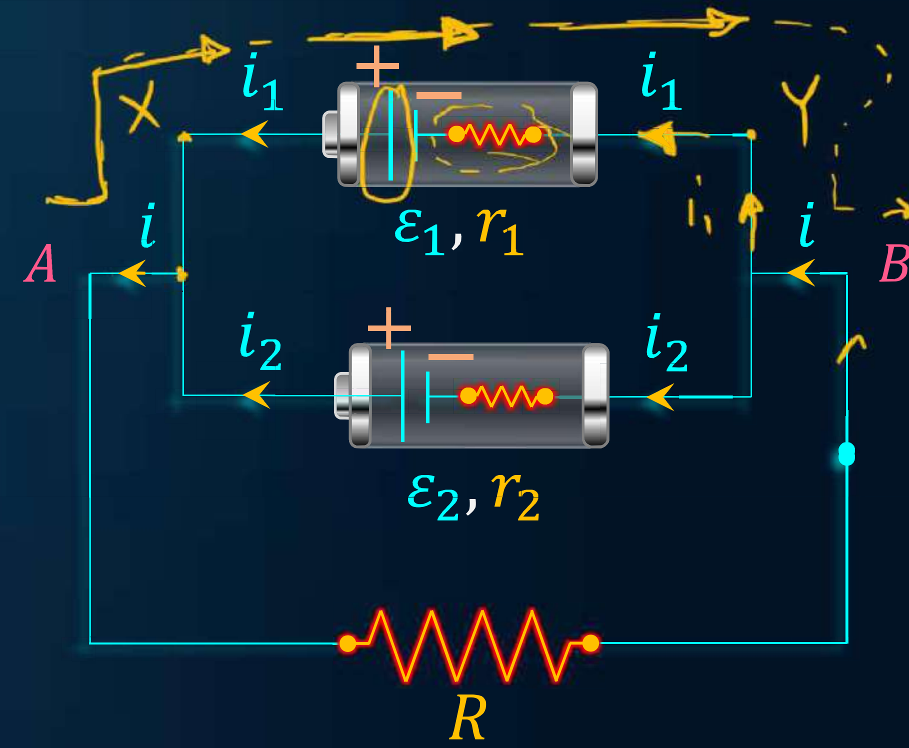
Apply KVL in path A X Y B

$$V_A - \varepsilon_1 + i_1 r_1 = V_B$$

$$V_A - V_B = \varepsilon_1 - i_1 r_1$$

$$V = \varepsilon_1 - i_1 r_1$$

$$i_1 = \frac{\varepsilon_1 - V}{r_1} \quad \text{—— (2)}$$



Same polarity cells

Parallel Combination of Cells

$i = i_1 + i_2$ ....(1)

$i_1 = \dfrac{\varepsilon_1 - V}{r_1}$ ....(2)

A P Q B

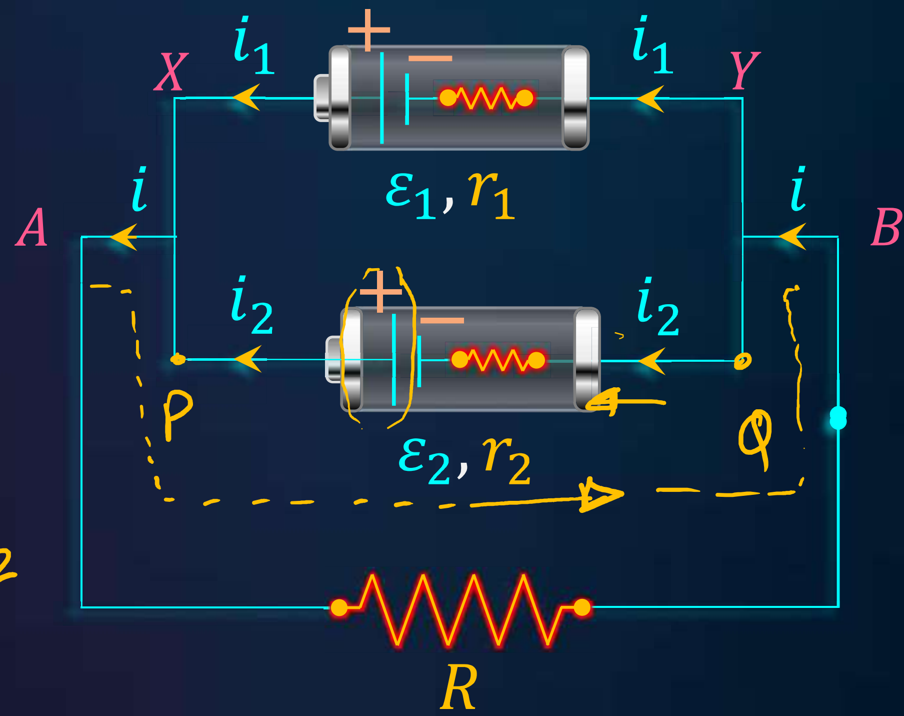
$V_A - \varepsilon_2 + i_2 r_2 = V_B$

$V_A - V_B = \varepsilon_2 - i_2 r_2$

$V = \varepsilon_2 - i_2 r_2$

$i_2 = \dfrac{\varepsilon_2 - V}{r_2}$ — (3)

$V = \varepsilon_{eq} - I r_{eq}$

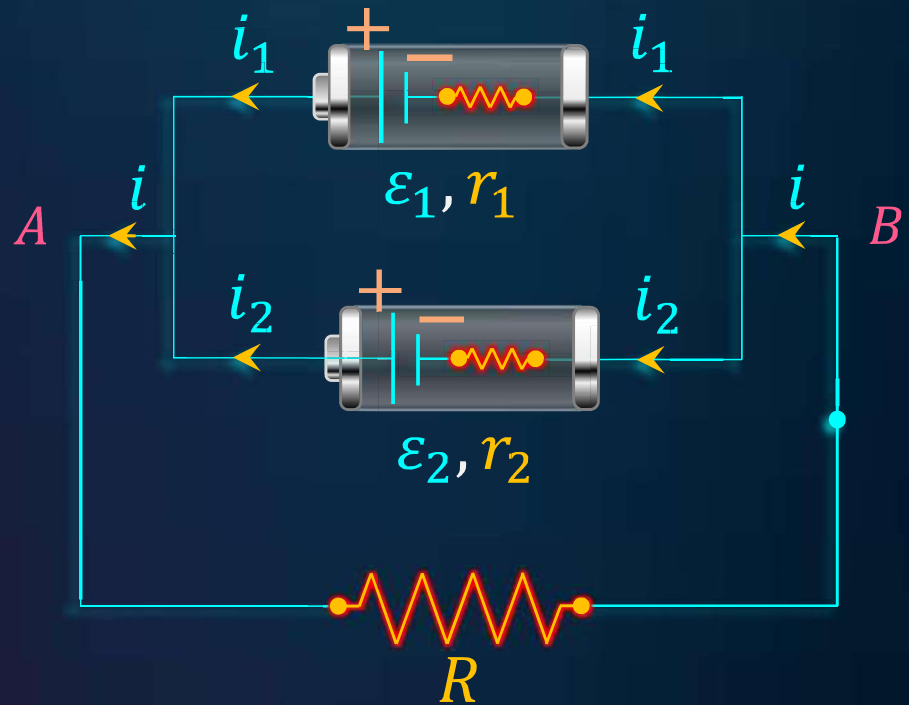


Parallel Combination of Cells

$$i = i_1 + i_2 \quad \text{.........}(1)$$

$$i_1 = \frac{\varepsilon_1 - V}{r_1} \quad \text{.........}(2)$$

$$i_2 = \frac{\varepsilon_2 - V}{r_2} \quad \text{.........}(3)$$



$$\underline{V = \varepsilon - Ir}$$

$$i = \frac{\varepsilon_1}{r_1} - \frac{V}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{V}{r_2}$$

$$i = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

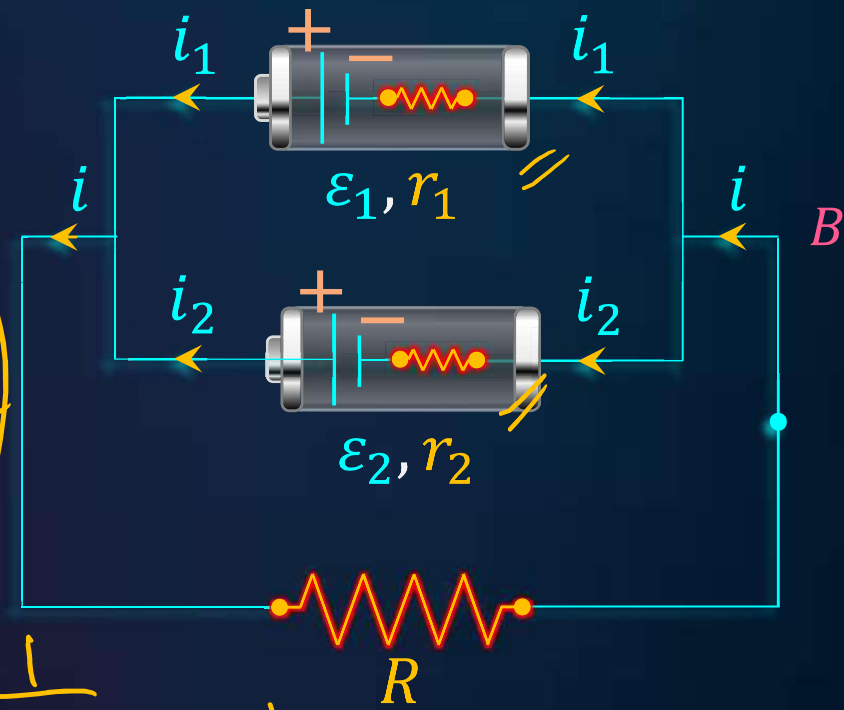
$$V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - i$$

Parallel Combination of Cells

$$V\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}-i$$

$$\underline{\underline{V}} = \underline{\underline{\varepsilon_{eq}}} - I\,\underline{\underline{r_{eq}}}$$

$$V = \frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} - i\left(\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}}\right)$$



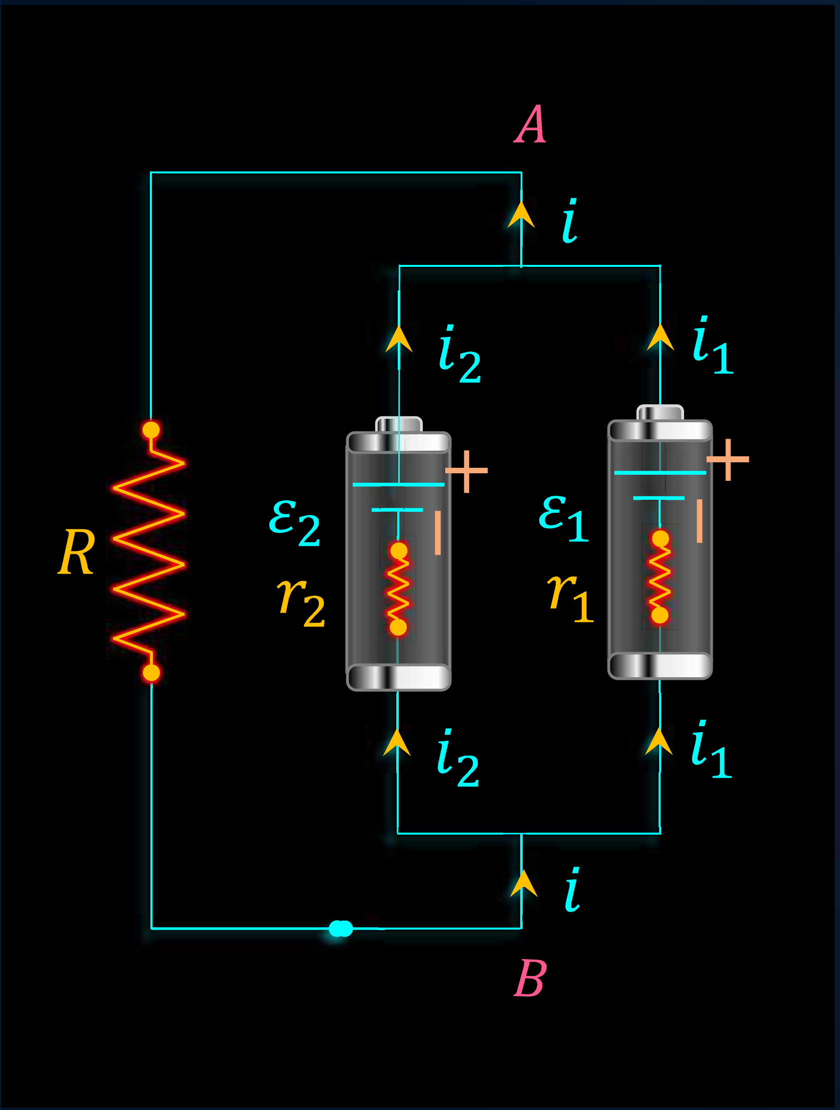
$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1}+\frac{1}{r_2}$$

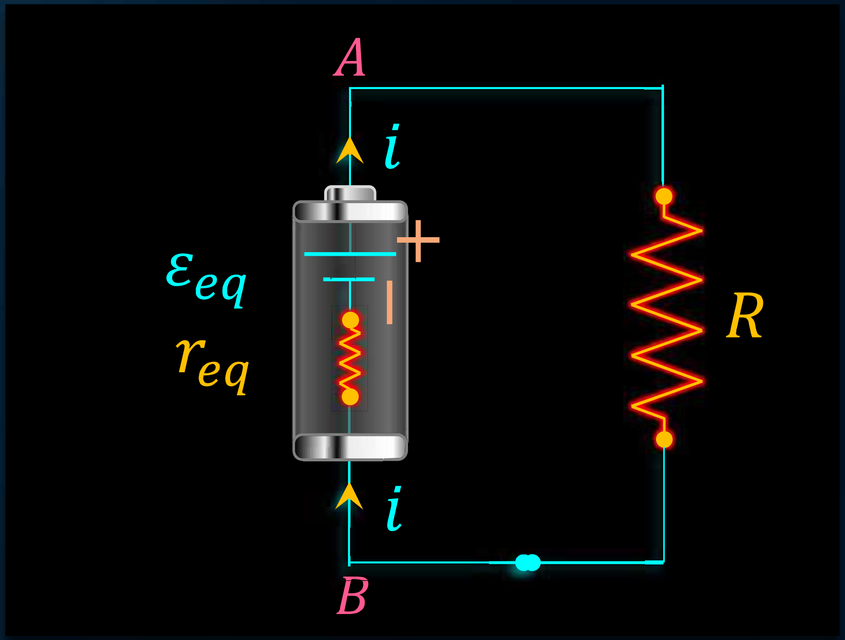
$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1}+\frac{1}{r_2}$$

Parallel Combination of Cells

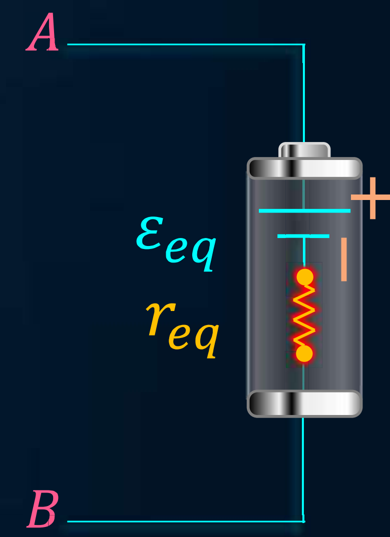


- Same polarity
- $i = i_1 + i_2$
- $$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$
- $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$



Parallel Combination of Cells

- n non-identical cells are connected in parallel with same polarity



$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3} + \cdots \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \frac{1}{r_n}}$$

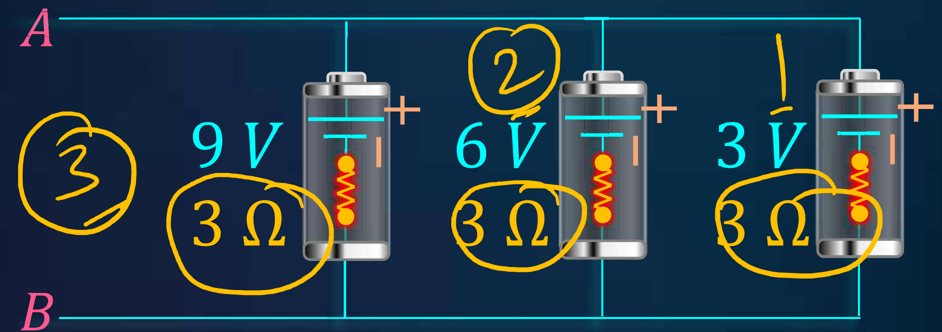
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \frac{1}{r_n}$$

$$\boxed{\varepsilon_{eq} = \frac{\sum_{i=1}^{n} \frac{\varepsilon_i}{r_i}}{\sum_{i=1}^{n} \frac{1}{r_i}}}$$

$$\boxed{\frac{1}{r_{eq}} = \sum_{i=1}^{n} \frac{1}{r_i}}$$

Parallel Combination of Cells

$$\boxed{\begin{aligned}\varepsilon_{eq} &= 6\,V \\ r_{eq} &= 1\,\Omega\end{aligned}}$$



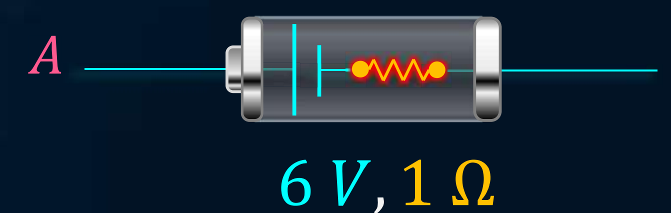
$$\frac{r}{n}$$

$$\frac{3}{3} = 1\,\Omega$$

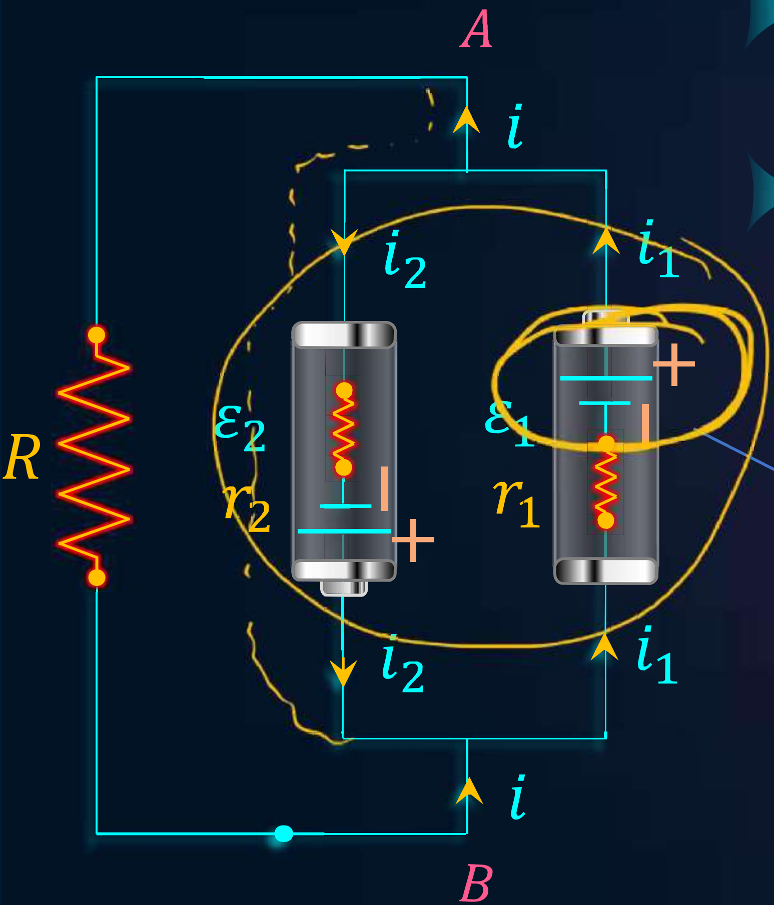
$$\varepsilon_{eq} = \frac{\dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2} + \dfrac{\varepsilon_3}{r_3}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}} \rightarrow 1$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$r_{eq} = 1\,\Omega$$



Parallel Combination of Cells

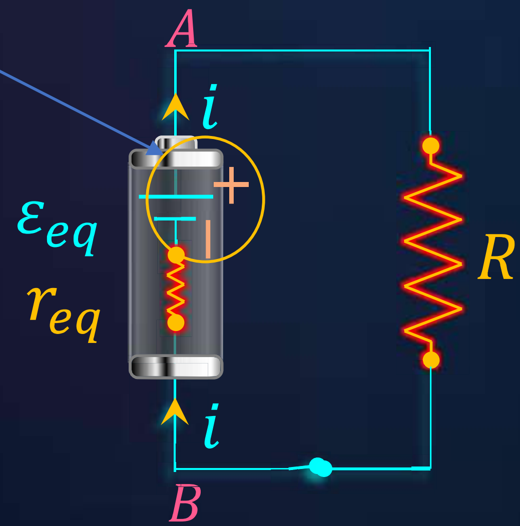


- Opposite polarity
- Suppose $\varepsilon_1 > \varepsilon_2$

$i = i_1 - i_2$

$$\varepsilon_{eq} = V = \frac{\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

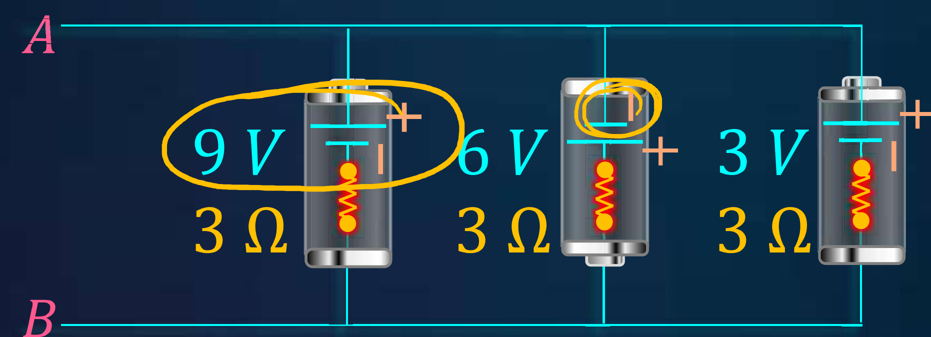
Note: — Direction of current depends on ε_1 and ε_2.



$$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}}$$

$$\boxed{\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}}$$

Parallel Combination of Cells

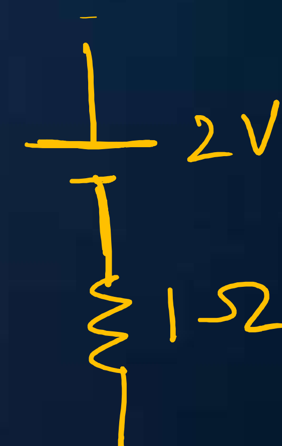


$r_{eq} = 1\ \Omega$

$$\varepsilon_{eq} = \frac{\dfrac{\varepsilon_1}{r_1} - \dfrac{\varepsilon_2}{r_2} + \dfrac{\varepsilon_3}{r_3}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}}$$

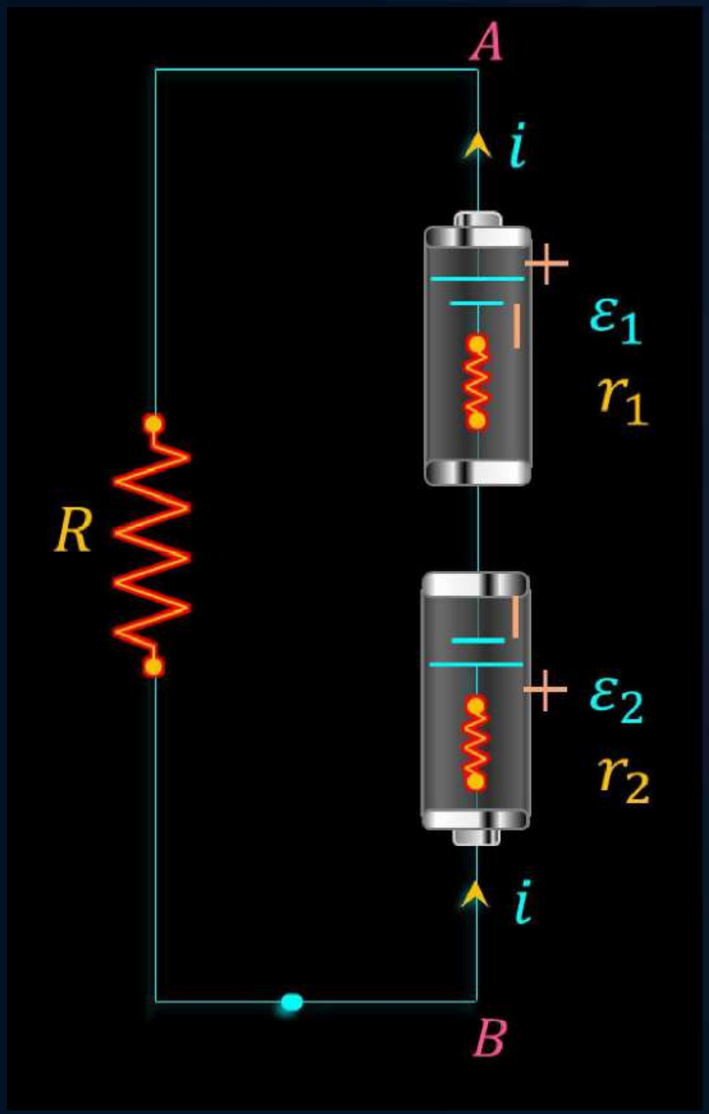
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

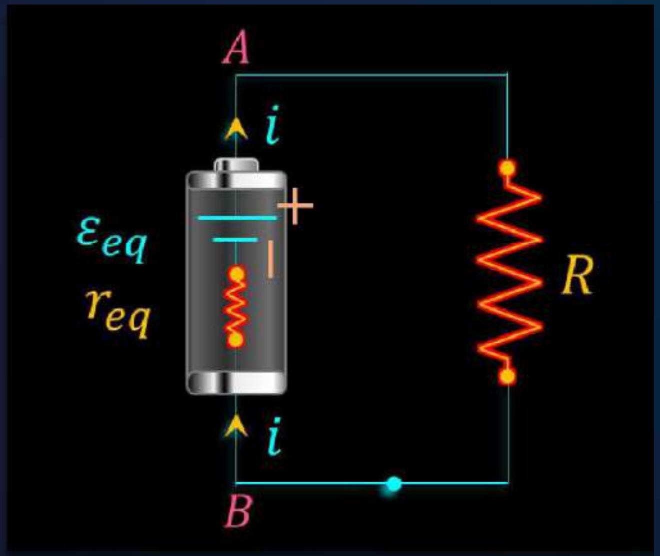
$3 - 2 + 1 = \underline{\underline{2\ V}}$



Recap

Series combination





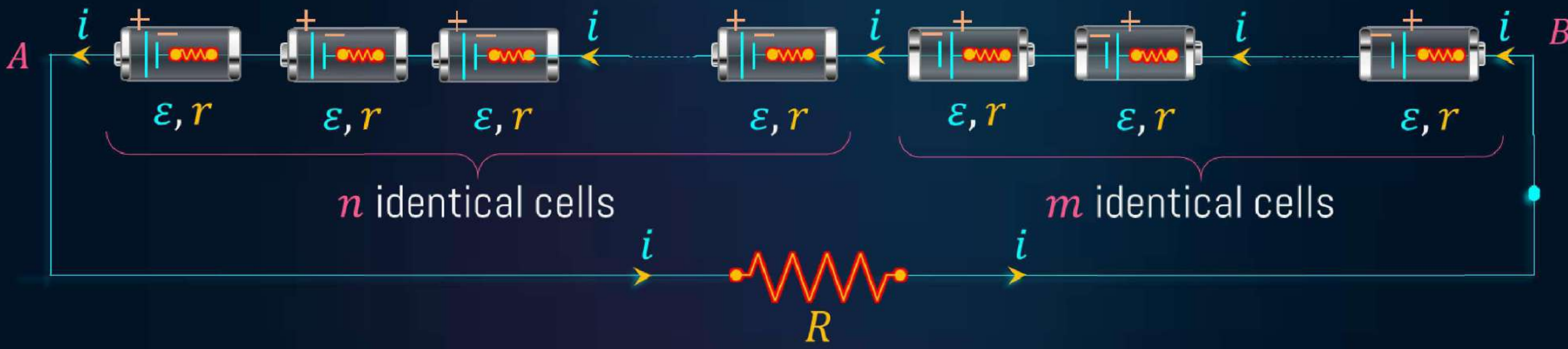
- Opposite polarity
- Same current flow through all cells

$$\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$$

$$r_{eq} = r_1 + r_2$$

Recap

Series combination



$$\varepsilon_{eq} = n\varepsilon - m\varepsilon$$

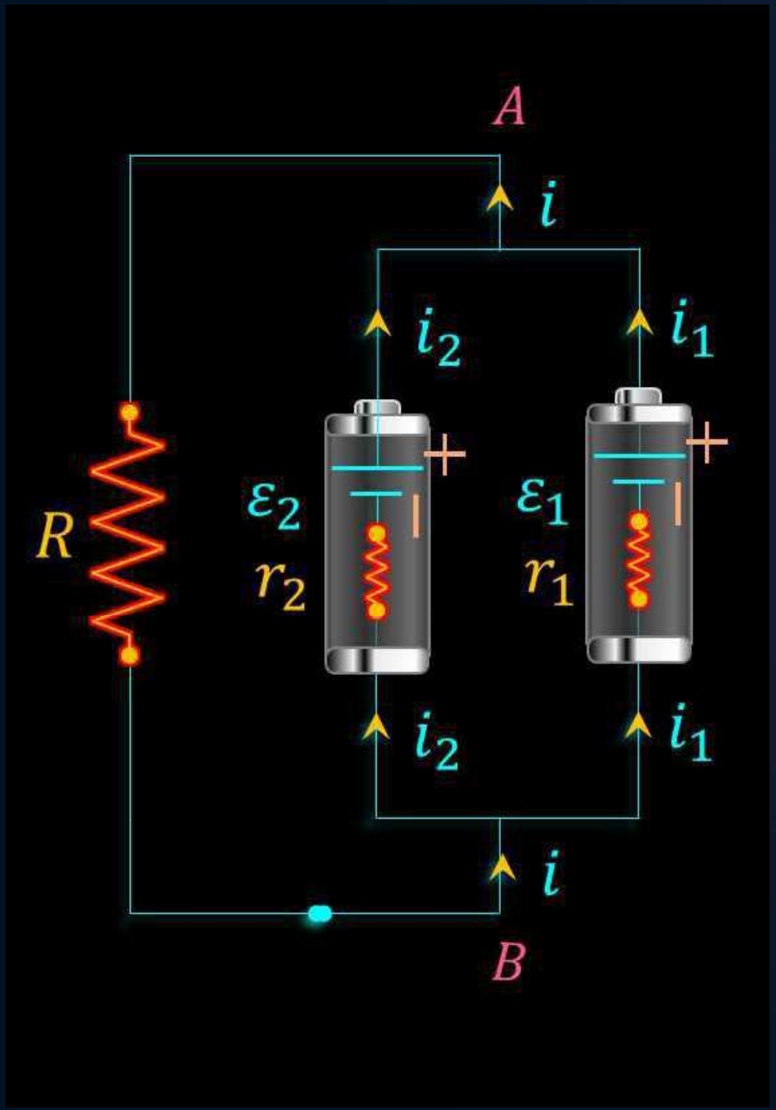
$$r_{eq} = (n+m)r$$

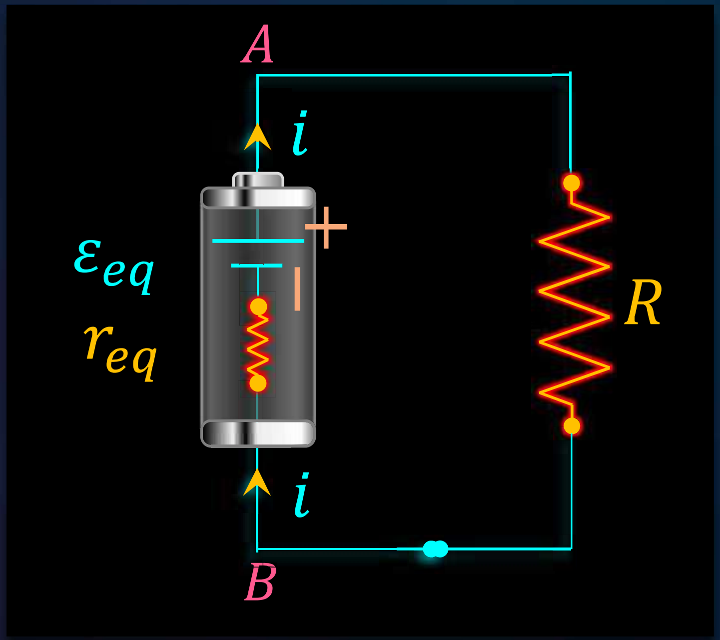
$$R_{eq} = R + (n+m)r$$

$$i = \frac{(n-m)\varepsilon}{R + (n+m)r}$$

Recap

Parallel combination





- Same polarity
- $i = i_1 + i_2$
- $$\boxed{\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}}$$
- $$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}}$$

Recap

Parallel combination

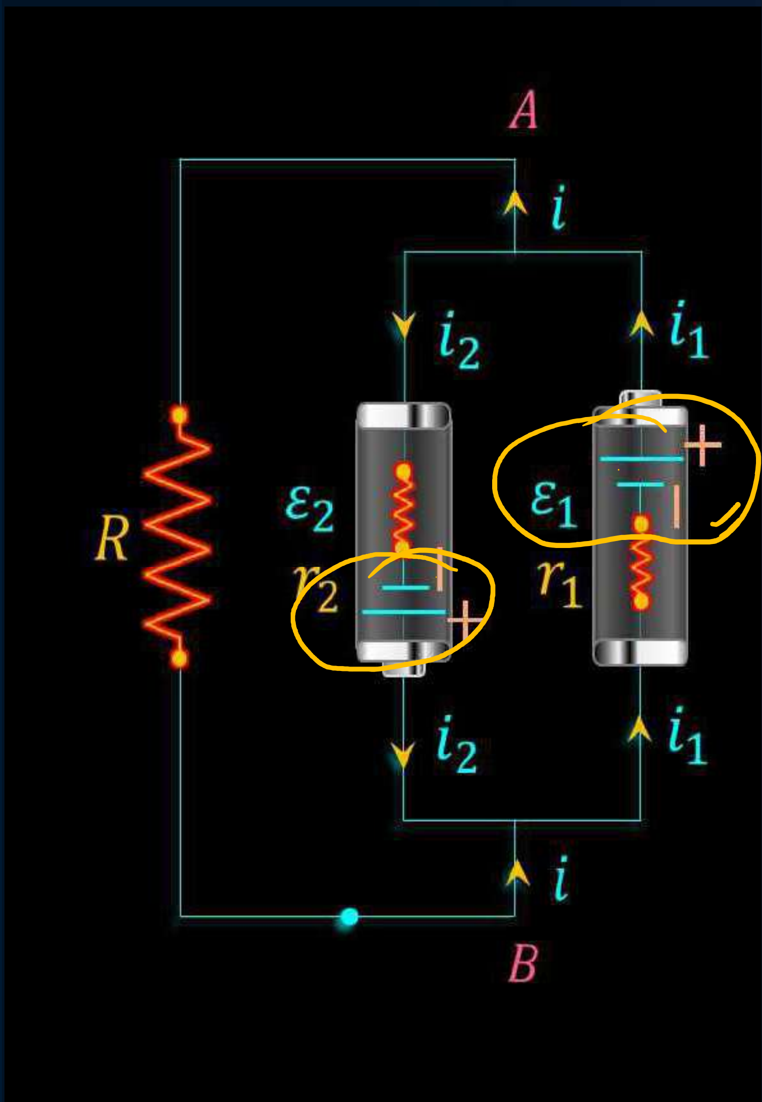
- n non-identical cells are connected in parallel with same polarity

$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3} + \cdots \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \frac{1}{r_n}}$$

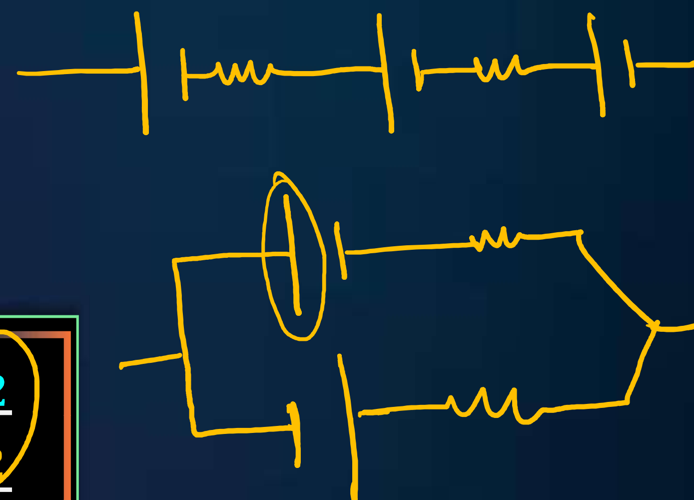
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \frac{1}{r_n}$$

Recap

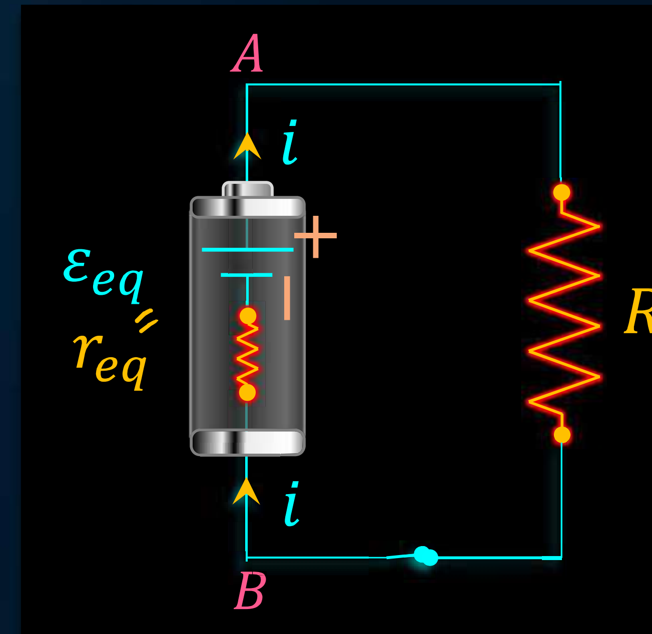
Parallel combination



- Opposite polarity
- $i = i_1 - i_2$



- $$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$
- $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$



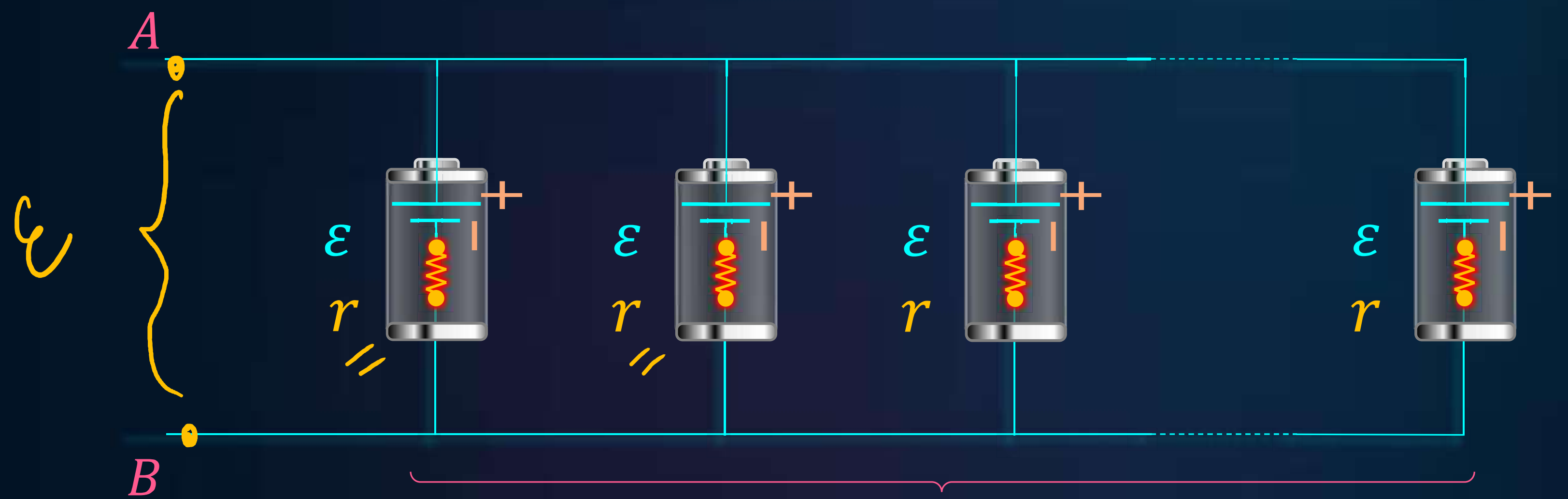
If the value of ε_{eq} comes out positive, then the equivalent cell will have same polarity as that of ε_1. If it comes out to be negative, then the polarity will be opposite.

Parallel Combination of Cells

- n identical cells are connected in parallel with same polarity

$$\mathcal{E}_{eq} = \mathcal{E}$$

$$r_{eq} = \frac{r}{n}$$



n identical cells

$$\mathcal{E}_{eq} = \frac{\frac{\mathcal{E}}{r} + \frac{\mathcal{E}}{r} \cdots}{\frac{1}{r} + \frac{1}{r} \cdots} = \frac{\cancel{n}\,\frac{\mathcal{E}}{\cancel{r}}}{\frac{\cancel{n}}{\cancel{r}}}$$

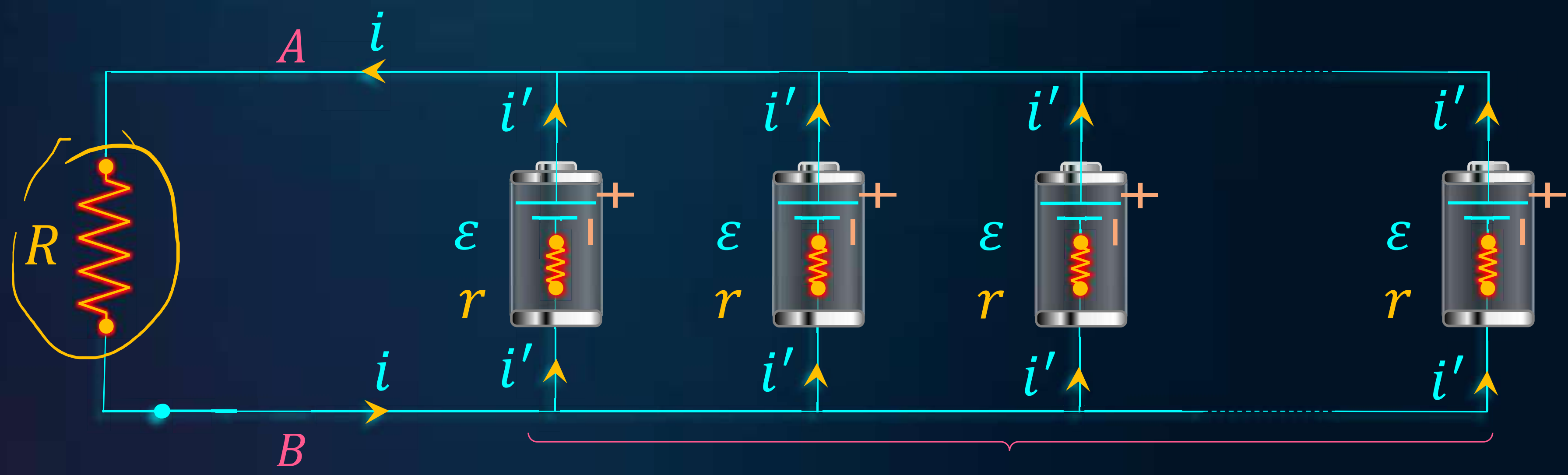
$$\boxed{\varepsilon_{eq} = \frac{\frac{n\varepsilon}{r}}{\frac{n}{r}} = \varepsilon}$$

$$\boxed{r_{eq} = \frac{r}{n}}$$

Parallel Combination of Cells

$$\boxed{\varepsilon_{eq} = \frac{\frac{n\varepsilon}{r}}{\frac{n}{r}} = \varepsilon}$$

$$\boxed{r_{eq} = \frac{r}{n}}$$



$$i = \frac{\varepsilon_{eq}}{R_{eq}}$$

$$R_{eq} = R + r_{eq}$$

$$R_{eq} = R + \frac{r}{n}$$

External resistance is in series connection with net internal resistance of all cells

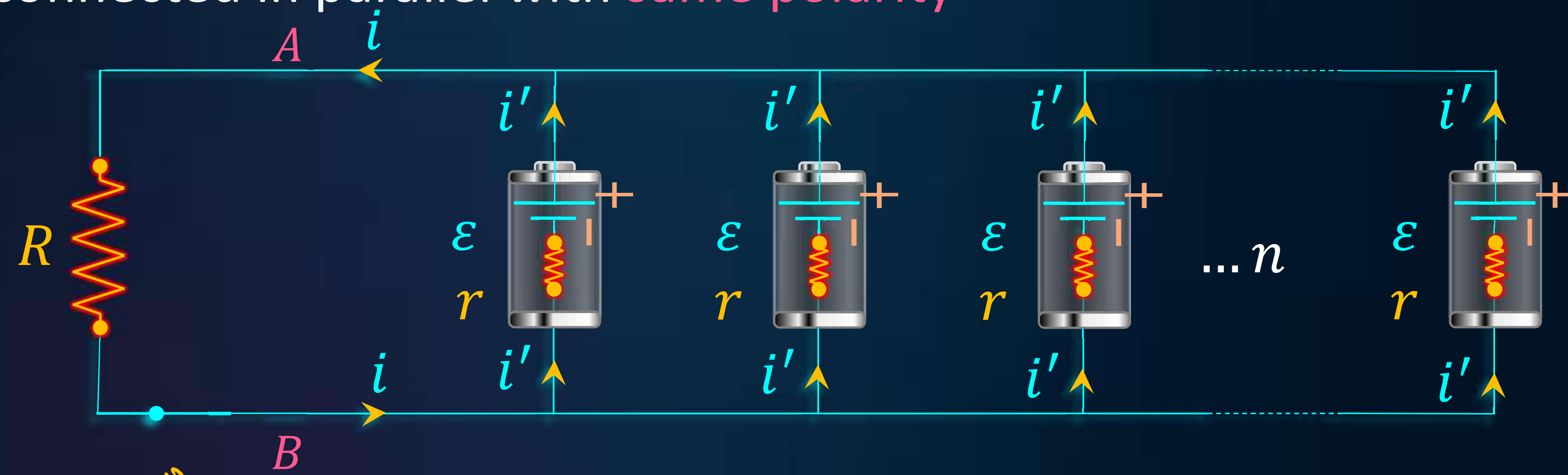
$$i = \frac{\varepsilon}{R + \frac{r}{n}}$$

$$\boxed{R_{eq} = R + \frac{r}{n}}$$

$$\boxed{i = \frac{\varepsilon}{R + \frac{r}{n}}}$$

Parallel Combination of Cells

- n identical cells are connected in parallel with same polarity



$$\boxed{i = \frac{\varepsilon}{R + \frac{r}{n}}}$$

If $R >> \frac{r}{n} \Rightarrow i = \frac{\varepsilon}{R}$ | $i = \frac{\varepsilon}{r}$

- $R \gg \frac{r}{n} \Rightarrow \boxed{i = \frac{\varepsilon}{R}}$

If $\frac{r}{n} >> R \Rightarrow i = \frac{n\varepsilon}{r}$

- $\frac{r}{n} \gg R \Rightarrow \boxed{i = \frac{n\varepsilon}{r}}$

Parallel Combination of Cells

- n identical cells are connected in parallel with same polarity

$$\boxed{i = \frac{n\varepsilon}{r}}$$

Current increases n times than current in single cell

$$\boxed{\varepsilon_{eq} = \varepsilon}$$

No change in e.m.f. than single cell

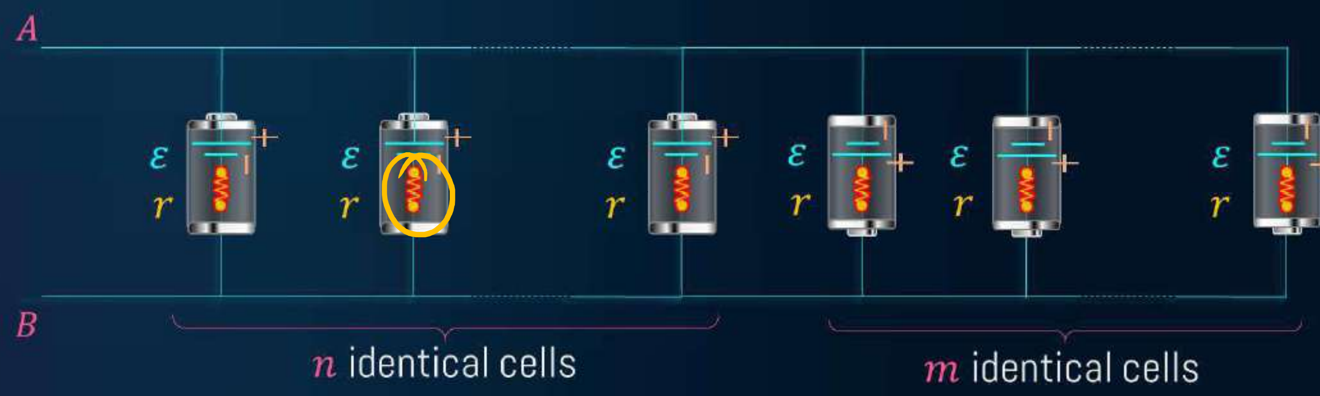
Operating voltage 12 V

$$\frac{r}{n} \gg R$$

Parallel Combination of Cells

- n identical cells with same polarity and m same cells with opposite polarity

$$\varepsilon_{eq} = \frac{\left(\frac{\varepsilon}{r}+\frac{\varepsilon}{r}+\frac{\varepsilon}{r}\dots\frac{\varepsilon}{r}\right)_n - \left(\frac{\varepsilon}{r}+\frac{\varepsilon}{r}+\frac{\varepsilon}{r}\dots\frac{\varepsilon}{r}\right)_m}{\left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}\dots\frac{1}{r}\right)_n + \left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}\dots\frac{1}{r}\right)_m}$$



$$\varepsilon_{eq} = \frac{n\frac{\varepsilon}{r} - m\frac{\varepsilon}{r}}{\frac{n}{r}+\frac{m}{r}} = \frac{(n-m)}{(n+m)}\varepsilon$$

$$r_{eq} = \frac{r}{(n+m)}$$

$$\boxed{\varepsilon_{eq} = \frac{(n-m)}{(n+m)}\varepsilon}$$

$$\boxed{r_{eq} = \frac{r}{n+m}}$$

Combination of Cells

- n identical cells are connected with same polarity

Series combination

$$\varepsilon_{eq} = n\varepsilon \qquad r_{eq} = nr$$

$$i = \frac{n\varepsilon}{R + nr}$$

For increase in e.m.f.

Parallel combination

$$\varepsilon_{eq} = \varepsilon \qquad r_{eq} = \frac{r}{n}$$

$$i = \frac{\varepsilon}{R + \frac{r}{n}} \Rightarrow i = \frac{n\varepsilon}{nR + r}$$

For increase in current

Combination of Cells

Example:

Consider Operational voltage for car is 12 V

$\therefore$ 10 cells of e.m.f 1.2 V are connected in series

$$\varepsilon_{eq} = n\varepsilon = 10 \times 1.2 = 12\ V$$

For increase in potential we use series combination

$1.2 \times 10 = 12V$

$0.1 \times 10 = 1A$

To get the desired voltage value, we need 10 cells of 1.2 V in series whereas to get the desired current, we need such 10 combinations with each cell giving out 0.1 A in parallel. So in total we require, 10 x 10 = 100 1.2 V cells.

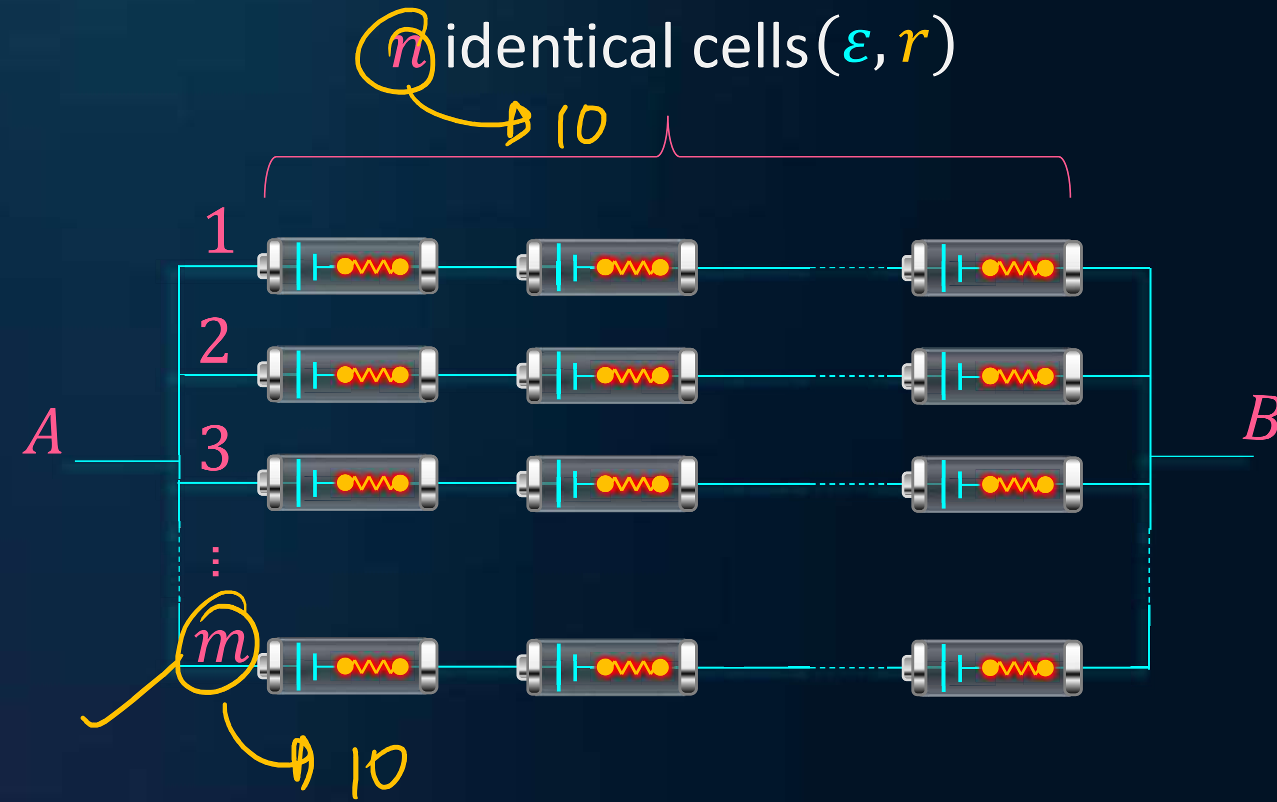
Combination of Cells

For increase in current we use parallel combination

For increase in potential we use series combination

Mixed combination of cell

Mixed Combination of Cells



- Batteries used in vehicles are mixed combination of cells

m rows each with n identical cells with same polarity are connected in parallel

Mixed Combination of Cells

n identical cells (ε, r) are in series

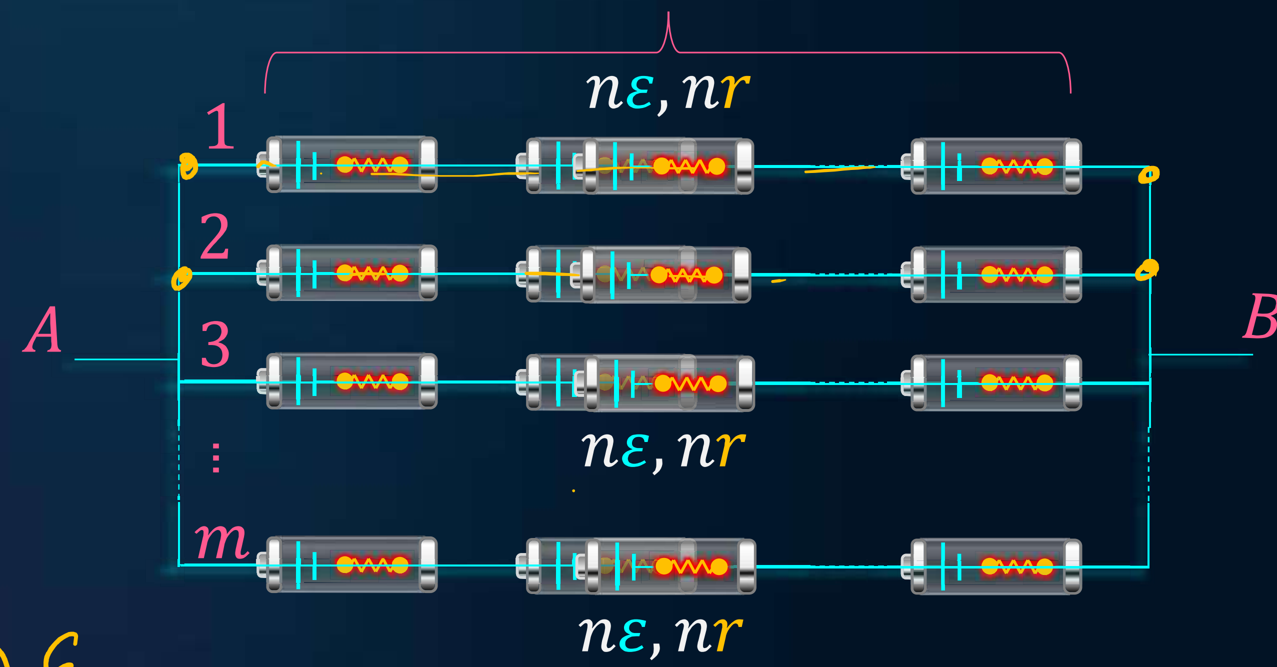
$\varepsilon_{eq_s} = n\varepsilon \qquad r_s = nr$

m identical cells $(n\varepsilon, nr)$ are in parallel

$\varepsilon_p = n\varepsilon \qquad \varepsilon_{eq} = n\varepsilon$

$r_p = \dfrac{nr}{m} \qquad r_{eq} = \dfrac{nr}{m}$

n identical cells (ε, r)



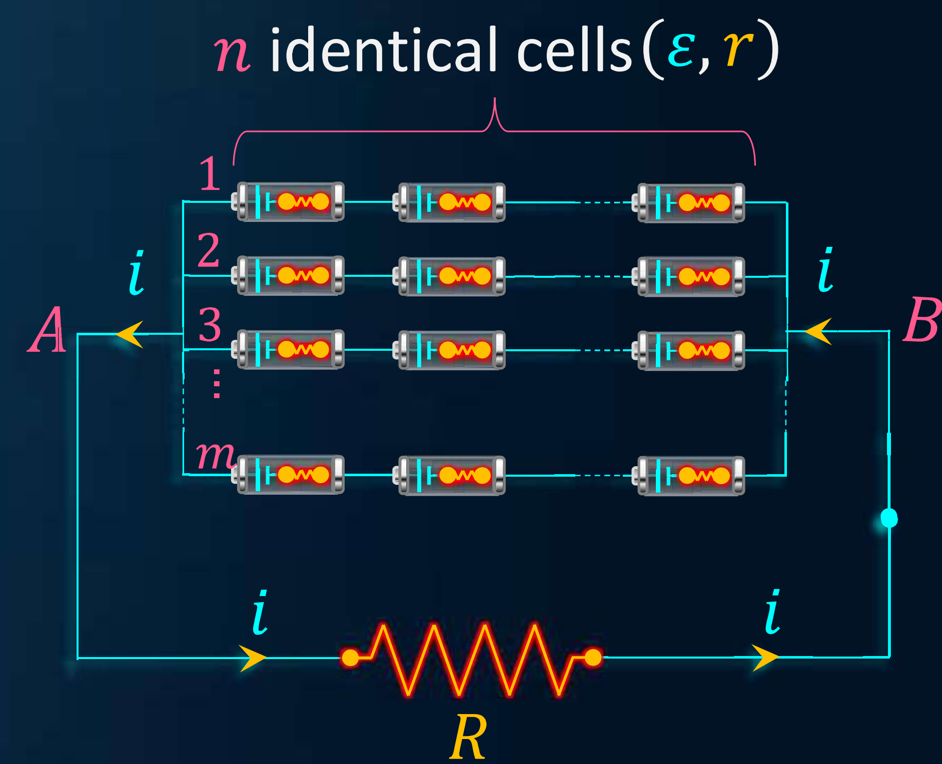
$$\boxed{\varepsilon_{eq} = n\varepsilon}$$

$$\boxed{r_{eq} = \frac{nr}{m}}$$

Mixed Combination of Cells

$$\varepsilon_{eq} = n\varepsilon$$

$$r_{eq} = \frac{nr}{m}$$



$$R_{eq} = R + r_{eq}$$

$$= R + \frac{nr}{m}$$

$$i = \frac{n\varepsilon}{R + \frac{nr}{m}} = \frac{nm\varepsilon}{mR + nR}$$

External resistance is in series connection with net internal resistance of all cells.

Number of cells in mixed combination $= mn$

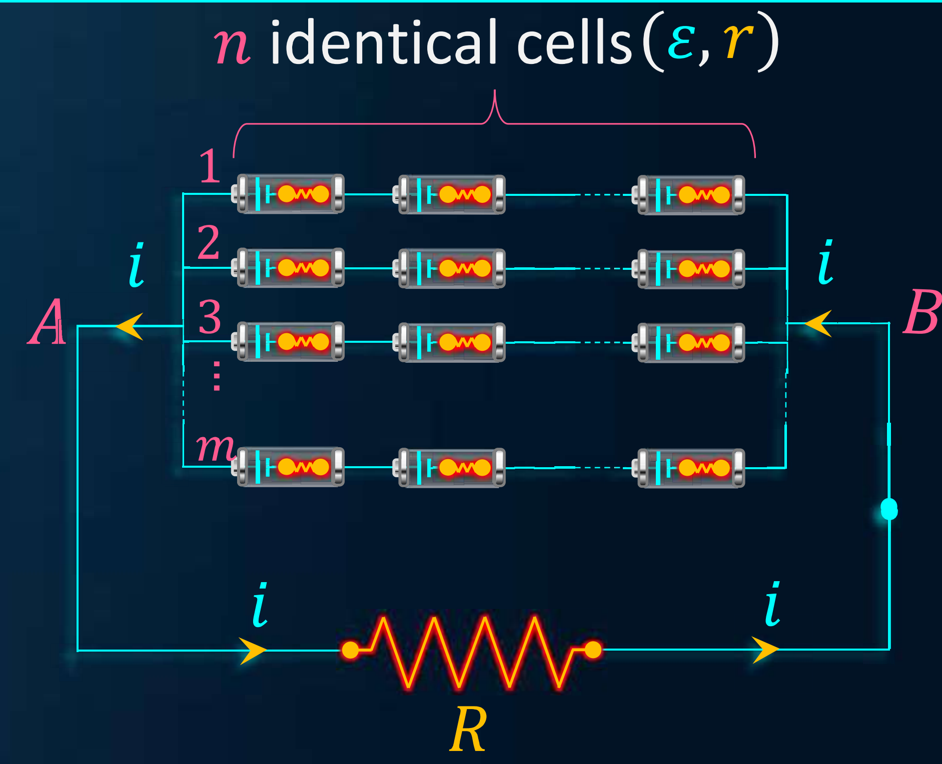
Mixed Combination of Cells

Condition for maximum current:

$$\boxed{i = \frac{nm\varepsilon}{mR + nr}}$$

$a = mR$

$b = nr$



$mR + nr$ Should be minimum

$$\left(\sqrt{a} - \sqrt{b}\right)^2 = a - 2\sqrt{ab} + b$$

$$\left(\sqrt{a} - \sqrt{b}\right)^2 + 2\sqrt{ab} = a + b$$

$$\therefore \quad mR + nr = \left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mRnr}$$

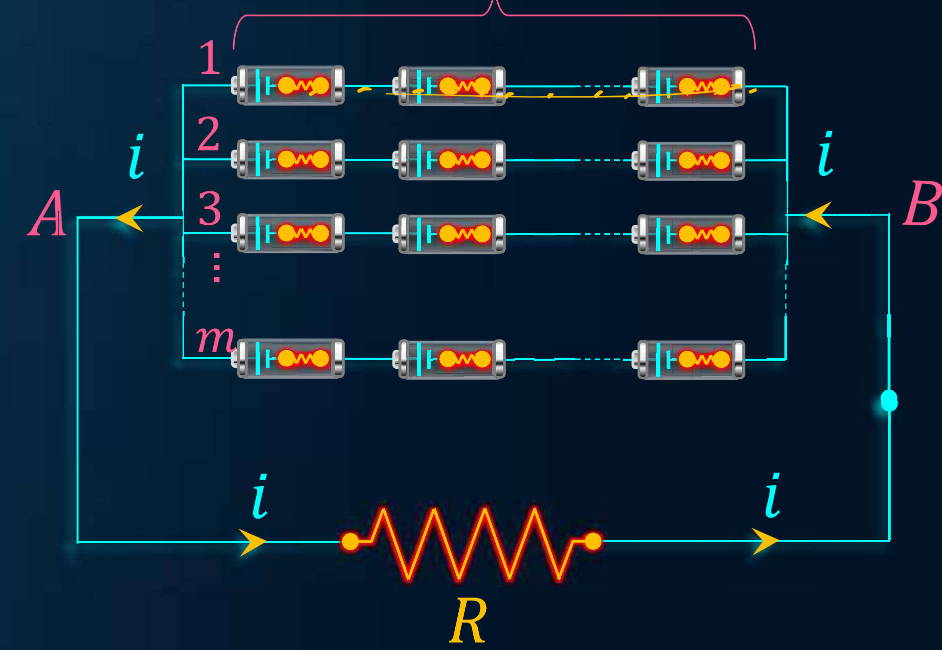
Mixed Combination of Cells

Condition for maximum current:

$$\boxed{i = \frac{nm\varepsilon}{mR + nr}}$$

$$\boxed{r_{eq} = \frac{nr}{m}}$$

n identical cells (ε, r)



$mR + nr$ Should be minimum

$$mR + nr = \left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mRnr}$$

$\rightarrow$ zero.

$$\sqrt{mR} = \sqrt{nr}$$

$$mR = nr$$

$$\boxed{R = \frac{nr}{m}}$$

$$\frac{R}{r} = \frac{n}{m} = \frac{\text{Number of cells in series}}{\text{Number of rows in parallel}}$$

$$\boxed{mR = nr}$$

$$\boxed{R = \frac{nr}{m}}$$

Mixed Combination of Cells

Condition for maximum current:

$$\boxed{i = \frac{nm\varepsilon}{mR + nr}}$$

$$\boxed{mR = nr}$$

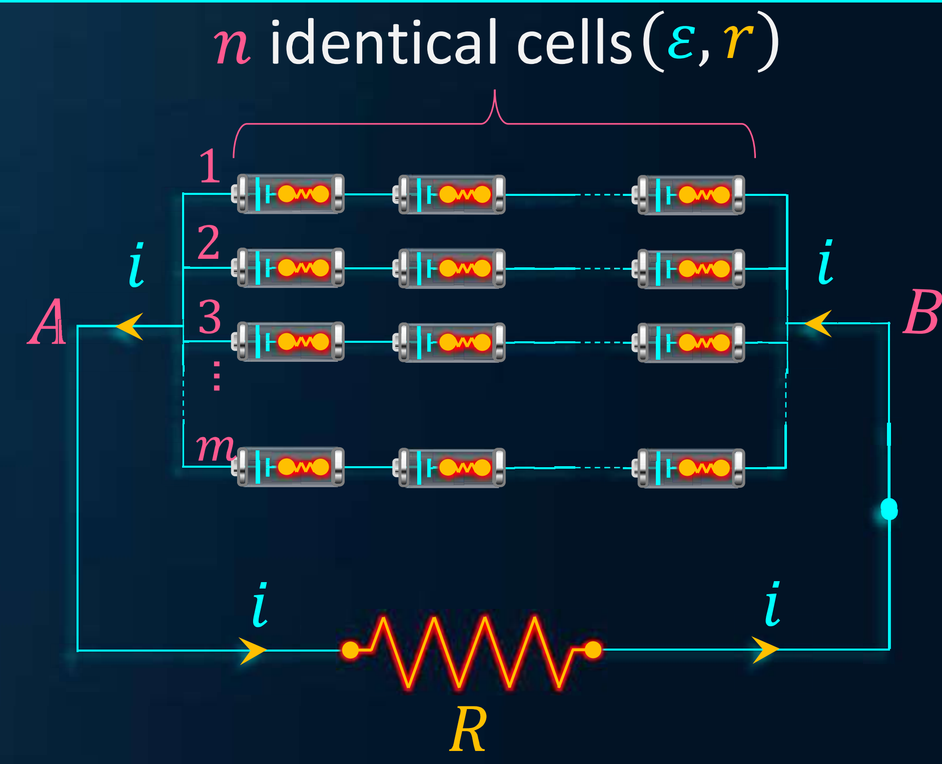
$$\boxed{R = \frac{nr}{m}}$$

$$\frac{nr}{m} = r_{eq}$$

For single cell $n = m = 1$

Condition for maximum current in single cell is $R = r$

When the load resistance becomes equal to the net internal resistance of the circuit then the voltage source delivers maximum power and this theorem is known as maximum power transfer theorem.



Mixed Combination of Cells

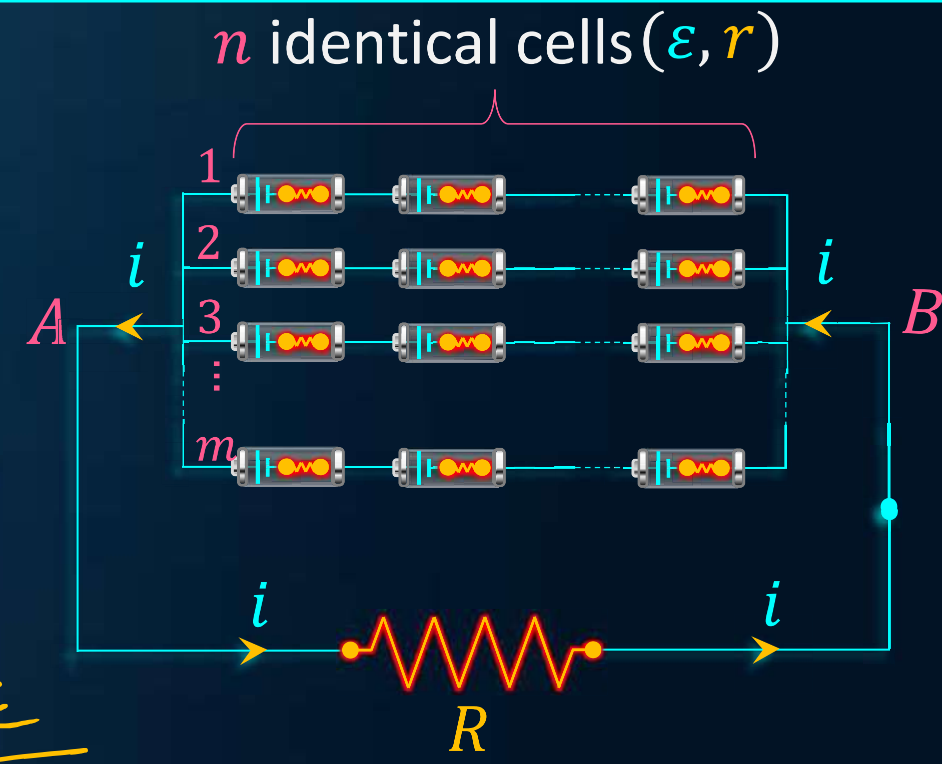
Condition for maximum current:

$$\boxed{i = \frac{nm\varepsilon}{mR + nr}}$$

R

$$\boxed{mR = nr}$$

i_{max}



$$i = \frac{nm\varepsilon}{mR + mR} = \frac{n\cancel{m}\varepsilon}{2\cancel{m}R} = \frac{n\varepsilon}{2R}$$

$$i = \frac{nm\varepsilon}{nr + nr} = \frac{m\varepsilon}{2r}$$

$$\boxed{i_{\max} = \frac{m\varepsilon}{2r} = \frac{n\varepsilon}{2R}}$$

Question

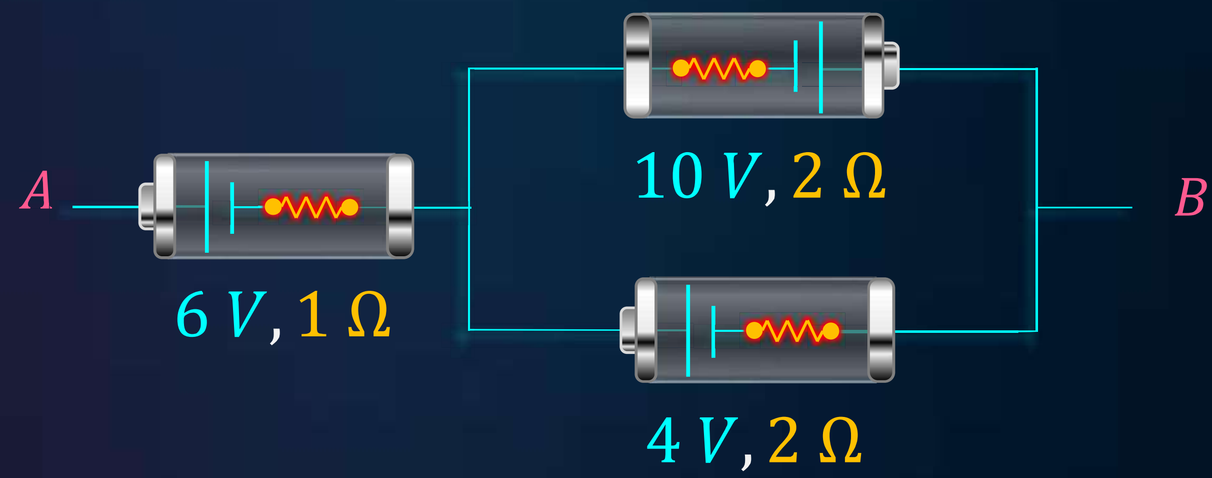
Find equivalent e.m.f and equivalent resistance of the given combination of cells.

(a) $3\ V, 2\ \Omega$

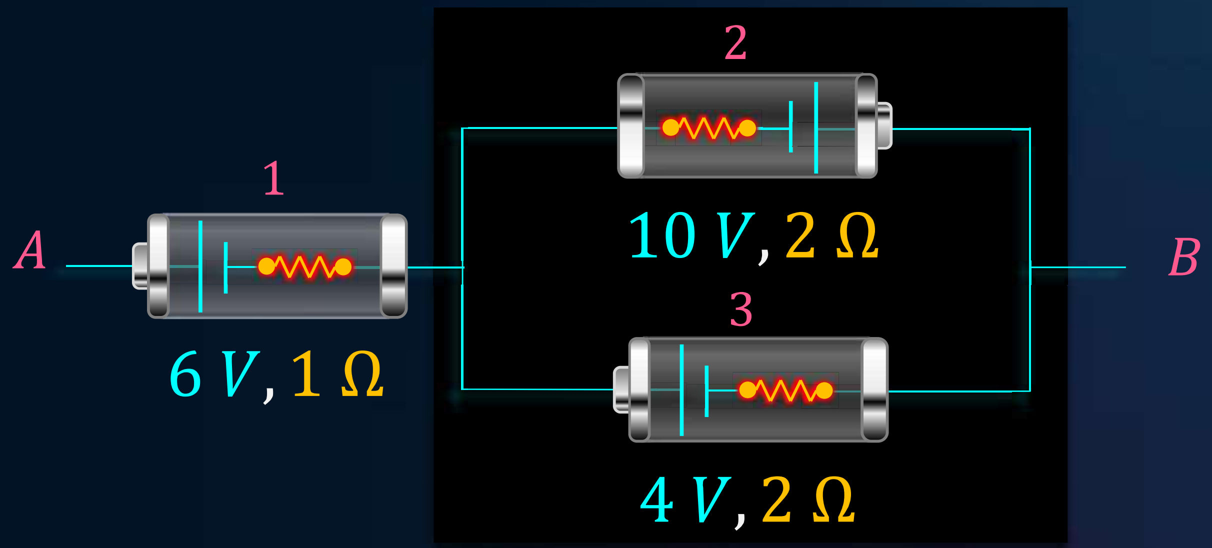
(b) $6\ V, 4\Omega$

(c) $3\ V, 3\ \Omega$

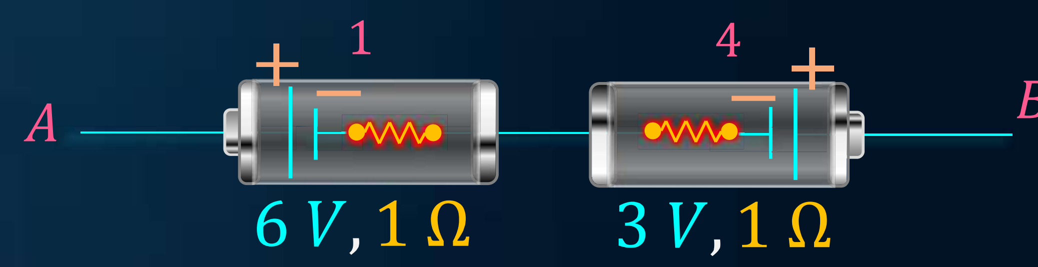
(d) $4\ V, 2\ \Omega$



Summary



$\Rightarrow$



Cells 2 and 3 are in parallel with opposite polarity

$$\varepsilon_{eq} = \frac{\frac{\varepsilon_3}{r_3} - \frac{\varepsilon_2}{r_2}}{\frac{1}{r_3} + \frac{1}{r_2}} = \frac{\frac{4}{2} - \frac{10}{2}}{\frac{1}{2} + \frac{1}{2}} = -3\ V$$

$$\frac{1}{r_{eq}} = \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{2} + \frac{1}{2} = 1\ \Omega$$

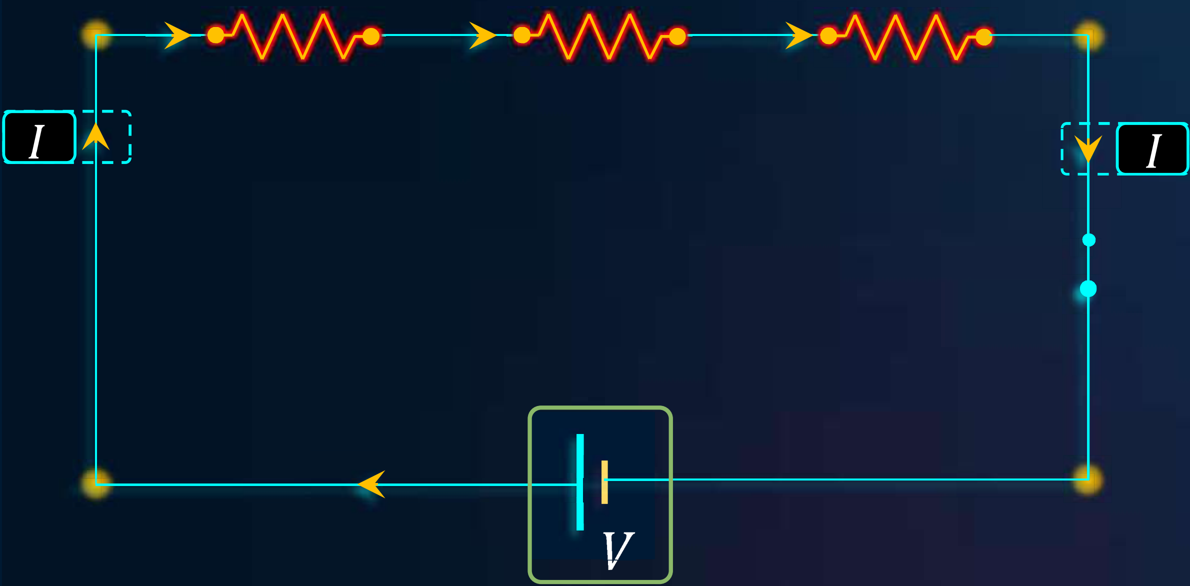
Cells 1 and 4 are in series with opposite polarity

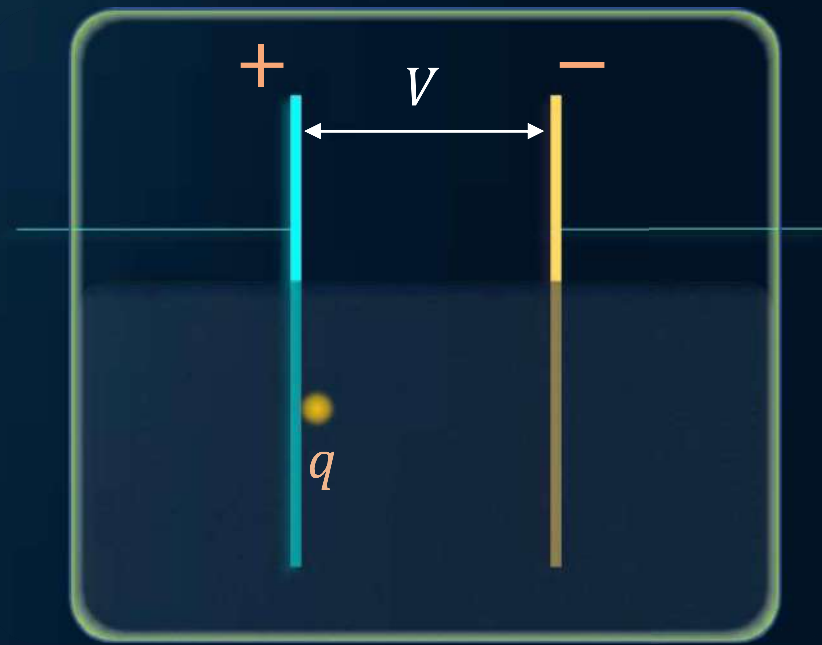
$$\varepsilon_{eq} = \varepsilon_1 - \varepsilon_4 = 6 - 3 = 3\ V$$

$$r_{eq} = r_1 + r_4 = 1 + 1 = 2\ \Omega$$

Thus, option (a) is the correct answer.

Power





Cell is required to do work to move charge.

$$\boxed{W_{cell} = qV}$$

$$V = \frac{W_{cell}}{q}$$

$$P = \frac{qV}{t} = VI$$

Work done by cell

$$W_{cell} = q \times V$$

Here, $q = -e$

$V = -V$

$W_{cell} = eV$

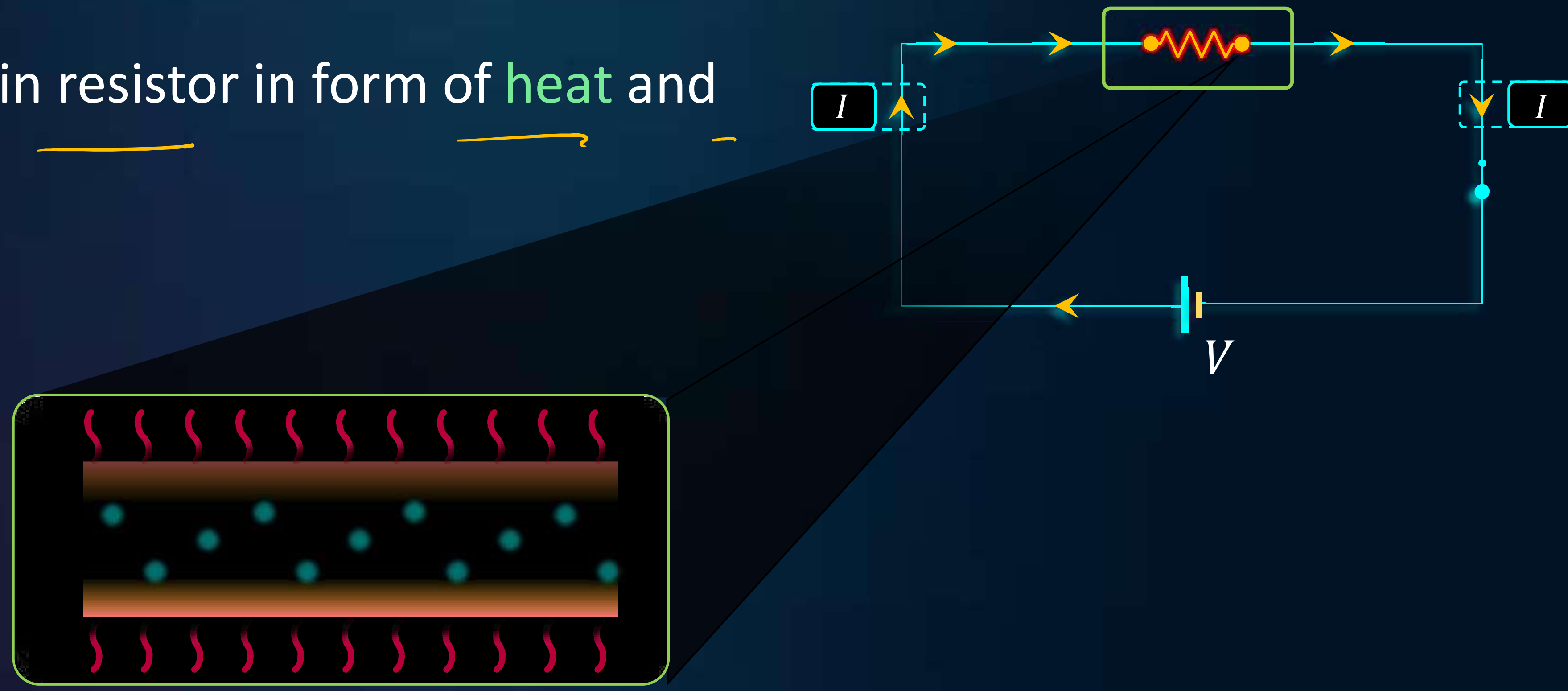
Energy supplied by cell= Work done by cell

Power delivered by cell

$$P = \frac{\text{Work done by cell}}{\text{Time}} = \frac{q \times V}{t} = VI$$

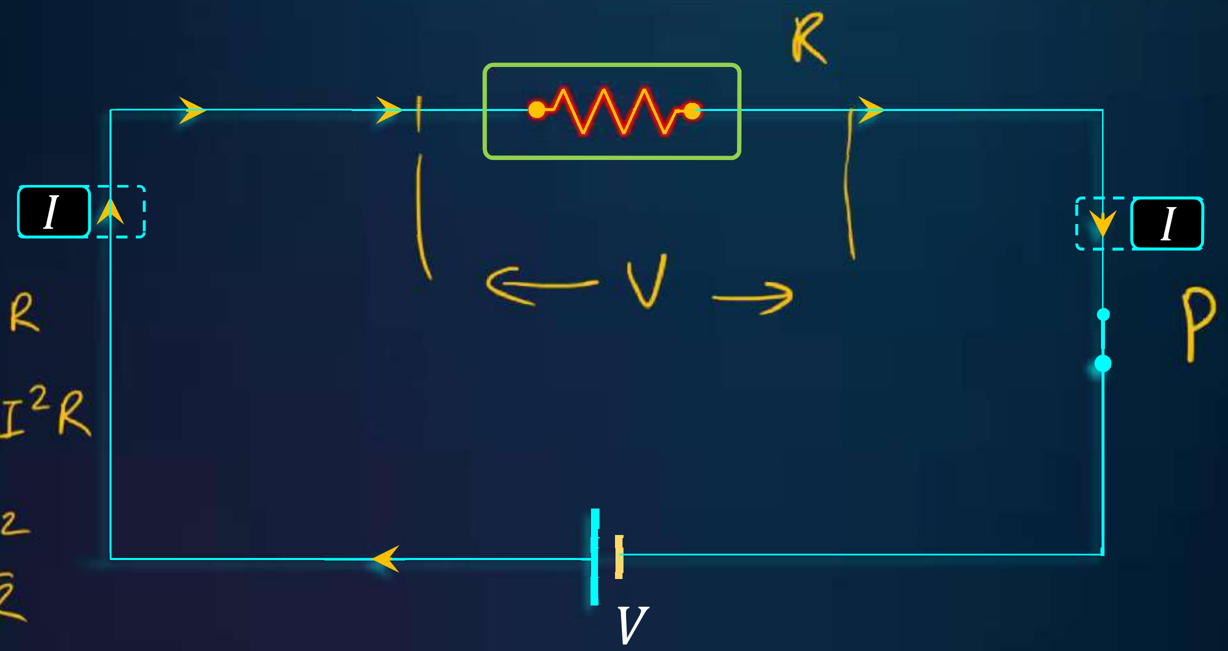
Power

- Power always gets dissipated in resistor in form of heat and light
- Resistor is a passive device.



- S.I unit of power is $\frac{Joule}{sec}$ or Watt.

Power



$P_{cell} = P_{diss}$

VI

$V = IR$

$P_{diss} = I^2R$

$P = \frac{V^2}{R}$

$P = VI = \frac{V^2}{R} = I^2R$

Power dissipated by resistor = Power delivered by cell

$$P = VI$$

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

$$V = IR$$

$$I = \frac{V}{R}$$

$$\boxed{P = VI = \frac{V^2}{R} = I^2R}$$

Energy dissipated by resistor in time (t) = (Power dissipated by resistor in time (t)) $\times\ t$

$$P = \frac{E}{t}$$

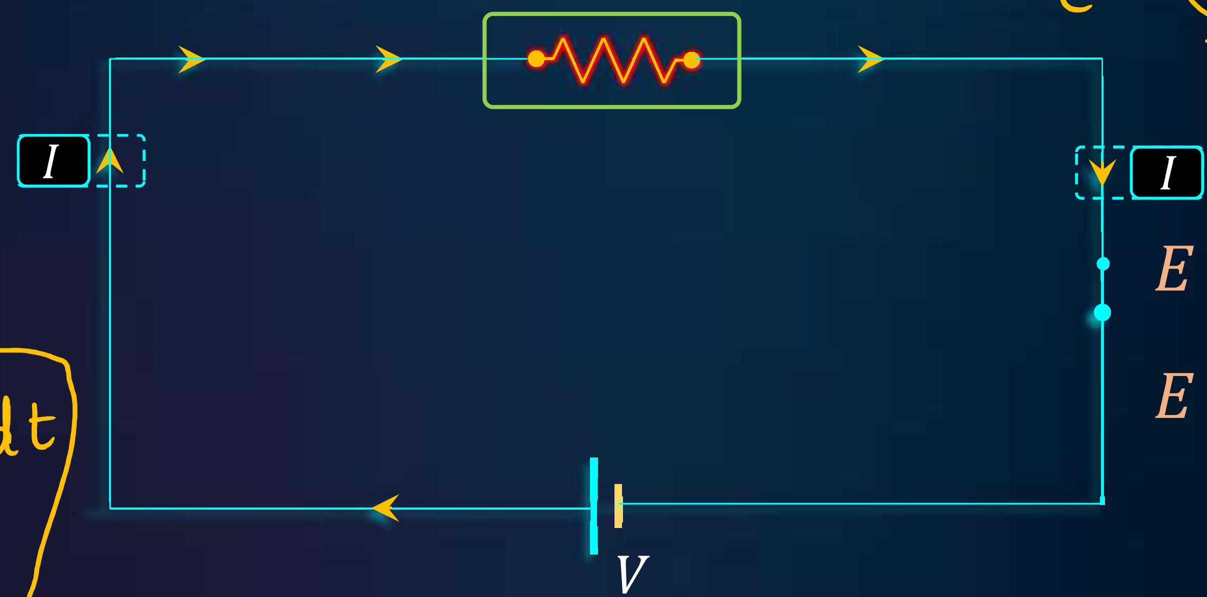
$$E = P \times t$$

$$E = VI\,t$$

$$\boxed{\int dE = \int I^2 R\,dt}$$

$$E = I^2 R t$$

$$E = \frac{V^2 t}{R}$$



$E = P \times t$ | $P = VI$

$E = VIt$ | $V = IR$

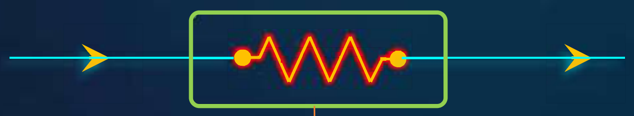
$\int dE = \int I^2 R\,dt$ (If I is varying with time t)

$E = I^2 R t$ (If I is constant)

$E = \dfrac{V^2 t}{R}$ | $I = \dfrac{V}{R}$

$$\boxed{E = VIt = \frac{V^2 t}{R} = I^2 R t}$$

Energy



$$P = VI$$

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

$$E = P \times t$$

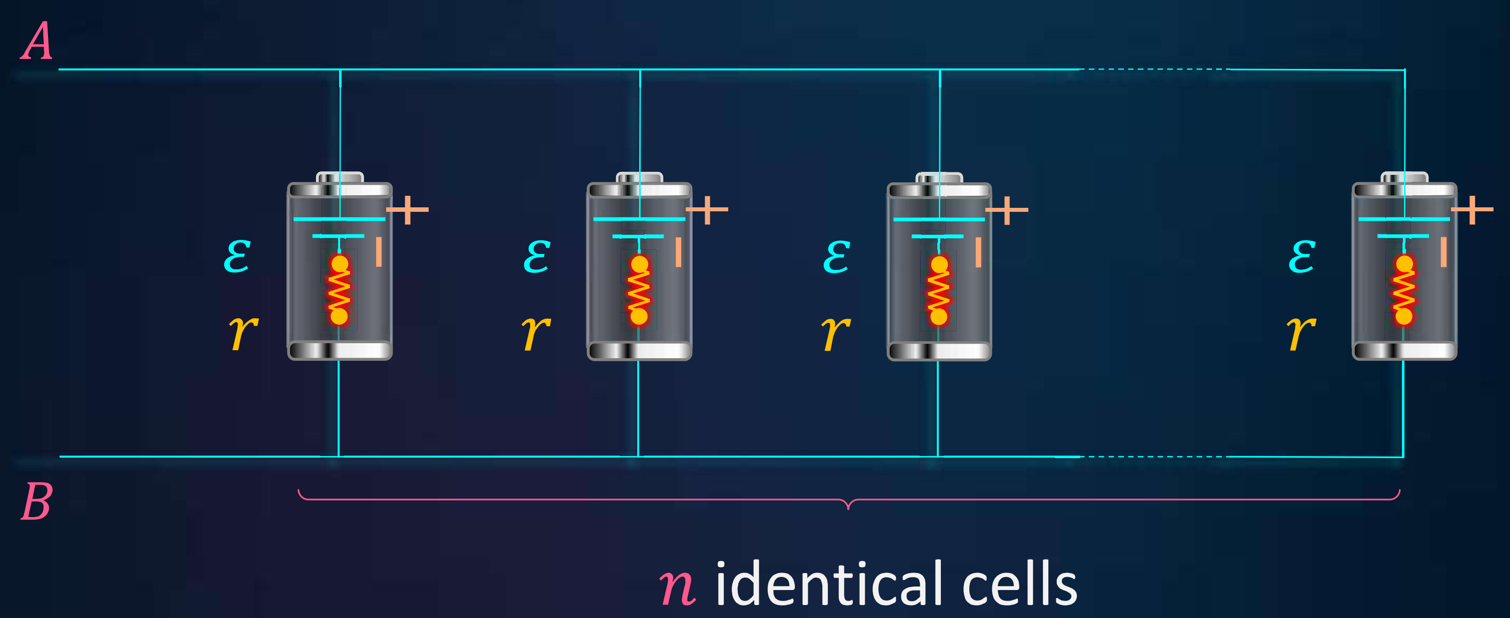
$$E = VIt$$

$$E = I^2Rt$$

$$E = \frac{V^2t}{R}$$

Recap

Parallel combination of cells



$$\varepsilon_{eq} = \frac{\frac{n\varepsilon}{r}}{\frac{n}{r}} = \varepsilon$$

$$r_{eq} = \frac{r}{n}$$

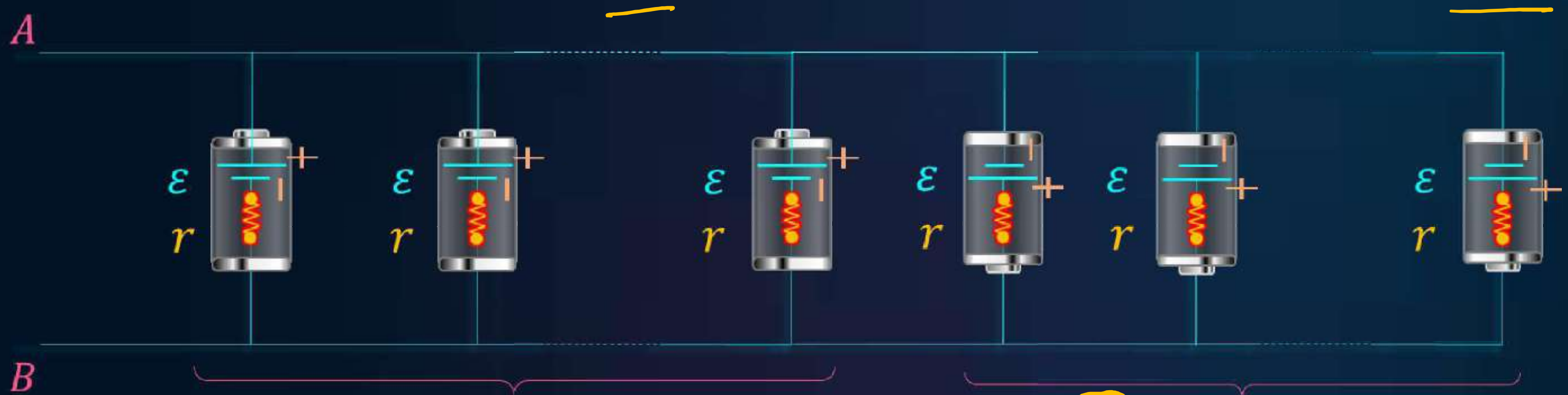
Recap

Parallel combination of cells

n identical cells with same polarity and m same cells with opposite polarity

$$\varepsilon_{eq} = \frac{(n-m)}{(n+m)}\varepsilon$$

$$r_{eq} = \frac{r}{n+m}$$



$$\varepsilon_{eq} = \frac{\left(\frac{\varepsilon}{r}+\frac{\varepsilon}{r}+\frac{\varepsilon}{r}\dots\frac{\varepsilon}{r}\right)_n - \left(\frac{\varepsilon}{r}+\frac{\varepsilon}{r}+\frac{\varepsilon}{r}\dots\frac{\varepsilon}{r}\right)_m}{\left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}\dots\frac{1}{r}\right)_n + \left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}\dots\frac{1}{r}\right)_m}$$

Recap

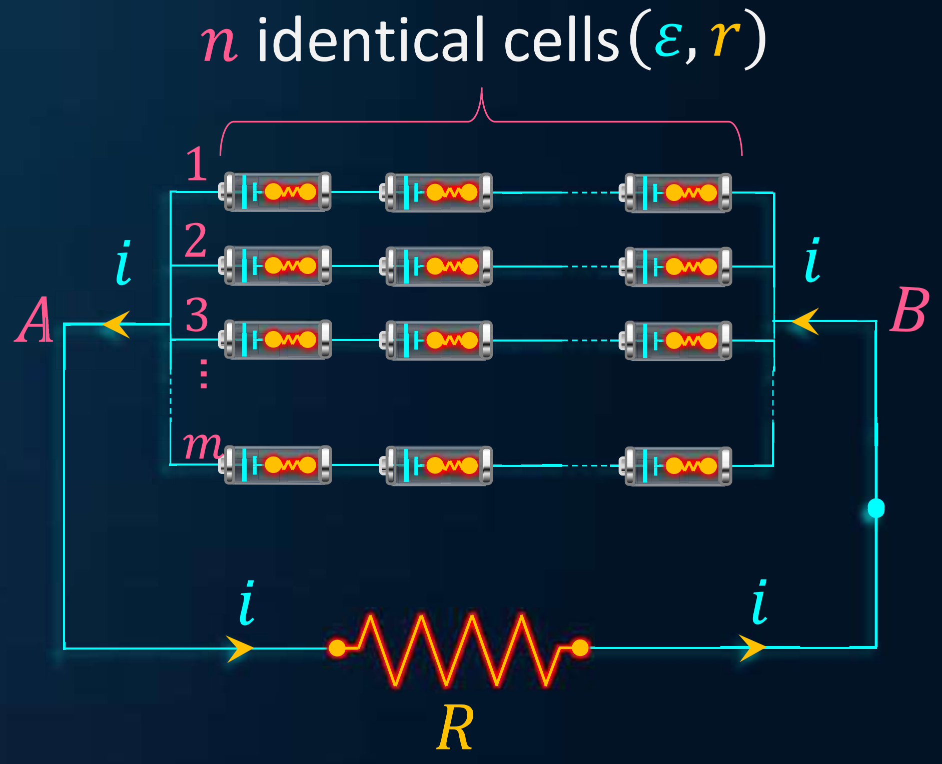
Mixed combination of cells

$$\varepsilon_{eq} = n\varepsilon$$

$$r_{eq} = \frac{nr}{m}$$

$$R_{eq} = R + \frac{nr}{m}$$

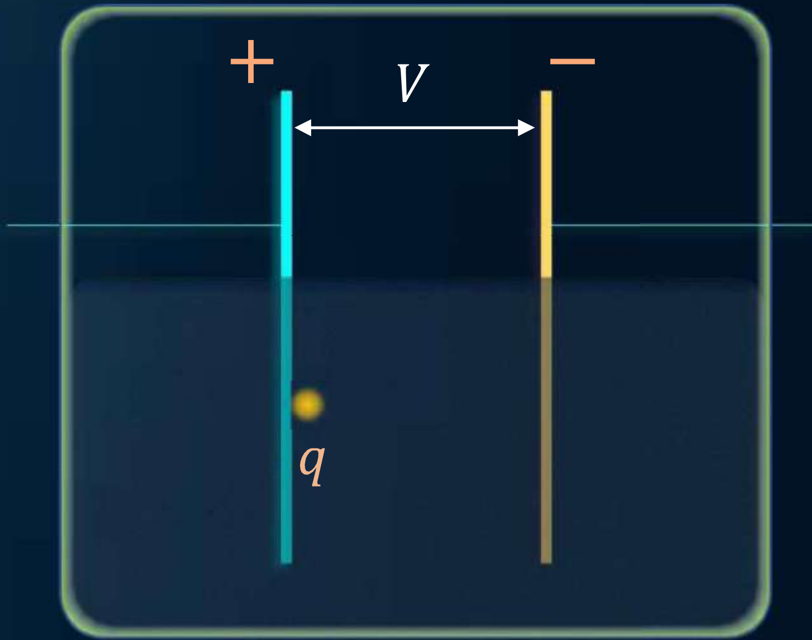
$$i = \frac{n\varepsilon}{R + \frac{nr}{m}}$$



Number of cells in mixed combination $= mn$

Recap

Power



Work done by cell: $W_{cell} = q \times V$

Power delivered by cell: $P = \dfrac{\text{Work done by cell}}{\text{Time}} = \dfrac{q \times V}{t} = VI$

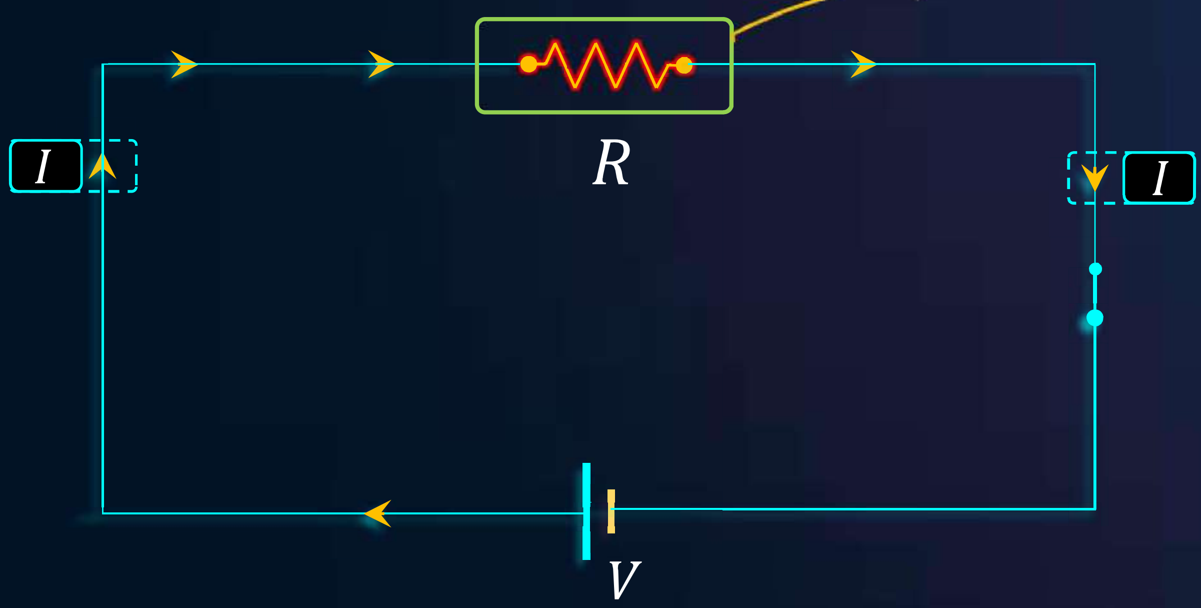
Recap

Power

- Power always gets dissipated in resistor in form of heat and light

- S.I unit of power is $\frac{Joule}{sec}$ or Watt.

$V = IR$

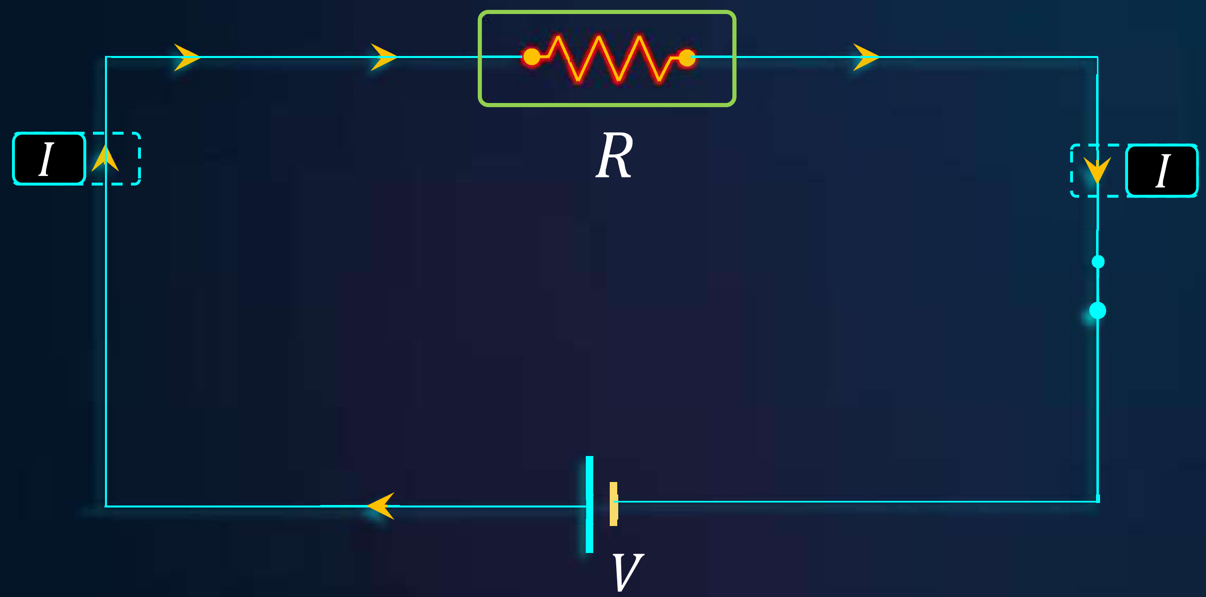


$$P = VI = \frac{V^2}{R} = I^2R$$

Recap

Energy

Energy dissipated by resistor in time (t) = (Power dissipated by resistor in time (t)) $\times t$

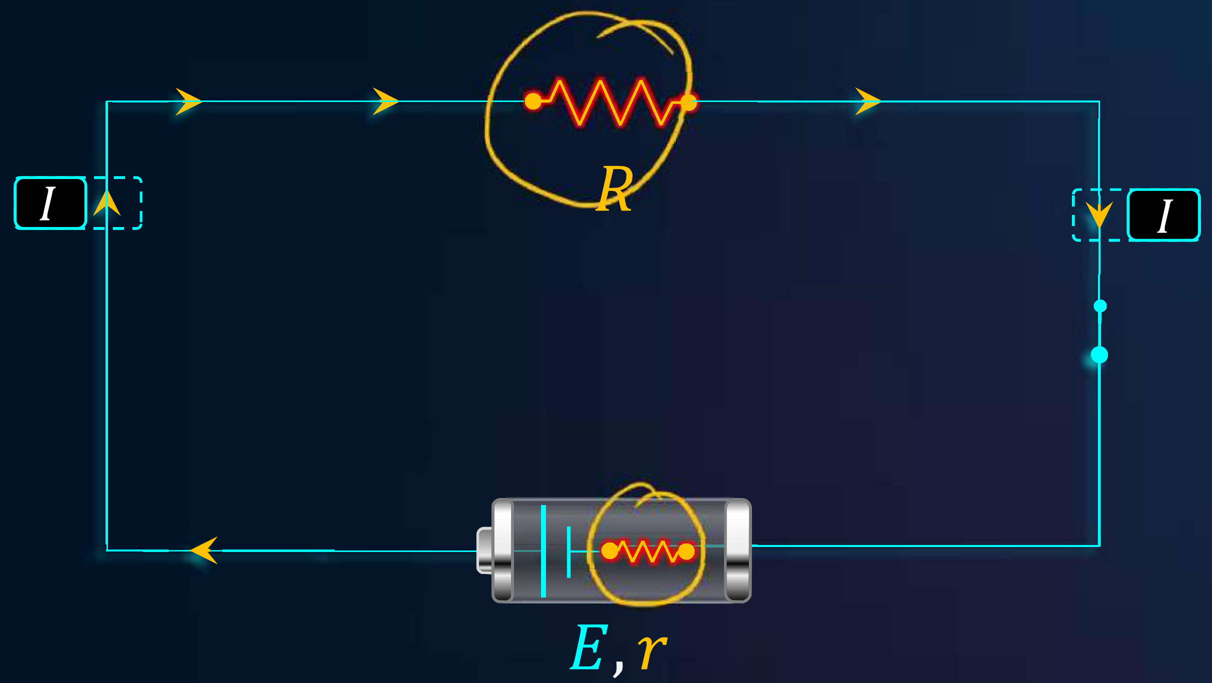


$$E = VIt = \frac{V^2 t}{R} = I^2 R t$$

$$\int dE = \int I^2 R \, dt$$ (If I is varying with time t)

$$E = I^2 R t$$ (If I is constant)

Maximum power transfer theorem



$P = 0$, when $R = 0$

$R = \infty$

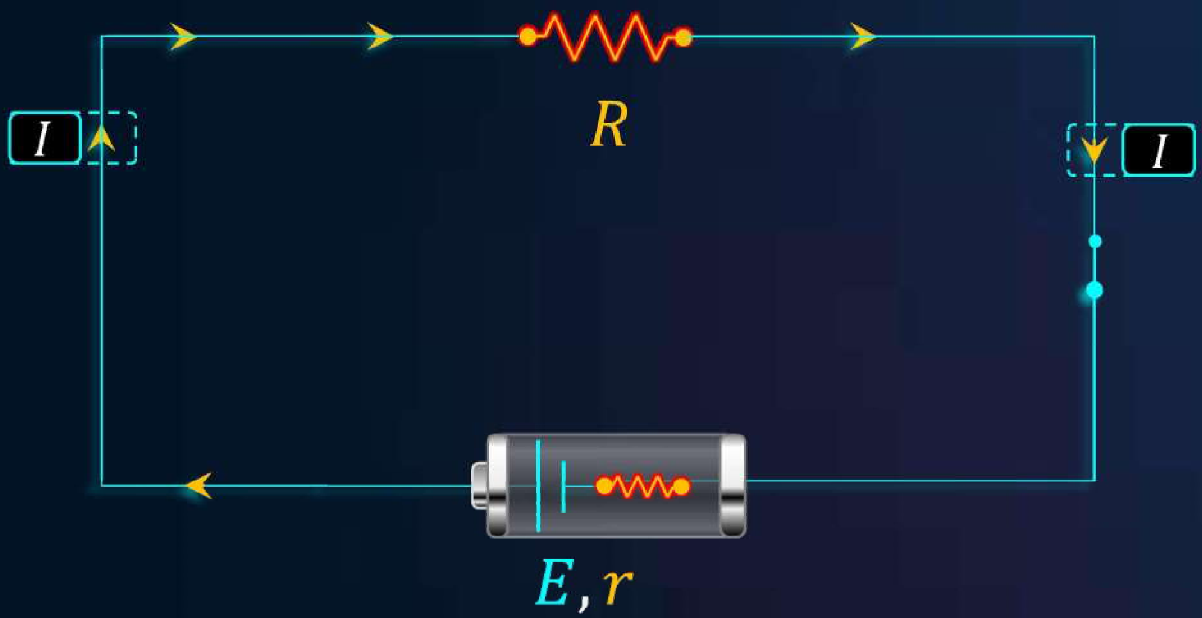
Power dissipated across R is given by:

$$P = i^2 R$$

$$i = \frac{E}{R + r}$$

$$P = \frac{E^2 R}{(R+r)^2} = \frac{E^2}{R\left(1 + \frac{r}{R}\right)^2}$$

Maximum power transfer theorem



$r^2 = R^2$

$\boxed{r = R}$

$$P = \frac{E^2 R}{(R+r)^2} = \frac{E^2}{R\left(1+\frac{r}{R}\right)^2}$$

For maximum power across R :

$$\frac{dP}{dR} = 0$$

$$\cancel{E^2}\frac{d}{dR}\left[\frac{R}{(R+r)^2}\right] = 0$$

$$\frac{(R+r)^2 - 2R(R+r)}{\cancel{(R+r)^4}} = 0$$

$$\Rightarrow (R+r)^2 = 2R(R+r)$$

$R^2 + r^2 + \cancel{2Rr} = 2R^2 + \cancel{2Rr}$

$$\therefore r = R$$

The power dissipated across the external resistance is maximum when the value of external resistance is equal to the effective internal resistance .This theorem is known as maximum power transfer theorem.

Power

Maximum power transfer theorem

$$P = \frac{E^2 R}{(R+r)^2} = \frac{E^2}{R\left(1+\frac{r}{R}\right)^2}$$

$r = R$ (Condition for maximum power)

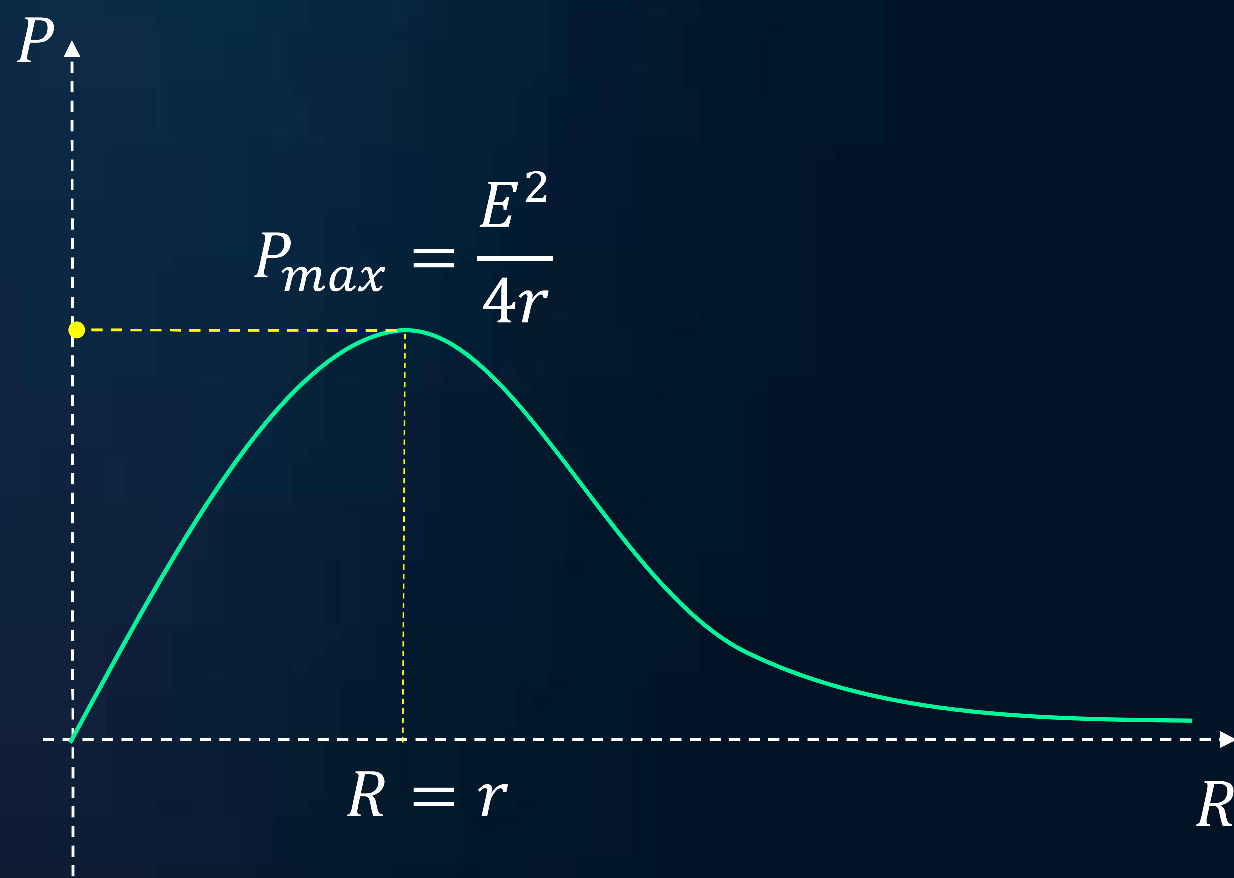
$$P = \frac{E^2 R}{(R+r)^2}$$

$$P = \frac{E^2 \cancel{R}}{4R^{\cancel{2}}}$$

Maximum power:

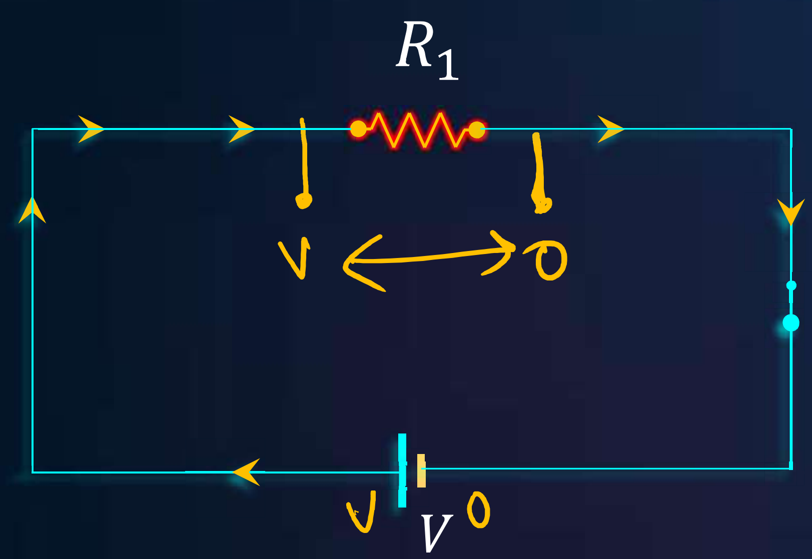
$$\boxed{P = \frac{E^2}{4R}}$$

$$\boxed{P = \frac{E^2}{4r}}$$

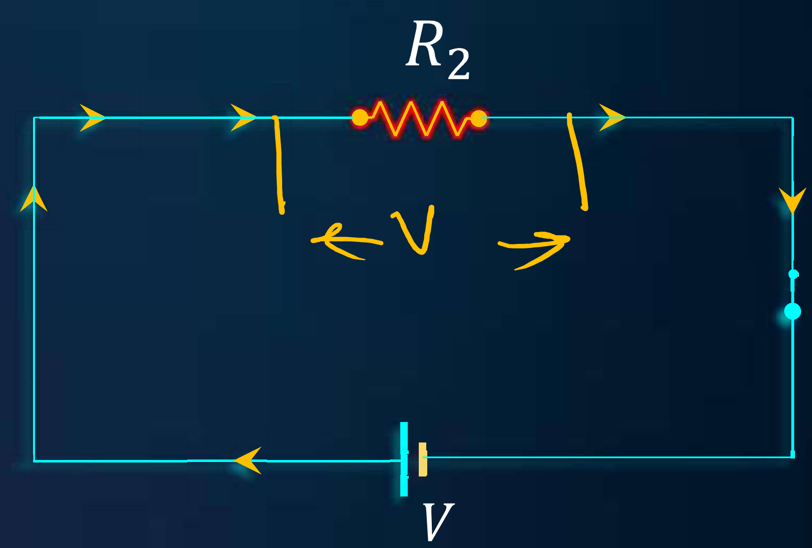


Equivalent power

Consider two resistors R_1 and R_2 connected separately across same voltage source V



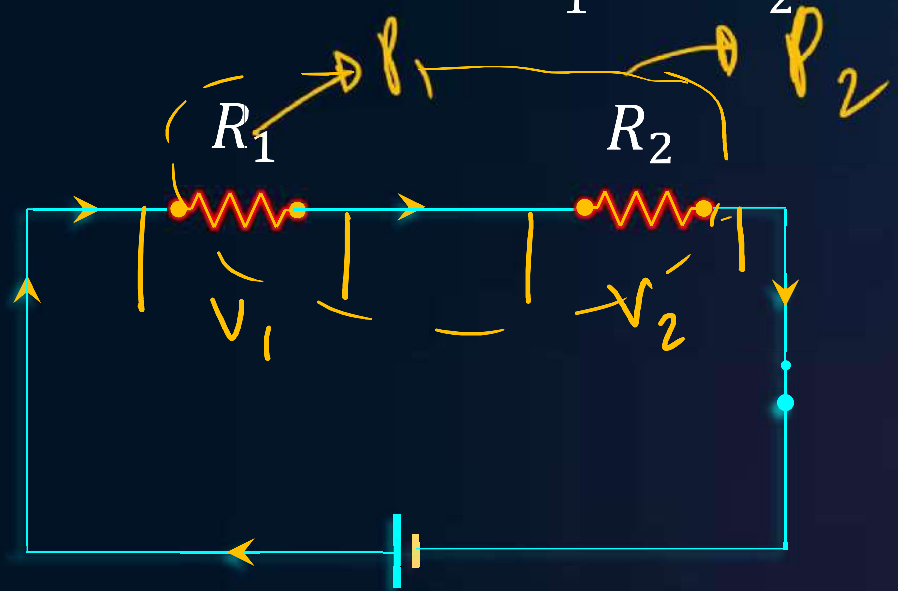
$$P_1 = \frac{V^2}{R_1}$$

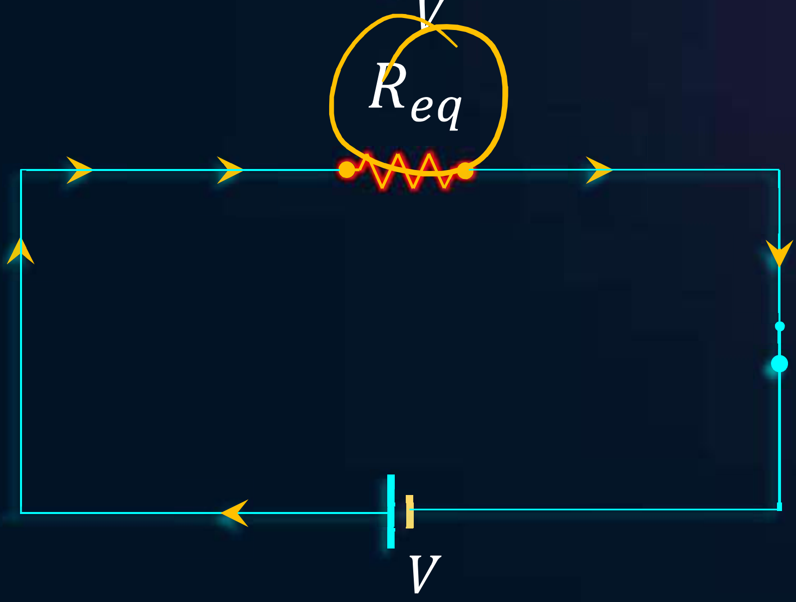


$$P_2 = \frac{V^2}{R_2}$$

Equivalent power

The two resistors R_1 and R_2 are now connected in series to same voltage source V





$$R_{eq} = R_1 + R_2$$

$$P_{eq} = \frac{V^2}{R_{eq}}$$

$$\frac{1}{P_{eq}} = \frac{R_{eq}}{V^2} = \frac{R_1 + R_2}{V^2} = \frac{R_1}{V^2} + \frac{R_2}{V^2}$$

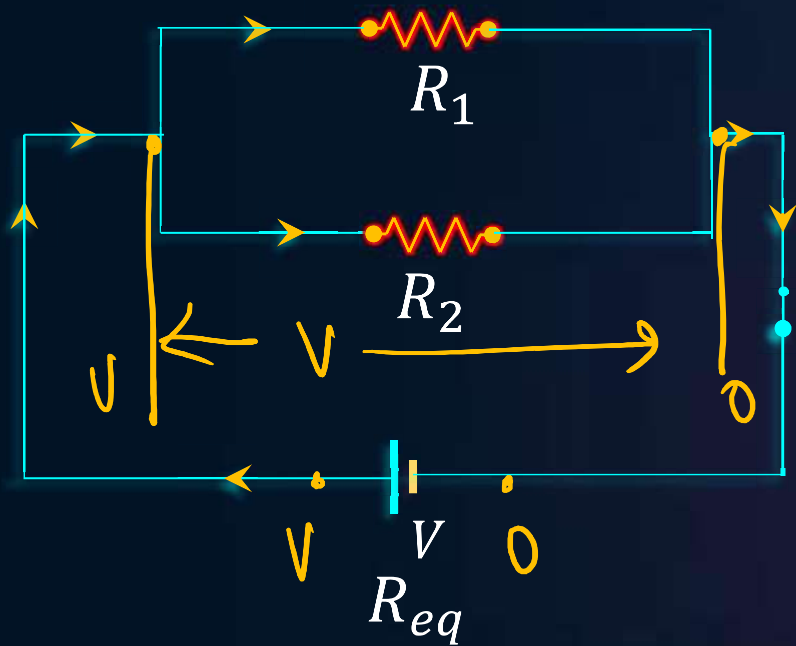
$$\boxed{P_1 = \frac{V^2}{R_1}}$$

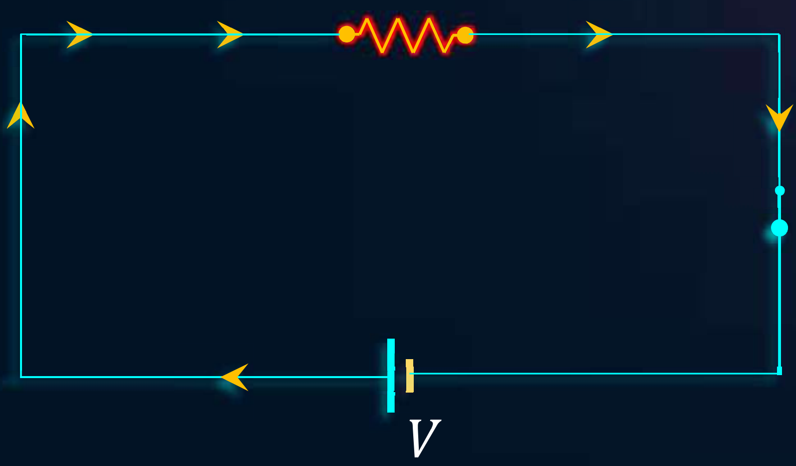
$$\boxed{P_2 = \frac{V^2}{R_2}}$$

$$\boxed{\frac{1}{P_{eq}} = \frac{1}{P_1} + \frac{1}{P_2}}$$

Equivalent power

The two resistors R_1 and R_2 are now connected in parallel to same voltage source V





$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad P_{eq} = P_1 + P_2$$

$$P_{eq} = \frac{V^2}{R_{eq}} = V^2 \times \frac{1}{R_{eq}}$$

$$P_{eq} = V^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$P_{eq} = \frac{V^2}{R_1} + \frac{V^2}{R_2}$$

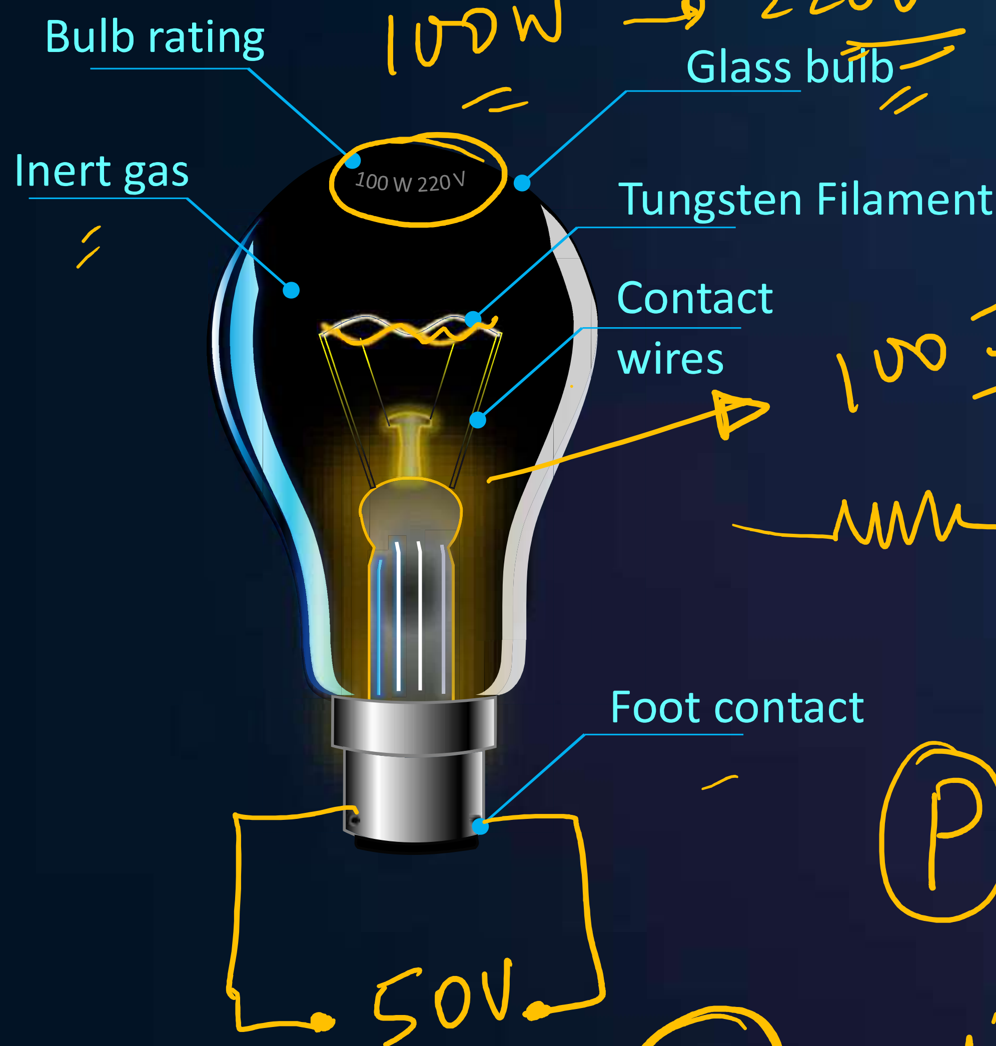
$$\boxed{P_1 = \frac{V^2}{R_1}}$$

$$\boxed{P_2 = \frac{V^2}{R_2}}$$

$$\boxed{P_{eq} = P_1 + P_2}$$

Bulb

$100\,W \rightarrow 220\,V$



$100\ J/s$

Circuit diagram symbol

$$P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{(220)^2}{100}$$

Bulb

Bulb rating (100 W, 220 V)

At $220\ V$, bulb will dissipate $100\ W$ energy

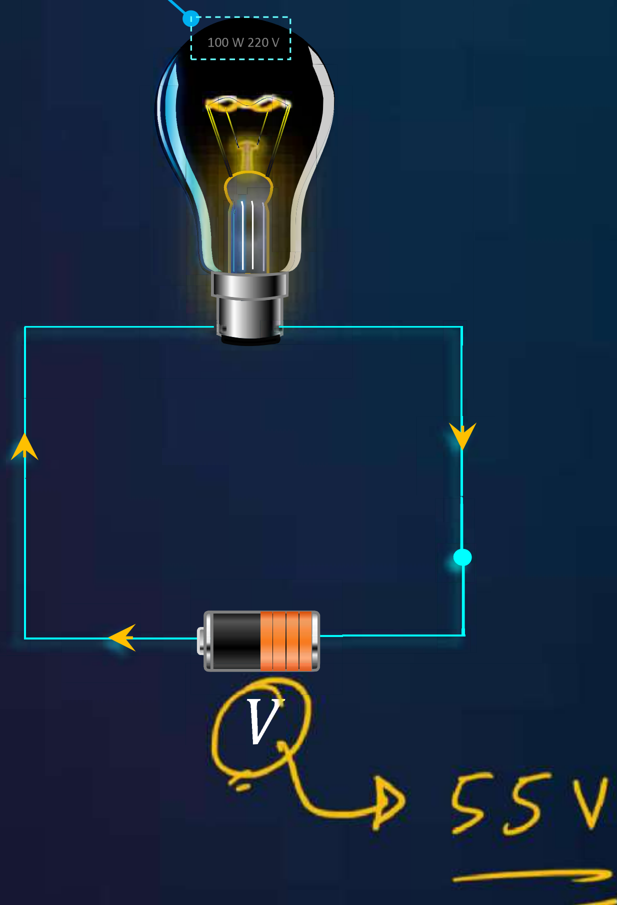
- Resistance of filament is constant

$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$

$R = 484\ \Omega$

- Power dissipated at $55\ V$

$P = \frac{V^2}{R} = \frac{(55)^2}{484} = 6.25 W$



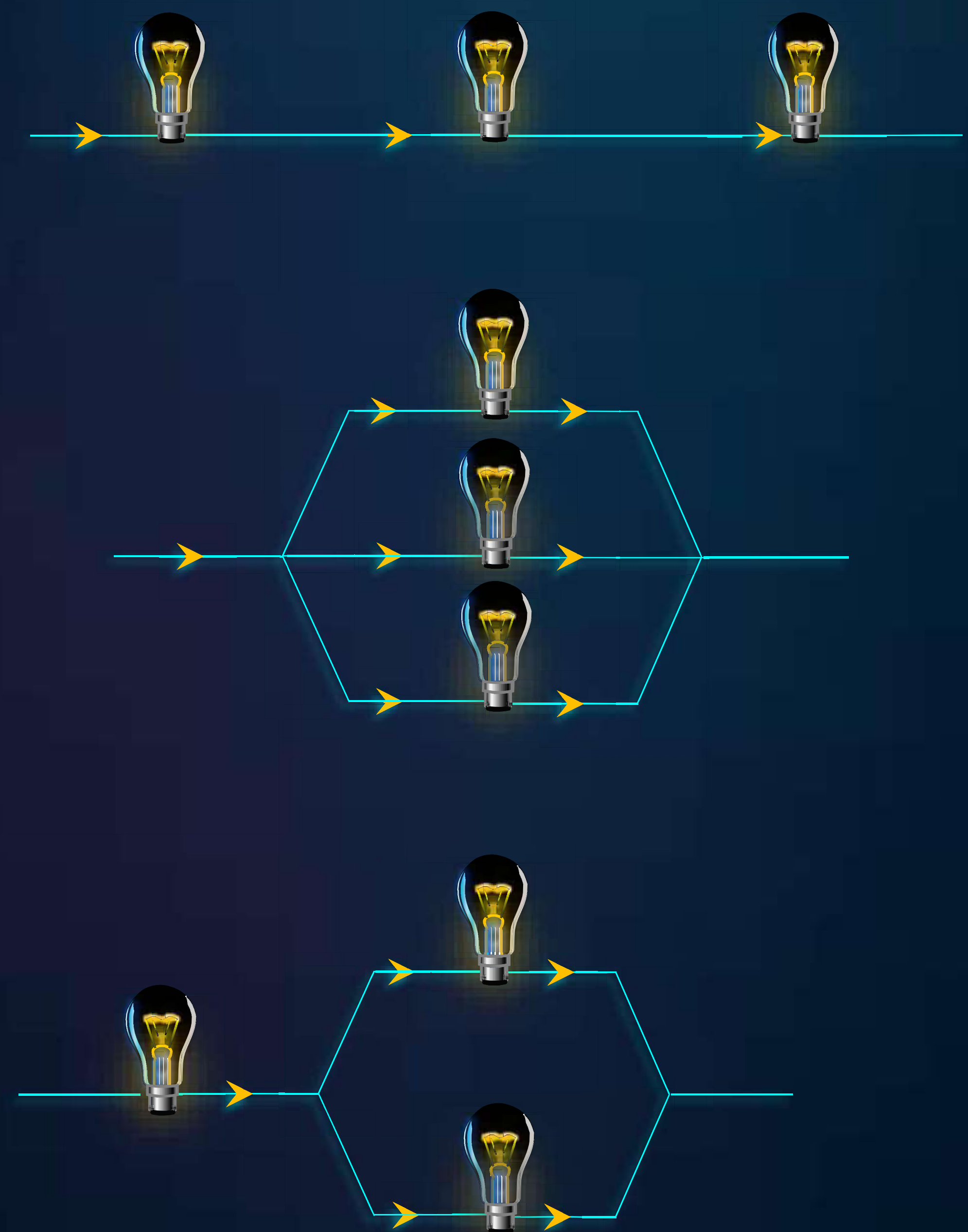
$$P = \frac{V^2}{R} = I^2R = VI$$

$$E = \frac{V^2}{R} \times t = I^2R \times t = VI \times t$$

$\boxed{\text{Brightness} \propto \text{Power}}$

Combination of Bulbs

- Series Combination
- Parallel combination
- Mixed combination



P, V

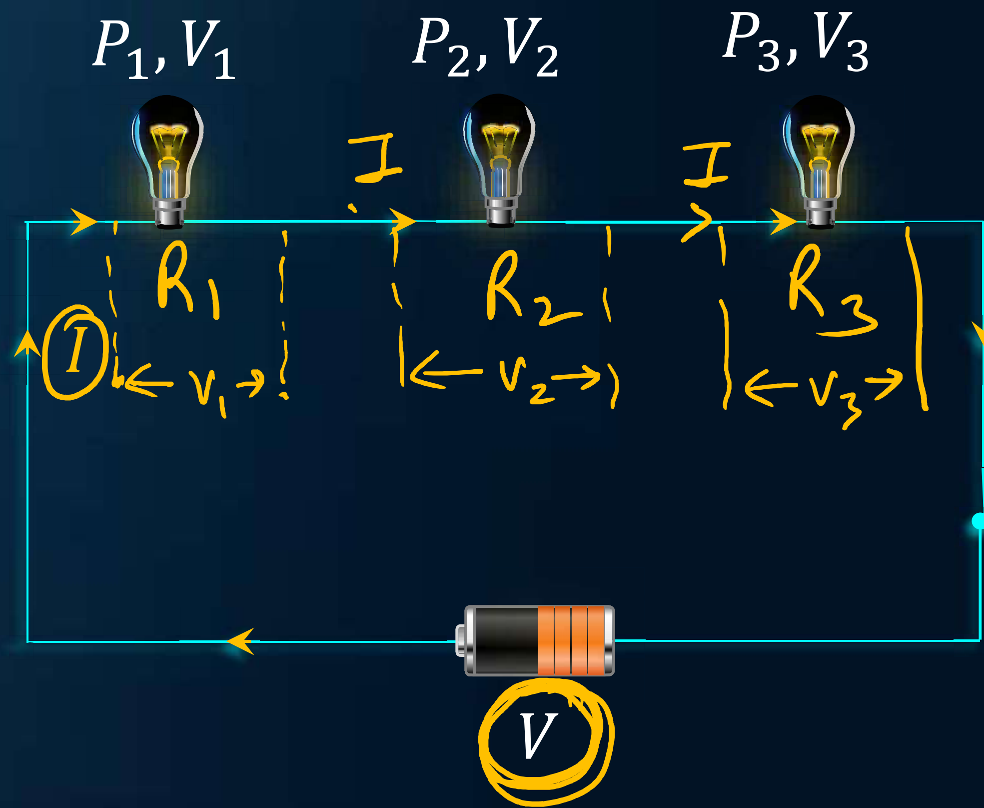
$$R = \frac{V^2}{P}$$

$$\uparrow P \propto B\uparrow$$

Combination of Bulbs

Series Combination

$$R_1 = \frac{V_1^2}{P_1} \; ; \; R_2 = \frac{V_2^2}{P_2} \; ; \; R_3 = \frac{V_3^2}{P_3}$$



$$P = \frac{V^2}{R} = \boxed{I^2 R} = VI$$

$$\uparrow P \propto R \uparrow$$

$$\uparrow P \propto B \uparrow$$

$$P_1 = I^2 R_1 \; ; \; P_2 = I^2 R_2$$

$$P_3 = I^2 R_3$$

$$P\uparrow \; R\uparrow \; B\uparrow$$

Combination of Bulbs

Series Combination

$$R_1 = \frac{V_1^2}{P_1} \quad\Big|\quad R_2 = \frac{V_2^2}{P_2} \quad\Big|\quad R_3 = \frac{V_3^2}{P_3}$$

$$P = \frac{V^2}{R} = I^2 R = VI$$

$$P \propto R \ \ (I = \text{Const.})$$

More resistance ⇒ More brighter bulb