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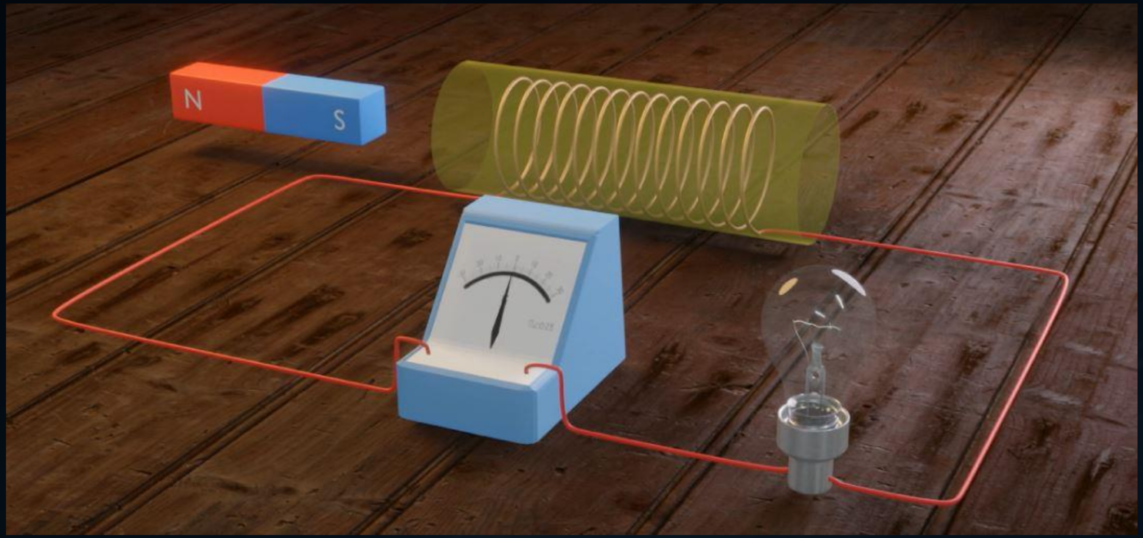
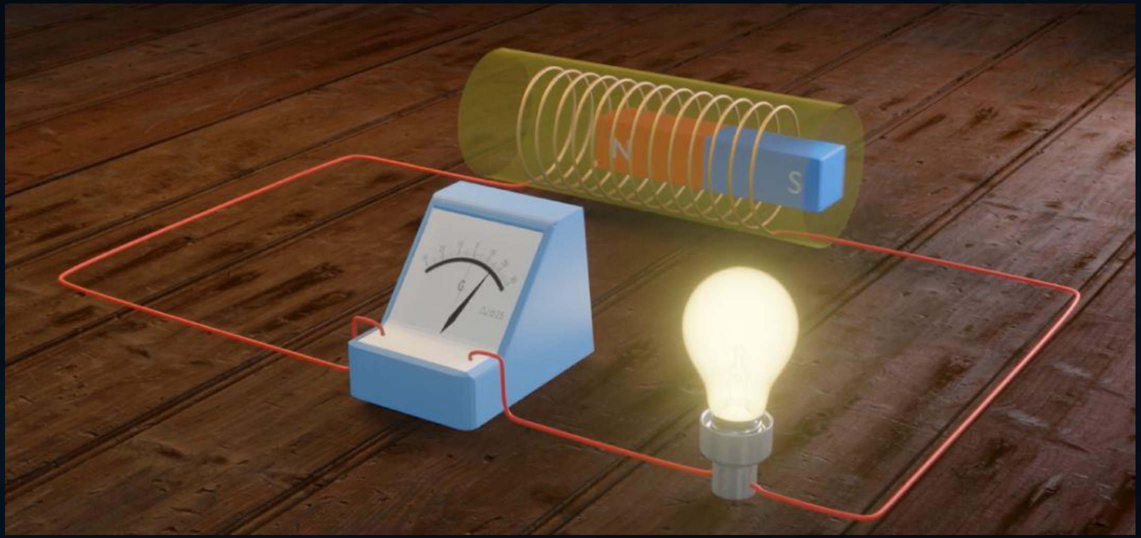
BYJU'S CLASSES

ELECTROMAGNETIC INDUCTION

Change in Magnetic Field → Induced Current





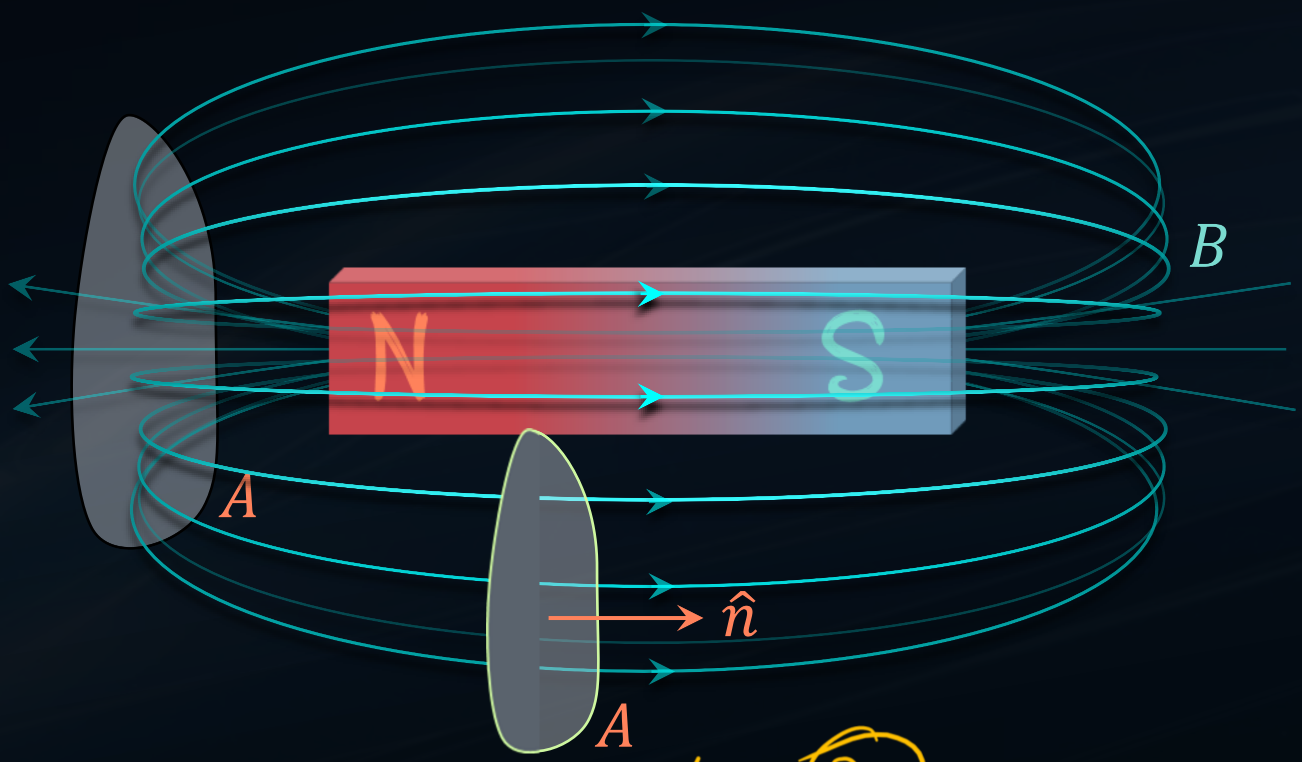
Faraday thought that if there's a magnetic field due to electric current then the reverse will also be true. He demonstrated through his experiments that electric current can be generated through change in magnetic field.

He observed that, the relative motion between magnet and conductor coil causes the current flow through the coil. He also observed that the direction of current also changes depending on the direction of the motion between the magnet and the coil.

MAGNETIC FLUX ϕ_B

The number of magnetic field lines passing normally through a given surface.

$\phi_B \propto B$



$\phi_B = 4$

$\phi_B = 3$

ϕ_B depends on:

$\boxed{\phi_B \propto B}$ $\boxed{\phi_B \propto A}$

MAGNETIC FLUX

$\boxed{\phi_B \propto B}$ $\boxed{\phi_B \propto A}$

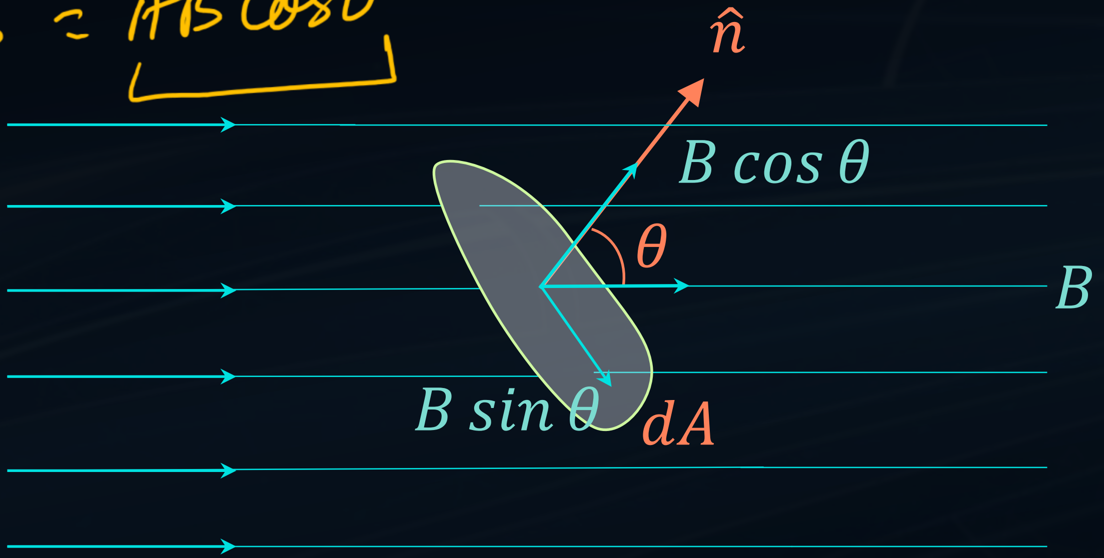
$$\phi_B = |\vec{B}| \cos\theta \, |\vec{A}|$$

$$\boxed{\phi_B = \vec{B}.\vec{A}}$$

SI Unit : Weber (Wb)

$$\boxed{\phi_B = \int B dA \cos\theta}$$

$\vec{A} \cdot \vec{B} = \underbrace{AB\cos\theta}$



$\phi_B = B\cos\theta \cdot A$

$\phi_B = BA\cos\theta$

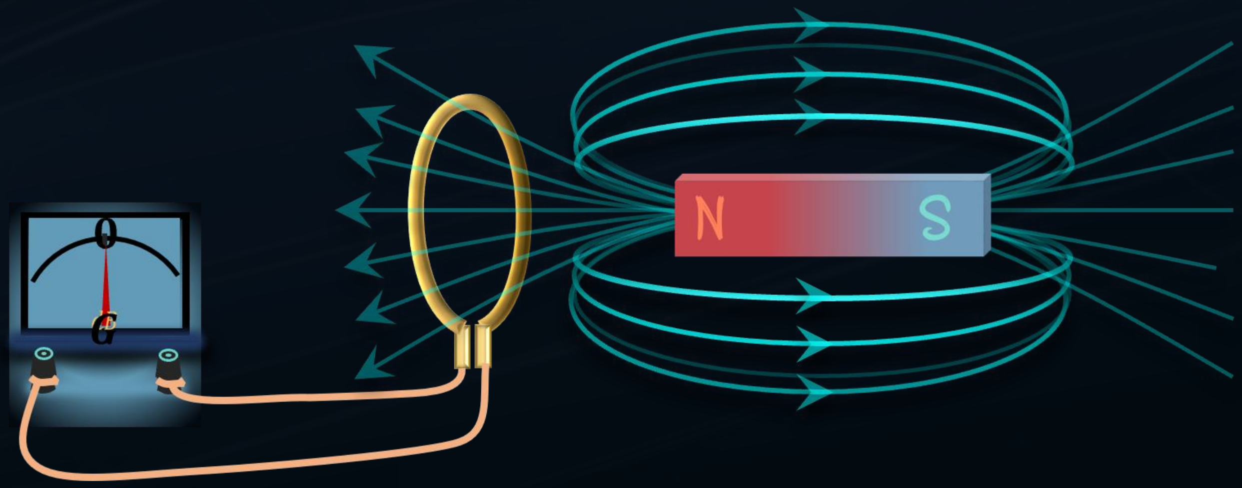
$\phi_B = \underset{\downarrow \, T}{\vec{B}} \cdot \underset{\downarrow \, m^2}{\vec{A}}$

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Faraday's experimental observations

Varying magnetic field strength

$\vec{B} = \vec{B}(t)$

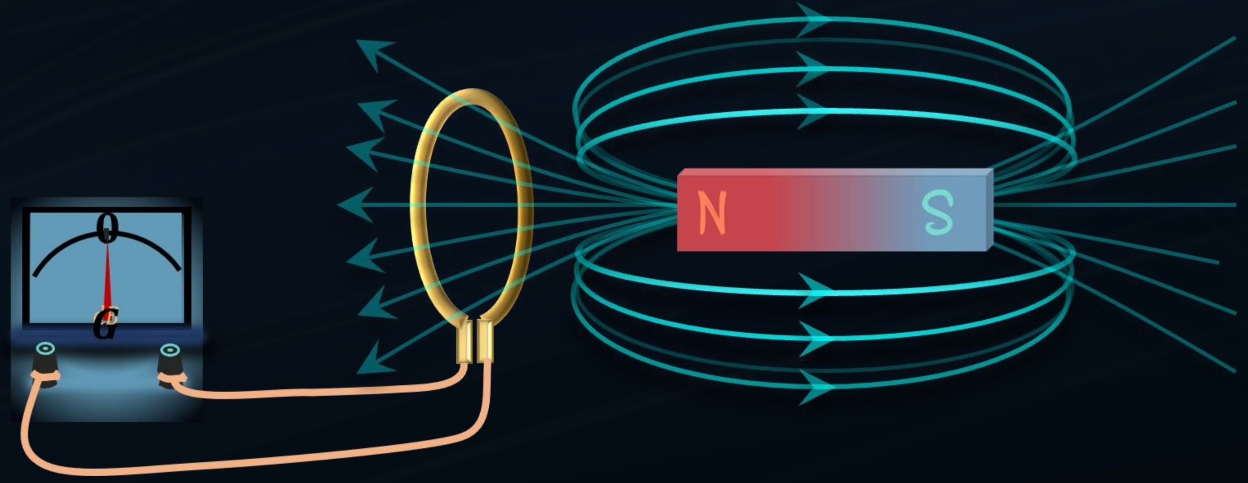


As we vary the magnetic field, the number of field lines passing through the coil (flux) also changes. Hence, current will get induced if we vary the magnetic field.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Faraday's experimental observations

Varying area of coil



If the area of coil is varied with time, the number of field lines passing through the coil (flux) also change. Hence, current will get induced if we vary the area of the loop/coil.

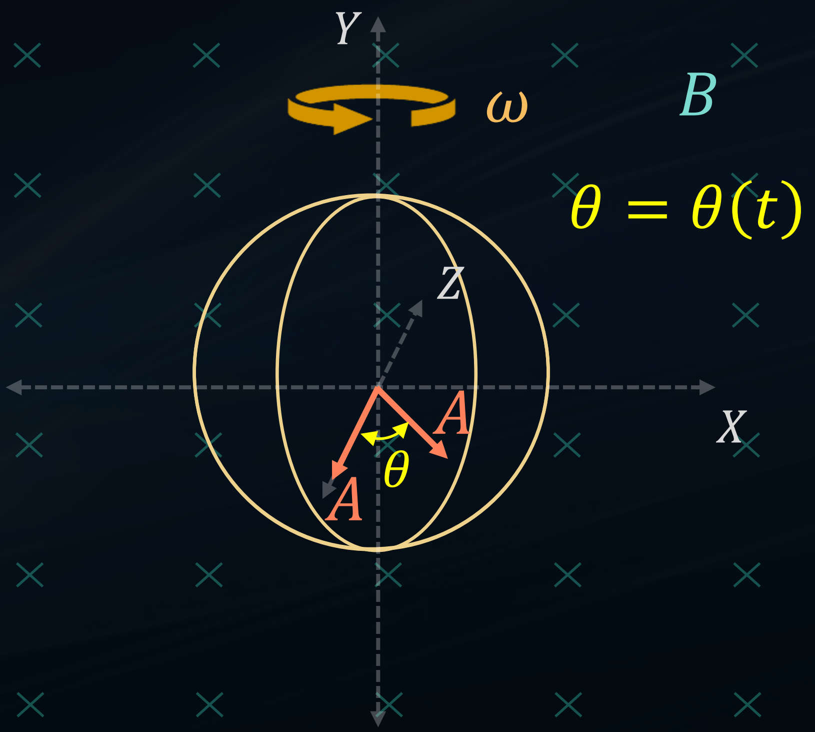
FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Faraday's experimental observations

Rotating coil

$$\phi_B = BA\cos\theta$$

Flux through the coil changes with time.



If we rotate the coil in magnetic field, the net area that is exposed to the magnetic field at particular angle will also vary. This will cause change in flux through the coil.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Faraday's First Law

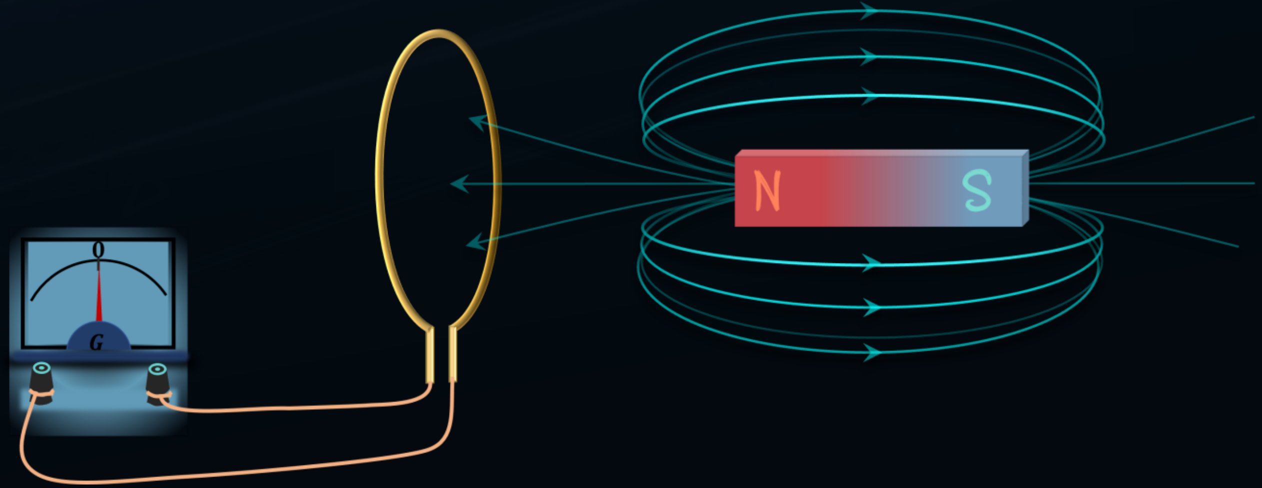
Whenever there is a change in magnetic flux linked with a conductor, an emf is induced in conductor. If it is a closed circuit, induced current will flow through it.

Faraday's Second Law

The magnitude of the induced emf in a conducting coil is proportional to the rate at which the magnetic flux through that coil changes with time.

$$\varepsilon \propto \left|\frac{\Delta\phi_B}{\Delta t}\right|$$

$$\varepsilon \propto \left|\frac{\Delta\phi_B}{\Delta t}\right|$$



FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

$$\varepsilon \propto \left|\frac{\Delta\phi_B}{\Delta t}\right|$$

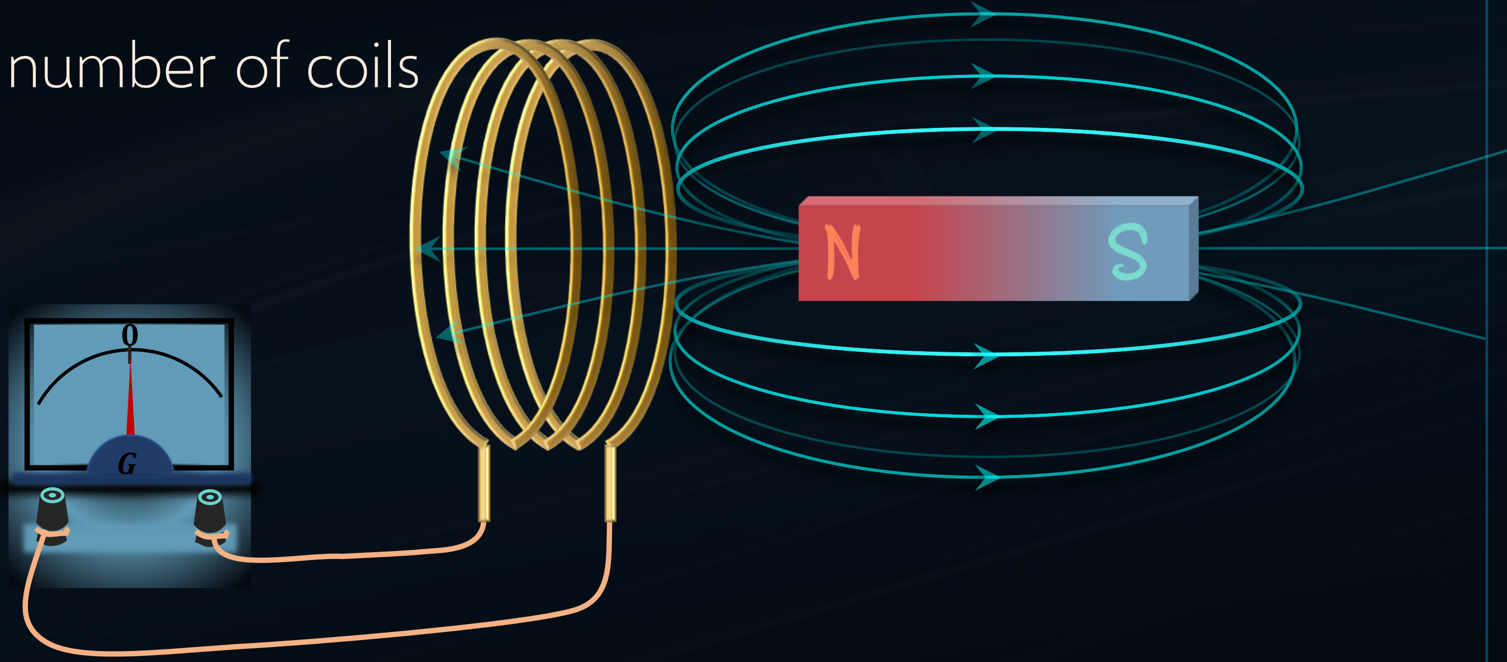
Average emf (ε_{Avg})

$$i_{avg} = \frac{\varepsilon_{Avg}}{r} = N\left|\frac{\Delta\phi_B}{\Delta t}\right| \times \frac{1}{r}$$

Instantaneous emf (ε_{Ins})

$$\varepsilon_{Ins} = N\left|\frac{d\phi_B}{dt}\right|$$

N number of coils



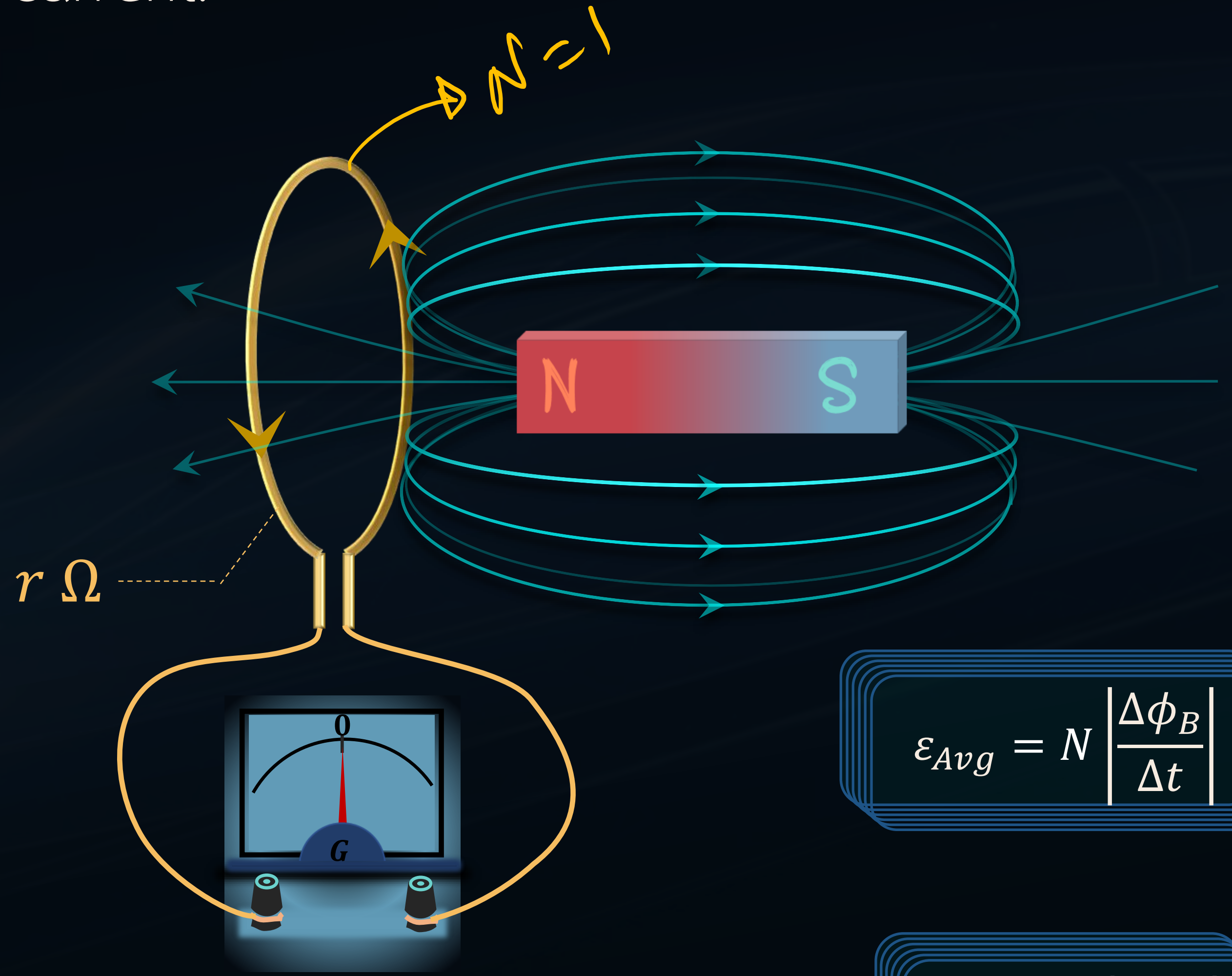
$$i_{in} = \frac{\varepsilon_{in}}{r}$$

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Magnitude of Induced current:

$$i_{Avg} = \frac{\varepsilon_{Avg}}{r} = \frac{N\left|\frac{\Delta\phi_B}{\Delta t}\right|}{r}$$

$$i_{Ins} = \frac{\varepsilon_{Ins}}{r} = \frac{N\left|\frac{d\phi_B}{dt}\right|}{r}$$



$$\varepsilon_{Avg} = N\left|\frac{\Delta\phi_B}{\Delta t}\right|$$

$$\varepsilon_{Ins} = N\left|\frac{d\phi_B}{dt}\right|$$

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Total charge flow $\longrightarrow \Delta q$

$$i = \frac{dq}{dt}$$

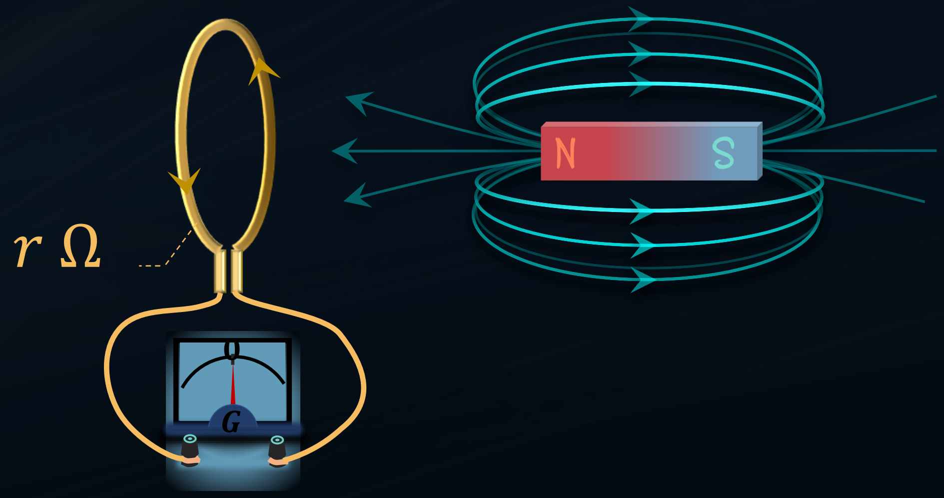
(i) $\frac{\Delta q}{\Delta t} \longrightarrow \frac{dq}{dt}$

$$\boxed{\varepsilon_{Avg} = N\left|\frac{\Delta\phi_B}{\Delta t}\right|} \qquad \boxed{\varepsilon_{Ins} = N\left|\frac{d\phi_B}{dt}\right|}$$

$$i_{Avg} = \frac{\varepsilon_{Avg}}{r} = \frac{N\left|\frac{\Delta\phi_B}{\Delta t}\right|}{r} \qquad i_{Ins} = \frac{\varepsilon_{Ins}}{r} = \frac{N\left|\frac{d\phi_B}{dt}\right|}{r}$$

$$i = N\frac{\left|\frac{d\phi_B}{dt}\right|}{r} = \frac{dq}{dt}$$

$$\boxed{\Delta q = N\frac{\Delta\phi_B}{r}}$$



$$\frac{\Delta q}{\cancel{\Delta t}} = \frac{N}{r}\left|\frac{\Delta\phi_B}{\cancel{\Delta t}}\right|$$

$$\Delta q = \frac{N\Delta\phi_B}{r}$$

$$dq = \frac{N\,d\phi_B}{r}$$

A coil of resistance $400\ \Omega$ is placed in a magnetic field. If the magnetic flux $\phi\ (Wb)$ linked with the coil varies with time $t(s)$ as $\phi = 50t^2 + 4$, the current in the coil at $t = 2\ s$ is.......

a. $0.5\ A$

b. $0.1\ A$

c. $2\ A$

d. $1\ A$

$r = 400\ \Omega$

$\phi(t) = 50t^2 + 4$

at $t = 2\ s$

$i = ?$

Solution

NEET

Given : $\phi = 50t^2 + 4$

$r = 400\ \Omega$

$t = 2\ s$

$$\left(\varepsilon_{Ins} = N\left|\frac{d\phi_B}{dt}\right|\right)$$

$$\varepsilon_{Ins} = \left|\frac{d\phi_B}{dt}\right| = \left|\frac{d(50t^2+4)}{dt}\right| = 100t$$

$$\frac{d}{dt}t^n \rightarrow nt^{n-1}$$

$$i_{Ins} = \frac{\varepsilon_{Ins}}{r} = \frac{100t}{r}$$

$$i_{Ins} = \frac{100 \times 2}{400} = 0.5\ A$$

Thus, option a is the correct answer.

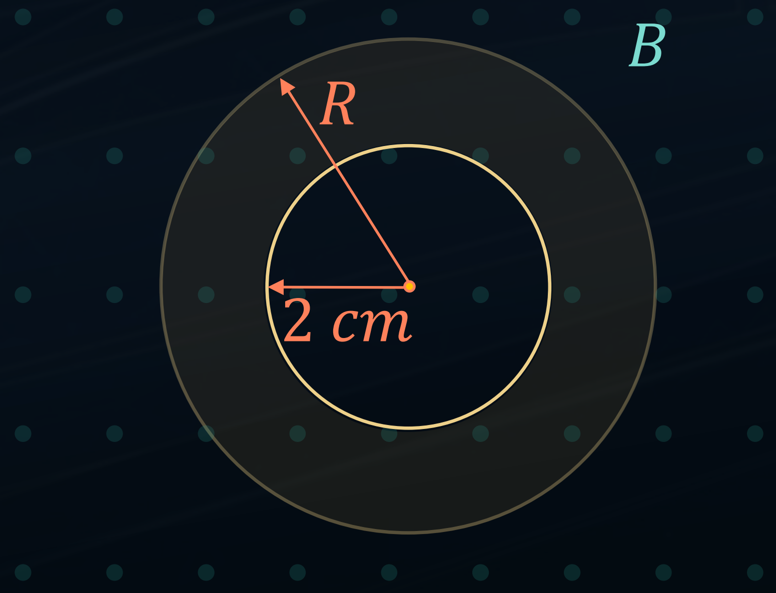
A conducting circular loop is placed in uniform magnetic field, $B = 0.025\ T$ with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of $1\ mm/s$. The induced emf when radius is $2\ cm$, is

a) $2\ \mu V$

b) $2\pi\ \mu V$

c) $\pi\ \mu V$

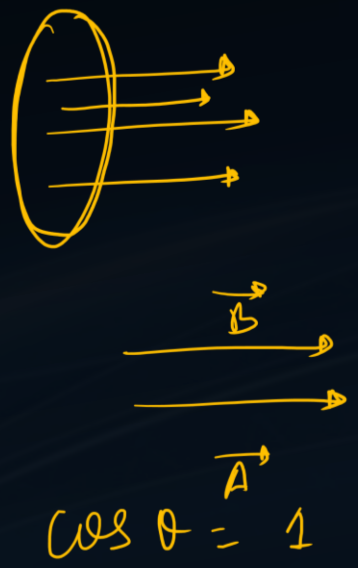
d) $\frac{\pi}{2}\ \mu V$

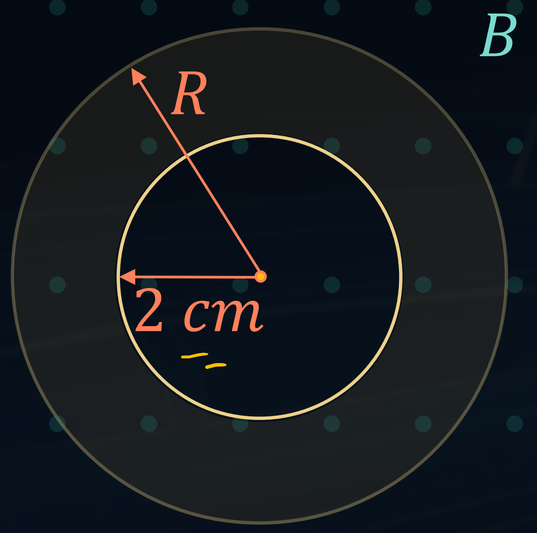


Solution

Given : $B = 0.025\,T$

$$\frac{dR}{dt} = 1\,mm/s$$





$$\phi_B = BA\cos 0^\circ = BA$$

$$\phi_B = B \times \pi R^2$$

$$\varepsilon_{Ins} = \left|\frac{d\phi_B}{dt}\right| = \frac{d(B \times \pi R^2)}{dt} = B(2\pi R)\left(\frac{dR}{dt}\right)$$

B R dR/dt

$$\varepsilon_{Ins} = B(2\pi R)\left(\frac{dR}{dt}\right) = 0.025 \times 2\pi \times 0.02 \times 10^{-3}\,V$$

$$\varepsilon_{Ins} = \pi \times 10^{-6}\,V = \pi\,\mu V$$

Thus, option c is the correct answer.

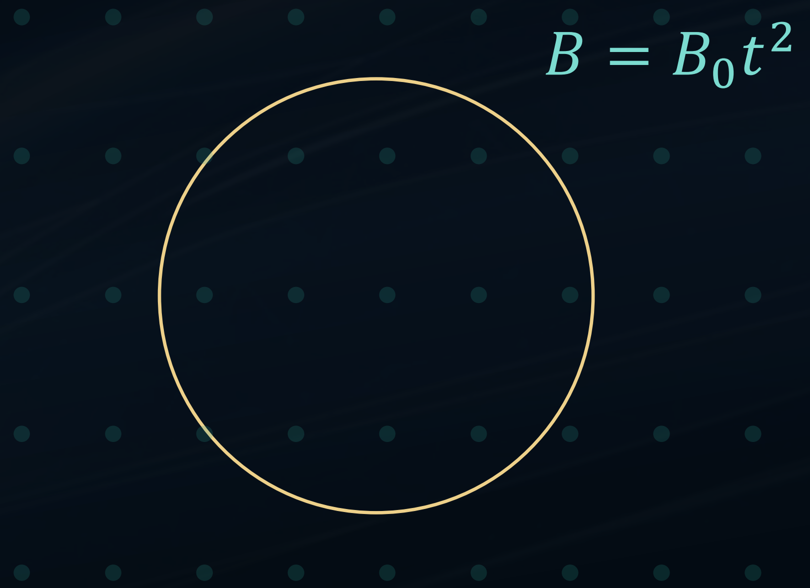
A coil of radius R and resistance r kept in a magnetic field $B = B_0 t^2$ as shown in figure. Find total charge that flows from $t = 0$ to $t = t_0$.

a) $\dfrac{B_0 \pi R^2 t_0^2}{r}$

b) $\dfrac{B_0 \pi R^2 t_0}{r}$

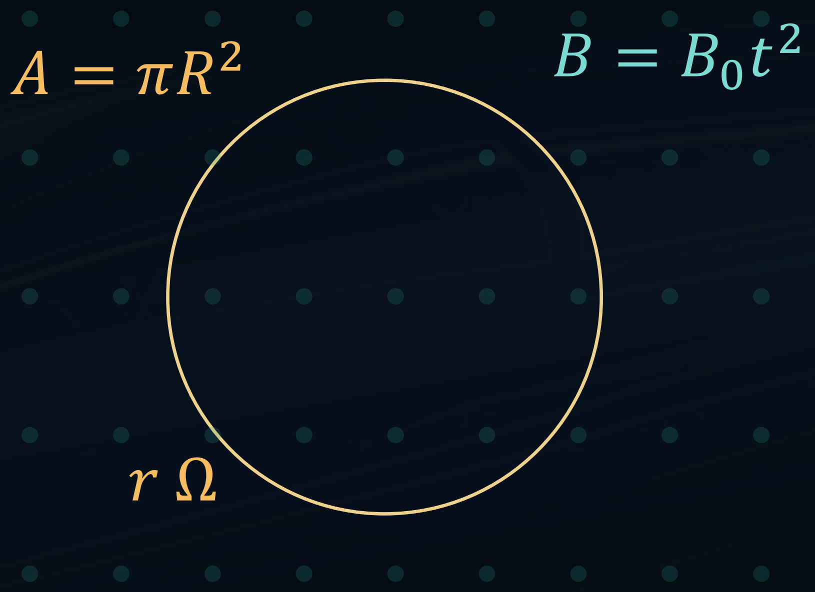
c) $\dfrac{B_0 2 t_0 \pi R^2}{r}$

d) $\dfrac{B_0 \pi R^2 t_0^2}{2r}$



Solution

$$\phi_B = BA\cos 0° = B_0 t^2 \pi R^2$$



$$\Delta q = \frac{\Delta \phi_B}{r}$$

$$\Delta q = \frac{\phi_{B_{t_0}} - \phi_{B_0}}{r}$$

$$\Delta q = \frac{B_0 t_0^2 \pi R^2 - 0}{r}$$

$$\Delta q = \frac{B_0 \pi R^2 t_0^2}{r}$$

Thus, option a is the correct answer.

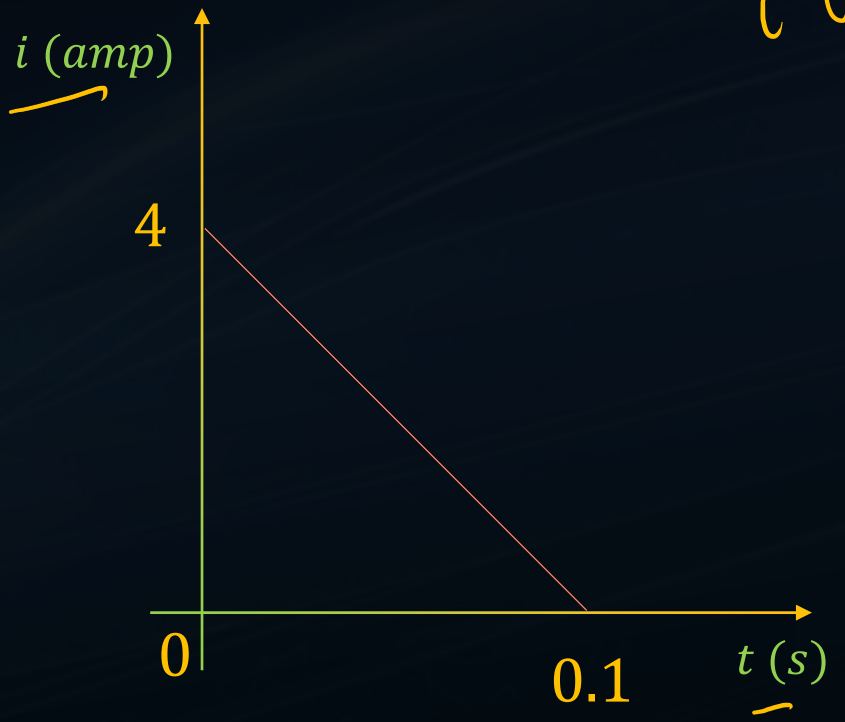
In a coil of resistance $10\ \Omega$, the induced current developed by changing magnetic flux through it is shown in figure as a function of time. The magnitude of flux through the coil in Weber is......

a) 8

b) 2

c) 6

d) 4



$\phi_B = ?$

i vs t

$i - t$

Solution

Given : Resistance $(r = 10\ \Omega)$

$$\Delta q = \frac{\Delta\phi_B}{r}$$

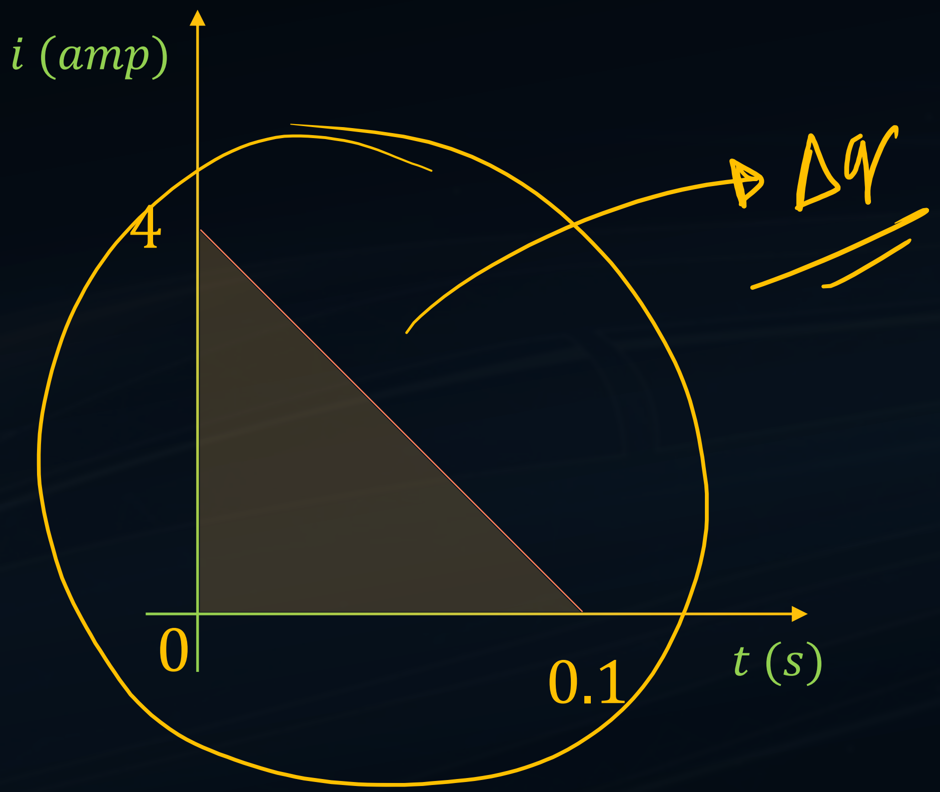
$\Delta q =$ Area under $i - t$ graph

Area under $i - t$ graph $= \frac{1}{2} \times 4 \times 0.1 = 0.2\ C$

$$\Delta\phi_B = r\Delta q$$

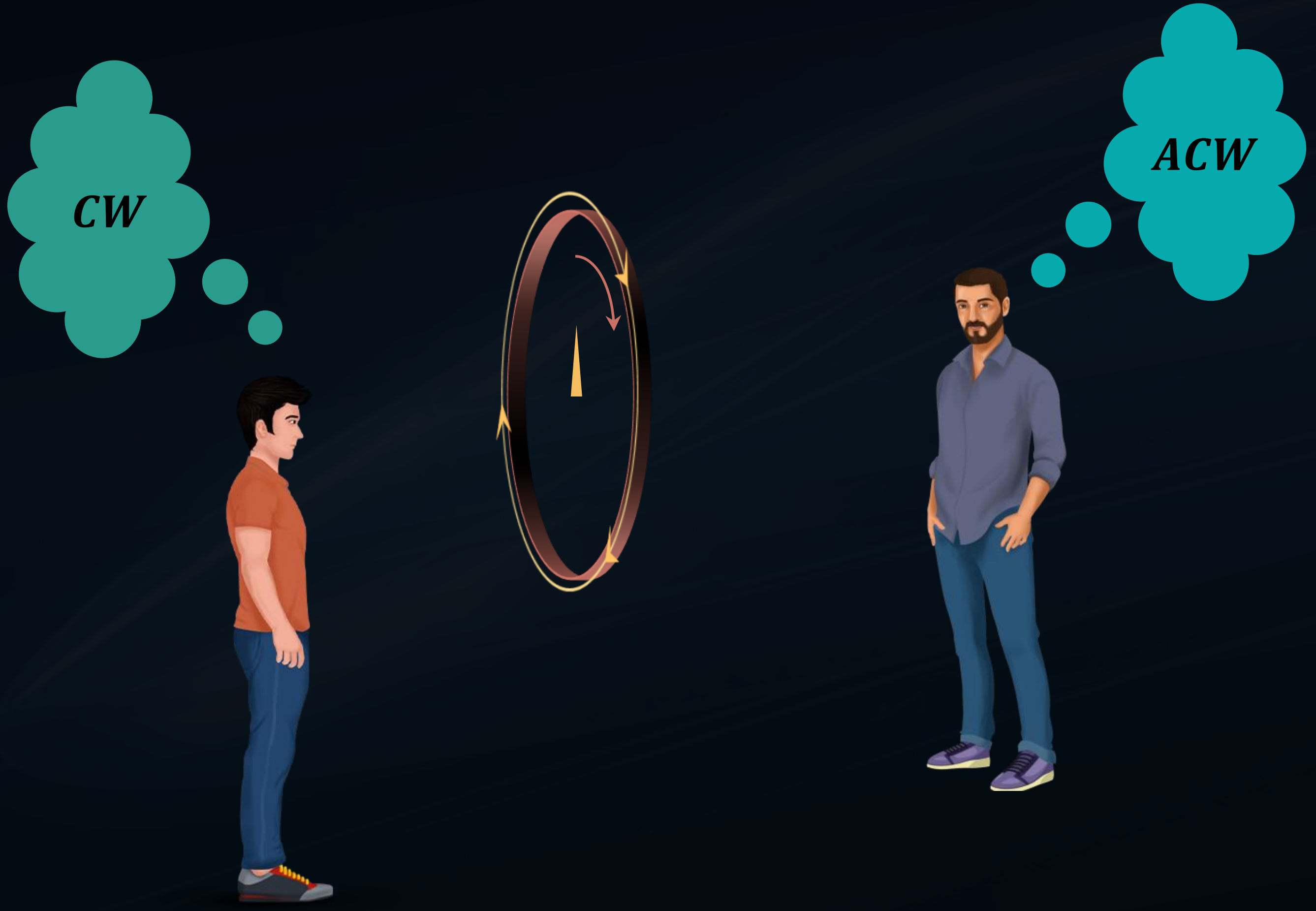
$$\Delta\phi_B = 10 \times 0.2 = 2 \text{ Weber}$$

Thus, option b is the correct answer.



The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

$$\varepsilon = -N\frac{d\phi_B}{dt}$$



effect
cause

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

opposition

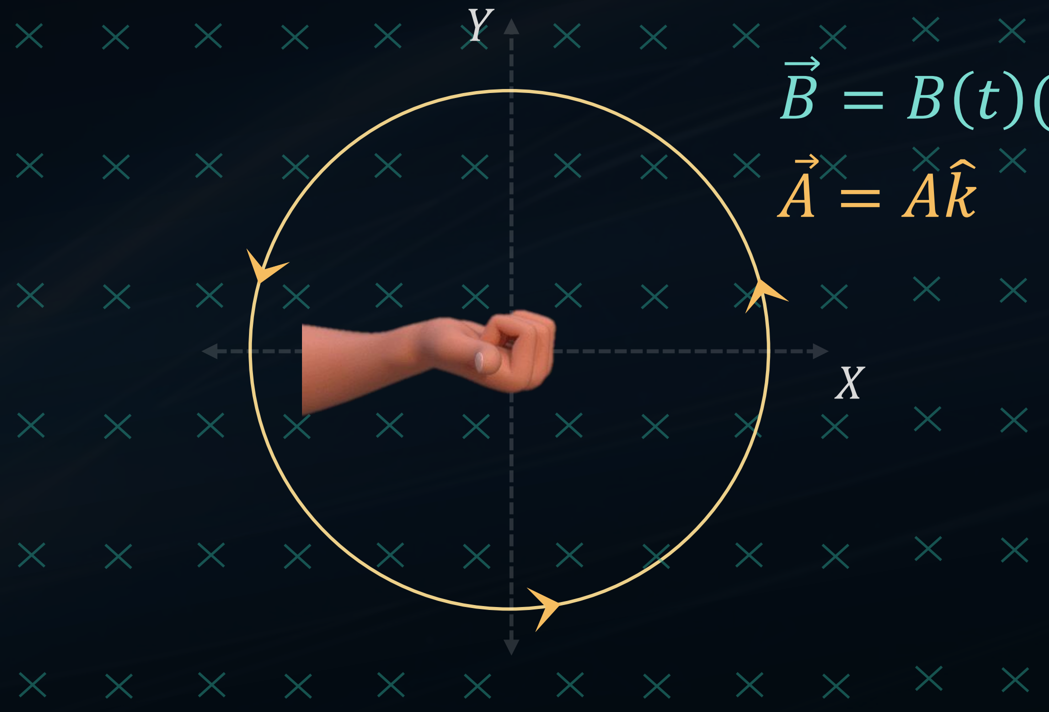
Relative motion of magnet and coil

LENZ'S LAW

Coil in changing magnetic field

Cause : Increase of $\vec{B}$ in inward direction

$$\varepsilon = -N\frac{d\phi_B}{dt}$$



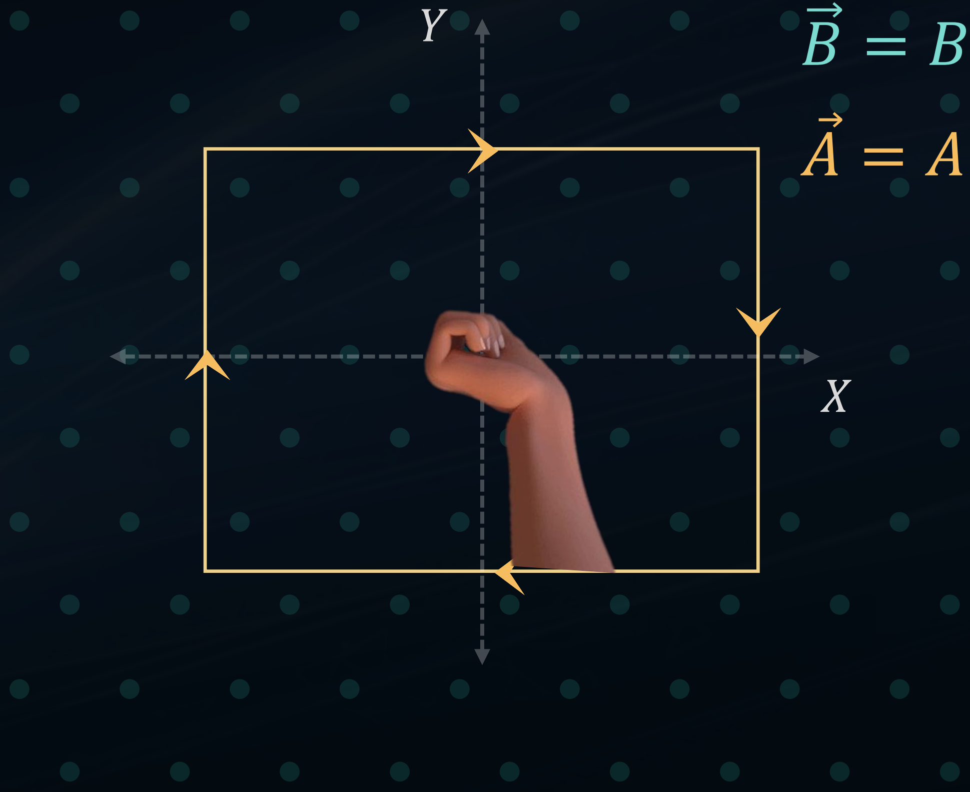
$$\vec{B} = B(t)(-\hat{k})$$

$$\vec{A} = A\hat{k}$$

Coil in changing magnetic field

Cause : Increase of $\vec{B}$ in outward direction

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

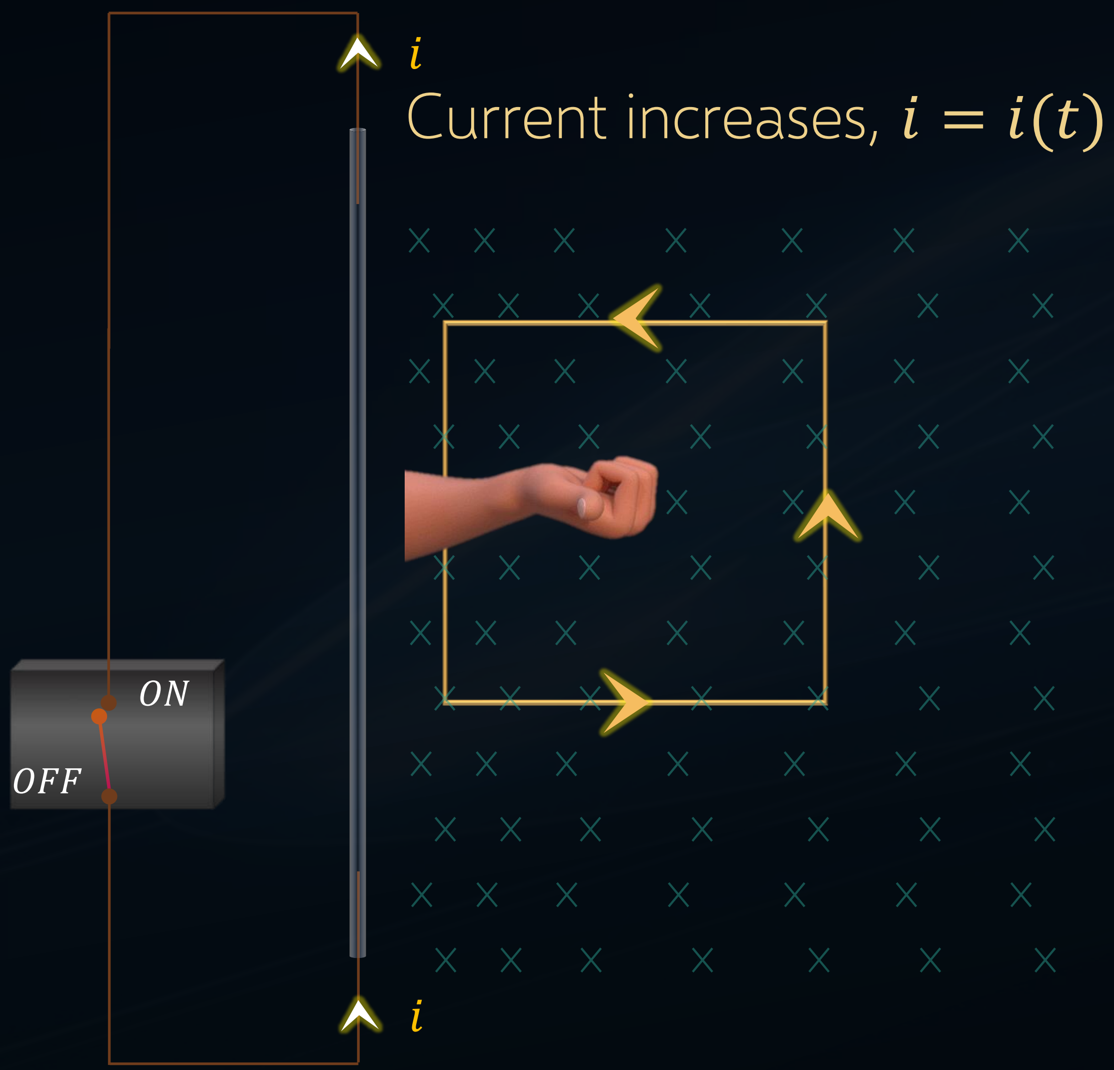


$\vec{B} = B(t)\hat{k}$

$\vec{A} = A(-\hat{k})$

LENZ'S LAW

Current carrying wire and coil



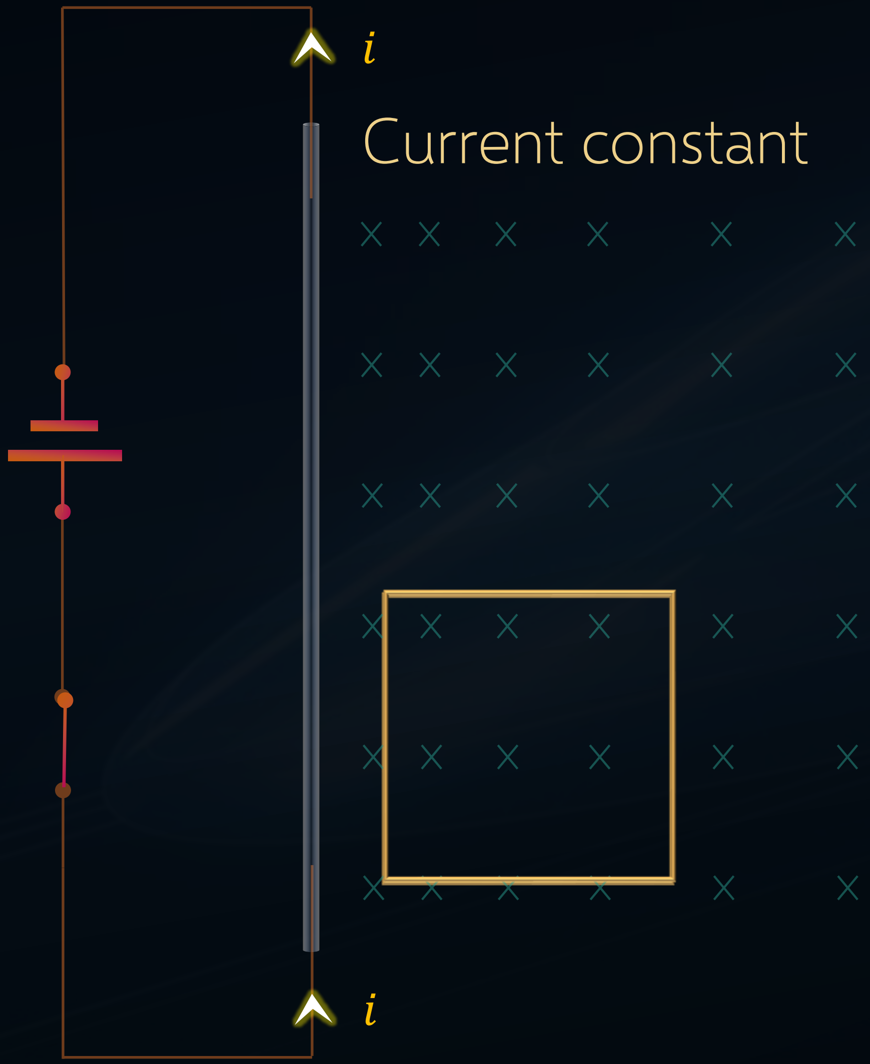
Flux inside the coil increases with time.

LENZ'S LAW

Current carrying wire and coil

Flux inside the coil decreases when coil move away from current.

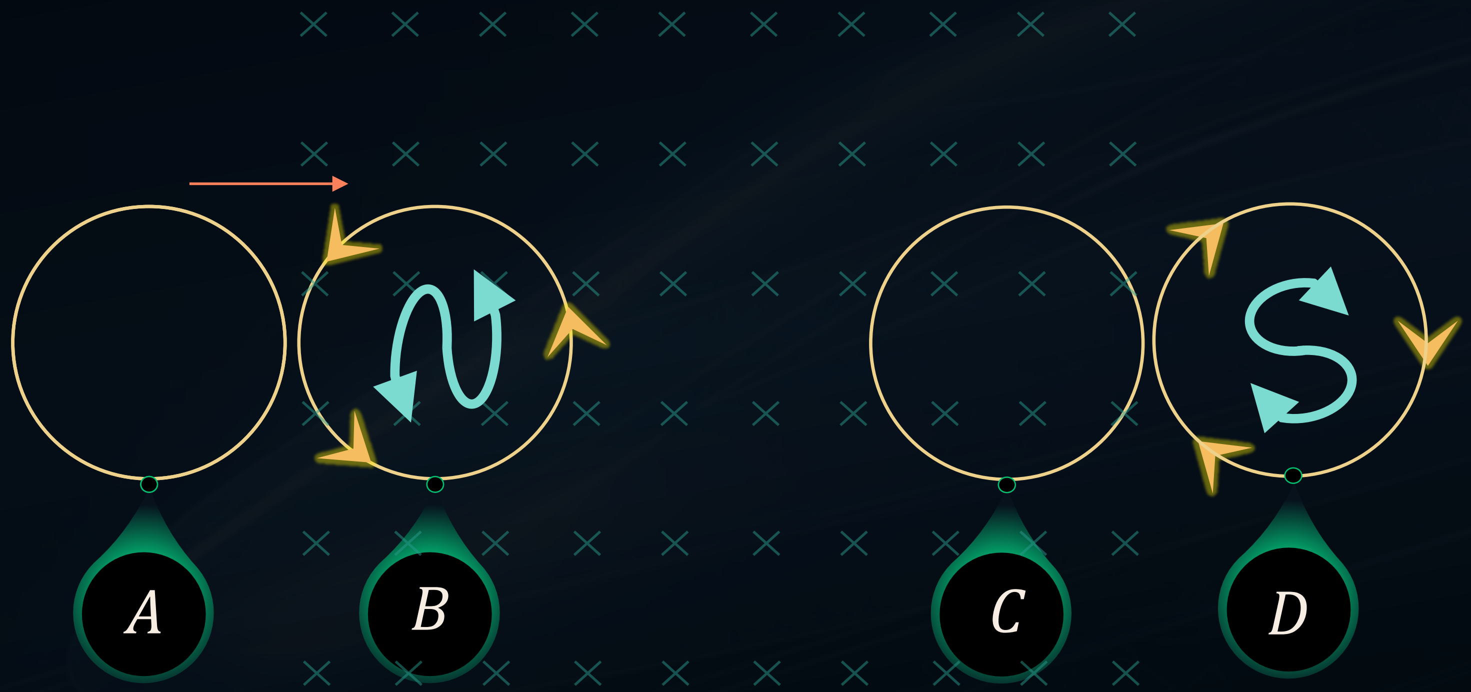
Current carrying wire and coil



Flux inside the coil is constant when it move parallel to current carrying wire.

LENZ'S LAW

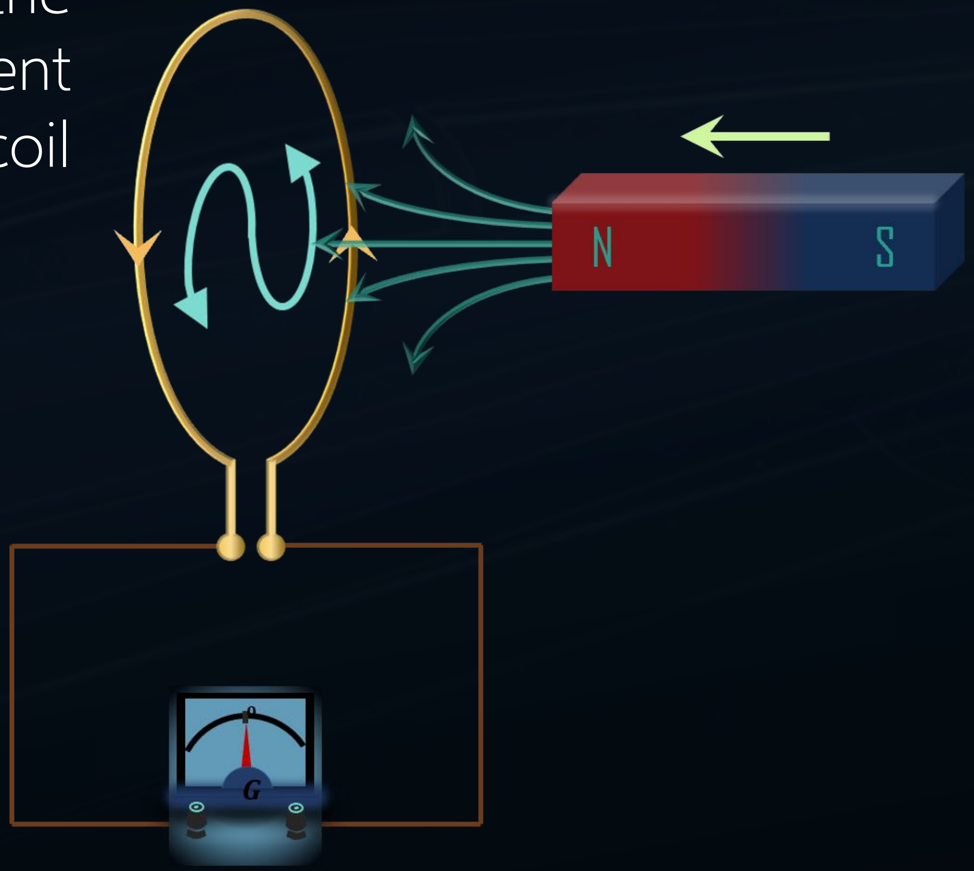
Coil moving in a magnetic field.



At	ϕ	i
A	0	0
A to B	increase	ACW
B to C	constant	0
C to D	decrease	CW

Lenz's law as a statement of conservation of energy

EMF generated in the coil will always try to resist the change in magnetic flux. The mechanical energy spent on the relative motion between the magnet and coil gets converted into the electrical energy.



Thus, Lenz's law is in accordance with conservation of energy

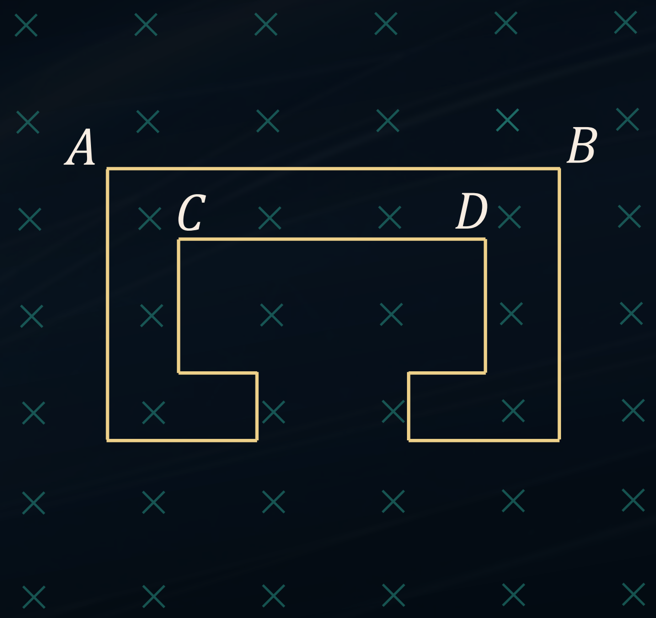
A wire is bent to form the double loop $ABDCA$. There is a time dependent magnetic field directed into the plane of the loop. If the magnitude of this field is decreasing, current will flow from

a) A to B and C to D

b) B to A and D to C

c) A to B and D to C

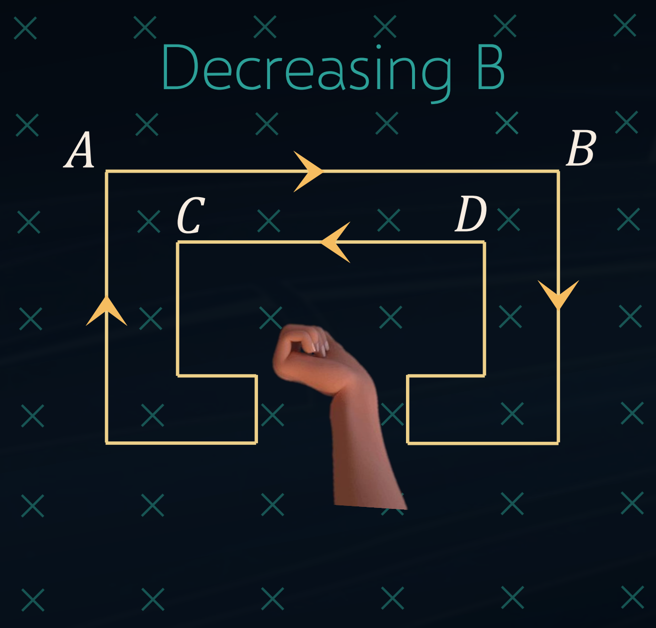
d) B to A and C to D



Solution

In this case, as the magnetic field going inside the plane is decreasing, the emf will generate in such a way that the will generate magnetic field going inside the loop to counter the decreasing magnetic field.

As the magnetic field is going inside the plane using right hand rule we can find that the current will flow from A to B and then D to C. The generated emf/current will always try to resist the change in magnetic flux.



$A \to B \to D \to C$

A to B and D to C

Thus, option c is the correct answer.

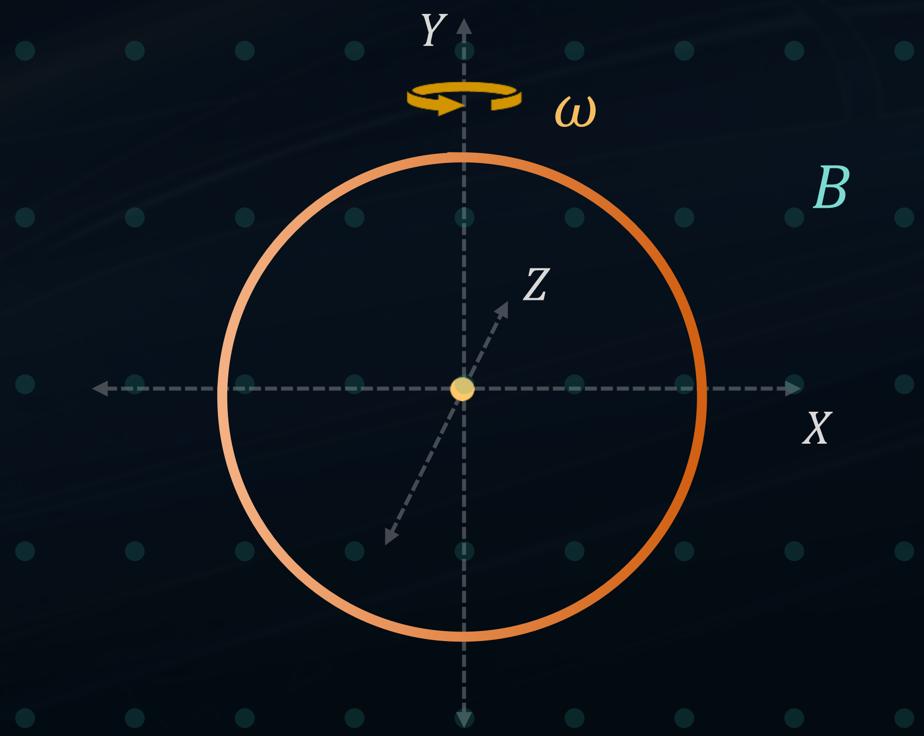
Find the average induced current and it's direction in the coil shown in figure, if it rotates $60°$ in $0.2\ sec$. Radius and resistance of coil are R and r respectively.

a) $\dfrac{5B\pi R^2}{2r}$ and ACW

b) $\dfrac{2B\pi R^2}{5r}$ and CW

c) $\dfrac{5B\pi R^2}{4r}$ and ACW

d) $\dfrac{5B\pi R^2}{2r}$ and CW



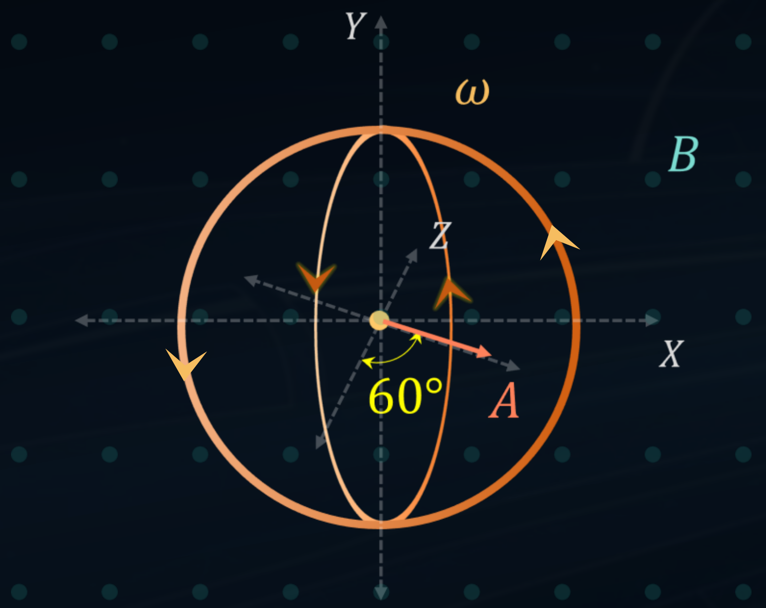
Solution

NEET

$$\phi_B = \vec{B}.\vec{A}$$

$$\phi_{B_i} = BA\cos 0^\circ = B\pi R^2$$

$$\phi_{B_f} = BA\cos 60^\circ = \frac{B\pi R^2}{2}$$



$$|\varepsilon| = \left|\frac{\Delta\phi}{\Delta t}\right| = \left|\frac{\phi_{B_f} - \phi_{B_i}}{\Delta t}\right| \Rightarrow |\varepsilon| = \left|\frac{\frac{B\pi R^2}{2} - B\pi R^2}{0.2}\right| = \frac{B\pi R^2}{2\times 0.2} = \frac{5B\pi R^2}{2}$$

$$i = \frac{\varepsilon}{r} \Rightarrow i = \frac{5B\pi R^2}{2r}\ ACW$$

Thus, option a is the correct answer.

A wire loop is rotated in a magnetic field. The frequency of change of direction of the induced emf is

a once per revolution

b twice per revolution

c four times per revolution

d six times per revolution

Summary NEET

Let in a time $'t'$ the coil rotates by an angle θ.

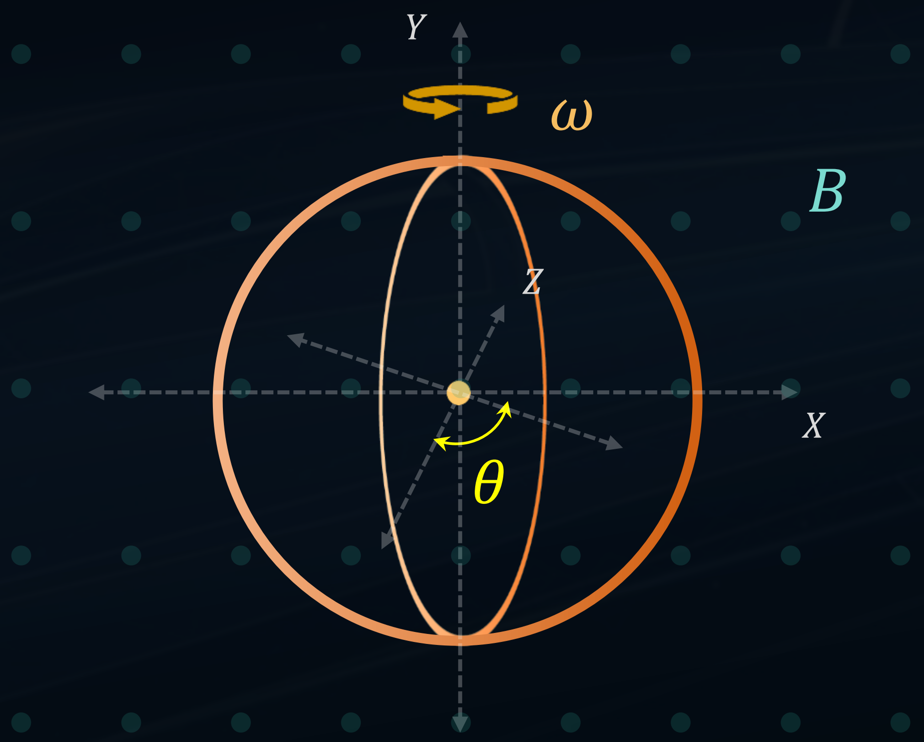
$$\phi_B = BA\cos\theta$$

$$\phi_B = BA\cos\omega t \qquad (\theta = \omega t)$$

$$\varepsilon = -\frac{d(BA\cos\omega t)}{dt} \qquad \left(\varepsilon = -N\frac{d\phi_B}{dt}\right)$$

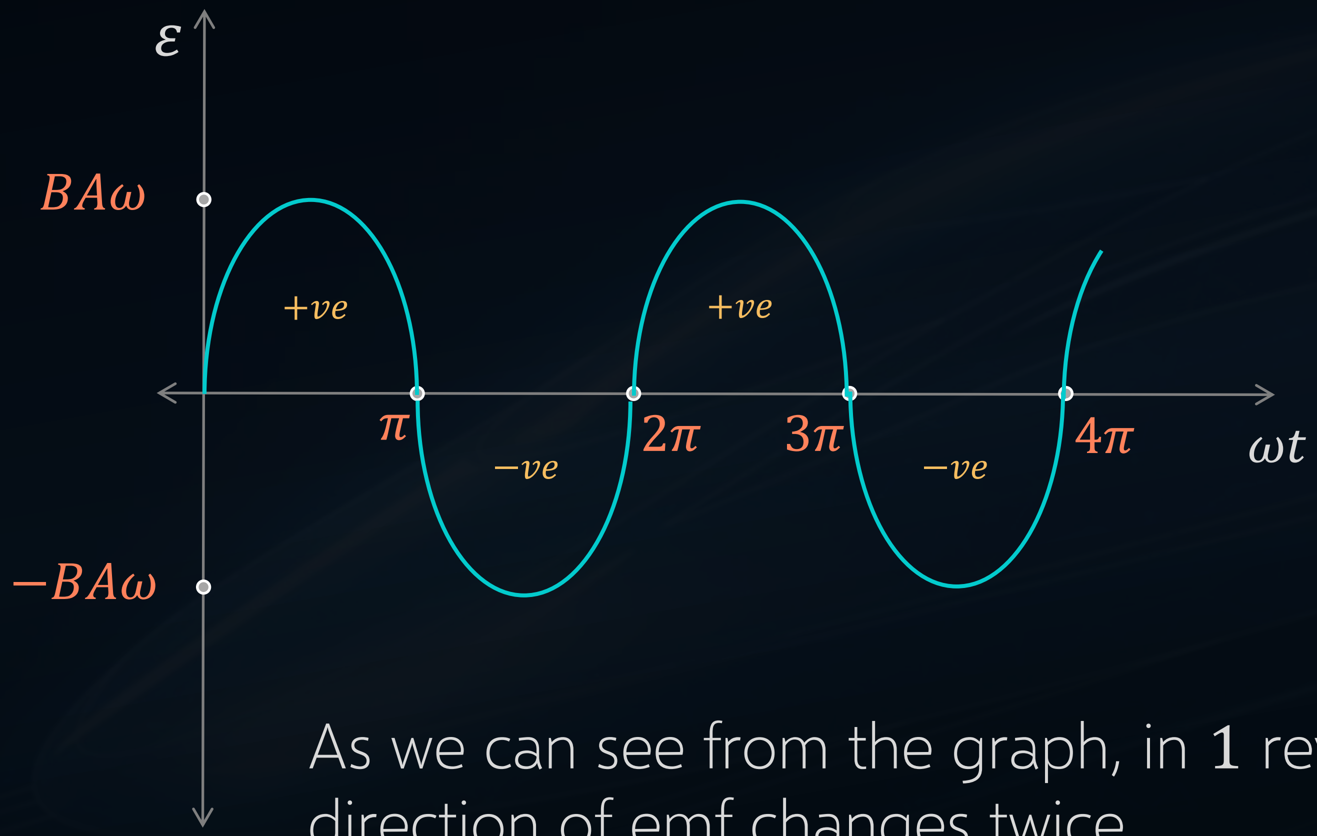
$$\varepsilon = -BA\frac{d(\cos\omega t)}{dt} = -BA(-\omega\sin\omega t)$$

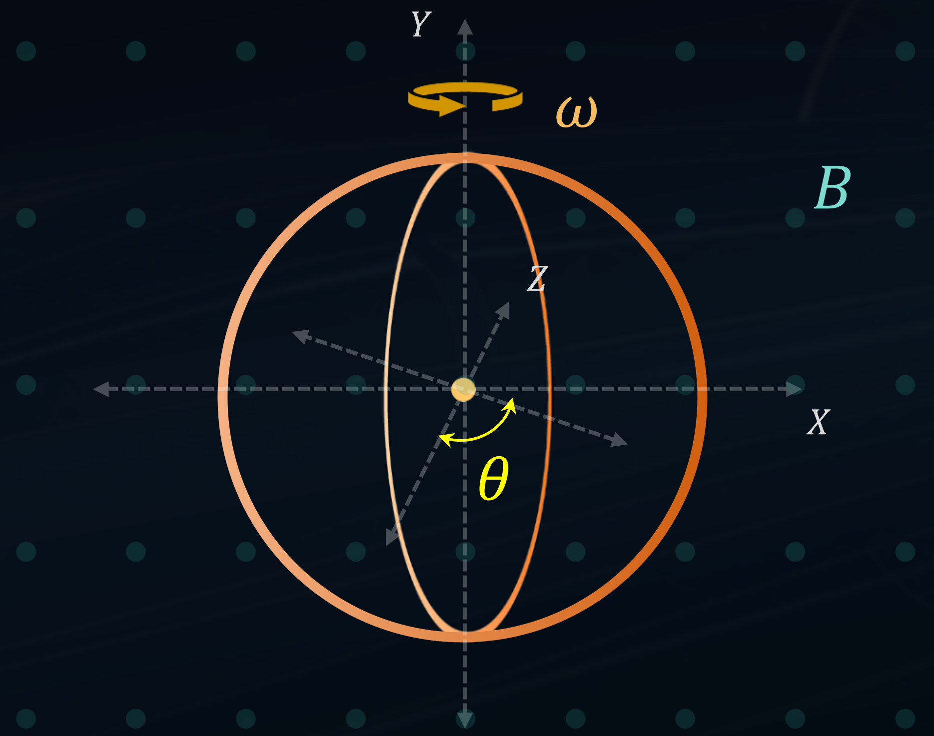
$$\varepsilon = BA\omega\sin\omega t$$



Solution

$$\varepsilon = BA\omega \sin \omega t$$





As we can see from the graph, in **1** revolution i.e., **360°** rotation direction of emf changes twice.

Thus, option b is the correct answer.

The magnetic field B is directed into the plane of the paper. ACDA is a semicircular conducting loop of radius R with the centre at O. The loop is now made to rotate clockwise with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The resistance of the loop is r. Obtain an expression for the magnitude of the induced current in the loop. Plot a graph between the induced current i and ωt, for two periods of rotation.

Summary

Given : Radius (R), Resistance (r)

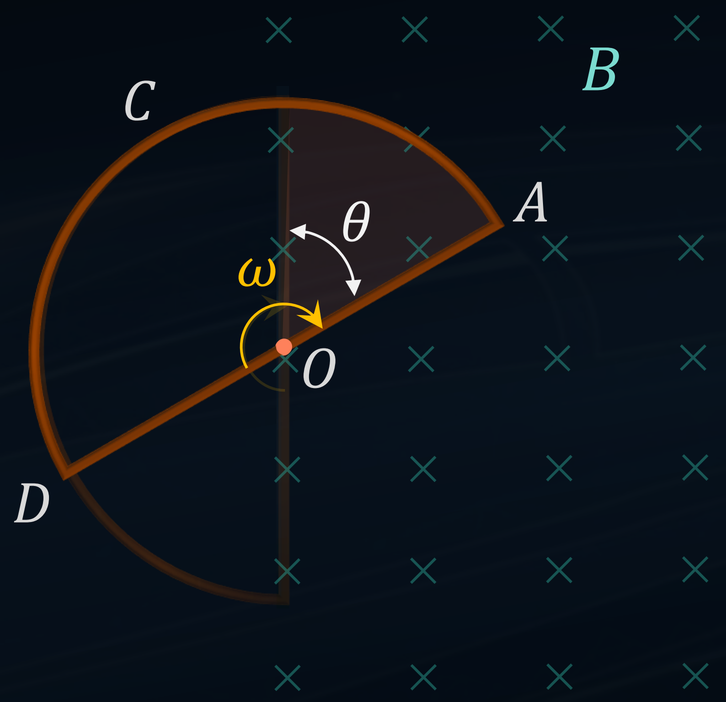
Area, $A = \dfrac{\theta}{2\pi} \times \pi R^2 = \dfrac{1}{2} R^2 \theta$

ϕ_B through coil at time $'t'$

$\phi_B = BA\cos 0° \; or \; BA\cos 180°$

$$\phi_B = \pm \frac{BR^2\theta}{2}$$

$$|\varepsilon| = (1)\left|\frac{d\left(\frac{BR^2\theta}{2}\right)}{dt}\right| \qquad |\varepsilon| = N\left|\frac{d\phi_B}{dt}\right|$$



Summary

Given : Radius (R), Resistance (r)

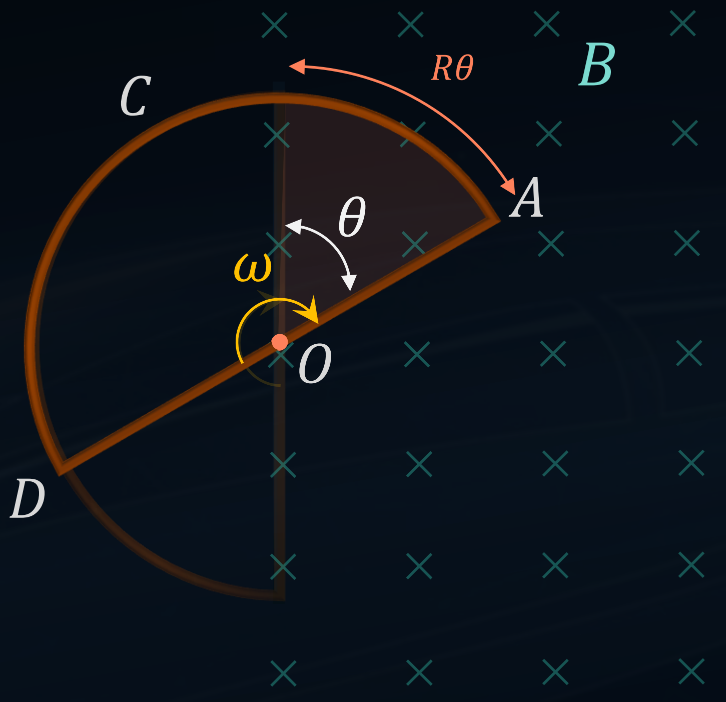
$$|\varepsilon| = (1)\left|\frac{d\left(\frac{BR^2\theta}{2}\right)}{dt}\right|$$

$$\varepsilon = \frac{BR^2}{2}\frac{d\theta}{dt} \qquad \frac{d\theta}{dt} = \omega$$

$$\varepsilon = \frac{BR^2}{2}\omega$$

$$i = \frac{\varepsilon}{r}$$

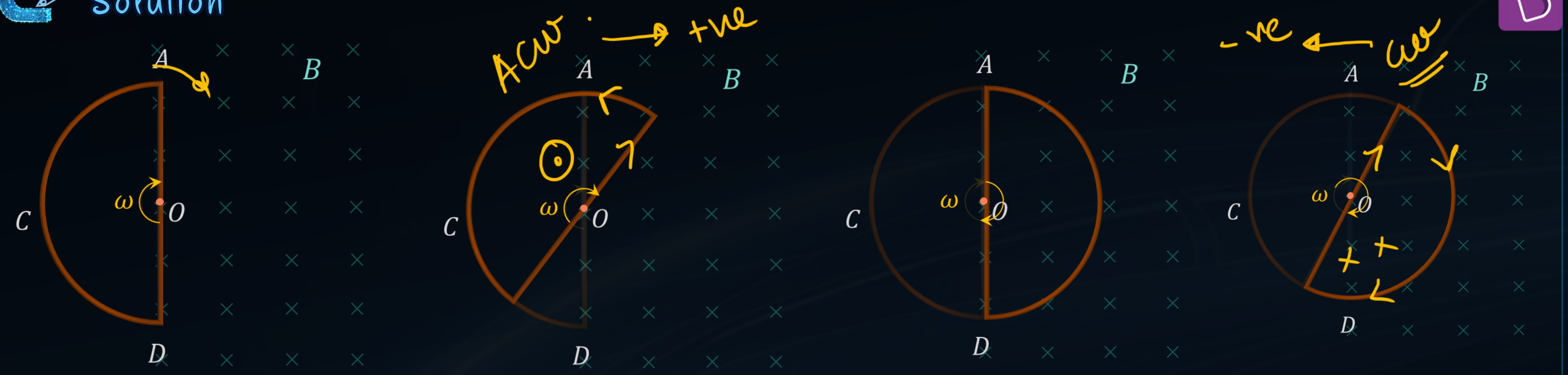
$$i = \frac{B\omega R^2}{2r}$$

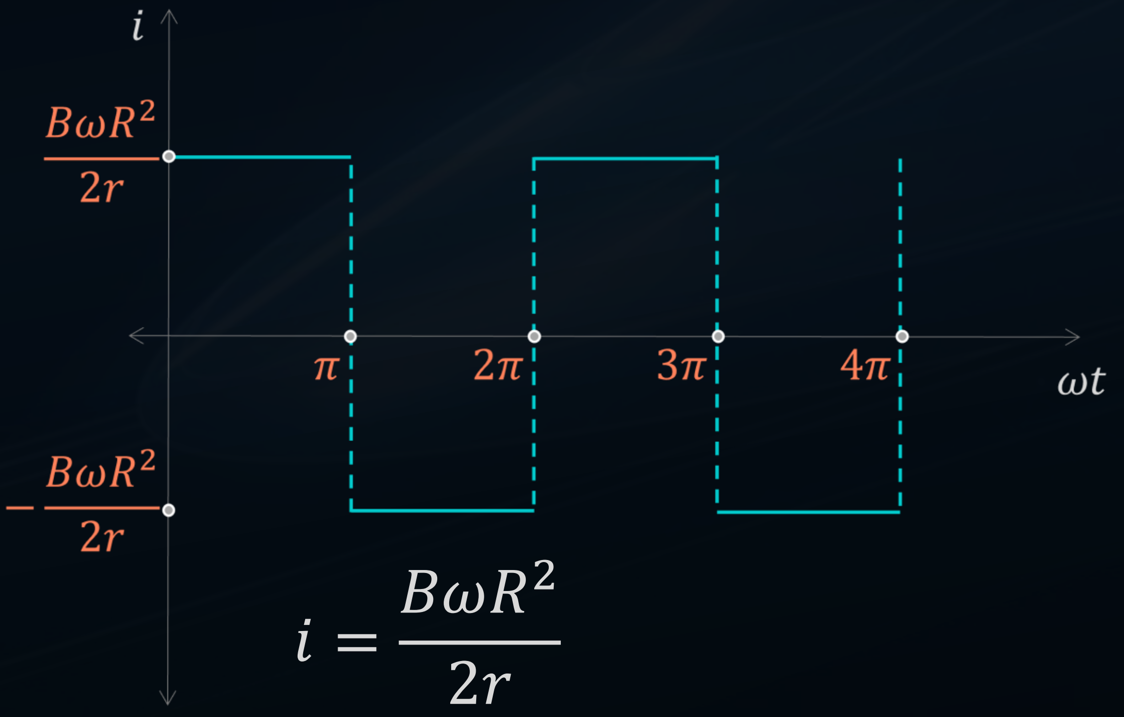


ACW + ve

CW → − ve

Solution



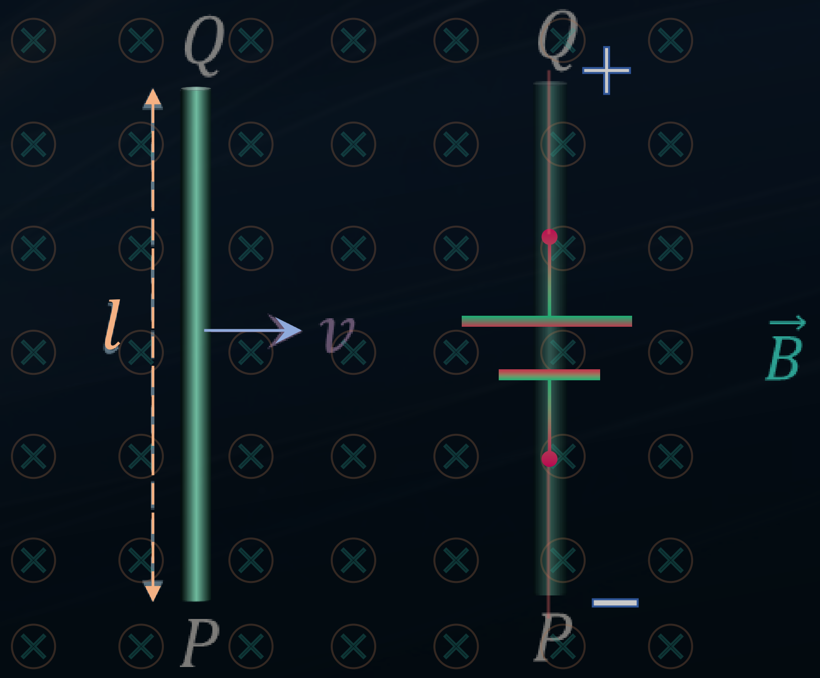


$$i = \frac{B\omega R^2}{2r}$$

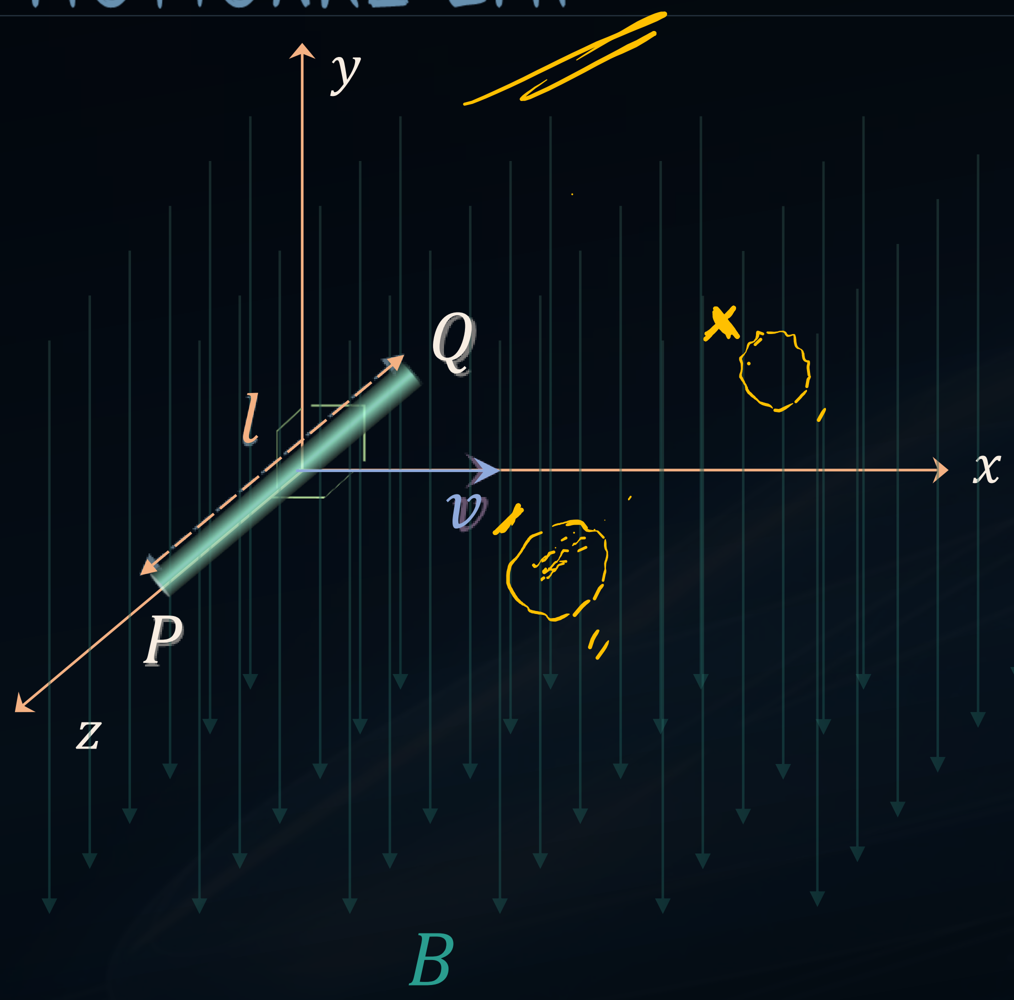
- As the coil rotates and goes into the magnetic field, the area that defines flux increases till it completes half revolution. The area starts decreasing when the coil completes next half of the revolution and come out of field.
- Current will be ACW in first half and it will be CW in next half of revolution as per Lenz's law and this cycle will continue as the coil rotates further.

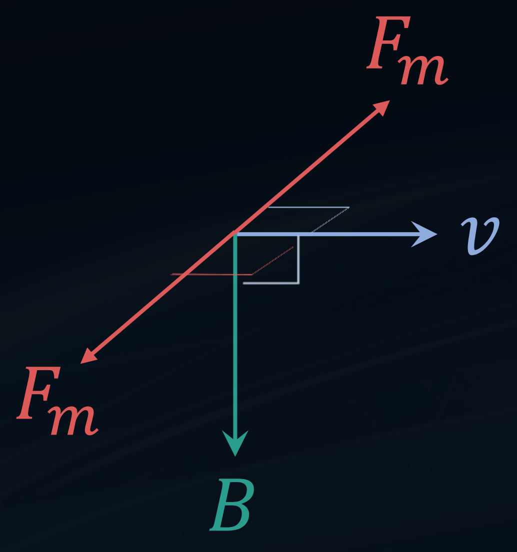
MOTIONAL EMF

When a conductor starts moving in a magnetic field, emf gets induced in it. It will start acting as a cell/battery. A potential difference gets developed between the ends of this conductor. This EMF developed due to motion of conductor in magnetic field is called as motional EMF.



MOTIONAL EMF

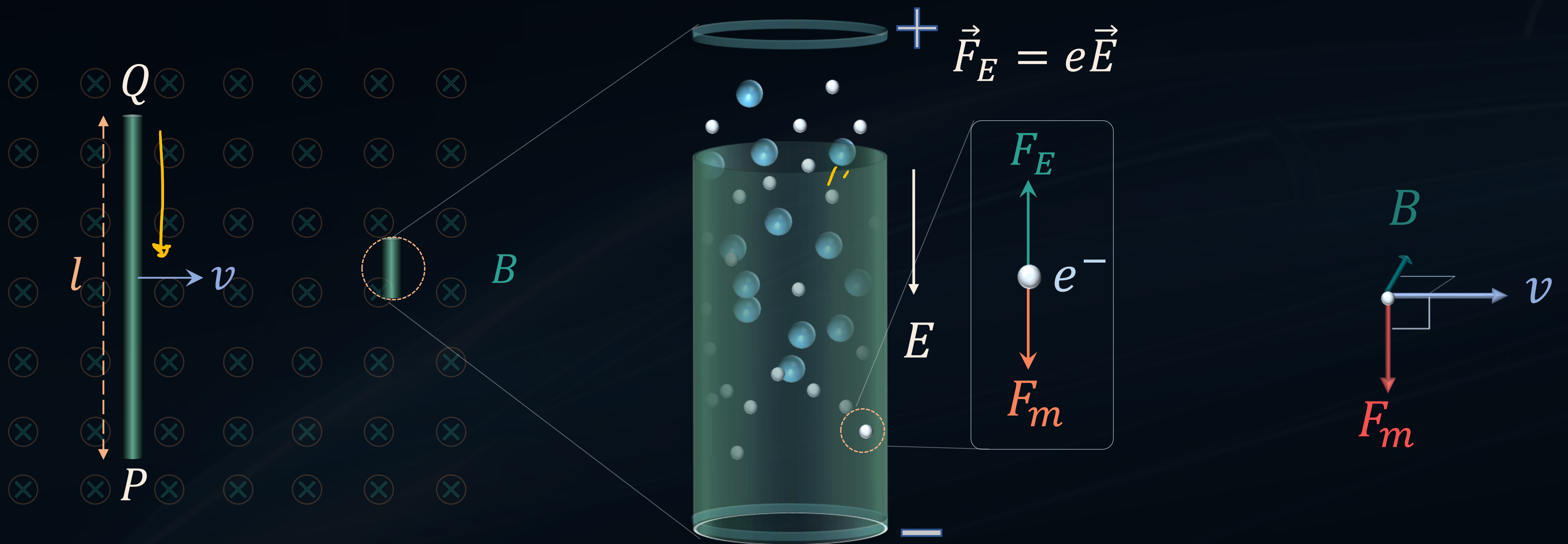




- Conductors have electrons in them. These electrons start moving when the conductor undergo motion in magnetic field.
- As the electrons are negatively charged particle, the direction of force will be in opposite direction of the magnetic force we took in case of positive particles.

For e^- Magnetic Force $(\vec{F}_m)$ is in opposite direction.

MOTIONAL EMF

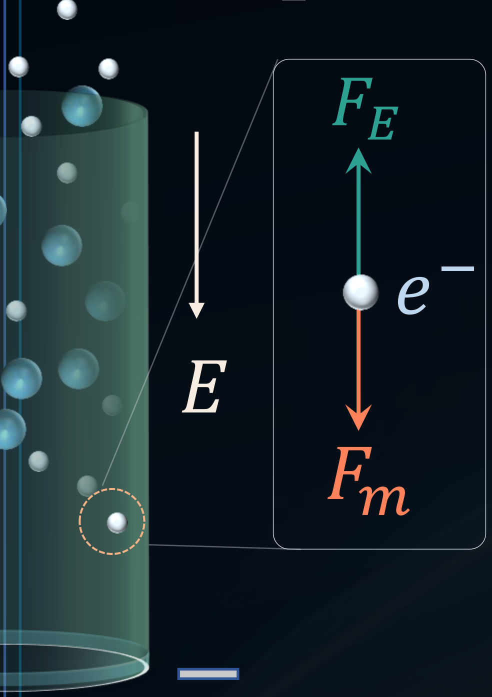


$$\vec{F}_m = -e\left(\vec{v} \times \vec{B}\right) \Rightarrow \left|\vec{F}_m\right| = evB$$

Electrons will start accumulating at the end *P*. Because of the accumulation, electric field develops in the *QP* direction. At steady state the magnetic force on the electrons become equal to the electric force.

MOTIONAL EMF

$\vec{F}_E = e\vec{E}$



$\left|\vec{F}_m\right| = evB$

Initially $\vec{F}_m > \vec{F}_E$

When, $\left|\vec{F}_E\right| = \left|\vec{F}_m\right|$

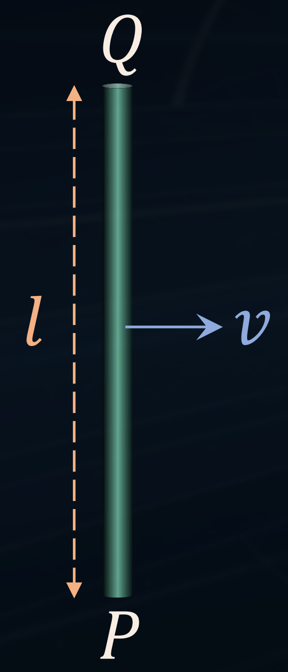
$\cancel{e}E = \cancel{e}vB$

$$\boxed{E = vB}$$

$\Delta V = El$

$\Delta V = vBl$

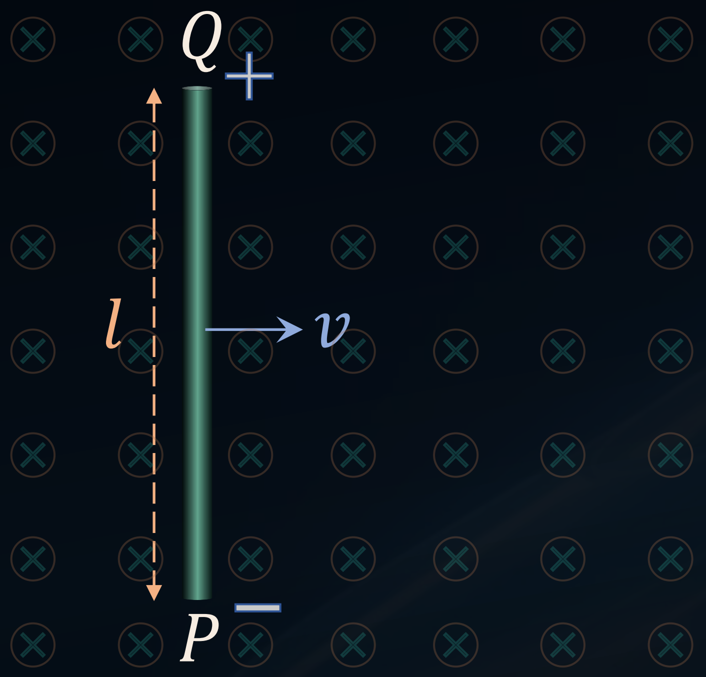
$$\boxed{\varepsilon = vBl}$$



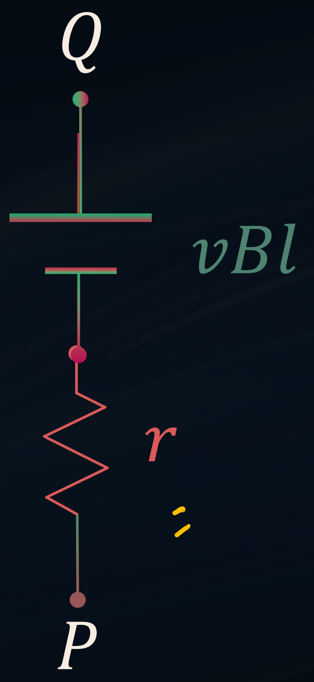
Potential difference across the two ends of the conductor of length l moving in a magnetic field of intensity B with velocity v is, $\varepsilon = vBl$.

MOTIONAL EMF AS EQUIVALENT BATTERY

$$\varepsilon = vBl$$



$\equiv$



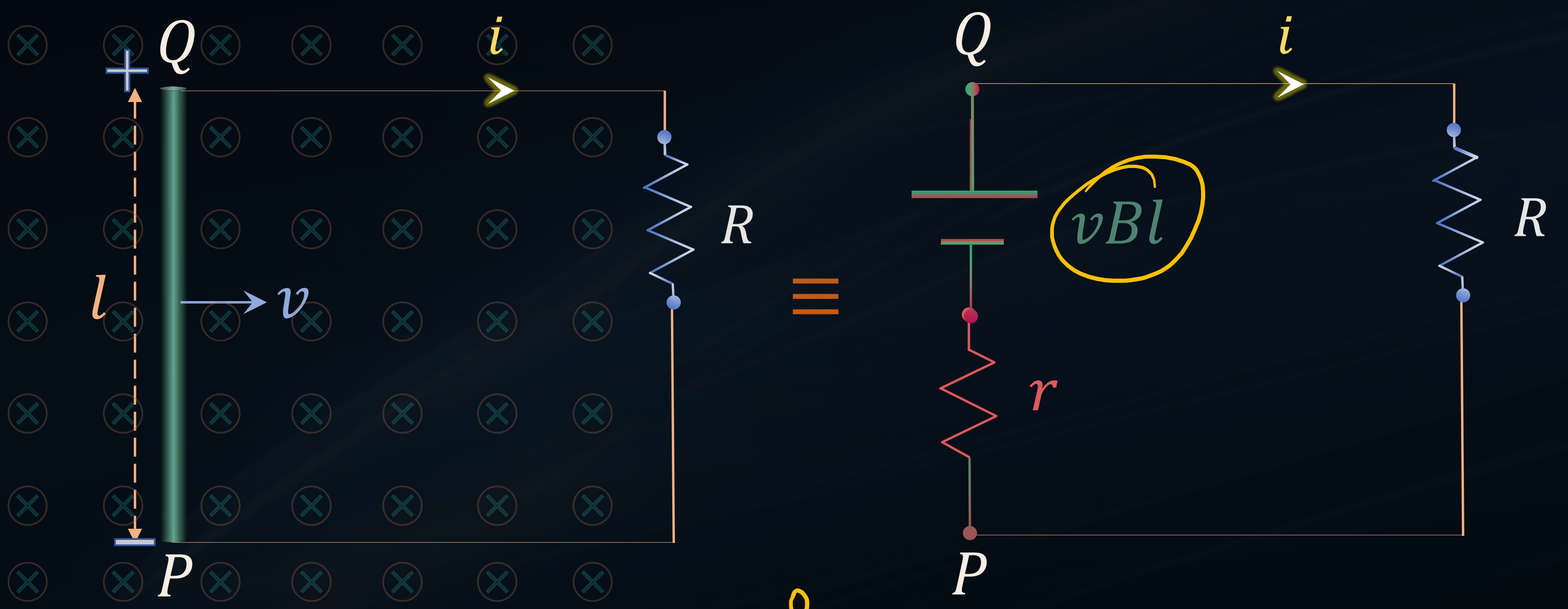
This rod can also be considered as a battery as EMF is developed in it. We can consider it as a battery with internal resistance equal to the resistance of the rod.

Rod $PQ \equiv$ Battery ($\varepsilon = vBl$, $r =$ resistance of rod)

MOTIONAL EMF AS EQUIVALENT BATTERY

If we connect this rod to an external circuit with resistance R ,the current through the circuit because of the motional emf can be calculated as follow:

$$\varepsilon = vBl$$

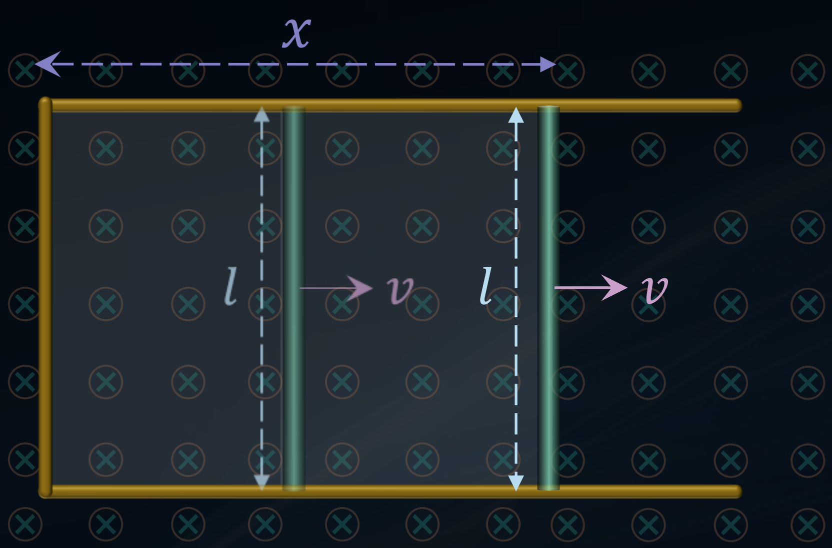


$$i = \frac{\varepsilon}{R+r} = \frac{vBl}{R+r}$$

$$i = \frac{vBl}{R+r}$$

Induced current

MOTIONAL EMF BY FARADAY'S LAW



$$\phi = B.A = BA\cos\theta$$

$$\phi = BA = Blx$$

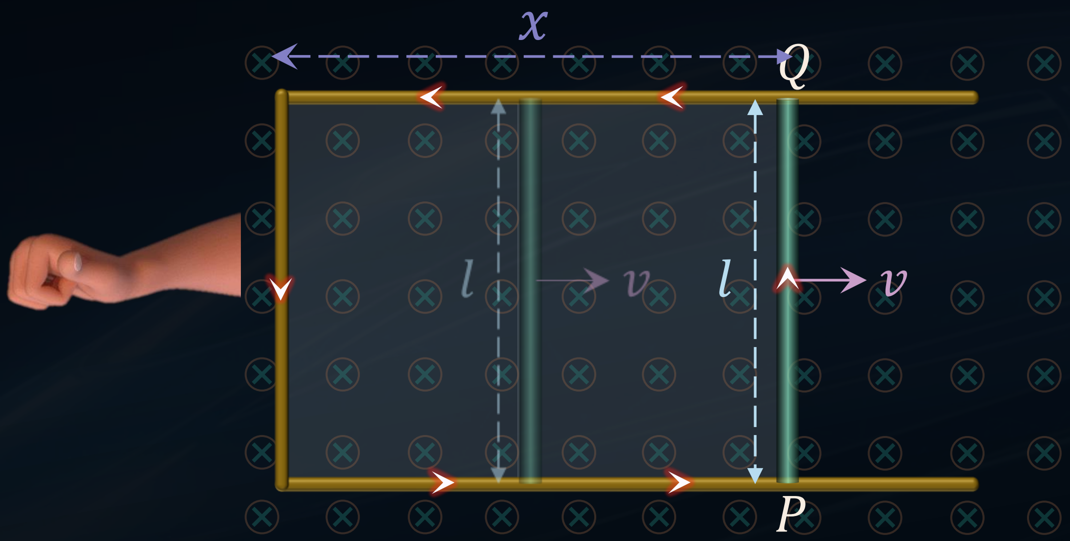
$$\varepsilon = \left|\frac{d\phi}{dt}\right| = Bl\frac{dx}{dt}$$

$$\boxed{\varepsilon = vBl}$$

MOTIONAL EMF BY FARADAY'S LAW

Direction - By Lenz's law

$\varepsilon = vBl$

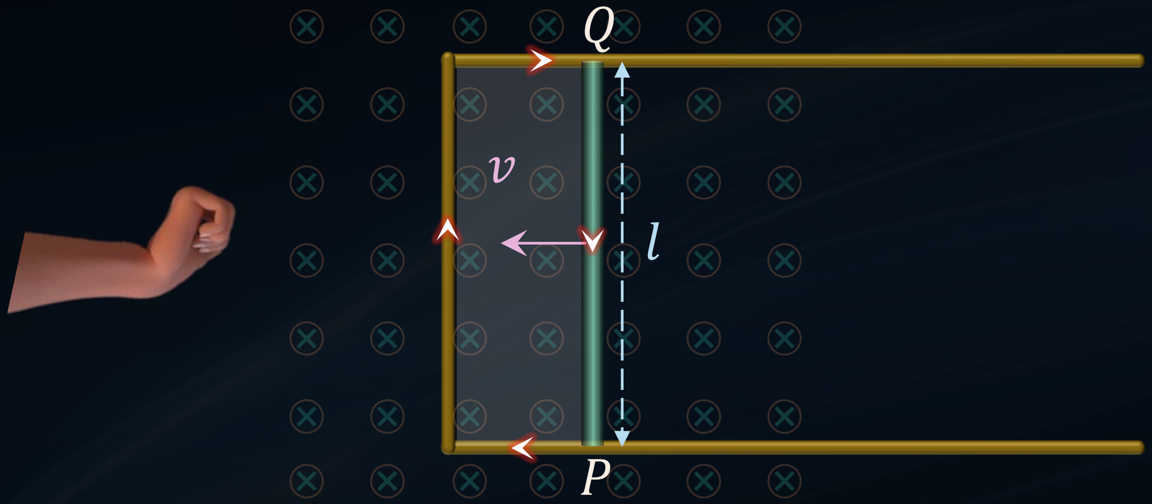


- ϕ increasing ($\phi \propto A$)
- Direction of induced current- anti-clockwise

MOTIONAL EMF BY FARADAY'S LAW

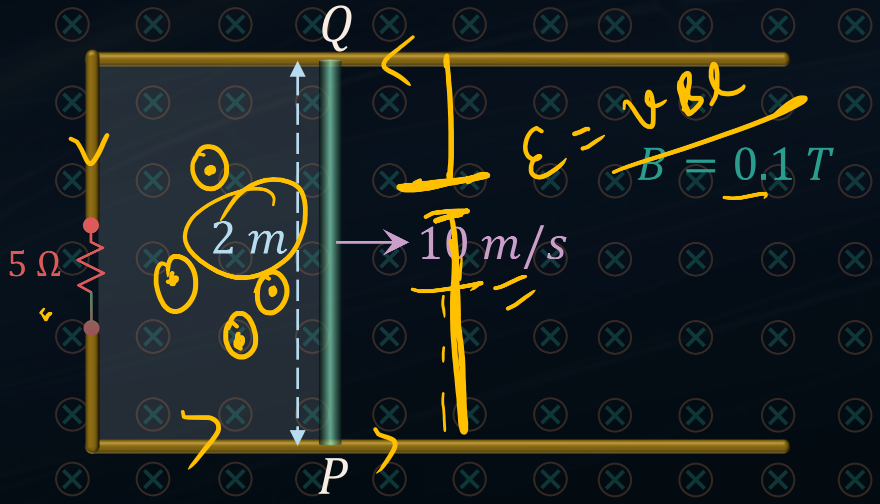
Direction - By Lenz's law

$$\varepsilon = vBl$$



- ϕ decreasing ($\phi \propto A$)
- Direction of induced current- clockwise

Find the current in 5 Ω resistance



a) 0.5 A

b) 0.4 A

c) 1 A

d) 0.2 A

Solution

$$\boxed{\varepsilon = vBl}$$

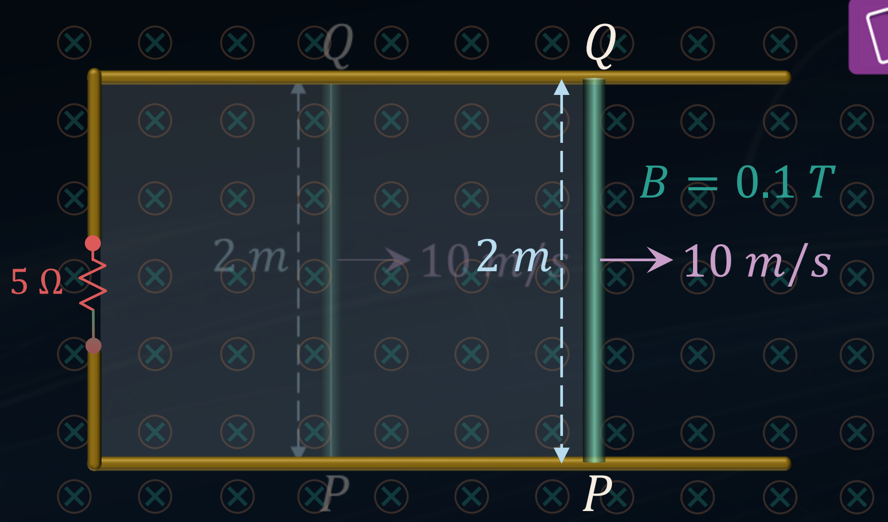
$$\varepsilon = 10 \times 0.1 \times 2$$

$$\varepsilon = 2\ V$$

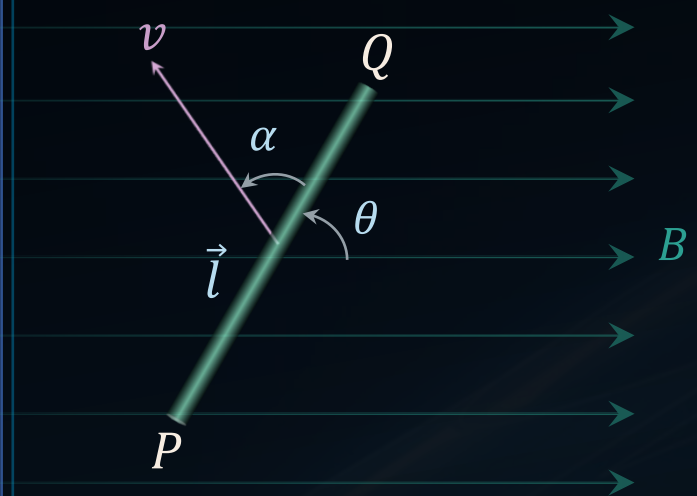
$$i = \frac{\varepsilon}{R}$$

$$i = \frac{2}{5} = 0.4\ A$$

Thus, option b is the correct answer.



MOTIONAL EMF (at different angle)

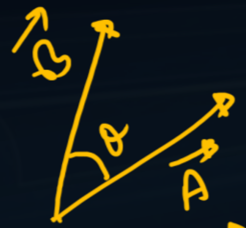


$$dV = -\vec{E}\,.\,d\,\vec{l}$$

$$\int dV = \int (\vec{v} \times \vec{B})\,.\,d\vec{l}$$

$$\Delta V = (\vec{v} \times \vec{B}).\vec{l}$$

$$\boxed{\varepsilon = (\vec{v} \times \vec{B}).\vec{l}}$$



$\vec{C} = \vec{A} \times \vec{B}$

$$\vec{F}_E = -\vec{F}_m$$

$$e\vec{E} = -e(\vec{v} \times \vec{B})$$

$$\boxed{\vec{E} = -(\vec{v} \times \vec{B})}$$

Electrons will start accumulating at the end P because of the accumulation electric field develops in the QP direction. At steady state, the magnetic force on the electrons become equal to the electric force.

MOTIONAL EMF (Different cases)

$$\varepsilon = (\vec{v} \times \vec{B}).\vec{l}$$

$$\varepsilon = (vB\sin 90°).\vec{l}$$

$$\varepsilon = vBl\cos 0°$$

$$\varepsilon = vBl$$

$$\boxed{\vec{B} \perp \vec{v} \perp \vec{l}}$$

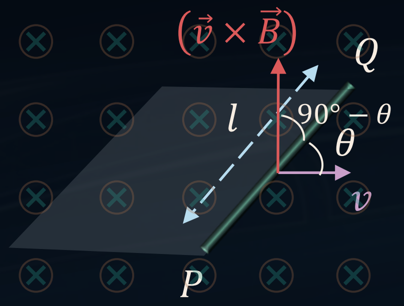
$$\varepsilon = (\vec{v} \times \vec{B}).\vec{l}$$

$$\varepsilon = (vB\sin 90°).\vec{l}$$

$$\varepsilon = vBl\cos 90°$$

$$\varepsilon = 0$$

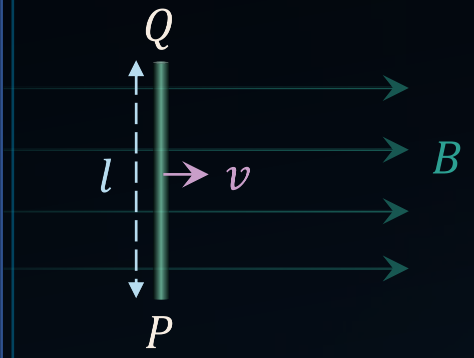
$$\boxed{\vec{v} \parallel \vec{l}}$$

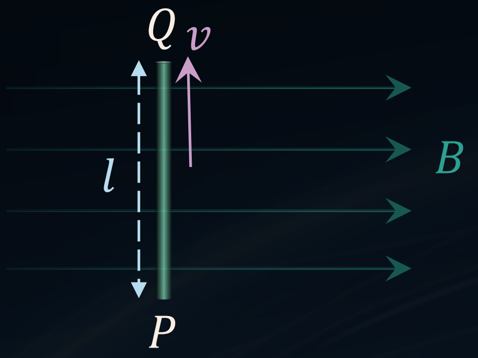


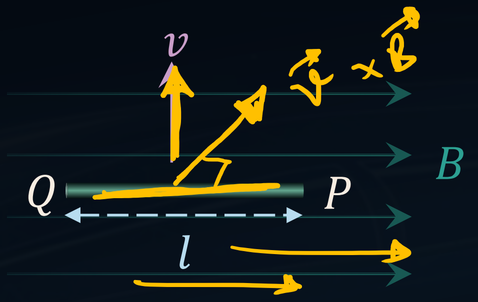
$$\boxed{\varepsilon = (\vec{v} \times \vec{B}).\vec{l}}$$

$$\varepsilon = vBl\sin\theta$$

MOTIONAL EMF (Different cases)







$\varepsilon = (\vec{v} \times \vec{B}).\vec{l}$

$\varepsilon = (vB \sin 0°).\vec{l}$

$\varepsilon \Rightarrow 0$

$$\boxed{\vec{v} \parallel \vec{B}}$$

$\varepsilon = (\vec{v} \times \vec{B}).\vec{l}$

$\varepsilon = (vB \sin 90°).\vec{l}$

$\varepsilon = vBl \cos 90°$

$\varepsilon = 0$

$$\boxed{\vec{v} \parallel \vec{l}}$$

$\varepsilon = (\vec{v} \times \vec{B}).\vec{l}$

$\varepsilon = (vB \sin 90°).\vec{l}$

$\varepsilon = vBl \underline{\cos 90°}$

$\varepsilon = 0$

$$\boxed{\vec{l} \parallel \vec{B}}$$

MOTIONAL EMF (Different cases)

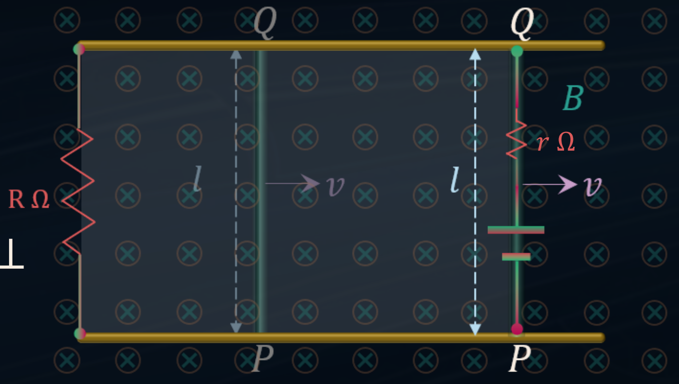
NEET

- If $\vec{B}, \vec{v}$ & $\vec{l}$ are mutually perpendicular. $\boxed{\varepsilon = vBl}$
- If any two of $\vec{B}, \vec{v}$ & $\vec{l}$ are parallel. $\boxed{\varepsilon = 0}$
- If $\vec{B}, \vec{v}$ & $\vec{l}$ are at some angle. $\boxed{\varepsilon = (\vec{v} \times \vec{B}).\vec{l}}$

PROBLEMS ON MOTIONAL EMF

BOARDS NEET

- A rod of length l having resistance r is sliding on frictionless rails which have *zero* resistance



Find induced emf and current in rod.

Induced emf, $\varepsilon = (\vec{v} \times \vec{B}) \cdot l$

$= vBl$

$\vec{B}, \vec{v}$ and $\vec{l}$ are mutually $\perp$

Current in rod, $i = \dfrac{\varepsilon}{R + r}$

$= \dfrac{vBl}{R + r}$

$\varepsilon = vBl$

PROBLEMS ON MOTIONAL EMF

- A rod of length l having resistance r is sliding on frictionless rails which have *zero* resistance

Find external force required to keep the rod moving with constant velocity v.

$$\vec{F}_m = i(\vec{l} \times \vec{B})$$

$$F_m = ilB \sin 90^\circ$$

$$= ilB$$

$$= \frac{B^2 l^2 v}{R + r}$$

$$i = \frac{vBl}{R + r}$$

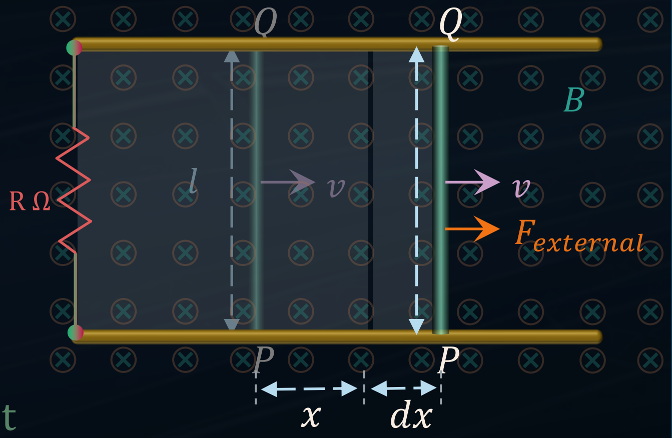
$$F_{ext} = \frac{B^2 l^2 v}{R + r}$$

$$F_m = F_{ext}$$

PROBLEMS ON MOTIONAL EMF

- A rod of length l having resistance r is sliding on frictionless rails which have *zero* resistance

Find input power



Work done by external force

$$dW = F_{ext}dx$$

$$dW = F_{ext}dx$$

$$F_{ext} = \frac{B^2l^2v}{R+r} = \text{Constant}$$

Input Power

$$P_i = \frac{dW}{dt} = F_{ext}\frac{dx}{dt}$$

$$\frac{dx}{dt} = v$$

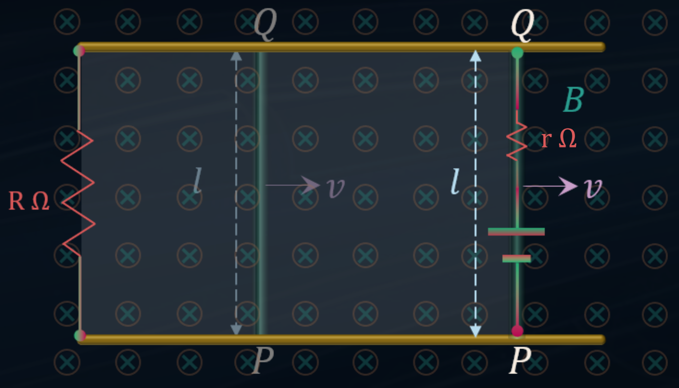
$$P_i = \frac{B^2l^2v^2}{R+r}$$

PROBLEMS ON MOTIONAL EMF

- A rod of length l having resistance r is sliding on frictionless rails which have *zero* resistance

Find output power

$$P_i = \frac{B^2 l^2 v^2}{R + r}$$



$$P_O = i^2 (R + r)$$

$$= \left(\frac{vBl}{R + r}\right)^2 (R + r)$$

$$= \frac{B^2 l^2 v^2}{R + r}$$

Power Input = Power Output

Mechanical Work → Electrical Energy

PROBLEMS ON MOTIONAL EMF

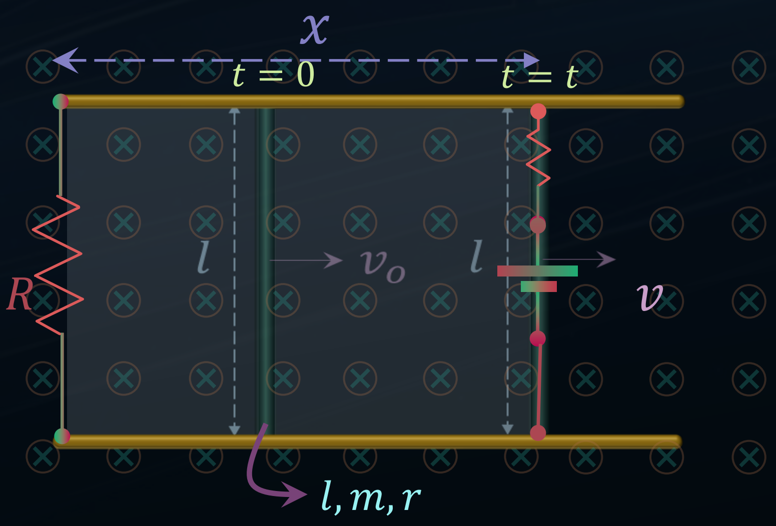
- A rod of length l having resistance r is sliding on frictionless rails which have *zero* resistance without any external force.

Find induced emf and current when velocity of rod is $'v'$

$$\varepsilon = vBl$$

$$i = \frac{\varepsilon}{R+r} = \frac{vBl}{R+r}$$

$$\boxed{i = \frac{vBl}{R+r}}$$



PROBLEMS ON MOTIONAL EMF

Find acceleration of the rod at this instant

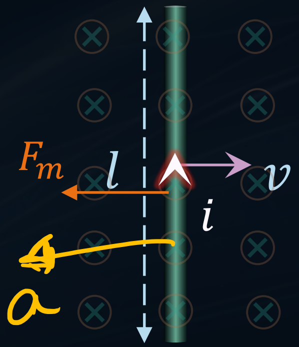
$$F_m = iBl$$

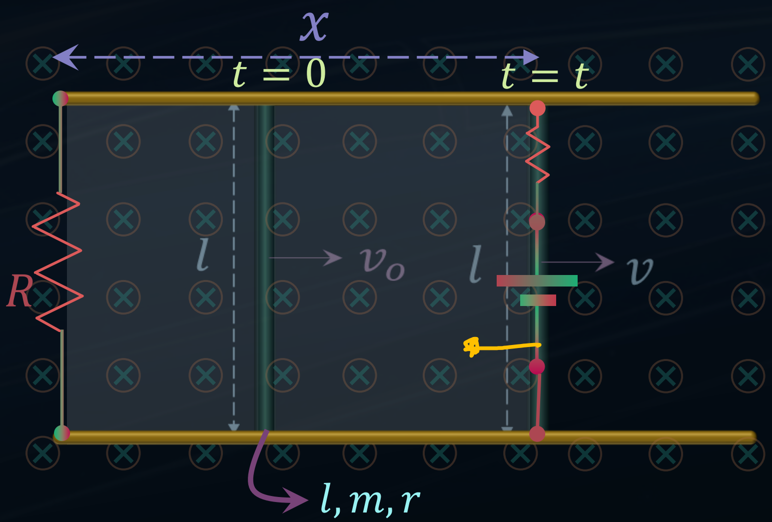
$$F_m = \frac{vBl}{R+r} \times Bl$$

$$F_m = \frac{B^2 l^2 v}{R+r}$$

$$a = \frac{F_m}{m} = \frac{\left(\frac{B^2 l^2 v}{R+r}\right)}{m}$$

$$\boxed{a = -\frac{B^2 l^2 v}{m(R+r)}}$$





$-ve$ sign implies retardation

PROBLEMS ON MOTIONAL EMF

Write $'v'$ as a function of $'t'$

$$a = -\frac{B^2 l^2 v}{m(R+r)}$$

$$\frac{dv}{dt} = -\frac{B^2 l^2 v}{m(R+r)}$$

$$\int_{v_o}^{v} \frac{dv}{v} = \int_0^t -\frac{B^2 l^2}{m(R+r)}\, dt$$

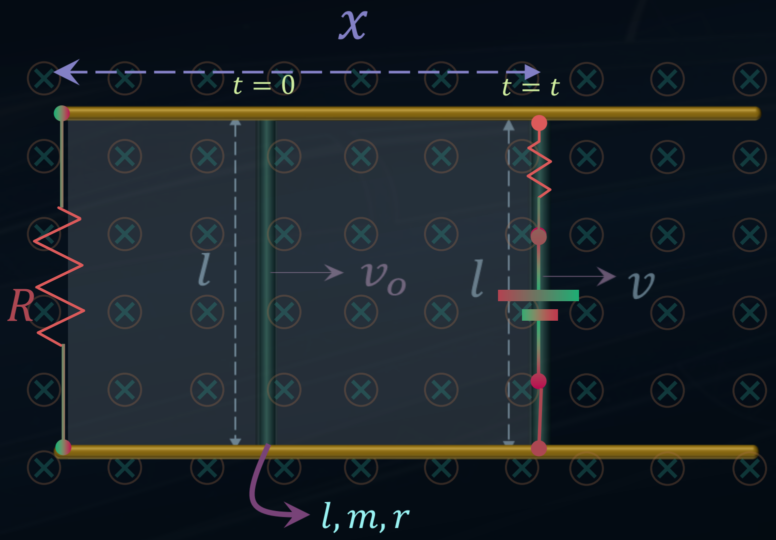
$$[\log_e v]^v_{v_o} = -\frac{B^2 l^2 (t)}{m(R+r)} \Rightarrow \log_e\left(\frac{v}{v_o}\right) = -\frac{B^2 l^2 (t)}{m(R+r)}$$

$$\boxed{v = v_o e^{-\frac{B^2 l^2 t}{m(R+r)}}}$$

$$\int_{x_1}^{x_2} \frac{1}{x} dx = [\log_e x]_{x_1}^{x_2}$$

$$\Rightarrow \log_e x_2 - \log_e x_1$$

$$= \log_e \frac{x_2 \to v}{x_1 \to v_0}$$



PROBLEMS ON MOTIONAL EMF

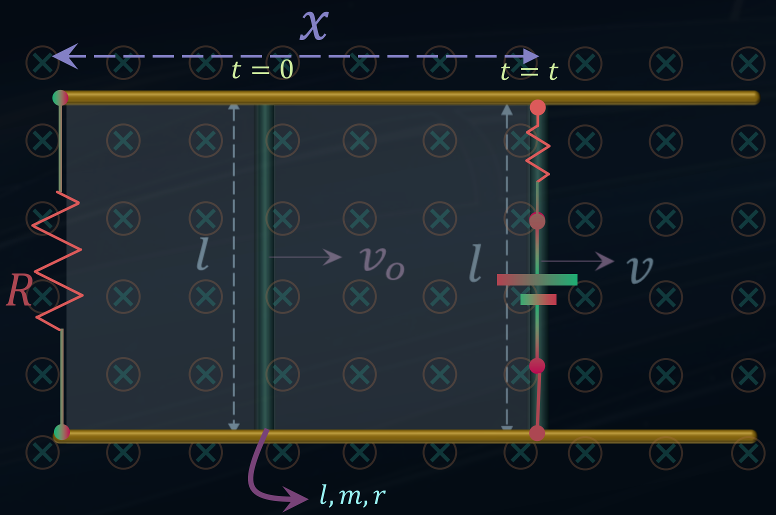
Write $'v'$ as a function of displacement $'x'$

$$a = -\frac{B^2 l^2 v}{m(R+r)}$$

$$\frac{v\,dv}{dx} = -\frac{B^2 l^2 v}{m(R+r)}$$

$$\int_{v_o}^{v} dv = \int_0^x -\frac{B^2 l^2}{m(R+r)}\,dx$$

$$v - v_o = -\frac{B^2 l^2}{m(R+r)}x$$



$$\boxed{v = v_o - \frac{B^2 l^2}{m(R+r)}x}$$

PROBLEMS ON MOTIONAL EMF

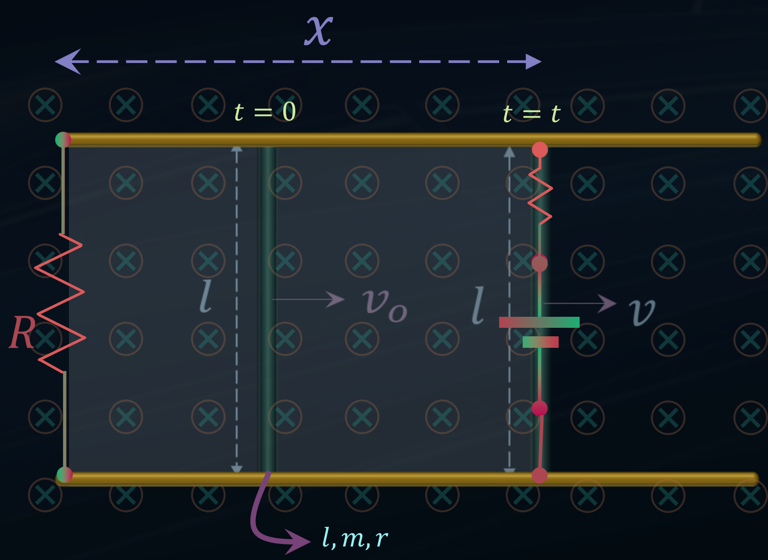
Find the distance covered by the rod before it stops.

$$v = v_o - \frac{B^2 l^2}{m(R+r)} x$$

$v = 0$ when rod stops

$$0 = v_o - \frac{B^2 l^2}{m(R+r)} x$$

$$\boxed{x = \frac{v_o m(R+r)}{B^2 l^2}}$$



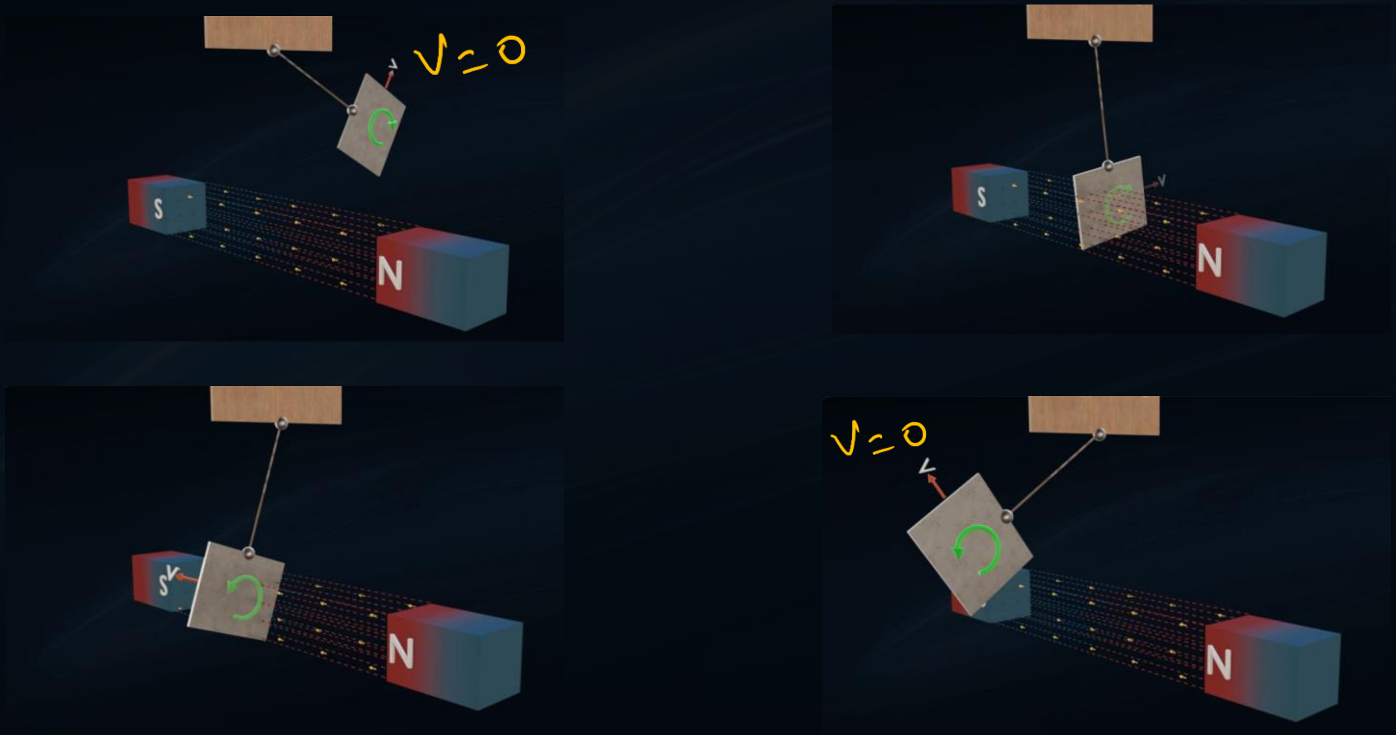
EDDY CURRENT

Eddy currents are loops of electric current induced within conductors by a changing magnetic field in the conductor according to Faraday's law of induction.

- Eddy currents are induced currents in the body of conductor when subjected to changing magnetic flux.
- They are also known as Foucault current after the name of the scientist Foucault.
- They are known as eddy currents as they are in the pattern of eddies in the water.
- Eddy currents are capable of generating heat in the conductor.
- Eddy currents ae capable of generating a force on the conductor in accordance with Faraday's laws and Lenz's law.

EDDY CURRENT

Consider a conductor tied to a string and moving in a varying magnetic field. Every time it passes the field, the direction of currents generated in it changes. The varying magnetic field causes the localized currents in the conductor.



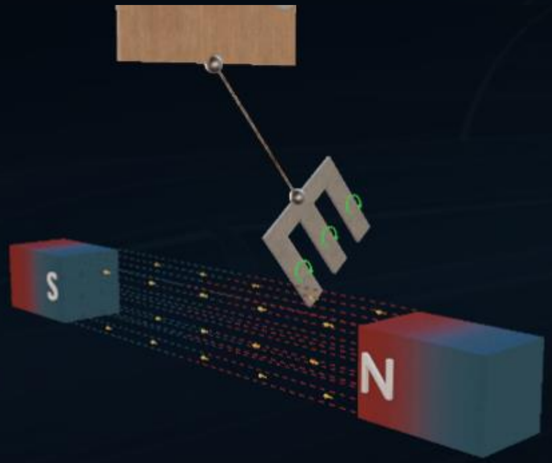
EDDY CURRENT

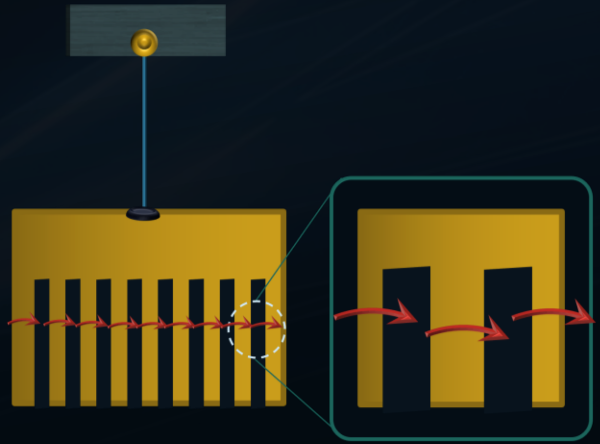
Localized currents induced in a conductor due to changing magnetic flux.

Eddy currents are only produced in electrical conducting materials and that too in presence of changing magnetic flux.

EDDY CURRENT

- By introducing slots in the plate one can reduce the area available for the generation of eddy currents.
- This reduces the intensity of damping.



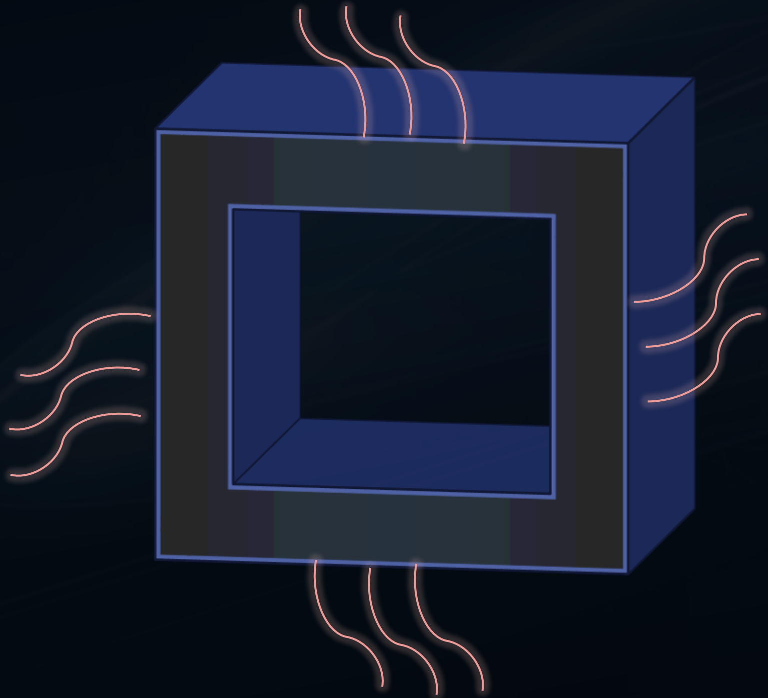


Eddy Current

By introducing slots in the plate, we are increasing the length through which current is traversing and reducing the area available for the currents. As resistance is directly proportional to the length and inversely proportional to the area, this will result in increase in resistance of the conductor. Which will cause lower current and lower heat loss due to eddy currents.

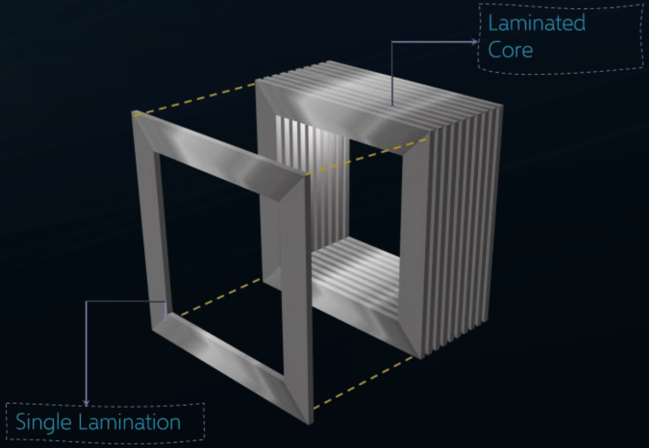
EDDY CURRENT: DISADVANTAGES

Dissipate electrical energy in the form of heat. Overheating of metallic cores of transformers, electric motors and other such devices.



EDDY CURRENT: DISADVANTAGES

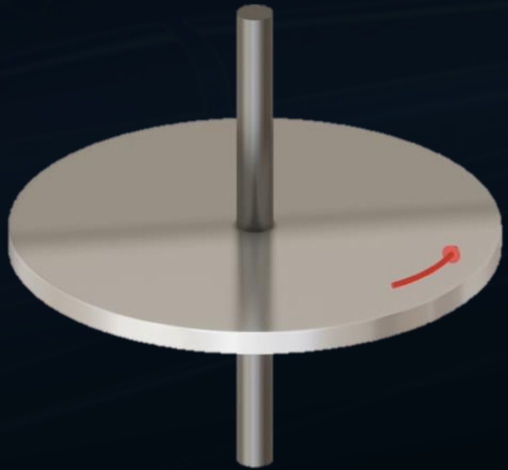
- In order to avoid the overheating of the core, metallic sheets are taken in the form of very thin sheets. These sheets are electrically insulated.
- The cores are insulated with some insulating material. As a result, heat won't get transferred to surroundings. These cores are known as laminated cores.
- By using these cores, eddy currents are reduced, as a result heat dissipation is reduced.



EDDY CURRENT : APPLICATION

- Electromagnetic brakes – To control the speed of fast-moving electric trains.

In this case, magnetic flux is passed in direction perpendicular to the rotating direction of wheels. This results in eddy currents flowing in opposite direction of rotation of the wheel which generates opposing force to slow down the wheels.
This type of breaking is very efficient as it reduces the damages due to traditional friction based braking system.



- Electromagnetic damping – Electromagnetic damping in galvanometers helps to reduce oscillations around equilibrium positions.

In which of the following devices, the eddy current effect is not used?

Solution NEET

a Electromagnet

b Electric heater

c Induction furnace

d Magnetic braking in train

Electric heater – It uses Joule's heating effect to convert the electrical energy to heat energy. It does not use eddy currents to produce heat. Whereas, electromagnets, induction furnace and magnetic breaking in train involves eddy currents.

Eddy currents are produced when

Solution

NEET

a A metal is kept in varying magnetic field

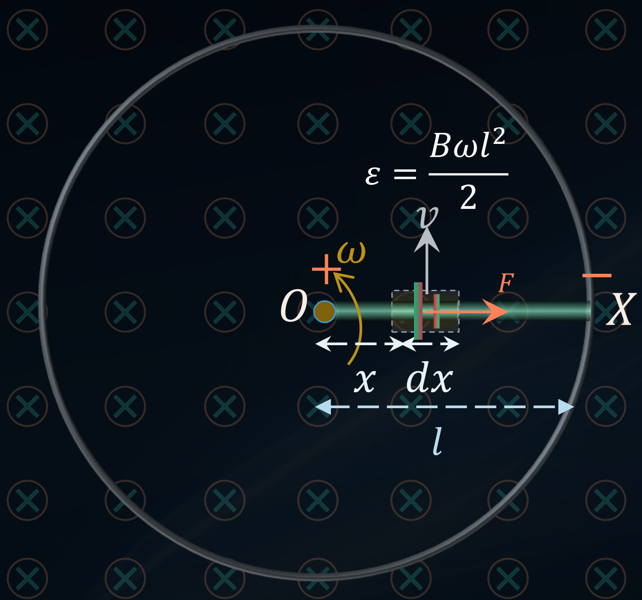
b A metal is kept in steady magnetic field

c A circular coil is placed in a magnetic field

d Through a circular coil, current is passed

A metal kept in varying magnetic field – Eddy currents are produced only when there's change in magnetic flux through a conductor. This is possible only in case of option a.

MOTIONAL EMF IN ROTATING CONDUCTING ROD



Consider a uniform conducting rod rotating with constant angular speed ω in uniform magnetic field going into the plane of motion of the rod.

$$v = x\omega$$

Consider an element of thickness dx at a distance x from O. Thus, EMF induced in the small element dx = $d\varepsilon$

$$d\varepsilon = Bvdx$$

$= \cdots \to \omega x$

$$\int d\varepsilon = \int Bvdx = \int_0^l B\omega x dx$$

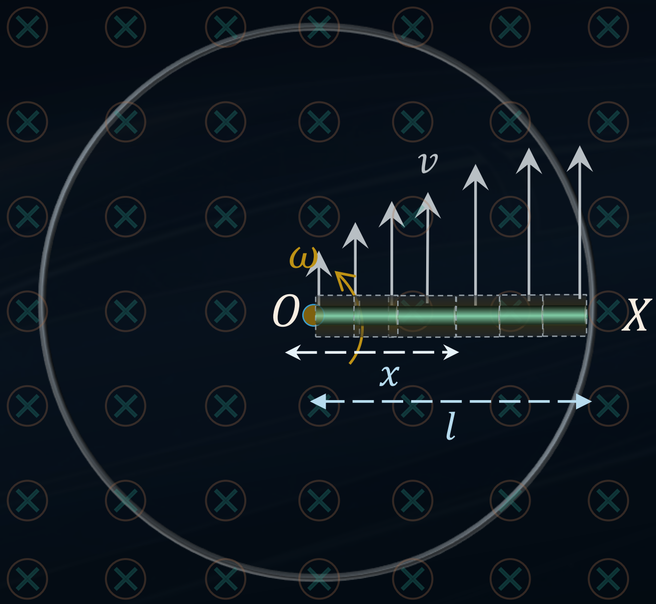
$\int x^n dx = \frac{x^{n+1}}{n+1}$

$\left[\frac{x^2}{2}\right]_0^l$

$$\boxed{\varepsilon = \frac{B\omega l^2}{2}}$$

- As $\vec{v} \times \vec{B}$ is pointing towards $'O'$, Thus $'O'$ is $(+)ve$ and $'X'$ is $(-)ve$.
- We neglect the force due to rotation (Centripetal).

MOTIONAL EMF IN ROTATING CONDUCTING ROD



$$v = x\omega$$

$$\varepsilon = \frac{B\omega l^2}{2}$$

$$V_o - V_X = \frac{B\omega l^2}{2}$$

EMF increases as we move from centre to the far end.

A metal rod of length l rotates about an end with a uniform angular velocity ω. A uniform magnetic field B exists in the direction of the axis of rotation. Calculate the emf induced between point X & Y $(V_Y - V_X)$. Neglect the centripetal force acting on the free electrons as they move in circular paths.

Solution

$$V_O - V_X = \frac{B\omega l^2}{2} \quad \text{— (I)}$$

$$V_O - V_Y = \frac{B\omega\left(\frac{l}{2}\right)^2}{2}$$

$$V_O - V_Y = \frac{B\omega l^2}{8} \quad \text{— (II)}$$

(I) − (II)

$$(V_O - V_X) - (V_O - V_Y) = V_Y - V_X$$

$$V_Y - V_X = \frac{B\omega l^2}{2} - \frac{B\omega l^2}{8}$$

$$V_Y - V_X = \frac{3B\omega l^2}{8}$$

$$\varepsilon = \frac{B\omega l^2}{2}$$

ALTERNATIVE METHOD:-

$$\int d\varepsilon = \varepsilon = \int_{\frac{l}{2}}^{l} B\omega x\,dx$$

$$V_Y - V_X = \left[\frac{B\omega x^2}{2}\right]_{\frac{l}{2}}^{l} = \left[\frac{B\omega l^2}{2}\right] - \left[\frac{B\omega (l/2)^2}{2}\right]$$

$$V_Y - V_X = \left[\frac{B\omega l^2}{2}\right] - \left[\frac{B\omega l^2}{8}\right]$$

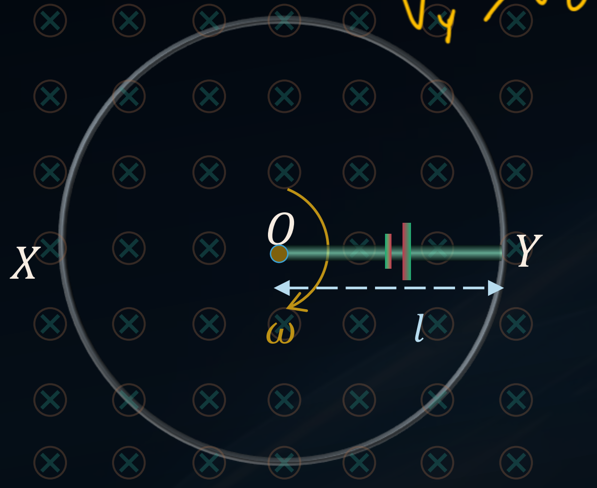
$$V_Y - V_X = \frac{3B\omega l^2}{8}$$

Find $V_Y - V_X$?

Solution NEET

Lets break the given rod in two parts as shown below.

$V_y > V_o$



$V_x > V_o$

$$V_Y - V_O = \frac{B\omega l^2}{2}$$

$$V_X - V_O = \frac{B\omega l^2}{2}$$

Combining two parts, we get:

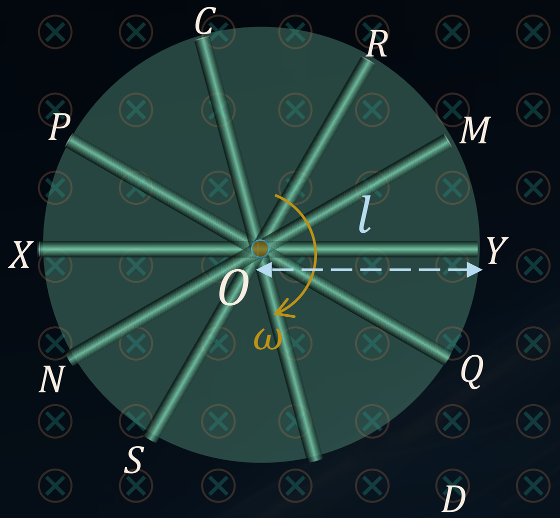
KVL from Y to X

$$V_Y - \cancel{\frac{B\omega l^2}{2}} + \cancel{\frac{B\omega l^2}{2}} = V_X$$

$$\boxed{V_Y - V_X = 0}$$

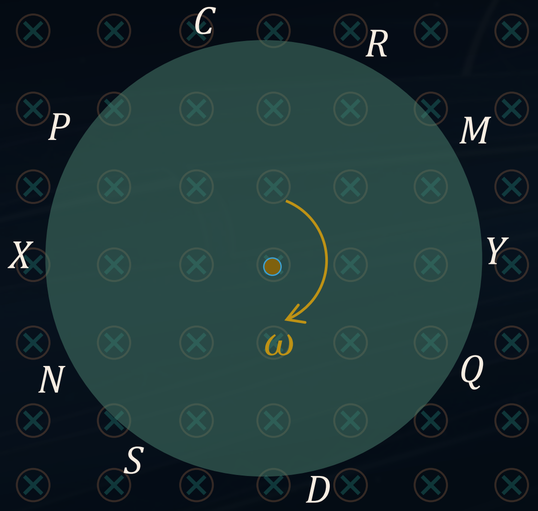
MOTIONAL EMF IN ROTATING CONDUCTING DISC

BOARDS



Combination of n number of rods

A conducting disc is formed



$$V_X = V_Y = \frac{B\omega l^2}{2} (Assume\ V_O = 0)$$

$$V_C = V_D = \frac{B\omega l^2}{2} \Rightarrow V_X = V_Y = V_C = V_D = V_P = V_Q$$

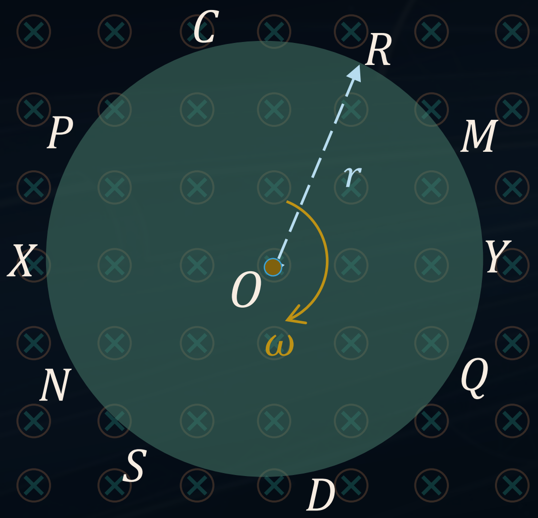
- Each point on the circumference of the disc is equipotential points. Hence, the potential difference between two points on the circumference is zero.

MOTIONAL EMF IN ROTATING CONDUCTING DISC

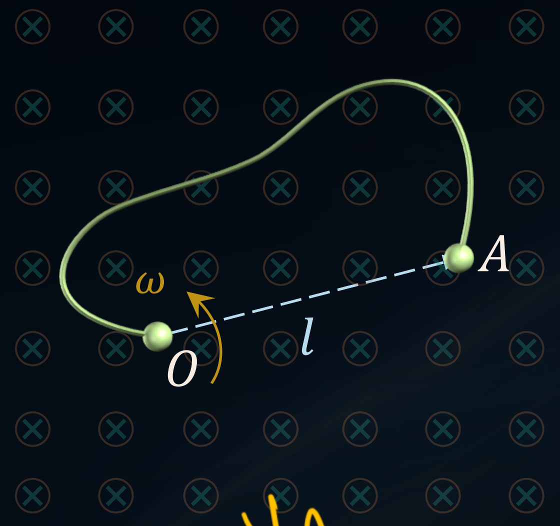
$$V_R - V_N = \frac{B\omega r^2}{2} - \frac{B\omega r^2}{2} = 0$$

$$V_X - V_P = \frac{B\omega r^2}{2} - \frac{B\omega r^2}{2} = 0$$

$$V_R - V_O = \frac{B\omega r^2}{2}$$



MOTIONAL EMF IN A ROTATING ARBITARY SHAPED CONDUCTING WIRE



$V_o > V_A$

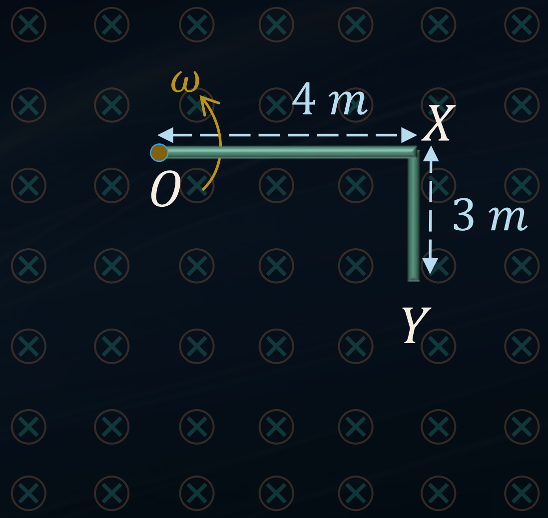
$$\boxed{\varepsilon = \frac{B\omega l^2}{2}}$$

l – Distance between the ends of wire

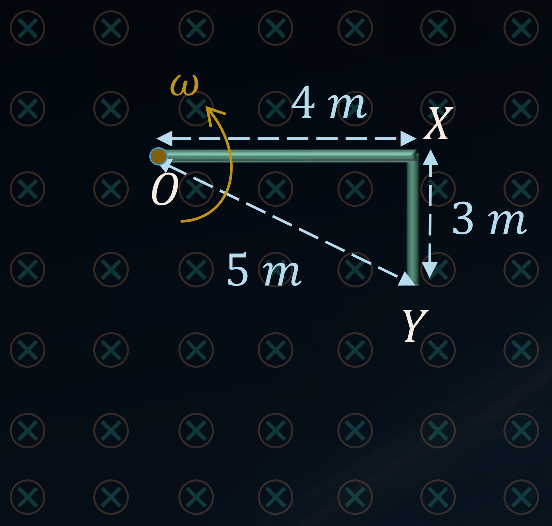
$$V_O - V_A = \frac{B\omega l^2}{2}$$

The right hand thumb rule suggests that the positive terminal of the battery having EMF equivalent to the EMF induced in the wire will be towards point O. Therefore, $V_O > V_A$.

Find $V_X - V_Y$?



Solution



For the motion of the rod OX, the right hand thumb rule suggests that the positive terminal of the battery having EMF equivalent to the EMF induced in the rod will be towards point O. Therefore, $V_O > V_X$.

$$\varepsilon = \frac{B\omega l^2}{2}$$

$$V_O - V_X = \frac{1}{2}B\omega(4)^2 = 8\,B\omega$$

Similarly,

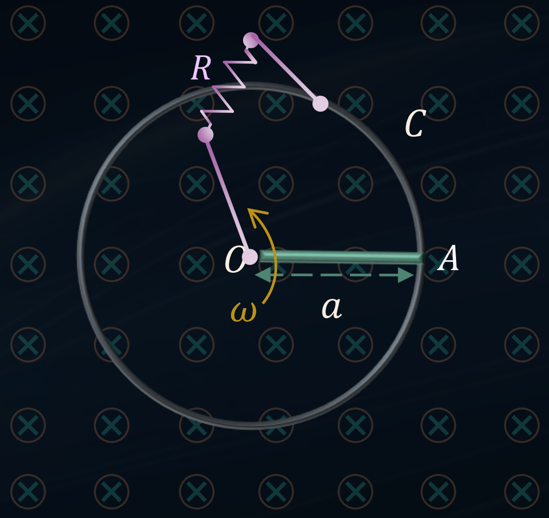
$$V_O - V_Y = \frac{B\omega(5)^2}{2} = \frac{25B\omega}{2}$$

Therefore,

$$V_X - V_Y = (V_O - V_Y) - (V_O - V_X) = \frac{25}{2}B\omega - 8\,B\omega = \frac{9}{2}B\omega$$

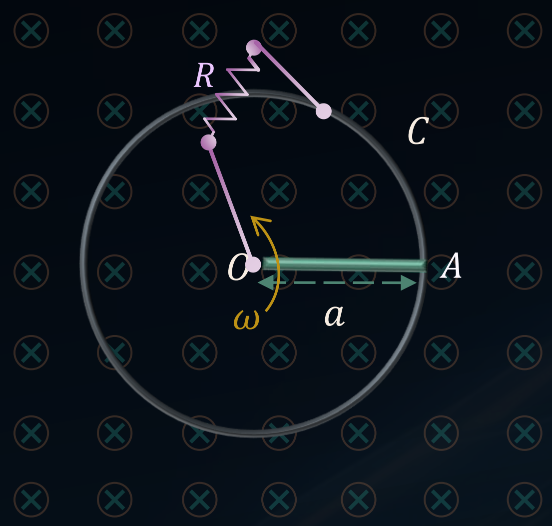
$$\boxed{V_X - V_Y = \frac{9}{2}B\omega}$$

Find current passing through the rod (circular ring is conducting).



$r \rightarrow$ Resistance of rotating rod

Solution



$V_0 > V_A$

$$V_0 - V_A = \frac{B\omega a^2}{2}$$

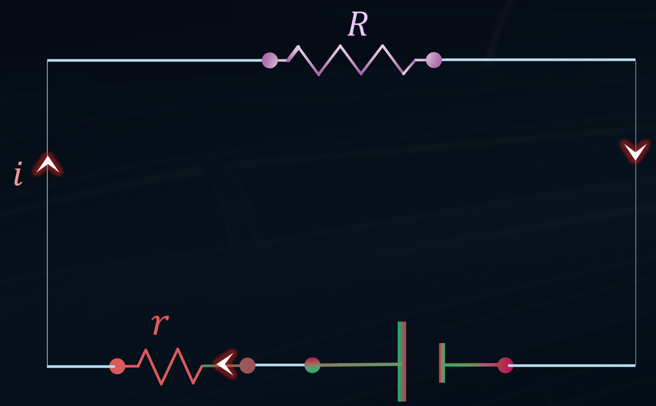
$r \rightarrow$ Resistance of rotating rod

Equivalent resistance of the circuit:

$R_e = R + r$

Current passing through the rod:

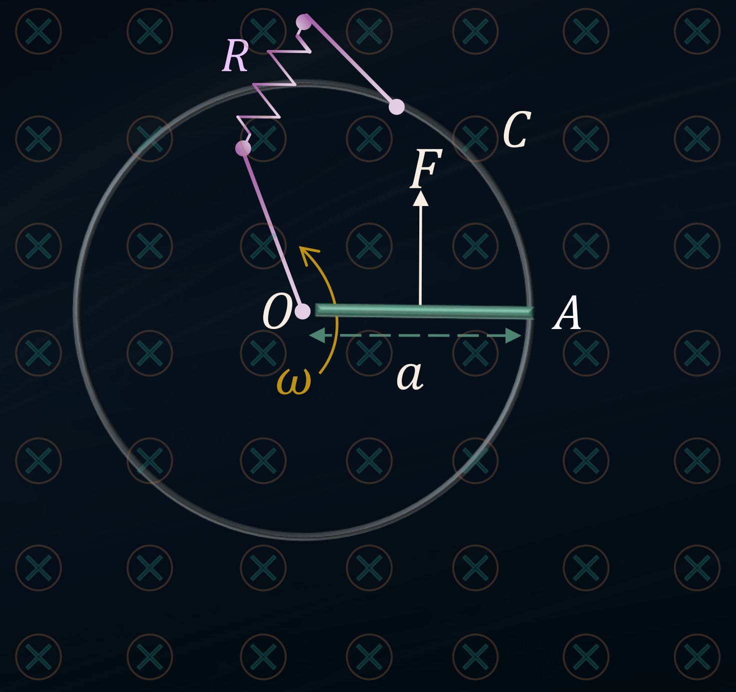
$$i = \frac{\varepsilon}{R_e} = \frac{B\omega a^2}{2(R+r)}$$



$$\varepsilon = \frac{B\omega a^2}{2}$$

$$\boxed{i = \frac{B\omega a^2}{2(R+r)}}$$

If a force "F" is applied on the mid-point of the rod so that the rod rotates with constant ω, find F ?



Solution

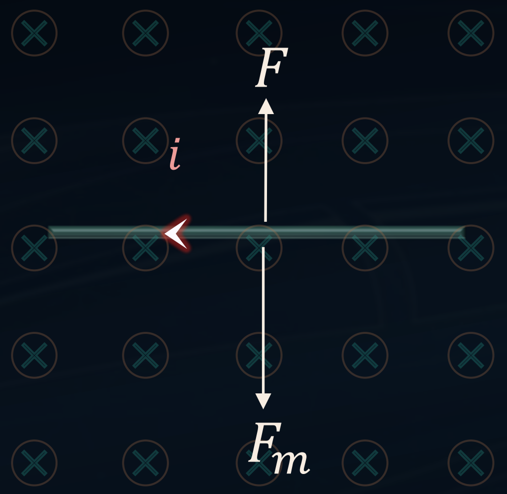
NEET

Given, $\omega = \text{Constant} \Rightarrow \alpha = 0 \Rightarrow \tau_{net} = 0$

As the net torque is zero, torque due to magnetic force and external fore is equal.

$$F_m . \frac{a}{2} = F . \frac{a}{2}$$

$$\Rightarrow F = F_m = iBa$$



$$F_m = iBl \sin 90° \Rightarrow iBa$$

Current, $i = \dfrac{\epsilon}{R_{circuit}} = \dfrac{\frac{B\omega a^2}{2}}{R + r}$

$$\Rightarrow F = \frac{B\omega a^2}{2(R+r)} Ba$$

$$\boxed{i = \frac{B\omega a^2}{2(R+r)}}$$

$$\boxed{F = \frac{B^2\omega a^3}{2(R+r)}}$$

ELECTRICAL COMPONENTS

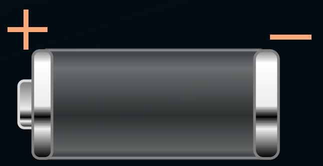
Classification of electrical components

Active electrical component

1) Active elements generate energy for any device. It is the core component to operate the device.

2) Active components control the charge flow in electrical or electronic circuits.

Example:



Battery

Passive electrical component

1) A passive element is an electrical component that does not generate power but instead dissipates, stores and/or releases it.

Example:

Resistor

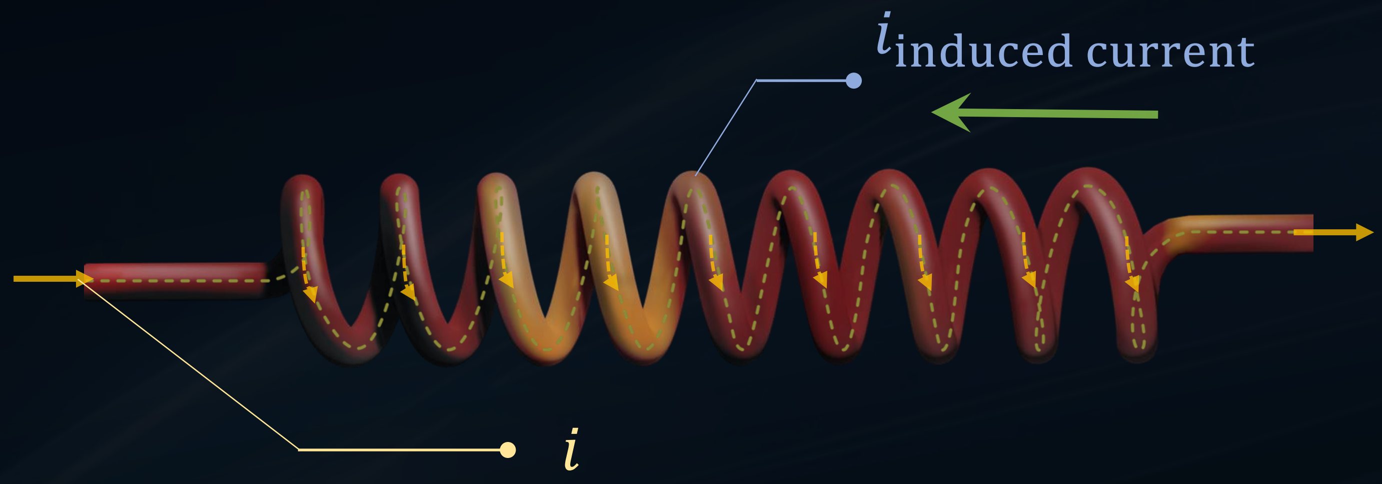
Capacitor

Inductor

SELF-INDUCTANCE

Consider current i is flowing through a conductor coil,

Case 1 : When i is increasing

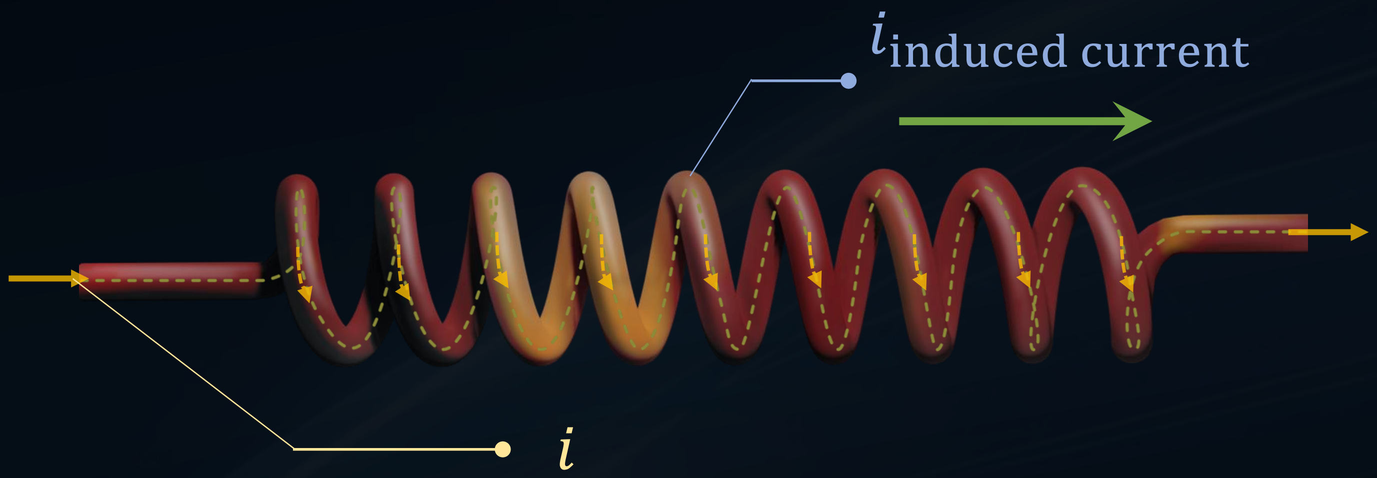


Induced current always opposes the growth in current.

SELF-INDUCTANCE

Consider current i is flowing through a conductor coil,

Case 2 : When i is decreasing



Induced current always supports the decaying current.

SELF-INDUCTANCE

1) Self-inductance is the property of the current-carrying coil that resists or opposes the change in current flowing through it.

2) This occurs mainly due to the self-induced emf produced in the coil.

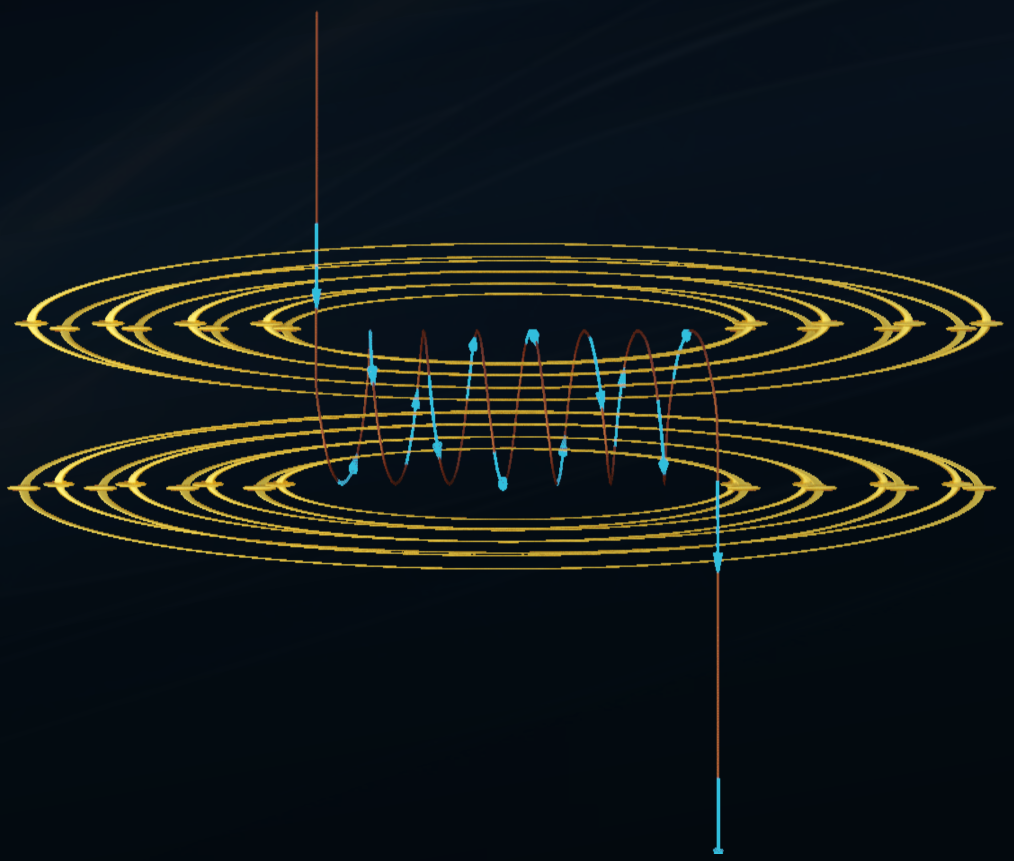
3) The self-induced emf in the coil will resist the rise of current when the current increases and vice versa.

4) This property is applicable for a time – varying current and not for the direct or steady current.

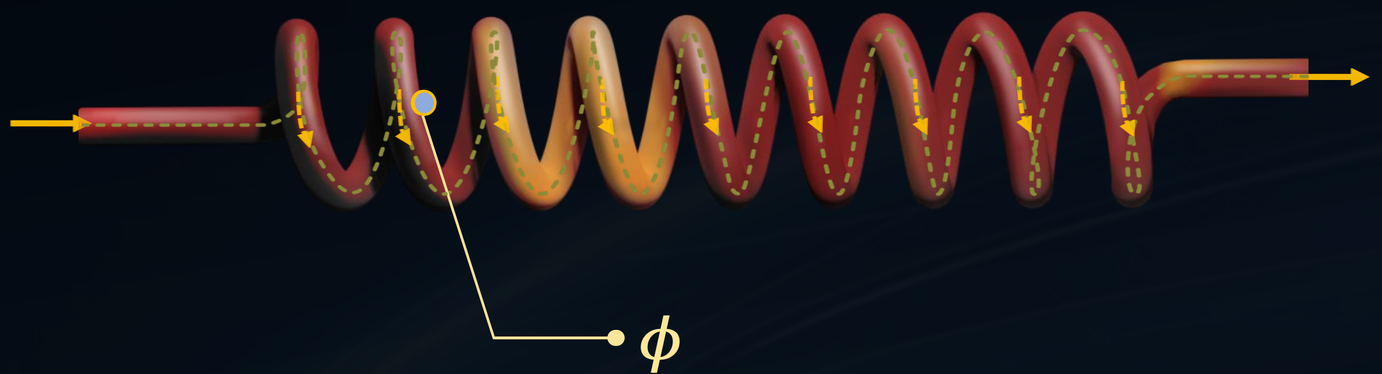
INDUCTANCE

The induced emf across a coil is directly proportional to the rate of change of current through it.

$$V \propto \frac{di}{dt}$$



COEFFICIENT OF SELF-INDUCTANCE



ϕ = Magnetic flux linked with one coil

ϕ_T = Total flux = $N\phi$

$\phi \propto i$

$\phi_T = Li$

$N\phi = Li$

$$\boxed{L = \frac{N\phi}{i}}$$

Coefficient of self-inductance

Unit = $\frac{\text{Weber}}{\text{Ampere}}$ Or Henry (H)

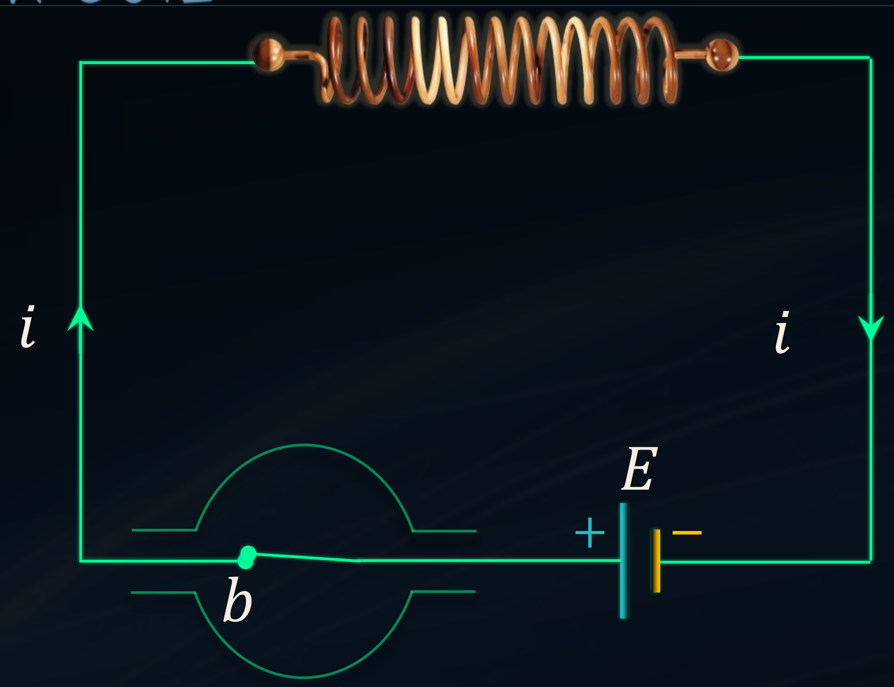
Magnetic Flux $(\phi) = [ML^2T^{-2}A^{-1}]$

Current $(i) = [A]$

$$L = \frac{[ML^2T^{-2}A^{-1}]}{[A]}$$

$$\boxed{L = [ML^2T^{-2}A^{-2}]}$$

INDUCED EMF IN A COIL



$$L = \frac{|\varepsilon|}{\left|\frac{di}{dt}\right|}$$

Induced EMF:

$$\varepsilon = -\frac{d\phi_T}{dt}$$

$$\varepsilon = -\frac{d(Li)}{dt}$$

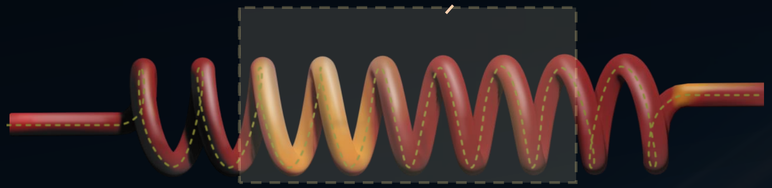
$$\varepsilon = -L\frac{di}{dt}$$

$$|\varepsilon| = \left|L\frac{di}{dt}\right|$$

If the rate of change current $\left(\frac{di}{dt}\right)$ is $1\ ampere/sec$, then the self inductance of a coil will be equal to the induced emf.

Where, $\frac{di}{dt}$ = Rate of change of current

FACTORS ON WHICH SELF-INDUCTANCE OF A COIL DEPENDS



Does, $L \propto \phi$ and $L \propto \frac{1}{i}$?

It depends on:

- area of cross-section of the coil.
- number of turns per unit length in the coil.
- length of the coil.
- permeability of the core material.

Find induced emf and $(V_A - V_B)$ in the given situation.

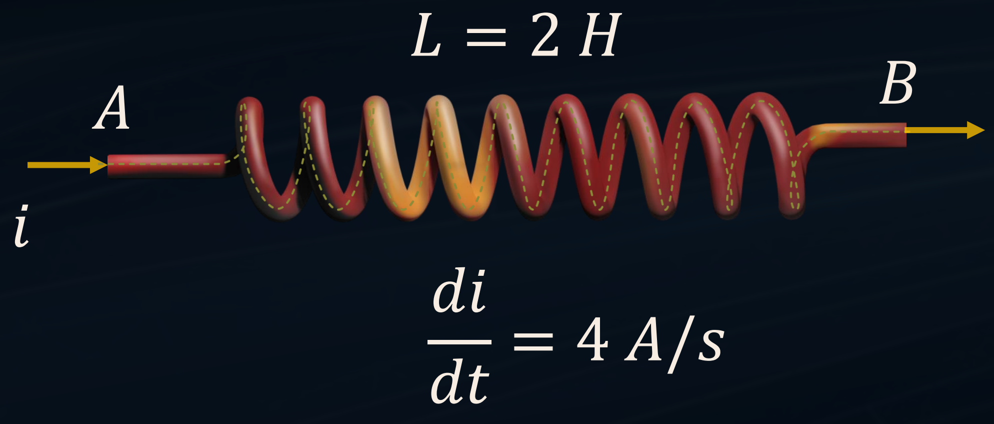
Solution

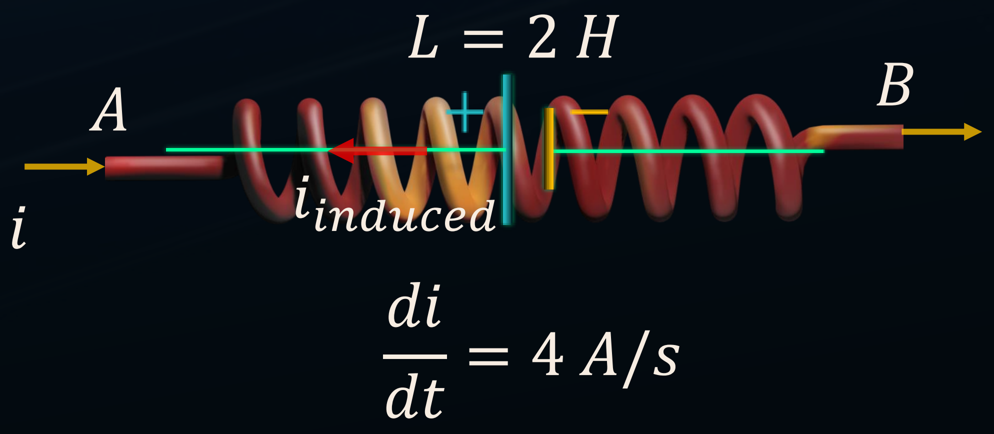
$$\varepsilon = L\left|\frac{di}{dt}\right| \Rightarrow \varepsilon = 8\ V$$

Here, $\frac{di}{dt} = 4\ A/s$ (increasing)

As the current is increasing, induced current will flow opposite to it i.e., from B to A. Thus, point A is at higher potential. And we know that potential difference magnitude is $8\ V$.

$$\therefore V_A - V_B = 8\ V$$





Find induced emf and $(V_P - V_Q)$ in the given situation.

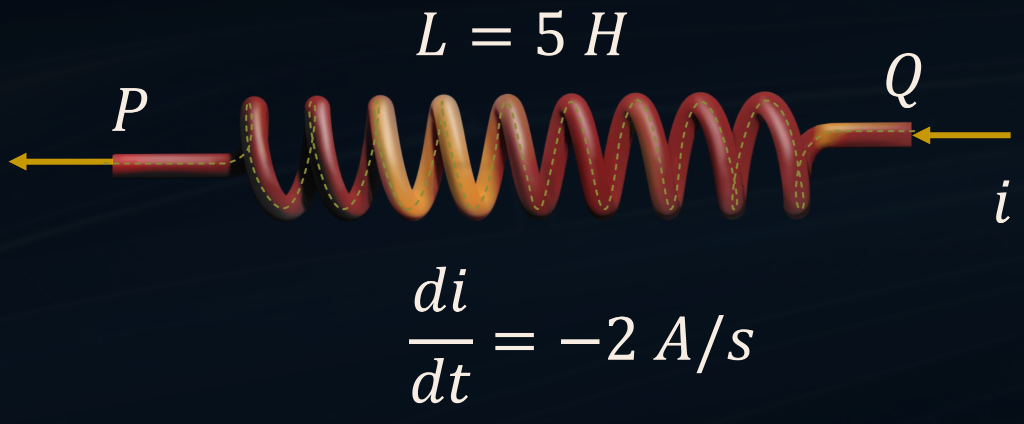
Solution

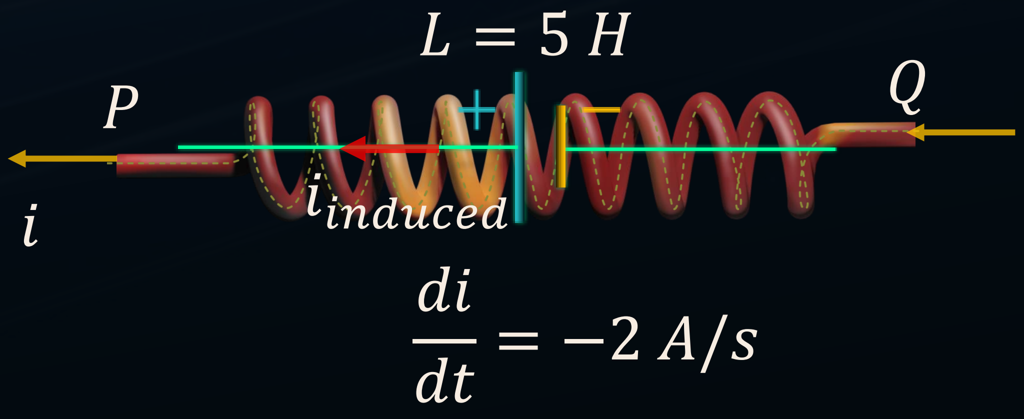
$$\varepsilon = L\left|\frac{di}{dt}\right| \Rightarrow \varepsilon = 10\ V$$

Here, $\frac{di}{dt} = -2\ A/s$ (decreasing)

As the current is decreasing, induced current will flow in same direction to it i.e., from Q to P. Thus, point P is at higher potential. And we know that potential difference magnitude is $10\ V$.

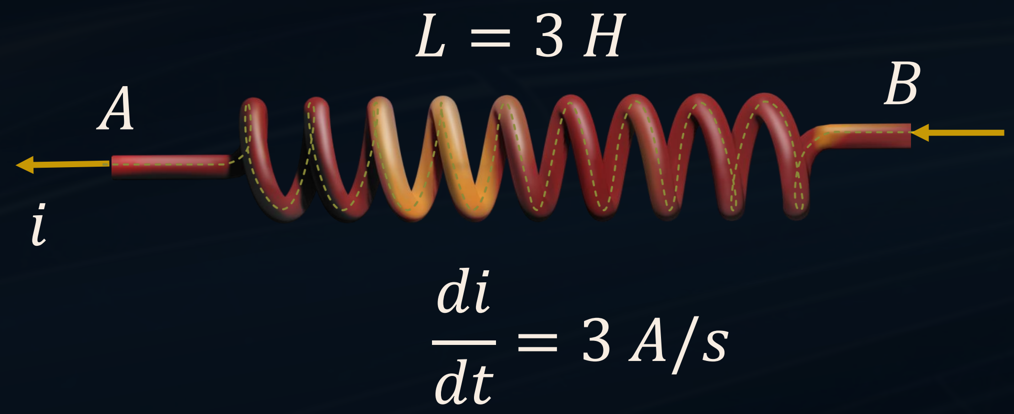
$\therefore V_P - V_Q = 10\ V$





Find induced emf and $(V_A - V_B)$.

Solution



Using KVL from B to A, we can write

$$V_B - L\frac{di}{dt} = V_A$$

$$V_B - 3(3) = V_A$$

$$\therefore V_B - V_A = 9\ V$$

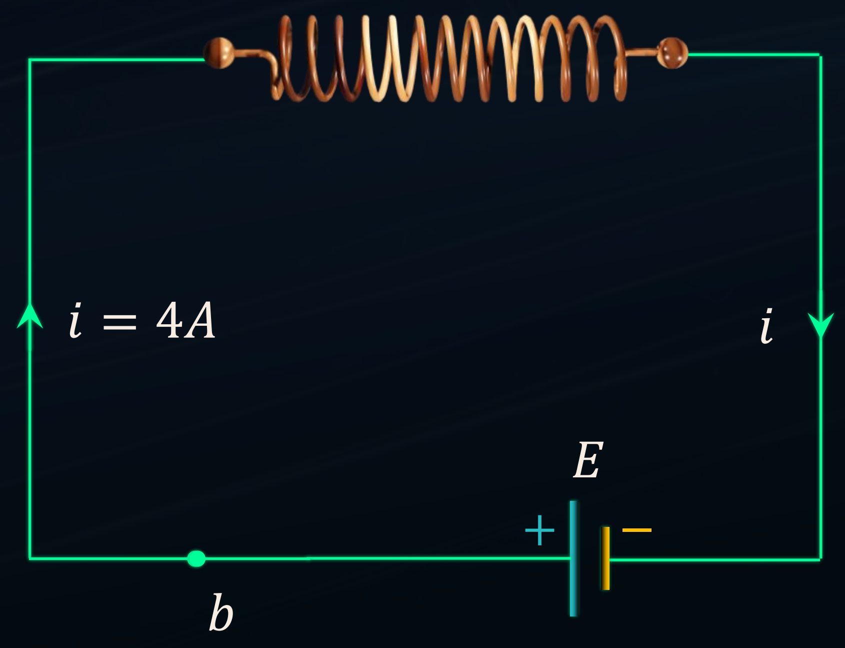
A long solenoid has 1000 turns. When a current of $4\ A$ flows through it, the magnetic flux linked with each turn of the solenoid is $4 \times 10^{-3}\ Wb$. The self-inductance of the solenoid is:

a) $2\ H$

b) $1\ H$

c) $4\ H$

d) $3\ H$



Solution NEET

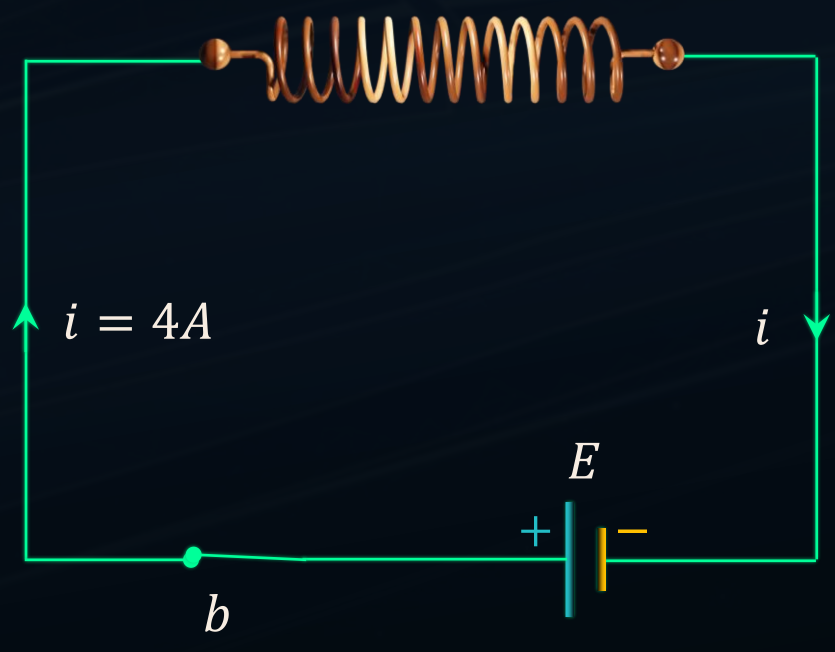
Given: $N = 1000;\ i = 4\ A;\ \phi = 4 \times 10^{-3}\ Wb$

Self Inductance, $\boxed{L = \dfrac{N\phi}{i}}$

$$L = \frac{1000 \times \cancel{4} \times 10^{-3}}{\cancel{4}}$$

$$L = 1000 \times 1 \times 10^{-3}$$

$$L = 1\ H$$



What is the self-inductance of a coil which produces $5\ V$ when the current changes from $3\ A$ to $2\ A$ in one millisecond?

a. $5000\ H$

b. $5\ mH$

c. $50\ H$

d. $5\ H$

Solution

NEET

Given: $e = 5\ V,\ \ t = 10^{-3}\ s,\ I_0 = 3\ A,\ \ I_1 = 2\ A$

Induced e.m.f., $\boxed{L = -\dfrac{\varepsilon}{dI/dt}}$

$$L = -\frac{\varepsilon}{\dfrac{(I_1 - I_o)}{t_1 - t_o}}$$

$$L = -\frac{5}{(2-3)/10^{-3}}$$

$$L = 5\ mH$$

For a coil of $L = 2\,mH$, current flowing through it is t^2e^{-t}. The time at which emf becomes zero is

a) $2\,sec$

b) $1\,sec$

c) $4\,sec$

d) $3\,sec$

Solution NEET

Given: $L = 2\,mH, i = t^2 e^{-t}$

Magnitude of Induced emf is,

$$\boxed{|e| = \left|L\frac{di}{dt}\right|}$$

Since $L \neq 0$, for emf to be zero, $\frac{di}{dt}$ should be zero.

$$\frac{di}{dt} = \frac{d}{dt}(t^2 e^{-t})$$

$$t^2(-1)e^{-t} + 2t\,e^{-t}$$

$$\Rightarrow 2t - t^2 = 0$$

$$\Rightarrow t(2-t) = 0 \quad \Rightarrow \quad t = 0 \text{ or } t = 2\,sec$$

$i = t^2 e^{-t}$, $u = t^2$, $v = e^{-t}$

$\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{di}{dt} = u.\frac{dv}{dt} + v.\frac{du}{dt}$

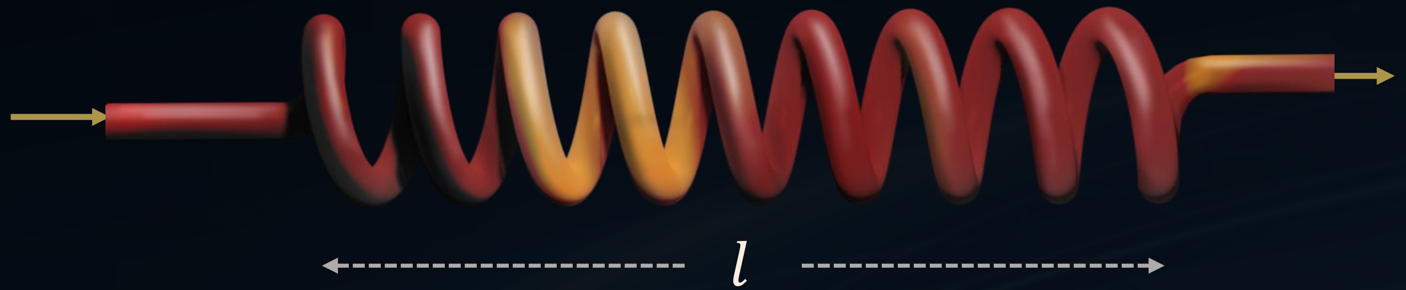
$= t^2(-1)e^{-t} + e^{-t}(2t)$

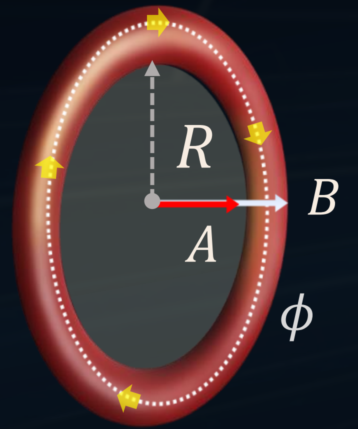
EMF is zero at 0 and 2 seconds.

Thus, option a is the correct answer.

COEFFICIENT OF SELF-INDUCTANCE FOR A LONG SOLENOID

BOARDS NEET





Magnetic Field inside the solenoid, $B = \mu_o ni$

Flux, $\phi = \vec{B}.\vec{A}$

$\implies BA\cos 0^o = BA$ [$\theta = 0°$ here as $\vec{B}$ and $\vec{A}$ are along same direction]

$\phi = \mu_o niA$ or $\phi = \mu_o ni\pi R^2$

Where,

l = Length of coil

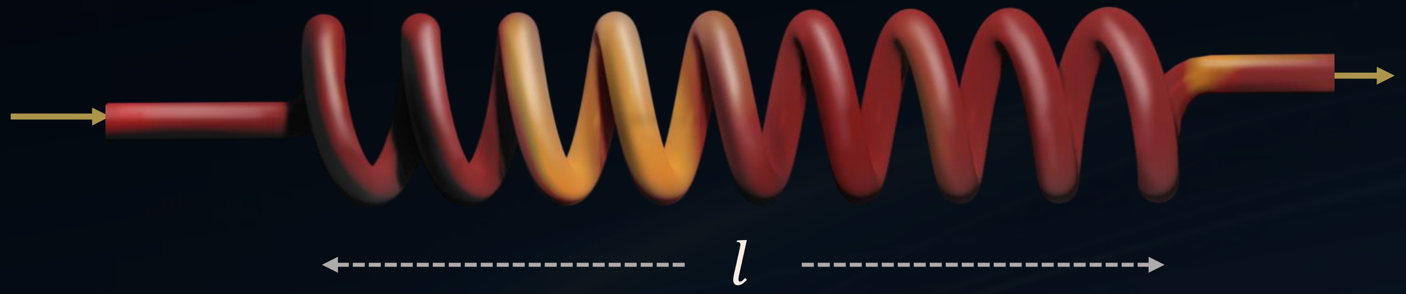
R = Radius of each coil

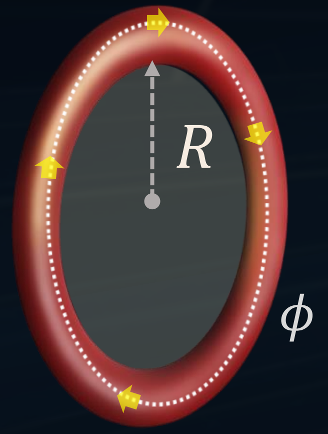
N = Total no. of turns

ϕ = Flux

n = No. of turns per unit length

COEFFICIENT OF SELF-INDUCTANCE FOR A LONG SOLENOID





$$\phi = \mu_o n i \pi R^2$$

Total Flux $(\phi_T) = N\phi = (nl)(\mu_o n i \pi R^2)$

$$\phi_T = \mu_0 n^2 \pi R^2 l i$$

$$L\cancel{i} = \mu_0 n^2 \pi R^2 l\cancel{i}$$

$$\boxed{L = \mu_0 n^2 \pi R^2 l}$$

$\because n = \dfrac{N}{l}$

$\because \phi = Li$

If the number of turns per unit length of a coil of solenoid is doubled, the self-inductance of solenoid will

a. remain unchanged

b. be halved

c. be doubled

d. become four times

Solution NEET

Coefficient of self-inductance, $\boxed{L = \mu_0 n^2 A l}$ A = Area of the solenoid

As, $L \propto n^2$

$$\frac{L_2}{L_1} = \left(\frac{n_2}{n_1}\right)^2$$

Given: $n_2 = 2n_1$

$$\frac{L_2}{L_1} = \left(\frac{2n_1}{n_1}\right)^2$$

$$L_2 = 4L_1$$

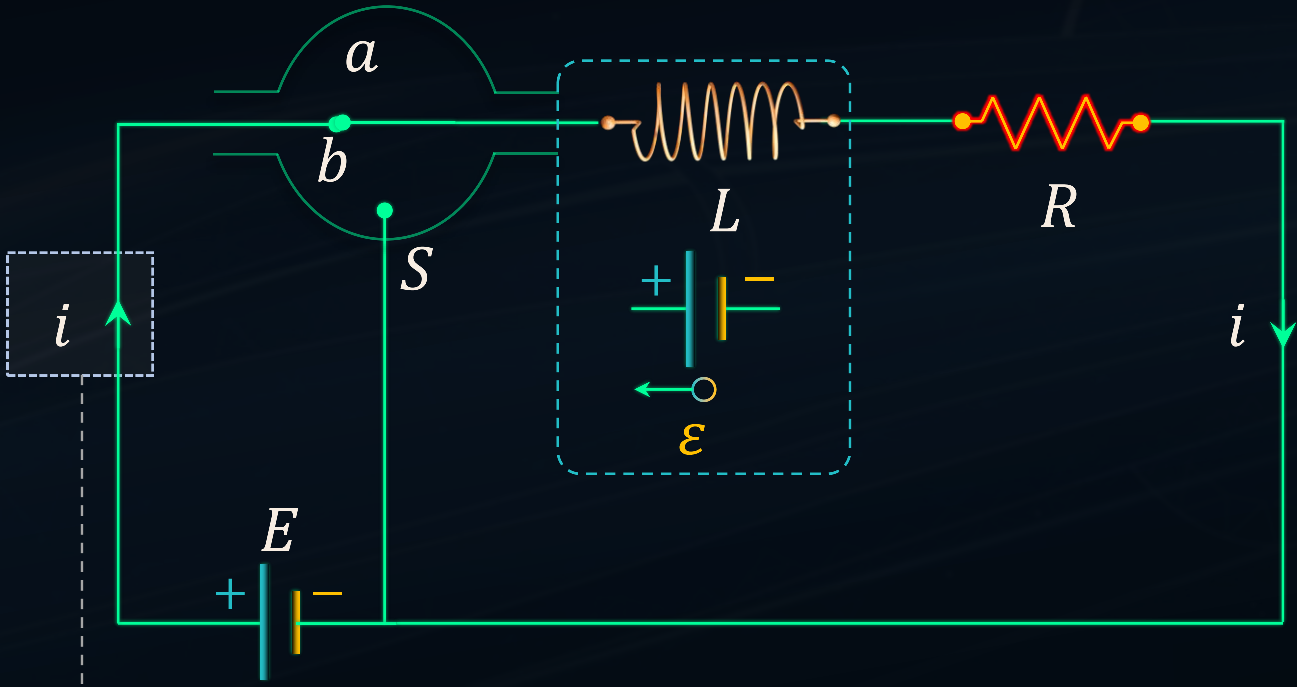
Thus, option d is the correct answer.

GROWTH OF CURRENT IN L-R CIRCUIT

$t = 0$

Switch $(a \to b)$

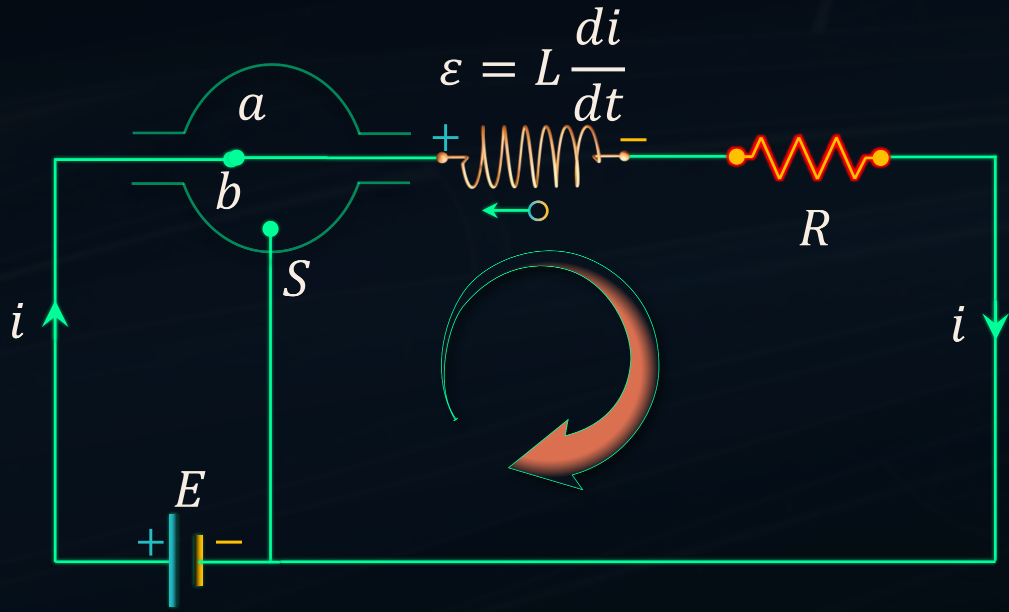
$$\left[i \neq \frac{E}{R}\right]_{t=0}$$



The current in the circuit does not attain the maximum steady state value (E/R) at once because the induced emf ε produced across the inductor opposes the growth of current.

GROWTH OF CURRENT IN L-R CIRCUIT

$t = 0$ @t Applying Kirchhoff's law:



$$+E - \frac{Ldi}{dt} - iR = 0 \Rightarrow E - \frac{Ldi}{dt} = iR$$

$$\int_0^t \frac{dt}{L} = \int_0^i \frac{di}{E - iR}$$

$$\frac{t}{L} = -\frac{1}{R}[\log_e(E - iR)]_0^i$$

$$\left(\because \int \frac{dx}{a - bx} = -\frac{1}{b}(\log_e(a - bx))\right)$$

$$\frac{-Rt}{L} = \log_e(E - iR) - \log_e(E)$$

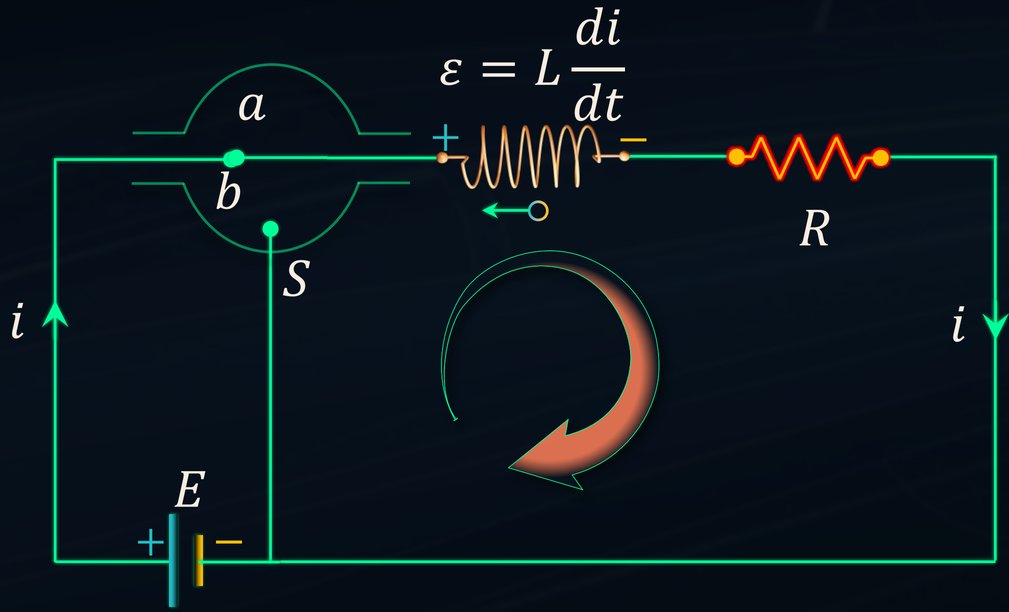
$\log_e a - \log_e b$
$= \log_e \frac{a}{b}$

$$\frac{-Rt}{L} = \log_e\left[1 - \frac{iR}{E}\right]$$

GROWTH OF CURRENT IN L-R CIRCUIT

$t = 0$

$@t$ Applying Kirchhoff's law:



$$\frac{-Rt}{L} = \log_e\left[1 - \frac{iR}{E}\right]$$

$$\frac{iR}{E} = \left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]$$

$$\boxed{i = \frac{E}{R}\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]}$$

GROWTH OF CURRENT IN L-R CIRCUIT

$t = 0$

$\left[i \neq \frac{E}{R}\right]_{t=0}$

$@t$

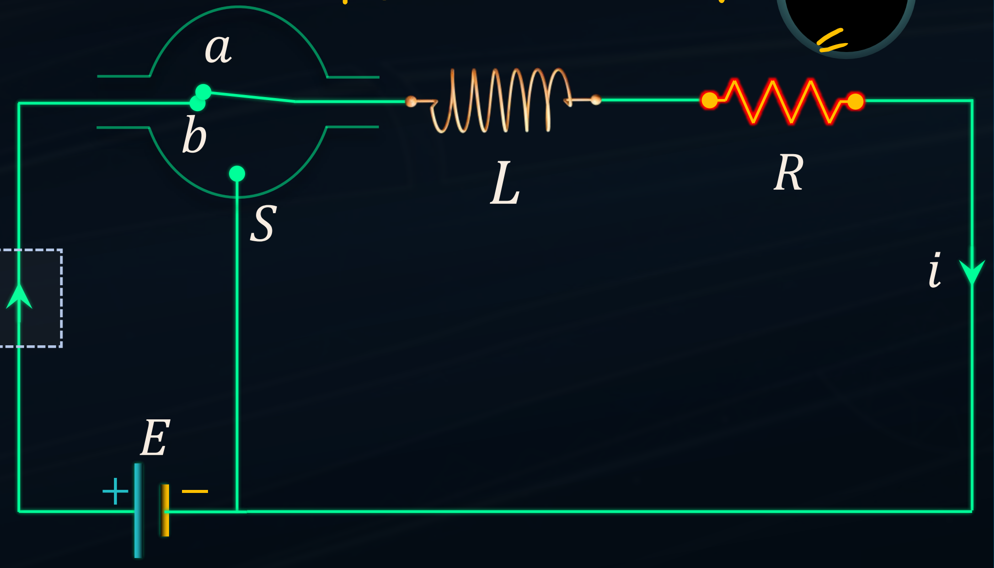
$$i = \frac{E}{R}\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]$$

$i = i_{max.}$ or $i_0 = E/R$

$$\boxed{i = i_0\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]}$$

$t \to \infty$

$i_{max} = i_0 = \frac{E}{R}$



$$\boxed{i = i_0\left[1 - e^{-\frac{Rt}{L}}\right]}$$

GROWTH OF CURRENT IN L-R CIRCUIT

$t = 0$

$$\left[i \neq \frac{E}{R}\right]_{t=0}$$

open

$@t$

$$i = \frac{E}{R}\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]$$

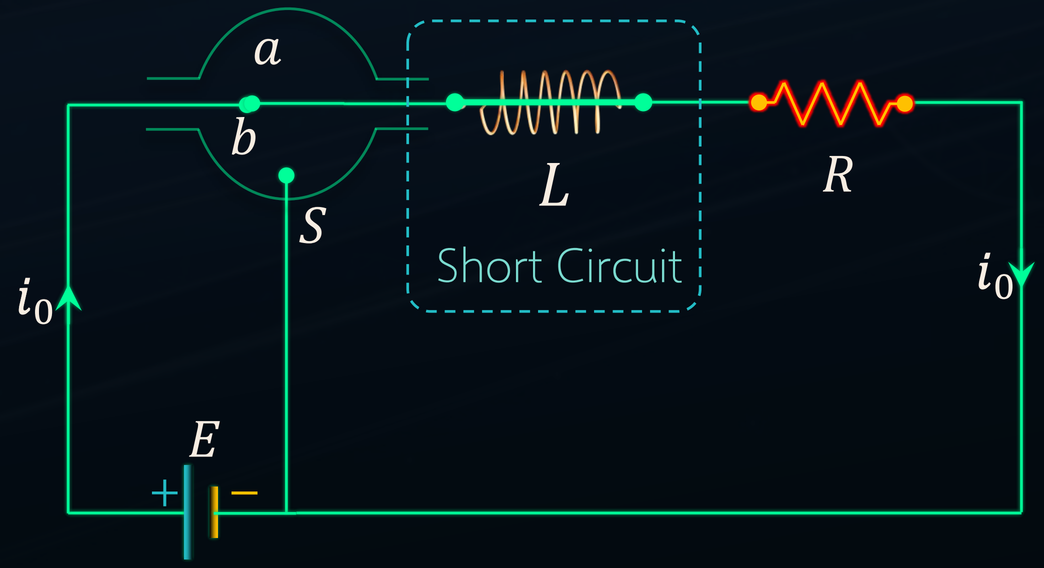
$t \to \infty$

short

$$i = i_{\text{max.}} \text{ or } i_0 = E/R$$

At steady state ($t \to \infty$):

- The rate of increase of current becomes zero.
- Current in the circuit does not depend on self inductance.



At steady state ($t \to \infty$)

GROWTH OF CURRENT IN L-R CIRCUIT

$t = 0$

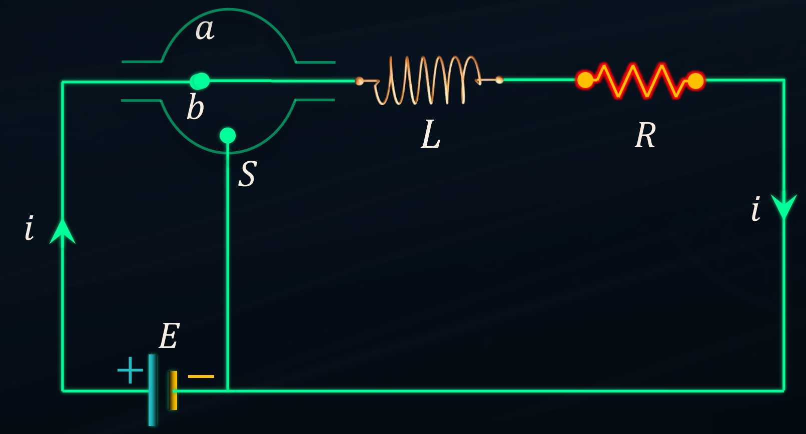
$\left[i \neq \frac{E}{R}\right]_{t=0}$

$@t$

$i = \frac{E}{R}\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]$

$t \to \infty$

$i = i_{max}$ or $i_0 = E/R$



Time constant of an $L - R$ circuit (τ):

$\tau = L/R$

$i = i_0\left[1 - e^{(-t/\tau)}\right]$

$@t = \tau$

$i = i_0(1 - e^{(-1)})$

$i = 0.63 \times i_0$

Time constant of an $L - R$ circuit (τ) is the time taken by the current to grow from *zero* to $0.632\ i_0$ or 63.2% of its final steady value.

GROWTH OF CURRENT IN L-R CIRCUIT

$t = 0$

$$\left[i \neq \frac{E}{R}\right]_{t=0}$$

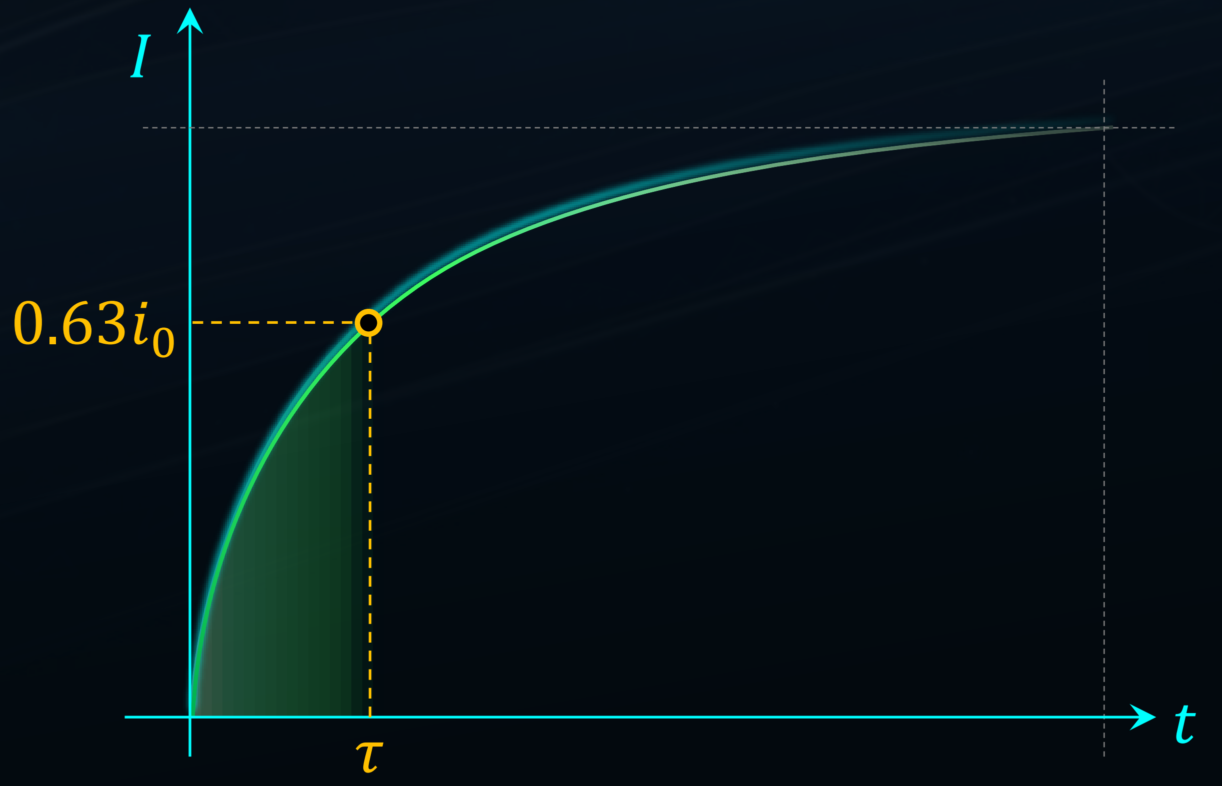
$@t$

$$i = \frac{E}{R}\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]$$

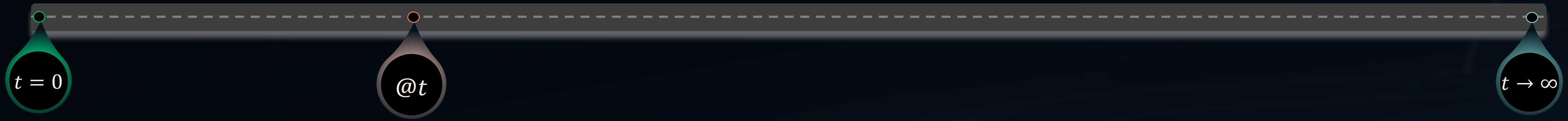
$t \to \infty$

$i = i_{max}$ or $i_0 = E/R$

- In one time constant, the current i is increased by 63% of its steady state value.
- It takes around 5 time constants, to reach steady state.



GROWTH OF CURRENT IN L-R CIRCUIT



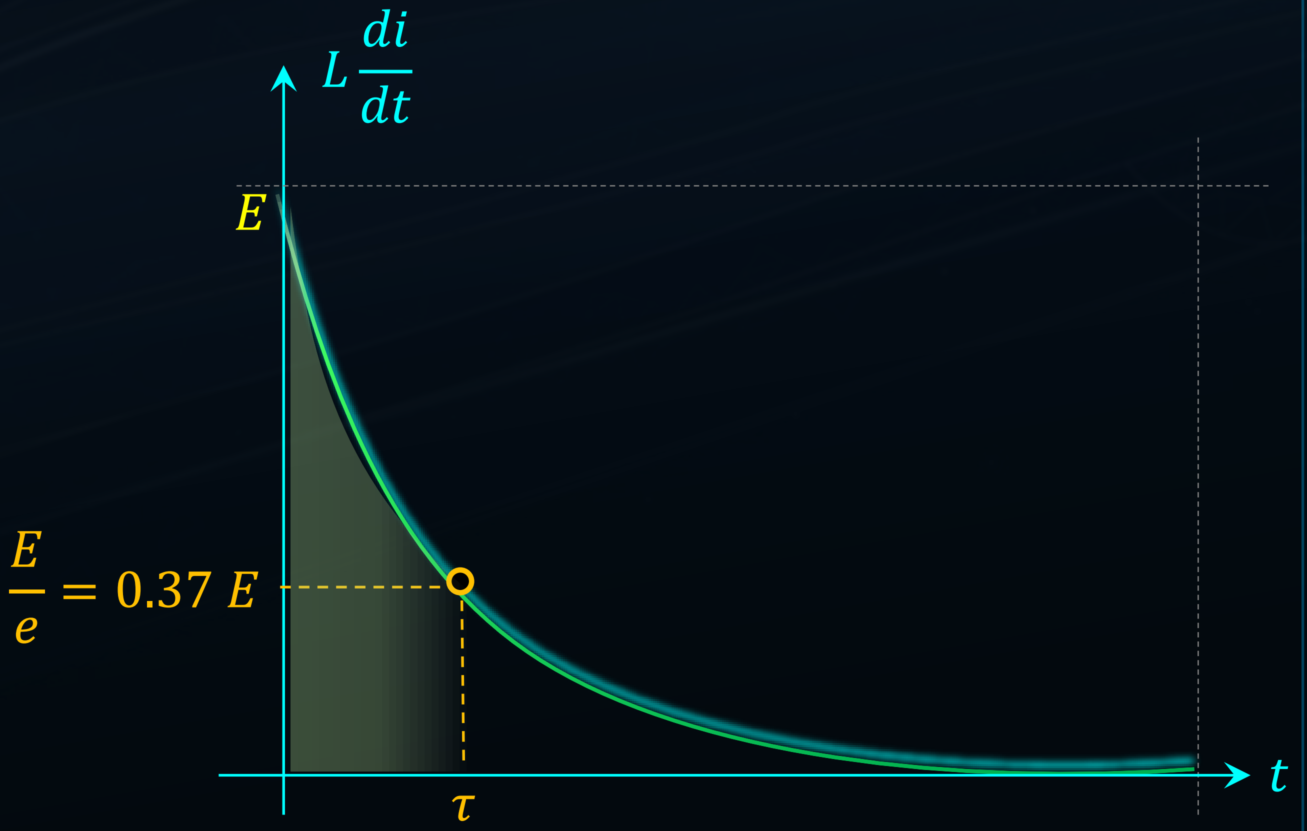
$$i = \frac{E}{R}\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]$$

Differentiating, $$\frac{di}{dt} = \frac{E}{R}\left(0 - \left(-\frac{R}{L}\right)e^{-\left(\frac{Rt}{L}\right)}\right)$$

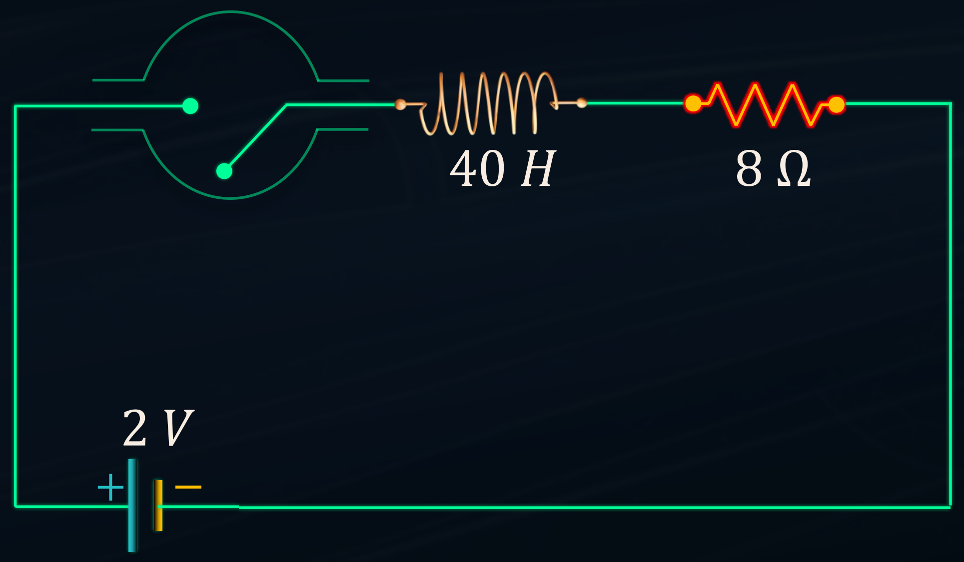
$$\boxed{L\frac{di}{dt} = E.e^{-\left(\frac{Rt}{L}\right)}}$$

At $t = 0$, $L\frac{di}{dt} = E$

In one time constant, the emf is decreased by 63% from its initial value.



A coil of inductance $40\,H$ is connected in series with a resistance of $8\,\Omega$ and the combination is joined to the terminals of a $2\,V$ battery. The time constant of the circuit is



a) $40\ seconds$

b) $20\ seconds$

c) $8\ seconds$

d) $5\ seconds$

Solution NEET

Given:

$L = 40\ H \mid R = 8\ \Omega \mid V = 2\ volt$

Time constant $(\tau) = L/R$

$\tau = 40/8 = 5\ seconds$

Thus, option d is the correct answer.

$$\boxed{\begin{aligned} &* \; i = i_0\left[1 - e^{-\frac{t}{\tau}}\right] \\ &* \; L\frac{di}{dt} = E\, e^{-\frac{t}{\tau}} \end{aligned}}$$

$* \; \tau = \frac{L}{R}$

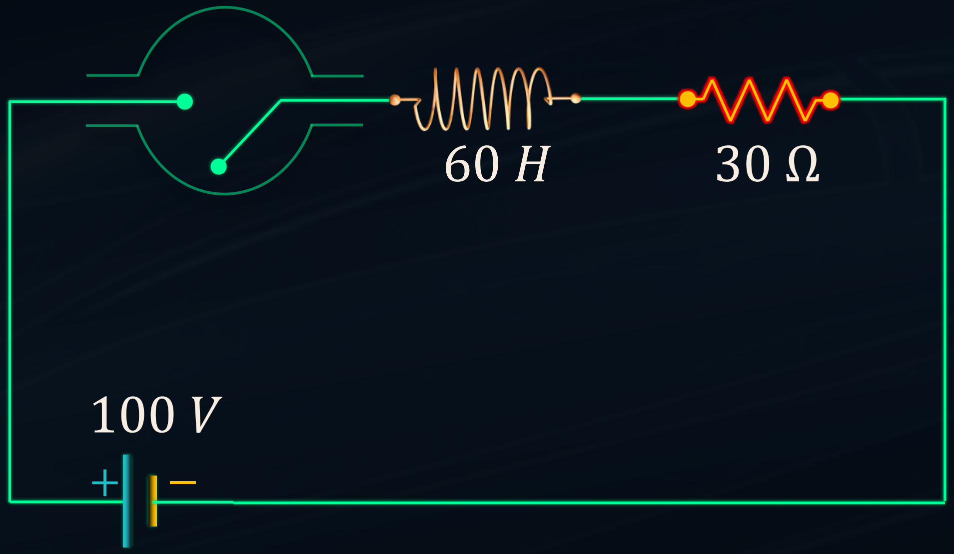
A solenoid has an inductance of $60\ H$ and a resistance of $30\ ohm$. It is connected to a $100\ volt$ battery. How long will it take for the current to reach $(e-1)/e \approx 63.2\ \%$ of its final value

a) $1\ second$

b) $2\ seconds$

c) $e\ seconds$

d) $2e\ seconds$



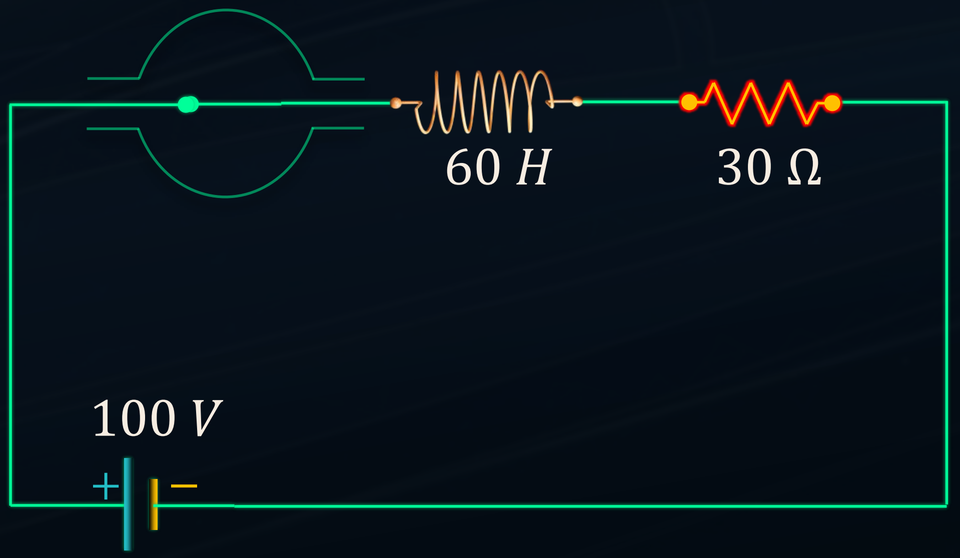
Solution NEET

$$i = i_0\left[1 - e^{(-t/\tau)}\right]$$

For current to reach $(e-1)/e \approx 63.2\ \%$ of final value,

$$i_0(e-1)/e = i_0\left[1 - e^{(-t/\tau)}\right]$$

$$i_0[1 - e^{(-1)}] = i_0\left[1 - e^{(-t/\tau)}\right]$$



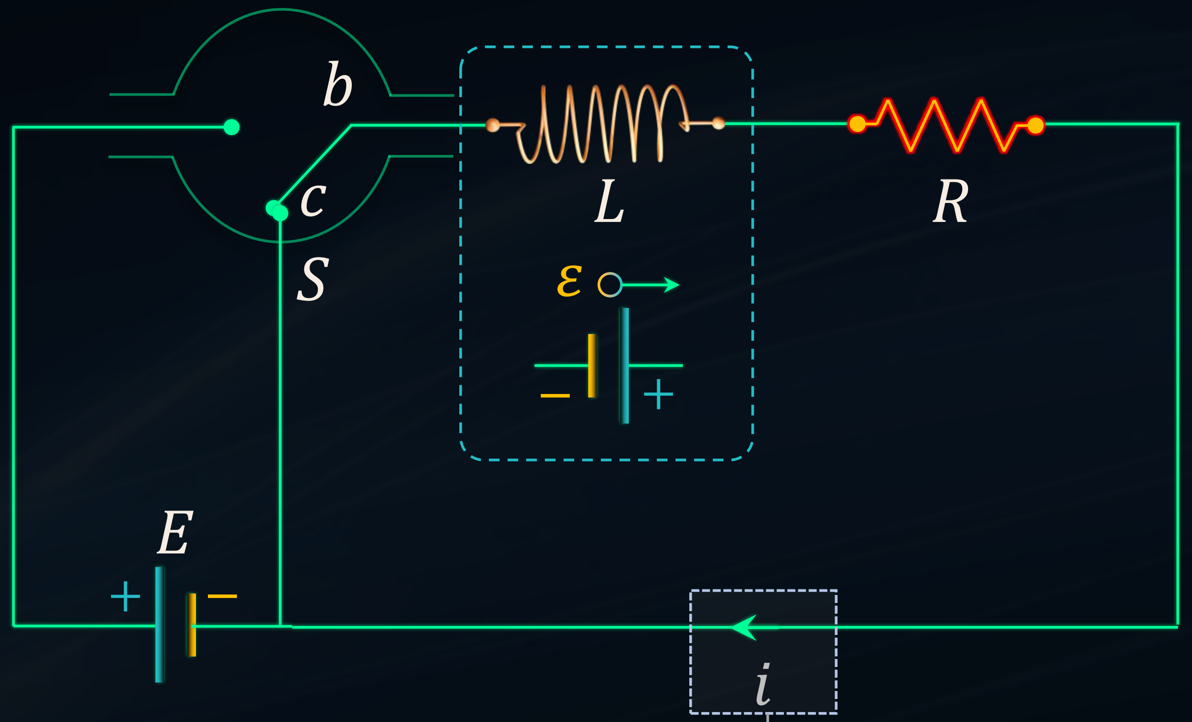
Comparing both side:

$$-\frac{t}{\tau} = -1 \Rightarrow t = \tau$$

$$\tau = \frac{L}{R} = \frac{60}{30} = 2\ sec$$

Thus, option b is the correct answer.

DECAY OF CURRENT IN L-R CIRCUIT

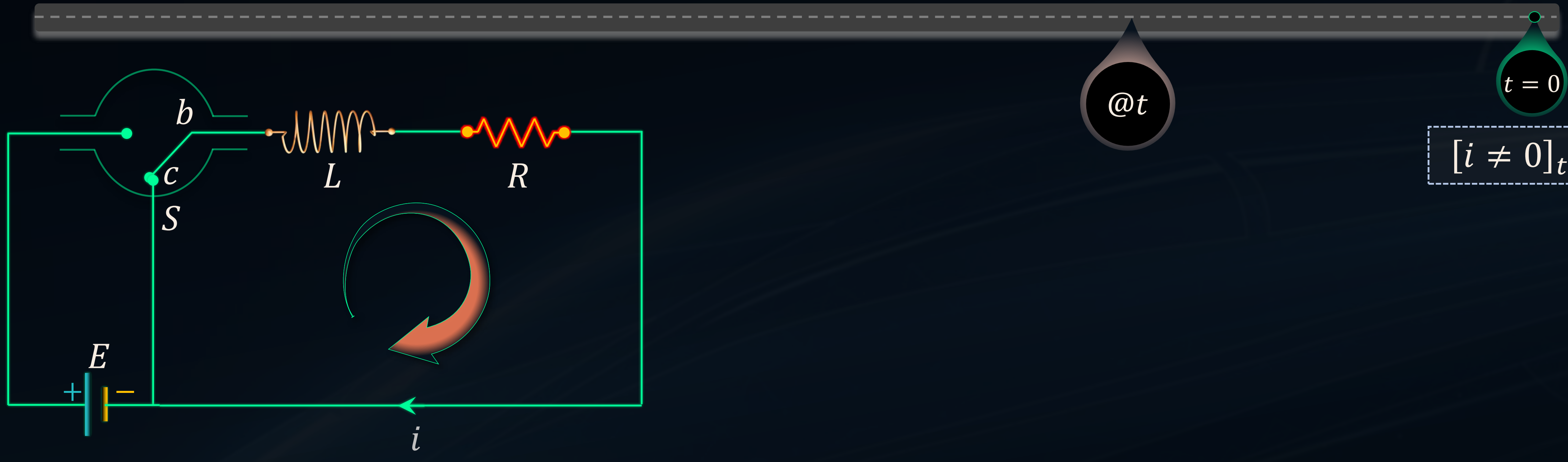


Switch ($b \to c$)

$[i \neq 0]_{t=0}$

The current does not suddenly fall to zero, the inductor will oppose the decay of current.

DECAY OF CURRENT IN L-R CIRCUIT



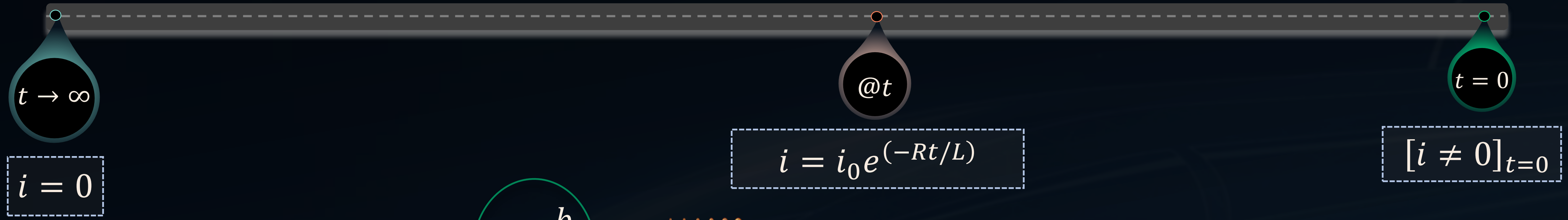
$$[i \neq 0]_{t=0}$$

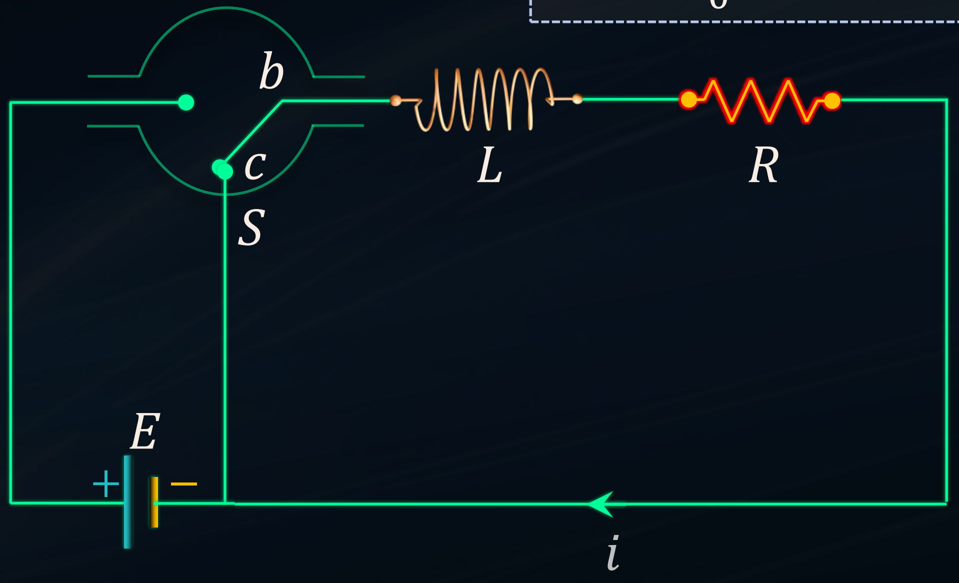
Applying Kirchhoff's law:

$$-\frac{Ldi}{dt} = iR \Rightarrow \int_{i_0}^{i} \frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

$$\Rightarrow \log_e\left(\frac{i}{i_0}\right) = -\frac{Rt}{L} \Rightarrow \boxed{i = i_0 e^{(-Rt/L)}}$$

DECAY OF CURRENT IN L-R CIRCUIT

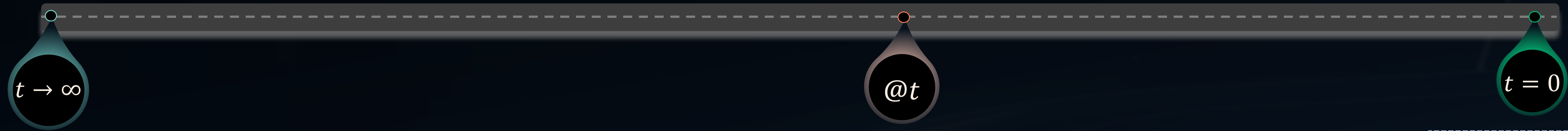




At steady state ($t \to \infty$):

$i = 0$, the current in the circuit will become zero.

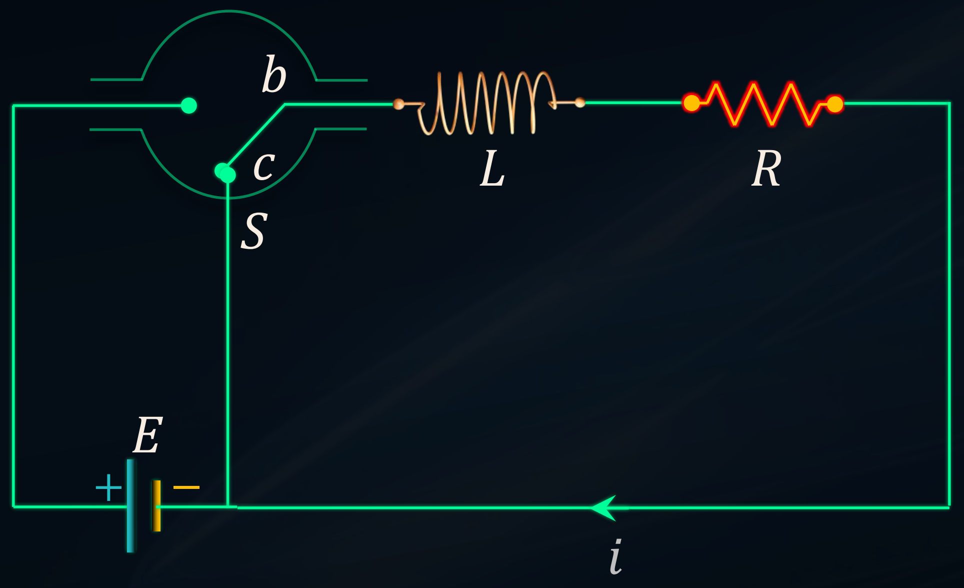
DECAY OF CURRENT IN L-R CIRCUIT



$i = 0$

$i = i_0 e^{(-Rt/L)}$

$[i \neq 0]_{t=0}$



Time constant of an $L - R$ circuit (τ):

$$i = i_0\left[e^{(-t/\tau)}\right] \quad (\because \tau = L/R)$$

@$t = \tau$

$$i = i_0(e^{(-1)})$$

$$i = 0.37 \times i_0$$

Time taken by the current to decay from i_0 to $0.37i_0$ or 37% of its initial steady value.

DECAY OF CURRENT IN L-R CIRCUIT

$t \to \infty$

$i = 0$

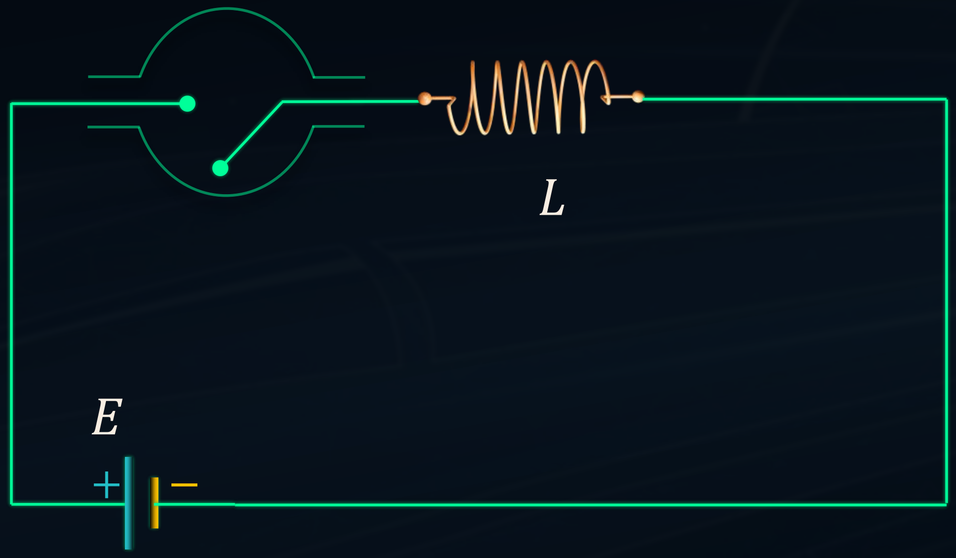
@t

$i = i_0 e^{(-Rt/L)}$

$t = 0$

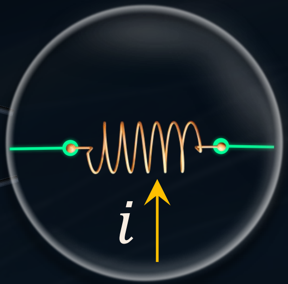
$[i \neq 0]_{t=0}$

MAGNETIC ENERGY STORED IN AN INDUCTOR



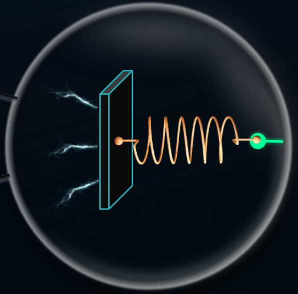
Switch ON

Current through inductor
$(0 \rightarrow i_0)$



Back EMF $\varepsilon = -\dfrac{Ldi}{dt}$

Inductor opposes
the flow of current



Battery needs to work
against back EMF

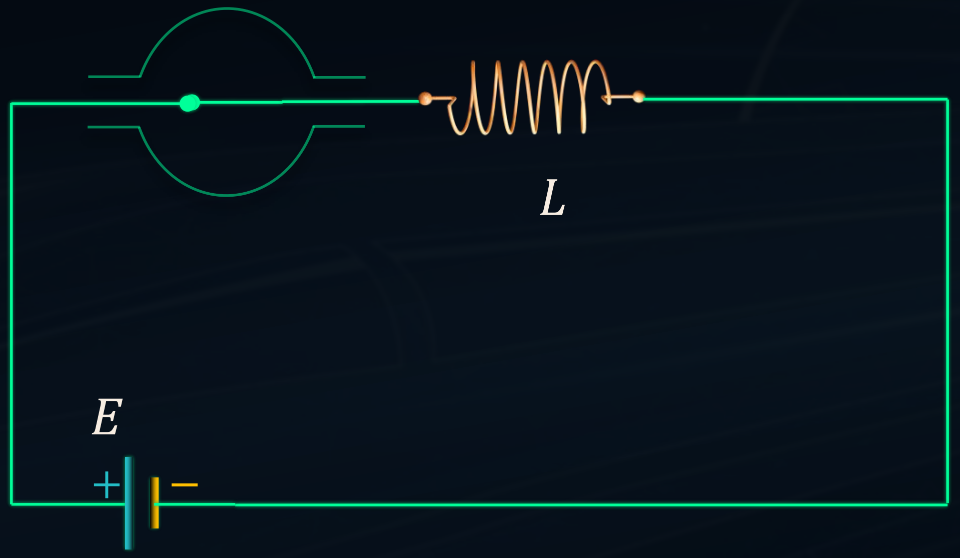
MAGNETIC ENERGY STORED IN AN INDUCTOR

Rate of work done is given by:

$$\frac{dW}{dt} = -\varepsilon i \Rightarrow dW = -\varepsilon i dt$$

$$dW = -\left(-\frac{Ldi}{dt}\right) idt \Rightarrow dW = iLdi$$

$$\int_0^W dW = \int_0^i iLdi \Rightarrow \boxed{W = \frac{1}{2}Li^2}$$



This energy is stored in the magnetic field generated in the inductor due to the flow of current.

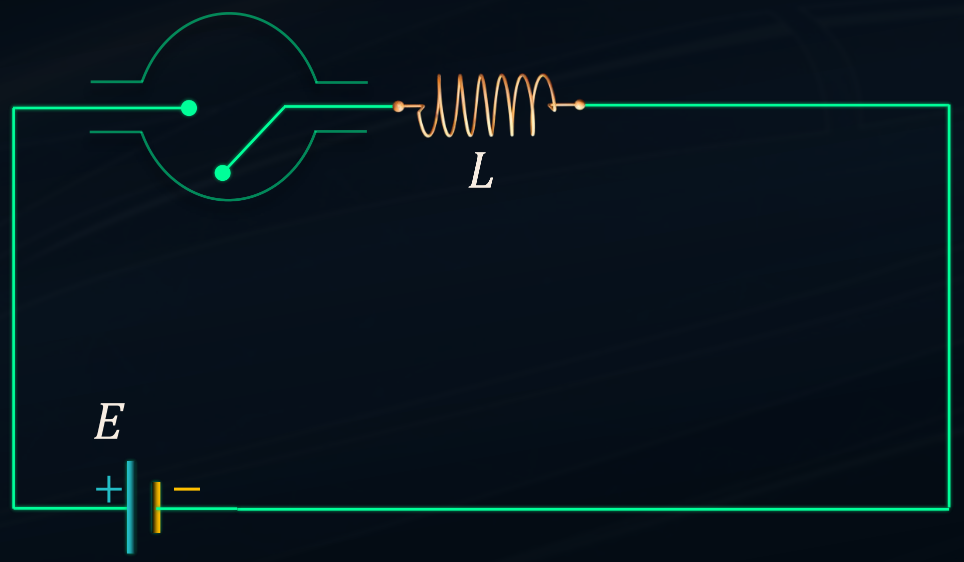
A magnetic potential energy stored in a certain inductor is $25\ mJ$, when the current in the inductor is $60\ mA$. The value of inductance is......

a) $1.389\ H$

b) $0.138\ H$

c) $13.89\ H$

d) $138.88\ H$



Solution NEET

$$\boxed{W = \frac{1}{2}Li^2}$$

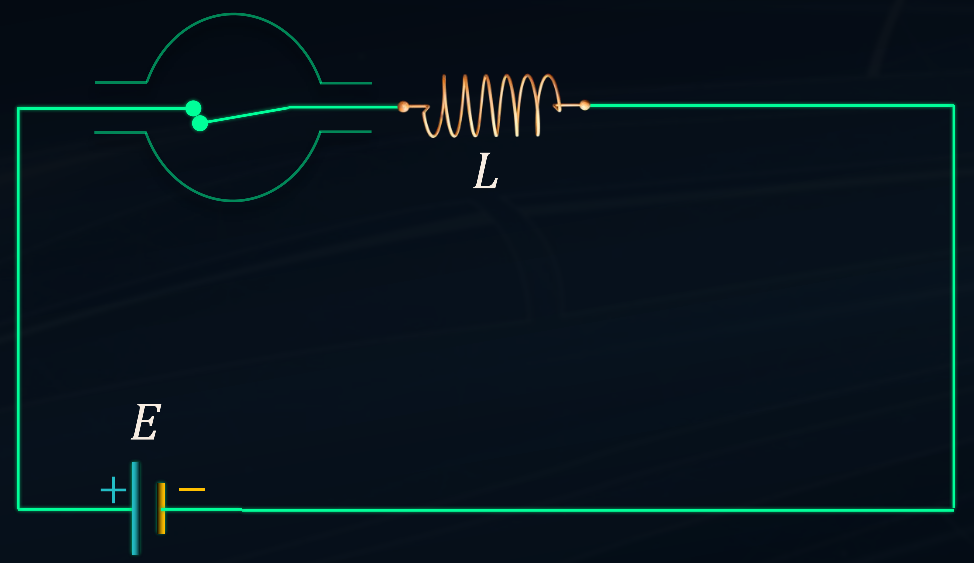
$$25 \times 10^{-3}\,J = \frac{1}{2} \times L \times (60 \times 10^{-3}\,A)^2$$

$$L = \frac{2 \times 25 \times 10^{-3}}{60 \times 60 \times 10^{-6}}$$

$$L = \frac{500}{36} = \frac{125}{9}$$

$$L = 13.89\,H$$

Thus, option c is the correct answer.



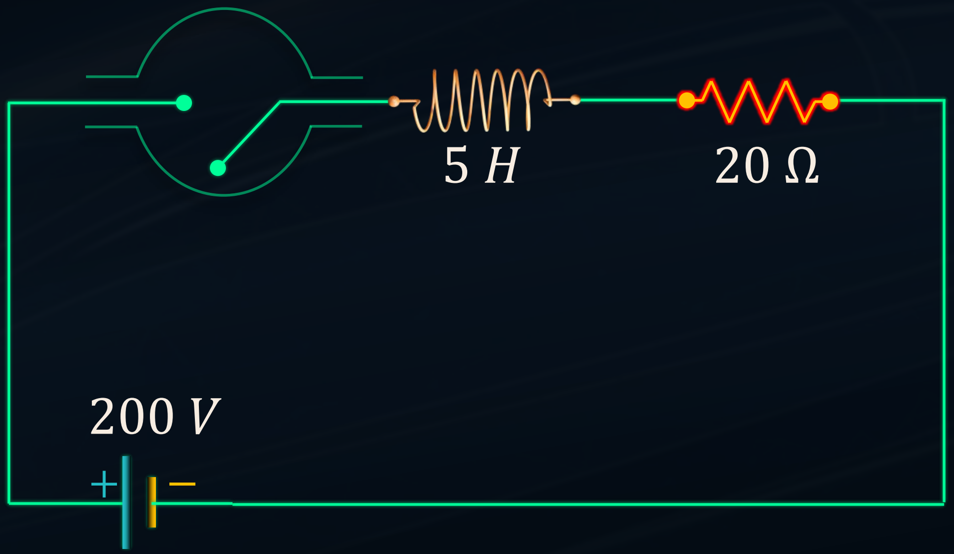
A coil of resistance of $20\ \Omega$ and inductance $5\ H$ has been connected to a $200\ V$ battery. The maximum energy in the coil is:

a) $250\ J$

b) $125\ J$

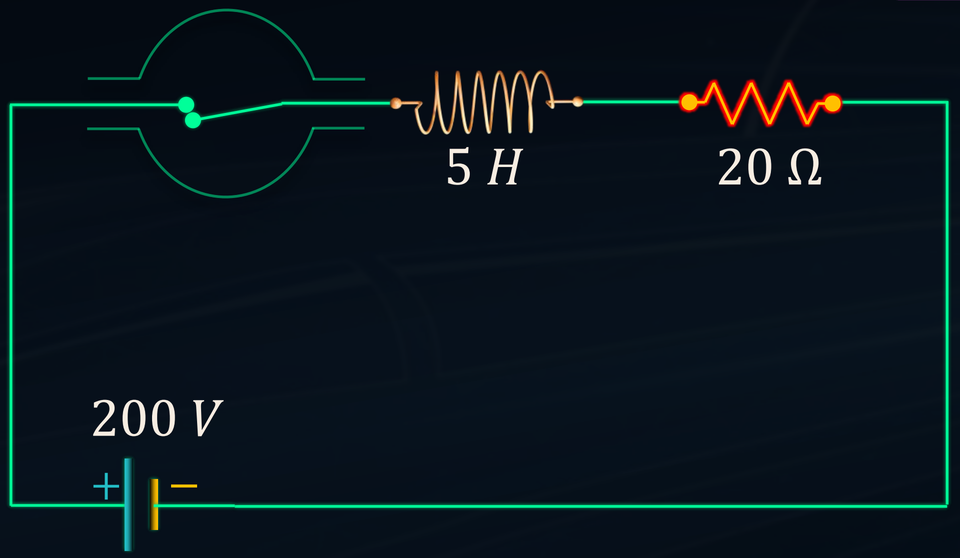
c) $500\ J$

d) $100\ J$



Solution NEET

Maximum energy will be at maximum current (i_0)



$$E = i_0 R \Rightarrow i_0 = \frac{200}{20} = 10\ A$$

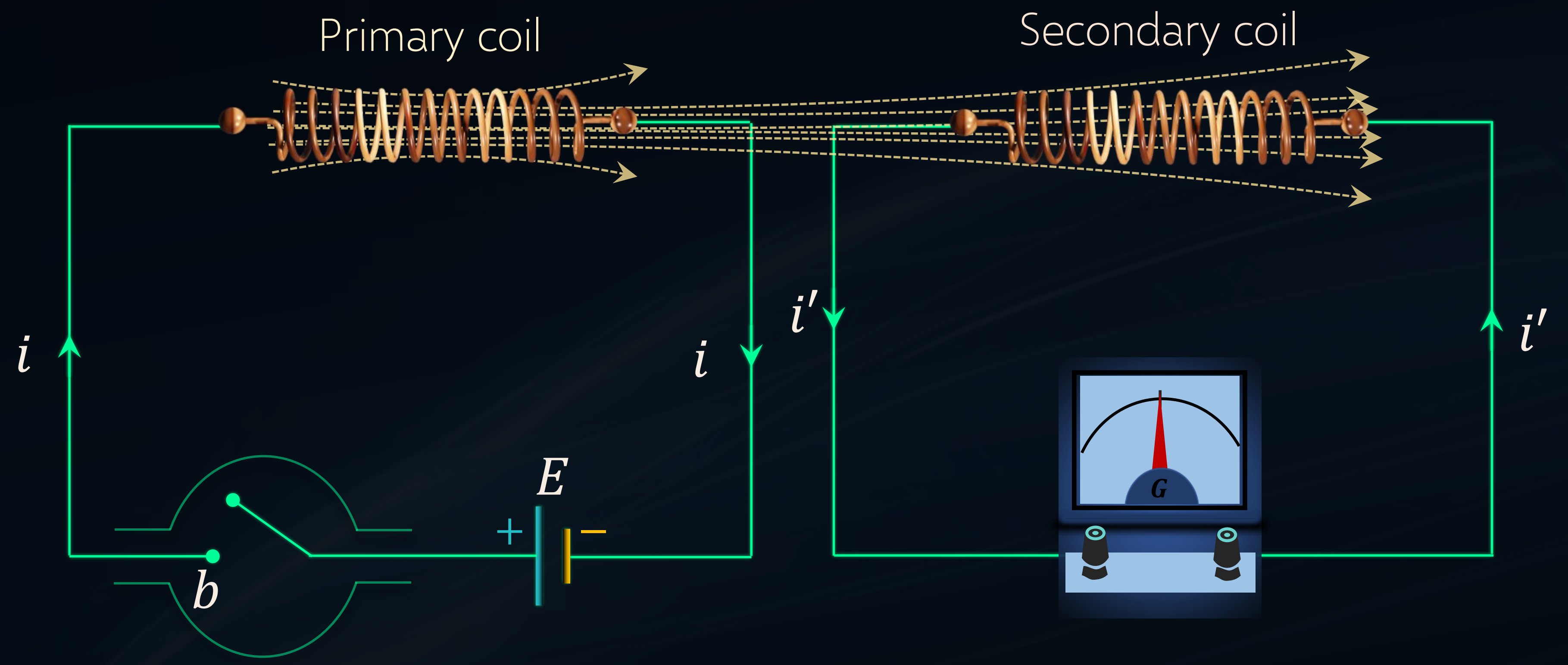
$$\boxed{W = \frac{1}{2} L i^2}$$

$$W = \frac{1}{2} \times 5 \times 10^2 = \frac{500}{2}$$

$$W = 250\ J$$

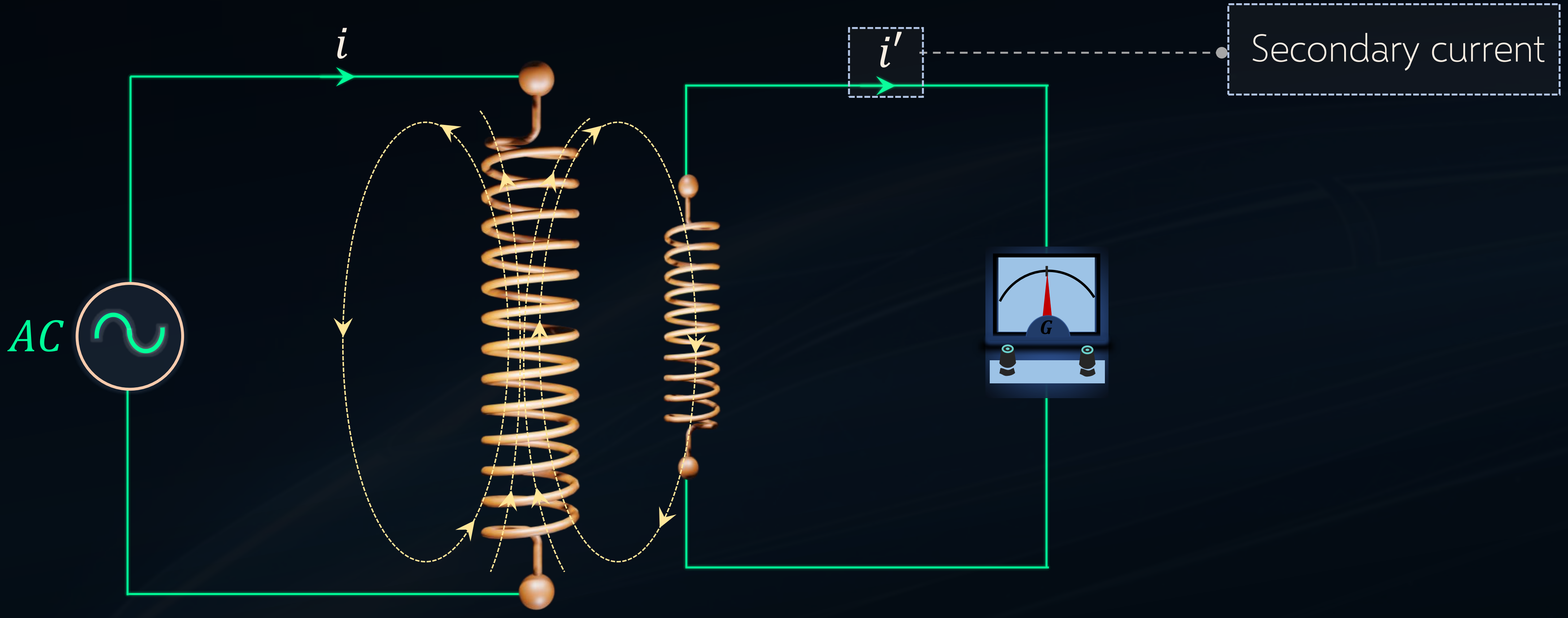
Thus, option a is the correct answer.

MUTUAL INDUCTANCE



When two coils are brought in proximity with each other, the magnetic field in one of the coils tend to link with the other. This further leads to the generation of voltage in the second coil. This property of a coil which affects or changes the current and voltage in a secondary coil is called mutual inductance.

MUTUAL INDUCTANCE

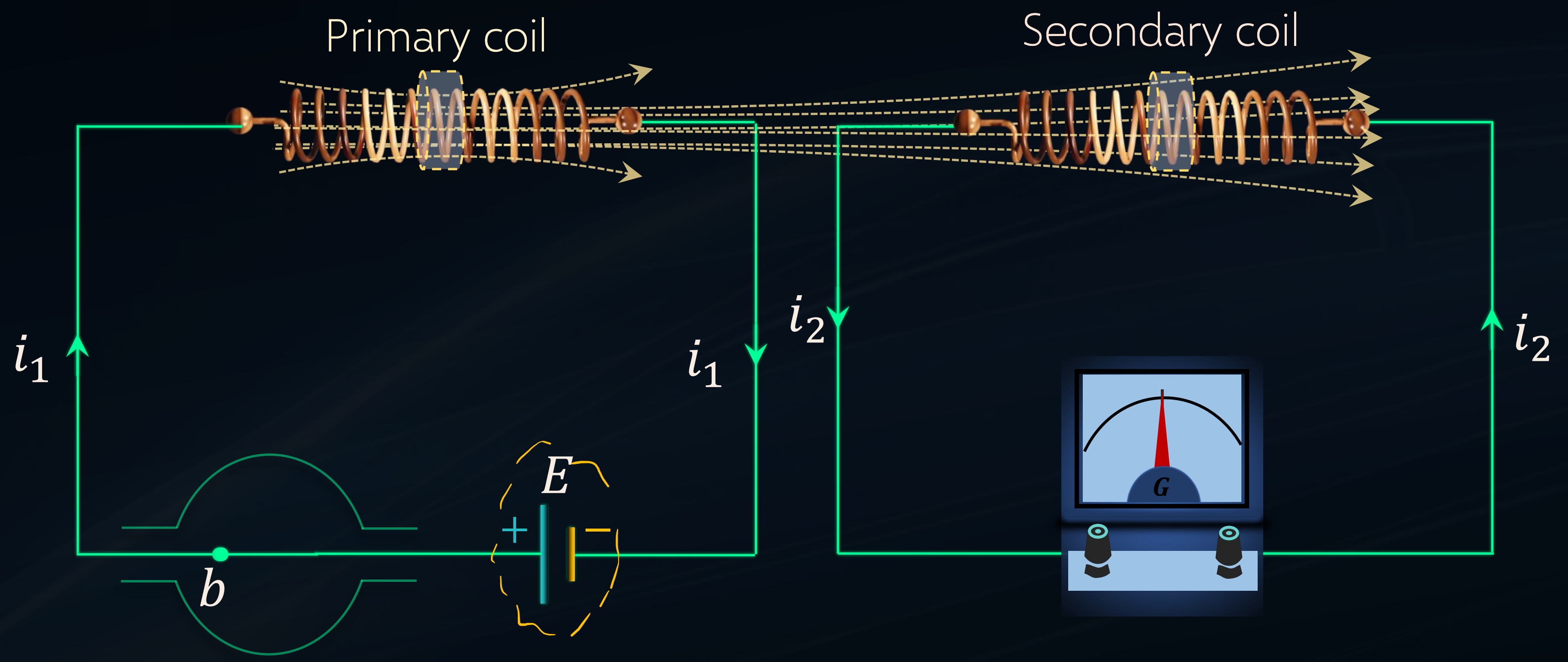


$$i_{(\text{primary})} \rightarrow B_{(\text{primary})} \rightarrow B_{(\text{secondary})} \rightarrow \phi_{(\text{secondary})} \rightarrow e.m.f_{(\text{induced})}$$

$$\downarrow$$

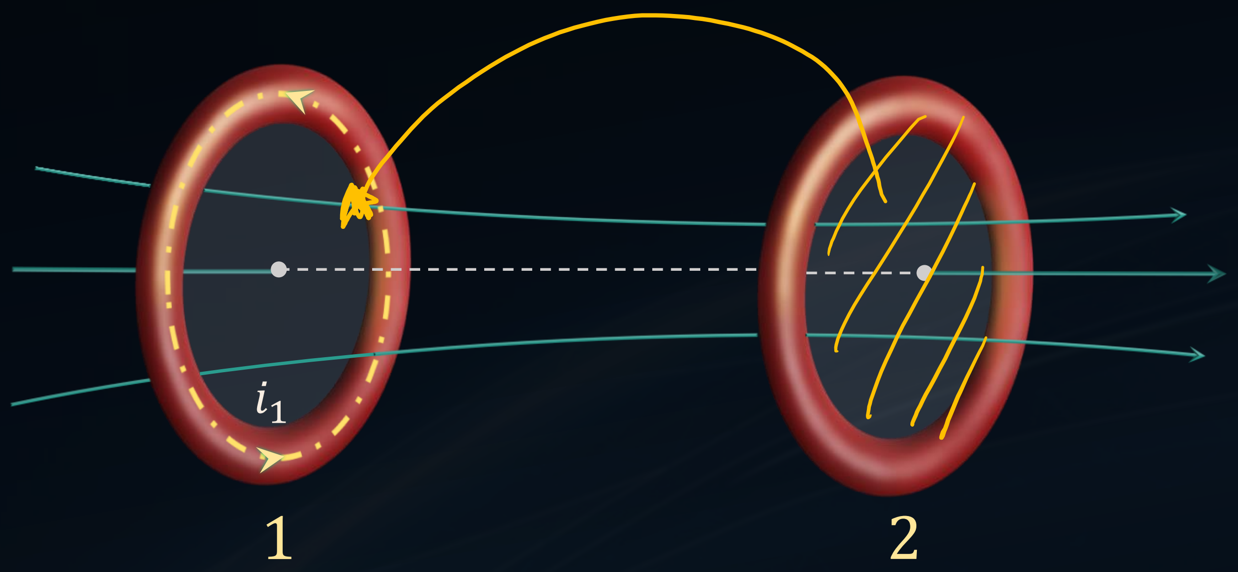
$$i_{(\text{secondary})}$$

COEFFICIENT OF MUTUAL INDUCTANCE



When two coils are brought in proximity with each other the magnetic field in one of the coils (in primary coil) tend to link with the other. This further leads to the generation of voltage in the secondary coil. This property of a coil which affects or changes the current and voltage in a secondary coil is called mutual inductance.

COEFFICIENT OF MUTUAL INDUCTANCE



$\phi_{21} \propto i_1$

$\phi_{21} = M i_1$

$M = \dfrac{\phi_{21}}{i_1}$

ϕ_{21}: Flux linked with coil **2** due to magnetic field generated by coil **1**

M: Coefficient of mutual inductance

$$M = \frac{\phi_{21}}{i_1}$$

Unit = $\dfrac{\text{Weber}}{\text{Ampere}}$ Or

Dimension: $[ML^2T^{-2}A^{-2}]$

The coefficient of mutual inductance of two coils is $6\ mH$. If the current flowing through one is $2\ A$, the induced emf in the second coil will be

a. Zero

b. $3\ V$

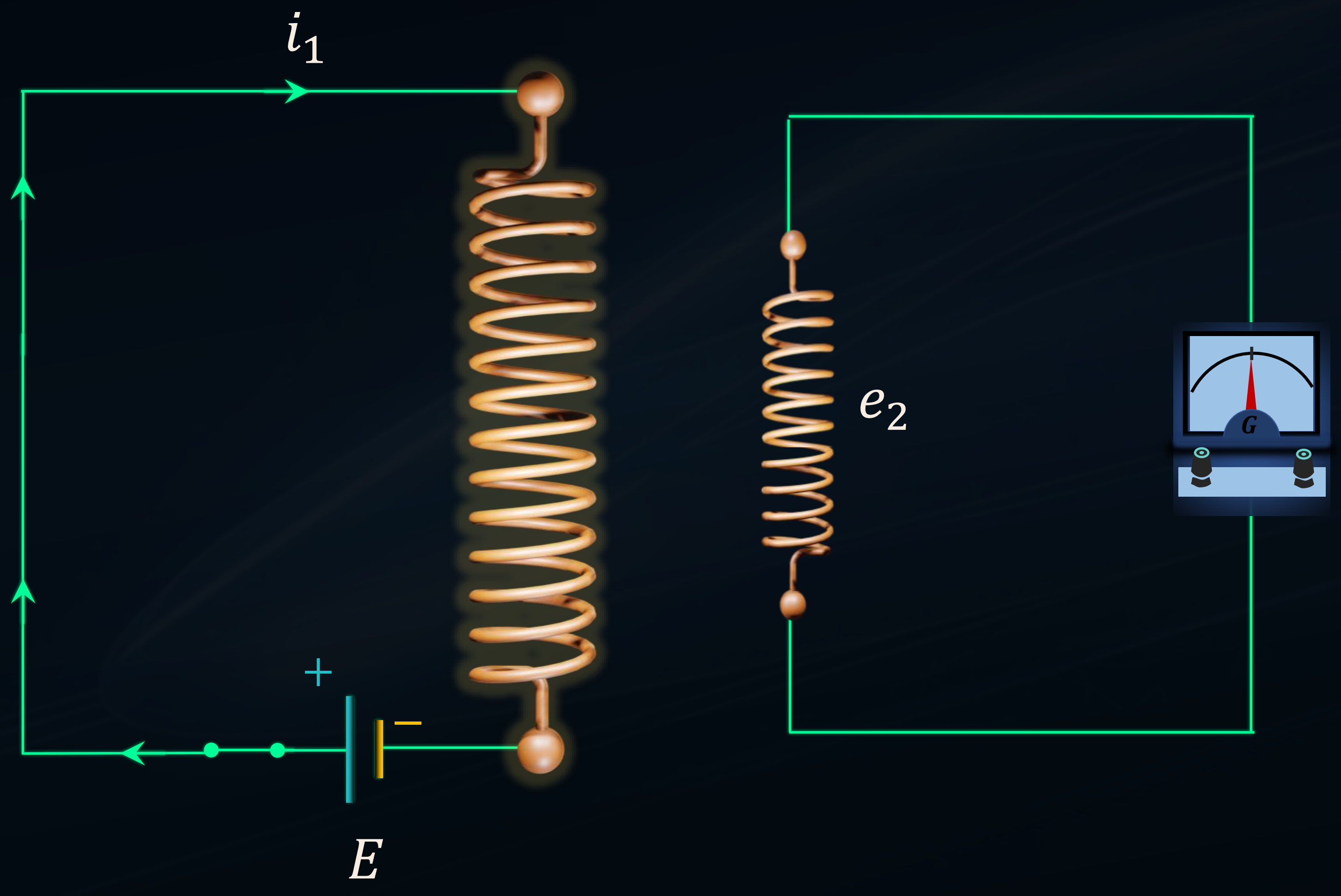
c. $3\ mV$

d. $2\ mV$

Solution

NEET

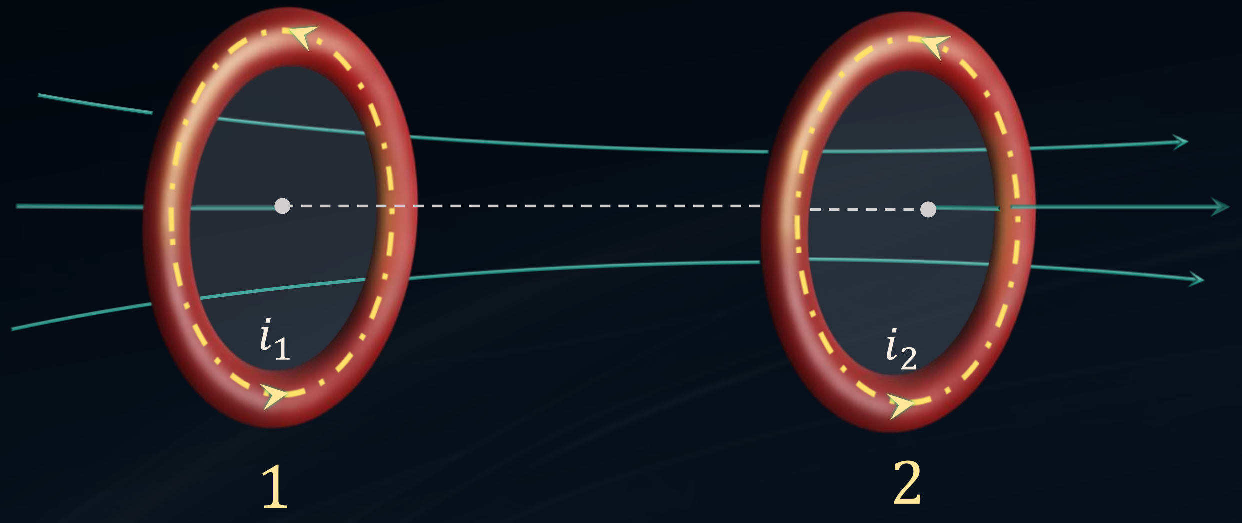
Given: $M = 6\, mH, i_1 = 2\ A$



- E.M.F. is induced in the secondary coil only when current in primary coil changes.
- $i_1 = 2\ A$
- $e_2 =$ zero

Thus, option a is the correct answer.

RECIPROCITY THEOREM



$$\phi_{12} \propto i_2 \qquad \phi_{21} \propto i_1$$

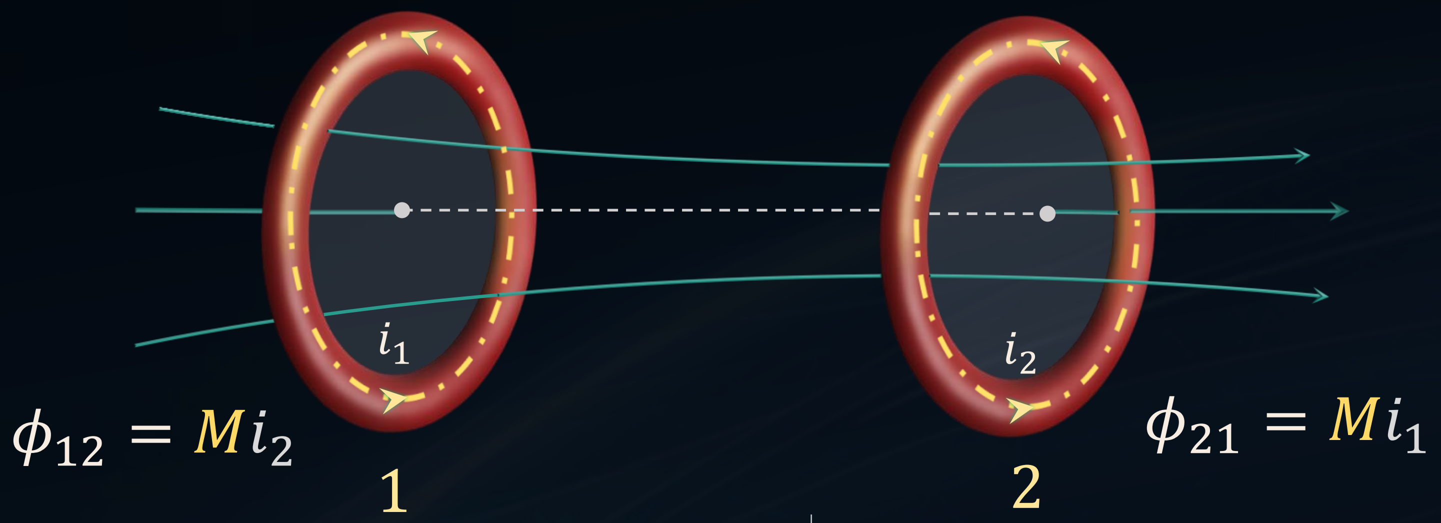
$$\phi_{12} = M_{12} i_2 \qquad \phi_{21} = M_{21} i_1$$

$$\boxed{M_{12} = M_{21} = M}$$

$$\phi_{12} = M i_2 \qquad \phi_{21} = M i_1$$

The theorem states that the two constants M_{12} and M_{21} are equal in the absence of material medium between the two coils.

INDUCED EMF IN SECONDARY COIL



Induced EMF in the coil 2 due to coil 1:

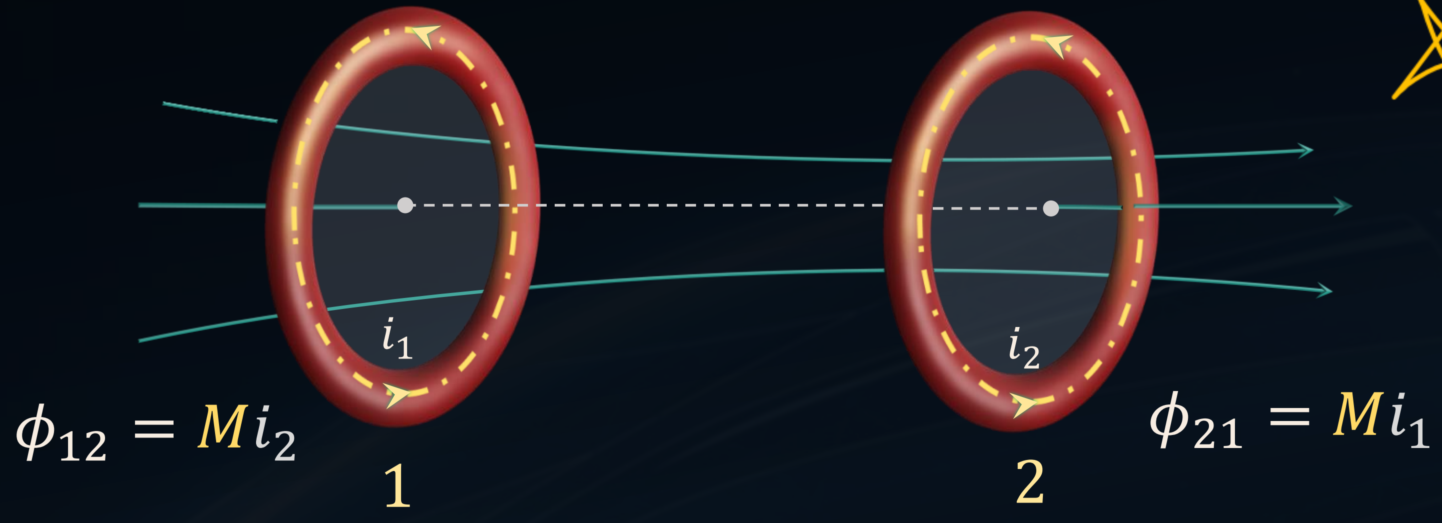
$$\varepsilon_2 = -\frac{d\phi_{21}}{dt}$$

$$\varepsilon_2 = -\frac{d(Mi_1)}{dt} \Rightarrow \varepsilon_2 = -M\frac{di_1}{dt}$$

$$\boxed{|\varepsilon_2| = M\left|\frac{di_1}{dt}\right|}$$

- It states that the magnitude of induced EMF in the secondary coil depends upon the rate of change of current in the primary coil.

INDUCED EMF IN SECONDARY COIL



$$|\varepsilon_2| = M\left|\frac{di_1}{dt}\right|$$

Induced EMF in the coil 1 due to coil 2:

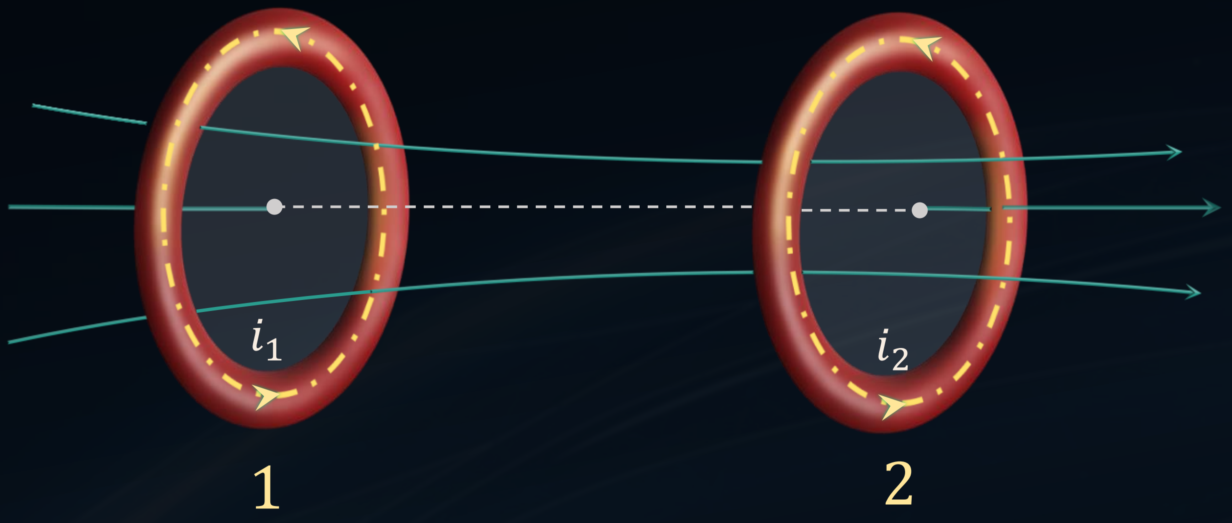
$$\varepsilon_1 = -\frac{d\phi_{12}}{dt}$$

$$\varepsilon_1 = -\frac{d(Mi_2)}{dt} \Rightarrow \varepsilon_1 = -M\frac{di_2}{dt}$$

$$|\varepsilon_1| = M\left|\frac{di_2}{dt}\right|$$

- It states that the magnitude of induced EMF in the secondary coil depends upon the rate of change of current in the primary coil (the coil with which a battery is connected).

INDUCED EMF IN SECONDARY COIL



$$|\varepsilon_1| = M\left|\frac{di_2}{dt}\right|$$

$$|\varepsilon_2| = M\left|\frac{di_1}{dt}\right|$$

If the rate of change of current in the primary coil $\left(\frac{di_1}{dt}\right)$ is $1\ Ampere/sec$, then the coefficient of mutual-inductance of the two coils, will equals to the induced EMF in a secondary coil.

Two coils have a mutual inductance $0.005\,H$. The current changes in the first coil according to the equation $I = I_m \sin \omega t$ where, $I_m = 10\,A$ and $\omega = 100\pi\ rad\ s^{-1}$. The maximum vale of the e.m.f induced in the second coil is

a π

b 2π

c 4π

d 5π

Solution NEET

Given: $M = 0.005\,H, I = I_m \sin \omega t$, $I_m = 10\,A$, $\omega = 100\pi\, rad\, s^{-1}$

Induced E.M.F. $\boxed{\varepsilon_2 = M\left|\frac{di}{dt}\right|}$

$$\varepsilon_2 = M\left|\frac{d(I_m \sin \omega t)}{dt}\right|$$

$$\varepsilon_2 = M I_m \omega \cos \omega t$$

$$(\varepsilon_2)_{Max} = M I_m \omega$$

(When $\cos \omega t = 1$)

$$(\varepsilon_2)_{Max} = M I_m \omega$$

$$(\varepsilon_2)_{Max} = 0.005 \times 10 \times 100\pi$$

$$(\varepsilon_2)_{Max} = 5\pi$$

Thus, option d is the correct answer.

$\frac{d(\sin 2\theta)}{d\theta}$

$2 \cos 2\theta$.

max

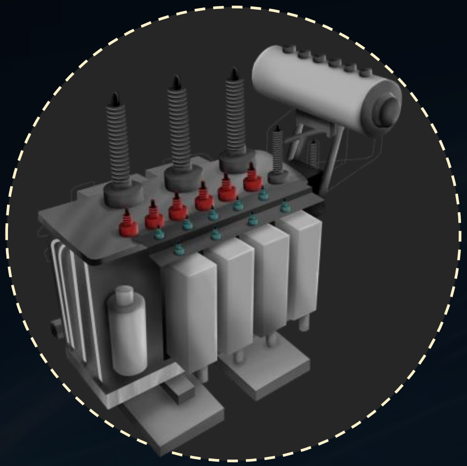
$\varepsilon_2 = M . \frac{d}{dt}(I_m \sin \omega t)$ 1

$\varepsilon_2 = M I_m . \omega \cos \omega t$

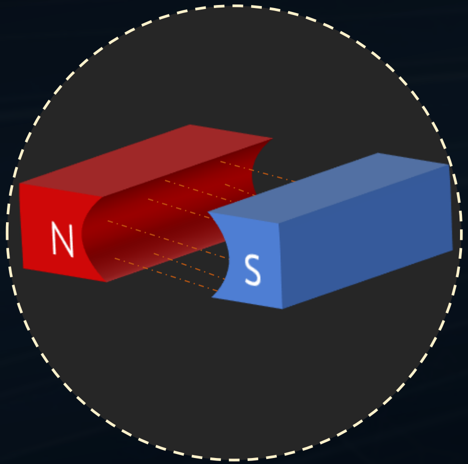
APPLICATION OF MUTUAL INDUCTANCE

- Some applications are:

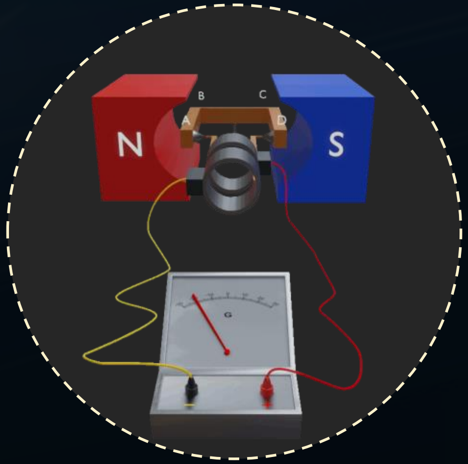
- Transformer



- All the electric devices, which work with a magnetic field

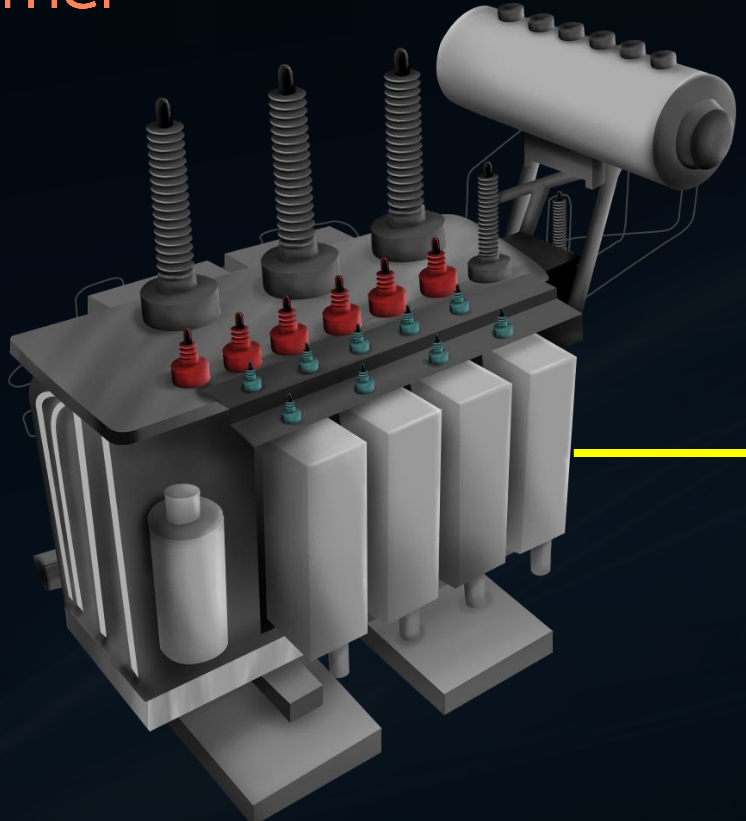


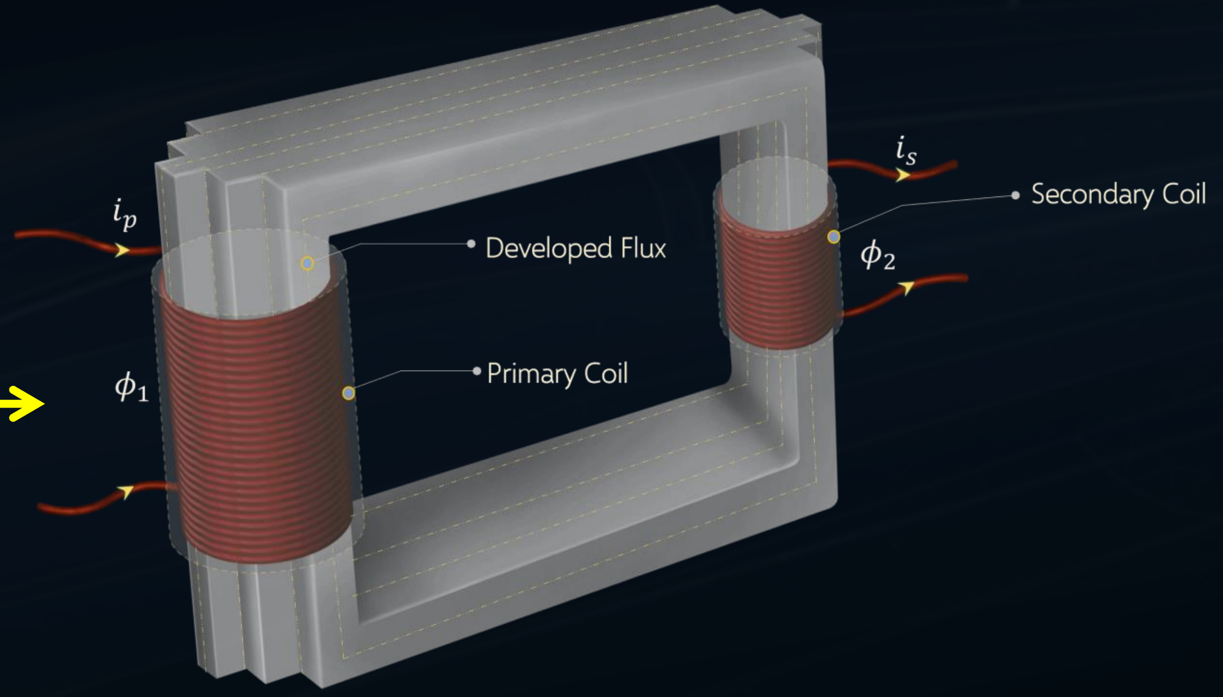
- Electric Generator



APPLICATION OF MUTUAL INDUCTANCE

Transformer

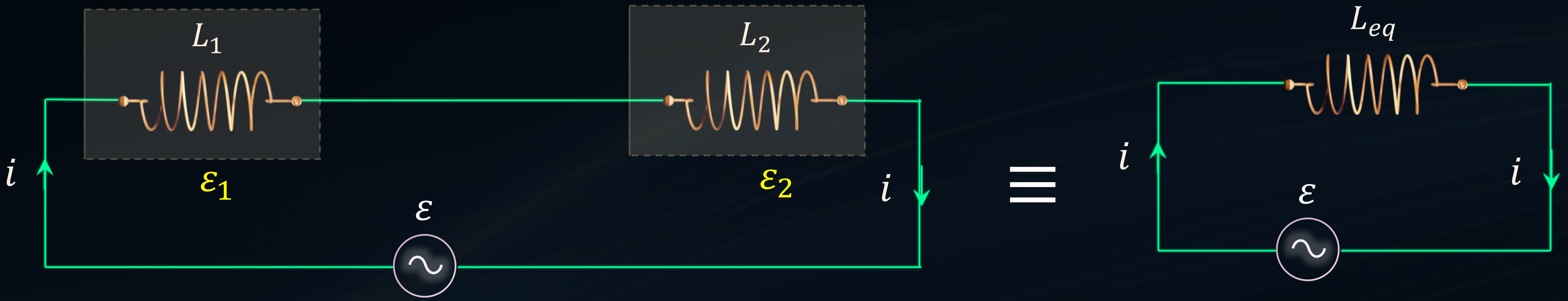




Transformer is a widely used application of mutual induction. There are usually two coils - primary coil and secondary coil on the transformer core. The core laminations are joined in the form of strips. The two coils have high mutual inductance. When an alternating current pass through the primary coil it creates a varying magnetic flux. As per Faraday's law of electromagnetic induction, this change in magnetic flux induces an emf (electromotive force) in the secondary coil which is linked to the core having a primary coil.

SERIES AND PARALLEL COMBINATION OF INDUCTORS

Series Combination



$$\varepsilon_1 = -L_1 \frac{di}{dt} \qquad \varepsilon_2 = -L_2 \frac{di}{dt}$$

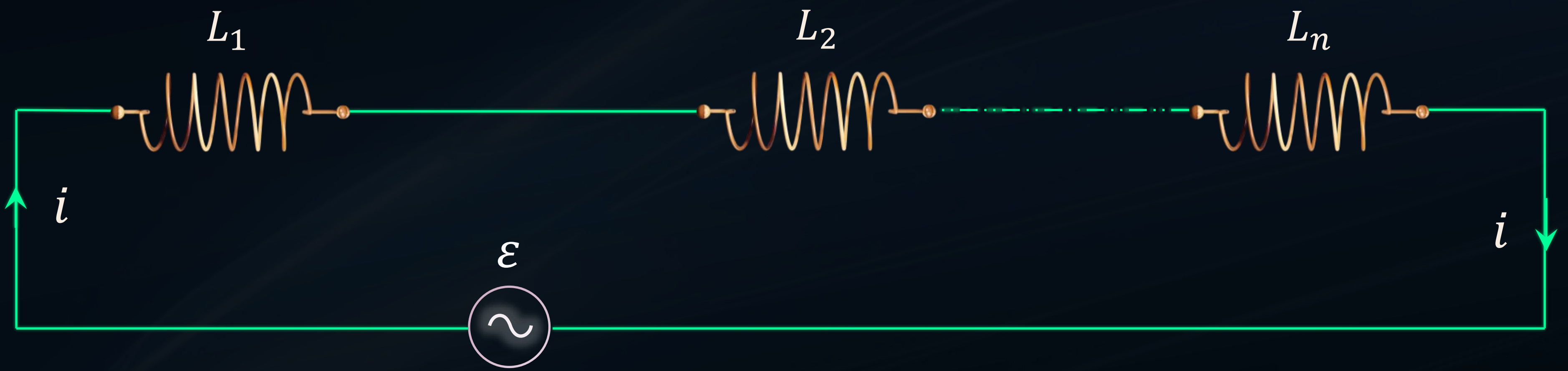
$$\varepsilon = \varepsilon_1 + \varepsilon_2$$

$$-L_{eq} \cancel{\frac{di}{dt}} = -L_1 \cancel{\frac{di}{dt}} + \left(-L_2 \cancel{\frac{di}{dt}}\right) \Rightarrow \boxed{L_{eq} = L_1 + L_2}$$

SERIES AND PARALLEL COMBINATION OF INDUCTORS

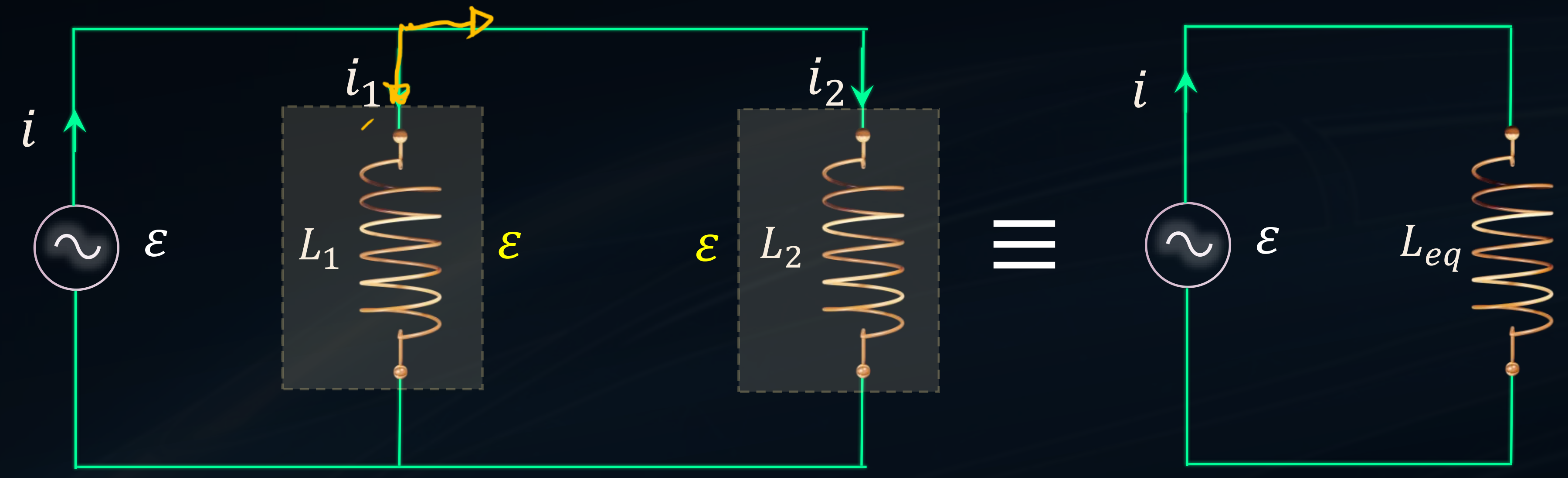
$$L_{eq} = L_1 + L_2$$

For series combination of $\boldsymbol{n}$ inductors



$$L_{eq} = L_1 + L_2 + \cdots L_n$$

Parallel Combination



$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \text{..........} \quad (1)$$

$$\varepsilon = -L_1 \frac{di_1}{dt} = -L_2 \frac{di_2}{dt}$$

$$\frac{di_1}{dt} = -\frac{\varepsilon}{L_1}, \quad \frac{di_2}{dt} = -\frac{\varepsilon}{L_2} \quad \text{..........} \quad (2)$$

Using (2) in (1), we get

$$-\frac{\cancel{\varepsilon}}{L_{eq}} = -\frac{\cancel{\varepsilon}}{L_1} + \left(-\frac{\cancel{\varepsilon}}{L_2}\right)$$

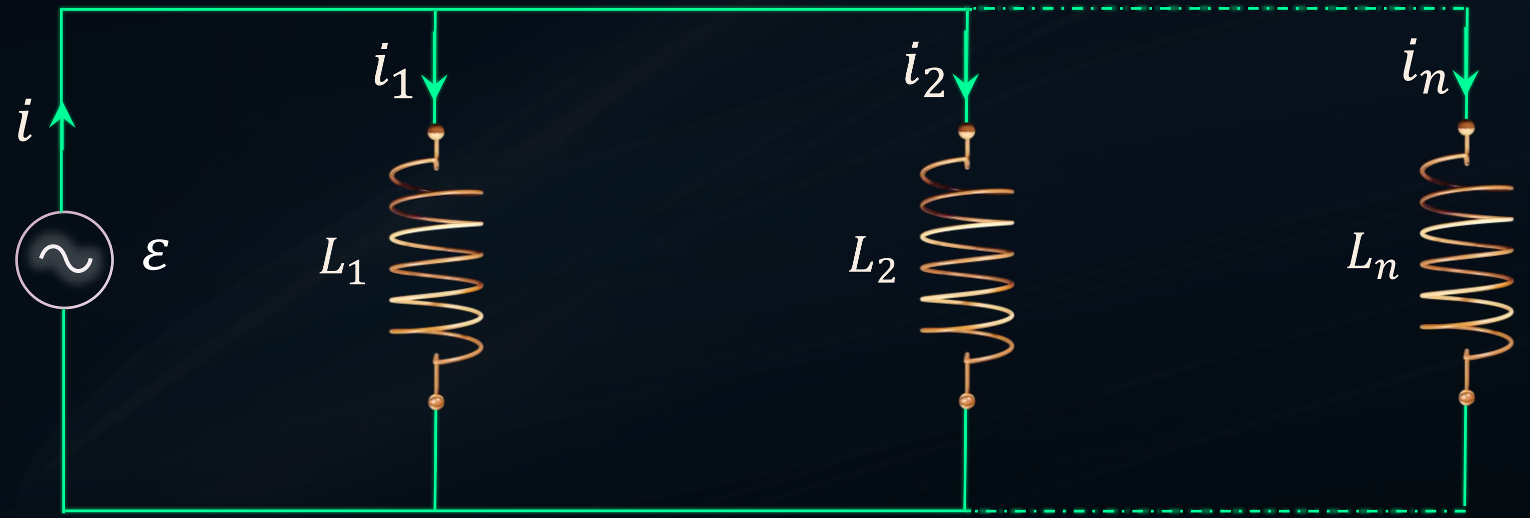
$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

SERIES AND PARALLEL COMBINATION OF INDUCTORS

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

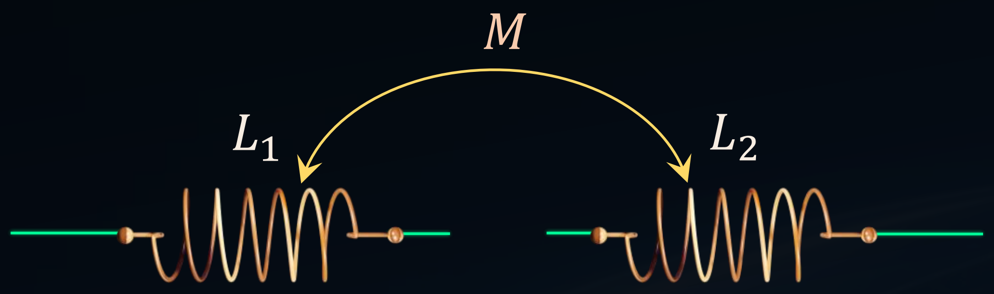
$$\boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$$

For parallel combination of n inductors



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_n}$$

COUPLING CONSTANT



In general, $M = K\sqrt{L_1 L_2}$

Where K is the coupling constant

$$0 \leq K \leq 1$$

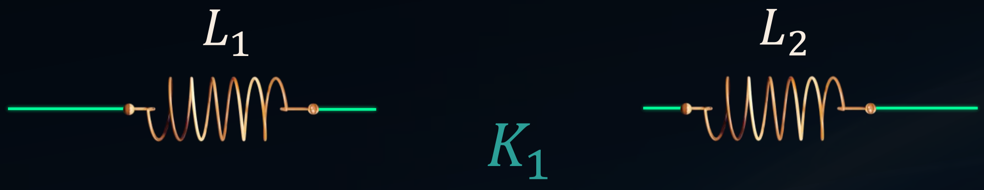
$K = 1$ when there is 100% flux linkage

COUPLING CONSTANT

$$\boxed{M = K\sqrt{L_1 L_2}}$$

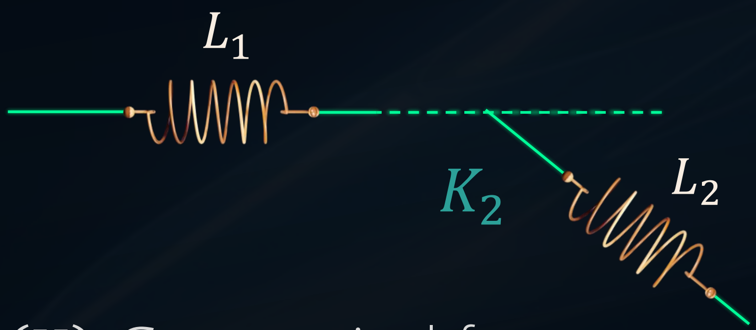
K depends on :

(I) Distance between coil



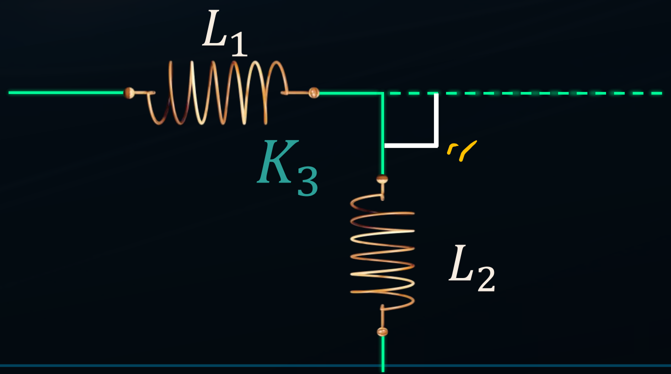
$$\boxed{K_1 < K}$$

(II) Relative orientation of the coils



$$\boxed{K_2 < K}$$

(II) Geometrical factors



$$\boxed{K_3 = 0}$$

Two coils of self-inductance $2\,mH$ and $8\,mH$ are placed so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is

a) $16\,mH$

b) $10\,mH$

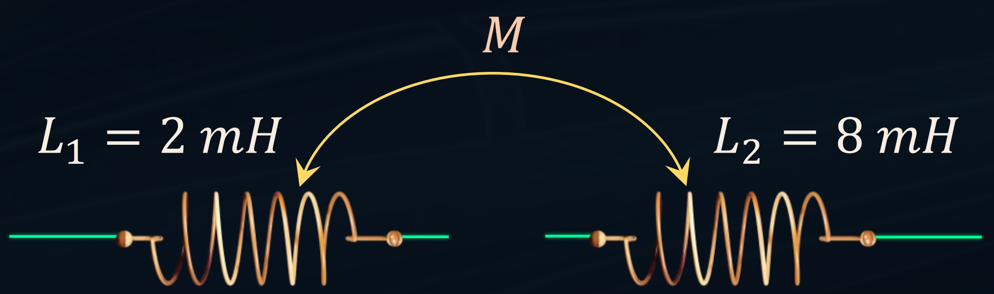
c) $6\,mH$

d) $4\,mH$

Solution NEET

In general, $M = K\sqrt{L_1 L_2}$

$K = 1$ when there is 100% flux linkage



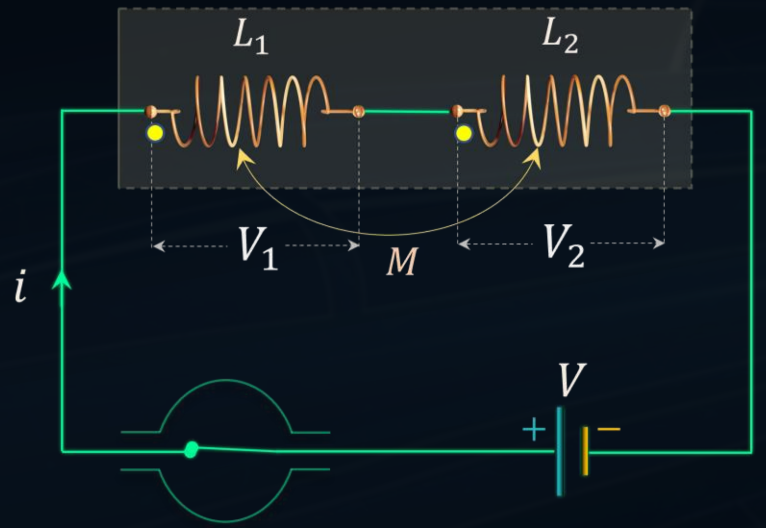
$\therefore M = \sqrt{2 \times 8} = 4\ mH$

Thus, option d is the correct answer.

SERIES AND PARALLEL COMBINATION OF INDUCTORS

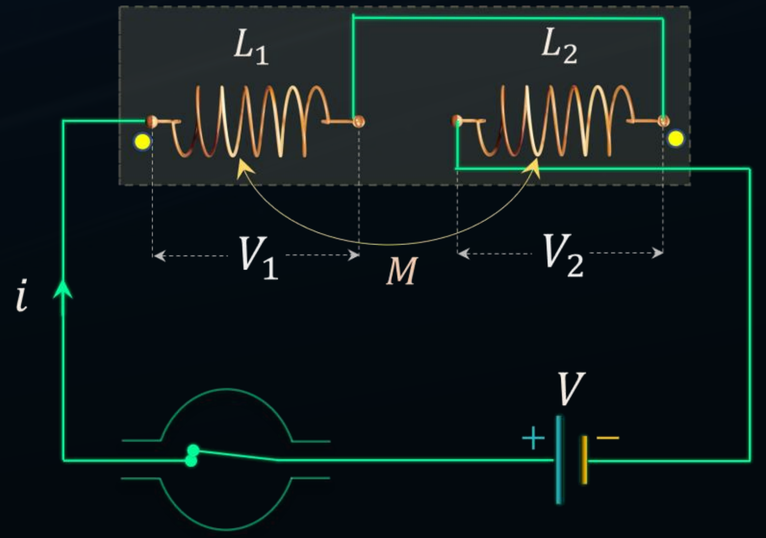
Series Combination (Aiding)

$$L_{eq} = L_1 + L_2 + 2M$$



Series Combination (Opposing)

$$L_{eq} = L_1 + L_2 - 2M$$



L_1 L_2

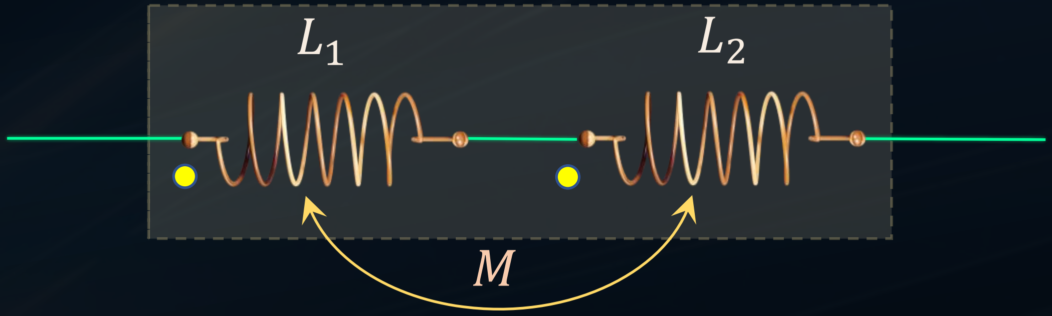
Two coils connected in series have a self-inductance of $20\ mH$ and $60\ mH$ respectively. The total inductance of the combination was found to be $100\ mH$. Determine the amount of mutual inductance that exists between the two coils assuming that they are aiding each other.

a $20\ mH$

b $30\ mH$

c $15\ mH$

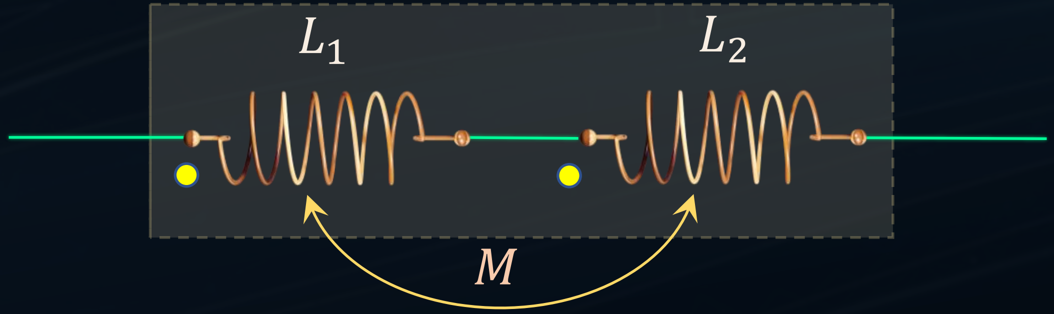
d $10\ mH$



Solution NEET

Given: $L_1 = 20\ mH, L_2 = 60\ mH, L_{eq} = 100\ mH$

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$



$$100 = 20 + 60 + 2M$$

$$2M = 100 - 20 - 60$$

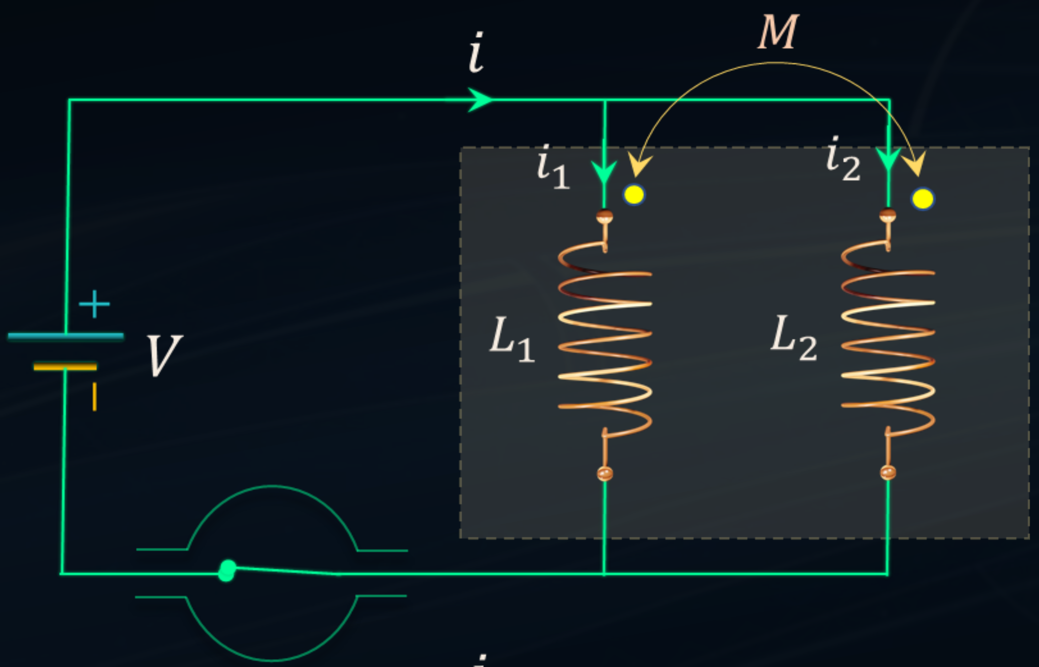
$$M = 10\ mH$$

Thus, option d is the correct answer.

SERIES AND PARALLEL COMBINATION OF INDUCTORS

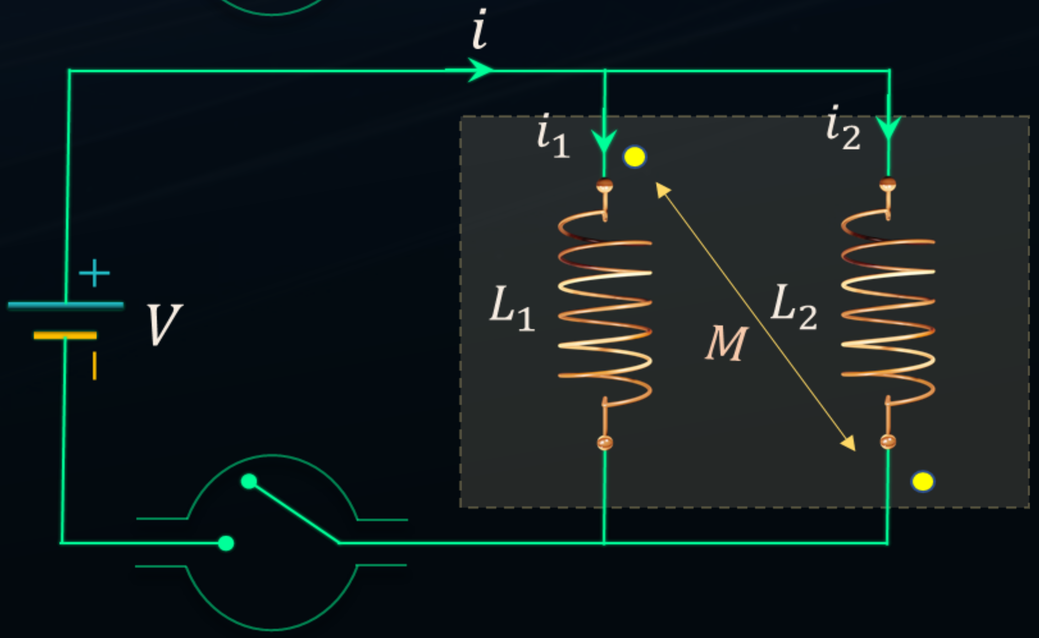
Parallel Combination (Aiding)

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



Parallel Combination (Opposing)

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



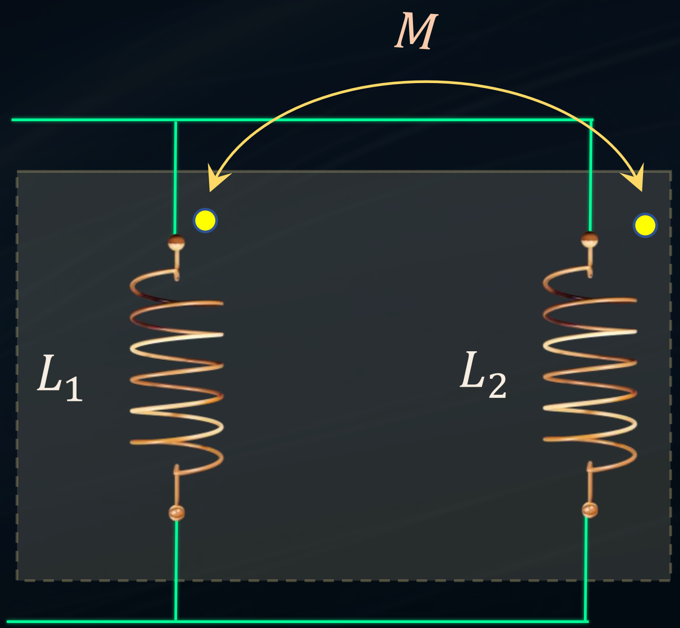
Two inductors whose self-inductance are of $75\,mH$ and $55\,mH$ respectively are connected together in parallel aiding. Their mutual inductance is given as $22.5\,mH$. Calculate the total inductance of the parallel combination.

a) $42.57\,mH$

b) $44.61\,mH$

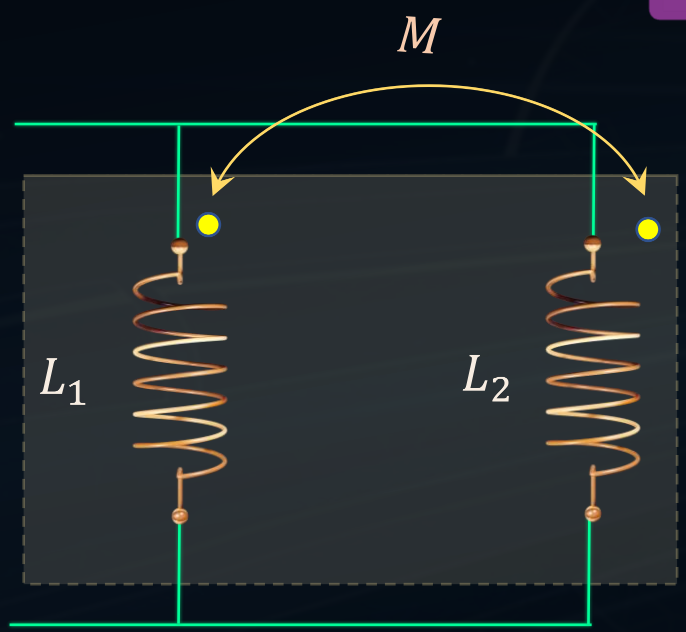
c) $45.08\,mH$

d) $40.80\,mH$



Solution NEET

Given: $L_1 = 75\ mH, L_2 = 55\ mH, M = 22.5\ mH$



$$\boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

$$L_{eq} = \frac{75 \times 55 - (22.5)^2}{75 + 55 - 2 \times 22.5}$$

$$L_{eq} = \frac{3618.75}{85}$$

$$L_{eq} = 42.57\ mH$$

Thus, option a is the correct answer.

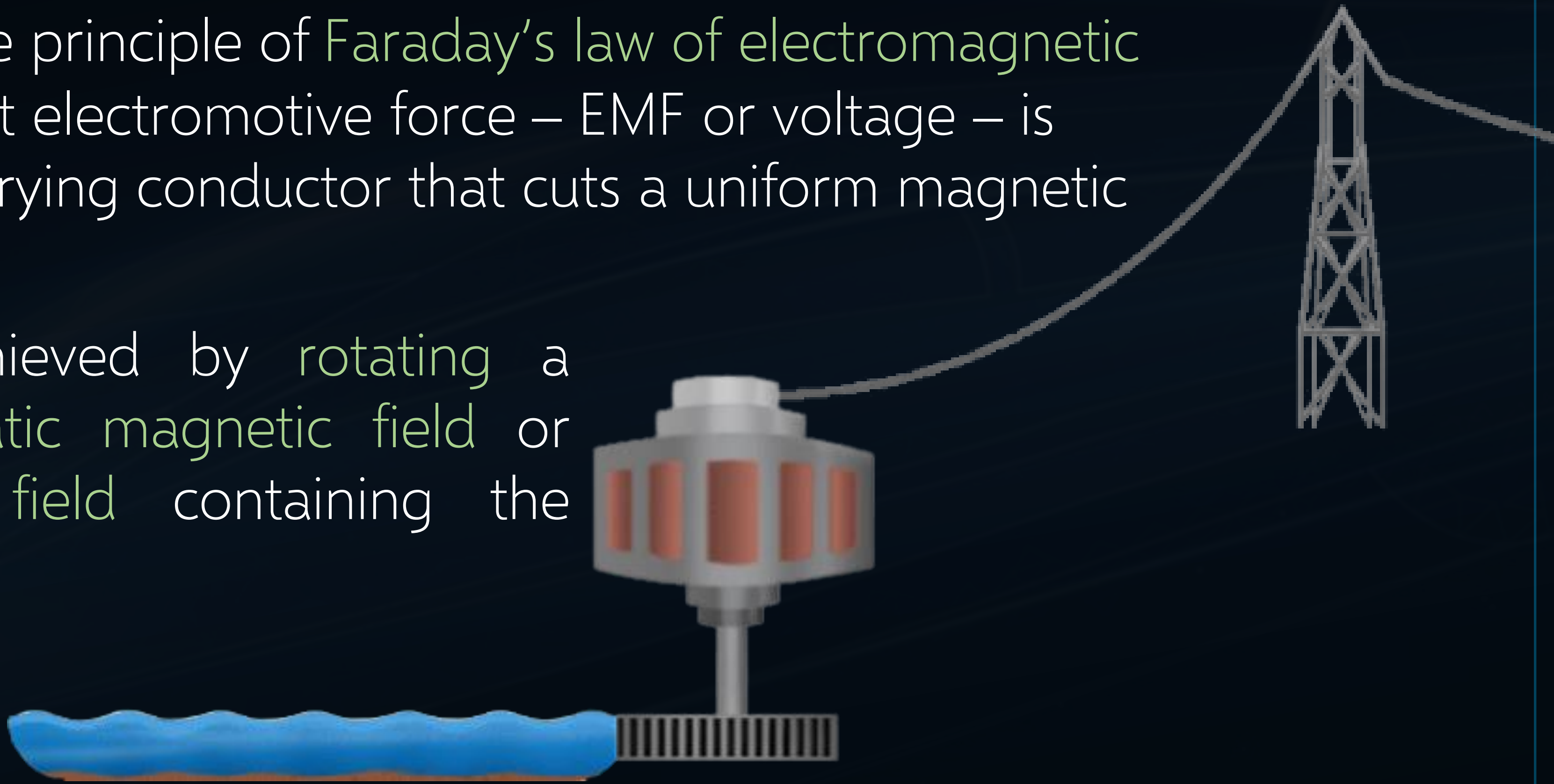
Generator

A device that converts mechanical energy into electric energy.

The AC Generator's input supply is mechanical energy supplied by steam turbines, gas turbines and combustion engines. The output is alternating electrical power in the form of alternating voltage and current.

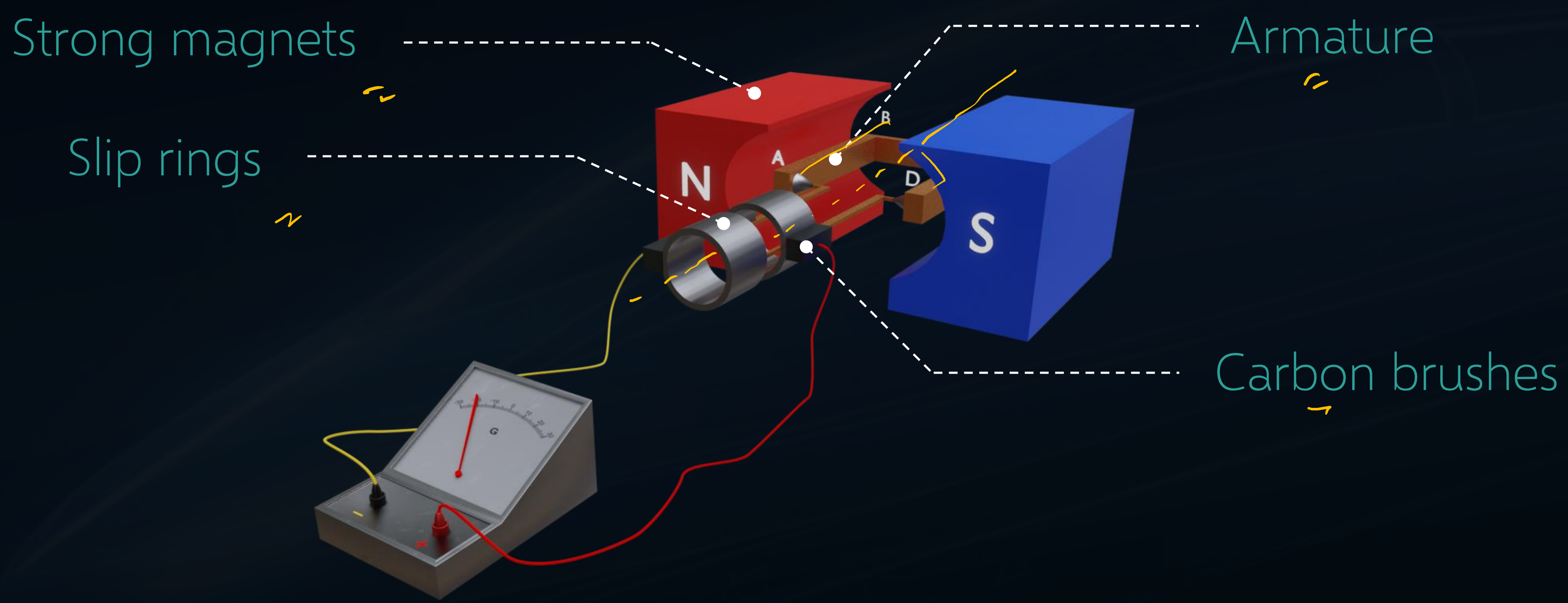
Generator

- AC generators work on the principle of Faraday's law of electromagnetic induction, which states that electromotive force – EMF or voltage – is generated in a current-carrying conductor that cuts a uniform magnetic field.
- This can either be achieved by rotating a conducting coil in a static magnetic field or rotating the magnetic field containing the stationary conductor.



- Alternating current is a type of electric current that changes direction with time periodically.
- Most appliances in our day-to-day life use AC.
- The current we get in our houses is $220\,V, 50\,Hz$ AC.

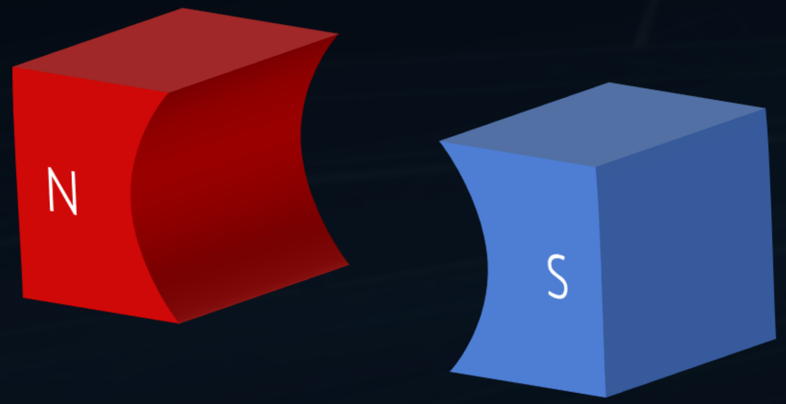
AC Generator- Construction



AC Generator- Parts

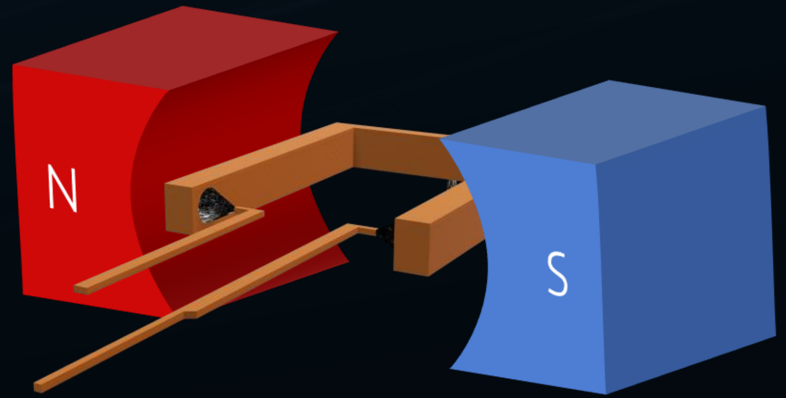
Magnets:

- Permanent magnet or an electromagnet used to produce a strong uniform magnetic field.



Armature:

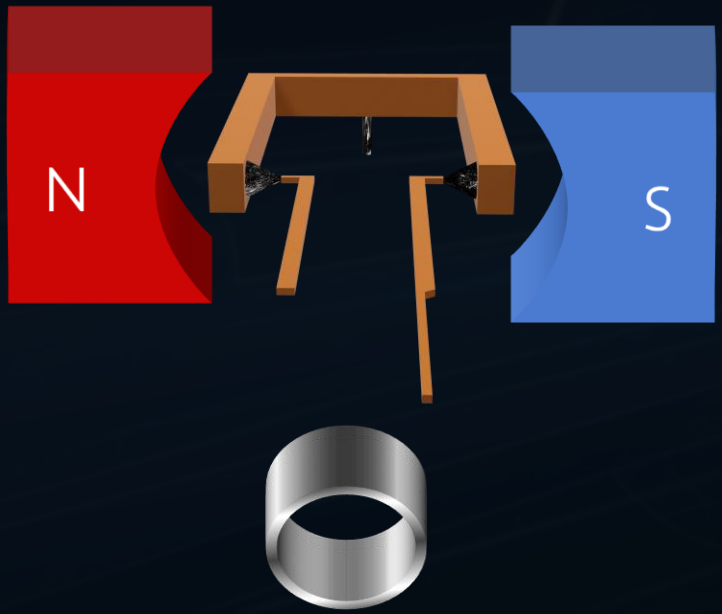
- A coil wound over soft-iron core. The two ends of the coil are connected to two slip rings.
- Axis of rotation is in the plane of the coil but perpendicular to the magnetic field.



AC Generator- Parts

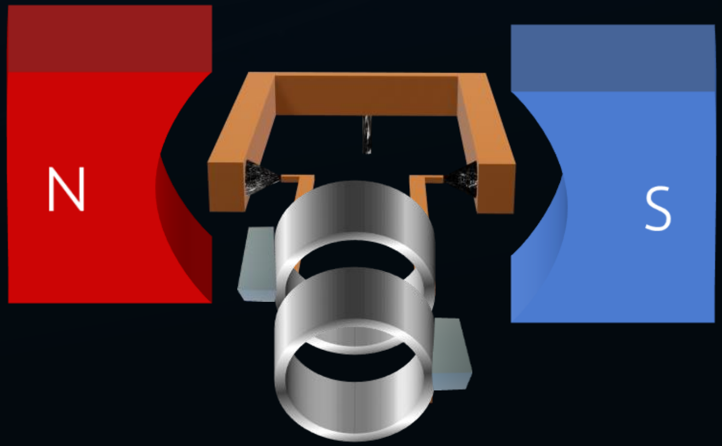
Slip rings:

- Two small rings slip against the brushes so that the contact is maintained all the time



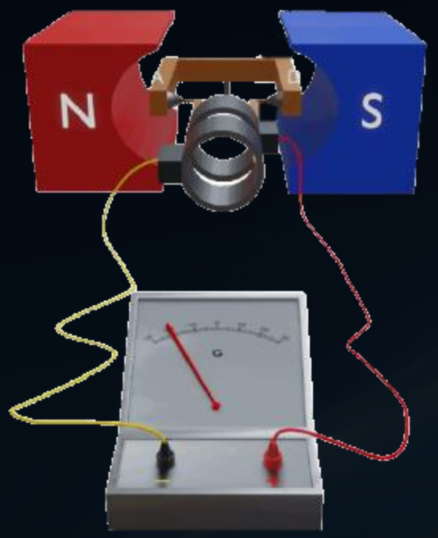
Carbon brushes:

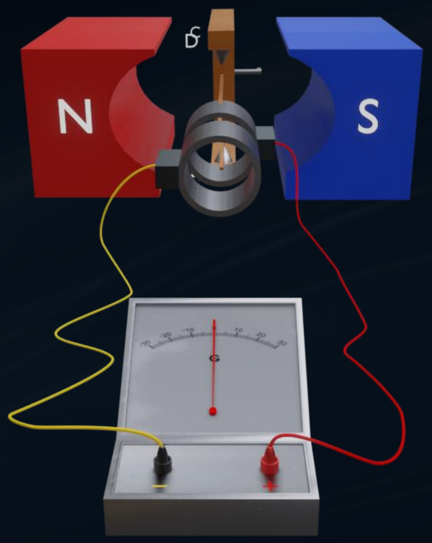
- Two graphite brushes permanently touch the slip rings.
- These brushes are connected to terminals of the circuit.



AC Generator- Working

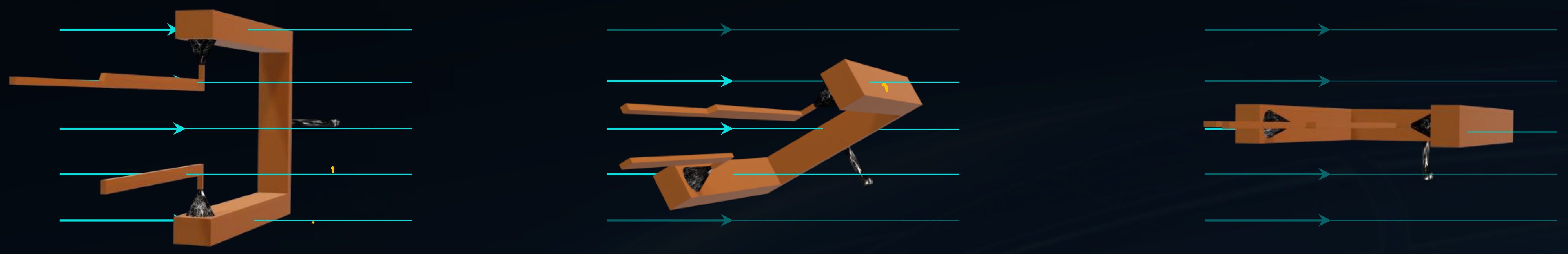
The generated EMF depends on the number of armature coil turns, magnetic field strength, and the speed of the rotating field.





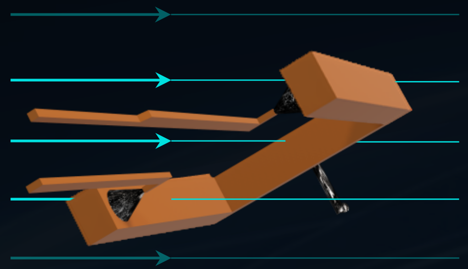
When the armature rotates between the poles of the magnet upon an axis perpendicular to the magnetic field, the flux linkage of the armature changes continuously. As a result, an electric current flows through the galvanometer and the slip rings and brushes. The galvanometer swings between positive and negative values. This indicates that there is an alternating current flowing through the galvanometer.

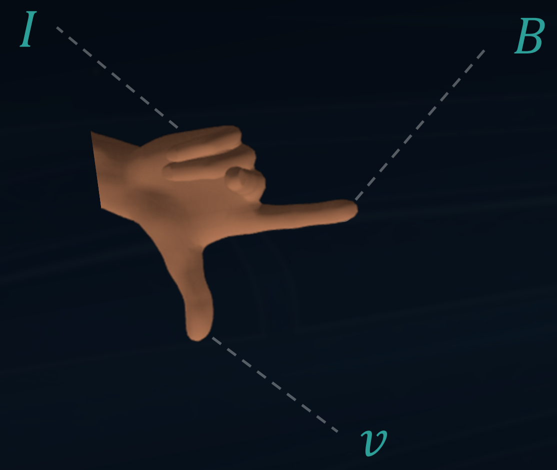
AC Generator- Working Principle



- Magnetic flux changes as the armature moves.
- Faraday's Law tells us that this generates emf in the coil.
- The direction of the induced current can be identified using Fleming's right-hand rule.

AC Generator- Working Principle



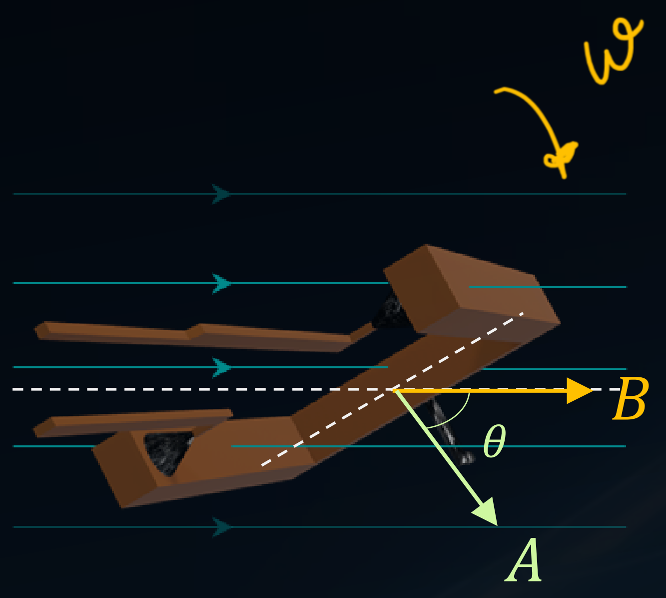


- The direction of induced current can be found by Fleming's Right-hand rule

- Current is going in on one side of armature and coming out of another side.

Hold the right-hand forefinger, middle finger and the thumb at right angles to each other. If the forefinger represents the direction of the magnetic field, the thumb points in the direction of motion or applied force, then the middle finger points in the direction of the induced current.

AC Generator- Working Principle



Magnitude of Induced emf :

For an armature moving at a constant angular speed ω

$$\theta = \omega t$$

The flux (ϕ) passing through the armature at any point

$$\boxed{\phi = BA\cos\omega t}$$

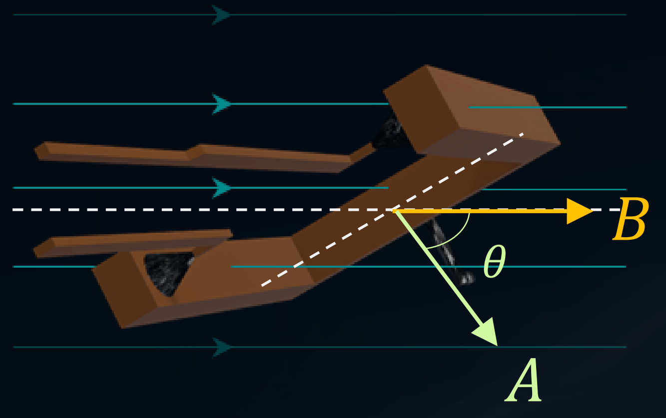
Where,

B = Magnetic field due to external magnets

A = Area of the armature coil

$\omega = \frac{\theta}{t} \rightarrow \theta = \omega t$

AC Generator- Working Principle



Magnitude of Induced emf :

$$\boxed{\phi = BA\cos\omega t}$$

$$\varepsilon = -\frac{d\phi}{dt} = BA\omega\sin\omega t$$

$$\frac{d}{dt}(\cos 2\theta) = -\sin 2\theta . 2$$

For N turns

$$\varepsilon = NBA\omega\sin\omega t$$

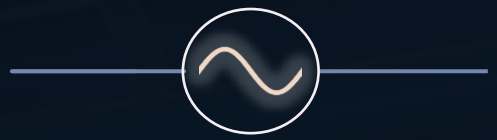
$$\boxed{\varepsilon = \varepsilon_0\sin\omega t}$$

$$\varepsilon_0 = NBA\omega$$

AC Generator- Working Principle

R: resistance of the coil

Circuit symbol of AC generator



- Emf varies sinusoidally with time
- Angular frequency: ω
- Time period: $T = \frac{2\pi}{\omega}$.
- Peak emf: $\varepsilon_{max} = \varepsilon_0$

Magnitude of Induced current :

$$\boxed{\varepsilon = \varepsilon_0 \sin \omega t}$$

$$iR = \varepsilon_0 \sin \omega t$$

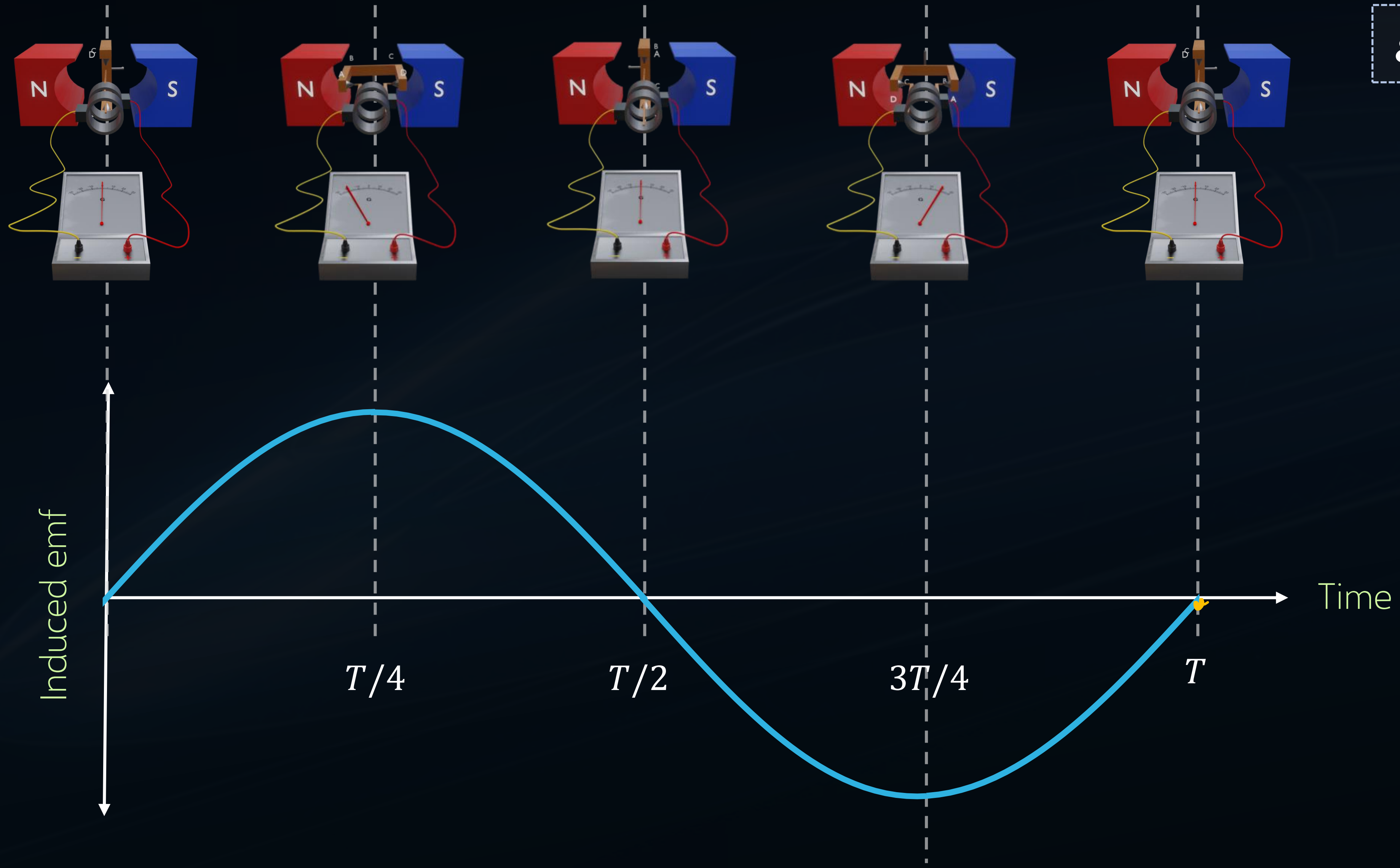
$$i = \frac{\varepsilon_0}{R} \sin \omega t$$

i_0

$$\boxed{i = i_0 \sin \omega t}$$

AC Generator- Working Principle

$$\varepsilon = \varepsilon_0 \sin \omega t$$



AC Generator- Working Principle

- By repeating this cycle, we get AC current of a certain frequency.
- In India, the frequency of these cycles is 50 Hz.

In a region of uniform magnetic induction $B = 10^{-2}$ Tesla, a circular coil of radius $30\ cm$ and resistance $\pi^2\ \Omega$ is rotated about an axis which is perpendicular to the direction of B and which forms a diameter of the coil. If the coil rotates at $200\ rpm$, the amplitude of the alternating current induced in the coil is

a) $4\pi^2\ mA$

b) $30\ mA$

c) $6\ mA$

d) $200\ mA$

Solution NEET

$$\boxed{\varepsilon = \varepsilon_0 \sin \omega t}$$

$$\varepsilon_0 = NBA\omega$$

$$I_0 = \frac{\varepsilon_0}{R} = \frac{NBA\omega}{R}$$

$$I_0 = \frac{2\pi\times\left(\frac{200}{60}\right)\times1\times10^{-2}\times\pi(0.3)^2}{\pi^2}$$

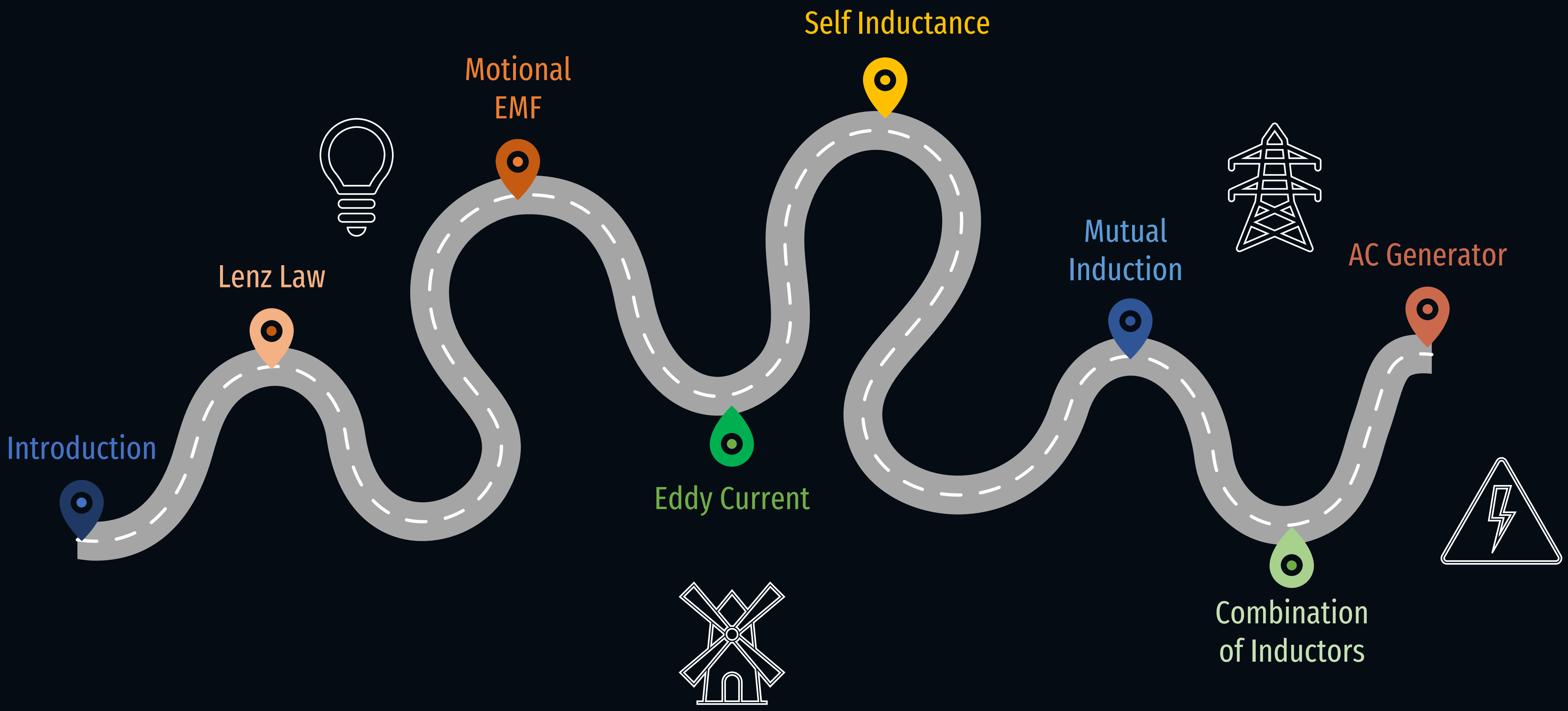
$$= 6\times10^{-3}A = 6\ mA$$

Thus, option c is the correct answer.

$$I_0 = 6\ mA$$

ELECTROMAGNETIC INDUCTION

Session wise content



Magnetic flux:

The number of magnetic field lines passing normally through a given surface.

$\boxed{\phi_B \propto B}$ $\boxed{\phi_B \propto A}$ SI Unit : Weber (Wb)

$$\phi_B = |\vec{B}| \cos\theta \, |\vec{A}| \qquad \boxed{\phi_B = \vec{B}.\vec{A}} \qquad \phi_B = \int B dA \cos\theta$$

Faraday's first law:

Whenever there is a change in magnetic flux linked with a conductor, an emf is induced in conductor. If it is a closed circuit, induced current will flow through it.

Faraday's second law:

The magnitude of the induced emf in a conducting coil is proportional to the rate at which the magnetic flux through that coil changes with time.

$$\varepsilon \propto \left|\frac{\Delta\phi_B}{\Delta t}\right|$$

Induced EMF:

Average emf $(\varepsilon_{Avg}) = N\left|\dfrac{\Delta\phi_B}{\Delta t}\right|$ Instantaneous emf $(\varepsilon_{Ins}) = N\left|\dfrac{d\phi_B}{dt}\right|$

Induced current:

Average current $(i_{Avg}) = \dfrac{\varepsilon_{Avg}}{r} = \dfrac{N\left|\dfrac{\Delta\phi_B}{\Delta t}\right|}{r}$ Instantaneous current $(i_{Ins}) = \dfrac{N\left|\dfrac{d\phi_B}{dt}\right|}{r}$

Total charge flow:

$$\Delta q = N\frac{\Delta\phi_B}{r}$$

Lenz's law:

The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

cause

effect

$$\boxed{\varepsilon = -N\frac{d\phi_B}{dt}}$$

opposition

Motional EMF:

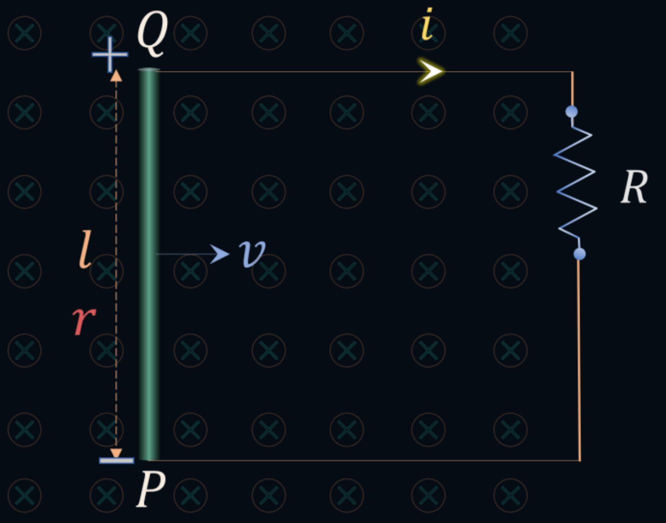
The EMF developed due to motion of conductor in magnetic field is called as motional EMF.

$$\vec{F}_m = -e(\vec{v} \times \vec{B}) \Rightarrow |\vec{F}_m| = evB \qquad \vec{F}_E = e\vec{E}$$

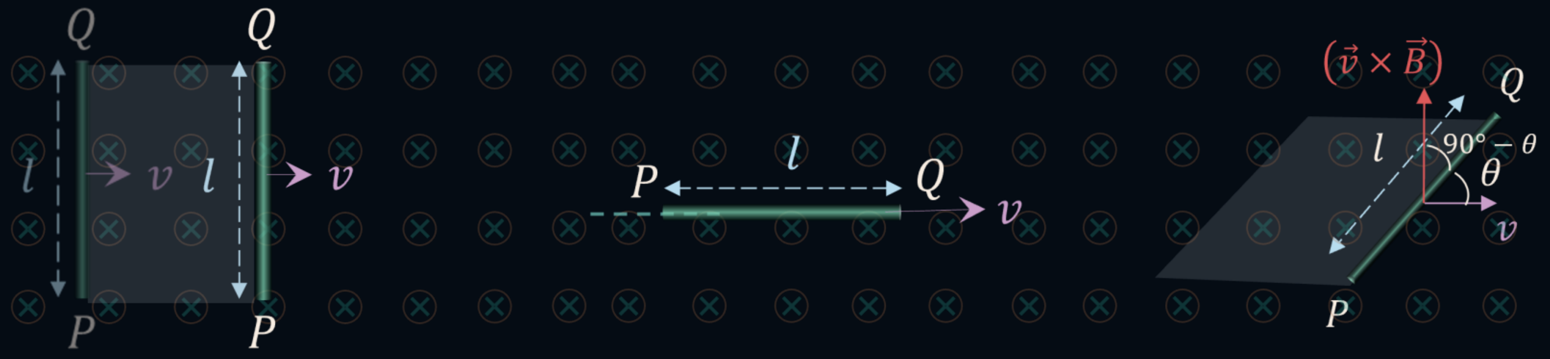
$$\varepsilon = vBl \qquad i = \frac{vBl}{R + r}$$

Direction: By Lenz's law

$\vec{B}, \vec{v}$ and $\vec{l}$ are mutually $\perp$



Different cases of motional EMF:



$\boxed{\vec{B} \perp \vec{v} \perp \vec{l}}$

$\varepsilon = vBl$

$\boxed{\vec{v} \parallel \vec{l}}$

$\varepsilon = 0$

$\boxed{\varepsilon = (\vec{v} \times \vec{B}).\vec{l}}$

$\varepsilon = vBl \sin\theta$

$\boxed{\vec{l} \parallel \vec{B}}$

OR

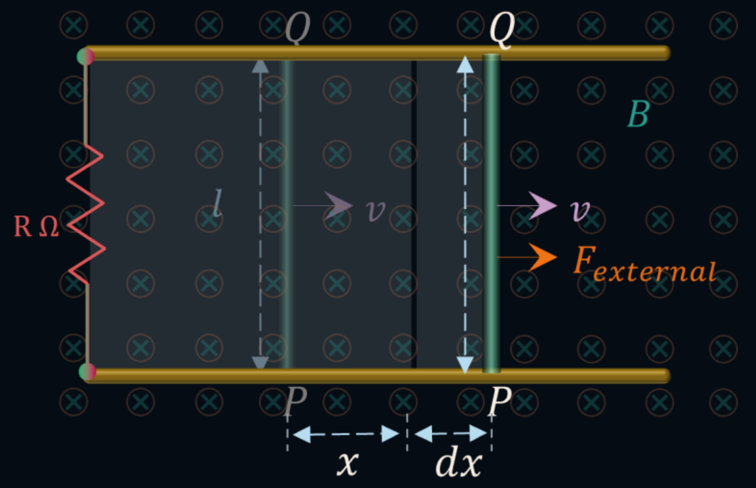
$\boxed{\vec{v} \parallel \vec{B}}$

$\varepsilon = 0$

Motional EMF:

A rod of length l having resistance r is sliding on frictionless rails which have $zero$ resistance.

External force required to keep the rod moving with constant velocity v.



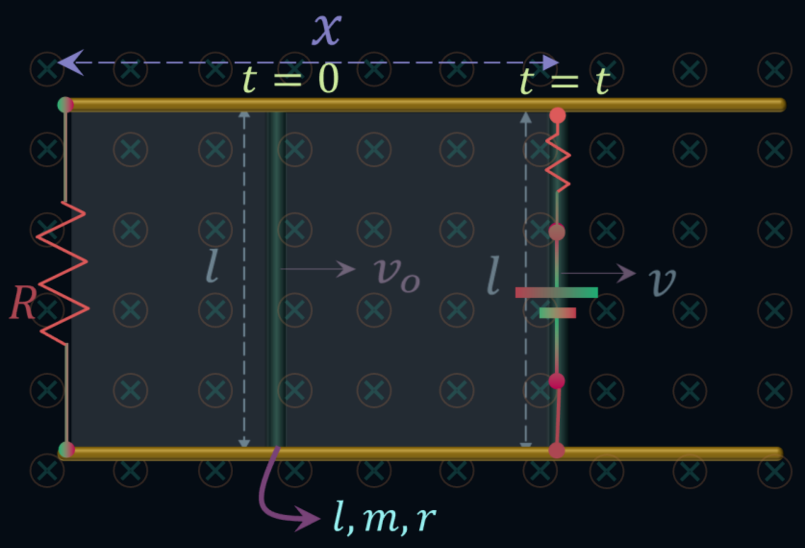
$$F_{ext} = \frac{B^2 l^2 v}{R + r}$$

Power $$P_i = P_o = \frac{B^2 l^2 v^2}{R + r}$$

Acceleration of the rod

$$a = -\frac{B^2 l^2 v}{m(R + r)}$$

$-ve$ sign implies retardation



$'v'$ as a function of displacement $'x'$

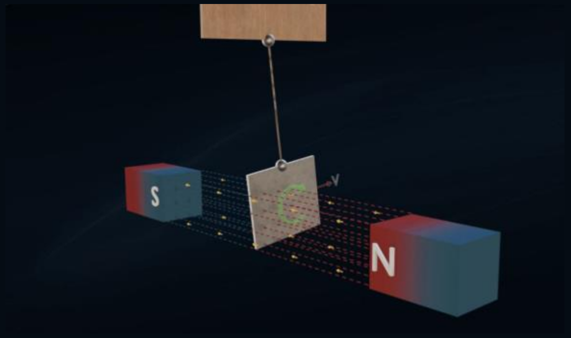
$$v = v_o - \frac{B^2 l^2}{m(R + r)} x$$

Distance covered by the rod before it stops

$$x = \frac{v_o m(R + r)}{B^2 l^2}$$

Eddy currents:

Eddy currents are loops of electrical current induced within conductors by a changing magnetic field in the conductor according to Faraday's law of induction.



Advantages: Electromagnetic brakes, electromagnetic damping in galvanometers

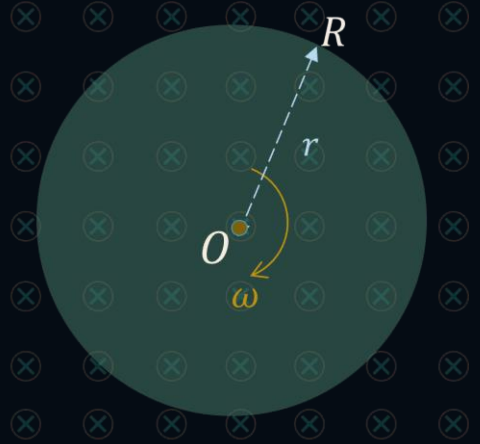
Disadvantages: Overheating of metallic cores of electric devices

By introducing slots in the conducting plate one can reduce the area available for the generation of eddy currents.

Motional EMF in rotating conducting rod:

$$\varepsilon = \frac{B\omega l^2}{2} \qquad V_o - V_X = \frac{B\omega l^2}{2}$$

Motional EMF in rotating conducting disc:



$$V_R - V_O = \frac{B\omega r^2}{2}$$

Motional EMF in a rotating arbitrary shaped conducting wire

$$\varepsilon = \frac{B\omega l^2}{2} \qquad V_O - V_A = \frac{B\omega l^2}{2}$$

l: Distance between the ends of wire

Electrical components

Active electrical component

Active elements generate energy for any device. It is the core component to operate the device. Active components control the charge flow in electrical or electronic circuits.

Example : Battery

Passive electrical component

A passive element is an electrical component that does not generate power but instead dissipates, stores and/or releases it.

Example : Resistor, capacitor, inductor

Inductance

The induced emf across a coil is directly proportional to the rate of change of current through it.

$$V \propto \frac{di}{dt} \qquad \text{Induced EMF: } \varepsilon = -L\frac{di}{dt}$$

Self inductance

Self-inductance is the property of the current-carrying coil that resists or opposes the change in current flowing through it.

Coefficient of self inductance = $L = \frac{N\phi}{i}$ Unit = $\frac{\text{Weber}}{\text{Ampere}}$ Or Henry (H) $L = [ML^2T^{-2}A^{-2}]$

Self inductance depends on:

- area of cross-section of the coil.
- length of the coil.
- number of turns per unit length in the coil.
- permeability of the core material.

Coefficient self inductance for a long solenoid

$B = \mu_o ni$ $\phi = \mu_o ni\pi R^2$ $\phi = Li$

$L = \mu_0 n^2 \pi R^2 l$

l = Length of coil R = Radius of each coil

N = Total no. of turns ϕ = Flux

n = No. of turns per unit length

Growth of current in LR circuit

$@t = 0$

$$\left[i \neq \frac{E}{R}\right]_{t=0}$$

$@t = t$

$$i = \frac{E}{R}\left[1 - e^{\left(\frac{-Rt}{L}\right)}\right]$$

$@t \to \infty$

$i = i_{\text{max.}}$ or $i_0 = E/R$

Decay of current in LR circuit

$@t \to \infty$

$i = 0$

$@t = t$

$i = i_0 e^{(-Rt/L)}$

$@t = 0$

$[i \neq 0]_{t=0}$

Time constant of LR circuit

Time constant (τ): L/R

In case of growth of current

$$i = \frac{E}{R}[1 - e^{\left(\frac{-Rt}{L}\right)}] \quad @t = \tau \quad i = i_0(1 - e^{(-1)}) = 0.63 \times i_0$$

Time taken by the current to grow from *zero* to $0.632\ i_0$ or 63.2% of its final steady value.

In case of decay of current

$$i = i_0[e^{(-t/\tau)}] \quad @t = \tau \quad i = i_0(e^{(-1)}) = 0.37 \times i_0$$

Time taken by the current to decay from i_0 to $0.37\ i_0$ or 37% of its initial steady value.

Magnetic energy stored in an inductor

$$W = \frac{1}{2} L i^2$$

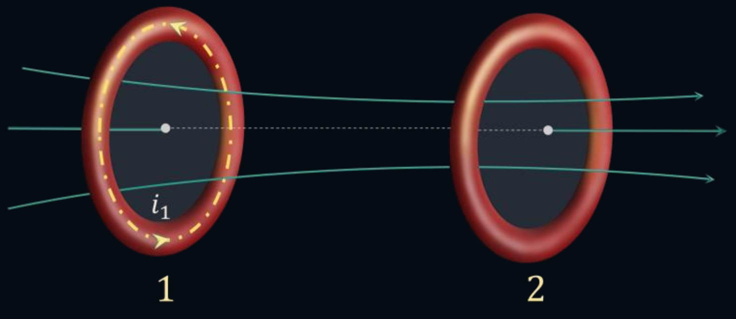
Mutual inductance

$i_{(\text{primary})} \rightarrow \boxed{B_{(\text{primary})}} \rightarrow \boxed{B_{(\text{secondary})}} \rightarrow \boxed{\phi_{(\text{secondary})}} \rightarrow \boxed{e.m.f_{(\text{induced})}} \rightarrow \boxed{i_{(\text{secondary})}}$

$\phi_{21} \propto i_1$ ϕ_{21}: Flux linked with 2 due to magnetic field of 1

$\phi_{21} = M i_1$ M: Coefficient of mutual inductance

$M = \dfrac{\phi_{21}}{i_1}$ Dimension: $[ML^2T^{-2}A^{-2}]$ Unit = $\dfrac{\text{Weber}}{\text{Ampere}}$ Or Henry (H)



Reciprocity theorem

Experiments and calculations that combine Ampere's law and Biot-Savart law confirm that the two constants, M_{12} and M_{21} are equal in the absence of material medium between the two coils.

$M_{12} = M_{21} = M$

Induced EMF in second coil

$$\varepsilon_2 = -M\frac{di_1}{dt} \qquad \varepsilon_1 = -M\frac{di_2}{dt}$$

Applications of mutual inductance: Transformers, electric generator etc.

Combination of inductors

For series combination of $\boldsymbol{n}$ inductors

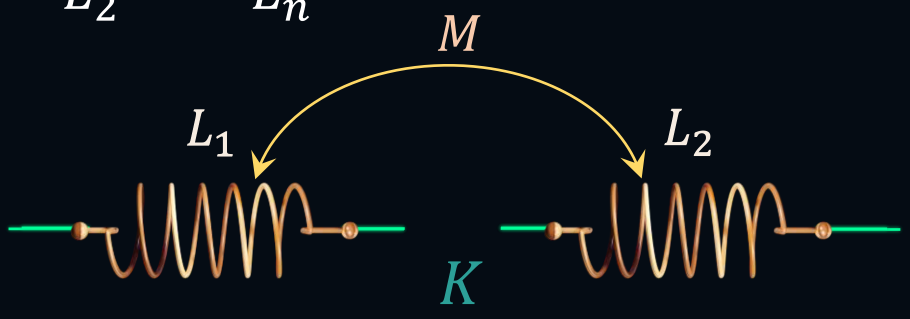
$$L_{eq} = L_1 + L_2 + \cdots L_n$$

For parallel combination of $\boldsymbol{n}$ inductors

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_n}$$

Coupling constant

In general, $M = K\sqrt{L_1 L_2}$ Where K is the coupling constant



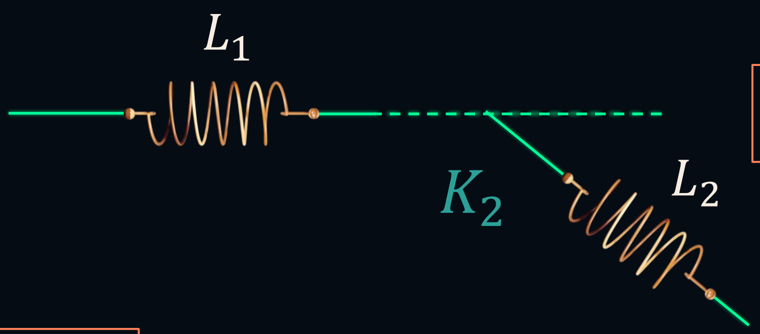
$0 \leq K \leq 1$ $K = 1$ when there is $100\ \%$ flux linkage

K depends on:

(I) Distance between coil

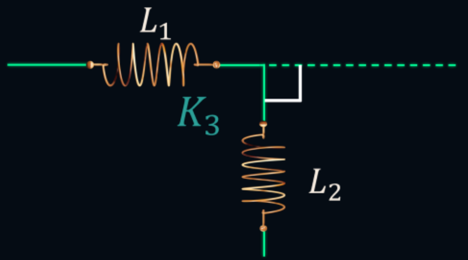
$$\boxed{K_1 < K}$$

(II) Relative orientation of the coils



$$\boxed{K_2 < K}$$

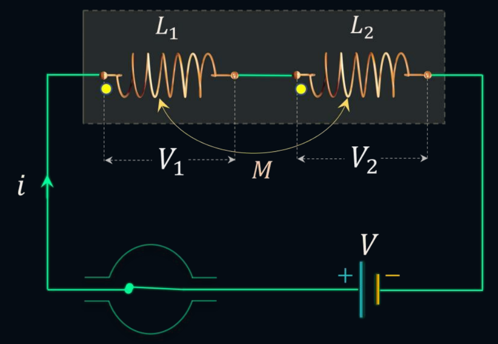
(III) Geometrical factors



$$\boxed{K_3 = 0}$$

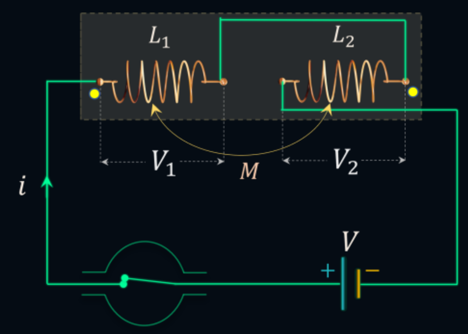
Combination of inductors

Series Combination (Aiding)



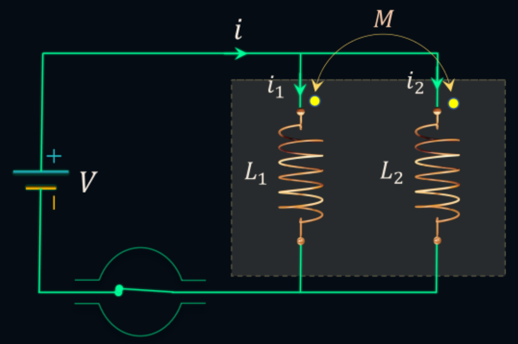
$$L_{eq} = L_1 + L_2 + 2M$$

Series Combination (Opposing)



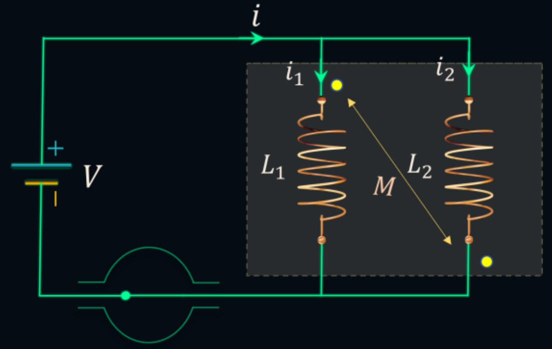
$$L_{eq} = L_1 + L_2 - 2M$$

Parallel Combination (Aiding)



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Parallel Combination (Opposing)

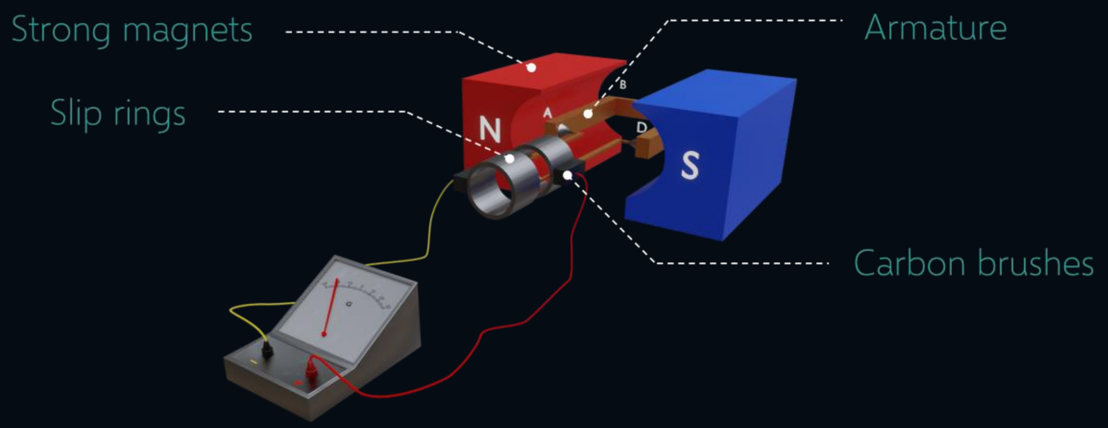


$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

AC generator

AC generators work on the principle of Faraday's law of electromagnetic induction, which states that electromotive force – EMF or voltage – is generated in a current-carrying conductor that cuts a uniform magnetic field.

The current we get in our houses is $220\ V, 50\ Hz$ AC



Magnitude of Induced emf :

$$\phi = BA\cos\omega t \qquad \varepsilon = -\frac{d\phi}{dt} = BA\omega\sin\omega t$$

For N turns, $\varepsilon = NBA\omega\sin\omega t$

$\varepsilon = \varepsilon_0 \sin\omega t$ Where, $\varepsilon_0 = NBA\omega$

Time period :

$$T = \frac{2\pi}{\omega}$$

Magnitude of Induced current :

$$i = \frac{\varepsilon_0}{R}\sin\omega t$$