

BYJU'S Study Planner for Board Term I

(CBSE Grade 12)

Date: 19/11/2021

Subject: Mathematics

Topic : Methods of Differentiation

Class: Standard XII

1. If $y = (2x^2 + 6x)(2x^3 + 5x^2)$, then $\frac{dy}{dx} =$

- A. $20x^4 + 80x^3 + 90x^2$
- B. $20x^4 + 88x^3 + 90x^2$
- C. $16x^4 + 88x^3 + 90x^2$
- D. $16x^4 + 80x^3 + 90x^2$

We have,

$$y = (2x^2 + 6x)(2x^3 + 5x^2)$$

Let,

$$u = 2x^2 + 6x \text{ and } v = 2x^3 + 5x^2$$

Using Product Rule:

$$\begin{aligned} \frac{d(uv)}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ \Rightarrow \frac{du}{dx} &= 4x + 6 \text{ and } \frac{dv}{dx} = 6x^2 + 10x \end{aligned}$$

Therefore

$$\begin{aligned} \frac{dy}{dx} &= \frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \\ &= (2x^2 + 6x)(6x^2 + 10x) + (2x^3 + 5x^2)(4x + 6) \\ &= 20x^4 + 88x^3 + 90x^2 \end{aligned}$$

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2. If $y = \frac{4x^2}{x^3 + 3}$, then $\frac{dy}{dx} =$

- A. $\frac{-4x^4 + 24x}{(x^3 + 3)}$
- B. $\frac{-4x^4 + 24x}{(x^3 + 3)^2}$
- C. $\frac{4x^3 + 24x}{(x^3 + 3)^2}$
- D. $\frac{-4x^4}{(x^3 + 3)}$

We have,

$$y = \frac{4x^2}{x^3 + 3}$$

Let,

$$u = 4x^2 \text{ and } v = x^3 + 3$$

$$\frac{du}{dx} = 8x \text{ and } \frac{dv}{dx} = 3x^2$$

Using quotient rule:

$$\begin{aligned}\frac{dy}{dx} &= \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \\ &= \frac{(x^3 + 3)(8x) - (4x^2)(3x^2)}{(x^3 + 3)^2} \\ &= \frac{8x^4 + 24x - 12x^4}{(x^3 + 3)^2} \\ &= \frac{-4x^4 + 24x}{(x^3 + 3)^2}\end{aligned}$$

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3. If $y = e^{\tan 3x}$, then $\frac{dy}{dx} =$

- A. $e^{\tan 3x} \times \sec^2 3x$
- B. $3e^{\tan 3x} \times \sec^2 3x$
- C. $3e^{\tan 3x} \times \tan 3x$
- D. $3e^{\tan 3x} \times \sec 3x$

We have,

$$y = e^{\tan 3x}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^{\tan 3x}) \\ &= e^{\tan 3x} \times \frac{d}{dx}(\tan 3x) \quad [\text{using chain rule}] \\ &= e^{\tan 3x} \times \sec^2 3x \times \frac{d}{dx}(3x) \\ &= 3e^{\tan 3x} \times \sec^2 3x\end{aligned}$$

4. If $y = 2^{x^3}$, then $\frac{dy}{dx} =$

- A. $3x^2(2^{x^3})$
- B. $3x^2(2^{x^3} \log 2)$
- C. $2^{x^3} \log 2$
- D. $6^{x^2} \log 2$

We have,

$$y = 2^{x^3}$$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}2^{x^3} \\ &= 2^{x^3} \times \log 2 \times \frac{d}{dx}(x^3) \quad [\text{Using chain rule}] \\ &= 3x^2 \times 2^{x^3} \times \log 2 \\ &= 3x^2(2^{x^3} \log 2)\end{aligned}$$

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5. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$, then $\frac{dy}{dx}$ at $x = 2$ is equal to

A. $2\sqrt{31}$

B. $4\sqrt{7}$

C. $4\sqrt{31}$

D. $2\sqrt{15}$

$$y = f(x^2) \Rightarrow \frac{dy}{dx} = f'(x^2) \cdot 2x$$

$$\therefore \frac{dy}{dx} = \sqrt{2x^4 - 1} \cdot 2x \quad [\because f'(x) = \sqrt{2x^2 - 1}]$$

at $x = 2$, $\frac{dy}{dx} = \sqrt{32 - 1} \cdot 4 = 4\sqrt{31}$

6. If $y = \log_7(2x - 3)$, then $\frac{dy}{dx} =$

A. $\frac{2}{(2x - 3) \log 7}$

B. $\frac{2}{(2x - 3)}$

C. $\frac{1}{(2x - 3) \log 7}$

D. $\frac{1}{(2x - 3)}$

We have,

$$y = \log_7(2x - 3)$$

$$\Rightarrow y = \frac{\log(2x - 3)}{\log 7} \quad [\because \log_a b = \frac{\log b}{\log a}]$$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\log 7} \frac{d}{dx}(\log(2x - 3)) \\ &= \frac{1}{\log 7} \times \frac{1}{(2x - 3)} \frac{d}{dx}(2x - 3) \quad [\text{Using chain rule}] \\ &= \frac{2}{(2x - 3) \log 7} \end{aligned}$$

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7. If $y = e^{\sin \sqrt{x}}$, then $\frac{dy}{dx} =$

- A. $\frac{e^{\cos \sqrt{x}} \sin \sqrt{x}}{2\sqrt{x}}$
- B. $\frac{e^{\sin \sqrt{x}}}{2\sqrt{x}}$
- C. $\frac{e^{\sin \sqrt{x}} \cos \sqrt{x}}{2\sqrt{x}}$
- D. $\frac{e^{\sin \sqrt{x}} \cos \sqrt{x}}{\sqrt{x}}$

We have,

$$y = e^{\sin \sqrt{x}}$$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} e^{\sin \sqrt{x}} \\ &= e^{\sin \sqrt{x}} \times \frac{d}{dx} (\sin \sqrt{x}) && [\text{Using chain rule}] \\ &= e^{\sin \sqrt{x}} \times \cos \sqrt{x} \times \frac{d}{dx} \sqrt{x} && [\text{Using chain rule}] \\ &= e^{\sin \sqrt{x}} \times \cos \sqrt{x} \times \frac{1}{2\sqrt{x}} \\ &= \frac{e^{\sin \sqrt{x}} \cos \sqrt{x}}{2\sqrt{x}} \end{aligned}$$

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8. If $y = 3^{\log_9(1+\tan^2 x)}$, $x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{dy}{dx} =$

- A. $\tan x$
- B. $\sec x$
- C. $\sec x \cdot \tan x$
- D. $\sec^2 x$

$$\begin{aligned}
y &= 3^{\log_9(1+\tan^2 x)} \\
&= 3^{\frac{1}{2}\log_3(1+\tan^2 x)} \\
&= 3^{\log_3(1+\tan^2 x)^{\frac{1}{2}}} \\
&= \sqrt{1 + \tan^2 x} \\
&= |\sec x| = \sec x \quad \left(\because x \in \left(0, \frac{\pi}{2}\right)\right) \\
\Rightarrow \frac{dy}{dx} &= \sec x \cdot \tan x
\end{aligned}$$

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9. If $g(x) = \frac{1}{x\sqrt{x^2 + 1}}$, then $g'(x) =$

A. $\frac{-2x^2 - 1}{x^2(x^2 + 1)^{\frac{5}{2}}}$

B. $\frac{2x^2 + 1}{x^2(x^2 + 1)^{\frac{3}{2}}}$

C. $\frac{-2x^2 - 1}{x^2(x^2 + 1)^{\frac{3}{2}}}$

D. $\frac{2x^2 + 1}{x^2(x^2 + 1)^{\frac{5}{2}}}$

$$g(x) = \frac{1}{x\sqrt{x^2 + 1}}$$

$$\left(\because \frac{d}{dx} \left(\frac{1}{f(x)} \right) = -\frac{1}{(f(x))^2} \times f'(x) \right)$$

$$\Rightarrow g'(x) = \frac{-1}{(x\sqrt{x^2 + 1})^2} \times \left(\sqrt{x^2 + 1} + x \cdot \frac{2x}{2\sqrt{x^2 + 1}} \right)$$

$$= \frac{-\left(\frac{x^2 + 1 + x^2}{\sqrt{x^2 + 1}} \right)}{(x\sqrt{x^2 + 1})^2}$$

$$= \frac{-2x^2 - 1}{x^2(x^2 + 1)^{\frac{3}{2}}}$$

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10. If $y = a \sin x + b \cos x$, (where a, b are constant), then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a

- A. function of x
- B. function of y
- C. function of x and y
- D. Constant

$$y = a \sin x + b \cos x$$

Differentiate it with respect to x ,

$$\frac{dy}{dx} = a \cos x - b \sin x$$

$$\begin{aligned} \left(\frac{dy}{dx}\right)^2 &= (a \cos x - b \sin x)^2 \\ &= a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cdot \cos x \end{aligned}$$

Now,

$$\begin{aligned} y^2 &= (a \sin x + b \cos x)^2 \\ &= a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cdot \cos x \\ \Rightarrow \left(\frac{dy}{dx}\right)^2 + y^2 &= a^2 (\sin^2 x + \cos^2 x) + b^2 (\sin^2 x + \cos^2 x) \end{aligned}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 + y^2 = (a^2 + b^2) = \text{constant}$$

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11. The derivative of $f(x) = |x|^3$ at $x = 0$ is

- A. 0
- B. 1
- C. -1
- D. Not defined

$$f(x) = |x|^3 \\ = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2, & x \geq 0 \\ -3x^2, & x < 0 \end{cases}$$

$$\therefore f'(x)|_{x=0} = 0$$

12. If $y = (1 + x^2) \tan^{-1} x - x$, then $\frac{dy}{dx}$ is equal to

- A. $\tan^{-1} x$
- B. $2x \tan^{-1} x$
- C. $2x \tan^{-1} x - 1$
- D. $\frac{2x}{\tan^{-1} x}$

$$y = (1 + x^2) \tan^{-1} x - x$$

By product rule

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= (1 + x^2) \frac{d}{dx}(\tan^{-1} x) + (\tan^{-1} x) \frac{d}{dx}(1 + x^2) - \frac{d}{dx}(x) \\ \Rightarrow \frac{dy}{dx} &= (1 + x^2) \cdot \frac{1}{(1 + x^2)} + \tan^{-1} x(2x) - 1 \\ \Rightarrow \frac{dy}{dx} &= 2x \tan^{-1} x \end{aligned}$$

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13. If $y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$, then $\frac{dy}{dx}$ is equal to

- A. $\frac{1}{2}$
- B. $-\frac{1}{2}$
- C. 1
- D. -1

Convert $\frac{\cos x}{1 + \sin x}$ in terms of tan and proceed.

$$\begin{aligned}
 & \because \frac{\cos x}{1 + \sin x} = \frac{\sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)} \\
 &= \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \\
 &\therefore \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) \\
 &\therefore x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\
 &\therefore \frac{\pi}{4} - \frac{x}{2} \in \left(0, \frac{\pi}{2}\right) \\
 &\Rightarrow y = \left(\frac{\pi}{4} - \frac{x}{2}\right) \\
 &\Rightarrow \frac{dy}{dx} = -\frac{1}{2}
 \end{aligned}$$

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14. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \dots}}}}$, then $\frac{dy}{dx} =$

- A. $\frac{y+x}{y^2 - 2x}$
- B. $\frac{y^3 - x}{2y^2 - 2xy - 1}$
- C. $\frac{y^3 + x}{2y^2 - x}$
- D. $\frac{y^2 - x}{2y^3 - 2xy - 1}$

We have,

$$\begin{aligned} y &= \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \dots}}}}, \\ \Rightarrow y^2 &= x + \sqrt{y + \sqrt{x + \sqrt{y + \dots}}} \\ \Rightarrow y^2 &= x + \sqrt{y+y} \\ \Rightarrow y^2 - x &= \sqrt{2y} \\ \Rightarrow (y^2 - x)^2 &= 2y \end{aligned}$$

On differentiating both sides w.r.t. x , we get

$$\begin{aligned} 2(y^2 - x) \left(2y \frac{dy}{dx} - 1 \right) &= 2 \frac{dy}{dx} \\ (y^2 - x) \left(2y \frac{dy}{dx} - 1 \right) &= \frac{dy}{dx} \\ \Rightarrow 2y(y^2 - x) \frac{dy}{dx} - (y^2 - x) &= \frac{dy}{dx} \\ \Rightarrow 2y(y^2 - x) \frac{dy}{dx} - \frac{dy}{dx} &= y^2 - x \\ \Rightarrow \frac{dy}{dx}(2y^3 - 2xy - 1) &= (y^2 - x) \\ \Rightarrow \frac{dy}{dx} &= \frac{y^2 - x}{(2y^3 - 2xy - 1)} \end{aligned}$$

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15. If $x = \frac{1-t}{1+t}$ and $y = \frac{2t}{1+t}$, then $\frac{d^2y}{dx^2}$ is equal to

- A. $\frac{2t}{(1+t)^2}$
- B. $\frac{1}{(1+t)^4}$
- C. $\frac{2t^2}{(1+t)^2}$
- D. 0

Given : $x = \frac{1-t}{1+t}$ and $y = \frac{2t}{1+t}$

We can write

$$x + y = 1$$

Differentiating w.r.t. x , we get

$$\Rightarrow 1 + \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -1$$

Differentiating w.r.t. x , we get

$$\therefore \frac{d^2y}{dx^2} = 0$$

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16. The derivative of $\ln x$ with respect to $\cot x$ is

A. $\frac{-\sin^2 x}{x}$

B. $\frac{\cos^2 x}{x}$

C. $\frac{-\sin^3 x}{x}$

D. $\frac{-\sin^2 x}{x^2}$

Let $y = \ln x$ and $z = \cot x$

Now,

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dz}{dx} = -\operatorname{cosec}^2 x$$

$$\Rightarrow \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{-1}{x \cdot \operatorname{cosec}^2 x}$$

$$\therefore \frac{dy}{dz} = \frac{-\sin^2 x}{x}$$

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17. If $y = a \cos(\ln x) + b \sin(\ln x)$, then $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} =$

- A. 0
- B. y
- C. $2y$
- D. $-y$

$$y = a \cos(\ln x) + b \sin(\ln x)$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = -a \sin(\ln x) \cdot \frac{1}{x} + b \cos(\ln x) \cdot \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = -a \sin(\ln x) + b \cos(\ln x)$$

Again differentiating w.r.t. x , we get

$$\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y$$

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18. If $x = a \cos \theta$, $y = b \sin \theta$, then $\frac{d^2y}{dx^2}$ is

- A. $-\frac{3b}{a^3} \operatorname{cosec}^4 \theta \cot^4 \theta$
- B. $\frac{b}{a^2} \operatorname{cosec}^2 \theta$
- C. $-\frac{b}{a^2} \operatorname{cosec}^3 \theta$
- D. None of these

We have $y = b \sin \theta$, $x = a \cos \theta$,

So,

$$\frac{dy}{d\theta} = b \cos \theta, \quad \frac{dx}{d\theta} = -a \sin \theta$$

$$\Rightarrow \frac{dy}{dx} = -\frac{b}{a} \cot \theta$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a} \operatorname{cosec}^2 \theta \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a} \operatorname{cosec}^2 \theta \times \frac{-1}{a \sin \theta}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{b}{a^2} \operatorname{cosec}^3 \theta$$

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19. If $x = \frac{e^t + e^{-t}}{2}$, $y = \frac{e^t - e^{-t}}{2}$, then $\frac{dx}{dy} =$

A. $-\frac{x}{y}$

B. $\frac{y}{x}$

C. $\frac{x}{y}$

D. $-\frac{y}{x}$

We have,

$$x = \frac{e^t + e^{-t}}{2},$$

$$\Rightarrow \frac{dx}{dt} = \frac{e^t - e^{-t}}{2} = y,$$

$$\text{and, } y = \frac{e^t - e^{-t}}{2}$$

$$\Rightarrow \frac{dy}{dt} = \frac{e^t + e^{-t}}{2} = x$$

$$\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}}$$

$$= y \times \frac{1}{x}$$

$$= \frac{y}{x}$$

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20. If $y^x = x^{\sin y}$, then $\frac{dy}{dx} =$

- A. $\frac{y}{x} \left[\frac{x \ln y - \sin y}{y \ln x \cdot \cos y - x} \right]$
- B. $\frac{y}{x} \left[\frac{x \ln y + \sin y}{y \ln x \cdot \cos y + x} \right]$
- C. $\frac{-y}{x} \left[\frac{x \ln y - \sin y}{y \ln x \cdot \cos y - x} \right]$
- D. $\frac{y}{x} \left[\frac{x \ln y - \sin y}{y \ln x \cdot \cos y + x} \right]$

Given,

$$y^x = x^{\sin y}$$

Taking ln on both sides, we get
 $x \ln y = \sin y \ln x$.

Differentiate w.r.t. x

$$\ln y + x \cdot \frac{1}{y} \cdot \frac{dy}{dx} = \sin y \cdot \frac{1}{x} + \ln x \cdot \cos y \cdot \frac{dy}{dx}$$

$$\Rightarrow \left(\ln y - \frac{\sin y}{x} \right) = \frac{dy}{dx} \left[\cos y \cdot \ln x - \frac{x}{y} \right]$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} \left[\frac{x \ln y - \sin y}{y \ln x \cdot \cos y - x} \right]$$