BYJU'S Study Planner for Board Term I
(CBSE Grade 12)

Date: 21/11/2021

Subject: Mathematics

Topic : Application of Derivative   Class: Standard XII

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1. The tangent to the curve $y = xe^{x^2}$ is passing through the point $(1, e)$ and also passes through the point :

   ✗ **A.** $(2, 3e)$

   ✗ **B.** $(3, 6e)$

   ✓ **C.** $\left(\frac{4}{3}, 2e\right)$

   ✗ **D.** $\left(\frac{5}{3}, 2e\right)$

   Given curve, $y = xe^{x^2}$

   And its slope of tangent $= \frac{dy}{dx}$

   $$\begin{aligned}\frac{dy}{dx} &= x \cdot 2x \cdot e^{x^2} + e^{x^2} \\ &= 2x^2e^{x^2} + e^{x^2} \\ &= e^{x^2}(1 + 2x^2)\end{aligned}$$

   At point $(1, e)$, $\frac{dy}{dx} = 3e$

   For given point $(1, e)$ equation of the tangent is

   $(y - e) = 3e(x - 1)$

   Hence, the point $\left(\frac{4}{3}, 2e\right)$ lies on the above line.

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2. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point:

A. $\left(-\frac{1}{2},-\frac{1}{2}\right)$

B. $\left(\frac{1}{2},\frac{1}{2}\right)$ ✔

C. $\left(\frac{1}{2},-\frac{1}{3}\right)$

D. $\left(\frac{1}{2},\frac{1}{3}\right)$

$$y=\frac{x+6}{(x-2)(x-3)}$$

Coordinates of point of intersection with y-axis is $(0,1)$

$$y=\frac{x+6}{x^2-5x+6}$$

$$\Rightarrow y'=\frac{(x^2-5x+6)-(2x-5)(x+6)}{(x^2-5x+6)^2}$$

$$\Rightarrow y'\Big|_{x=0}=\frac{6-(-30)}{36}=1$$

Then, slope of normal at $(0,1)$ is -1

Equation of normal passing through $(0,1)$ is $y-1=-1(x-0)$

i.e., $x+y=1$

Thus, the normal passes through $\left(\frac{1}{2},\frac{1}{2}\right)$.

3. Let C be a curve given by $y(x) = 1 + \sqrt{4x-3}, x > \frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is:

A. $(3, -4)$

B. $(1, 7)$ ✔

C. $(2, 3)$

D. $(4, -3)$

$y = 1 + \sqrt{4x-3}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4x-3}} \times 4 = \frac{2}{3}$

$\Rightarrow x = 3, y = 1 + 3 = 4$

Slope of the normal $= -\frac{3}{2}$

Equation of the normal is,

$y - 4 = -\frac{3}{2}(x - 3)$

$\Rightarrow 3x + 2y - 17 = 0$

$\Rightarrow$ Which passes through $(1, 7)$

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4. The approximate change in volume V of a cube of side x meters caused by increasing the side by 2% is:

A. $0.03x^3$ cubic meter

B. $0.04x^3$ cubic meter

C. $0.06x^3$ cubic meter ✔

D. $0.08x^3$ cubic meter

We have volume $V = x^3$

$\Rightarrow dV = \left(\frac{dV}{dx}\right)\triangle x$

$= (3x^2)\triangle x$

$= (3x^2)(0.02x)$

$= 0.06x^3m^3$

Thus the approximate change in volume V is $0.06x^3m^3$

5. The approximate value of $f(5.001)$ where $f(x) = x^3 - 7x^2 + 15$, is:

A. -34.995 ✔

B. 34.995

C. -35.005

D. 35.005

Firstly break the number 5.001 as $x = 5$ and $\triangle x = 0.001$ and use the relation $f(x + \triangle x) = f(x) + \triangle x f'(x)$

Consider, $f(x) = x^3 - 7x^2 + 15$

$\Rightarrow f'(x) = 3x^2 - 14x$

$\Rightarrow f(x + \triangle x) = f(x) + \triangle x f'(x)$

$= (x^3 - 7x^2 + 15) + \triangle x(3x^2 - 14x)$

$\Rightarrow f(5.001) = (5^3 - 7 \times 5^2 + 15) + (3 \times 5^2 - 14 \times 5)(0.001)$

$= (125 - 175 + 15) + (75 - 70)(0.001)$

$= -35 + (5)(0.001)$

$= -35 + 0.005 = -34.995$

6. A particle moves in a straight line according to the law $v^2 = 4a(x\sin x + \cos x)$ where v is the velocity of a particle at a distance x from the fixed point. Then the acceleration is

A. $2ax\sin x$

B. $ax\sin x$

C. $ax\cos x$

D. $2ax\cos x$ ✓

Given: $v^2 = 4a(x\sin x + \cos x)$

We know, Acceleration $= \dfrac{dv}{dt}$

$\Rightarrow 2v\dfrac{dv}{dt} = 4a(x\cos x\dfrac{dx}{dt} + \sin x\dfrac{dx}{dt} - \sin x\dfrac{dx}{dt})$

$\Rightarrow \dfrac{dv}{dt} = 2ax\cos x\left(\because \dfrac{dx}{dt} = v\right)$

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7. Coffee is coming out from a conical filter, with height and diameter both are 15 cm into a cylindrical coffee pot with a diameter 15 cm. The rate at which coffee comes out from the filter into the pot is $100 \ \text{cm}^3/\text{min}$.

The rate (in cm/min) at which the level of coffee in the pot is rising at the instance when it is 10 cm, is equal to

- ✔ **A.** $\frac{16}{9\pi}$
- ✖ **B.** $\frac{5}{3\pi}$
- ✖ **C.** $\frac{9}{16\pi}$
- ✖ **D.** $\frac{4}{3\pi}$

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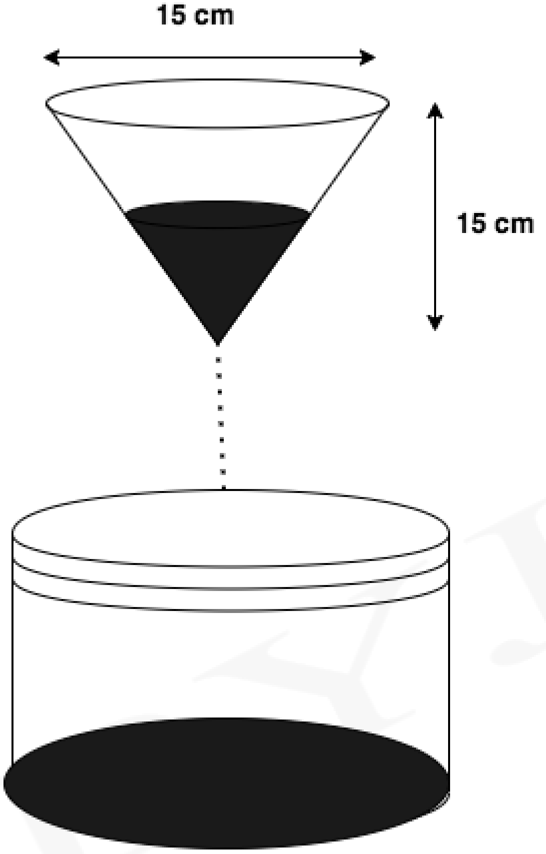
For a cylindrical pot, $V = \pi r^2 h$

$\Rightarrow \frac{dV}{dt} = \pi \left[r^2 \frac{dh}{dt} \right]$

$\{\because r = \text{constant}\}$



$\Rightarrow 100 = \pi r^2 \frac{dh}{dt}$

$\Rightarrow 100 = \pi \cdot \frac{225}{4} \cdot \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{16}{9\pi} \text{cm/min}$

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8. The approximate value of $(25)^{\frac{1}{3}}$ is

A. 2.921

B. 2.923

C. 2.924

D. 2.926

Let $f(x) = x^{\frac{1}{3}}$
$x = 27$ and $\triangle x = -2$
We know, $f(x + \triangle x) = f(x) + \triangle x f'(x)$
$f(27 - 2) = f(27) - 2.\frac{1}{3}.(27)^{-\frac{2}{3}}$
$f(25) = 3 - \frac{2}{27} = \frac{79}{27}$
$\therefore (25)^{\frac{1}{3}} = 2.926$

9. A particle moves on a line according to the law $s = at^2 + bt + c$. If the displacement after one second is 16 cm, the velocity after 2 seconds is 24 cm/sec and the acceleration is 8 cm/sec^2, then (a, b, c)

A. $(4, 8, 4)$

B. $(4, 4, 8)$

C. $(8, 4, 4)$

D. $(8, 8, 4)$

We have,
$s = at^2 + bt + c$
when $t = 1,\ a + b + c = 16$
$v = \frac{ds}{dt} = 2at + b$
when $t = 2,\ 4a + b = 24$
acceleration$= \frac{dv}{dt} = 2a = 8 \Rightarrow a = 4, b = 8, c = 4$

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10. If the rate of change in the volume of a sphere is equal to the rate of change in its radius, then the radius is equal to

- ✗ **A.** $\frac{1}{2\pi}$
- ✓ **B.** $\frac{1}{2\sqrt{\pi}}$
- ✗ **C.** $\frac{1}{\sqrt{2\pi}}$
- ✗ **D.** $\frac{2}{\pi}$

Volume of sphere, $V = \frac{4}{3}\pi r^3$

Given, $\frac{dV}{dt} = \frac{dr}{dt}$

$\Rightarrow \frac{4\pi}{3} \times 3r^2 \times \frac{dr}{dt} = \frac{dr}{dt}$

$\Rightarrow r^2 = \frac{1}{4\pi} \Rightarrow r = \frac{1}{2\sqrt{\pi}}$

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11. The normal to the curve $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$ at any point θ is such that

- ✗ **A.** it makes a constant angle with the $x-$ axis.
- ✗ **B.** it passes through origin.
- ✓ **C.** it is at a constant distance from the origin.
- ✗ **D.** none of these

$x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$

$$\frac{dy}{d\theta}=a\theta\sin\theta$$

$$\frac{dx}{d\theta}=a\theta\cos\theta$$

$$\Rightarrow \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\tan\theta$$

$\therefore$ Slope of the normal$=-\cot\theta$

Therefore, equation of the normal is

$$y-a(\sin\theta-\theta\cos\theta)=-\frac{\cos\theta}{\sin\theta}(x-a(\cos\theta+\theta\sin\theta))$$

$$y\sin\theta-a\sin^2\theta+a\theta\sin\theta\cos\theta=-x\cos\theta+a\cos^2\theta+a\theta\sin\theta\cos\theta$$

$$\Rightarrow x\cos\theta+y\sin\theta=a.$$

As θ varies, inclination is not constant. It does not pass through $(0,0)$

Its distance from the origin is $\left|\frac{a}{\sqrt{\cos^2\theta+\sin^2\theta}}\right|=a$ is constant

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12. If $3x + 2y = 1$ acts as a tangent to $y = f(x)$ at $x = \frac{1}{2}$ and if $p = \lim_{x \to 0} \frac{x(x-1)}{f\left(\frac{e^{2x}}{2}\right) - f\left(\frac{e^{-2x}}{2}\right)}$, then $\sum_{r=1}^{\infty} p^r =$______

- ✔ **A.** $\frac{1}{2}$
- ✘ **B.** $\frac{1}{3}$
- ✘ **C.** $\frac{1}{6}$
- ✘ **D.** $\frac{1}{7}$

Slope of $3x + 2y = 1$ is $\frac{-3}{2}$

$\Rightarrow f'\left(\frac{1}{2}\right) = \frac{-3}{2}$

Using L-Hospital's rule we will get

$$p = \lim_{x \to 0} \frac{2x - 1}{f'\left(\frac{e^{2x}}{2}\right) e^{2x} + f'\left(\frac{e^{-2x}}{2}\right) e^{-2x}} = \frac{-1}{\frac{-3}{2} - \frac{3}{2}}$$

$$= \frac{-1}{-3} = \frac{1}{3} \Rightarrow \sum_{r=1}^{\infty} p^r = \frac{\frac{1}{3}}{1 - \left(\frac{1}{3}\right)} = \frac{1}{2}$$

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13. The normal to the curve $x = a(1+\cos\theta), y = a\sin\theta$ at θ always passes through the fixed point

- ✔ **A.** $(a, 0)$
- ✖ **B.** $(0, a)$
- ✖ **C.** $(0, 0)$
- ✖ **D.** (a, a)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{\cos\theta}{\sin\theta}$$

Slope of normal= $\dfrac{\sin\theta}{\cos\theta}$

The equation of the normal at $[a(1+\cos\theta), a\sin\theta]$ is

$$(y - a\sin\theta) = \frac{\sin\theta}{\cos\theta}[x - a(1+\cos\theta)]$$
$$y\cos\theta - a\sin\theta\cos\theta = x\sin\theta - a\sin\theta - a\sin\theta\cos\theta$$
$$y\cos\theta = (x-a)\sin\theta$$

Using family of lines we get fixed point $= (a, 0)$

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14. If m is the slope of a tangent to the curve $e^y = 1 + x^2$, then

A. $|m| > 1$

B. $m > 1$

C. $m > -1$

D. $|m| \leq 1$ ✓

$\frac{dy}{dx} = m = \frac{2x}{1+x^2}$
$\Rightarrow |m| = \frac{2|x|}{1+x^2}$
$(1-x)^2 \geq 0$
$\Rightarrow 1 + x^2 \geq 2x$
$(1+x)^2 \geq 0$
$\Rightarrow 1 + x^2 \geq -2x$
$\therefore 1 + x^2 \geq 2|x|$
$\Rightarrow |m| \leq 1$

15. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, which is parallel to the $x-$ axis, is

A. $y = 8$

B. $y = 0$

C. $y = 3$ ✓

D. $y = 2$

$\frac{dy}{dx} = 0$
$\Rightarrow 1 - \frac{8}{x^3} = 0$
$\Rightarrow x = 2$
$y = 2 + \frac{4}{2^2}$
$\Rightarrow y = 3$
Equation of Tangent at $(2, 3)$ is
$y - 3 = 0(x - 2) \Rightarrow y = 3$

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16. The number of tangent(s) to the curve $y = \cos(x+y), -2\pi \le x \le 2\pi$, that is (are) perpendicular to the line $2x - y = 3$ is

A. 1

B. 2 ✔

C. 3

D. 4

$\frac{dy}{dx} = -\sin(x+y)\left(1 + \frac{dy}{dx}\right)$

Substituting $\frac{dy}{dx} = -\frac{1}{2}$ we get

$\sin(x+y) = 1$

$\Rightarrow y = \cos(x+y) = 0$

$\Rightarrow \sin x = 1 \Rightarrow x = \frac{\pi}{2}, \frac{-3\pi}{2}$

No. of points $= 2$

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17. If $2a + 3b + 6c = 0$, then atleast one root of the equation $ax^2 + bx + c = 0$ lies in the interval

- ✔ **A.** $(0, 1)$
- ✖ **B.** $(1, 2)$
- ✖ **C.** $(2, 3)$
- ✖ **D.** $(1, 3)$

Given, $2a + 3b + 6c = 0$
Let $f'(x) = ax^2 + bx + c$

$$f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$$

$$\Rightarrow f(x) = \frac{2ax^3 + 3bx^2 + 6cx + 6d}{6}$$

Now, $f(1) = \dfrac{2a + 3b + 6c + 6d}{6} = \dfrac{6d}{6} = d$

and $f(0) = \dfrac{6d}{6} = d$

$\therefore f(0) = f(1)$

$\Rightarrow f'(x)$ will vanish atleast once betweeen 0 and 1 (by Rolle's theorem)

$\therefore$ atleast one of the roots of the equation $ax^2 + bx + c = 0$ lies between 0 and 1

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18. If $f(x) = x^3 + bx^2 + ax$ satisfies the condition of Rolle's theorem on $[1, 3]$ with $c = 2 + \frac{1}{\sqrt{3}}$. Then $(a + b) =$

A. 0

B. 3

C. -4

D. 5 ✔

$f(x) = x^3 + bx^2 + ax$
$\Rightarrow f(1) = a + b + 1 \cdots (i)$
$f(3) = 3a + 9b + 27 \cdots (ii)$
$f(1) = f(3) \Rightarrow a + 4b + 13 = 0 \cdots (iii)$
$\Rightarrow f'(c) = 0,$
$\Rightarrow 3c^2 + 2bc + a = 0$
putting the value of c

$$\Rightarrow 3\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$$

$$\Rightarrow 13 + 4\sqrt{3} + 4b + \frac{2b}{\sqrt{3}} + a = 0$$

Comparing with (iii), we get

$$4\sqrt{3} + \frac{2b}{\sqrt{3}} = 0$$

$\Rightarrow b = -6$
Putting b in(iii)
$\Rightarrow a = 11$
$\therefore a + b = 5$

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19. Let f be differentiable for all x. If $f(1)=-2$ and $f'(x) \geq 2$ for all $x \in [1,6]$, then

A. $f(6)<8$

B. $f(6) \geq 8$ ✔

C. $f(6) \geq 5$

D. $f(6) \leq 5$

Given that $f(1)=-2$ and $f'(x) \geq 2 \;\forall\; x \in [1,6]$
Lagrange's mean value theorem states that if $f(x)$ be continuous on $[a,b]$ and differentiable on (a,b) then there exists some c between a and b such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

Given that f is differentiable for all x. Therefore Lagrange's mean value theorem can be applied.

Therefore, $f'(c)=\dfrac{f(6)-f(1)}{6-1} \geq 2$

$\Rightarrow f(6)-f(1) \geq 10$
$\Rightarrow f(6)-(-2) \geq 10$
$\Rightarrow f(6) \geq 10-2$
$\Rightarrow f(6) \geq 8$

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20. Given the function $f(x) = x^3 - 2x^2 - x + 1$. Then the value(s) of c satisfying the conditions of the mean value theorem for the function on the interval $[-2, 2]$, is

A. ± 1

B. $\frac{2}{3}$

C. $-\frac{2}{3}$ ✔

D. 2

The given function is a cubic polynomial and, hence, it is continuous and differentiable everywhere. Therefore the LMVT is applicable to this function.

$f(-2) = (-2)^3 - 2 \cdot (-2)^2 - (-2) + 1$
$f(-2) = -8 - 8 + 2 + 1 = -13$
$f(2) = 2^3 - 2 \cdot 2^2 - 2 + 1$
$f(2) = 8 - 8 - 2 + 1 = -1$
$f'(x) = 3x^2 - 4x - 1$

According to LMVT, we have:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 3c^2 - 4c - 1 = \frac{-1 - (-13)}{2 - (-2)},$$

$\Rightarrow 3c^2 - 4c - 1 = 3,$
$\Rightarrow 3c^2 - 4c - 4 = 0$

$$\Rightarrow (3c + 2)(c - 2) = 0 \Rightarrow c = -\frac{2}{3}, 2$$

Only one root $c_1 = -\frac{2}{3}$ belongs to the open interval $(-2, 2)$