BYJU'S Full Test for Board Term I
(CBSE Grade 12)
MATHEMATICS ANSWER KEYS and SOLUTIONS

ANSWER KEY

Q1	C	Q26	B
Q2	A	Q27	B
Q3	B	Q28	B
Q4	C	Q29	B
Q5	B	Q30	A
Q6	C	Q31	A
Q7	D	Q32	D
Q8	C	Q33	C
Q9	C	Q34	D
Q10	C	Q35	A
Q11	C	Q36	B
Q12	B	Q37	A
Q13	C	Q38	B
Q14	B	Q39	B
Q15	C	Q40	B
Q16	B	Q41	D
Q17	C	Q42	B
Q18	A	Q43	B
Q19	D	Q44	A
Q20	D	Q45	D
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SOLUTIONS

Q1	If $A=\begin{bmatrix} x & 1 & 3 \\ -1 & y & 4 \\ -3 & z & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the values of x, y, z respectively are A. $0, 0, 0$ B. $0, 0, 4$ C. $0, 0, -4$ D. $1, 3, 4$ **Answer:** $(C)\ 0, 0, -4$ **Solution:** Given: $A=\begin{bmatrix} x & 1 & 3 \\ -1 & y & 4 \\ -3 & z & 0 \end{bmatrix}$ A is skew symmetric, then $A^T=-A$ or, $a_{ij}=-a_{ji}$ for all i, j $\Rightarrow a_{11}=x=0$ and $a_{22}=y=0$ $a_{32}=-a_{23} \Rightarrow a_{32}=-4=z$ Hence, the values of x, y and z are $0, 0$ and -4 respectively.
Q2	Let $A=\mathbb{R}-\{3\}$ and $B=\mathbb{R}-\{1\}$. If $f: A \rightarrow B$ is defined by $f(x)=\frac{x-2}{x-3}$, then A. f is bijective B. f is one-one but not onto C. f is onto but not one-one D. f is not a function **Answer:** $(A)\ f$ is bijective **Solution:** For one-one: Let $f(x_1)=f(x_2)$ $\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$ $\Rightarrow x_1x_2-3x_1-2x_2+6=x_1x_2-2x_1-3x_2+6$ $\Rightarrow x_1=x_2$

	$\therefore f$ is one-one function. Now, let $y = \frac{x-2}{x-3}$ $\Rightarrow xy - 3y = x - 2$ $\Rightarrow xy - x = 3y - 2$ $\Rightarrow x = \dfrac{3y-2}{y-1}$ x is defined for all $y \in \mathbb{R} - \{1\}$ $\therefore$ Range of f is $\mathbb{R} - \{1\}$ which is equal to co-domain. $\therefore f$ is onto function. Hence, f is bijective.
Q3	If $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, x \neq 0 \\ a, \quad x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a is A. 4 B. 2 C. $\frac{1}{2}$ D. -2 **Answer:** (B) 2 **Solution:** Given: $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x \neq 0 \\ a, & x = 0 \end{cases}$ Since $f(x)$ is continuous at $x = 0$, $\therefore \lim\limits_{x \to 0} f(x) = f(0) = a$ $\Rightarrow a = \lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$ $\Rightarrow a = \lim\limits_{x \to 0} \dfrac{2\sin^2 x}{x^2}$ $\Rightarrow a = \lim\limits_{x \to 0} \left(\frac{\sin x}{x}\right)^2 \times 2 = 2 \quad \left(\because \lim\limits_{x \to 0} \frac{\sin x}{x} = 1\right)$ $\therefore a = 2$
Q4	The principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is A. $\frac{4\pi}{3}$ B. $\frac{\pi}{3}$ C. $\frac{5\pi}{6}$ D. $-\frac{\pi}{6}$

	Answer: $(C)\ \frac{5\pi}{6}$ **Solution:** We know that $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ for all $x \in [-1, 1]$. $\therefore \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\Rightarrow \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ We know that the range of principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\frac{5\pi}{6} \in [0, \pi]$. $\therefore$ Principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is $\frac{5\pi}{6}$.
Q5	If $\begin{bmatrix} 2 & x \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ z & p \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ z+3 & p-4 \end{bmatrix}$, then the value of x is A. -3 B. 3 C. 4 D. 5 **Answer:** (B) 3 **Solution:** Given: $\begin{bmatrix} 2 & x \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ z & p \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ z+3 & p-4 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 4 & x-3 \\ z+3 & p-4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ z+3 & p-4 \end{bmatrix}$ We know that corresponding entries of equal matrices are equal. $\therefore x = 3$
Q6	The equation of the normal to the curve $y = x^2 + x$ at $x = -2$ is A. $x = 3y + 8$ B. $y = 3x + 8$ C. $3y = x + 8$ D. $3x = y + 8$ **Answer:** $(C)\ 3y = x + 8$ **Solution:** $y = x^2 + x$ The point on the curve is $(-2, 2)$.

	Differentiating with respect to x, we get $\frac{dy}{dx} = 2x + 1$ At $x = -2$, $\frac{dy}{dx} = -4 + 1 = -3$ So, the slope of normal at $(-2, 2)$ is $\frac{1}{3}$. Now, equation of the normal at $(-2, 2)$ is $y - 2 = \frac{1}{3}(x + 2)$ $\therefore 3y = x + 8$
Q7	If $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at every point in its domain, then the value of $f(0)$ is A. 2 B. $-\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{1}{3}$ **Answer:** $(D)\ \frac{1}{3}$ **Solution:** For $f(x)$ to be continuous at every point in its domain, it must be continuous at $x = 0$. $\therefore$ We must have $\lim_{x \to 0} f(x) = f(0)$ $\Rightarrow f(0) = \lim_{x \to 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ Dividing numerator and denominator by x, we get $f(0) = \lim_{x \to 0} \frac{2 - \frac{\sin^{-1} x}{x}}{2 + \frac{\tan^{-1} x}{x}} = \frac{2 - 1}{2 + 1}$ $\therefore f(0) = \frac{1}{3}$

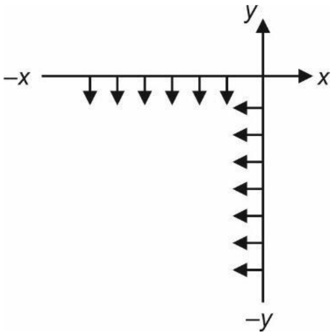
Q8	The derivative of $\tan^{-1}(\tan x)$ with respect to x where $x \in \left(0, \frac{\pi}{2}\right)$, is A. $\frac{1}{1+\tan^2 x}$ B. x C. 1 D. $\frac{\sec^2 x}{1+\tan x}$ **Answer:** $(C)\ 1$ **Solution:** Let $y = \tan^{-1}(\tan x)$ $\Rightarrow y = x \ \forall\, x \in \left(0, \frac{\pi}{2}\right)$ Differentiating with respect to x, we get $\frac{dy}{dx} = 1$
Q9	If $f(x) = x^2 - 2ax + 6$ is strictly increasing function for $x > 0$, then A. $a \in (1, 2)$ B. $a \in (0, \infty)$ C. $a \in (-\infty, 0]$ D. $a \in (0, 7)$ **Answer:** $(C)\ a \in (-\infty, 0]$ **Solution:** Given: $f(x) = x^2 - 2ax + 6$ Differentiating with respect to x, we get $f'(x) = 2x - 2a$ For strictly increasing, $f'(x) > 0$ $\Rightarrow\ 2x - 2a > 0$ $\Rightarrow\ x > a$ but $x > 0$ $\therefore a \in (-\infty, 0]$
Q10	The derivative of $\ln(\sec\theta + \tan\theta)$ with respect to $\sec\theta$ at $\theta = \frac{\pi}{4}$ is A. -1 B. 0 C. 1 D. $\sqrt{2}$ **Answer:** $(C)\ 1$

	Solution: Let $y = \ln(\sec\theta + \tan\theta)$ and $z = \sec\theta$ Now, $\frac{dy}{dz} = \frac{\frac{dy}{d\theta}}{\frac{dz}{d\theta}} \quad \cdots(1)$ We have, $y = \ln(\sec\theta + \tan\theta)$ Differentiating with respect to θ, we get $\frac{dy}{d\theta} = \frac{1}{\sec\theta + \tan\theta} \times \frac{d}{d\theta}(\sec\theta + \tan\theta)$ $\therefore \frac{dy}{d\theta} = \frac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} = \sec\theta$ and $z = \sec\theta$ Differentiating with respect to θ, we get $\frac{dz}{d\theta} = \sec\theta\tan\theta$ From (1), we get $\frac{dy}{dz} = \frac{\sec\theta}{\sec\theta\tan\theta}$ $\Rightarrow \frac{dy}{dz} = \frac{1}{\tan\theta}$ At $\theta = \frac{\pi}{4}$, $\frac{dy}{dz} = 1$
Q11	If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$, then $\frac{dy}{dx}$ is A. $y - x^n$ B. $y - \frac{x^n}{n}$ C. $y - \frac{x^n}{n!}$ D. $y - \frac{x^n}{(n-1)!}$ **Answer:** $(C)\ y - \frac{x^n}{n!}$ **Solution:** Given: $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$ Differentiating with respect to x, we get $\frac{dy}{dx} = 0 + \frac{1}{1!} + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \cdots + \frac{nx^{n-1}}{n!}$

	$\Rightarrow \frac{dy}{dx} = \left\{1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}\right\} - \frac{x^n}{n!}$ $\therefore \frac{dy}{dx} = y - \frac{x^n}{n!}$
Q12	The area of the triangle whose vertices are $A(-3,0)$, $B(3,0)$ and $C(0,3)$, is A. 18 square units B. 9 square units C. 6 square units D. 12 square units **Answer:** (B) 9 square units **Solution:** We know that the area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\Delta = \left\lvert \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right\rvert$ $\Rightarrow \Delta = \left\lvert \frac{1}{2} \begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & 3 & 1 \end{vmatrix} \right\rvert$ $\Rightarrow \Delta = \left\lvert \frac{1}{2}\{-3(-3) - 0 + 1(9-0)\} \right\rvert$ $\Rightarrow \Delta = 9$ square units
Q13	If $x = a\cos^3\theta$ and $y = a\sin^3\theta$ where $\theta \in \left(0, \frac{\pi}{2}\right)$ and a is constant, then $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is equal to A. $\tan^2\theta$ B. $\sec^2\theta$ C. $\sec\theta$ D. $-\sec\theta$ **Answer:** (C) $\sec\theta$ **Solution:** We have, $x = a\cos^3\theta$ Differentiating with respect to θ, we get $\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$

	and $y = a\sin^3\theta$ Differentiating with respect to θ, we get $\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$ Now, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\Rightarrow \frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$ $\Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\tan^2\theta = \sec^2\theta$ $\Rightarrow \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{\sec^2\theta} = \lvert\sec\theta\rvert$ $\therefore \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sec\theta \quad \left[\because \theta \in \left(0,\frac{\pi}{2}\right)\right]$
Q14	Adjoint of matrix $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ is A. $\begin{bmatrix}4 & 2\\3 & 1\end{bmatrix}$ B. $\begin{bmatrix}4 & -2\\-3 & 1\end{bmatrix}$ C. $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ D. $\begin{bmatrix}1 & -2\\-3 & 4\end{bmatrix}$ **Answer:** (B) $A = \begin{bmatrix}4 & -2\\-3 & 1\end{bmatrix}$ **Solution:** Given: $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ $\Rightarrow adj(A) = \begin{bmatrix}4 & -3\\-2 & 1\end{bmatrix}^T$ $\therefore adj(A) = \begin{bmatrix}4 & -2\\-3 & 1\end{bmatrix}$
Q15	Region represented by $x < 0$ and $y < 0$ is A. First quadrant B. Second quadrant C. Third quadrant D. Fourth quadrant **Answer:** (C) Third quadrant

Solution:



Clearly, this is third quadrant.

Q16

If $y = e^{\tan 3x}$, then $\frac{dy}{dx}$ is

A. $e^{\tan 3x} \times \sec^2 3x$

B. $3e^{\tan 3x} \times \sec^2 3x$

C. $3e^{\tan 3x} \times \tan 3x$

D. $3e^{\tan 3x} \times \sec 3x$

Answer: $(B)\ 3e^{\tan 3x} \times \sec^2 3x$

Solution:

Given: $y = e^{\tan 3x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = e^{\tan 3x} \times \frac{d}{dx}(\tan 3x)$$

$$\Rightarrow \frac{dy}{dx} = e^{\tan 3x} \times \sec^2 3x \times \frac{d}{dx}(3x)$$

$$\therefore \frac{dy}{dx} = 3e^{\tan 3x} \times \sec^2 3x$$

Q17

If the variable tangent to the curve $x^2 y = c^3$ makes intercepts a and b on x and y axes respectively, then the value of $a^2 b$ is

A. $27c^3$

B. $\frac{4}{27}c^3$

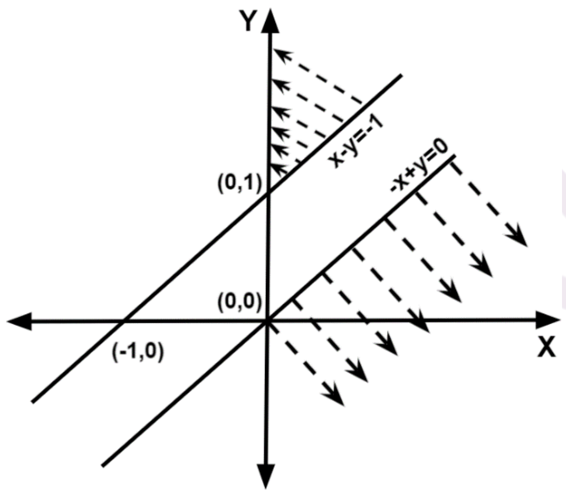
C. $\frac{27}{4}c^3$

D. $\frac{4}{9}c^3$

Answer: $(C)\ \frac{27}{4}c^3$

	Solution: Given curve: $x^2y = c^3$ Differentiating with respect to x, we get $x^2\dfrac{dy}{dx} + 2xy = 0$ $\Rightarrow \dfrac{dy}{dx} = -\dfrac{2y}{x}$ Equation of tangent at (h, k) is $y - k = -\dfrac{2k}{h}(x - h)$ At $y = 0,\ x = \frac{3h}{2} = a$ At $x = 0,\ y = 3k = b$ $\therefore a^2b = \dfrac{9h^2}{4} \times 3k = \dfrac{27h^2k}{4} = \dfrac{27}{4}c^3$
Q18	A relation R on the set of natural numbers $\mathbb{N}$ is defined as $xRy \Leftrightarrow x^2 - 4xy + 3y^2 = 0; x, y \in \mathbb{N}$. Then R is A. reflexive but neither symmetric nor transitive relation B. symmetric but neither reflexive nor transitive relation C. transitive but neither reflexive nor symmetric relation D. an equivalence relation **Answer:** (A) reflexive but neither symmetric nor transitive relation **Solution:** Given: $xRy \Leftrightarrow x^2 - 4xy + 3y^2 = 0; x, y \in \mathbb{N}$ $\Rightarrow x^2 - xy - 3xy + 3y^2 = 0$ $\Rightarrow x(x - y) - 3y(x - y) = 0$ $\Rightarrow (x - 3y)(x - y) = 0$ $\therefore\ (x, y) \in R$ iff $(x - 3y)(x - y) = 0$ As $(x - 3x)(x - x) = 0\ \forall\ x \in \mathbb{N}$, $\Rightarrow (x, x) \in R$ So, R is a reflexive relation. It can be observed that $(3, 1) \in R$ but $(1, 3) \notin R$ as $(1 - 9)(1 - 3) \neq 0$ So, R is not a symmetric relation. As $(3, 1)$ and $\left(1, \frac{1}{3}\right) \in R$ but $\left(3, \frac{1}{3}\right) \notin R$, so R is not a transitive relation.

Q19	A covered box of volume $72\ \mathrm{cm}^3$ and the base sides in a ratio of 1: 2 is to be made. The length of all sides so that the total surface area is the least possible, is A. 2, 4, 9 B. 8, 4, 3 C. 3, 6, 2 D. 6, 3, 4 **Answer:** (D) 6, 3, 4 **Solution:** Let l, b, h be the dimensions. Given, $l = 2b$ So, volume of box, $V = lbh = 2b^2h$ $\Rightarrow 72 = 2b^2h$ $\Rightarrow h = \frac{36}{b^2}$ Surface area, $S = 2(lb + bh + hl)$ $\Rightarrow S = 2\left(2b^2 + b\left(\frac{36}{b^2}\right) + 2b\left(\frac{36}{b^2}\right)\right)$ $\Rightarrow S = 2\left(2b^2 + \frac{108}{b}\right)$ $\Rightarrow S = 4\left(b^2 + \frac{54}{b}\right)$ Differentiating with respect to b, we get $\frac{dS}{db} = 4\left(2b - \frac{54}{b^2}\right)$ For maximum or minimum, $\frac{dS}{db} = 0$ $\Rightarrow b = 3$ And $\frac{d^2S}{db^2} = 4\left(2 + \frac{108}{b^3}\right)$ For $b = 3$, $\frac{d^2S}{db^2} > 0$ Hence, S is minimum when $b = 3$ So, the dimensions are $6, 3, \frac{36}{9}$ or, $6, 3, 4$

Q20	For the objective function $Z = x + 2y$, subject to constraints: $x - y \leq -1$, $-x + y \leq 0, x \geq 0$, which of the following is CORRECT? A. $Z_{\max} = 1$ B. $Z_{\min} = -1$ C. $Z_{\min} = 0$ D. No feasible solution is possible **Answer:** (D) No feasible solution is possible **Solution:** Given constraints: $x - y \leq -1, -x + y \leq 0, x \geq 0$ Plotting the graph for the feasible region:  From graph, there is no feasible region for the given constraints. So, no feasible solution is possible.
Q21	If $R = \{(x, y): x, y \in \mathbb{Z}, x^2 + y^2 \leq 4\}$ is a relation on $\mathbb{Z}$, then domain of R is A. $\{0, 1, 2\}$ B. $\{0, -1, -2\}$ C. $\{-2, -1, 0, 1, 2\}$ D. $\{-2, -1, 1, 2\}$ **Answer:** (C) $\{-2, -1, 0, 1, 2\}$ **Solution:** Given: $R = \{(x, y): x, y \in \mathbb{Z}, x^2 + y^2 \leq 4\}$ If $y = 0$, then $x = 0, \pm 1, \pm 2$ If $y = \pm 1$, then $x = 0, \pm 1$ If $y = \pm 2$, then $x = 0$

	$\therefore$ Domain of R is $\{-2, -1, 0, 1, 2\}$
Q22	If $y = (1 + x)(1 + x^2)(1 + x^3) \cdots\cdots (1 + x^n)$, then the value of $\frac{dy}{dx}$ at $x = 0$ is A. 1 B. -1 C. 0 D. 2 **Answer:** $(A)\ 1$ **Solution:** Given: $y = (1 + x)(1 + x^2)(1 + x^3) \cdots\cdots (1 + x^n)$ Taking $\ln$ on both sides, we get $\ln y = \ln(1 + x) + \ln(1 + x^2) + \ln(1 + x^3) + \cdots + \ln(1 + x^n)$ Differentiating with respect to x, we get $\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{3x^2}{1+x^3} + \cdots + \frac{nx^{n-1}}{1+x^n}$ At $x = 0$, we have $y = 1$ $\therefore \left.\frac{dy}{dx}\right\|_{x=0} = 1$
Q23	For real constants a and b, if $f(x) = \begin{cases} x^2 + b + 1, x < 0 \\ 3ax + 2, \ \ x \geq 0 \end{cases}$ is differentiable at $x = 0$, then the value of $a + b$ is A. 0 B. 1 C. -1 D. 2 **Answer:** $(B)\ 1$ **Solution:** If function is differentiable, then it is continuous also. $\therefore L.H.L. = R.H.L. = f(0) \ \ \cdots (1)$ $L.H.L. = \lim_{x \to 0^-} x^2 + b + 1$ $= b + 1$ $R.H.L. = \lim_{x \to 0^+} 3ax + 2$ $= 2$ From (1), we have $b = 1$

	Since $f(x)$ is differentiable at $x = 0$, $\therefore L.H.D. = R.H.D. \quad \cdots(2)$ $L.H.D. = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$ $= \lim_{h \to 0} \frac{(0-h)^2 + 2 - 2}{-h}$ $= \lim_{h \to 0} \frac{h^2}{-h} = 0$ $R.H.D. = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$ $= \lim_{h \to 0} \frac{3a(0+h) + 2 - 2}{h}$ $= 3a$ From (2), we have $a = 0$ $\therefore a + b = 1$
Q24	If $y^x = 2^x$ where $y > 0$, then $\frac{dy}{dx}$ is A. $\frac{y}{x} \ln\left(\frac{2}{y}\right)$ B. $\frac{x}{y} \ln\left(\frac{2}{y}\right)$ C. $\frac{y}{x} \ln\left(\frac{y}{2}\right)$ D. $\frac{x}{y} \ln\left(\frac{y}{2}\right)$ **Answer:** $(A)\ \frac{y}{x} \ln\left(\frac{2}{y}\right)$ **Solution:** Given: $y^x = 2^x$ Taking $\ln$ on both sides, we get $x \ln y = x \ln 2$ Differentiating with respect to x, we get $x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \ln y = \ln 2$ $\Rightarrow \frac{x}{y} \cdot \frac{dy}{dx} = \ln 2 - \ln y$ $\therefore \frac{dy}{dx} = \frac{y}{x} \ln\left(\frac{2}{y}\right)$

Q25

If $f(x) = (x+1)^3(x-3)^3$, then

A. $f(x)$ is strictly decreasing in $(3, \infty)$

B. $f(x)$ is strictly decreasing in $(1, 3)$

C. $f(x)$ is strictly increasing in $(-1, 1)$

D. $f(x)$ is strictly decreasing in $(-\infty, -1)$

Answer: (D) $f(x)$ is strictly decreasing in $(-\infty, -1)$

Solution:

$f(x) = (x+1)^3(x-3)^3$

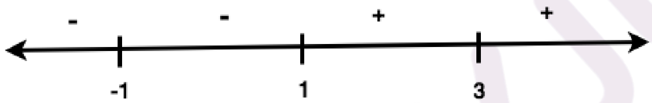
Differentiating with respect to x, we get

$f'(x) = 3(x+1)^2(x-3)^3 + 3(x+1)^3(x-3)^2$

$\Rightarrow f'(x) = 3(x+1)^2(x-3)^2[(x-3)+(x+1)]$

$\Rightarrow f'(x) = 6(x+1)^2(x-3)^2(x-1)$

Let us find out sign of $f'(x)$ using wavy curve method.



Hence, $f(x)$ is strictly decreasing in $(-\infty, -1)$.

Q26

Let $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. If $AB = I_3$, where I_3 is the identity matrix of order 3, then the value of $x + y$ is

A. 1

B. 0

C. -1

D. 4

Answer: (B) 0

Solution:

Given: $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

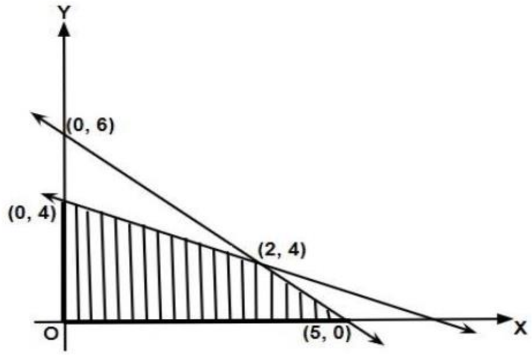
$$AB = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore x + y = 0$

Q27

The feasible region of an LPP is shown in the figure. If $Z = 5x + 2y$, then the maximum value of Z occurs at



A. $(0, 0)$

B. $(5, 0)$

C. $(2, 4)$

D. $(0, 4)$

Answer: (B) $(5, 0)$

Solution:

The table of values at corner points for objective function $Z = 5x + 2y$ is given below:

Corner point: (x, y)	Value: $Z = 5x + 2y$
$(0, 0)$	$5 \times 0 + 2 \times 0 = 0$
$(5, 0)$	$5 \times 5 + 2 \times 0 = 25$ (maximum)
$(2, 4)$	$5 \times 2 + 2 \times 4 = 18$
$(0, 4)$	$5 \times 0 + 2 \times 4 = 8$

Hence, maximum value of Z occurs at $(5, 0)$.

Q28

Cofactor of 2 in matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

A. -6

B. 6

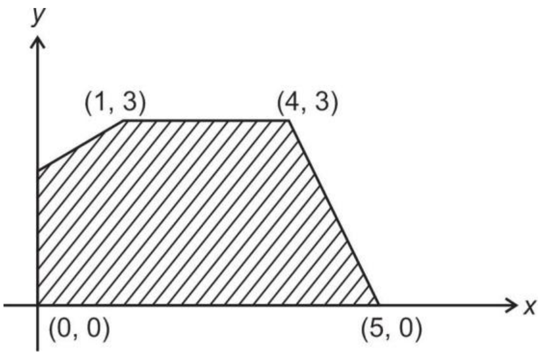
C. 8

D. -8

Answer: (B) 6

Solution:

	$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ Cofactor of 2, $C_{12} = (-1)^{1+2} \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} = -1(36 - 42)$ $\therefore C_{12} = 6$
Q29	If $f: [1, \infty) \to B$ defined by $f(x) = x^2 - 2x + 6$ is a surjective function, then B is equal to A. $[1, \infty)$ B. $[5, \infty)$ C. $[6, \infty)$ D. $[2, \infty)$ **Answer:** (B) $[5, \infty)$ **Solution:** $f(x) = x^2 - 2x + 6$ $\Rightarrow f(x) = (x-1)^2 + 5 \geq 5$ for $x \geq 1$ $\therefore$ Range of f in domain $[1, \infty)$ is $[5, \infty)$. $\therefore B = [5, \infty)$
Q30	The value of λ for which the matrix $A = \begin{bmatrix} \lambda & 2 & 2 \\ -3 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ is not invertible, is A. -5 B. 5 C. 0 D. -1 **Answer:** (A) -5 **Solution:** Given: $A = \begin{bmatrix} \lambda & 2 & 2 \\ -3 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ Since matrix A is not invertible, therefore $\|A\| = 0$ $\Rightarrow \begin{vmatrix} \lambda & 2 & 2 \\ -3 & 0 & 4 \\ 1 & -1 & 1 \end{vmatrix} = 0$ $\Rightarrow \lambda(0 + 4) - 2(-3 - 4) + 2(3 - 0) = 0$ $\Rightarrow 4\lambda + 14 + 6 = 0$ $\therefore \lambda = -5$

Q31	The feasible region for an LPP is shown shaded in the figure. Let $Z = 6x - 4y$ be the objective function. The minimum of Z occurs at  A. $(1, 3)$ B. $(0, 0)$ C. $(4, 3)$ D. $(5, 0)$ **Answer:** (A) $(1, 3)$ **Solution:** The table of values at corner points for objective function $Z = 6x - 4y$ is given below: Corner point: (x, y) — Value: $Z = 6x - 4y$ $(0, 0)$ — $6 \times 0 - 4 \times 0 = 0$ $(1, 3)$ — $6 \times 1 - 4 \times 3 = -6$ (minimum) $(4, 3)$ — $6 \times 4 - 4 \times 3 = 12$ $(5, 0)$ — $6 \times 5 - 4 \times 0 = 30$ Hence, the minimum value of Z occurs at $(1, 3)$.
Q32	If $x^m + y^m = 1$ where m is a constant such that $\frac{dy}{dx} = -\frac{x}{y}$ $\forall\, x, y \in \mathbb{R} - \{0\}$, then the value of m is A. -1 B. 0 C. 1 D. 2 **Answer:** (D) 2 **Solution:** Given: $\frac{dy}{dx} = -\frac{x}{y} \quad \cdots (1)$

and $x^m + y^m = 1$

Differentiating with respect to x, we get

$$mx^{m-1} + my^{m-1} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow x^{m-1} + y^{m-1} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^{m-1}}{y^{m-1}} \quad \cdots (2)$$

From (1) and (2), we get

$$\frac{x^{m-1}}{y^{m-1}} = \frac{x}{y}$$

$$\Rightarrow m - 1 = 1$$

$$\therefore m = 2$$

Q33

The local maximum value of $f(x) = \frac{x}{1+4x+x^2}$ is

A. $\frac{1}{2}$

B. $-\frac{1}{4}$

C. $\frac{1}{6}$

D. $\frac{1}{5}$

Answer: $(C)\ \frac{1}{6}$

Solution:

$$f(x) = \frac{x}{1+4x+x^2}$$

Differentiating with respect to x, we get

$$f'(x) = \frac{(1+4x+x^2)\cdot 1 - x\cdot(2x+4)}{(1+4x+x^2)^2}$$

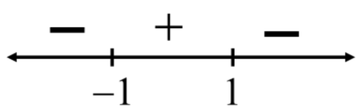
$$\Rightarrow f'(x) = \frac{x^2+4x+1-2x^2-4x}{(1+4x+x^2)^2}$$

$$\Rightarrow f'(x) = \frac{(1-x)(1+x)}{(1+4x+x^2)^2}$$

For maximum or minimum, $f'(x) = 0$

$$\Rightarrow \frac{(1-x)(1+x)}{(1+4x+x^2)^2} = 0$$

$$\Rightarrow x = -1, 1$$

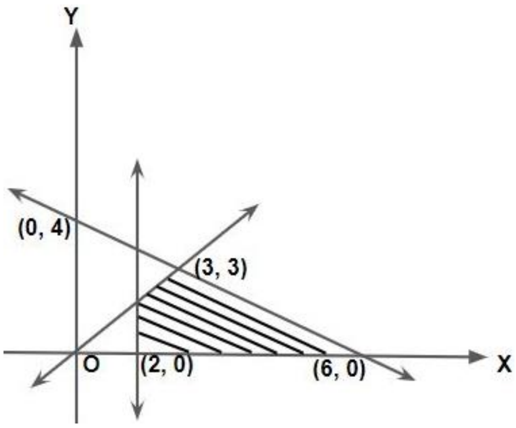


Since sign of $f'(x)$ changes from positive to negative as x crosses 1 from left to right, therefore $x = 1$ is a point of local maximum.

Therefore, $f(x)$ has local maximum value at $x = 1$

and the maximum value, $f(1) = \frac{1}{1+4+1} = \frac{1}{6}$

Q34 The shaded region in the figure is the solution set of the inequations



A. $4x + 6y \leq 24,\ x \leq 2,\ y \leq x,\ x, y \geq 0$

B. $4x + 6y \geq 24,\ x \geq 2,\ y \geq x,\ x, y \geq 0$

C. $4x + 6y \geq 24,\ x \leq 2,\ y \geq x,\ x, y \geq 0$

D. $4x + 6y \leq 24,\ x \geq 2,\ y \leq x,\ x, y \geq 0$

Answer: $(D)\ 4x + 6y \leq 24,\ x \geq 2,\ y \leq x,\ x, y \geq 0$

Solution:

The line joining $(6, 0)$ and $(0, 4)$ is $\frac{x}{6} + \frac{y}{4} = 1$ i.e., $4x + 6y = 24$.

The line joining $(0, 0)$ and $(3, 3)$ is $y = x$.

On observation, we can conclude that option (D) is the correct set of inequations which represents the shaded region.

Q35 The range of the function $f(x) = \sin^{-1}\left(\frac{x^2}{1+x^2}\right),\ x \in \mathbb{R}$ is equal to

A. $\left[0, \frac{\pi}{2}\right)$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $\left(0, \frac{\pi}{2}\right)$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: $(A)\ \left[0, \frac{\pi}{2}\right)$

	Solution: We have, $\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$ For $x \in \mathbb{R}$, $x^2 \in [0, \infty)$ $\Rightarrow 1 + x^2 \in [1, \infty)$ $\Rightarrow \frac{1}{1+x^2} \in (0, 1]$ $\Rightarrow \frac{x^2}{1+x^2} \in [0, 1)$ Taking $\sin^{-1}$ on both sides, we get $\sin^{-1}\left(\frac{x^2}{1+x^2}\right) \in \left[0, \frac{\pi}{2}\right)$ $\therefore$ Range of f is $\left[0, \frac{\pi}{2}\right)$
Q36	If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then A. $\alpha = a^2 + b^2, \beta = ab$ B. $\alpha = a^2 + b^2, \beta = 2ab$ C. $\alpha = ab, \beta = a^2 + b^2$ D. $\alpha = 2ab, \beta = a^2 + b^2$ **Answer:** $(B)\ \alpha = a^2 + b^2, \beta = 2ab$ **Solution:** Given: $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ $\Rightarrow A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & ab + ab \\ ab + ab & a^2 + b^2 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ $\therefore \alpha = a^2 + b^2$ and $\beta = 2ab$
Q37	The domain of the function $f(x) = \cot^{-1}(x)$ is A. $\mathbb{R}$ B. $\mathbb{R} - (-1, 1)$ C. $(0, \pi)$ D. $[-1, 1]$ **Answer:** $(A)\ \mathbb{R}$ **Solution:** $f(x) = \cot^{-1}(x)$

	f is defined for all $x \in \mathbb{R}$ $\therefore$ Domain of f is $\mathbb{R}$.
Q38	If $A = \begin{bmatrix} 0 & -3 & 6 \\ 3 & 0 & 9 \\ -6 & -9 & 0 \end{bmatrix}$, then $A + 3A^T$ is equal to A. A^T B. $2A^T$ C. $-A^T$ D. $-2A^T$ **Answer:** $(B)\ 2A^T$ **Solution:** Given: $A = \begin{bmatrix} 0 & -3 & 6 \\ 3 & 0 & 9 \\ -6 & -9 & 0 \end{bmatrix}$ $a_{ij} = -a_{ji}$ for all i, j. $\therefore$ A is skew-symmetric. $\Rightarrow A^T = -A$ $\Rightarrow A + A^T = O$ $\Rightarrow A + 3A^T = 2A^T$
Q39	If the slope of the tangent to the curve $x^2y + ax + by = 2$ at $(1, 1)$ is 2, then (a, b) is A. $(6, 5)$ B. $(6, -5)$ C. $(-2, 3)$ D. $(-2, -3)$ **Answer:** $(B)\ (6, -5)$ **Solution:** Given curve: $x^2y + ax + by = 2$ Point $(1, 1)$ satisfies the curve. $\Rightarrow a + b = 1 \ \cdots (1)$ Differentiating the given curve with respect to x, we get $2xy + x^2\frac{dy}{dx} + a + b\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\left(\frac{a + 2xy}{x^2 + b}\right)$ So, slope of the tangent at $(1, 1)$ is

	$\frac{dy}{dx} = -\left(\frac{a+2}{1+b}\right)$ $\Rightarrow -\left(\frac{a+2}{1+b}\right) = 2$ $\Rightarrow a + 2b = -4 \quad \cdots (2)$ Solving (1) and (2), we get $a = 6$ and $b = -5$
Q40	If A is a square matrix of order 3 such that $\det(A) = 2$, then the value of $\det(\text{adj } A)$ is A. 2 B. 4 C. 8 D. $\frac{1}{2}$ **Answer:** (B) 4 **Solution:** Given: $\|A\| = 2$ We know that $\|\text{adj } A\| = \|A\|^{n-1}$, where n is the order of A. $\therefore \|\text{adj } A\| = 2^{3-1} = 4$
Q41	The value of $\sin^{-1}(2) + \cos^{-1}(2)$ is A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$ C. π D. not defined **Answer:** (D) Not defined **Solution:** We have, $\sin^{-1}(2) + \cos^{-1}(2)$ Domain of $\sin^{-1} x$ and $\cos^{-1} x$ are same, i.e., $[-1, 1]$ and $2 \notin [-1, 1]$ $\therefore \sin^{-1}(2) + \cos^{-1}(2)$ is not defined.
Q42	The derivative of $\cos^{-1}(x^2)$ with respect to x is A. $\frac{2x}{\sqrt{1-x^2}}$ B. $\frac{-2x}{\sqrt{1-x^4}}$ C. $\frac{-1}{\sqrt{1-x^4}}$ D. $\frac{-x}{\sqrt{1-x^4}}$

	Answer: $(B)\ \frac{-2x}{\sqrt{1-x^4}}$ **Solution:** Let $y = \cos^{-1}(x^2)$ Differentiating with respect to x, we get $\frac{dy}{dx} = -\frac{1}{\sqrt{1-(x^2)^2}} \times \frac{d}{dx}(x^2)$ $\therefore\ \frac{dy}{dx} = -\frac{2x}{\sqrt{1-x^4}}$
Q43	If a matrix has 4 elements, then total number of possible orders that matrix can have, is A. 2 B. 3 C. 1 D. 4 **Answer:** $(B)\ 3$ **Solution:** Number of elements $= 4$ Then possible orders of matrices are $1 \times 4, 4 \times 1, 2 \times 2$ Hence, number of matrices is 3.
Q44	The function $f: \mathbb{N} \to \mathbb{N}$ given by $f(x) = x^3 - 1$ is A. one-one but not onto B. onto but not one-one C. bijective D. many-one only **Answer:** (A) one-one but not onto **Solution:** For one-one: Let $f(x_1) = f(x_2)$ $\Rightarrow x_1^3 - 1 = x_2^3 - 1$ $\Rightarrow x_1^3 = x_2^3$ $\Rightarrow x_1 = x_2$ $\therefore\ f$ is one-one.

For onto:

1 does not have a pre-image in $\mathbb{N}$ (domain)

$\Rightarrow$ Range $\neq$ Co-domain

$\therefore f$ is into.

Q45

If $Z = x + 2y$ subject to the following constraints: $x + y \leq 5, x + 2y \geq 6, x \geq 3,$ $y \geq 0$, then sum of the maximum and minimum values of Z is

A. 7

B. 5

C. 12

D. 13

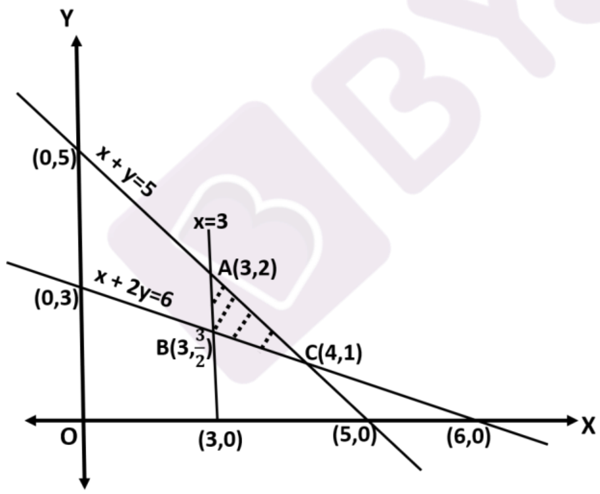
Answer: (D) 13

Solution:

Given: objective function, $Z = x + 2y$

Constraints: $x + y \leq 5, x + 2y \geq 6, x \geq 3, y \geq 0$

Plotting the graph of feasible region:



From graph, the feasible region is ABC.

Corner point: (x, y)	Value: $Z = x + 2y$
$A(3,2)$	$1 \times 3 + 2 \times 2 = 7$
$B\left(3, \frac{3}{2}\right)$	$1 \times 3 + 2 \times \frac{3}{2} = 6$
$C(4,1)$	$1 \times 4 + 2 \times 1 = 6$

So, $Z_{\max} = 7$ and $Z_{\min} = 6$

Hence, $Z_{\max} + Z_{\min} = 13$

The electricity cost per hour for running an electric car is proportional to the square of its speed it generates in km/hr. The electricity costs Rs. 12 per hour at speed of 4 km per hour and the fixed charges to run the car amounts to Rs. 300 per hour. Assume the speed of car is u km/hr.	
Q46	Given that the electricity cost per hour is k times the square of the speed the car generates in km/hr, then value of k is A. $\frac{3}{16}$ B. $\frac{3}{4}$ C. $\frac{4}{3}$ D. $\frac{16}{3}$ **Answer:** $(B)\ \frac{3}{4}$ **Solution:** Electricity cost $= k(u)^2$, where k is proportionality constant. $\Rightarrow 12 = k(4)^2$ $\therefore k = \frac{3}{4}$
Q47	If the car has travelled a distance of 500 km, then the total cost of running the car is given by function A. $375u - \frac{150000}{u}$ B. $375u + \frac{150000}{u}$ C. $750u - \frac{150000}{u}$ D. $750u + \frac{150000}{u}$ **Answer:** $(B)\ 375u + \frac{150000}{u}$ **Solution:** Time taken to cover 500 km is $\frac{500}{u}$ hours Fixed charges: Given that fixed charges to run the car amounts to Rs. 300 per hour. $\Rightarrow$ Fixed charges for $\frac{500}{u}$ hours $= 300 \times \frac{500}{u} = \frac{150000}{u}$ Running charges: Electricity cost per hour $= k(u)^2 = \frac{3}{4}u^2$

	$\Rightarrow$ Electricity cost for $\frac{500}{u}$ hours $= \frac{3}{4}u^2 \times \frac{500}{u} = 375u$ Now, Total cost $=$ Fixed charges $+$ Running charges $\therefore$ Total cost of running the car $= 375u + \frac{150000}{u}$
Q48	The most economical speed to run the car is A. 80 km/hr B. 10 km/hr C. 20 km/hr D. 40 km/hr **Answer:** (C) 20 km/hr **Solution:** We have, Total cost of running the car, $C = 375u + \frac{150000}{u}$ Differentiating with respect to u, we get $\frac{dC}{du} = 375 - \frac{150000}{u^2}$ For most economical speed, $\frac{dC}{du} = 0$ $\Rightarrow 375 - \frac{150000}{u^2} = 0$ $\Rightarrow u^2 = \frac{150000}{375} = 400$ $\therefore u = 20$ km/hr
Q49	The electricity cost for car to travel 500 km at the most economical speed is A. $\text{Rs.}\,1835$ B. $\text{Rs.}\,7500$ C. $\text{Rs.}\,3500$ D. $\text{Rs.}\,15000$ **Answer:** (B) $\text{Rs.}\,7500$ **Solution:** We have, Electricity cost $= 375u$ $\therefore$ Electricity cost at the most economical speed $= 375 \times 20 =$ $\text{Rs.}\,7500$

Q50	The total cost for the car to travel 500 km at the most economical speed is A. $\text{Rs.}\,7500$ B. $\text{Rs.}\,15000$ C. $\text{Rs.}\,8000$ D. $\text{Rs.}\,21000$ **Answer:** (B) $\text{Rs.}\,15000$ **Solution:** We have, Total cost of running the car $= 375u + \frac{150000}{u}$ $\Rightarrow$ Total cost of running the car at the most economical speed $= 375 \times 20 + \frac{150000}{20}$ $= \text{Rs.}\,15000$