Exercise 1.2 Page: 8

1.State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

Solution:

True

Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.

i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, $\sqrt{2}$, $\sqrt{5}$, π, 0.102….

Real numbers - The collection of both rational and irrational numbers are known as real numbers.

i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, π, 0.102…

$\therefore$ Every irrational number is a real number, however, every real numbers are not irrational numbers.

(ii) Every point on the number line is of the form $\sqrt{m}$ where m is a natural number.

Solution:

False

The statement is false since as per the rule, a negative number cannot be expressed as square roots.

E.g., $\sqrt{9} = 3$ is a natural number.

But $\sqrt{2} = 1.414$ is not a natural number.

Similarly, we know that there are negative numbers on the number line but when we take the root of a negative number it becomes a complex number and not a natural number.

E.g., $\sqrt{-7} = 7i$, where $i = \sqrt{-1}$

$\therefore$ The statement that every point on the number line is of the form $\sqrt{m}$, where m is a natural number is false.

(iii) Every real number is an irrational number.

Solution:

False

The statement is false, the real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.

Real numbers - The collection of both rational and irrational numbers are known as real numbers.

i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, π, 0.102…

Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.

i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, $\sqrt{2}$, $\sqrt{5}$, π, 0.102….

$\therefore$ Every irrational number is a real number, however, every real number is not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:

No, the square roots of all positive integers are not irrational.

For example,

$\sqrt{4} = 2$ is rational.

$\sqrt{9} = 3$ is rational.

Hence, the square roots of positive integers 4 and 9 are not irrational. (2 and 3, respectively).

3. Show how √5 can be represented on the number line.

Solution:

Step 1: Let line AB be of 2 unit on a number line.

Step 2: At B, draw a perpendicular line BC of length 1 unit.

Step 3: Join CA

Step 4: Now, ABC is a right-angled triangle. Applying Pythagoras theorem,

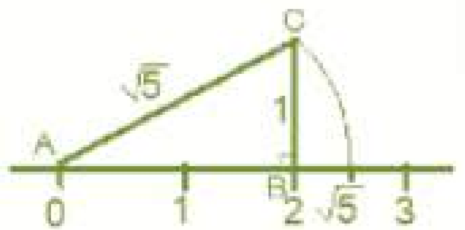
$AB^2+BC^2 = CA^2$

$2^2+1^2 = CA^2 \Rightarrow CA^2 = 5$

$\Rightarrow CA = \sqrt{5}$. Thus, CA is a line of length √5 unit.

Step 5: Taking CA as a radius and A as a center draw an arc touching the number line. The point at which number line get intersected by arc is at √5 distance from 0 because it is a radius of the circle whose center was A.

Thus, √5 is represented on the number line as shown in the figure.



4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP1 of unit length. Draw a line segment P1P2 perpendicular to OP_1 of unit length (see Fig. 1.9). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in Fig. 1.9 :

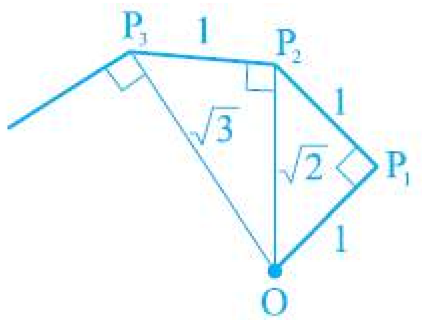
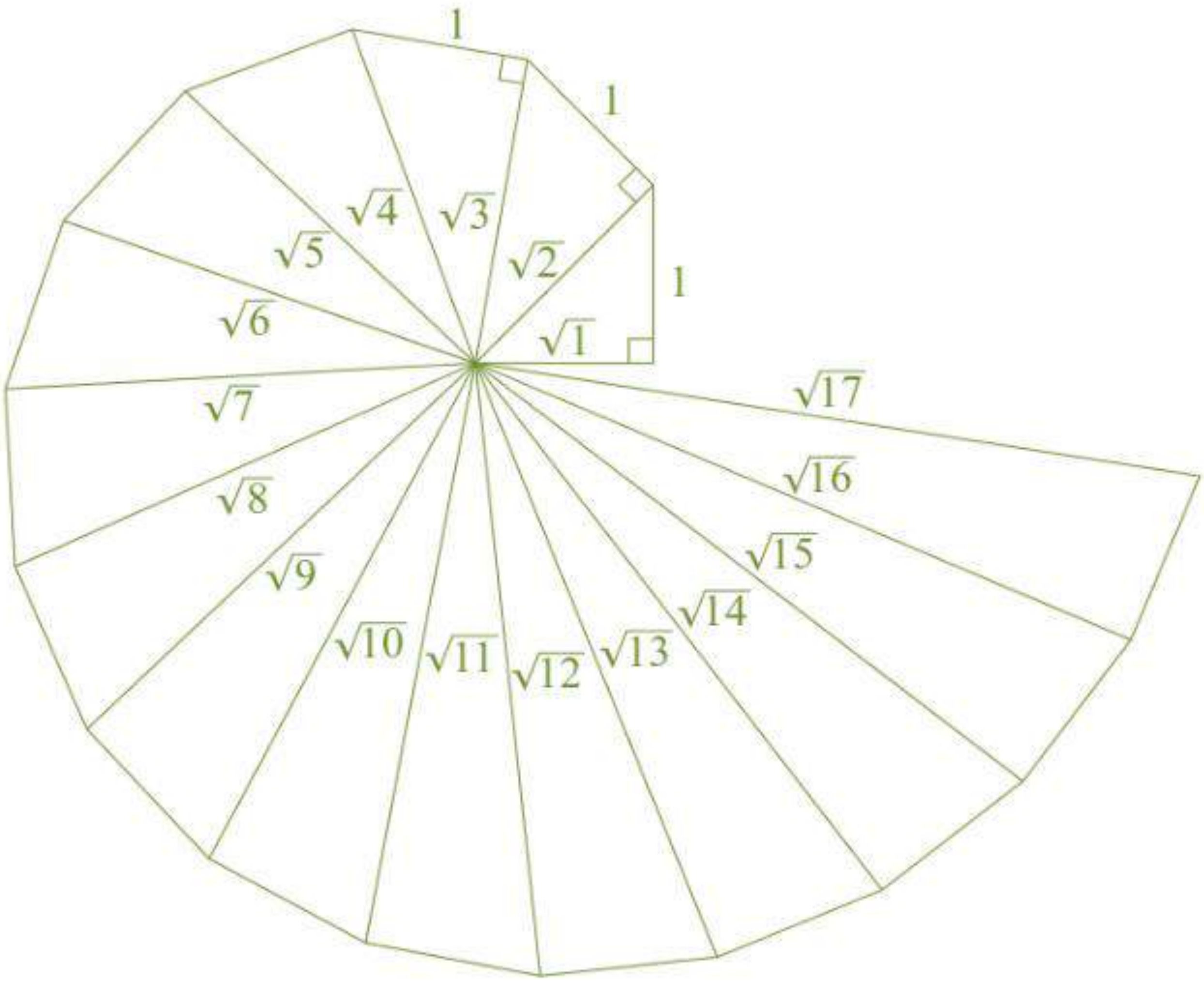


Fig. 1.9 : Constructing square root spiral

Constructing this manner, you can get the line segment $P_{n-1}Pn$ by square root spiral drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points P_2, P_3,....,Pn,... ., and joined them to create a beautiful spiral depicting √2, √3, √4, ...

Solution:



Step 1: Mark a point O on the paper. Here, O will be the center of the square root spiral.
Step 2: From O, draw a straight line, OA, of 1cm horizontally.
Step 3: From A, draw a perpendicular line, AB, of 1 cm.
Step 4: Join OB. Here, OB will be of $\sqrt{2}$
Step 5: Now, from B, draw a perpendicular line of 1 cm and mark the end point C.
Step 6: Join OC. Here, OC will be of $\sqrt{3}$
Step 7: Repeat the steps to draw $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$….