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1. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) x+1
Solution:
    x+1= 0
    ⇒x = −1
    ∴Remainder:
    p(−1) = $(−1)^3+3(−1)^2+3(−1)+1$
         = −1+3−3+1
         = 0

(ii) x−1/2
Solution:
    x-1/2 = 0
    ⇒ x = 1/2
    ∴Remainder:
    p(1/2) = $(1/2)^3+3(1/2)^2+3(1/2)+1$
         = (1/8)+(3/4)+(3/2)+1
         = 27/8

(iii) x
Solution:
    x = 0
    ∴Remainder:
    p(0) = $(0)^3+3(0)^2+3(0)+1$
       = 1

(iv) x+π
Solution:
    x+π = 0
    ⇒ x = −π
    ∴Remainder:
    p(0) = $(−π)^3 +3(−π)^2+3(−π)+1$
       = $−π^3+3π^2−3π+1$

(v) 5+2x
Solution:
    5+2x=0
    ⇒ 2x = −5
    ⇒ x = -5/2
    ∴Remainder:
    $(-5/2)^3+3(-5/2)^2+3(-5/2)+1$ = (-125/8)+(75/4)-(15/2)+1
    = -27/8

2. Find the remainder when x³−ax²+6x−a is divided by x-a.

Solution:

Let $p(x) = x^3 - ax^2 + 6x - a$

$x - a = 0$

$\therefore x = a$

Remainder:

$p(a) = (a)^3 - a(a^2) + 6(a) - a$

$\quad\quad = a^3 - a^3 + 6a - a = 5a$

3. Check whether 7+3x is a factor of 3x³+7x.

Solution:

$7 + 3x = 0$

$\Rightarrow 3x = -7$

$\Rightarrow x = -7/3$

$\therefore$Remainder:

$3(-7/3)^3 + 7(-7/3) = -(343/9) + (-49/3)$

$\quad\quad\quad\quad\quad\quad\quad = (-343 - (49)3)/9$

$\quad\quad\quad\quad\quad\quad\quad = (-343 - 147)/9$

$\quad\quad\quad\quad\quad\quad\quad = -490/9 \neq 0$

$\therefore 7 + 3x$ is not a factor of $3x^3 + 7x$