Exercise 2.4

1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x) = x^3+x^2+x+1$

The zero of x+1 is -1. [x+1 = 0 means x = -1]

$p(-1) = (-1)^3+(-1)^2+(-1)+1$

$= -1+1-1+1$

$= 0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

Solution:

Let $p(x)= x^4+x^3+x^2+x+1$

The zero of x+1 is -1. . [x+1= 0 means x = -1]

$p(-1) = (-1)^4+(-1)^3+(-1)^2+(-1)+1$

$= 1-1+1-1+1$

$= 1 \neq 0$

∴By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

Let $p(x)= x^4+3x^3+3x^2+x+1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1 \neq 0$

∴By factor theorem, x+1 is not a factor of $x^4+3x^3+3x^2+x+1$

(iv) $x^3 - x^2- (2+\sqrt{2})x +\sqrt{2}$

Solution:

Let $p(x) = x^3-x^2-(2+\sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(-1) = (-1)^3-(-1)^2-(2+\sqrt{2})(-1) + \sqrt{2} = -1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2} \neq 0$

∴By factor theorem, x+1 is not a factor of $x^3-x^2-(2+\sqrt{2})x +\sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3+x^2-2x-1$, $g(x) = x+1$

Solution:

$p(x) = 2x^3+x^2-2x-1$, $g(x) = x+1$

$g(x) = 0$

$\Rightarrow x+1 = 0$

$\Rightarrow x = -1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1) = 2(-1)^3+(-1)^2-2(-1)-1$

$= -2+1+2-1$

$= 0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x+2$

Solution:

$p(x) = x^3+3x^2+3x+1$, $g(x) = x+2$

$g(x) = 0$

$\Rightarrow x+2 = 0$

$\Rightarrow x = -2$

$\therefore$ Zero of g(x) is -2.

Now,

$p(-2) = (-2)^3+3(-2)^2+3(-2)+1$

$= -8+12-6+1$

$= -1 \neq 0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, $g(x) = x-3$

Solution:

$p(x) = x^3-4x^2+x+6$, $g(x) = x -3$

$g(x) = 0$

$\Rightarrow x-3 = 0$

$\Rightarrow x = 3$

$\therefore$ Zero of g(x) is 3.

Now,

$p(3) = (3)^3-4(3)^2+(3)+6$

$= 27-36+3+6$

$= 0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x–1 is a factor of p(x) in each of the following cases:

(i) $p(x) = x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow (1)^2+(1)+k = 0$

$\Rightarrow 1+1+k = 0$

$\Rightarrow 2+k = 0$

$\Rightarrow k = -2$

(ii) $p(x) = 2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2} = 0$

$\Rightarrow 2+k+\sqrt{2} = 0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x) = kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k = \sqrt{2}-1$

(iv)$p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k = 0$

$\Rightarrow k-3+k = 0$

$\Rightarrow 2k-3 = 0$

$\Rightarrow k= 3/2$

4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -7 and product =1×12 = 12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4 = 12]

$12x^2-7x+1= 12x^2-4x-3x+1$

$= 4x(3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum = 7 and product = 2×3 = 6
We get 6 and 1 as the numbers [6+1 = 7 and 6×1 = 6]

$2x^2+7x+3 = 2x^2+6x+1x+3$
$= 2x(x+3)+1(x+3)$
$= (2x+1)(x+3)$

(iii) $\mathbf{6x^2+5x-6}$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum = 5 and product = 6×-6 = -36
We get -4 and 9 as the numbers [-4+9 = 5 and -4×9 = -36]

$6x^2+5x-6 = 6x^2+9x-4x-6$
$= 3x(2x+3)-2(2x+3)$
$= (2x+3)(3x-2)$

(iv) $\mathbf{3x^2-x-4}$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum = -1 and product = 3×-4 = -12
We get -4 and 3 as the numbers [-4+3 = -1 and -4×3 = -12]

$3x^2-x-4 = 3x^2-4x+3x-4$
$= x(3x-4)+1(3x-4)$
$= (3x-4)(x+1)$

5. Factorize:

(i) $\mathbf{x^3-2x^2-x+2}$

Solution:

Let $p(x) = x^3-2x^2-x+2$
Factors of 2 are ±1 and ± 2
Now,
$p(x) = x^3-2x^2-x+2$
$p(-1) = (-1)^3-2(-1)^2-(-1)+2$
$= -1-2+1+2$
$= 0$
Therefore, (x+1) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2-3x+2 \\
\hline
x+1 & x^3-2x^2-x+2 \\
 & x^3+x^2 \\
 & - \quad - \\
\hline
 & -3x^2-x+2 \\
 & -3x^2-3x \\
 & + \quad + \\
\hline
 & 2x+2 \\
 & 2x+2 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

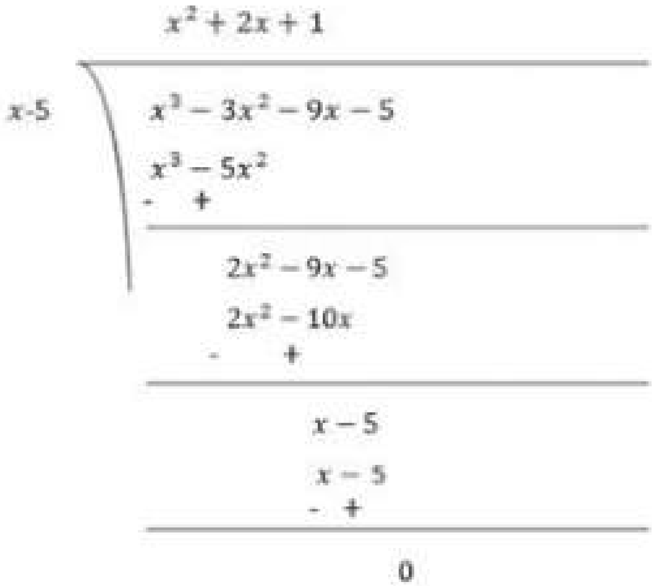
Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2)$ $= (x+1)(x^2–x–2x+2)$
$= (x+1)(x(x−1)−2(x−1))$
$= (x+1)(x−1)(x+2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$
Factors of 5 are ±1 and ±5
By trial method, we find that
$p(5) = 0$
So, $(x-5)$ is factor of $p(x)$
Now,
$p(x) = x^3–3x^2–9x–5$
$p(5) = (5)^3–3(5)^2–9(5)–5$
$= 125−75−45−5$
$= 0$
Therefore, $(x-5)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$

$= (x-5)(x(x+1)+1(x+1))$

$= (x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$p(x)= x^3+13x^2+32x+20$

$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$

$= -1+13-32+20$

$= 0$

Therefore, $(x+1)$ is the factor of $p(x)$

$$\begin{array}{r|l} & x^2+12x+20 \\ \hline x+1 & x^3+13x^2+32x+20 \\ & x^3+x^2 \\ & -\quad - \\ \hline & 12x^2+32x+20 \\ & 12x^2+12x \\ & -\quad\quad - \\ \hline & 20x+20 \\ & 20x+20 \\ & -\quad - \\ \hline & 0 \end{array}$$

Now, Dividend = Divisor × Quotient +Remainder

$$\begin{aligned}(x+1)(x^2+12x+20) &= (x+1)(x^2+2x+10x+20) \\ &= (x+1)x(x+2)+10(x+2) \\ &= (x+1)(x+2)(x+10)\end{aligned}$$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors = $2\times(-1)= -2$ are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, (y-1) is factor of p(y)

Now,

$$\begin{aligned}p(y) &= 2y^3+y^2-2y-1 \\ p(1) &= 2(1)^3+(1)^2-2(1)-1 \\ &= 2+1-2 \\ &= 0\end{aligned}$$

Therefore, (y-1) is the factor of p(y)

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2 - 2y - 1 \\
 & \quad 3y^2 - 3y \\
 & \quad - \quad + \\
\hline
 & \qquad y - 1 \\
 & \qquad y - 1 \\
 & \qquad - \; + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(y-1)(2y^2+3y+1) &= (y-1)(2y^2+2y+y+1) \\
&= (y-1)(2y(y+1)+1(y+1)) \\
&= (y-1)(2y+1)(y+1)
\end{aligned}
$$