

Exercise 8.1

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Solution:

Let the common ratio between the angles be = x.

We know that the sum of the interior angles of the quadrilateral = 360°

Now,

$$3x + 5x + 9x + 13x = 360^\circ$$

$$\Rightarrow 30x = 360^\circ$$

$$\Rightarrow x = 12^\circ$$

\therefore , Angles of the quadrilateral are:

$$3x = 3 \times 12^\circ = 36^\circ$$

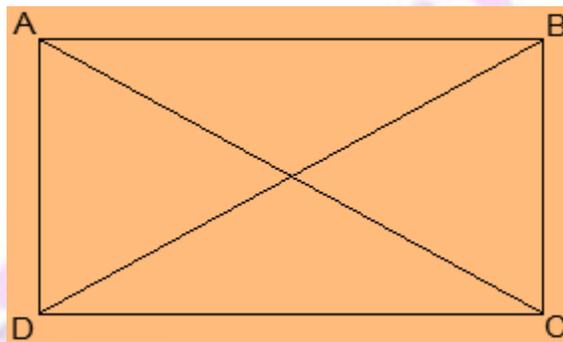
$$5x = 5 \times 12^\circ = 60^\circ$$

$$9x = 9 \times 12^\circ = 108^\circ$$

$$13x = 13 \times 12^\circ = 156^\circ$$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution:



Given that,

$$AC = BD$$

To show that, ABCD is a rectangle if the diagonals of a parallelogram are equal

To show ABCD is a rectangle we have to prove that one of its interior angles is right angled.

Proof,

In $\triangle ABC$ and $\triangle BAD$,

$$AB = BA \text{ (Common)}$$

$$BC = AD \text{ (Opposite sides of a parallelogram are equal)}$$

$$AC = BD \text{ (Given)}$$

Therefore, $\triangle ABC \cong \triangle BAD$

[SSS congruency]

$$\angle A = \angle B$$

[Corresponding parts of Congruent Triangles]

also,

$$\angle A + \angle B = 180^\circ \text{ (Sum of the angles on the same side of the transversal)}$$

$$\Rightarrow 2\angle A = 180^\circ$$

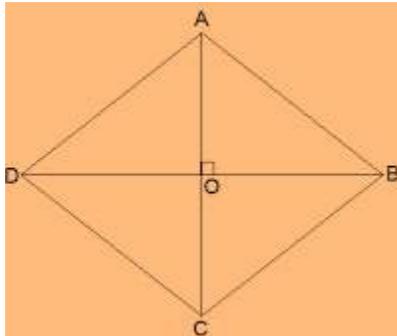
$$\Rightarrow \angle A = 90^\circ = \angle B$$

\therefore , ABCD is a rectangle.

Hence Proved.

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution:



Let ABCD be a quadrilateral whose diagonals bisect each other at right angles.

Given that,

$$OA = OC$$

$$OB = OD$$

$$\text{and } \angle AOB = \angle BOC = \angle OCD = \angle ODA = 90^\circ$$

To show that,

if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

i.e., we have to prove that ABCD is parallelogram and $AB = BC = CD = AD$

Proof,

In $\triangle AOB$ and $\triangle COB$,

$$OA = OC \text{ (Given)}$$

$$\angle AOB = \angle COB \text{ (Opposite sides of a parallelogram are equal)}$$

$$OB = OB \text{ (Common)}$$

Therefore, $\triangle AOB \cong \triangle COB$ [SAS congruency]

Thus, $AB = BC$ [CPCT]

Similarly we can prove,

$$BC = CD$$

$$CD = AD$$

$$AD = AB$$

$$\therefore, AB = BC = CD = AD$$

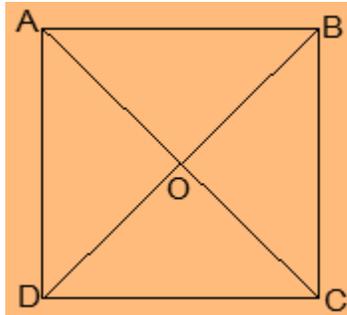
Opposites sides of a quadrilateral are equal hence ABCD is a parallelogram.

\therefore , ABCD is rhombus as it is a parallelogram whose diagonals intersect at right angle.

Hence Proved.

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:



Let ABCD be a square and its diagonals AC and BD intersect each other at O.

To show that,

$$AC = BD$$

$$AO = OC$$

and $\angle AOB = 90^\circ$

Proof,

In $\triangle ABC$ and $\triangle BAD$,

$$BC = BA \text{ (Common)}$$

$$\angle ABC = \angle BAD = 90^\circ$$

$$AC = AD \text{ (Given)}$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SAS congruency}]$$

Thus,

$$AC = BD \quad [\text{CPCT}]$$

diagonals are equal.

Now,

In $\triangle AOB$ and $\triangle COD$,

$$\angle BAO = \angle DCO \text{ (Alternate interior angles)}$$

$$\angle AOB = \angle COD \text{ (Vertically opposite)}$$

$$AB = CD \text{ (Given)}$$

$$\therefore \triangle AOB \cong \triangle COD \quad [\text{AAS congruency}]$$

Thus,

$$AO = CO \quad [\text{CPCT}].$$

\therefore , Diagonal bisect each other.

Now,

In $\triangle AOB$ and $\triangle COB$,

$$OB = OB \text{ (Given)}$$

$$AO = CO \text{ (diagonals are bisected)}$$

$$AB = CB \text{ (Sides of the square)}$$

$$\therefore \triangle AOB \cong \triangle COB \quad [\text{SSS congruency}]$$

also, $\angle AOB = \angle COB$

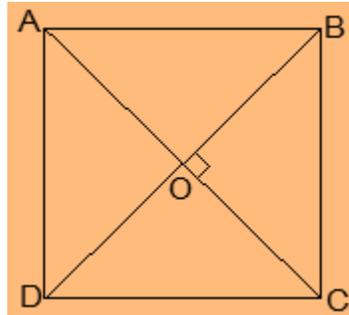
$$\angle AOB + \angle COB = 180^\circ \text{ (Linear pair)}$$

Thus, $\angle AOB = \angle COB = 90^\circ$

\therefore , Diagonals bisect each other at right angles

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution:



Given that,

Let ABCD be a quadrilateral and its diagonals AC and BD bisect each other at right angle at O.

To prove that,

The Quadrilateral ABCD is a square.

Proof,

In $\triangle AOB$ and $\triangle COD$,

$AO = CO$ (Diagonals bisect each other)

$\angle AOB = \angle COD$ (Vertically opposite)

$OB = OD$ (Diagonals bisect each other)

$\therefore \triangle AOB \cong \triangle COD$ [SAS congruency]

Thus,

$AB = CD$ [CPCT] --- (i)

also,

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\Rightarrow AB \parallel CD$

Now,

In $\triangle AOD$ and $\triangle COB$,

$AO = CO$ (Diagonals bisect each other)

$\angle AOD = \angle COB$ (Vertically opposite)

$OD = OB$ (Common)

$\therefore \triangle AOD \cong \triangle COB$ [SAS congruency]

Thus,

$AD = CB$ [CPCT] --- (ii)

also,

$AD = BC$ and $AD = CD$

$\Rightarrow AD = BC = CD = AB$ --- (ii)

also, $\angle ADC = \angle BCD$ [CPCT]

and $\angle ADC + \angle BCD = 180^\circ$ (co-interior angles)

$\Rightarrow 2\angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 90^\circ$ --- (iii)

One of the interior angles is right angle.

Thus, from (i), (ii) and (iii) given quadrilateral ABCD is a square.

Hence Proved.

6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig. 8.19). Show that

(i) it bisects $\angle C$ also,

(ii) ABCD is a rhombus.

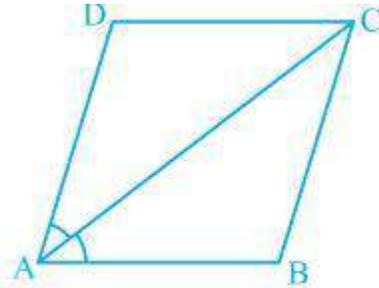


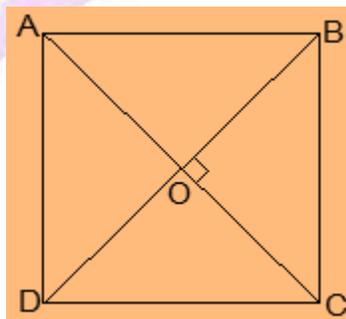
Fig. 8.19

Solution:

- (i) In $\triangle ADC$ and $\triangle CBA$,
 $AD = CB$ (Opposite sides of a parallelogram)
 $DC = BA$ (Opposite sides of a parallelogram)
 $AC = CA$ (Common Side)
 $\therefore \triangle ADC \cong \triangle CBA$ [SSS congruency]
 Thus,
 $\angle ACD = \angle CAB$ by CPCT
 and $\angle CAB = \angle CAD$ (Given)
 $\Rightarrow \angle ACD = \angle BCA$
 Thus,
 AC bisects $\angle C$ also.
- (ii) $\angle ACD = \angle CAD$ (Proved above)
 $\Rightarrow AD = CD$ (Opposite sides of equal angles of a triangle are equal)
 Also, $AB = BC = CD = DA$ (Opposite sides of a parallelogram)
 Thus,
 ABCD is a rhombus.

7. ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Solution:



Given that,

ABCD is a rhombus.
 AC and BD are its diagonals.

Proof,

$AD = CD$ (Sides of a rhombus)
 $\angle DAC = \angle DCA$ (Angles opposite of equal sides of a triangle are equal.)

also, $AB \parallel CD$

$\Rightarrow \angle DAC = \angle BCA$ (Alternate interior angles)

$$\Rightarrow \angle DCA = \angle BCA$$

\therefore , AC bisects $\angle C$.

Similarly,

We can prove that diagonal AC bisects $\angle A$.

Following the same method,

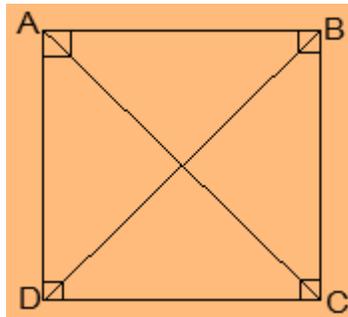
We can prove that the diagonal BD bisects $\angle B$ and $\angle D$.

8. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

(i) ABCD is a square

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.

Solution:



- (i) $\angle DAC = \angle DCA$ (AC bisects $\angle A$ as well as $\angle C$)
 $\Rightarrow AD = CD$ (Sides opposite to equal angles of a triangle are equal)
 also, $CD = AB$ (Opposite sides of a rectangle)
 $\therefore AB = BC = CD = AD$
 Thus, ABCD is a square.

- (ii) In $\triangle BCD$,
 $BC = CD$
 $\Rightarrow \angle CDB = \angle CBD$ (Angles opposite to equal sides are equal)
 also, $\angle CDB = \angle ABD$ (Alternate interior angles)
 $\Rightarrow \angle CBD = \angle ABD$
 Thus, BD bisects $\angle B$
 Now,
 $\angle CBD = \angle ADB$
 $\Rightarrow \angle CDB = \angle ADB$
 Thus, BD bisects $\angle B$ as well as $\angle D$

9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$ (see Fig. 8.20). Show that:

- (i) $\triangle APD \cong \triangle CQB$
 (ii) $AP = CQ$
 (iii) $\triangle AQB \cong \triangle CPD$
 (iv) $AQ = CP$
 (v) APCQ is a parallelogram

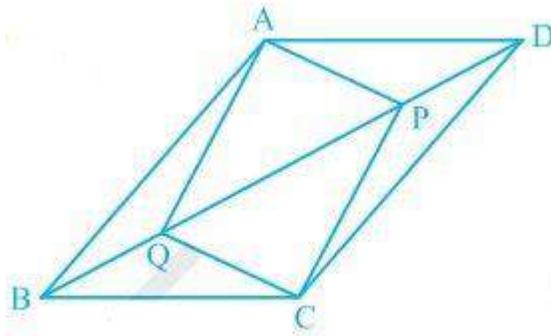


Fig. 8.20

Solution:

- (i) In $\triangle APD$ and $\triangle CQB$,
 $DP = BQ$ (Given)
 $\angle ADP = \angle CBQ$ (Alternate interior angles)
 $AD = BC$ (Opposite sides of a parallelogram)
 Thus, $\triangle APD \cong \triangle CQB$ [SAS congruency]
- (ii) $AP = CQ$ by CPCT as $\triangle APD \cong \triangle CQB$.
- (iii) In $\triangle AQB$ and $\triangle CPD$,
 $BQ = DP$ (Given)
 $\angle ABQ = \angle CDP$ (Alternate interior angles)
 $AB = CD$ (Opposite sides of a parallelogram)
 Thus, $\triangle AQB \cong \triangle CPD$ [SAS congruency]
- (iv) As $\triangle AQB \cong \triangle CPD$
 $AQ = CP$ [CPCT]
- (v) From the questions (ii) and (iv), it is clear that APCQ has equal opposite sides and also has equal and opposite angles. \therefore , APCQ is a parallelogram.

10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that

- (i) $\triangle APB \cong \triangle CQD$
- (ii) $AP = CQ$

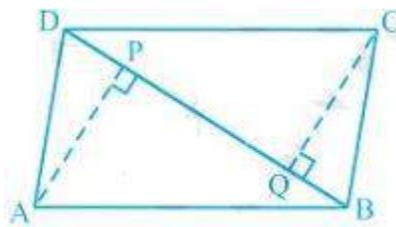


Fig. 8.21

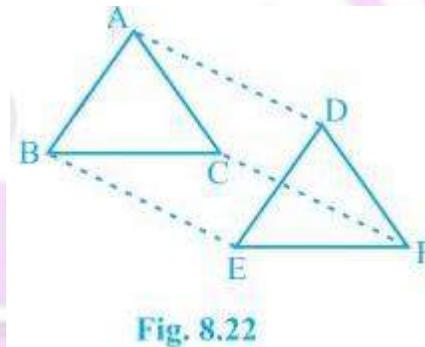
Solution:

- (i) In $\triangle APB$ and $\triangle CQD$,
 $\angle ABP = \angle CDQ$ (Alternate interior angles)
 $\angle APB = \angle CQD (= 90^\circ)$ as AP and CQ are perpendiculars
 $AB = CD$ (ABCD is a parallelogram)
 $\therefore \triangle APB \cong \triangle CQD$ [AAS congruency]
- (ii) As $\triangle APB \cong \triangle CQD$.
 $\therefore AP = CQ$ [CPCT]

11. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22).

Show that

- (i) quadrilateral ABED is a parallelogram
 (ii) quadrilateral BEFC is a parallelogram
 (iii) $AD \parallel CF$ and $AD = CF$
 (iv) quadrilateral ACFD is a parallelogram
 (v) $AC = DF$
 (vi) $\triangle ABC \cong \triangle DEF$.



Solution:

- (i) $AB = DE$ and $AB \parallel DE$ (Given)
 Two opposite sides of a quadrilateral are equal and parallel to each other.
 Thus, quadrilateral ABED is a parallelogram
- (ii) Again $BC = EF$ and $BC \parallel EF$.
 Thus, quadrilateral BEFC is a parallelogram.
- (iii) Since ABED and BEFC are parallelograms.
 $\Rightarrow AD = BE$ and $BE = CF$ (Opposite sides of a parallelogram are equal)
 $\therefore AD = CF$.
 Also, $AD \parallel BE$ and $BE \parallel CF$ (Opposite sides of a parallelogram are parallel)
 $\therefore AD \parallel CF$
- (iv) AD and CF are opposite sides of quadrilateral ACFD which are equal and parallel to each other. Thus, it is a parallelogram.
- (v) Since ACFD is a parallelogram
 $AC \parallel DF$ and $AC = DF$
- (vi) In $\triangle ABC$ and $\triangle DEF$,

$$\begin{aligned} AB &= DE \text{ (Given)} \\ BC &= EF \text{ (Given)} \\ AC &= DF \text{ (Opposite sides of a parallelogram)} \\ \therefore, \Delta ABC &\cong \Delta DEF \quad \text{[SSS congruency]} \end{aligned}$$

12. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see Fig. 8.23). Show that

- (i) $\angle A = \angle B$
- (ii) $\angle C = \angle D$
- (iii) $\Delta ABC \cong \Delta BAD$
- (iv) diagonal $AC =$ diagonal BD

[Hint : Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

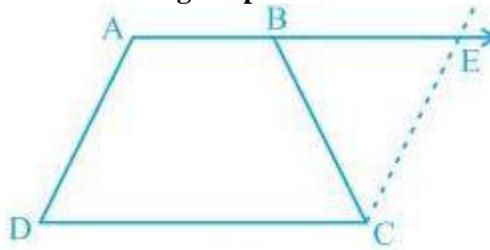


Fig. 8.23

Solution:

To Construct: Draw a line through C parallel to DA intersecting AB produced at E.

- (i) $CE = AD$ (Opposite sides of a parallelogram)

$$AD = BC \text{ (Given)}$$

$$\therefore, BC = CE$$

$$\Rightarrow \angle CBE = \angle CEB$$

also,

$$\angle A + \angle CBE = 180^\circ \text{ (Angles on the same side of transversal and } \angle CBE = \angle CEB)$$

$$\angle B + \angle CBE = 180^\circ \text{ (As Linear pair)}$$

$$\Rightarrow \angle A = \angle B$$

- (ii) $\angle A + \angle D = \angle B + \angle C = 180^\circ$ (Angles on the same side of transversal)

$$\Rightarrow \angle A + \angle D = \angle A + \angle C \text{ (} \angle A = \angle B)$$

$$\Rightarrow \angle D = \angle C$$

- (iii) In ΔABC and ΔBAD ,

$$AB = AB \text{ (Common)}$$

$$\angle DBA = \angle CBA$$

$$AD = BC \text{ (Given)}$$

$$\therefore, \Delta ABC \cong \Delta BAD \quad \text{[SAS congruency]}$$

- (iv) Diagonal $AC =$ diagonal BD by CPCT as $\Delta ABC \cong \Delta BAD$.