Exercise 8.1 Page: 146

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Solution:

Let the common ratio between the angles be = x.

We know that the sum of the interior angles of the quadrilateral = 360°

Now,

$3x+5x+9x+13x = 360°$

$\Rightarrow \quad 30x = 360°$

$\Rightarrow \quad x = 12°$

∴, Angles of the quadrilateral are:

$3x = 3\times12° = 36°$

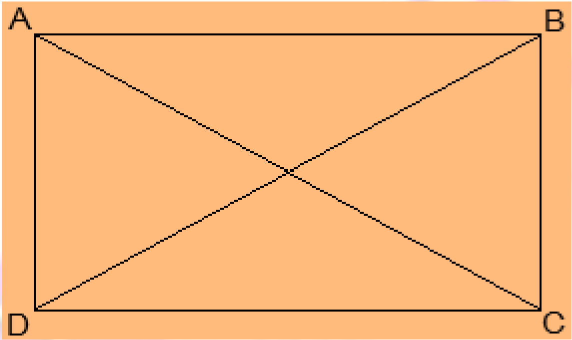
$5x = 5\times12° = 60°$

$9x = 9\times12° = 108°$

$13x = 13\times12° = 156°$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution:



Given that,

AC = BD

To show that, ABCD is a rectangle if the diagonals of a parallelogram are equal

To show ABCD is a rectangle we have to prove that one of its interior angles is right angled.

Proof,

In ΔABC and ΔBAD,

AB = BA (Common)

BC = AD (Opposite sides of a parallelogram are equal)

AC = BD (Given)

Therefore, ΔABC ≅ ΔBAD [SSS congruency]

∠A = ∠B [Corresponding parts of Congruent Triangles]

also,

∠A+∠B = 180° (Sum of the angles on the same side of the transversal)

⇒ 2∠A = 180°

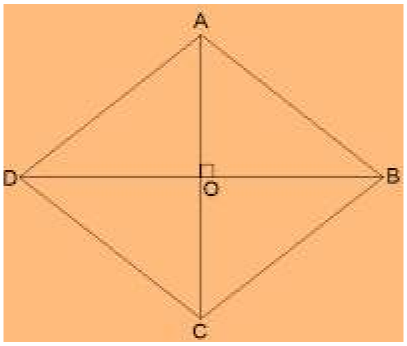
⇒ ∠A = 90° = ∠B

∴ , ABCD is a rectangle.

Hence Proved.

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution:



Let ABCD be a quadrilateral whose diagonals bisect each other at right angles.

Given that,

OA = OC

OB = OD

and ∠AOB = ∠BOC = ∠OCD = ∠ODA = 90°

To show that,

if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

i.e., we have to prove that ABCD is parallelogram and AB = BC = CD = AD

Proof,

In ΔAOB and ΔCOB,

OA = OC (Given)

∠AOB = ∠COB (Opposite sides of a parallelogram are equal)

OB = OB (Common)

Therefore, ΔAOB ≅ ΔCOB [SAS congruency]

Thus, AB = BC [CPCT]

Similarly we can prove,

BC = CD

CD = AD

AD = AB

∴, AB = BC = CD = AD

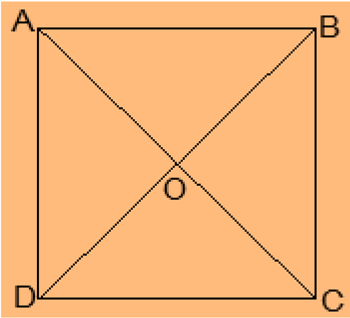
Opposites sides of a quadrilateral are equal hence ABCD is a parallelogram.

∴, ABCD is rhombus as it is a parallelogram whose diagonals intersect at right angle.

Hence Proved.

4. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:



Let ABCD be a square and its diagonals AC and BD intersect each other at O.

To show that,

AC = BD

AO = OC

and ∠AOB = 90°

Proof,

In ΔABC and ΔBAD,

BC = BA (Common)

∠ABC = ∠BAD = 90°

AC = AD (Given)

∴ ΔABC ≅ ΔBAD [SAS congruency]

Thus,

AC = BD [CPCT]

diagonals are equal.

Now,

In ΔAOB and ΔCOD,

∠BAO = ∠DCO (Alternate interior angles)

∠AOB = ∠COD (Vertically opposite)

AB = CD (Given)

∴, ΔAOB ≅ ΔCOD [AAS congruency]

Thus,

AO = CO [CPCT].

∴, Diagonal bisect each other.

Now,

In ΔAOB and ΔCOB,

OB = OB (Given)

AO = CO (diagonals are bisected)

AB = CB (Sides of the square)

∴, ΔAOB ≅ ΔCOB [SSS congruency]

also, ∠AOB = ∠COB

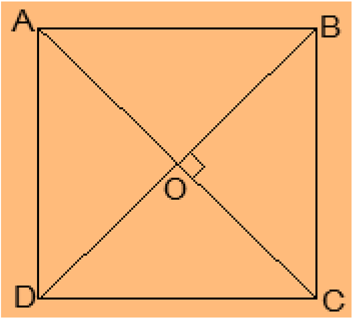
∠AOB+∠COB = 180° (Linear pair)

Thus, ∠AOB = ∠COB = 90°

∴ , Diagonals bisect each other at right angles

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution:



Given that,
Let ABCD be a quadrilateral and its diagonals AC and BD bisect each other at right angle at O.

To prove that,
The Quadrilateral ABCD is a square.

Proof,

In ΔAOB and ΔCOD,

AO = CO (Diagonals bisect each other)
∠AOB = ∠COD (Vertically opposite)
OB = OD (Diagonals bisect each other)

∴, ΔAOB ≅ ΔCOD [SAS congruency]

Thus,

AB = CD [CPCT] --- (i)

also,

∠OAB = ∠OCD (Alternate interior angles)
⇒ AB || CD

Now,

In ΔAOD and ΔCOD,

AO = CO (Diagonals bisect each other)
∠AOD = ∠COD (Vertically opposite)
OD = OD (Common)

∴, ΔAOD ≅ ΔCOD [SAS congruency]

Thus,

AD = CD [CPCT] --- (ii)

also,

AD = BC and AD = CD
⇒ AD = BC = CD = AB --- (ii)

also, ∠ADC = ∠BCD [CPCT]

and ∠ADC+∠BCD = 180° (co-interior angles)

⇒ 2∠ADC = 180°
⇒∠ADC = 90° --- (iii)

One of the interior angles is right angle.

Thus, from (i), (ii) and (iii) given quadrilateral ABCD is a square.

Hence Proved.

6. Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig. 8.19). Show that
(i) it bisects ∠C also,
(ii) ABCD is a rhombus.

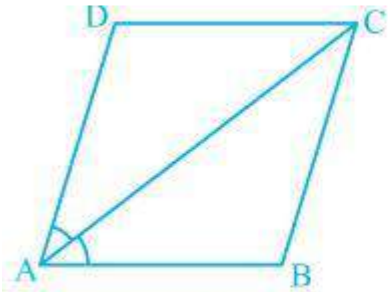


Fig. 8.19

Solution:

(i) In ΔADC and ΔCBA,

AD = CB (Opposite sides of a parallelogram)

DC = BA (Opposite sides of a parallelogram)

AC = CA (Common Side)

∴, ΔADC ≅ ΔCBA [SSS congruency]

Thus,

∠ACD = ∠CAB by CPCT

and ∠CAB = ∠CAD (Given)

⇒ ∠ACD = ∠BCA

Thus,

AC bisects ∠C also.

(ii) ∠ACD = ∠CAD (Proved above)

⇒ AD = CD (Opposite sides of equal angles of a triangle are equal)

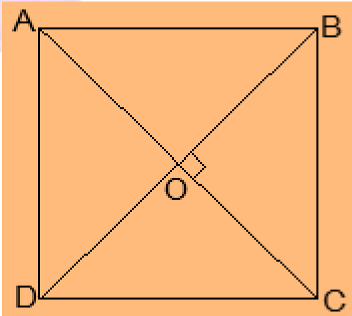
Also, AB = BC = CD = DA (Opposite sides of a parallelogram)

Thus,

ABCD is a rhombus.

7. ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Solution:



Given that,

ABCD is a rhombus.

AC and BD are its diagonals.

Proof,

AD = CD (Sides of a rhombus)

∠DAC = ∠DCA (Angles opposite of equal sides of a triangle are equal.)

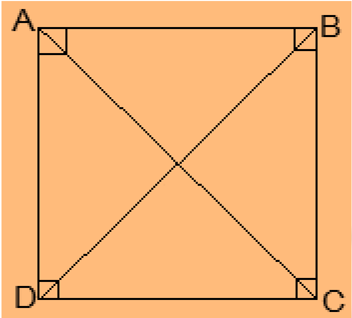
also, AB || CD

⇒∠DAC = ∠BCA (Alternate interior angles)

⇒∠DCA = ∠BCA
∴, AC bisects ∠C.
Similarly,
We can prove that diagonal AC bisects ∠A.
Following the same method,
We can prove that the diagonal BD bisects ∠B and ∠D.

8. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:
(i) ABCD is a square
(ii) Diagonal BD bisects ∠B as well as ∠D.

Solution:



(i) ∠DAC = ∠DCA (AC bisects ∠A as well as ∠C)
⇒ AD = CD (Sides opposite to equal angles of a triangle are equal)
also, CD = AB (Opposite sides of a rectangle)
∴,AB = BC = CD = AD
Thus, ABCD is a square.

(ii) In ΔBCD,
BC = CD
⇒ ∠CDB = ∠CBD (Angles opposite to equal sides are equal)
also, ∠CDB = ∠ABD (Alternate interior angles)
⇒ ∠CBD = ∠ABD
Thus, BD bisects ∠B
Now,
∠CBD = ∠ADB
⇒ ∠CDB = ∠ADB
Thus, BD bisects ∠B as well as ∠D

9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.20). Show that:
(i) ΔAPD ≅ ΔCQB
(ii) AP = CQ
(iii) ΔAQB ≅ ΔCPD
(iv) AQ = CP
(v) APCQ is a parallelogram

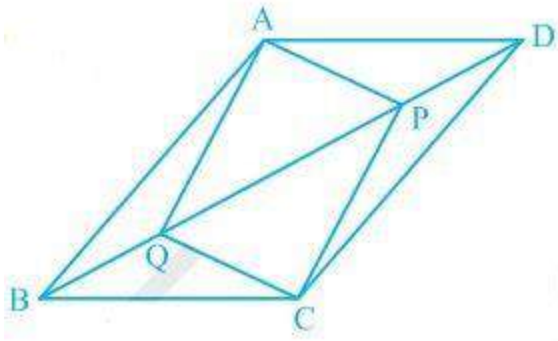


Fig. 8.20

Solution:

(i) In ΔAPD and ΔCQB,
DP = BQ (Given)
∠ADP = ∠CBQ (Alternate interior angles)
AD = BC (Opposite sides of a parallelogram)
Thus, ΔAPD ≅ ΔCQB [SAS congruency]

(ii) AP = CQ by CPCT as ΔAPD ≅ ΔCQB.

(iii) In ΔAQB and ΔCPD,
BQ = DP (Given)
∠ABQ = ∠CDP (Alternate interior angles)
AB = CD (Opposite sides of a parallelogram)
Thus, ΔAQB ≅ ΔCPD [SAS congruency]

(iv) As ΔAQB ≅ ΔCPD
AQ = CP [CPCT]

(v) From the questions (ii) and (iv), it is clear that APCQ has equal opposite sides and also has equal and opposite angles. ∴, APCQ is a parallelogram.

10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that
(i) ΔAPB ≅ ΔCQD
(ii) AP = CQ

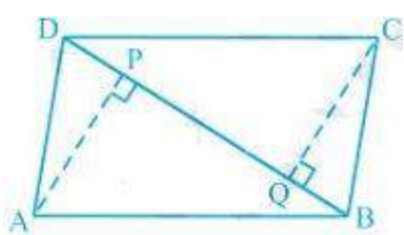


Fig. 8.21

Solution:

(i) In ΔAPB and ΔCQD,
∠ABP = ∠CDQ (Alternate interior angles)
∠APB = ∠CQD (= 90º as AP and CQ are perpendiculars)
AB = CD (ABCD is a parallelogram)
∴, ΔAPB ≅ ΔCQD [AAS congruency]

(ii) As ΔAPB ≅ ΔCQD.
∴, AP = CQ [CPCT]

11. In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22).

Show that

(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) ΔABC ≅ ΔDEF.

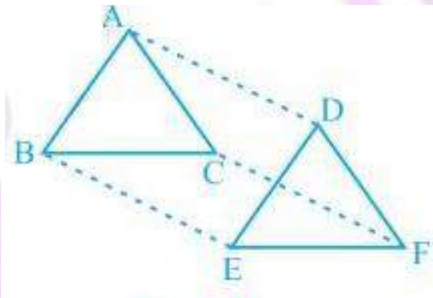


Fig. 8.22

Solution:

(i) AB = DE and AB || DE (Given)
Two opposite sides of a quadrilateral are equal and parallel to each other.
Thus, quadrilateral ABED is a parallelogram

(ii) Again BC = EF and BC || EF.
Thus, quadrilateral BEFC is a parallelogram.

(iii) Since ABED and BEFC are parallelograms.
⇒ AD = BE and BE = CF (Opposite sides of a parallelogram are equal)
∴, AD = CF.
Also, AD || BE and BE || CF (Opposite sides of a parallelogram are parallel)
∴, AD || CF

(iv) AD and CF are opposite sides of quadrilateral ACFD which are equal and parallel to each other. Thus, it is a parallelogram.

(v) Since ACFD is a parallelogram
AC || DF and AC = DF

(vi) In ΔABC and ΔDEF,

AB = DE (Given)
BC = EF (Given)
AC = DF (Opposite sides of a parallelogram)
∴, ΔABC ≅ ΔDEF [SSS congruency]

12. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.23). Show that
(i) ∠A = ∠B
(ii) ∠C = ∠D
(iii) ΔABC ≅ ΔBAD
(iv) diagonal AC = diagonal BD
[Hint : Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

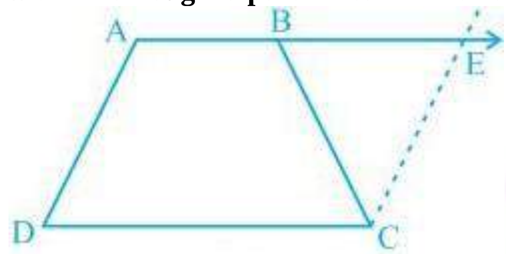


Fig. 8.23

Solution:

To Construct: Draw a line through C parallel to DA intersecting AB produced at E.
(i) CE = AD (Opposite sides of a parallelogram)
AD = BC (Given)
∴, BC = CE
⇒∠CBE = ∠CEB
also,
∠A+∠CBE = 180° (Angles on the same side of transversal and ∠CBE = ∠CEB)
∠B +∠CBE = 180° (As Linear pair)
⇒∠A = ∠B

(ii) ∠A+∠D = ∠B+∠C = 180° (Angles on the same side of transversal)
⇒∠A+∠D = ∠A+∠C (∠A = ∠B)
⇒∠D = ∠C

(iii) In ΔABC and ΔBAD,
AB = AB (Common)

∠DBA = ∠CBA
AD = BC (Given)
∴, ΔABC ≅ ΔBAD [SAS congruency]

(iv) Diagonal AC = diagonal BD by CPCT as ΔABC ≅ ΔBAD.

Exercise 8.2 Page: 150

1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that:
(i) SR || AC and SR = 1/2 AC
(ii) PQ = SR
(iii) PQRS is a parallelogram.

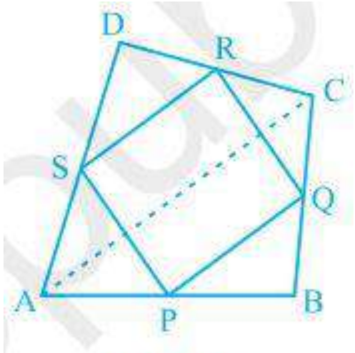


Fig. 8.29

Solution:

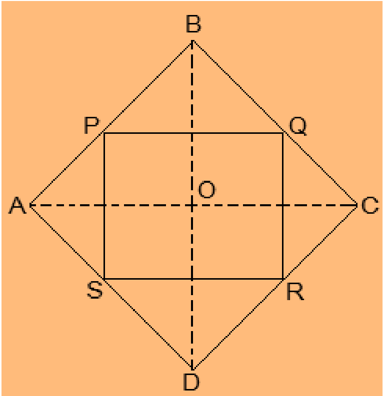
(i) In ΔDAC,
R is the mid point of DC and S is the mid point of DA.
Thus by mid point theorem, SR || AC and SR = ½ AC

(ii) In ΔBAC,
P is the mid point of AB and Q is the mid point of BC.
Thus by mid point theorem, PQ || AC and PQ = ½ AC
also, SR = ½ AC
∴, PQ = SR

(iii) SR || AC ---------------------- from question (i)
and, PQ || AC ---------------------- from question (ii)
⇒ SR || PQ - from (i) and (ii)
also, PQ = SR
∴, PQRS is a parallelogram.

2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Solution:



Given in the question,

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

To Prove,

PQRS is a rectangle.

Construction,

Join AC and BD.

Proof:

In ΔDRS and ΔBPQ,

DS = BQ (Halves of the opposite sides of the rhombus)

∠SDR = ∠QBP (Opposite angles of the rhombus)

DR = BP (Halves of the opposite sides of the rhombus)

∴, ΔDRS ≅ ΔBPQ [SAS congruency]

RS = PQ [CPCT]---------------------- (i)

In ΔQCR and ΔSAP,

RC = PA (Halves of the opposite sides of the rhombus)

∠RCQ = ∠PAS (Opposite angles of the rhombus)

CQ = AS (Halves of the opposite sides of the rhombus)

∴, ΔQCR ≅ ΔSAP [SAS congruency]

RQ = SP [CPCT]---------------------- (ii)

Now,

In ΔCDB,

R and Q are the mid points of CD and BC respectively.

⇒ QR || BD

also,

P and S are the mid points of AD and AB respectively.

⇒ PS || BD

⇒ QR || PS

∴, PQRS is a parallelogram.

also, ∠PQR = 90°

Now,

In PQRS,

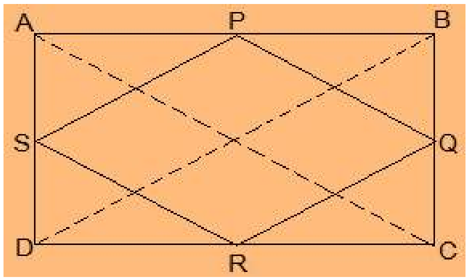
RS = PQ and RQ = SP from (i) and (ii)

∠Q = 90°

∴, PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Solution:



Given in the question,

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively.

Construction,

Join AC and BD.

To Prove,

PQRS is a rhombus.

Proof:

In ΔABC

P and Q are the mid-points of AB and BC respectively

∴, PQ || AC and PQ = ½ AC (Midpoint theorem) --- (i)

In ΔADC,

SR || AC and SR = ½ AC (Midpoint theorem) --- (ii)

So, PQ || SR and PQ = SR

As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.

∴, PS || QR and PS = QR (Opposite sides of parallelogram) --- (iii)

Now,

In ΔBCD,

Q and R are mid points of side BC and CD respectively.

∴, QR || BD and QR = ½ BD (Midpoint theorem) --- (iv)

AC = BD (Diagonals of a rectangle are equal) --- (v)

From equations (i), (ii), (iii), (iv) and (v),

PQ = QR = SR = PS

So, PQRS is a rhombus.

Hence Proved

4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.

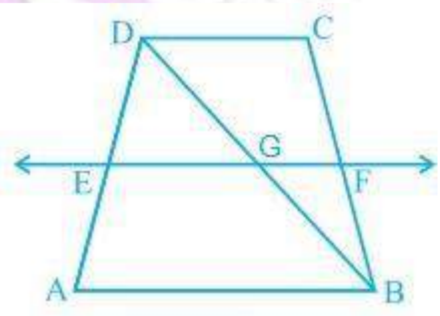


Fig. 8.30

Solution:

Given that,

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD.

To prove,

F is the mid-point of BC.

Proof,

BD intersected EF at G.

In ΔBAD,

E is the mid point of AD and also EG || AB.
Thus, G is the mid point of BD (Converse of mid point theorem)

Now,
In ΔBDC,
G is the mid point of BD and also GF || AB || DC.
Thus, F is the mid point of BC (Converse of mid point theorem)

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.31). Show that the line segments AF and EC trisect the diagonal BD.

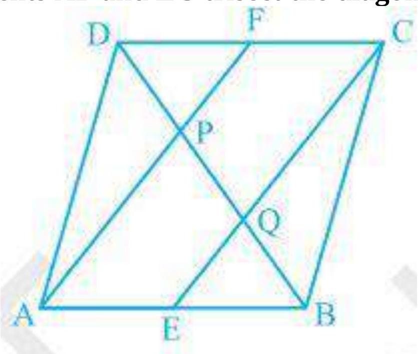


Fig. 8.31

Solution:

Given that,
ABCD is a parallelogram. E and F are the mid-points of sides AB and CD respectively.

To show,
AF and EC trisect the diagonal BD.

Proof,
ABCD is a parallelogram
∴, AB || CD
also, AE || FC

Now,
AB = CD (Opposite sides of parallelogram ABCD)
⇒½ AB = ½ CD
⇒ AE = FC (E and F are midpoints of side AB and CD)
AECF is a parallelogram (AE and CF are parallel and equal to each other)
AF || EC (Opposite sides of a parallelogram)

Now,
In ΔDQC,
F is mid point of side DC and FP || CQ (as AF || EC).
P is the mid-point of DQ (Converse of mid-point theorem)
⇒ DP = PQ --- (i)

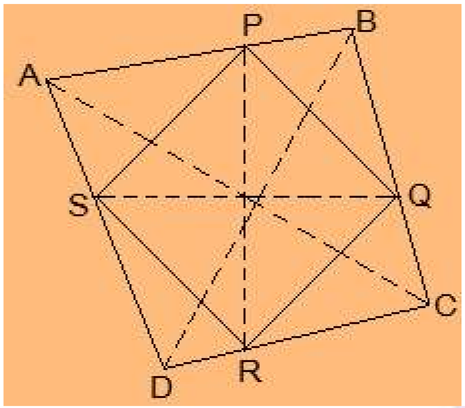
Similarly,
In ΔAPB,

E is midpoint of side AB and EQ || AP (as AF || EC).
Q is the mid-point of PB (Converse of mid-point theorem)
⇒ PQ = QB --- (ii)
From equations (i) and (i),
DP = PQ = BQ
Hence, the line segments AF and EC trisect the diagonal BD.

Hence Proved.

6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution:



Let ABCD be a quadrilateral and P, Q, R and S are the mid points of AB, BC, CD and DA respectively.

Now,

In ΔACD,

R and S are the mid points of CD and DA respectively.

∴, SR || AC.

Similarly we can show that,

PQ || AC,

PS || BD and

QR || BD

∴, PQRS is parallelogram.

PR and QS are the diagonals of the parallelogram PQRS. So, they will bisect each other.

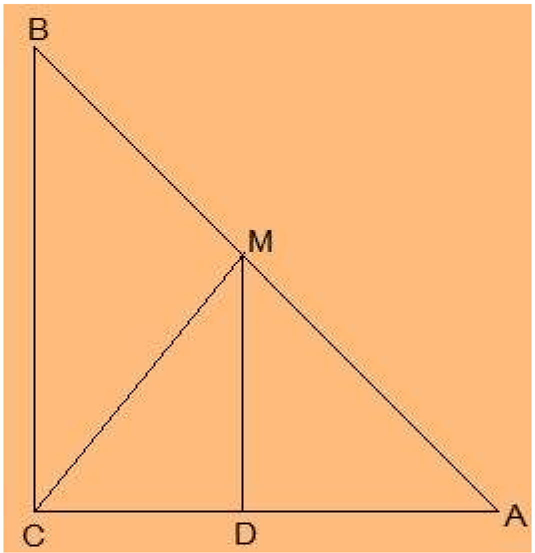
7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) CM = MA = ½ AB

Solution:



(i) In ΔACB,
M is the midpoint of AB and MD || BC
∴, D is the midpoint of AC (Converse of mid point theorem)

(ii) ∠ACB = ∠ADM (Corresponding angles)
also, ∠ACB = 90°
∴, ∠ADM = 90° and MD ⊥ AC

(iii) In ΔAMD and ΔCMD,
AD = CD (D is the midpoint of side AC)
∠ADM = ∠CDM (Each 90°)
DM = DM (common)
∴, ΔAMD ≅ ΔCMD [SAS congruency]
AM = CM [CPCT]
also, AM = ½ AB (M is midpoint of AB)
Hence, CM = MA = ½ AB