

CAT 2020 Question Paper with Solution Slot 3 QA

1. Let 'm' and 'n' be natural numbers such that n is even and $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$. Then, m - 2n equal

- A. 3
- B. 1
- C. 4
- D. 2

Answer: B

Solution:

$$0.2 < n/11 < 0.5$$

$$\Rightarrow 2.2 < n < 5.5$$

Since n is an even natural number, we know the following:

$$\therefore n = 4$$

$$0.2 < m/20 < 0.5$$

$$\Rightarrow 4 < m < 10 \dots\dots\dots(1)$$

$$0.2 < n/m < 0.5$$

$$\Rightarrow 0.2 < 4/m < 0.5$$

$$\Rightarrow 5 > m/4 > 2$$

$$\Rightarrow 20 > m > 8 \dots\dots\dots(2)$$

Using (1) and (2), we get the following:

$$m = 9$$

$$m - 2n = 9 - 2(4) = 1$$

###TOPIC###Quantitative Aptitude||Number System||Properties of Numbers###

2. If $\log_a 30 = A$, $\log_a(5/3) = -B$, and $\log_2 a = 1/3$, then $\log_3 a$ equals _____.

- A. $\frac{A+B-3}{2}$
- B. $\frac{A+B}{2} - 3$
- C. $\frac{A+B-3}{2}$
- D. $\frac{2}{A+B} - 3$

Answer: C

Solution:

$$\log_a 2 = 3 \dots\dots\dots(1)$$

$$\log_a 30 = A$$

$$\Rightarrow \log_a 2 + \log_a 3 + \log_a 5 = A$$

$$\Rightarrow \log_a 5 + \log_a 3 = A - 3 \dots\dots\dots(2)$$

$$\log_a \frac{5}{3} = -B$$

$$\Rightarrow \log_a 5 - \log_a 3 = -B \dots\dots\dots(3)$$

$$(3) - (2):$$

$$2\log_a 3 = A - 3 + B$$

$$\Rightarrow \log_a 3 = \frac{A+B-3}{2}$$

$$\Rightarrow \log_3 a = \frac{2}{A+B-3}$$

###TOPIC###Quantitative Aptitude||Algebra||Logarithm###

3. A batsman played $n + 2$ innings and got out on all occasions. His average score in these $n + 2$ innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first 'n' innings. In these 'n' innings, his average score was 30 runs and lowest score was 'x' runs. The smallest possible value of 'x' is ____.

- A. 4
- B. 1
- C. 2
- D. 3

Answer: C

Solution:

Sum of scores in $(n + 2)$ innings = $29 \times (n + 2) = 29n + 58$

Sum of scores in n innings = $30 \times n = 30n$

$30n + 38 + 15 = 29n + 58$ or $n = 5$

Sum of scores in first 5 innings = $5 \times 30 = 150$

To minimize the score in one inning, we need to maximize the score in the other 4 innings.

The maximum possible score in each of the four innings = 37

Sum of 4 innings = $4 \times 37 = 148$

Minimum possible score in one of the first five innings = $150 - 148 = 2$

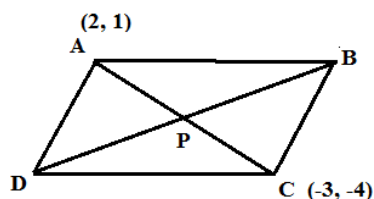
###TOPIC###Quantitative Aptitude||Arithmetic||Averages###

4. The points (2, 1) and (-3, -4) are opposite vertices of a parallelogram. If the other two vertices lie on the line $x + 9y + c = 0$, then c is ____.

- A. 12
- B. 13
- C. 14
- D. 15

Answer: C

Solution:



Diagonals of a parallelogram bisect each other.

\therefore P is the midpoint of AC. We can calculate the coordinates of P using the midpoint formula.

$$x = \frac{2-3}{2} = \frac{-1}{2}, \quad y = \frac{1-4}{2} = \frac{-3}{2}$$

Equation of BD is given as the following:

$$x + 9y + c = 0$$

Since P lies on BD, we can put $x = -1/2$ and $y = -3/2$ in the equation of BD.

$$\therefore \frac{-1}{2} + 9\left(\frac{-3}{2}\right) + c = 0$$

$$\Rightarrow c = 14$$

###TOPIC### Quantitative Aptitude||Geometry||Quadrilateral & Polygons###

5. In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is _____.

- A. 417
- B. 357
- C. 399
- D. 439

Answer: C

Solution:

Let the maximum marks in the exam = 100k

Mishnu's score = 52k

Asha's score = 64k

According to question, we have the following:

$$\Rightarrow 52k + 23 = 64k - 34$$

$$\Rightarrow 57 = 12k$$

$$\Rightarrow k = 19/4$$

Maximum marks = $100 \times 19/4 = 475$

Geeta's score = 84% of 475 = 399.

6. The vertices of a triangle are (0, 0), (4, 0), and (3, 9). The area of the circle passing through three points is _____.

- A. $\frac{14\pi}{3}$
- B. $\frac{123\pi}{7}$
- C. $\frac{12\pi}{5}$
- D. $\frac{205\pi}{9}$

Answer: D

Solution:

Let the equation of the circle be the following [considering the general format of circle]:

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots\dots\dots(1)$$

Substituting the point (0, 0) in the above equation, we get the following:

$$(0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0$$

$$c = 0 \dots\dots\dots(2)$$

Putting (4, 0) in the above equation, we get the following:

$$(4)^2 + (0)^2 + 2g(4) + 2f(0) + c = 0$$

$$16 + 8g + c = 0$$

$$g = -2 \dots\dots\dots(3)$$

Putting (3, 9) in the above equation, we get the following:

$$(3)^2 + (9)^2 + 2g(3) + 2f(9) + c = 0$$

$$9 + 81 + 6(-2) + 18f + 0 = 0$$

$$f = -13/3 \dots\dots\dots(4)$$

$$\text{Radius of circle} = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 169/9 - 0} = \frac{\sqrt{205}}{3}$$

$$\text{Area of circle} = \pi \left(\frac{\sqrt{205}}{3} \right)^2 = \frac{205\pi}{9}$$

###TOPIC### Quantitative Aptitude||Co-ordinate Geometry||Co-ordinate Geometry###

7. Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if _____.

- A. $k \neq e$
- B. $|k| = 2$
- C. $|k| \neq 2$
- D. $k = 2$

Answer: C

Solution:

$$Kx + y = 3$$

$$4x + ky = 4$$

For unique solution,

$$\frac{k}{4} \neq \frac{1}{k}$$

$$\Rightarrow |k| \neq 2$$

###TOPIC### Quantitative Aptitude||Algebra||Linear Equations###

8. Let m and n be positive integers. If $x^2 + mx + 2n = 0$ and $x^2 + 2nx + m = 0$ have real roots, then the smallest possible value of $m + n$ is ____.

- A. 5
- B. 8
- C. 7
- D. 6

Answer: D

Solution:

$x^2 + mx + 2n = 0$ have real roots.

$$m^2 - 4 \times 1 \times 2n \geq 0$$

$$\Rightarrow m^2 - 8n \geq 0 \dots\dots\dots(1)$$

$x^2 + 2nx + m = 0$ have real roots.

$$(2n)^2 - 4 \times 1 \times m \geq 0$$

$$\Rightarrow n^2 - m \geq 0 \dots\dots\dots(2)$$

The least combinations (m, n) which satisfy the first inequality are $(3, 1), (4, 1), (4, 2), (5, 1), (5, 2), (5, 3)$ and so on.

Among them, $(3, 1)$ and $(4, 1)$ do not satisfy the second inequality.

Hence, the least combination which satisfies both the given inequalities is $m = 4$ and $n = 2$.

Smallest value of $m + n = 4 + 2 = 6$

###TOPIC###Quantitative Aptitude||Algebra||Quadratic Equations###

9. A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to ____.

- A. 22
- B. 25
- C. 35
- D. 31

Answer: B

Solution:

Let the CP of 1 kg sugar be $100a$.

So, MP = $120a$

SP of 5 kg = $120a \times 5$

SP of 15 kg = $0.9 \times 120a = 108a$

Let the SP of the remaining 12 kg sugar be b .

Total SP = $5 \times 120a + 15 \times 108a + 3 \times 0 + 12 \times b = 2220a + 12b$

Total CP = $35 \times 100a = 3500a$

Given, $2220a + 12b = 1.15 \times 3500a$

$\Rightarrow 12b = 1805a$

$\Rightarrow b = 150.42a$ (approx)

Required percentage = $\frac{150.42a - 120a}{120a} \times 100 = 25.4\%$ (approx)

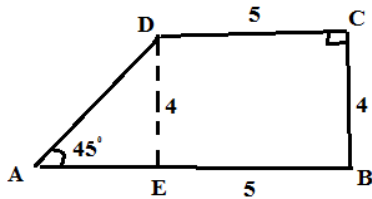
###TOPIC###Quantitative Aptitude||Arithmetic||Profit, Loss and Discount###

10. In a trapezium ABCD, AB is parallel to DC, BC is perpendicular to DC and $\angle BAD = 45^\circ$. If DC = 5 cm and BC = 4 cm, the area of the trapezium in sq cm is ____.

Answer: 28

Solution:

Using the given information, we can draw the following figure:



Draw DE perpendicular on AB.

In triangle AED,

$$\tan 45 = DE/AE$$

$$1 = DE/AE$$

$$AE = 4 \text{ cm}$$

$$\text{Area of trapezium} = \frac{1}{2} \times (5+9) \times 4 = 28 \text{ cm}^2$$

###TOPIC###Quantitative Aptitude||Geometry||Quadrilateral & Polygons###

11. If a, b, c are non-zero and $14^a = 36^b = 84^c$, then $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ is equal to ____.

Answer: 3

Solution:

Let $14^a = 36^b = 84^c = k$

$14^a = k$

$\Rightarrow a \log 14 = \log k$

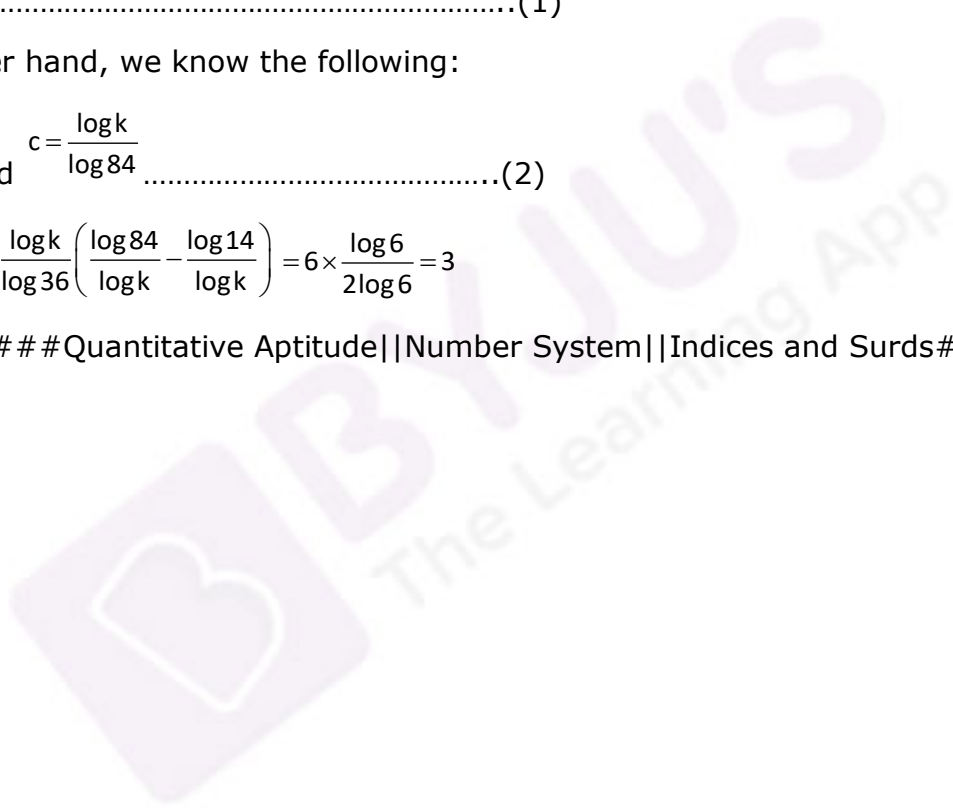
$\Rightarrow a = \frac{\log k}{\log 14}$ (1)

On the other hand, we know the following:

$b = \frac{\log k}{\log 36}$ and $c = \frac{\log k}{\log 84}$ (2)

$$6b\left(\frac{1}{c} - \frac{1}{a}\right) = 6 \frac{\log k}{\log 36} \left(\frac{\log 84}{\log k} - \frac{\log 14}{\log k}\right) = 6 \times \frac{\log 6}{2 \log 6} = 3$$

###TOPIC### Quantitative Aptitude||Number System||Indices and Surds###



12. A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became Rs. 18522. The amount, in rupees, that the person had invested is _____.

Answer: 16000

Solution:

Let the principal be P.

Rate = 10% p.a. = 5% half yearly

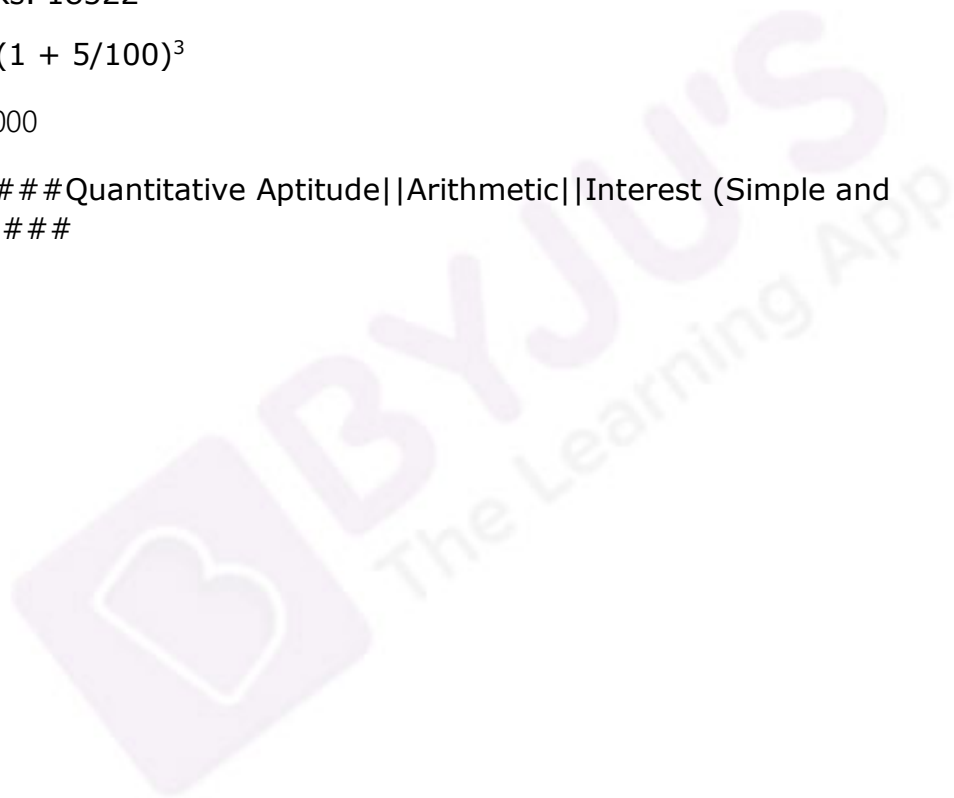
Time = 1.5 years = 3 'Half-years'

Amount = Rs. 18522

$$18522 = P (1 + 5/100)^3$$

$$\Rightarrow p = \text{Rs. } 16000$$

###TOPIC###Quantitative Aptitude||Arithmetic||Interest (Simple and Compound)###



13. How many of the integers 1, 2, ..., 120 are divisible by none of 2, 5, and 7?

- A. 43
- B. 42
- C. 41
- D. 40

Answer: C

Solution:

2 and 5 are prime factors of 120.

So, number of integers from 1 to 120 which are not divisible by 2 or 5

$$= 120 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{5}\right) = 48$$

Numbers from 1 to 120 which are multiple of 7 but not 2 or 5 are (7, 21, 49, 63, 77, 91, 119). There are seven such numbers.

Number of integers from 1 to 120 which are divisible by none of 2, 5, and 7 =
 $48 - 7 = 41$

###TOPIC### Quantitative Aptitude || Number System || Factors and their properties###

14. Let N , x , and y be positive integers such that $N = x + y$, $2 < x < 10$ and $14 < y < 23$. If $N > 25$, then how many distinct values are possible for N ?

Answer: 6

Solution:

$$2 < x < 10 \text{ and } 14 < y < 23$$

$$N = x + y$$

$$\therefore 3 + 15 \leq N \leq 9 + 22$$

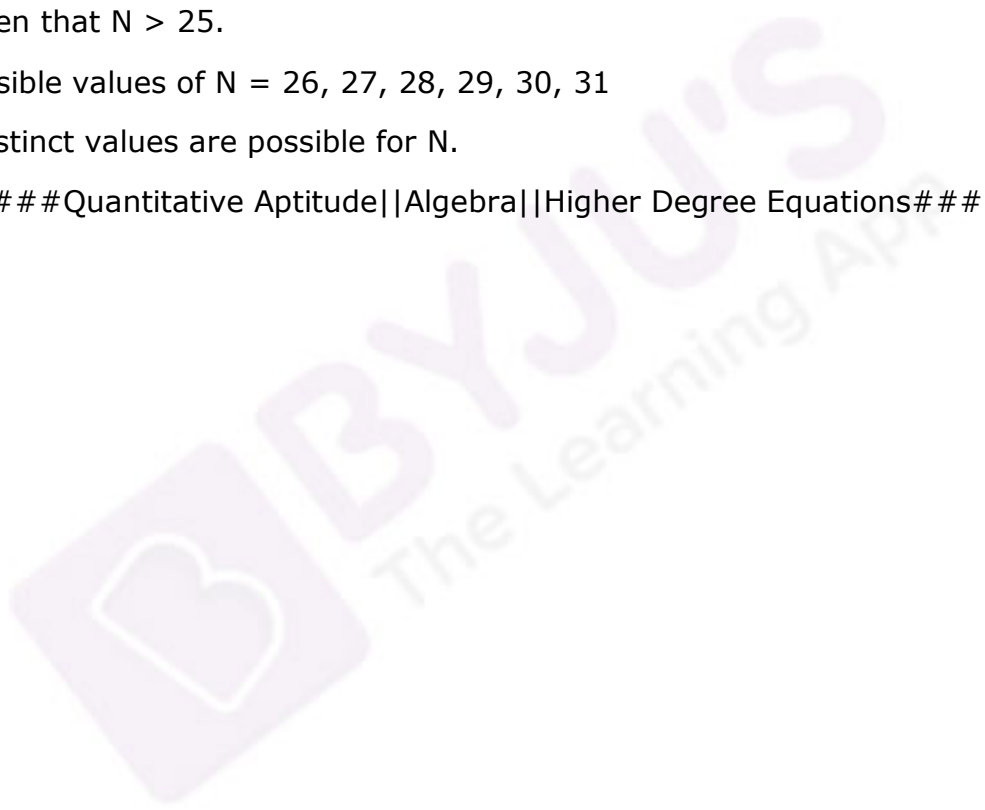
$$\Rightarrow 18 \leq N \leq 31$$

But it is given that $N > 25$.

So, the possible values of $N = 26, 27, 28, 29, 30, 31$

Hence, 6 distinct values are possible for N .

###TOPIC###Quantitative Aptitude||Algebra||Higher Degree Equations###



15. Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/h, 10 km/h, and 8 km/h, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point?

- A. 4.8
- B. 4.6
- C. 5.2
- D. 4.2

Answer: A

Solution:

Ratio of speeds of Anil and Sunil = 15:10 = 3:2

So, when Anil and Sunil meet for the first time at the starting point, Anil should have completed 3 rounds and Sunil should have completed 2 rounds.

Time taken by Anil to complete 2 rounds = $\frac{2 \times 3}{10} = \frac{3}{5}$ hrs

Distance covered by Ravi in $\frac{3}{5}$ h = $8 \times \frac{3}{5} = 4.8$ km

###TOPIC### Quantitative Aptitude||Arithmetic||Speed, Time and Distance###

16. Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is ____.

Answer: 18

Solution:

Let the age of Tom, Dick, and Harry be k , $3k$ and $6k$, respectively.

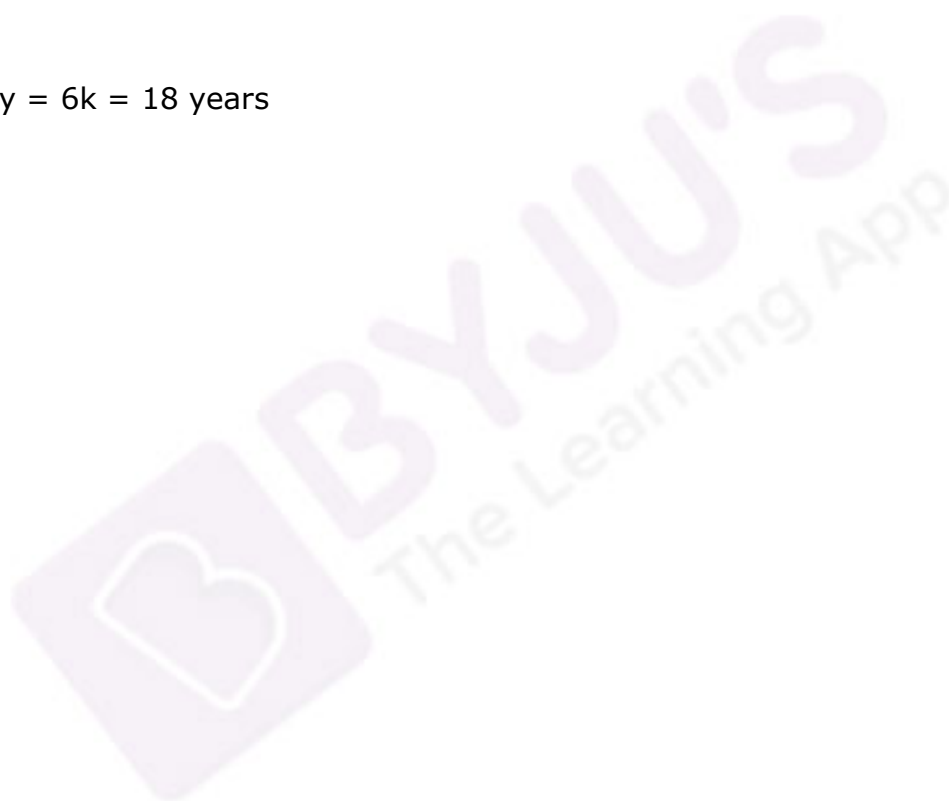
According to question, we can write the following:

$$3k = \frac{k + 3k + 6k}{3} - 1$$

$$\Rightarrow 9k = 10k - 3$$

$$\Rightarrow k = 3$$

Age of Harry = $6k = 18$ years



17. A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km of road had been completed. How many additional people would he need to employ in order to finish the work exactly on time?

Answer: 40

Solution:

Let's assume k additional persons are required.

$$N_1 = 140 \text{ persons} \quad N_2 = (140 + k) \text{ persons}$$

$$D_1 = 60 \text{ days} \quad D_2 = 200 - 60 = 140 \text{ days}$$

$$W_1 = 1.5 \text{ km} \quad W_2 = 6 - 1.5 = 4.5 \text{ km}$$

Since work done is directly proportional to the number of persons and number of days, we have the following:

$$\frac{N_1 D_1}{N_2 D_2} = \frac{W_1}{W_2}$$

$$\frac{140 \times 60}{(140 + k) \times 140} = \frac{1.5}{4.5}$$

$$\Rightarrow 180 = 140 + k$$

$$\Rightarrow 40 = k$$

Hence, 40 additional persons are required.

$$18. \frac{2 \times 4 \times 8 \times 16}{(\log_2 4)^2 (\log_4 8)^3 (\log_8 16)^4} \text{ equals } \underline{\hspace{2cm}}.$$

Answer: 24

Solution:

$$\begin{aligned} & \frac{2 \times 4 \times 8 \times 16}{(\log_2 4)^2 (\log_4 8)^3 (\log_8 16)^4} \\ &= \frac{2 \times 4 \times 8 \times 16}{\left(\frac{\log 4}{\log 2}\right)^2 \left(\frac{\log 8}{\log 4}\right)^3 \left(\frac{\log 16}{\log 8}\right)^4} \\ &= \frac{2 \times 4 \times 8 \times 16}{\left(\frac{2}{1}\right)^2 \left(\frac{3}{2}\right)^3 \left(\frac{4}{3}\right)^4} \\ &= \frac{2 \times 4 \times 8 \times 16}{\frac{128}{3}} = 24 \end{aligned}$$

###TOPIC###Quantitative Aptitude||Algebra||Logarithm###

19. If $f(x + y) = f(x)f(y)$ and $f(5) = 4$, then $f(10) - f(-10)$ is equal to _____.

- A. 3
- B. 0
- C. 15.9375
- D. 14.0625

Answer: C

Solution:

It is given that $f(x + y) = f(x) \times f(y)$(1)

Putting $x = 5$ and $y = 5$ in equation (1), we get the following:

$$f(10) = f(5) \times f(5) = 4 \times 4$$

$$\Rightarrow f(10) = 16 \text{(2)}$$

Putting $x = 0$ and $y = 5$ in equation (1), we get the following:

$$f(5) = f(0) \times f(5)$$

$$\Rightarrow 4 = f(0) \times 4$$

$$\Rightarrow f(0) = 1 \text{(3)}$$

Putting $x = 10$ and $y = -10$ in equation (1), we get the following:

$$f(0) = f(10) \times f(-10)$$

$$\Rightarrow 1 = 16 \times f(-10)$$

$$\Rightarrow f(-10) = 1/16 \dots\dots\dots(4)$$

$$\therefore f(10) - f(-10) = 16 - 1/16 = 15.9375$$

###TOPIC###Quantitative Aptitude||Functions and Graphs||Functions and Graphs###



20. How many pairs (a, b) of positive integers are there such that $a \leq b$ and $ab = 4^{2017}$?

- A. 2017
- B. 2019
- C. 2018
- D. 2020

Answer: C

Solution:

$$ab = 4^{2017}$$

$$\Rightarrow ab = (2^2)^{2017}$$

$$\Rightarrow ab = 2^{2034}$$

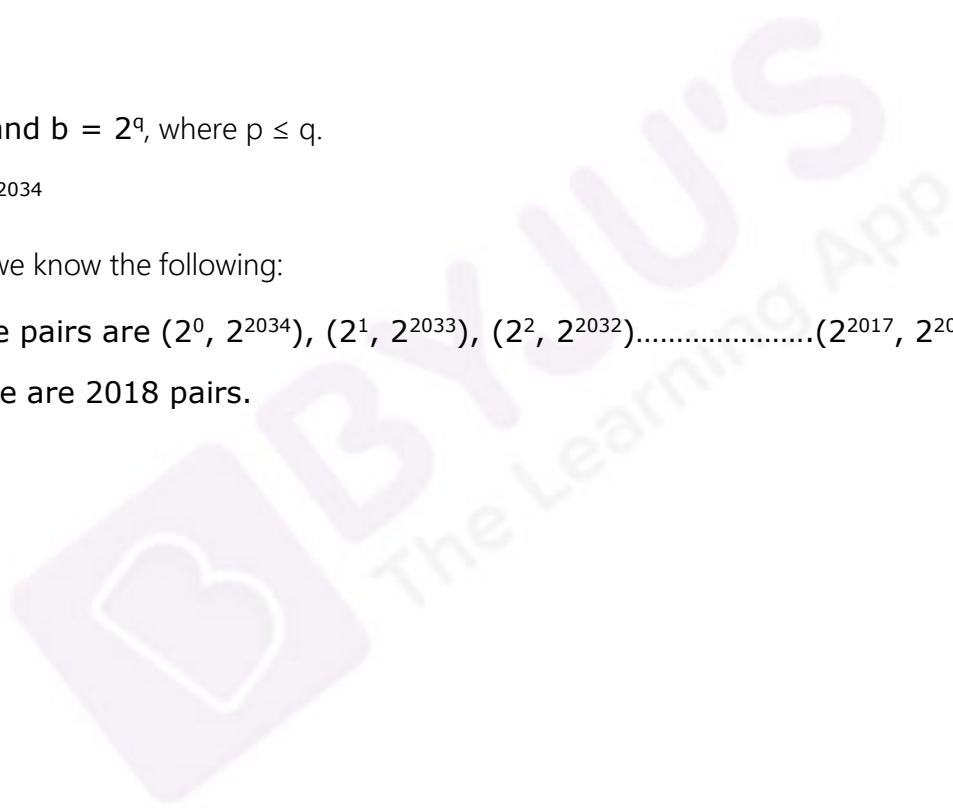
Let $a = 2^p$ and $b = 2^q$, where $p \leq q$.

$$2^p \times 2^q = 2^{2034}$$

Since $p \leq q$, we know the following:

The possible pairs are $(2^0, 2^{2034}), (2^1, 2^{2033}), (2^2, 2^{2032}) \dots (2^{2017}, 2^{2017})$.

Hence, there are 2018 pairs.



###TOPIC###Quantitative Aptitude||Number System||Factors and their properties###

21. A and B are two railway stations 90 km apart. A train leaves A at 9:00 a.m., heading towards B at a speed of 40 km/h. Another train leaves B at 10:30 a.m., heading towards A at a speed of 20 km/h. The trains meet each other at _____.

- A. 11:20 a.m.
- B. 10:45 a.m.
- C. 11:45 a.m.
- D. 11:00 a.m.

Answer: D

Solution:

Let A and B meet t hours after the first train started.

Distance covered by the 1st train in t hours + distance covered by 2nd train in $(t - 1.5)$ hours = 90

$$40(t) + 20(t - 1.5) = 90$$

$$60t - 30 = 90$$

$$t = 2$$

The two trains meet at 11:00 a.m.

###TOPIC###Quantitative Aptitude||Arithmetic||Speed, Time and Distance###

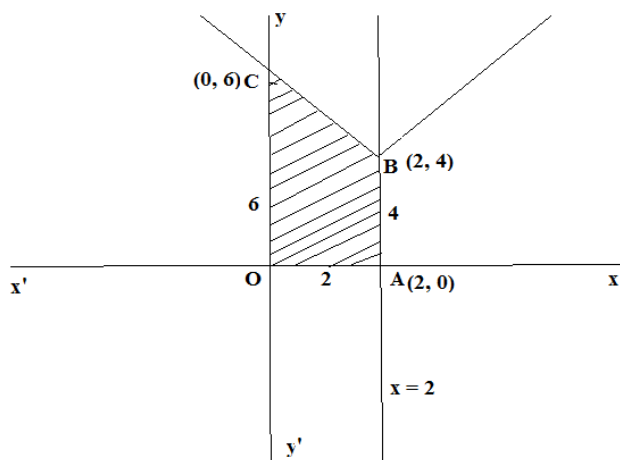
22. The area, in sq. units, enclosed by the lines $x = 2$, $y = |x - 2| + 4$, the X-axis, and the Y-axis is equal to _____.

- A. 6
- B. 8
- C. 12
- D. 10

Answer: D

Solution:

We can draw the following graph from the given equations.



Required Area = Area of Trapezium OABC

$$\begin{aligned} &= \frac{1}{2} \times (4+6) \times 2 = 10 \text{ sq. units} \end{aligned}$$

###TOPIC### Quantitative Aptitude || Co-ordinate Geometry || Co-ordinate Geometry###

23. If $x_1 = -1$ and $x_m = x_{m+1} + (m + 1)$ for every positive integer m , then x_{100} equals _____.

- A. -5150
- B. -5051
- C. -5050
- D. -5151

Answer: C

Solution:

$$x_m = x_{m+1} + (m + 1)$$

$$\Rightarrow x_{m+1} = x_m - m - 1 \dots\dots\dots(1)$$

Let $m = 1$

$$\Rightarrow x_2 = x_1 - 1 - 1 \dots\dots\dots(2)$$

Let $m = 2$

$$x_3 = x_2 - 2 - 1$$

$$\Rightarrow x_3 = x_1 - 1 - 1 - 2 - 1$$

$$\Rightarrow x_3 = x_1 - (1 + 1) - (1 + 2) \dots\dots\dots(3)$$

Let $m = 3$

$$x_4 = x_3 - 3 - 1$$

$$\Rightarrow x_4 = x_1 - (1 + 1) - (1 + 2) - 3 - 1$$

$$\Rightarrow x_4 = x_1 - (1 + 1 + 1) - (1 + 2 + 3) \dots\dots\dots(4)$$

Similarly, we can write the following:

$$x_{100} = x_1 - (1 + 1 + 1 \dots\dots\dots 99 \text{ times}) - (1 + 2 + 3 + \dots\dots\dots + 99)$$

$$\Rightarrow x_{100} = x_1 - 99 - 99 \times 100/2$$

$$\Rightarrow x_{100} = -1 - 99 - 4950$$

$$\Rightarrow x_{100} = -5050$$

###TOPIC###Quantitative Aptitude||Functions and Graphs||Functions and Graphs###

24. Two alcohol solutions, A and B, are mixed in the proportion 1:3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is _____.

- A. 89%
- B. 94%
- C. 90%
- D. 92%

Answer: D

Solution:

The initial mixture has 1 unit of A and 3 units of B.

Then the volume is doubled by adding A.

- ⇒ 4 units of A are added.
- ⇒ The final mixture has 5 units of A and 3 units of B.

Let solution B contain $k\%$ alcohol. Using the formula of weighted average, we get the following:

$$\frac{60 \times 5 + k \times 3}{5 + 3} = 72$$

$$\Rightarrow 300 + 3k = 576$$

$$\Rightarrow k = 92$$

Hence, B contains 92% alcohol.

###TOPIC###Quantitative Aptitude||Arithmetic||Mixtures and Alligations###

25. How many integers in the set $\{100, 101, 102, \dots, 999\}$ have at least one digit repeated?

Answer: 252

Solution:

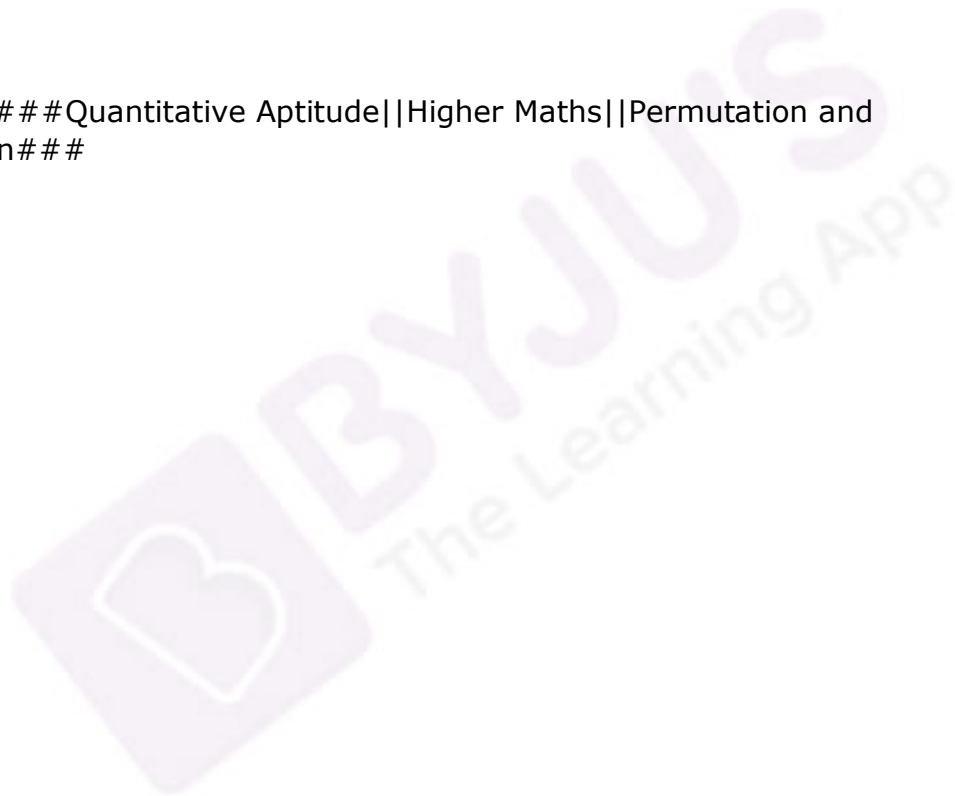
The set $\{100, 101, 102, \dots, 999\}$ is the set of all three-digit numbers.

Total number of 3-digit numbers = $9 \times 10 \times 10 = 900$

Number of 3-digit numbers having distinct digits = $9 \times 9 \times 8 = 648$

Number of 3-digit numbers having at least one digit repeated = $900 - 648 = 252$

###TOPIC###Quantitative Aptitude||Higher Maths||Permutation and Combination###



26. Vimla starts for office every day at 9 a.m. and reaches exactly on time if she drives at her usual speed of 40 km/h. She is 6 minutes late as she drives at 35 km/h. One day, she covers two-thirds of her distance to office in one-third of her usual time to reach office, and then stops for 8 minutes. The speed, in km/h, at which she should drive the remaining distance to reach office exactly on time is _____.

- A. 28
- B. 26
- C. 29
- D. 27

Answer: A

Solution:

Let the distance be 'd'.

According to the question, we can write the following:

$$\frac{d}{35} - \frac{d}{40} = \frac{6}{60}$$

$$\Rightarrow d = 28 \text{ km}$$

$$\text{Usual time} = \frac{28}{40} \text{ hrs} = 42 \text{ mins}$$

She covers $\frac{56}{3}$ km in 14 min and then takes a break of 8 min.

$$\text{Remaining distance} = \frac{28}{3} \text{ km}$$

$$\text{Remaining time} = 42 - 14 - 8 = 20 \text{ min} = \frac{1}{3} \text{ h}$$

$$\text{Required speed} = \frac{\frac{28}{3}}{\frac{1}{3}} = 28 \text{ km/hr}$$

###TOPIC### Quantitative Aptitude || Arithmetic || Speed, Time and Distance###

