Subject Code - 041

Marking Scheme
CLASS: XII
Session: 2021-22
Mathematics (Code-041)
Term - 2

SECTION – A

1.	Find: $\int \frac{logx}{(1+logx)^2}dx$ Solution: $\int \frac{logx}{(1+logx)^2}dx = \int \frac{logx+1-1}{(1+logx)^2}dx = \int \frac{1}{1+logx}dx - \int \frac{1}{(1+logx)^2}dx$ $=\frac{1}{1+logx} \times x - \int \frac{-1}{(1+logx)^2} \times \frac{1}{x} \times xdx - \int \frac{1}{(1+logx)^2}dx = \frac{x}{1+logx} + c$ OR Find: $\int \frac{sin2x}{\sqrt{9-cos^4x}}dx$ Solution: Put $cos^2x = t \Rightarrow -2cosxsinxdx = dt \Rightarrow sin2xdx = -dt$ The given integral $= -\int \frac{dt}{\sqrt{3^2-t^2}} = -\sin^{-1}\frac{t}{3} + c = -\sin^{-1}\frac{cos^2x}{3} + c$	1/2 1+1/2 1 1
2.	Write the sum of the order and the degree of the following differential equation: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$ Solution: Order = 2 Degree = 1 Sum = 3	1 1/2 ½
3.	If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $\lvert\hat{a}+\hat{b}\rvert = 2cos\frac{\theta}{2}$, where θ is the angle between them. Solution: $(\hat{a}+\hat{b}).(\hat{a}+\hat{b}) = \lvert\hat{a}\rvert^2 + \lvert\hat{b}\rvert^2 + 2(\hat{a}.\hat{b})$ $\lvert\hat{a}+\hat{b}\rvert^2 = 1 + 1 + 2cos\theta$ $= 2(1+cos\theta) = 4cos^2\frac{\theta}{2}$ $\therefore \lvert\hat{a}+\hat{b}\rvert = 2cos\frac{\theta}{2},$	1 1/2 ½
4.	Find the direction cosines of the following line: $\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$ Solution: The given line is $\frac{x-3}{1} = \frac{y-\frac{1}{2}}{1} = \frac{z}{4}$ Its direction ratios are <1, 1, 4> Its direction cosines are $\langle\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\rangle$	1 1/2 ½

5.	A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement. Solution: Let X be the random variable defined as the number of red balls. Then X = 0, 1 P(X=0) = $\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$ P(X=1) =$\frac{1}{4}\times\frac{3}{3}+\frac{3}{4}\times\frac{1}{3}=\frac{6}{12}=\frac{1}{2}$ Probability Distribution Table: X: 0, 1 P(X): $\frac{1}{2}$, $\frac{1}{2}$	1/2 1/2 1/2 ½
6.	Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack? Solution: The required probability = P((The first is a red jack card and The second is a jack card) or (The first is a red non-jack card and The second is a jack card)) $=\frac{2}{52}\times\frac{3}{51}+\frac{24}{52}\times\frac{4}{51}=\frac{1}{26}$	1 1
	SECTION – B	
7.	Find: $\int\frac{x+1}{(x^2+1)x}dx$ Solution: Let $\frac{x+1}{(x^2+1)x}=\frac{Ax+B}{x^2+1}+\frac{C}{x}=\frac{(Ax+B)x+C(x^2+1)}{(x^2+1)x}$ $\Rightarrow x+1=(Ax+B)x+C(x^2+1)$ (An identity) Equating the coefficients, we get B = 1, C = 1, A + C = 0 Hence, A = -1, B = 1, C = 1 The given integral = $\int\frac{-x+1}{x^2+1}dx+\int\frac{1}{x}dx$ $=\frac{-1}{2}\int\frac{2x-2}{x^2+1}dx+\int\frac{1}{x}dx$ $=\frac{-1}{2}\int\frac{2x}{x^2+1}dx+\int\frac{1}{x^2+1}dx+\int\frac{1}{x}dx$ $=\frac{-1}{2}\log(x^2+1)+\tan^{-1}x+log\|x\|+c$	1/2 ½ 1/2 1+1/2
8.	Find the general solution of the following differential equation: $x\frac{dy}{dx}=y-xsin(\frac{y}{x})$ Solution: We have the differential equation: $\frac{dy}{dx}=\frac{y}{x}-sin(\frac{y}{x})$ The equation is a homogeneous differential equation. Putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ The differential equation becomes $v+x\frac{dv}{dx}=v-sinv$ $\Rightarrow\frac{dv}{sinv}=-\frac{dx}{x}\Rightarrow cosecvdv=-\frac{dx}{x}$ Integrating both sides, we get	1 ½

	$log\|cosecv - cotv\| = -log\|x\| + logK, K > 0$ (Here, $logK$ is an arbitrary constant.) $\Rightarrow log\|(cosecv - cotv)x\| = logK$ $\Rightarrow \|(cosecv - cotv)x\| = K$ $\Rightarrow (cosecv - cotv)x = \pm K$ $\Rightarrow \left(cosec\frac{y}{x} - cot\frac{y}{x}\right)x = C$, which is the required general solution. OR Find the particular solution of the following differential equation, given that y = 0 when x = $\frac{\pi}{4}$: $\frac{dy}{dx} + ycotx = \frac{2}{1+sinx}$ Solution: The differential equation is a linear differential equation I F = $e^{\int cotxdx} = e^{logsinx} = sinx$ The general solution is given by $ysinx = \int 2\frac{sinx}{1+sinx}dx$ $\Rightarrow ysinx = 2\int \frac{sinx+1-1}{1+sinx}dx = 2\int [1 - \frac{1}{1+sinx}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{1+\cos\left(\frac{\pi}{2} - x\right)}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{2cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}]dx$ $\Rightarrow ysinx = 2\int [1 - \frac{1}{2}sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)]dx$ $\Rightarrow ysinx = 2[x + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)] + c$ Given that y = 0, when x = $\frac{\pi}{4}$, Hence, 0 = $2[\frac{\pi}{4} + tan\frac{\pi}{8}] + c$ $\Rightarrow c = -\frac{\pi}{2} - 2tan\frac{\pi}{8}$ Hence, the particular solution is $y = cosecx[2\left\{x + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\} - \left(\frac{\pi}{2} + 2tan\frac{\pi}{8}\right)]$	1 ½ 1 ½ 1 ½
9.	If $\vec{a} \neq \vec{0}, \vec{a}.\vec{b} = \vec{a}.\vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$. Solution: We have $\vec{a}.\left(\vec{b} - \vec{c}\right) = 0$ $\Rightarrow \left(\vec{b} - \vec{c}\right) = \vec{0}$ or $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$ $\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$ Also, $\vec{a} \times \left(\vec{b} - \vec{c}\right) = \vec{0}$ $\Rightarrow \left(\vec{b} - \vec{c}\right) = \vec{0}$ or $\vec{a} \parallel \left(\vec{b} - \vec{c}\right)$ $\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \parallel \left(\vec{b} - \vec{c}\right)$ $\vec{a}$ *can not be both perpendicular to* $\left(\vec{b} - \vec{c}\right)$ *and parallel to* $\left(\vec{b} - \vec{c}\right)$ Hence, $\vec{b} = \vec{c}$.	 1 1 1
10.	Find the shortest distance between the following lines: $\vec{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + s\left(2\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ $\vec{r} = \left(\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) + t\left(4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right)$	

	Solution: Here, the lines are parallel. The shortest distance = $\frac{\lvert(\vec{a_2}-\vec{a_1})\times\vec{b}\rvert}{\lvert\vec{b}\rvert}$	
	$= \frac{\lvert(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k})\rvert}{\sqrt{4+1+1}}$	1+1/2
	$(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 3\\ 2 & 1 & 1\end{vmatrix} = -3\hat{i}+6\hat{j}$	1
	Hence, the required shortest distance = $\frac{3\sqrt{5}}{\sqrt{6}}$ units	½
	OR	
	Find the vector and the cartesian equations of the plane containing the point $\hat{i}+2\hat{j}-\hat{k}$ and parallel to the lines $\vec{r} = (\hat{i}+2\hat{j}+2\hat{k}) + s(2\hat{i}-3\hat{j}+2\hat{k}) = 0$ and $\vec{r} = (3\hat{i}+\hat{j}-2\hat{k}) + t(\hat{i}-3\hat{j}+\hat{k}) = 0$	
	Solution: Since, the plane is parallel to the given lines, the cross product of the vectors $2\hat{i}-3\hat{j}+2\hat{k}$ and $\hat{i}-3\hat{j}+\hat{k}$ will be a normal to the plane	
	$(2\hat{i}-3\hat{j}+2\hat{k})\times(\hat{i}-3\hat{j}+\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 2\\ 1 & -3 & 1\end{vmatrix} = 3\hat{i}-3\hat{k}$	1
	The vector equation of the plane is $\vec{r}.(3\hat{i}-3\hat{k}) = (\hat{i}+2\hat{j}-\hat{k}).(3\hat{i}-3\widehat{k)}$	1
	or, $\vec{r}.(\hat{i}-\hat{k}) = 2$	
	and the cartesian equation of the plane is x – z – 2 = 0	1
	SECTION – C	
11.	Evaluate: $\int_{-1}^{2}\lvert x^3-3x^2+2x\rvert dx$	
	Solution: The given definite integral = $\int_{-1}^{2}\lvert x(x-1)(x-2)\rvert dx$	
	$= \int_{-1}^{0}\lvert x(x-1)(x-2)\rvert dx + \int_{0}^{1}\lvert x(x-1)(x-2)\rvert dx + \int_{1}^{2}\lvert x(x-1)(x-2)\rvert dx$	1+1/2
	$= -\int_{-1}^{0}(x^3-3x^2+2x)dx + \int_{0}^{1}(x^3-3x^2+2x)dx - \int_{1}^{2}(x^3-3x^2+2x)dx$	1/2
	$= -[\frac{x^4}{4}-x^3+x^2]_{-1}^{0} + [\frac{x^4}{4}-x^3+x^2]_{0}^{1} - [\frac{x^4}{4}-x^3+x^2]_{1}^{2}$	
	$= \frac{9}{4}+\frac{1}{4}+\frac{1}{4} = \frac{11}{4}$	2

12. Using integration, find the area of the region in the first quadrant enclosed by the line x + y = 2, the parabola $y^2 = x$ and the x-axis.

Solution: Solving x + y = 2 and $y^2 = x$ simultaneously, we get the points of intersection as (1, 1) and (4, -2). 1

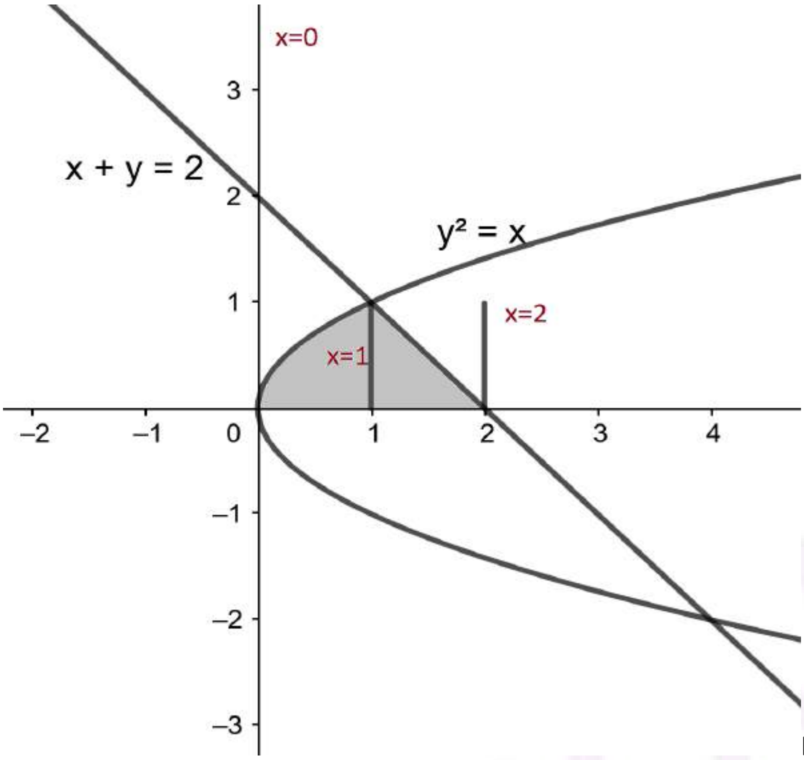

Fig 1 1

The required area = the shaded area = $\int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)dx$ 1

$= \frac{2}{3}[x^{\frac{3}{2}}]_0^1 + [2x - \frac{x^2}{2}]_1^2$

$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ square units 1

OR

Using integration, find the area of the region: $\{(x,y): 0 \le y \le \sqrt{3}x, x^2 + y^2 \le 4\}$

Solution: Solving $y = \sqrt{3}x \; and \; x^2 + y^2 = 4$, we get the points of intersection as (1, $\sqrt{3}$) and (-1, $-\sqrt{3}$) 1

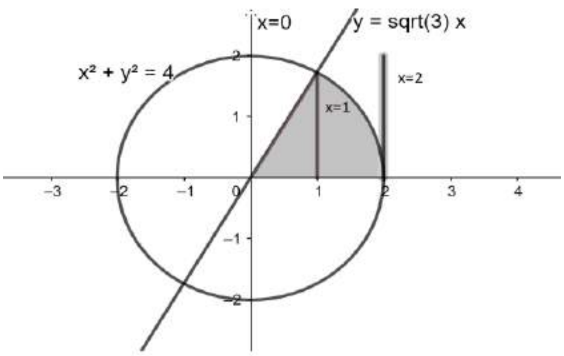

Fig 2 1

	The required area = the shaded area = $\int_0^1 \sqrt{3}x\,dx + \int_1^2 \sqrt{4-x^2}dx$ $= \frac{\sqrt{3}}{2}[x^2]_0^1 + \frac{1}{2}[x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}]_1^2$ $= \frac{\sqrt{3}}{2} + \frac{1}{2}[2\pi - \sqrt{3} - 2\frac{\pi}{3}]$ $= \frac{2\pi}{3}$ square units	1 1
13.	Find the foot of the perpendicular from the point (1, 2, 0) upon the plane x – 3y + 2z = 9. Hence, find the distance of the point (1, 2, 0) from the given plane. Solution: The equation of the line perpendicular to the plane and passing through the point (1, 2, 0) is $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z}{2}$ The coordinates of the foot of the perpendicular are $(\mu+1, -3\mu+2, 2\mu)$ for some μ These coordinates will satisfy the equation of the plane. Hence, we have $\mu + 1 - 3(-3\mu + 2) + 2(2\mu) = 9$ $\Rightarrow \mu = 1$ The foot of the perpendicular is (2, -1, 2). Hence, the required distance = $\sqrt{(1-2)^2 + (2+1)^2 + (0-2)^2} = \sqrt{14}\,units$	1 ½ 1 ½ 1

14. **CASE-BASED/DATA-BASED**

Fig 3

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the given information, answer the following questions.

	(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?	
	(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?	
	Solution: Let E_1 = The policy holder is accident prone. E_2 = The policy holder is not accident prone. E = The new policy holder has an accident within a year of purchasing a policy. (i) $P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$ $= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10} = \frac{7}{25}$ (ii) By Bayes' Theorem, $P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}$ $= \frac{\frac{20}{100} \times \frac{6}{10}}{\frac{280}{1000}} = \frac{3}{7}$	1 1 1 1
