## GATE 2019

Set-1

Mechanical Engineering

Questions \& Solutions

## SECTION: GENERAL APTITUDE

1. The sum and product of two integers are 26 and 165 respectively. The difference between these two integers is $\qquad$ .
A. 2
B. 4
C. 6
D. 3

Ans. B
Sol. Let the two integers be $x$ and $y$
According to the given condition,
$x+y=26$ and $x y=165$
using the identity,

$$
\begin{aligned}
(x+y)^{2} & =(x-y)^{2}-4 x y \\
\Rightarrow 26^{2}=(x-y)^{2}-4 & \times 65 \\
\Rightarrow 676 & =(x-y)^{2}-660
\end{aligned}
$$

Or $x-y=4$
So, $x=15$ and $y=11$
2. John Thomas, an $\qquad$ writer, passed away in 2018.
A. prominent
B. imminent
C. eminent
D. dominant

Ans. C
Sol. eminent means important, famous Imminent means something about to happen Eminent means a person who is famous and respected within a particular sphere Dominant means having power and influence over others.

Here, eminent is the most suitable word
3. The minister avoided any mention of the issue of women's reservation in the private sector. He was accused of $\qquad$ the issue.
A. skirting
B. tying
C. belting
D. collaring

Ans. A
Sol. Skirting means
to avoid discussing a subject or problem,
usually because there are difficulties that you do not want to deal with

Tying means to tie something or someone
Belting means to beat someone harshly
Collaring means to seize someone
4. A worker noticed that the hour hand on the factory clock had moved by 225 degrees during her say at the factory. For how long did she stay in the factory?
A. 7.5 hours
B. 4 hours and 15 mins
C. 3.75 hours
D. 8.5 hours

Ans. A
Sol. Hour hand moves 360 degrees in 12 hrs So, when the hour hand has moved 225 degrees that means hrs passed $=\frac{12 \times 225}{360}=7.5$
5. $\qquad$ I permitted him to leave, I wouldn't have had any problem with him being absent,
$\qquad$ I?
A. Have, wouldn't
B. Had, wouldn't
C. Have, would
D. Had, would

Ans. D
Sol. The correct sentence is
Had I permitted him to leave, I wouldn't have had any problem with him being absent, would I?
6. A firm hires employees at five different skill levels P, Q, R, S, T. The shares of employment at these skill levels of total employment in 2010 is given in the pie chart as shown. There were a total of 600 employees in 2010 and the total employment increased by 15\% from 2010 to 2016. The total employment at skill levels P, $Q$ and $R$ remained unchanged during this
period. If the employment at skill level S increased by $40 \%$ from 2010 to 2016, how many employees were there at skill level T in 2016?

A. 30
B. 72
C. 60
D. 35

Ans. C

## Sol. In 2010,

Total employees $=600$
$P=0.2 \times 600=120$
$\mathrm{Q}=0.25 \times 600=150$
$R=0.25 \times 600=150$
$S=0.25 \times 600=150$
$\mathrm{T}=0.05 \times 600=30$
Employees increased in 2016
= $15 \%$ = 90
Total employees in $2016=600+90$
$=690$
$P, Q$ and $R$ remained unchanged, so total of $P$,
$Q$ and $R$ in $2016=120$
$+150+150=420$
S increased by $40 \%=0.4 \times 150=60$
Total S employees in $2016=150+60=210$
Therefore, T in 2016
$=690-(420+210)=60$
7. $M$ and $N$ had four children $P, Q, R$ and $S$. Of them, only P and R were married. They had
children $X$ and $Y$ respectively. If $Y$ is a legitimate child of W, which one of the following statements is necessarily FALSE?
A. $W$ is the wife of $R$
B. $W$ is the wife of $P$
C. $M$ is the grandmother of $Y$
D. $R$ is the father of $Y$

Ans. B
Sol. $M$ and $N$ are two persons
$P, Q, R$ and $S$ are children of $M$ and $N$
$X$ is the child of $P$ and $Y$ is
the child of $R$
And it is also mentioned that $Y$ is the legitimate child of $W$ that means $R$ and $W$ are spouse Now checking the statements one by one
Option A says $W$ is the wife of $R$ which is possible

Option C says M is the grandmother of $Y$ which is also possible
Option $D$ says $R$ is the father of $Y$ which is also possible
Option B says $W$ is the wife of $P$ which is the only statement which is not possible and necessarily false
8. Congo was named by Europeans. Congo's dictator Mobuto later changed the name of the country and the river to Zaire with the objective of Africanising names of persons and spaces. However, the name Zaire was a Portuguese alteration of Nzadi o Nzere, a local African term meaning 'River that swallows Rivers'. Zaire was the Portuguese name for the Congo river in the 16th and 17th centuries. Which one of the following statements can be inferred from the paragraph above?
A. Mobuto was not entirely successful in Africanising the name of his country
B. The term Nzadi o Nzere was of Portuguese origin
C. As a dictator Mobuto ordered the Portuguese to alter the name of the river to Zaire
D. Mobuto's desire to Africanise names was prevented by the Portuguese
Ans. A
Sol. Option A can be clearly inferred from the above paragraph
9. A person divided an amount of Rs. 100,000 into two parts and invested in two different schemes. In one he got $10 \%$ profit and in the other he got $12 \%$. If the profit percentages are interchanged with these investments he would have got Rs. 120 less. Find the ratio between his investments in the two schemes.
A. $47: 53$
B. $9: 16$
C. $37: 63$
D. $11: 14$

Ans. A
Sol. Let the two portions be $x$ and 100000 - x
According to first condition, profit
$=0.1 x+0.12(100000-x)$
According to second condition, profit
$=0.12 \mathrm{x}+0.1(100000-\mathrm{x})$
Then, $0.1 x+0.12(100000-x)$
$-[0.12 x+0.1(100000-x)]=120$
On solving this, $x=47000$, and $100000-x$
$=53000$
Hence, ratio $=47: 53$
10. Under a certain legal system, prisoners are allowed to make one statement. If their statement turns out to be true then they are hanged. If the statement turns out to be false then they are shot. One prisoner made a statement and the judge had no option but to set him free. Which one of the following could be that statement?
A. I will be shot
B. I did not commit the crime
C. You committed the crime
D. I committed the crime

Ans. A
Sol. Option (A) is the only correct option. In all other cases, the victim would not be set free.

## MECHANICAL ENGINEERING

1. A solid cube of side 1 m is kept at a room temperature of $32^{\circ} \mathrm{C}$. The coefficient of linear thermal expansion of the cube material is $1 \times$ $10^{-5} /{ }^{\circ} \mathrm{C}$ and the bulk modulus is 200 GPa . If the cube is constrained all around and heated uniformly to $42^{\circ} \mathrm{C}$, then the magnitude of volumetric (mean) stress (in MPa) induced due to heating is $\qquad$ -.

Ans. 60
Sol. side of cube $=1 \mathrm{~m}$
Temperature rise $(\Delta T)=10^{\circ} \mathrm{C}$
Coefficient of linear thermal expansion $(\alpha)=1$ $\times 10^{-5} /{ }^{\circ} \mathrm{C}$

Bulk modulus $(\mathrm{K})=200 \mathrm{GPa}$

$$
\epsilon_{V}=\epsilon_{x}+\epsilon_{y}+\epsilon_{z}=\frac{1-2 \mu}{E}\left(\sigma_{x}+\sigma_{y}+\sigma_{z}\right)
$$

Since, the cube is symmetric, therefore, $\sigma_{x}=$ $\sigma_{y}=\sigma_{z}=\sigma$

$$
\epsilon_{V}=\frac{1-2 \mu}{E}(3 \sigma)
$$

This volumetric strain will be equal to $3 \alpha \Delta T$
So, $\frac{1-2 \mu}{E}(3 \sigma)=3 \alpha \Delta T$

$$
E=3 K(1-2 \mu)
$$

So, $\frac{1-2 \mu}{E}(3 \sigma)=\frac{3 \sigma}{3 K}=3 \alpha \Delta T$
Or, $\sigma=3 K \alpha \Delta T=3 \times 200 \times 10^{3} \times 10^{-5} \times 10=60$
2. Evaluation of $\int_{2}^{4} x^{3} d x$ using a 2-equalsegment trapezoidal rule gives a value of

Ans. 63
Sol. $h=\frac{4-2}{2}=1$

| $y$ | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| $x^{3}$ | 8 | 27 | 64 | | $\int y d x=\frac{h}{2}\left[\left(y_{2}+y_{4}\right)+2\left(y_{3}\right)\right]$ | $=\frac{1}{2}[(8+64)+2(27)]$ |
| ---: | :--- |
|  | $=63$ |

3. A spur gear with $20^{\circ}$ full depth teeth is transmitting 20 kW at 200 rad/s. The pitch circle diameter of the gear is 100 mm . The magnitude of the force applied on the gear in the radial direction is
A. 1.39 kN
B. 0.73 kN
C. 0.36 kN
D. 2.78 kN

Ans. B
Sol. $\mathrm{P}=20 \mathrm{~kW}, \omega=200 \mathrm{rad} / \mathrm{s}$

$$
D_{p}=100 \mathrm{~mm} ; r_{p}=50 \mathrm{~mm}
$$

Using $P=T \times \omega$

$$
\begin{gathered}
20 \times 10^{3}=T \times 200 \\
\Rightarrow T=100 \mathrm{~N}-\mathrm{m} \\
T=F_{t} \times r_{p} \\
\Rightarrow 100=F_{t} \times \frac{50}{1000} \\
F_{t}=2 \mathrm{kN} \\
\frac{F_{r}}{F_{t}}=\tan \theta \\
\Rightarrow F_{r}=F_{t} \tan \theta=2 \times \tan 20=0.73 \mathrm{kN}
\end{gathered}
$$

4. In a casting process, a vertical channel through which molten metal flows downward from pouring basin to runner for reaching the mold cavity is called
A. riser
B. pin hole
C. sprue
D. blister

Ans. C
Sol. The vertical channel through which molten metal flows downward from pouring basin to runner for reaching the mold cavity is called Sprue
5. Consider the stress-strain curve for an ideal elastic-plastic strain hardening metal as shown in the figure. The metal was loaded in uniaxial tension from O. Upon loading, the stress-strain curve passes through initial yield point at $P$, and
then strain hardens to point $Q$, where the loading was stopped. From point Q , the specimen was unloaded to point $R$, where the stress is zero. If the same specimen is reloaded in tension from point $R$, the value of stress at which the material yields again is $\qquad$ MPa.


Ans. 210
Sol. When the specimen is reloaded beyond the yield point, yield strength increases because of strain hardening. Therefore, the material will start yielding from R
6. The length, width and thickness of a steel sample are $400 \mathrm{~mm}, 40 \mathrm{~mm}$ and 20 mm , respectively. Its thickness needs to be uniformly reduced by 2 mm in a single pass by using horizontal slab milling. The milling cutter (diameter: 100 mm , width: 50 mm ) has 20 teeth and rotates at 1200 rpm . The feed per tooth is 0.05 mm . The feed direction is along the length of the sample. If the over-travel distance is the same as the approach distance, the approach distance and time taken to complete the required machining task are
A. $21 \mathrm{~mm}, 28.9 \mathrm{~s}$
B. $14 \mathrm{~mm}, 21.4 \mathrm{~s}$
C. $21 \mathrm{~mm}, 39.4 \mathrm{~s}$
D. $14 \mathrm{~mm}, 18.4 \mathrm{~s}$

Ans. B
Sol. Depth of cut (d) $=2 \mathrm{~mm}$ Milling cutter diameter (D) $=100 \mathrm{~mm}$
over-travel distance is the same as the approach distance $=\sqrt{d(D-d)}=\sqrt{2(100-2)}=$ 14 mm
Total distance travelled by tool $=$ length of sample + approach distance + overtravel $=400$ $+14+14=428 \mathrm{~mm}$

Feed per tooth $=0.05 \mathrm{~mm}$
No. of teeth $=20$
So, feed (f) $=20 \times 0.05=1 \mathrm{~mm} / \mathrm{rev}$
Rotation speed ( N ) $=1200 \mathrm{rpm}$
Therefore, machining time
$=\frac{\text { total length }}{\text { feed } \times N}=\frac{428}{1 \times\left(\frac{1200}{60}\right)}=21.4 \mathrm{sec}$
7. A block of mass 10 kg rests on a horizontal floor. The acceleration due to gravity is 9.81 $\mathrm{m} / \mathrm{s}^{2}$. The coefficient of static friction between the floor and the block is 0.2 . A horizontal force of 10 N is applied on the block as shown in the figure. The magnitude of force of friction (in N ) on the block is $\qquad$


Ans. 10
Sol. The Normal reaction is equal to the weight of the body
So, Normal reaction will be equal to mass $\times$

$$
g=10 \times 9.81=98.1 \mathrm{~N}
$$

The static friction force
$=\mu \times N=0.2 \times 98.1=19.62$
Now, since the mass is at rest, therefore the friction force will be equal to applied horizontal force
$=10 \mathrm{~N}$
8. The table presents the demand of a product. By simple three-months moving average method,
the demand-forecast of the product for the month of September is

| Month | Demand |
| :--- | :--- |
| January | 450 |
| February | 440 |
| March | 460 |
| April | 510 |
| May | 520 |
| June | 495 |
| July | 475 |
| August | 560 |

A. 510
B. 536.67
C. 490
D. 530

Ans. A
Sol. By simple three-months moving average method, the forecast for the month of September will be $\frac{495+475+560}{3}=510$
9. For a hydrodynamically and thermally fully developed laminar flow through a circular pipe of constant cross-section, the Nusselt number at constant wall heat flux $\left(\mathrm{Nu}_{\mathrm{q}}\right)$ and that at constant wall temperature $\left(\mathrm{Nu}_{\mathrm{T}}\right)$ are related as
A. $\mathrm{Nu}_{q}>\mathrm{Nu}_{\mathrm{T}}$
B. $\mathrm{Nu}_{q}<\mathrm{Nu}_{\mathrm{T}}$
C. $\mathrm{Nu}_{q}=\left(\mathrm{Nu}_{\mathrm{T}}\right)^{2}$
D. $\mathrm{Nu}_{q}=\mathrm{Nu}{ }_{\mathrm{T}}$

Ans. A
Sol. For a hydrodynamically and thermally fully developed laminar flow through a circular pipe of constant cross-section,
Nusselt number at constant wall heat flux $\left(\mathrm{Nu}_{\mathrm{q}}\right)$ $=4.36$

Nusselt number at constant wall temperature $(\mathrm{Nut})=3.66$
Hence, $\mathrm{Nu}_{\mathrm{q}}>\mathrm{Nu}_{\mathrm{T}}$
10. Air of mass 1 kg , initially at 300 K and 10 bar , is allowed to expand isothermally till it reaches
a pressure of 1 bar. Assuming air as an ideal gas with gas constant of $0.287 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$, the change in entropy of air (in $\mathrm{kJ} / \mathrm{kg} . \mathrm{K}$, round off to two decimal places) is $\qquad$
Ans. 0.66
Sol. $\Delta s=C_{p} \ln \frac{T_{2}}{T_{1}}-R \ln \frac{P_{2}}{P_{1}}$
As the expansion is isothermal, so
$T_{1}=T_{2}$

$$
\begin{aligned}
\Delta s & =-R \ln \frac{P_{2}}{P_{1}}=-0.287 \times \ln \frac{1}{10} \\
& =-0.287 \times-2.3026=0.661
\end{aligned}
$$

11. The natural frequencies corresponding to the spring-mass systems I and II are $\omega_{\text {I }}$ and $\omega_{\text {II, }}$ respectively. The ratio $\frac{\omega_{\mathrm{I}}}{\omega_{\mathrm{II}}}$ is

A. $\frac{1}{4}$
B. 4
C. $\frac{1}{2}$
D. 2

Ans.
Sol. natural frequency is given by $\sqrt{\frac{m}{k_{e q}}}$
For system I: It is a series connection For this, the equivalent $k_{e q}$ is given by

$$
k_{e q}=\frac{k_{1} k_{2}}{k_{1}+k_{2}}=\frac{k \cdot k}{k+k}=\frac{k}{2}
$$

For system II: It is a parallel connection
For this, the equivalent $k_{e q}$ is given by

$$
k_{e q}=k_{1}+k_{2}=k+k=2 k
$$

$$
\begin{gathered}
\omega_{I}=\sqrt{\frac{m}{k / 2}}=\sqrt{\frac{2 m}{k}} \\
\omega_{I I}=\sqrt{\frac{m}{2 k}}
\end{gathered}
$$

Therefore, $\frac{\omega_{I}}{\omega_{I I}}=\sqrt{\frac{2 m}{k} \cdot \frac{2 k}{m}}=\sqrt{4}=2$
12. As per common design practice, the three types of hydraulic turbines, in descending order of flow rate, are
A. Pelton, Francis, Kaplan
B. Francis, Kaplan, Pelton
C. Kaplan, Francis, Pelton
D. Pelton, Kaplan, Francis

Ans. C
Sol. Pelton turbine: High head and low discharge Francis turbine: Medium head and medium discharge

Kaplan turbine: Low head and high discharge
13. During a high cycle fatigue test, a metallic specimen is subjected to cyclic loading with a mean stress of +140 MPa , and a minimum stress of -70 MPa. The R-ratio (minimum stress to maximum stress) for this cyclic loading is
$\qquad$ (round off to one decimal place)

Ans. -0.2
Sol. Mean stress $=+140 \mathrm{MPa}$
Minimum stress $=-70 \mathrm{MPa}$

$$
\text { mean stress }=\frac{\text { max stress }+ \text { min stress }}{2}
$$

Therefore, max stress $=+350 \mathrm{MPa}$
So,
$\mathrm{R}-$ ratio (minimum stress to maximum stress)

$$
=\frac{\min \text { stress }}{\max \text { stress }}=\frac{-70}{+350}=-\frac{1}{5}=-0.2
$$

14. A cylindrical rod of diameter 10 mm and length 1.0 m is fixed at one end. The other end is twisted by an angle of $10^{\circ}$ by applying a torque. If the maximum shear strain in the rod is $p \times$
$10^{-3}$, then p is equal to $\qquad$ (round off to two decimal places).
Ans. 0.87
Sol. From the torsion equation,

$$
\frac{\tau_{\max }}{R}=\frac{G \theta}{L}
$$

Maximum shear strain

$$
=\frac{\tau_{\max }}{G}=\frac{R \theta}{L}=\frac{5 \times 10 \times \frac{\pi}{180}}{1000}=0.87 \times 10^{-3}
$$

On comparing with $p \times 10^{-3}$, we have $p=0.87$
15. A flat-faced follower is driven using a circular eccentric cam rotating at a constant angular velocity $\omega$. At time $t=0$, the vertical position of the follower is $y(0)=0$, and the system is in the configuration shown below.


The vertical position of the follower face, $y(t)$ is given by
A. $e(1+\cos 2 \omega t)$
B. $\mathrm{e}(1-\cos \omega \mathrm{t})$
C. e $\sin \omega t$
D. e $\sin 2 \omega t$

Ans. B


Sol. Vertical displacement $=A B-A C$

$$
\begin{aligned}
& A B=D B^{\prime} \\
& \text { So, Vertical displacement }=D B^{\prime}-A C \\
& \text { Now, } D B^{\prime}=O B^{\prime}-O D=R-e \cos \theta \\
& A C=O C-O A=R-e
\end{aligned}
$$

So, Vertical displacement
$=(R-e \cos \theta)-(R-e)=e-e \cos \theta$
$=\mathrm{e}(1-\cos \theta)=\mathrm{e}(1-\cos \omega t)$
16. The lengths of a large stock of titanium rods follow a normal distribution with a mean ( $\mu$ ) of 440 mm and a standard deviation ( $\sigma$ ) of 1 mm . What is the percentage of rods whose lengths lie between 438 mm and 441 mm ?
A. $99.75 \%$
B. $81.85 \%$
C. $86.64 \%$
D. $68.4 \%$

Ans. B
Sol. $\mu=440 ; \sigma=1$;


Standard normal Variable, $Z=\frac{x-\mu}{\sigma}$
For, $X=438$
$Z=\frac{X-\mu}{\sigma}=\frac{438-440}{1}=-2$
$Z=\frac{X-\mu}{\sigma}=\frac{441-440}{1}=1$
Now, form the normal distribution table, $P(-2)=0.4772+0.3414=81.86 \%$
17. Consider the matrix

$$
P=\left[\begin{array}{lll}
1 & 1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{array}\right]
$$

The number of distinct eigenvalues of $P$ is
A. 1
B. 0
C. 3
D. 2

Ans.
Sol. The characteristic equation of matrix $P$ which satisfies $|P-\lambda I|=0$

$$
\left.\begin{aligned}
& \text { So, } \left\lvert\,\left[\begin{array}{lll}
1 & 1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{array}\right]-\lambda\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]\right.
\end{aligned} \right\rvert\,=0, \begin{array}{r}
\text { Or }\left|\begin{array}{ccc}
1-\lambda & 1 & 0 \\
0 & 1-\lambda & 1 \\
0 & 0 & 1-\lambda
\end{array}\right|=0 \\
\text { Or, }(1-\lambda)[(1-\lambda)(1-\lambda)-0]=0 \\
\Rightarrow \lambda=1,1,1
\end{array}
$$

Therefore, no. of distinct eigen values = 1
18. During a non-flow thermodynamic process (1-2) executed by a perfect gas, the heat interaction is equal to the work interaction ( $\mathrm{Q}_{1-2}=\mathrm{W}_{1-2}$ ) when the process is
A. Adiabatic
B. Isentropic
C. Isothermal
D. Polytropic

Ans.
Sol. According to ${ }^{\text {st }}$ law of thermodynamics

$$
\delta Q=d U+\delta W
$$

If the heat interaction is equal to the work interaction ( $\mathrm{Q}_{1-2}=\mathrm{W}_{1-2}$ )
Then, $d U=0$ or Internal energy $(U)$ is constant which is possible only in the case of isothermal process
19. Which one of the following welding methods provides the highest heat flux ( $\mathrm{W} / \mathrm{mm}^{2}$ ) ?
A. Laser beam welding
B. Oxy-acetylene gas welding
C. Plasma arc welding
D. Tungsten inert gas welding

Ans. A
Sol. Laser Beam welding provides the highest heat flux

| Welding | Heat density <br> $\left(\mathrm{W} / \mathrm{cm}^{2}\right)$ | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ |
| :---: | :---: | :---: |
| Gas welding | $10^{2}-10^{3}$ | $2500-3500$ |
| Shielded <br> metal arc <br> welding | $10^{4}$ | $>6000$ |
| Gas metal <br> arc welding | $10^{5}$ | $8000-10000$ |
| Plasma arc <br> welding | $10^{6}$ | $15000-30000$ |
| Electron <br> beam <br> welding | $>10^{7}-10^{8}$ | $20000-30000$ |
| Laser beam <br> welding | $>30000$ |  |

20. For the equation $\frac{d y}{d x}+7 x^{2} y=0$, if $y(0)=3 / 7$, then the value of $y(1)$ is
A. $\frac{3}{7} e^{-3 / 7}$
B. $\frac{7}{3} e^{-7 / 3}$
C. $\frac{3}{7} e^{-7 / 3}$
D. $\frac{7}{3} e^{-3 / 7}$

Ans. C
Sol. The given equation is $\frac{d y}{d x}+7 x^{2} y=0$,
Or, $\frac{d y}{d x}=-7 x^{2} y$
Or $\frac{d y}{y}=-7 x^{2} d x$
Integrating both sides,

$$
\ln y=-\frac{7}{3} x^{3}+C
$$

Or $y=e^{\left(-\frac{7}{3} x^{3}+c\right)}$
Using $\mathrm{y}(0)=3 / 7$

$$
3 / 7=e^{(0+c)}=e^{c}
$$

Or, $C=\ln \frac{3}{7}$
Therefore, $\ln y=-\frac{7}{3} x^{3}+\ln \frac{3}{7}$
Or, $\ln y-\ln \frac{3}{7}=-\frac{7}{3} x^{3}$

$$
\ln \frac{7 y}{3}=-\frac{7}{3} x^{3}
$$

Or $\frac{7 y}{3}=e^{-\frac{7}{3} x^{3}}$
Or, $y=\frac{3}{7} e^{-\frac{7}{3} x^{3}}$
Therefore, $y(1)=\frac{3}{7} e^{-\frac{7}{3}}$
21. A slender rod of length $L$, diameter $d(L \gg d)$ and thermal conductivity $k_{1}$ is joined with another rod of identical dimensions, but of thermal conductivity $\mathrm{k}_{2}$, to form a composite cylindrical rod of length 2 L . The heat transfer in radial direction and contact resistance are negligible. The effective thermal conductivity of the composite rod is
A. $k_{1}+k_{2}$
B. $\frac{2 \mathrm{k}_{1} \mathrm{k}_{2}}{\mathrm{k}_{1}+\mathrm{k}_{2}}$
C. $\sqrt{k_{1} k_{2}}$
D. $\frac{k_{1} k_{2}}{k_{1}+k_{2}}$

Ans. B
Sol. $R_{t h}=\frac{\Delta T}{Q}=\frac{L}{k A}$
This will be the series connection

$$
\begin{aligned}
& R_{t h_{e q}}=R_{t h_{1}}+R_{t h_{2}} \\
& \frac{2 L}{k_{e q} A}=\frac{L}{k_{1} A}+\frac{L}{k_{2} A}
\end{aligned}
$$

Or,

$$
\begin{array}{r}
\frac{2}{k_{e q}}=\frac{1}{k_{1}}+\frac{1}{k_{2}} \\
\Rightarrow k_{e q}=\frac{2 k_{1} k_{2}}{k_{1}+k_{2}}
\end{array}
$$

22. A parabola $x=y^{2}$ with $0 \leq x \leq 1$ is shown in the figure. The volume of the solid of rotation obtained by rotating the shaded area by $360^{\circ}$ around the x -axis is

A. $\frac{\pi}{2}$
B. $2 \pi$
C. $п$
D. $\frac{\pi}{4}$

Ans. A
Sol. Volume of the solid of rotation obtained by rotating the shaded area by $360^{\circ}$ around the $x$ axis is given by

$$
\text { volume }(V)=\int_{0}^{1} \pi y^{2} d x
$$

Or, $V=\int_{0}^{1} \pi x d x=\pi\left(\frac{x^{2}}{2}\right)_{0}^{1}=\frac{\pi}{2}$
23. Consider an ideal vapor compression refrigeration cycle. If the throttling process is replaced by an isentropic expansion process, keeping all the other processes unchanged, which one of the following statements is true for the modified cycle?
A. Coefficient of performance is the same as that of the original cycle.
B. Coefficient of performance is lower than that of the original cycle.
C. Coefficient of performance is higher than that of the original cycle.
D. Refrigerating effect is lower than that of the original cycle.

Ans. C
Sol. When the throttling process is replaced by an isentropic expansion process, keeping all the other processes unchanged, then the heat absorbed by the evaporator (desired effect) will increase, therefore, the COP will increase
24. The position vector $\overrightarrow{O P}$ of point $P(20,10)$ is rotated anti-clockwise in $\mathrm{X}-\mathrm{Y}$ plane by an angle $\theta=30^{\circ}$ such that point $P$ occupies position $Q$, as shown in the figure. The coordinates ( $x, y$ ) of $Q$ are

A. $(18.66,12.32)$
B. $(22.32,8.26)$
C. $(12.32,18.66)$
D. $(13.40,22.32)$

Ans. C
Sol. Rotational matrix for a point in 2D coordinated by $\theta$ in anti-clockwise rotation.
$R=\left[\begin{array}{cc}\cos \theta & -\sin \theta \\ \sin \theta & \cos \theta\end{array}\right]$
Point $_{Q}=R\left\{\begin{array}{l}20 \\ 10\end{array}\right\}=\left[\begin{array}{cc}\cos \theta & -\sin \theta \\ \sin \theta & \cos \theta\end{array}\right]\left\{\begin{array}{l}20 \\ 10\end{array}\right\}$
Point $_{Q}=\left[\begin{array}{cc}\cos 30 & -\sin 30 \\ \sin 30 & \cos 30\end{array}\right]\left\{\begin{array}{l}20 \\ 10\end{array}\right\}$
Point $_{Q}=\left[\begin{array}{cc}0.866 & -0.5 \\ 0.5 & 0.866\end{array}\right]\left\{\begin{array}{l}20 \\ 10\end{array}\right\}=\left\{\begin{array}{l}12.32 \\ 18.66\end{array}\right\}$
Point $_{Q}=(12.32,18.66)$
25. Water flows through a pipe with a velocity given by $\vec{V}=\left(\frac{4}{t}+x+y\right) \hat{j} \mathrm{~m} / \mathrm{s}$, where $\hat{j}$ is the unit vector in the $y$ direction, $t(>0)$ is in seconds, and $x$ and $y$ are in meters. The magnitude of total acceleration at the point $(x, y)=(1,1)$ at $\mathrm{t}=2 \mathrm{~s}$ is $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$.

Ans. 3
Sol. $\vec{a}=a_{x} \hat{\imath}+a_{y} \hat{\jmath}+a_{z} \hat{k}$

$$
\vec{V}=f(x, y, z, t)
$$

Here, $u=V_{x}=0 ; v=V_{y}=\frac{4}{t}+x+y$;

$$
\begin{gathered}
w=V_{z}=0 \\
a_{x}=u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}+w \frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}=0 \\
a_{y}=u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}+w \frac{\partial v}{\partial z}+\frac{\partial v}{\partial t}=\frac{4}{t}+x+y-\frac{4}{t^{2}} \\
a_{z}=u \frac{\partial w}{\partial x}+v \frac{\partial w}{\partial y}+w \frac{\partial w}{\partial z}+\frac{\partial w}{\partial t}=0
\end{gathered}
$$

So, $\vec{a}=a_{x} \hat{\imath}+a_{y} \hat{\jmath}+a_{z} \hat{k}$

$$
=0+\frac{4}{t}+x+y-\frac{4}{t^{2}}+0
$$

At $(x, y)=(1,1)$ at $t=2 s$,

$$
\vec{a}=0+\frac{4}{2}+1+1-\frac{4}{4}=2+1+1-1=3
$$

26. In a UTM experiment, a sample of length 100 mm , was loaded in tension until failure. The failure load was 40 kN . The displacement, measured using the cross-head motion, at failure, was 15 mm . The compliance of the UTM is constant and is given by $5 \times 10^{-8} \mathrm{~m} / \mathrm{N}$. The strain at failure in the sample is
$\qquad$ \%.
Ans. 2
Sol. UTM constant $=\frac{\delta_{\text {failure }}}{F}=5 \times 10^{-8}$
Strain at failure $=\frac{\delta_{\text {failure }}}{L}=\frac{U T M \text { Cons } \times F}{100}$
Strain at failure $=\frac{5 \times 10^{-8} \times 40 \times 10^{3}}{\frac{100}{1000}}=2 \%$
27. At a critical point in a component, the state of stress is given as $\sigma_{x x}=100 \mathrm{MPa}, \sigma_{y y}=220 \mathrm{MPa}$, $\sigma_{x y}=\sigma_{y x}=80 \mathrm{MPa}$ and all other stress components are zero. the yield strength of the material is 468 MPa . The factor of safety on the basis of maximum shear stress theory is
$\qquad$ (round off to one decimal place).

Ans. 1.8
Sol. $\sigma_{x x}=100 \mathrm{MPa}, \sigma_{y y}=220 \mathrm{MPa}$,
$\sigma_{x y}=\sigma_{y x}=80 \mathrm{MPa}$

$$
\begin{gathered}
\sigma_{1,2}=\frac{1}{2}\left[\left(\sigma_{x}+\sigma_{y}\right) \pm \sqrt{\left(\sigma_{x}-\sigma_{y}\right)^{2}+4 \sigma x y^{2}}\right] \\
=\frac{1}{2}\left[(100+220) \pm \sqrt{(100-220)^{2}+4(80)^{2}}\right] \\
\sigma_{1}=260, \sigma_{2}=60
\end{gathered}
$$

According to maximum shear stress theory, Larger of $\left[\left|\frac{\sigma_{1}-\sigma_{2}}{2}\right|,\left|\frac{\sigma_{1}}{2}\right|,\left|\frac{\sigma_{2}}{2}\right|\right]=\frac{s_{y t}}{2 \times F O S}$
So, Larger of $\left[\left\lfloor\frac{260-60}{2}\left|,\left|\frac{260}{2}\right|,\left|\frac{60}{2}\right|\right]=\frac{468}{2 \times F O S}\right.\right.$
Or, $130=\frac{468}{2 \times F O S} \Rightarrow F O S=1.8$
28. A single block brake with a short shoe and torque capacity of $250 \mathrm{~N}-\mathrm{m}$ is shown. The cylindrical brake drum rotates anticlockwise at 100 rpm and the coefficient of friction is 0.25 . The value of $a$, in mm (round off to one decimal place), such that maximum actuating force $P$ is 2000 N, is $\qquad$


Ans.
Sol. FBD of drum


$$
F_{t}=\mu R
$$

FBD of block


Taking moments about O, we get

$$
\begin{aligned}
P \times 2.5 a & =F_{t} \times a / 4+R \times a \\
2000 \times 2.5 a & =0.25 R \times a / 4+R \times a
\end{aligned}
$$

On solving this, we get $R=4705.88 N$

$$
\begin{gathered}
T=F_{t} \times \text { radius }=F_{t} \times a=\mu R \times a \\
250=0.25 \times 4705.88 \times a \\
\Rightarrow a=0.2125 \mathrm{~m} \text { or } 212.5 \mathrm{~mm}
\end{gathered}
$$

29. Five jobs ( $J 1, J 2, J 3, J 4$ and $J 5$ ) need to be processed in a factory. Each job can be assigned to any of the five different machines (M1, M2, M3, M4 and M5). The time durations taken (in minutes) by the machines for each of the jobs, are given in the table. However, each job is assigned to a specific machine in such a way that the total processing time is minimum. The total processing time is $\qquad$ minutes.

|  | M1 | M2 | M3 | M4 | M5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| J1 | 40 | 30 | 50 | 50 | 58 |
| J2 | 26 | 38 | 60 | 26 | 38 |
| J3 | 40 | 34 | 28 | 24 | 30 |
| J4 | 28 | 40 | 40 | 32 | 48 |
| J5 | 28 | 32 | 38 | 22 | 44 |

Ans. 146
Sol. Rewriting the matrix using row minimum

| 10 | 0 | 20 | 20 | 28 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 12 | 34 | 0 | 12 |
| 16 | 10 | 4 | 0 | 6 |
| 0 | 12 | 12 | 4 | 20 |
| 6 | 10 | 16 | 0 | 22 |

Again, rewriting the matrix using column minimum

| 10 | 0 | 16 | 20 | 22 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 12 | 30 | 0 | 6 |
| 16 | 10 | 0 | 0 | 0 |
| 0 | 12 | 8 | 4 | 14 |
| 6 | 10 | 12 | 0 | 16 |

Start assigning the zeros, first row-wise and then column-wise

| 10 | 0 | 16 | 20 | 22 |
| :---: | :---: | :---: | :---: | :---: |
| $x$ | 12 | 30 | $X$ | 6 |
| 16 | 10 | 0 | $\infty$ | $x$ |
| 0 | 12 | 8 | 4 | 14 |
| 6 | 10 | 12 | 0 | 16 |

Now, since after assigning, we get only 4 assignments and since this is a $5 \times 5$ matrix, so we have to create one more zero We do it using the Hungarian method

| 17 | - | 16 | 20 | 22 |
| :---: | :---: | :---: | :---: | :---: |
| < | 12 | 30 | $\times$ | 6 |
| 1 | 10 | 0 | * | $\rightarrow$ |
| ( | 12 | 8 |  | 14 |
| 1 | 10 | 12 | (0) | 16 |

Now, $\theta$ (minimum value) among all the elements through which no line pass is 6 Now updating the matrix using $\theta$

| 16 | 0 | 16 | 26 | 22 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 6 | 24 | 0 | 0 |
| 22 | 10 | 0 | 6 | 0 |
| 0 | 6 | 2 | 4 | 8 |
| 6 | 4 | 6 | 0 | 10 |

Now making the assignments row-wise and column-wise, we get

| 16 | 0 | 16 | 26 | 22 |
| :---: | :---: | :---: | :---: | :---: |
| $\not 2$ | 6 | 24 | $\mathfrak{Z}$ | $\square$ |
| 22 | 10 | 0 | 6 | $\not ্$ |
| 0 | 6 | 2 | 4 | 8 |
| 6 | 4 | 6 | 0 | 10 |

This is the optimal assignment, since we get 5 assignments
So, the assignments will be as follows

$$
\begin{aligned}
& J_{1} \rightarrow M_{2} \\
& J_{2} \rightarrow M_{5} \\
& J_{3} \rightarrow M_{3} \\
& J_{4} \rightarrow M_{1} \\
& J_{5} \rightarrow M_{4}
\end{aligned}
$$

And the minimum processing time will be equal to $30+38+28+28+22=146$
30. Consider a prismatic straight beam of length $L$ $=n \mathrm{~m}$, pinned at two ends as shown in the figure. The beam has a square cross-section of side $p=6 \mathrm{~mm}$. The Young's modulus $E=200$ GPa, and the coefficient of thermal expansion a $=3 \times 10^{-6} \mathrm{~K}^{-1}$. The minimum temperature rise required to cause Euler buckling of the beam is
$\qquad$ K.


Ans. 1
Sol. According to the given condition,

$$
\begin{gathered}
R_{\text {th }}=R_{\text {buckling }} \\
\alpha \Delta T E A=\frac{\pi^{2} E I}{L^{2}} \\
\Rightarrow 3 \times 10^{-6} \times \Delta T \times 36 \times 10^{-6}=\frac{\pi^{2} \times \frac{1}{12} \times\left(6 \times 10^{-3}\right)^{4}}{\pi^{2}} \\
\Rightarrow \Delta T=1 K
\end{gathered}
$$

31. Two immiscible. Incompressible, viscous fluids having same densities but different viscosities are contained between two infinite horizontal parallel plates, 2 m apart as shown below. The bottom plate is fixed and the upper plate moves to the right with a constant velocity of $3 \mathrm{~m} / \mathrm{s}$. With the asusmptions of Newtonian fluid, steady, and fully developed laminar flow with zero pressure gradient in all directions, the momentum equations simplify to
$\frac{d^{2} u}{d y^{2}}=0$.
If the dynamic viscosity of the lower fluid, $\mu_{2}$, is twice that of the upper fluid, $\mu_{1}$, then the velocity at the interface (round off to two decimal places) is $\qquad$ $\mathrm{m} / \mathrm{s}$.


Ans. 1
Sol. Since, the fluid is Newtonian, therefore

$$
\tau \propto \frac{d u}{d y}=\text { constant }
$$

Or, $\tau_{1}=\tau_{2}$

$$
\begin{aligned}
\mu_{2} \frac{(V-0)}{1} & =\mu_{1} \frac{(3-V)}{1} \\
\mu_{2} & =2 \mu_{1}
\end{aligned}
$$

So, $2 V=3-V$ or $3 V=3$ or $V=1$
32. If one mole of $\mathrm{H}_{2}$ gas occupies a rigid container with a capacity of 1000 litres and the temperature is raised from $27^{\circ} \mathrm{C}$ to $37^{\circ} \mathrm{C}$, the change in pressure of the contained gas (round off to two decimal places), assuming ideal gas behaviour, is $\qquad$ Pa. $(R=8.314$ J/mol.K)
Ans.
Sol. From ideal gas equation, $\mathrm{pV}=\mathrm{mRT}$
Or, pV $=\mathrm{n} \bar{R} T$
So, $p \times 1=1 \times 8.314 \times 300 \quad[1000$ litres $=$ $1 \mathrm{~m}^{3} / \mathrm{s}$ ]

So, $\mathrm{p}=2494.2 \mathrm{~Pa}$
Container is rigid, so $\Delta V=V_{2}=V_{1}$
Using, $\frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}}$

$$
\Rightarrow P_{2}=\frac{2494.2 \times 310}{300}=2577.34 \mathrm{~Pa}
$$

Therefore,

$$
\Delta P=P_{2}-P_{1}=2577.34-2494.2=
$$ 83.14 Pa

33. A car having weight W is moving in the direction as shown in the figure. The center of gravity (CG) of the car is located at height $h$ from the ground, midway between the front and rear wheels. The distance between the front and rear wheels is $I$. The acceleration of the car is a, and acceleration due to gravity is g . The reactions on the front wheels ( $R_{f}$ ) and rear wheels $\left(R_{r}\right)$ are given by

A. $R_{f}=R_{r}=\frac{W}{2}+\frac{W}{g}\left(\frac{h}{l}\right) a$
B. $\mathrm{R}_{\mathrm{f}}=\frac{\mathrm{W}}{2}+\frac{\mathrm{W}}{\mathrm{g}}\left(\frac{\mathrm{h}}{\mathrm{l}}\right) \mathrm{a} ; \mathrm{R}_{\mathrm{r}}=\frac{\mathrm{W}}{2}-\frac{\mathrm{W}}{\mathrm{g}}\left(\frac{\mathrm{h}}{\mathrm{l}}\right) \mathrm{a}$
C. $R_{f}=\frac{W}{2}-\frac{W}{g}\left(\frac{h}{l}\right) a ; R_{r}=\frac{W}{2}+\frac{W}{g}\left(\frac{h}{l}\right) a$
D. $R_{f}=R_{r}=\frac{W}{2}-\frac{W}{g}\left(\frac{h}{l}\right) a$

Ans.
Sol. The car is moving in the forward direction, so the pseudo force will act in the backward direction


Taking moments about $A$ and equating it to 0 , we get

$$
R_{f} \times l+m a \times h-W \times \frac{l}{2}=0
$$

Where, $\mathrm{m}=$ mass of the car $=\frac{W}{g}$
Substituting this in the above equation,

$$
\begin{gathered}
R_{f} \times l+\frac{W}{g} a \times h-W \times \frac{l}{2}=0 \\
\Rightarrow R_{f}=\frac{W}{2}-\frac{W}{g}\left(\frac{h}{l}\right) a
\end{gathered}
$$

Similarly, if we take moment about point $B$ and equate it to 0 , we get

$$
R_{r}=\frac{W}{2}+\frac{W}{g}\left(\frac{h}{l}\right) a
$$

34. In a four bar planar mechanism shown in the figure, $A B=5 \mathrm{~cm}, A D=4 \mathrm{~cm}$ and $D C=2 \mathrm{~cm}$. In the configuration shown, both $A B$ and $D C$ are perpendicular to $A D$. The bar $A B$ rotates with an angular velocity of $10 \mathrm{rad} / \mathrm{s}$. The magnitude of angular velocity (in rad/s) of bar DC at this instant is

A. 0
B. 15
C. 10
D. 25

Ans. D
Sol. $V_{B}=V_{C}$

$$
\begin{aligned}
\omega_{A B} \times A B= & \omega_{D C} \times D C \\
& 10 \times 5=\omega_{D C} \times 2 \\
& \omega_{D C}=\frac{50}{2}=25 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

35. The variable $x$ takes a value between 0 and 10 with uniform probability distribution. The variable y takes a value between 0 and 20 with uniform probability distribution.
The probability of the sum of variables $(x+y)$ being greater than 20 is
A. 0.33
B. 0
C. 0.25
D. 0.50

Ans. C
Sol:


$$
\operatorname{Area}(\triangle P Q R)=\frac{1}{2} \times 10 \times 10=50
$$

Total area $=20 \times 10=200$
probability $=\frac{50}{200}=0.25=25 \%$
36. The value of the following definite integral is
$\qquad$ (round off to three decimal places)

$$
\int_{1}^{e}(x \ln x) d x
$$

Ans. 2.097
Sol. we have to use integration by parts in this

$$
\int u v d x=u \int v d x-\int\left[\frac{d u}{d x} \int v d x\right] d x
$$

Using the ILATE formula,
We have $u=\ln x$ and $v=x$

$$
\begin{gathered}
\int x \ln x d x=\ln x \int x d x-\int \frac{1}{x} \int x d x d x \\
\left(\frac{x^{2}}{2} \ln x-\frac{x^{2}}{4}\right)_{1}^{e}=\frac{e^{2}+1}{4}=2.097
\end{gathered}
$$

37. In orthogonal turning of a cylindrical tube of wall thickness 5 mm , the axial and the tangential cutting forces were measured as 1259 N and 1601 N , respectively. The measured chip thickness after machining was found to be 0.3 mm . The rake angle was $10^{\circ}$ and the axial feed was $100 \mathrm{~mm} / \mathrm{min}$. the rotational speed of the spindle was 1000 rpm. Assuming the material to be perfectly and Merchant's first solution, the shear strength of the material is closest to
A. 722 MPa
B. 200 MPa
C. 920 MPa
D. 875 MPa

Ans. A
Sol. $\mathrm{N}=1000 \mathrm{rpm}$, rake angle $(\alpha)=10^{\circ}$
Feed $(f)=100 \mathrm{~mm} / \mathrm{min}=100 / 1000$
$=0.1 \mathrm{~mm} / \mathrm{rev}=\mathrm{t}$
Chip thickness ratio (r)
$=\frac{\text { uncut chip thickness }}{\text { cut chip thickness }}=\frac{0.1}{0.3}=\frac{1}{3}$

Shear strength is given by

$$
\tau_{s}=\frac{F_{s} \sin \phi}{b t}
$$

Where, $F_{s}=$ shear force

$$
\begin{gathered}
\phi=\text { Shear angle } \\
b=\text { width }=5 \mathrm{~mm}
\end{gathered}
$$

Using $\tan \phi=\frac{r \cos \alpha}{1-r \sin \alpha}=\frac{0.33 \cos 10}{1-0.33 \sin 10}=0.345$

$$
\begin{aligned}
& \text { Or, } \phi=19.02 \\
& \qquad F_{s}=F_{c} \cos \phi-F_{t} \sin \phi
\end{aligned}
$$

$$
\begin{aligned}
& =1606 \cos 19.02-1259 \sin 19.02 \\
& =1103.29 \mathrm{~N}
\end{aligned}
$$

So,

$$
\tau_{s}=\frac{1103.29 \sin 19.02}{5 \times 0.1}=719.12 \mathrm{MPa}
$$

38. Match the following sand mold casting defects with their respective causes.

| Defect |  | Cause |  |
| :---: | :---: | :---: | :---: |
| P | Blow hole | 1 | Poor <br> collapsibility |
| Q | Misrun | 2 | Mold erosion |
| R | Hot tearing | 3 | Poor <br> permeability |
| S | Wash | 4 | Insufficient <br> fluidity |

A. P-4, Q-3, R-1, S-2
B. $\mathrm{P}-2, \mathrm{Q}-4, \mathrm{R}-1, \mathrm{~S}-3$
C. $\mathrm{P}-3, \mathrm{Q}-4, \mathrm{R}-1, \mathrm{~S}-2$
D. $\mathrm{P}-3, \mathrm{Q}-4, \mathrm{R}-2, \mathrm{~S}-1$

Ans. C
Sol. Tiny gas bubbles are called porosities, but larger gas bubbles are called blowholes or blisters. Such defects can be caused by air entrained in the melt, steam or smoke from the casting sand, or other gasses from the melt or mold. This is caused by poor permeability

A misrun occurs when the liquid metal does not completely fill the mold cavity, leaving an unfilled portion. This is caused by insufficient fluidity

Hot tears is the metallurgical defect. This happens because the metal is weak when it is hot and the residual stresses in the material can cause the casting to fail as it cools. This is caused by poor collapsibility

Wash is the casting defect of areas of excess metal. These appear when the molten metal erodes the molding sand.
39. A circular shaft having diameter $65.00_{-0.05}^{+0.01}$ mm is manufactured by turning process. A 50 $\mu \mathrm{m}$ thick coating of TiN is deposited on the shaft. Allowed variation in TiN film thickness is $\pm 5 \mu \mathrm{~m}$. The minimum hole diameter (in mm ) to just provide clearance fit is
A. 64.95
B. 65.12
C. 65.10
D. 65.01

Ans. B
Sol. For clearance fit, shaft maximum possible diameter should be less than bore inner diameter.

Dia $a_{\text {min }}=\operatorname{Max}\left(65.00_{-0.05}^{+0.01}\right)+2 \times \operatorname{Max}\left(0.05_{-0.005}^{+0.005}\right)$
Dia min $=65+0.01+2 \times(0.05+0.005)$

$$
D i a_{\min }=65.12
$$

40. Three slabs are joined together as shown in the figure. There is no thermal contact resistance at interfaces. The center slab experiences a non-uniform internal heat generation with an average value equal to $10000 \mathrm{Wm}^{-3}$, while the left and right slabs have no internal heat generation. All slabs have thickness equal to 1 m and thermal conductivity of each slab is equal to $5 \mathrm{Wm}^{-1} \mathrm{~K}^{-1}$. The two extreme faces are exposed to fluid with heat transfer coefficient
$100 \mathrm{Wm}^{-2} \mathrm{~K}^{-1}$ and bulk temperature $30^{\circ} \mathrm{C}$ as shown. The heat transfer in the slabs is assumed to be one dimensional and steady, and all properties are constant. If the left extreme face temperature $T_{1}$ is measured to be $100^{\circ} \mathrm{C}$, the right extreme face temperature $\mathrm{T}_{2}$ is
$\qquad$ ${ }^{\circ} \mathrm{C}$.


Ans.
Sol. Total heat generated $=$ Heat convected from both the sides to the ambient

$$
\begin{gathered}
\dot{q} \times A \times L=h \times A \times(100-30)+h \times A \times\left(T_{2}-30\right) \\
10000 \times 1=100 \times 70+100 \times\left(T_{2}-30\right) \\
\Rightarrow T_{2}-30=30
\end{gathered}
$$

Or, $T_{2}=60^{\circ} \mathrm{C}$
41. A cube of side 100 mm is placed at the bottom of an empty container on one of its faces. The density of the cube is $800 \mathrm{~kg} / \mathrm{m}^{3}$, Liquid of density $1000 \mathrm{~kg} / \mathrm{m}^{3}$ is now poured into the container. The minimum height to which the liquid needs to be poured into the container for the cube to just lift up is $\qquad$ mm .
Ans. 80
Sol. According to the condition of equilibrium,

Buoyancy force $=$ Weight of cube
Weight of liquid displaced
$=$ Volume $\times$ density $\times g$
Area $\times \mathrm{h} \times \rho_{\text {liquid }} \times \mathrm{g}=$ Volume $\times \rho_{\text {cube }} \times g$
$(100)^{2} \times h \times 1000=(100)^{3} \times 800$
So, $h=800 / 10=80 \mathrm{~mm}$
42. Consider an elastic straight beam of length $L=$ $10 п \mathrm{~m}$, with square cross-section of side $\mathrm{a}=5$ mm , with Young's modulus $\mathrm{E}=200 \mathrm{GPa}$. This straight beam was bent in such a way that the two ends meet, to form a circle of mean radius R. Assuming that Euler-Bernoulli beam theory is applicable to this bending problem, the maximum tensile bending stress in the bent beam is $\qquad$ MPa.


Ans. 100
Sol. The beam of length $L=10 \pi \mathrm{~m}$ is bent to form a circle of mean radius R .

Then, $L=2 \pi R$

$$
\Rightarrow 10 \pi=2 \pi R
$$

Or, $\mathrm{R}=5 \mathrm{~m}$
$y=2.5 \mathrm{~mm}$ [since cross section is square with side 5 mm ]
According to Euler-Bernoulli beam theory

$$
\frac{M}{I}=\frac{\sigma}{y}=\frac{E}{R}
$$

Using, $\sigma=\frac{E y}{R}=\frac{200 \times 10^{3} \times 2.5}{5 \times 10^{3}}=100 \mathrm{MPa}$
43. The set of equations
$x+y+z=1$
$a x-a y+3 z=5$
$5 x-3 y+a x=6$
has infinite solutions, if $\mathrm{a}=$
A. 4
B. -3
C. -4
D. 3

Ans. A
Sol. The coefficient matrix is given by $\left[\begin{array}{ccc}1 & 1 & 1 \\ a & -a & 3 \\ 5 & -3 & a\end{array}\right]$ And the augmented matrix is given by $\left(\begin{array}{ccc|c}1 & 1 & 1 & 1 \\ a & -a & 3 & 5 \\ 5 & -3 & a & 6\end{array}\right)$

For infinite solution, Rank(coefficient matrix) = Rank(augmented matrix) < Order of matrix(3)

$$
\begin{aligned}
&\left(\begin{array}{ccc|c}
1 & 1 & 1 & 1 \\
a & -a & 3 & 5 \\
5 & -3 & a & 6
\end{array}\right) \\
& R_{2} \rightarrow R_{2}-a R_{1}, R_{3} \rightarrow R_{3}-5 R_{1} \\
&\left(\begin{array}{ccc|c}
1 & 1 & 1 & 1 \\
0 & -2 a & 3-a & 5-a \\
0 & -8 & a-5 & 1
\end{array}\right) \\
& R_{3} \rightarrow a R_{3}-4 R_{2} \\
&\left(\begin{array}{ccc|c}
1 & 1 & 1 & 1 \\
0 & -2 a & 3-a & 5-a \\
0 & 0 & a^{2}-a-12 & 5 a-20
\end{array}\right)
\end{aligned}
$$

For Rank(coefficient matrix)
$=\operatorname{Rank}$ (augmented matrix $=2$

$$
a^{2}-a-12=0 \text { or } a=-3,4
$$

And $5 a-20=0$ or $a=4$
So, $a=4$ is the correct answer
44. A gas is heated in a duct as it flows over a resistance heater. Consider a 101 kW electric heating system. The gas enters the heating section of the duct at 100 kPa and $27^{\circ} \mathrm{C}$ with a volume flow rate of $15 \mathrm{~m}^{3} / \mathrm{s}$. If heat is lost from the gas in the duct to the surroundings at a rate of 51 kW , the exit temperature of the gas is (Assume constant pressure, ideal gas, negligible change in kinetic and potential energies and constant specific heat; $C p=1$ $\mathrm{kJ} / \mathrm{kg} \mathrm{K} ; \mathrm{R}=0.5 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ )
A. $37^{\circ} \mathrm{C}$
B. $53^{\circ} \mathrm{C}$
C. $32^{\circ} \mathrm{C}$
D. $76^{\circ} \mathrm{C}$

Ans. C
Sol. From ideal gas equation, $\mathrm{p} \dot{V}=\dot{m} \mathrm{R} T$

$$
\begin{gathered}
100 \times 15=\dot{m} \times 0.5 \times 300 \\
\Rightarrow \dot{m}=10 \mathrm{~kg} / \mathrm{s}
\end{gathered}
$$

Using SFEE,

$$
\begin{gathered}
\dot{m} h_{1}+\dot{Q_{i n}}+Q_{o u t} \dot{m} h_{2}+\dot{W_{c v}} \\
\Rightarrow \dot{m} h_{1}+101+(-51)=\dot{m} h_{2} \\
\dot{m}\left(h_{2}-h_{1}\right)=50
\end{gathered}
$$

Or, $\dot{m} C_{p}\left(T_{2}-T_{1}\right)=50$

$$
\Rightarrow 10 \times 1\left(T_{2}-300\right)=50
$$

Or, $\left(T_{2}-300\right)=5$
$\Rightarrow T_{2}=305 \mathrm{~K}=305-273=32^{\circ} \mathrm{C}$
45. A harmonic function is analytic if it satisfies the Laplace equation.
If $u(x, y)=2 x^{2}-2 y^{2}+4 x y$ is a harmonic function, then its conjugate harmonic function $v(x, y)$ is
A. $4 y^{2}-4 x y+$ constant
B. $-4 x y+2 y^{2}-2 x^{2}+$ constant
C. $2 x^{2}-2 y^{2}+x y+$ constant
D. $4 x y-2 x^{2}+2 y^{2}+$ constant

Ans. D
Sol. Since the harmonic function is analytic, so it satisfies the Cauchy-Riemann (C-R) equations Therefore, $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$
Given $u=2 x^{2}-2 y^{2}+4 x y$

$$
\Rightarrow \frac{\partial u}{\partial x}=4 x+4 y=\frac{\partial v}{\partial y}
$$

So, on integrating, $v=4 x y+2 y^{2}+f(x)$
Now, $\frac{\partial v}{\partial x}=4 y+f^{\prime}(x)=-\frac{\partial u}{\partial y}=4 y-4 x$
So, $f^{\prime}(x)=-4 x$
Or, $f(x)=-2 x^{2}+$ constant
Therefore,
$v=4 x y+2 y^{2}-2 x^{2}+$ constant b
46. A gas turbine with air as the working fluid has an isentropic efficiency of 0.70 when operating at a pressure ratio of 3 . Now, the pressure ratio of the turbine is increased to 5, while maintaining the same inlet conditions. Assume air as a perfect gas with specific heat ratio $\gamma=$ 1.4. If the specific work output remains the same for both the cases, the isentropic efficiency of the turbine at the pressure ratio of 5 is $\qquad$ (round off to two decimal places)

Ans. 0.51

Sol. Given:
$\left(r_{p}\right)_{1}=3,\left(r_{p}\right)_{2}=5$
$\left(\eta_{\text {isen }}\right)_{1}=0.70$
$\gamma=1.4$
$T_{1}=$ turbine inlet temperature is same for both the cases

Actual work done in case 1

$$
\begin{array}{r}
=\left(h_{1}-h_{2}\right)_{1}=m C_{p} \eta_{1}\left(T_{1}-T_{2}\right) \\
W_{1}=m C_{p} T_{1} \eta_{1}\left(1-\frac{1}{\left(r_{p}\right)_{1}^{\frac{\gamma-1}{\gamma}}}\right)
\end{array}
$$

Actual work done in case 2

$$
\begin{gathered}
=\left(h_{1}-h_{2}\right)_{2}=m C_{p} \eta_{2}\left(T_{1}-T_{2}\right) \\
W_{2}=C_{p} T_{1} \eta_{2}\left(1-\frac{1}{\left(r_{p}\right)_{2}^{\frac{\gamma-1}{\gamma}}}\right)
\end{gathered}
$$

Now, $W_{1}=W_{2}$

$$
\begin{gathered}
m C_{p} T_{1} \eta_{1}\left(1-\frac{1}{\left(r_{p}\right)_{1}^{\frac{\gamma-1}{\gamma}}}\right)=m C_{p} T_{1} \eta_{2}\left(1-\frac{1}{\left(r_{p}\right)_{2}^{\frac{\gamma-1}{\gamma}}}\right) \\
\eta_{1}\left(1-\frac{1}{\left(r_{p}\right)_{1}^{\frac{\gamma-1}{\gamma}}}\right)=\eta_{2}\left(1-\frac{1}{\left(r_{p}\right)_{2}^{\frac{\gamma-1}{\gamma}}}\right)
\end{gathered}
$$

Substituting the values of
$\left(r_{p}\right)_{1}=3,\left(r_{p}\right)_{2}=5$
$\left(\eta_{\text {isen }}\right)_{1}=0.70$ in the above equation, we get $\eta_{2}=0.51$
47. A plane-strain compression (forging) of a block is shown in the figure. The strain in the $z-$ direction is zero. The yield strength ( $\mathrm{S}_{y}$ ) in uniaxial tension/compression of the material of the block is 300 MPa and it follows the Tresca (maximum shear stress) criterion. Assume that the entire block has started yielding. At a point where $\sigma_{x}=40 \mathrm{MPa}$ (compressive) and $\mathrm{T}_{\mathrm{xy}}=0$, the stress component $\sigma_{y}$ is

A. 340 MPa (tensile)
B. 260 MPa (compressive)
C. 260 MPa (tensile)
D. 340 MPa (compressive)

Ans. D
Sol. For plane strain,

$$
\epsilon_{z}=\frac{\sigma_{z}}{E}-\mu\left(\frac{\sigma_{x}}{E}+\frac{\sigma_{y}}{E}\right)=0
$$

So, $\sigma_{z}=\mu\left(\sigma_{x}+\sigma_{y}\right)=\mu\left(-40+\sigma_{y}\right)$
Maximum shear stress
$=$ larger of $\left|\left[\frac{\sigma_{x}-\sigma_{y}}{2}\right],\left[\frac{\sigma_{y}-\sigma_{z}}{2}\right],\left[\frac{\sigma_{z}-\sigma_{x}}{2}\right]\right|$
Maximum shear stress
$=$ larger of $\left|\left[\frac{-40-\sigma_{y}}{2}\right],\left[\frac{\sigma_{y}-\mu\left(\sigma_{y}-40\right)}{2}\right],\left[\frac{\mu\left(-40+\sigma_{y}\right)+40}{2}\right]\right|$
From the above equation, $\left[\frac{-40-\sigma_{y}}{2}\right]$ will be maximum
So, $\left|\frac{-40-\sigma_{y}}{2}\right| \leq \frac{S_{y t}}{2 F O S}$
or $-40-\sigma_{y}=300$

$$
\Rightarrow \sigma_{y}=-340 M P a=340(\text { compression })
$$

Or $-40-\sigma_{y}=-300$

$$
\Rightarrow \sigma_{y}=260 M P a=260(\text { compression })
$$

But in forging, force cannot be tensile, hence the correct answer must be 340 MPa (compressive)
48. $A$ truss is composed of members $A B, B C, C D$, $A D$ and $B D$, as shown in the figure. $A$ vertical load of 10 kN is applied at point D . The magnitude of force (in kN ) in the member BC is


Ans. 5
Sol.


Due to symmetry, $R_{A}=R_{C}=5$
Considering, joint C


$$
\begin{aligned}
\sum F_{y} & =0 ; F_{C D} \sin 45=5 \\
& \Rightarrow F_{C D}=5 \sqrt{2}
\end{aligned}
$$

Now, $\sum F_{B C}=F_{C D} \cos 45=5 \sqrt{2} \times \frac{1}{\sqrt{2}}=5$
49. A project consists of six activities. The immediate predecessor of each activity and the estimated duration is also provided in the table below:

| Activity | Immediate <br> predecessor | Estimated duration <br> (weeks) |
| :---: | :---: | :---: |
| P | - | 5 |
| Q | - | 1 |
| R | Q | 2 |
| S | $\mathrm{P}, \mathrm{R}$ | 4 |
| T | P | 6 |
| U | $\mathrm{S}, \mathrm{T}$ | 3 |

If all activities other than $S$ take the estimated amount of time, the maximum duration (in
weeks) of the activity $S$ without delaying the completion of the project is $\qquad$
Ans. 6
Sol. The network of the given project will be as shown below


Activity $S$ can be delayed upto 2 weeks Normal duration of activity $S$ is 4 weeks Hence, maximum duration of activity $S$ can be 6 weeks
50. The wall of a constant diameter pipe of length 1 m is heated uniformly with flux $\mathrm{q}^{\prime \prime}$ by wrapping a heater coil around it. The flow at the inlet to the pipe is hydrodynamically fully developed. The fluid is incompressible and the flow is assumed to be laminar and steady all through the pipe. The bulk temperature of the fluid is equal to $0^{\circ} \mathrm{C}$ at the inlet and $50^{\circ} \mathrm{C}$ at the exit. The wall temperatures are measured at three locations, $\mathrm{P}, \mathrm{Q}$ and R as shown in the figure. The flow thermally develops after some distance from the inlet. The following measurements are made:


##  <br> Constant wall flux

Among the locations $P, Q$ and $R$, the flow is thermally developed at
A. P, Q and R
B. P and Q only
C. $Q$ and $R$ only
D. R only

Ans. C
Sol.

$T_{b}(0.2 m)=10^{\circ} \mathrm{C}$
$T_{b}(P)=20^{\circ} \mathrm{C} \quad T_{w}(P)=50^{\circ} \mathrm{C}$
$T_{b}(Q)=30^{\circ} \mathrm{C} \quad T_{w}(Q)=80^{\circ} \mathrm{C}$
$T_{b}(R)=40^{\circ} \mathrm{C} \quad T_{w}(R)=90^{\circ} \mathrm{C}$
$\Delta T=T_{w}-T_{b}$ at $P=50-20=30^{\circ} \mathrm{C}$
$\Delta T=T_{w}-T_{b}$ at $Q=80-30=50^{\circ} \mathrm{C}$
$\Delta T=T_{w}-T_{b}$ at $R=90-40=50^{\circ} \mathrm{C}$
In the fully developed region, $\Delta T=T_{w}-T_{b}$ must be constant, which is satisfied by $Q$ and $R$ only, hence only $Q$ and $R$ must be in the fully developed region
51. A steam power cycle with regeneration as shown below on the T-s diagram employs a single open feedwater heater for efficiency improvement. The fluids mix with each other in an open feedwater heater. The turbine is isentropic and the input (bleed) to the feedwater heater from the turbine is at state 2 as shown in the figure. Process 3-4 occurs in the condenser. The pump work is negligible. The input to the boiler is at state 5 . The following information is available from the steam tables:

| State | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Enthalpy <br> $(\mathrm{kJ} / \mathrm{kg})$ | 3350 | 2800 | 2300 | 175 | 700 | 1000 |



The mass flow rate of steam bled from the turbine as a percentage of the total mass flow rate at the inlet to the turbine at state 1 is
$\qquad$ -.
Ans. 20
Sol. For open feed water heater, using the energy balance

Let $x$ be the fraction of steam bled from the turbine

$$
\begin{gathered}
x h_{2}+(1-x) h_{4}=h_{5} \\
\text { or, } 2800 x+(1-x) 175=700
\end{gathered}
$$

Or, $x=0.2=20 \%$
52. A uniform thin disk of mass 1 kg and radius 0.1 $m$ is kept on a surface as shown in the figure. A spring of stiffness $k_{1}=400 \mathrm{~N} / \mathrm{m}$ is connected to the disk center $A$ and another spring of stiffness $k_{2}=100 \mathrm{~N} / \mathrm{m}$ is connected at point $B$ just above point $A$ on the circumference of the disk. Initially, both the springs are unstretched. Assume pure rolling of the disk. For small disturbance from the equilibrium, the natural frequency of vibration of the system is _ rad/s (round off to one decimal place).


Ans. 23.09
Sol. Consider the point at which the circle touches the bottom as ' $\mathrm{O}^{\prime}$


Using the Energy method,

$$
\begin{gathered}
I_{o} \ddot{\theta}+\left(k_{1} R \theta\right) R+\left(k_{2} 2 R \theta\right) 2 R=0 \\
I_{o}=\frac{M R^{2}}{2}+M R^{2}=\frac{3}{2} M R^{2}
\end{gathered}
$$

Substituting this value

$$
\frac{3}{2} M R^{2} \ddot{\theta}+\left(k_{1} R^{2}+4 k_{2} R^{2}\right) \theta=0
$$

Or,

$$
\frac{3}{2} M \ddot{\theta}+\left(k_{1}+4 k_{2}\right) \theta=0
$$

Substituting the values of $M, k_{1}$ and $k_{2}$ in above equation

$$
\frac{3}{2} \ddot{\theta}+(400+4 \times 100) \theta=0
$$

Or,

$$
\begin{gathered}
\frac{3}{2} \ddot{\theta}+(800) \theta=0 \\
3 \ddot{\theta}+(1600) \theta=0 \\
\ddot{\theta}+\frac{1600}{3} \theta=0
\end{gathered}
$$

Comparing it with standard equation,

$$
\ddot{\theta}+\omega_{n}^{2} \theta=0
$$

We get, $\omega_{n}=\sqrt{\frac{1600}{3}}=23.09$
53. Taylor's tool life equation is given by $V T^{n}=C$, where V is in $\mathrm{m} / \mathrm{min}$ and T is in min. In a turning operation, two tools $X$ and $Y$ are used. For tool $\mathrm{X}, \mathrm{n}=0.3$ and $\mathrm{C}=60$ and for tool $\mathrm{Y}, \mathrm{n}=0.6$ and $C=90$. Both the tools will have the same tool life for the cutting speed (in $\mathrm{m} / \mathrm{min}$, round off to one decimal place) of $\qquad$

Ans. 39.99
Sol. According to Taylor's tool life equation

$$
V T^{n}=C
$$

For tool $X, n=0.3$ and $C=60$
So, $V T^{0.3}=60$

$$
\Rightarrow T=\left(\frac{60}{V}\right)^{1 / 0.3}
$$

For tool $\mathrm{Y}, \mathrm{n}=0.6$ and $\mathrm{C}=90$
So, $V T^{0.6}=90$

$$
\Rightarrow T=\left(\frac{90}{V}\right)^{1 / 0.6}
$$

Tool life is same,
Therefore

$$
\left(\frac{60}{V}\right)^{1 / 0.3}=\left(\frac{90}{V}\right)^{1 / 0.6}
$$

On solving this, we get

$$
V=39.99 \mathrm{~m} / \mathrm{min}
$$

54. The rotor of a turbojet engine of an aircraft has a mass 180 kg and polar moment of intertia 10 $\mathrm{kg} . \mathrm{m}^{2}$ about the rotor axis. The rotor rotates at a constant speed of $1100 \mathrm{rad} / \mathrm{s}$ in the clockwise direction when viewed from the front of the aircraft. The aircraft while flying at a speed of 800 km per hour takes a turn with a radius of 1.5 km to the left. The gyroscopic moment exerted by the rotor on the aircraft structure and the direction of motion of the nose when the aircraft turns, are
A. 162.9 N.m and the nose goes down
B. 1629.6 N.m and the nose goes up
C. 1629.6 N.m and the nose goes down
D. 162.9 N.m and the nose goes up

Ans. C
Sol. $m=180 \mathrm{~kg} ; I=10 \mathrm{~kg}-\mathrm{m}^{2}$

$$
\begin{gathered}
\omega=1100 \mathrm{rad} / \mathrm{s} \\
\omega_{p}=\frac{V}{R}=\frac{800 \times 5 / 18}{1.5 \times 1000}=0.148 \mathrm{rad} / \mathrm{s}
\end{gathered}
$$

Gyroscopic couple $=I \omega \omega_{p}=10 \times 1100 \times 0.148=$ 1629.6 N.m

When rotor rotates in the clockwise direction when viewed from the front of the aircraft, the noes goes down
55. In ASA system, the side cutting and end cutting edge angles of a sharp turning tool are $45^{\circ}$ and $10^{\circ}$, respectively. The feed during cylindrical turning is $0.1 \mathrm{~mm} / \mathrm{rev}$. The center line average surface roughness (in $\mu \mathrm{m}$, round off to one decimal place) of the generated surface is

Ans. 3.74
Sol. $\operatorname{SCEA}=45^{\circ}$ and ECEA $=10^{\circ}$
Feed $(f)=0.1 \mathrm{~mm} / \mathrm{rev}$
center line average surface roughness $R_{a}=$

$$
\begin{aligned}
& \frac{f}{4(\tan S C E A+\cot E C E A)}=\frac{0.1}{4\left(\tan 45^{\circ}+\cot 10^{\circ}\right)} \\
= & \frac{0.1}{4(1+5.67)}=\frac{0.1}{26.68}=3.74 \times 10^{-3} \mathrm{~mm}=3.74 \mu \mathrm{~m}
\end{aligned}
$$

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