## GATE 2019

Set-2

Mechanical Engineering

Questions \& Solutions

## SECTION: GENERAL APTITUDE

1. A final examination is the $\qquad$ of $a$ series of evaluations that a student has to go through.
A. desperation
B. insinuation
C. consultation
D. culmination

Ans. D
Sol. A final examination is the culmination of a series of evaluations that a student has to go through.

Meaning of culmination is the highest or climactic point of something, especially as attained after a long time

Hence, it is the most appropriate word
2. If $\mathrm{IMHO}=$ JNIP; IDK $=$ JEL; and $\mathrm{SO}=\mathrm{TP}$, then IDC = $\qquad$ .
A. JDC
B. $J C D$
C. JDE
D. JED

Ans. D
Sol. $\mathrm{I}+1=\mathrm{J}, \mathrm{M}+1=\mathrm{N}, \mathrm{H}+1=\mathrm{I}, \mathrm{O}+1=\mathrm{P} ; \mathrm{I}$ $+1=J, D+1=E, K+1=L ; S+1=T, O+$ $1=P$

Hence, IDC will be written as JED
$I+1=J, D+1=E, C+1=D$
3. Are there enough seats here? There are
$\qquad$ people here than I expected.
A. many
B. most
C. more
D. least

Ans. C
Sol. Comparison is made here between the number of people and number of seats, hence, more is the most appropriate word
4. Once the team of analysts identify the problem, we $\qquad$ in a better position to comment on the issue.

Which one of the following choices CANNOT fill the given blank?
A. might be
B. are going to be
C. will be
D. were to be

Ans. C
Sol. Future is being discussed here, hence will be is the most appropriate phrase
5. The product of three integers $X, Y$ and $Z$ is 192 . $Z$ is equal to 4 and $P$ is equal to the average of $X$ and $Y$. What is the minimum possible value of $P$ ?
A. 7
B. 6
C. 8
D. 9.5

Ans. A
Sol. $X Y Z=192$ and $Z=4$
So, $X Y=48$

$$
P=\frac{X+Y}{2}
$$

For finding the minimum possible of $P, X+Y$ should be minimum

Now, the possible values of $X$ and $Y$ are as follows

| $\mathbf{X}$ | $\mathbf{Y}$ |
| :---: | :---: |
| 48 | 1 |
| 24 | 2 |
| 16 | 3 |
| 12 | 4 |
| 8 | 6 |

So, from this we can clearly see that $X+Y$ will be minimum only when $X=8$ and $Y=6$ or viceversa
So minimum value of $\mathrm{P}=\frac{8+6}{2}=7$
6. $X$ is an online media provider. By offering unlimited and exclusive online content at attractive prices for a loyalty membership, X is almost forcing its customers towards its loyalty membership. If its loyalty membership
continues to grow at its current rate, within the next eight years more households will be watching $X$ than cable television.
Which one of the following statements can be inferred from the above paragraph?
A. Most households that subscribe to X's loyalty membership discontinue watching cable television
B. Non-members prefer to watch cable television
C. The $X$ is cancelling accounts of nonmembers
D. Cable television operators don't subscribe to X's loyalty membership

Ans. A
Sol. option A is the most appropriate solution
7. Fiscal deficit was $4 \%$ of the GDP in 2015 and that increased to $5 \%$ in 2016. If the GDP increased by $10 \%$ from 2015 to 2016, the percentage increase in the actual fiscal deficit is $\qquad$ .
A. 35.70
B. 10.00
C. 37.50
D. 25.00

Ans. C
Sol. Let the GDP in $2015=x$
Then, the GDP in 2016 will be
$=1.1 \mathrm{x}$
So, fiscal deficit in 2015
$=0.04 \times \mathrm{x}=0.04 \mathrm{x}$
And the fiscal deficit in 2016
$=0.05 \times 1.1 \mathrm{x}=0.055 \mathrm{x}$
Increase in fiscal deficit
$=0.055 \mathrm{x}-0.04 \mathrm{x}=0.015 \mathrm{x}$
So, the percentage increase in fiscal deficit $=$ $\frac{0.015 \mathrm{x}}{0.04 \mathrm{x}} \times 100=37.5 \%$
8. Two pipes $P$ and $Q$ can fill a tank in 6 hours and 9 hours respectively, while a third pipe $R$ can empty the tank in 12 hours. Initially, P and R are open for 4 hours. Then P is closed and Q is opened. After 6 more hours R is closed. The total time taken to fill the tank (in hours) is
$\qquad$ .
A. 16.50
B. 14.50
C. 15.50
D. 13.50

Ans. B
Sol. 1 hour work of $P=1 / 6$
1 hour work of $\mathrm{Q}=1 / 9$
1 hour work of $R=-1 / 12$ (negative sign indicates that its emptying the tank)
Tank filled when P and R works initially for 4 hours
$=4(1 / 6-1 / 12)=1 / 3$
Tank filled by when Q and R works for another 6 hours
$=6(1 / 9-1 / 12)=1 / 6$
Total tank filled in these 10 hours $=1 / 3+1 / 6=$ 0.5

So, the remaining half tank will be filled by Q alone in 4.5 hours (Since Q can fill the complete tank in 9 hours)
Therefore, total time required to fill the tank $=$ $4+6+4.5=14.5$ hours
9. While teaching a creative writing class in India, I was surprised at receiving stories from the students that were all set in distant places: in the American West with cowboys and in Manhattam penthouses with clinking ice cubes. This was, till an eminent cAribbean writer gave the writers in the once-colonised countries the confidence to see the shabby lives around them as worthy of being "told".

The writer of this passage is surprised by the creative writing assignments of his students, because $\qquad$ .
A. Some of the students had written about ice cubes and cowboys
B. None of the students had written about ice cubes and cowboys
C. Some of the students had written stories set in foreign places
D. None of the students had written stories set in India

Ans. C
Sol. option C is the most appropriate solution
10. Mola is a digital platform for taxis in a city. It offers three types of rides - Pool, Mini and Prime. The Table below presents the number of rides for the past four months. The platform earns one US dollar per ride. What is the percentage share of revenue contributed by Prime to the total revenues of Mola, for the entire duration?

| Type | Month |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | January | February | March | April |
| Pool | 170 | 320 | 215 | 190 |
| Mini | 110 | 220 | 180 | 70 |
| Prime | 75 | 180 | 120 | 90 |

A. 38.74
B. 23.97
C. 25.86
D. 16.24

Ans. B
Sol. Revenue contributed by Pool $=170+320+$ $215+190=895$

Revenue contributed by Mini $=110+220+$ $180+70=580$
Revenue contributed by Prime $=75+180+$ $120+90=465$

Total revenue of Mola $=895+580+465=$ 1940
So, the percentage share $=\frac{465}{1940} \times 100=23.97 \%$

## MECHANICAL ENGINEERING

1. A wire of circular cross-section of diameter 1.0 mm is bent into a circular arc of radius 1.0 m by application of pure bending moments at its ends. The Young's modulus of the material of the wire is 100 GPa . The maximum tensile stress developed in the wire is $\qquad$ MPa.

Ans. 50
Sol. Tensile stress

$$
\begin{aligned}
\sigma=\frac{\mathrm{E}}{\mathrm{R}} \mathrm{y}= & \frac{100 \times 10^{3}}{1}\left(\frac{1}{1000 \times 2}\right) \\
& =50 \mathrm{MPa}
\end{aligned}
$$

2. In an electrical discharge machining process, the breakdown voltage across inter electrode gap (IEG) is 200 V and the capacitance of the RC circuit is $50 \mu \mathrm{~F}$. The energy (in J) released per spark across the IEG is $\qquad$
Ans. 1
Sol: $\mathrm{E}=\frac{1}{2} \times$ Capacitance $\times$ voltage $^{2}$

$$
E=0.5 \times 50 \times 10^{-6} \times 200^{2}=1 \mathrm{~J}
$$

3. If $x$ is the mean of data $3, x, 2$ and 4 , then the mode is $\qquad$
Ans. 3
Sol. $x=\frac{3+x+2+4}{4}$
$4 \times x=3+x+2+4$
$4 \times \mathrm{x}-\mathrm{x}=3+2+4=9$
$\mathrm{x}=3$
Mode is 3 .
(Maximum repeated value)
4. The cold forming process in which a hardened tool is pressed against a workpiece (when there is relative motion between the tool and the workpiece) to produce a roughened surface with a regular pattern is
A. Strip rolling
B. Knurling
C. Roll forming
D. Chamfering

Ans. B
Sol. We provide special pattern to prevent the relative motion between contact surface. This special pattern is known as Knurling.
5. Endurance limit of a beam subjected to pure bending decreases with
A. increase in the surface roughness and increase in the size of the beam
B. decrease in the surface roughness and decrease in the size of the beam
C. decrease in the surface roughness and increase in the size of the beam
D. increase in the surface roughness and decrease in the size of the beam

Ans. A
Sol. Formula for endurance limit
$\sigma_{e}=K_{a} \times K_{b} \times K_{c} \times K_{d} \times \sigma_{e}^{\prime}$
Where
$\mathrm{K}_{\mathrm{a}}$ - Size factor
$\mathrm{K}_{\mathrm{b}}$ - Surface factor
$\mathrm{K}_{\mathrm{c}}$ - load factor
$K_{d}$ - Temperature factor
$\sigma_{\mathrm{e}}^{\prime}$ - Endurance Strength
Form above formula it is clear that the Endurance limit increases with increase in size, surface, load, temp and endurance strength.
6. In matrix equation $[A]\{X\}=\{R\}$.
$[A]=\left[\begin{array}{ccc}4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15\end{array}\right],\{X\}=\left\{\begin{array}{l}2 \\ 1 \\ 4\end{array}\right\}$ and
$\{R\}=\left\{\begin{array}{l}32 \\ 16 \\ 64\end{array}\right\}$.

One of the eigenvalues of matrix $[\mathrm{A}]$ is
A. 4
B. 15
C. 8
D. 16

Ans.
Sol. $[A]\{X\}=\{R\}$.

## We know

$$
\begin{gathered}
\text { AX }=\lambda X ; \text { Where } \lambda \text { - Eigen Value } \\
\qquad\left[\begin{array}{ccc}
4 & 8 & 4 \\
8 & 16 & -4 \\
4 & -4 & 15
\end{array}\right]\left\{\begin{array}{l}
2 \\
1 \\
4
\end{array}\right\}=\left\{\begin{array}{l}
32 \\
16 \\
64
\end{array}\right\} \\
{\left[\begin{array}{ccc}
4 & 8 & 4 \\
8 & 16 & -4 \\
4 & -4 & 15
\end{array}\right]\left\{\begin{array}{l}
2 \\
1 \\
4
\end{array}\right\}=16\left\{\begin{array}{l}
2 \\
1 \\
4
\end{array}\right\}}
\end{gathered}
$$

Hence, $\lambda=16$
7. Water enters a circular pipe of length $L=5.0$ m and diameter $\mathrm{D}=0.20 \mathrm{~m}$ with Reynolds number $R_{e d}=500$. The velocity profile at the inlet of the pipe is uniform while it is parabolic at the exit. The Reynolds number at the exit of the pipe is $\qquad$ .
Ans. 500
Sol. Reynold's Number
$\operatorname{Re}=\frac{\rho V_{a v} \mathbf{D}}{\mu}$
Where: -
$\rho$ - Fluid Density
$\mathrm{V}_{\mathrm{av}}$ - Average velocity
D - Dimeter of Pipe
$\mu$ - Dynamic viscosity of Fluid
$\operatorname{Re}=\left(\frac{\rho V_{\mathrm{av}} \mathrm{D}}{\mu}\right)=\left(\frac{\rho\left(\frac{\mathrm{Q}_{\text {pipe }}}{A_{\text {pipe }}}\right) \mathrm{D}}{\mu}\right)$
From above expiration, Reynold's Number is independent of flow profile. Hence, Reynold's Number for parabolic flow profile is 500 .
8. For a simple compressible system, $v, s, p$ and T are specific volume, specific entropy, pressure and temperature, respectively. As per Maxwell's relations, $\quad\left(\frac{\partial v}{\partial s}\right)_{p}$ is equal to
A. $\left(\frac{\partial \mathrm{p}}{\partial \mathrm{v}}\right)_{\mathrm{T}}$
B. $\left(\frac{\partial \mathbf{s}}{\partial \mathrm{T}}\right)_{\mathrm{p}}$
C. $\left(\frac{\partial T}{\partial \mathrm{p}}\right)_{\mathrm{s}}$
D. $-\left(\frac{\partial T}{\partial V}\right)_{p}$

Ans. C
Sol. From mathematical relation expiration

$$
\mathrm{dx}=\mathrm{Mdy}+\mathrm{Ndz}
$$

$$
\Rightarrow\left(\frac{\partial \mathrm{M}}{\partial \mathrm{z}}\right)_{\mathrm{y}}=\left(\frac{\partial \mathrm{N}}{\partial \mathrm{y}}\right)_{\mathrm{z}}
$$

From Maxwell relation
$\mathrm{dh}=\mathrm{Tds}+\mathrm{Vdp}$
$\therefore\left(\frac{\partial \mathrm{T}}{\partial \mathrm{p}}\right)_{\mathrm{s}}=\left(\frac{\partial \mathrm{V}}{\partial \mathrm{S}}\right)_{\mathrm{p}}$
9. Hardenability of steel is a measure of
A. the ability to harden when it is cold worked
B. the maximum hardness that can be obtained when it is austenitized and then quenched
C. the ability to retain its hardness when it is heated to elevated temperatures
D. the depth to which required hardening is obtained when it is austenitized and then quenched
Ans. D
Sol. The depth to which required hardening is obtained when it is austenitized and then quenched.
10. The directional derivative of the function $f(x, y)$ $=x^{2}+y^{2}$ along a line directed from $(0,0)$ to $(1,1)$, evaluated at the point $x=1, y=1$ is
A. 2
B. $\sqrt{ } 2$
C. $4 \sqrt{ } 2$
D. $2 \sqrt{ } 2$

Ans. D
Sol. Directional derivative of function along the line is the scalar value of derivative along the line. i.e. we have to calculate value of derivative of function in the direction of given line vector
$\nabla \mathrm{f}=\hat{\imath}\left(\frac{\partial \mathrm{f}}{\mathrm{dx}}\right)+\hat{\jmath}\left(\frac{\partial \mathrm{f}}{\mathrm{dy}}\right)$
$\nabla \mathrm{f}=2 \hat{\imath}+2 \hat{\jmath}$
Now, calculating the value of $\nabla \mathrm{f}$ in $1 \hat{\imath}+1 \hat{\jmath}$
D.D of given function is

$$
(2 \hat{\imath}+2 \hat{\jmath}) \cdot \frac{(\hat{\imath}+\hat{\jmath})}{|\hat{\imath}+\hat{\jmath}|}=2 \sqrt{2}
$$

11. A spur gear has pitch circle diameter $D$ and number of teeth $T$. The circular pitch of the gear is
A. $\frac{2 \pi D}{T}$
B. $\frac{T}{D}$
C. $\frac{\pi D}{T}$
D. $\frac{D}{T}$

Ans. C
Sol. circular pitch $=\frac{\pi D}{T}$
12. A rigid triangular body, $P Q R$, with sides of equal length of 1 unit moves on a flat plane. At the instant shown, edge QR is parallel to the $x$-axis, and the body moves such that velocities of points $P$ and $R$ are $V_{P}$ and $V_{R}$, in the $x$ and $y$ directions, respectively. The magnitude of the angular velocity of the body is

A. $V_{R} / \sqrt{3}$
B. $V_{p} / \sqrt{3}$
C. $2 \mathrm{~V}_{\mathrm{R}}$
D. $2 \mathrm{~V}_{\mathrm{P}}$

Ans. C
Sol. The given body is a rigid body; Hence the given body will not have any deformation, and the possible motion is translation and angular motion. we know that the angular motion of the object is always defined as the motion about the point. Hence, first we have to find a point parallel to both velocity $\mathrm{V}_{\mathrm{P}}$ and $\mathrm{V}_{\mathrm{R}}$.

$\therefore$ Angular velocity,

$$
\begin{gathered}
\omega=\frac{\text { Velocity }}{\text { Perpendicular distance }} \\
\omega=\frac{\mathrm{V}_{\mathrm{p}}}{\mathrm{PS}}=\frac{\mathrm{V}_{\mathrm{R}}}{\mathrm{SR}}
\end{gathered}
$$

PS $=\frac{\sqrt{3}}{2} \times$ Side of triangle $=\frac{\sqrt{3}}{2} \times 1=\frac{\sqrt{3}}{2}$
$\mathrm{SR}=\frac{1}{2} \times$ Side of triangle $=\frac{1}{2} \times 1=\frac{1}{2}$

$$
\omega=\frac{2 \mathrm{~V}_{\mathrm{p}}}{\sqrt{3}}=\frac{2 \mathrm{~V}_{\mathrm{R}}}{1}
$$

13. Sphere 1 with a diameter of 0.1 m is completely enclosed by another sphere 2 of diameter 0.4 m . The view factor $\mathrm{F}_{12}$ is
A. 0.25
B. 0.0625
C. 1.0
D. 0.5

Ans. C
Sol. $\mathrm{F}_{12}=1$
14. The transformation matrix for mirroring a point in $x-y$ plane about the line $y=x$ is given by
A. $\left[\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}\right]$
B. $\left[\begin{array}{cr}0 & -1 \\ -1 & 0\end{array}\right]$
C. $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$
D. $\left[\begin{array}{cc}-1 & 0 \\ 0 & 1\end{array}\right]$

Ans. C
Sol. The transformation matrix for mirroring a point in $x-y$ plane about the line $y=x$ is given by $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$
15. The state of stress at a point in a component is represented by a Mohr's circle of radius 100 MPa centered at 200 MPa on the normal stress axis. On a plane passing through the same point, the normal stress is 260 MPa . The magnitude of the shear stress on the same plane at the same point is $\qquad$ MPa.

Ans. 80
Sol.

$\tau=\sqrt{100^{2}-60^{2}}=80 \mathrm{MPa}$
16. One-dimensional steady state heat conduction takes place through a solid whose crosssectional area varies linearly in the direction of heat transfer. Assume there is no heat generation in the solid and the thermal conductivity of the material is constant and independent of temperature. The temperature distribution in the solid is
A. Quadratic
B. Exponential
C. Logarithmic
D. Linear

Ans. C
Sol. Let the given body cross-sectional area varies linearly, and heat is flowing from left to right.


If there is no heat generation,
$\frac{\mathrm{d}}{\mathrm{dx}}\left(-\mathrm{KA} \frac{\mathrm{dT}}{\mathrm{dX}}\right)=0$
$\int \frac{\mathrm{d}}{\mathrm{dx}}\left(-K A \frac{\mathrm{dT}}{\mathrm{dX}}\right)=\int 0$
$\left(-K A \frac{d T}{d X}\right)=C$
$-K\left(A_{0}+A_{c} x\right) \frac{d T}{d X}=C$
$d T=\frac{-C d X}{K\left(A_{0}+A_{c} x\right)}$
$\int_{1}^{2} d T=\int_{1}^{2} \frac{-C d X}{K\left(A_{0}+A_{c} x\right)}$
$\mathrm{T}_{2}-\mathrm{T}_{1}=-\frac{\mathrm{C}}{\mathrm{K}} \ln \left(\frac{\mathrm{A}_{\mathrm{c}} \mathrm{x}_{2}+\mathrm{A}_{0}}{\mathrm{~A}_{\mathrm{c}} \mathrm{x}_{1}+\mathrm{A}_{0}}\right)$
17. An analytic function $f(z)$ of complex variable $z$ $=x+I y$ may be written as $f(z)=u(x, y)+i v$ ( $x, y$ ). Then, $u(x, y)$ and $v(x, y)$ must satisfy
A. $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$
B. $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$
C. $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$
D. $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$

Ans. C
Sol. If $f(z)=u(x, y)+i v(x, y)$ is analytic function then it must satisfy the following relations.
$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$
$\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$
18. A thin vertical flat plate of height $L$, and infinite width perpendicular to the plane of the figure, is losing heat to the surroundings by natural convection. The temperatures of the plate and the surroundings, and the properties of the surrounding fluid, are constant. The relationship between the average Nusselt and Rayleigh numbers is given as $N u=K \operatorname{Ra} 1 / 4$, where K is a constant. The length scales for Nusselt and Rayleigh numbers are the height of the plate. The height of the plate is increased to 16 L keeping all other factors constant.


If the average heat transfer coefficient for the first plate is $h_{1}$ and that for the second plate is $h_{2}$, the value of the ratio $h_{1} / h_{2}$ is $\qquad$ .
Ans. 2
Sol. $N_{u}=K\left(R_{\mathrm{a}}\right)^{\frac{1}{4}}$
We know that $\mathrm{R}_{\mathrm{a}}=\mathrm{G}_{\mathrm{r}} \mathrm{P}_{\mathrm{r}}$
$N_{u}=K\left(G_{r} P_{r}\right)^{\frac{1}{4}}$
$\frac{\mathrm{hL}}{\mathrm{K}} \propto\left(\mathrm{G}_{\mathrm{r}}\right)^{\frac{1}{4}} \propto\left(\frac{\mathrm{~g} \beta \Delta \mathrm{LL}^{3}}{\mathrm{~V}^{2}}\right)^{\frac{1}{4}}$
$h L \propto L^{\frac{3}{4}}$
$h \propto L^{-\frac{1}{4}}$
$\frac{\mathrm{h}_{2}}{\mathrm{~h}_{1}} \propto\left(\frac{\mathrm{~L}_{1}}{\mathrm{~L}_{2}}\right)^{\frac{1}{4}}=>\frac{\mathrm{h}_{1}}{\mathrm{~h}_{2}} \propto\left(\frac{\mathrm{~L}_{2}}{\mathrm{~L}_{1}}\right)^{\frac{1}{4}}=(16)^{\frac{1}{4}}=2$
19. Which one of the following modifications of the simple ideal Rankine cycle increases the thermal efficiency and reduces the moisture content of the steam at the turbine outlet?
A. Decreasing the condenser pressure.
B. Increasing the boiler pressure.
C. Decreasing the boiler pressure.
D. Increasing the turbine inlet temperature.

Ans. D
Sol. Due to increase of the turbine inlet temperature, quality of the stream at outlet of the turbine improves. Which increases the thermal efficiency and reduces the moisture content of the steam at the turbine outlet.
20. Consider a linear rectangular thin sheet of metal, subjected to uniform uniaxial tensile stress of 100 MPa along the length direction. Assume plane stress conditions in the plane normal to the thickness. The Young's modulus $\mathrm{E}=200 \mathrm{MPa}$ and Poisson's ratio $\mathrm{v}=0.3$ are given. The principal strains in the plane of the sheet are
A. $(0.5,0.0)$
B. $(0.35,-0.15)$
C. $(0.5,-0.5)$
D. $(0.5,-0.15)$

Ans.

Sol. $\epsilon_{1}=\frac{\sigma_{1}}{\mathrm{E}}-\mu \frac{\sigma_{2}}{\mathrm{E}}=\frac{100}{200}-0.3 \times \frac{0}{200}=0.5$

$$
\begin{aligned}
\epsilon_{2}=\frac{\sigma_{2}}{\mathrm{E}}-\mu \frac{\sigma_{1}}{\mathrm{E}}=\frac{0}{200}- & 0.3 \times \frac{100}{200} \\
= & -0.15
\end{aligned}
$$

21. The figure shows an idealized plane truss. If a horizontal force of 300 N is applied at point A, then the magnitude of the force produced in member CD is $\qquad$ N.


Ans. 0
Sol.


The momentum about the point A must be zero, because there is no deformation of any truss.
Hence, $\mathrm{F}_{\mathrm{CD}}=0\left[\mathrm{~F}_{\mathrm{CD}}\right.$ is not passing through the point A]
22. The fluidity of molten metal of cast alloys (without any addition of fluxes) increases with increase in
A. freezing range
B. surface tension
C. degree of superheat
D. viscosity

Ans. C
Sol. Fluidity of molten metal of cast alloys increases with increase in degree of superheat.
23. The most common limit gage used for inspecting the hole diameter is
A. Ring gage
B. Snap gage
C. Plug gage
D. Master gage

Ans. C
Sol. The most common limit gage used for inspecting the hole diameter is Plug gage.
24. The differential equation $\frac{d y}{d x}+4 y=5$ is valid in the domain $0 \leq x \leq 1$ with $y(0)=2.25$. The solution of the differential equation is
A. $y=e^{-4 x}+1.25$
B. $y=e^{4 x}+5$
C. $y=e^{4 x}+1.25$
D. $y=e^{-4 x}+5$

Ans. A
Sol. The given equation is.
$\frac{d y}{d x}+4 y=5$
If Integration factor $=e^{\int 4 d x}=e^{4 x}$

$$
\begin{array}{ll}
\Rightarrow & y \times I F=\int 5 \times I F d x+C \\
\Rightarrow & y \times e^{4 x}=5 \times \frac{e^{4 x}}{4}+C \\
\Rightarrow & y \times e^{4 x}=5 \times \frac{e^{4 x}}{4}+C \ldots .(1)  \tag{1}\\
& \text { Now for at } x=0, \\
\Rightarrow & 2.25=\frac{5}{4}+C \\
\Rightarrow & C=1 \\
\Rightarrow & \text { Using the value of C in equ (1) } \\
\Rightarrow & y \times \frac{e^{4 x}}{e^{4 x}}=\frac{5}{e^{4 x}} \times \frac{e^{4 x}}{4}+\frac{C}{e^{4 x}} \\
\Rightarrow & y=1.25+e^{-4 x}
\end{array}
$$

25. A two-dimensional incompressible frictionless flow field is given by $\vec{u}=x \hat{i}-y \hat{j}$. If $\rho$ is the density of the fluid, the expression for pressure gradient vector at any point in the flow field is given as
A. $-\rho\left(x^{2} \hat{i}+y^{2} \hat{j}\right)$
B. $-\rho(x \hat{i}+y \hat{j})$
C. $\rho(x \hat{i}+y \hat{j})$
D. $\rho(x \hat{i}-y \hat{j})$

Ans. B
Sol. Euler's Equation of motion in 2D.
$\frac{\partial u}{\partial t}+u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}+w \frac{\partial u}{\partial z}=f_{x}-\frac{1}{\rho} \frac{\partial p}{\partial x}$
$\frac{\partial v}{\partial t}+u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}+w \frac{\partial v}{\partial z}=f_{y}-\frac{1}{\rho} \frac{\partial p}{\partial y}$
As there is no extra force acting on the fluid.
$\mathrm{f}_{\mathrm{x}}$ and $\mathrm{f}_{\mathrm{y}}=0$
Now form the given expiration,
$u=x \& v=y$
$\therefore \frac{\partial \mathrm{x}}{\partial \mathrm{t}}+\mathrm{x} \frac{\partial \mathrm{x}}{\partial \mathrm{x}}+\mathrm{y} \frac{\partial \mathrm{x}}{\partial \mathrm{y}}+\mathrm{w} \frac{\partial \mathrm{x}}{\partial \mathrm{z}}=0-\frac{1}{\rho} \frac{\partial \mathrm{p}}{\partial \mathrm{x}}$
$\frac{\partial y}{\partial t}+x \frac{\partial y}{\partial x}+y \frac{\partial y}{\partial y}+w \frac{\partial v}{\partial z}=0-\frac{1}{\rho} \frac{\partial p}{\partial y}$
$\therefore 0+\mathrm{x} \times 1+0+0=0-\frac{1}{\rho} \frac{\partial \mathrm{p}}{\partial \mathrm{x}}$
$0+0+\mathrm{y} \times 1+0=0-\frac{1}{\rho} \frac{\partial \mathrm{p}}{\partial \mathrm{y}}$
$\therefore$ Press Grad Vec $=\frac{\partial \mathrm{p}}{\partial \mathrm{x}} \hat{\imath}+\frac{\partial \mathrm{p}}{\partial \mathrm{y}} \hat{\jmath}$
$=-\rho(x \hat{i}+y \hat{y})$
26. A short shoe external drum brake is shown in the figure. The diameter of the brake drum is 500 mm . The dimensions $\mathrm{a}=1000 \mathrm{~mm}, \mathrm{~b}=$ 500 mm and $\mathrm{c}=200 \mathrm{~mm}$. The coefficient of friction between the drum and the shoe is 0.35 . The force applied on the lever $\mathrm{F}=100 \mathrm{~N}$ as shown in the figure. The drum is rotating anticlockwise. The braking torque on the drum is
$\qquad$ N.m (round off to two decimal places).


Ans. 20.34
Sol.


Applying momentum about point O .
$\sum \mathrm{M}_{\mathrm{H}}=0$
$\mathrm{F} \times \mathrm{a}+\mu \mathrm{R}_{\mathrm{N}} \times \mathrm{c}-\mathrm{R}_{\mathrm{N}} \times \mathrm{b}=0$
$\mathrm{R}_{\mathrm{N}}=232.55$
Breaking Torque $=\mu R_{N} R_{N}=20.34$
27. A uniform disc with radius $r$ and a mass of $m$ kg is mounted centrally on a horizontal axle of negligible mass and length of $1.5 r$. The disc spins counter-clockwise about the axle with angular speed $\omega$, when viewed from the righthand side bearing, Q . The axle precesses about a vertical axis at $\omega_{p}=\omega / 10$ in the clockwise direction when viewed from above. Let Rp and $\mathrm{R}_{\mathrm{Q}}$ (positive upwards) be the resultant reaction forces due to the mass and the gyroscopic effect, at bearings $P$ and $Q$, respectively. Assuming $\omega^{2} r=300 \mathrm{~m} / \mathrm{s}^{2}$ and $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$, the ratio of the larger to the smaller bearing reaction force (considering appropriate signs) is $\qquad$ .


Ans. -3
Sol. Gyroscope couple (GC) $=\mathrm{I} \omega \omega_{\mathrm{p}}$
Using: $\omega_{\mathrm{p}}=\frac{\omega}{10}$ and $\omega^{2} \mathrm{r}=300$
$\mathrm{GC}=\frac{\mathrm{mr}^{2}}{2} \frac{\omega^{2}}{10}=15 \mathrm{mr}$
Reaction at P and Q due to gravity
$R_{p 1}+R_{q 1}=m g=10 m$
$\mathrm{R}_{\mathrm{p} 1}=5 \mathrm{~m}$
$\therefore \mathrm{R}_{\mathrm{q} 1}=5 \mathrm{~m}$
Reaction at P and Q due to GC
Taking momentum about point p .
$\mathrm{R}_{\mathrm{q} 2} \times 1.5 \mathrm{r}-15 \mathrm{mr}=0$
$\mathrm{R}_{\mathrm{q} 2}=\frac{15 \mathrm{mr}}{1.5 \mathrm{r}}=10 \mathrm{~m}$
$\therefore \mathrm{R}_{\mathrm{p} 2}=-10 \mathrm{~m}$
Now,
$\frac{\mathrm{R}_{\mathrm{Q}}}{\mathrm{R}_{\mathrm{p}}}=\frac{\mathrm{R}_{\mathrm{q} 1}+\mathrm{R}_{\mathrm{q} 2}}{\mathrm{R}_{\mathrm{P} 1}+\mathrm{R}_{\mathrm{P} 2}}=\frac{5+10}{5-10}=-3$
28. The figure shows a pouring arrangement for casting of a metal block. Frictional losses are negligible. The acceleration due to gravity is $9.81 \mathrm{~m} / \mathrm{s}^{2}$. The time (in s , round off to two decimal places) to fill up the mold cavity (of size $40 \mathrm{~cm} \times 30 \mathrm{~cm} \times 15 \mathrm{~cm}$ ) is $\qquad$


Ans. 29.84
Sol. Mold + filling+time $+\frac{\mathrm{V}_{\mathrm{m}}}{\overline{\mathrm{Ag} \cdot \sqrt{2 \mathrm{gh}}}}$
Time $=\frac{0.40 \times 0.30 \times 0.15}{\pi \frac{0.02^{2}}{4} \cdot \sqrt{2 \times 9.81 \times 0.20}} 28.94 \mathrm{Sec}$
29. The activities of a project, their duration and the precedence relationships are given in the table. For example, in a precedence relationship " $X<Y, Z$ " means that $X$ is predecessor of activities $Y$ and $Z$. The time to complete the activities along the critical path is
$\qquad$ weeks.

| Activity | Duration <br> (Weeks) | Precedence <br> Relationship |
| :---: | :---: | :---: |
| A | 5 | $\mathrm{~A}<\mathrm{B}, \mathrm{C}, \mathrm{D}$ |$|$| $\mathrm{B}<\mathrm{E}, \mathrm{F}, \mathrm{G}$ |
| :---: |
| B |
| C |
| D |
| E |
| F |
| G |
| H |
| I |
| 7 |

A. 21
B. 25
C. 23
D. 17

Ans. C
Sol.

30. The crank of a slider-crank mechanism rotates counter-clockwise (CCW) with a constant angular velocity $\omega$, as shown. Assume the length of the crank to be r.


Using exact analysis, the acceleration of the slider in the $y$-direction, at the instant shown, where the crank is parallel to x-axis, is given by
A. $-\omega^{2} r$
B. $2 \omega^{2} r$
C. $\omega^{2} r$
D. $-2 \omega^{2} r$

Ans. C
Sol.


Velocity of point $A, V_{A}=r \times \omega$
The radial acceleration of point $A, a_{o a}=r \omega^{2}$
Since, the inclination of line $A B$ is 45 degree, the acceleration of the point $B$ will be equal to the acceleration of point A. i.e. $\mathrm{r} \omega^{2}$
31. A gas tungsten arc welding operation is performed using a current of 250 A and an arc voltage of 20 V at a welding speed of $5 \mathrm{~mm} / \mathrm{s}$. Assuming that the arc efficiency is $70 \%$, the net heat input per unit length of the weld will be $\qquad$ $\mathrm{kJ} / \mathrm{mm}$ (round off to one decimal place).

Ans. 0.7

Sol. Heat input $=\eta \times$ Power $_{\text {elec }}$
Heat input $=0.7 \times 250 \times 20$

$$
=3500 \text { Watt }
$$

Heat input per unit length $=\frac{3500}{5}=700 \frac{\mathrm{~J}}{\mathrm{~mm}}$ $=0.7 \mathrm{KJ} / \mathrm{mm}$
32. Water flows through two different pipes $A$ and $B$ of the same circular cross-section but at different flow rates. The length of pipe $A$ is 1.0 m and that of pipe $B$ is 2.0 m . The flow in both the pipes is laminar and fully developed. If the frictional head loss across the length of the pipes is same, the ratio of volume flow rates $\mathrm{Q}_{\mathrm{B}} / \mathrm{Q}_{A}$ is $\qquad$ (round off to two decimal places).
Ans. 0.5
Sol. Frictional losses of pipe
$h f \propto \mathbf{f L} \frac{\mathbf{V}^{2}}{\mathbf{D}} \propto \mathbf{f L} \frac{\left(\frac{\mathbf{Q}}{\mathbf{A}}\right)^{2}}{\mathbf{D}} \propto \mathbf{f L} \frac{\mathbf{Q}^{2}}{\mathbf{D}^{5}}$
$\frac{h_{f B}}{h f A}=\frac{f_{B} L_{B} Q_{B}^{2}}{f_{A} L_{A} Q_{A}^{2}}$; Where $f=\frac{64}{\operatorname{Re}}$
$\frac{Q_{B}^{2}}{Q_{A}^{2}}=\frac{h_{f B} L_{A} f_{A}}{h_{A} L_{B} f_{B}}=\frac{L_{A} f_{A}}{L_{B} f_{B}}=\frac{L_{A} Q_{B}}{L_{B} Q_{A}}$
$\frac{\mathrm{Q}_{\mathrm{B}}}{\mathrm{Q}_{\mathrm{A}}}=\frac{\mathrm{L}_{\mathrm{A}}}{\mathrm{L}_{\mathrm{B}}}=0.5$
33. Consider two concentric circular cylinders of different materials $M$ and $N$ in contact with each other at $r=b$, as shown below. The interface at $\mathrm{r}=\mathrm{b}$ is frictionless. The composite cylinder system is subjected to internal pressure $P$. Let $\left(\mathrm{u}_{r}^{M}, \mathrm{u}_{\theta}^{M}\right)$ and $\left(\sigma_{r r}^{M}, \sigma_{\theta \theta}^{M}\right)$ denote the radial and tangential displacement and stress components, respectively, in material $M$ Similarly, $\left(u_{r}^{N}, u_{\theta}^{N}\right)$ and $\left(\sigma_{r r}^{N}, \sigma_{\theta \theta}^{N}\right)$ denote the radial and tangential displacement and stress components, respectively, in material N. The boundary conditions that need to be satisfied
at the frictionless interface between the two cylinders are :

A. $u_{r}^{M}=u_{r}^{N}$ and $\sigma_{r r}^{M}=\sigma_{r r}^{N}$ and $u_{\theta}^{M}=u_{\theta}^{N} \quad$ and $\sigma_{\theta \theta}^{M}=\sigma_{\theta \theta}^{N}$
B. $\sigma_{r r}^{M}=\sigma_{r r}^{N}$ and $\sigma_{\theta \theta}^{M}=\sigma_{\theta \theta}^{N}$ only
C. $u_{\theta}^{M}=u_{\theta}^{N}$ and $\sigma_{\theta \theta}^{M}=\sigma_{\theta \theta}^{N}$ only
D. $u_{r}^{M}=u_{r}^{N}$ and $\sigma_{r r}^{M}=\sigma_{r r}^{N}$ only

Ans. D
Sol.


Internal pressure causes the stress in radial as well as circumferential direction.

At $r=b$ i.e. interference of the cylinder can be frictionless if Stress in radial direction must be equal. Velocity in radial direction must be equal. (This condition will make the normal force Zero)

As radius of the cylinder increases due to inside pressure, there will be relative velocity between the contact surfaces in the circumferential direction.
34. The binary phase diagram of metals $P$ and $Q$ is shown in the figure. An alloy $X$ containing $60 \%$ P and $40 \%$ Q (by weight) is cooled from liquid to solid state. The fractions of solid and liquid (in weight percent) at $1250^{\circ} \mathrm{C}$, respectively, will be

A. $22.2 \%$ and $77.8 \%$
B. $68.0 \%$ and $32.0 \%$
C. $77.8 \%$ and $22.2 \%$
D. $32.0 \%$ and $68.0 \%$

Ans. A
Sol. Mass fraction of liquid $=\frac{C_{\alpha}-C_{0}}{C_{\alpha}-C_{L}}$
$M_{L}=\frac{68-40}{68-32}=0.778=77.8 \%$
$M_{S}=1-M_{L}=0.222=22.2 \%$
35. The annual demand of valves per year in a company is 10,000 units. The current order quantity is 400 valves per order. The holding cost is Rs. 24 per valve per year and the ordering cost is Rs. 400 per order. If the current order quantity is changed to Economic Order Quantity, then the saving in the total cost of inventory per year will be Rs.
$\qquad$ (round off to two decimal places).
Ans. 943.59
Sol. Annual Demand $(D)=10000$ nos
Order Quantities (Q) $=400$ / order
Holding cost $\left(C_{h}\right)=24 /$ valve/year

Ordering Cost ( $\mathrm{C}_{0}$ ) $=$ RS 400 /order
TIC $=\frac{\mathrm{D}}{\mathrm{Q}} \mathrm{C}_{0}+\frac{\mathrm{Q}}{2} \mathrm{C}_{\mathrm{h}}$
TIC $=\frac{10000}{400} 400+\frac{10000}{2} 24=14800$
TIC $*=\sqrt{2 \mathrm{DC}_{0} \mathrm{C}_{\mathrm{h}}}$
TIC $*=13856.41$
Saving $=14800-13856.41=$ Rs. 943.59
36. A slender uniform rigid bar of mass $m$ is hinged at $O$ and supported by two springs, with stiffnesses 3 k and k , and a damper with damping coefficient $c$, as shown in the figure. For the system to be critically damped, the ratio $\mathrm{c} / \sqrt{ } \mathrm{km}$ should be

A. 4
B. 2
C. $4 \sqrt{ } 7$
D. $2 \sqrt{ } 7$

Ans. C
Sol. $\frac{\mathrm{C}}{\sqrt{\mathrm{km}}}$

$$
=? \text {; When System is critically damped }
$$

Suppose that we apply a external force downward at horizontal road at spring constant 3 K and leave it freely for to and fro motion.
Therefore, moment about point " O "; $\sum \mathrm{M}_{0}=0$

$$
\begin{gathered}
\mathrm{I}_{0} \ddot{\theta}+\mathrm{K}\left(\frac{3 \mathrm{~L}}{4} \theta\right)\left(\frac{3 \mathrm{~L}}{4}\right)+3 \mathrm{~K}\left(\frac{\mathrm{~L}}{4} \theta\right)\left(\frac{\mathrm{L}}{4}\right)+\mathrm{C}\left(\frac{\mathrm{~L}}{4} \dot{\theta}\right)\left(\frac{\mathrm{L}}{4}\right) \\
=0 \\
\mathrm{I}_{0} \ddot{\theta}+\mathrm{K}\left(\frac{9 \mathrm{~L}^{2}}{16} \theta\right)+3 \mathrm{~K}\left(\frac{\mathrm{~L}^{2}}{16} \theta\right)+\mathrm{C}\left(\frac{\mathrm{~L}^{2}}{16} \dot{\theta}\right)=0
\end{gathered}
$$

Now, moment of inertia of rod about the center of the rod is

$$
\mathrm{I}_{\mathrm{C}}=\frac{\mathrm{mL}^{2}}{12}
$$

And moment of inertia about the hinged point is $\mathrm{I}_{0}=\mathrm{I}_{\mathrm{C}}+\mathrm{mr}^{2}=\frac{\mathrm{mL}^{2}}{12}+\mathrm{m}\left(\frac{\mathrm{L}}{4}\right)^{2}=\frac{7}{48} \mathrm{~mL}^{2}$
$\frac{7}{48} \mathrm{~mL}^{2} \ddot{\theta}+\frac{1}{16} \mathrm{CL}^{2} \dot{\theta}+\frac{12}{16} \mathrm{KL}^{2} \theta=0$
As we know that for critical dumping, the root of the quadratic equation should be real and equal.
$\therefore B^{2}-4 A C=0$
$\left(\frac{1}{16} \mathrm{CL}^{2}\right)^{2}-4\left(\frac{7}{48} \mathrm{~mL}^{2}\right)\left(\frac{12}{16} \mathrm{KL}^{2}\right)=0$
$C=\sqrt{4\left(\frac{7}{1} \mathrm{~m}\right)\left(\frac{4}{1} \mathrm{~K}\right)}=4 \sqrt{7 \mathrm{~km}}$
$\therefore \frac{\mathrm{C}}{\sqrt{\mathrm{km}}}=4 \sqrt{7}$
37. The probability that a part manufactured by a company will be defective is 0.05 . If 15 such parts are selected randomly and inspected, then the probability that at least two parts will be defective is $\qquad$ (round off to two decimal places).

Ans. 0.17
Sol. Total no. of parts $=15$
$P[$ defective $(p)]=0.05$
$P[$ non-defective(q)]
$=1-0.05=0.95$
P [at least 2 defective $]=1-[\mathrm{P}($ no
defective $)+\mathrm{P}(1$ defective $)]$
Using the binomial distribution,
P[at least 2 defective]
$=1-\left[{ }^{15} \mathrm{C}_{0} \mathrm{p}^{0} \mathrm{q}^{15}+{ }^{15} \mathrm{C}_{1} \mathrm{p}^{1} \mathrm{q}^{14}\right]$
$=1-\left[0.05^{0} 0.95^{15}\right.$
$\left.+15 \times 0.05 \times 0.95^{14}\right]$
$=1-[0.4639+0.3657]$
$=1-0.8296=0.1704$
38. A four bar mechanism is shown in the figure. The link numbers are mentioned near the links. Input link 2 is rotating anti-clockwise with a
constant angular speed $\omega_{2}$. Length of different links are :

$$
\mathrm{O}_{2} \mathrm{O}_{4}=\mathrm{O}_{2} \mathrm{~A}=\mathrm{L}
$$

$$
\mathrm{AB}=\mathrm{O}_{4} \mathrm{~B} \quad \sqrt{2} \mathrm{~L}
$$

The magnitude of the angular speed of the output link 4 is $\omega_{4}$ at the instant when link 2 makes an angle of $90^{\circ}$ with $\mathrm{O}_{2} \mathrm{O}_{4}$ as shown. The ratio $\frac{\omega_{4}}{\omega_{2}}$ is $\qquad$ (round off to two decimal places).


Ans. 0.79
Sol. Using Instantaneous center method

$\mathrm{I}_{24} \mathrm{I}_{12} \omega_{2}=\mathrm{I}_{24} \mathrm{I}_{41} \omega_{4}$
$\mathrm{I}_{24} \mathrm{I}_{12}=\mathrm{L} \tan 75^{\circ}$ (from $\Delta \mathrm{I}_{12} \mathrm{I}_{23} \mathrm{I}_{24}$ )
$\mathrm{I}_{24} \mathrm{I}_{41}=\mathrm{L}+\mathrm{L} \tan 75^{\circ}$
$\therefore \frac{\omega_{4}}{\omega_{2}}=\frac{\mathrm{I}_{24} \mathrm{I}_{12}}{\mathrm{I}_{24} \mathrm{I}_{41}}=\frac{\mathrm{L} \tan 75^{\circ}}{\mathrm{L}+\mathrm{L} \tan 75^{\circ}}=0.788$
39. Two masses $A$ and $B$ having mass $m_{a}$ and $m_{b}$, respectively, lying in the plane of the figure shown, are rigidly attached to a shaft which revolves about an axis through O perpendicular
to the figure. The radii of rotation of the masses $m_{a}$ and $m_{b}$ are $r_{a}$ and $r_{b}$, respectively. The angle between lines $O A$ and $O B$ is $90^{\circ}$. If $m_{a}=10 \mathrm{~kg}$, $m_{b}=20 \mathrm{~kg}, r_{a}=200 \mathrm{~mm}$ and $\mathrm{r}_{\mathrm{b}}=400 \mathrm{~mm}$, then the balance mass to be placed at a radius of 200 mm is $\qquad$ kg (round off to two decimal places).


Ans. 41.23
Sol. Let the balance mass be m

$$
\sum \mathrm{F}_{\mathrm{x}}=\mathrm{m}_{\mathrm{A}} \mathrm{r}_{\mathrm{A}} \cos 0^{\circ}+\mathrm{m}_{\mathrm{B}} \mathrm{r}_{\mathrm{B}} \cos 90^{\circ}
$$

$+\mathrm{mrcos} \theta=0----(1)$

$$
\begin{equation*}
\sum \mathrm{F}_{\mathrm{y}}=\mathrm{m}_{\mathrm{A}} \mathrm{r}_{\mathrm{A}} \sin 0^{\circ}+\mathrm{m}_{\mathrm{B}} \mathrm{r}_{\mathrm{B}} \sin 90^{\circ} \tag{2}
\end{equation*}
$$

$+\operatorname{mrsin} \theta=0$
From (1),

$$
m r \cos \theta=-\left[m_{A} r_{A} \cos 0^{\circ}+m_{B} r_{B} \cos 90^{\circ}\right]
$$

Or, $\operatorname{mrcos} \theta=-[10 \times 200]=-2000$
From (2),

$$
m r \sin \theta=-\left[\mathrm{m}_{\mathrm{A}} \mathrm{r}_{\mathrm{A}} \sin 0+\mathrm{m}_{\mathrm{B}} \mathrm{r}_{\mathrm{B}} \sin 90^{\circ}\right]
$$

Or, $\operatorname{mrsin} \theta=-[20 \times 400]=-8000$
Using $\left.\sqrt{(m r \cos \theta)^{2}+(m r s i n} \theta\right)^{2}=m r$

$$
=\sqrt{2000^{2}+8000^{2}}=8246.21
$$

Now, since the radius of the balance mass is 200 mm , therefore $\mathrm{m}=8246.21 / 200=41.23 \mathrm{~kg}$
40. A prismatic, straight elastic, cantilever beam is subjected to a linearly distributed transverse load as shown below. If the beam length is $L$, Young's modulus E, and area moment of inertia $I$, the magnitude of the maximum deflection is

A. $\frac{\mathrm{qL}^{4}}{60 E \mathrm{I}}$
B. $\frac{\mathrm{qL}^{4}}{30 \mathrm{EI}}$
C. $\frac{\mathrm{qL}^{4}}{10 \mathrm{EI}}$
D. $\frac{\mathrm{qL}^{4}}{15 E I}$

Ans. B
Sol. The maximum deflection of the cantilever beam subjected to uniformly varying load (UVL) is given by $\frac{q L^{4}}{30 E I}$
41. Given a vector $\overrightarrow{\mathrm{u}}=\frac{1}{3}\left(-y^{3} \hat{\mathrm{i}}+\mathrm{x}^{3} \hat{\mathrm{j}}+\mathrm{z}^{3} \hat{\mathrm{k}}\right)$ and $\hat{n}$ as the unit normal vector to the surface of the hemisphere $\left(x^{2}+y^{2}+z^{2}=1 ; z \geq 0\right)$, the value of integral $\int(\nabla \times \overrightarrow{\mathrm{u}}) \bullet \hat{\mathrm{n}} \mathrm{dS}$ evaluated on the curved surface of the hemisphere $S$ is
A. $п$
B. $\frac{\pi}{3}$
C. $\frac{\pi}{2}$
D. $-\frac{\pi}{2}$

Ans. C
Sol. Hemisphere

$$
\left(x^{2}+y^{2}+z^{2}=1 ; z \geq 0\right)
$$



To find the integration of the given expiration, it is easy if we are using the stoke's theorem. Surface integral will become line integral.
$\int(\nabla \times \overrightarrow{\mathrm{u}}) \cdot \hat{\mathrm{n}} \mathrm{d} S=\int \overrightarrow{\mathrm{u}} \cdot \mathrm{dr}$

Now, putting the value of $u$ in the above equation.

$$
\begin{aligned}
& \int(\nabla \times \overrightarrow{\mathrm{u}}) \cdot \hat{\mathrm{nd}} \mathrm{~d}=\int_{\mathrm{c}} \frac{1}{3}\left(-\mathrm{y}^{3} \mathrm{dx}+\mathrm{x}^{3} \mathrm{dy}+\mathrm{z}^{3} \mathrm{dz}\right) \\
& \quad=\oint_{\mathrm{c}} \frac{1}{3}\left(-\mathrm{y}^{3} \mathrm{dx}+\mathrm{x}^{3} \mathrm{dy}\right) \because \mathrm{Z}=0 \text { at base circle } \\
& =\oint_{\mathrm{c}}\left(\mathrm{y}^{2} \mathrm{dxdy}+\mathrm{x}^{2} \mathrm{dxdy}\right) \text { [using green theorem] } \\
& =\iint\left(\mathrm{y}^{2}+\mathrm{x}^{2}\right) \mathrm{dxdy}
\end{aligned}
$$

Now converting the above equation into polar coordinate
$x^{2}+y^{2}=r^{2}$
$\int_{\theta=0}^{2 \pi} \int_{r=0}^{1} r^{2} r d r d \theta=\int_{\theta=0}^{2 \pi}\left(\int_{r=0}^{1} r^{3} d r\right) d \theta=\frac{\pi}{2}$
42. The thickness of a sheet is reduced by rolling (without any change in width) using 600 mm diameter rolls. Neglect elastic deflection of the rolls and assume that the coefficient of friction at the roll-workpiece interface is 0.05 . The sheet enters the rotating rolls unaided. If the initial sheet thickness is 2 mm , the minimum possible final thickness that can be produced by this process in a single pass is $\qquad$ mm (round off to two decimal places).
Ans. 1.25
Sol. Draft $=(\Delta H)_{\max }=\mu^{2} R$

$$
=0.05^{2} \times 300=0.75 \mathrm{~mm}
$$

Hence, final thickness of the sheet $=2-0.75$ $=1.25 \mathrm{~mm}$
43. A ball of mass 3 kg moving with a velocity of 4 $\mathrm{m} / \mathrm{s}$ undergoes a perfectly-elastic directcentral impact with a stationary ball of mass $m$. After the impact is over, the kinetic energy of the 3 kg ball is 6 J . The possible value ( s ) of m is/are
A. 6 kg only
B. $1 \mathrm{~kg}, 9 \mathrm{~kg}$
C. 1 kg only
D. $1 \mathrm{~kg}, 6 \mathrm{~kg}$

Ans. D
Sol: Conservation of momentum gives $m_{1} u_{1}+m_{2} u_{2}$

$$
\begin{aligned}
& =m_{1} v_{1}+m_{2} v_{2} \\
& m_{1}=3 \mathrm{~kg}, \mathrm{u}_{1}=4 \mathrm{~m} / \mathrm{s}, \mathrm{~m}_{2}=\mathrm{mkg}, \\
& \mathrm{u}_{2}=0
\end{aligned}
$$

Substituting in the above equation,

$$
3 \times 4+\mathrm{m} \times 0=3 \times \mathrm{v}_{1}+\mathrm{m} \times \mathrm{v}_{2}
$$

Or, $12=3 \mathrm{v}_{1}+\mathrm{mv}_{2}------(1)$
For perfectly-elastic impact,

$$
\mathrm{e}=\frac{\mathrm{v}_{2}-\mathrm{v}_{1}}{\mathrm{u}_{1}-\mathrm{u}_{2}}=1
$$

Or, $\mathrm{v}_{2}-\mathrm{v}_{1}=\mathrm{u}_{1}-\mathrm{u}_{2}=4 \mathrm{~m} / \mathrm{s}----$ (2)
Conservation of kinetic energy gives

$$
\frac{1}{2} m_{1} u_{1}^{2}+\frac{1}{2} m_{2} u_{2}^{2}=\frac{1}{2} m_{1} v_{1}^{2}+\frac{1}{2} m_{2} v_{2}^{2}
$$

Or,

$$
\begin{equation*}
\frac{1}{2} 3(4)^{2}+0=6+\frac{1}{2} m_{2} v_{2}^{2} \tag{3}
\end{equation*}
$$

Or, $\mathrm{mv}_{2}^{2}=36$
From equation (2), $\mathrm{v}_{1}=\mathrm{v}_{2}-4$
Substituting in equation (1),

$$
12=3\left(v_{2}-4\right)+\mathrm{mv}_{2}
$$

Or $12=3 \mathrm{v}_{2}-12+\mathrm{mv}_{2}$
Or $\mathrm{v}_{2}(3+\mathrm{m})=24$
Or $\mathrm{v}_{2}=\frac{24}{3+\mathrm{m}}$
Substituting in equation (3)

$$
\begin{gathered}
\mathrm{m}\left(\frac{24}{3+\mathrm{m}}\right)^{2}=36 \\
576 \mathrm{~m}=36(3+\mathrm{m})^{2} \\
\Rightarrow 16 \mathrm{~m}=9+\mathrm{m}^{2}+6 \mathrm{~m} \\
\mathrm{~m}^{2}-10 \mathrm{~m}+9=0
\end{gathered}
$$

On solving, we get

$$
\mathrm{m}=1,9
$$

Therefore, possible values of m are 1 kg and 9 kg .
44. The derivative of $f(x)=\cos (x)$ can be estimated using the
approximation $\mathrm{f}^{\prime}(\mathrm{x})=\frac{\mathrm{f}(\mathrm{x}+\mathrm{h})-\mathrm{f}(\mathrm{x}-\mathrm{h})}{2 \mathrm{~h}}$.
The percentage error is calculated as $\left(\frac{\text { Exact value-Approximate value }}{\text { Exact value }}\right) \times 100$. The percentage error in the derivative of $f(x)$ at $x=$ $\pi / 6$ radian, choosing $h=0.1$ radian, is
A. > $1 \%$ and $<5 \%$
B. $<0.1 \%$
C. $>0.1 \%$ and $<1 \%$
D. $>5 \%$

Ans. C
Sol. $f(x)=\cos (x)$
$\Rightarrow f^{\prime}(x)=-\sin (x)$
$=-\sin (\pi / 6)=-0.5$ (exact value)
Approximate value

$$
\begin{gathered}
=f^{\prime}(x)=\frac{f(x+h)-f(x-h)}{2 h} . \\
f^{\prime}(x)=\frac{\cos (x+h)-\cos x-h}{2 h} \\
\cos (x+h)=\cos x \cosh -\sin x \sinh \\
\cos (x-h)=\cos x \cosh +\sin x \sinh \\
\text { So, }
\end{gathered}
$$

$$
\mathrm{f}^{\prime}(\mathrm{x})=\frac{\cos x \cosh -\sin x \sinh -[\cos x \cosh +\sin x \sinh ]}{2 h}
$$

$$
=\frac{-2 \sin x \sinh }{2 h}=\frac{-\sin x \sinh }{h}
$$

$$
\text { Or, } \mathrm{f}^{\prime}(\mathrm{x})=\frac{-\sin \left(\frac{\pi}{6}\right) \sin (0.1)}{0.1}
$$

$$
=\frac{-0.5 \times 0.0998}{0.1}=-0.4992
$$

So, percentage error
$=\frac{-0.5-(-0.4992)}{-0.5} \times 100=0.166$
45. The aerodynamic drag on a sports car depends on its shape. The car has a drag coefficient of 0.1 with the windows and the roof closed. With the windows and the roof open, the drag coefficient becomes 0.8 . The car travels at 44 $\mathrm{km} / \mathrm{h}$ with the windows and roof closed. For the same amount of power needed to overcome
the aerodynamic drag, the speed of the car with the windows and roof open (round off to two decimal places), is $\qquad$ $\mathrm{km} / \mathrm{h}$ (the density of air and frontal area may be assumed to be constant).

Ans. 22
Sol. The power needed to overcome the aerodynamic drag is given by $\mathrm{F}_{\mathrm{d}} \times \mathrm{V}$ Where, $\mathrm{F}_{\mathrm{d}}$ is the drag force given by

$$
\mathrm{F}_{\mathrm{d}}=1 / 2 \mathrm{C}_{\mathrm{d}} \rho A V^{2}
$$

For the power to remain same in both the cases of windows closed and open, then

$$
\left(F_{d} \times V\right)_{\text {windows closed }}=\left(F_{d} \times V\right)_{\text {windows open }}
$$

Or,
$\left(1 / 2 C_{d} \rho A V^{3}\right)_{\text {windows closed }}=\left(1 / 2 C_{d} \rho A V^{3}\right)_{\text {windows open }}$
Or, $0.1 \times 44^{3}=0.8 \times \mathrm{V}^{3}$
Or $\mathrm{V}=22 \mathrm{~km} / \mathrm{h}$
46. An idealized centrifugal pump (blade outer radius of 50 mm ) consumes 2 kW power while running at 3000 rpm . The entry of the liquid into the pump is axial and exit from the pump is radial with respect to impeller. If the losses are neglected, then the mass flow rate of the liquid through the pump is $\qquad$ kg/s (round off to two decimal places).

Ans. 8.106
Sol. Diameter
$=2 \times$ radius of blade $=2 \times 50=100 \mathrm{~mm}=0.1 \mathrm{~m}$

$$
\mathrm{u}=\frac{\pi \mathrm{DN}}{60}=\frac{\pi \times 0.1 \times 3000}{60}=15.71 \mathrm{~m} / \mathrm{sec}
$$

Now, power $=$ mass flow rate $\times \mathrm{u}^{2}$

$$
\Rightarrow 2000=\dot{\mathrm{m}} \times(15.71)^{2}
$$

Or, $\dot{\mathrm{m}}=8.106$
47. The figure shows a heat engine (HE) working between two reservoirs. The amount of heat
$\left(\mathrm{Q}_{2}\right)$ rejected by the heat engine is drawn by a heat pump (HP). The heat pump receives the entire work output (W) of the heat engine. If temperatures.
$\mathrm{T}_{1}>\mathrm{T}_{3}>\mathrm{T}_{2}$, then the relation between the efficiency ( $\eta$ ) of the heat engine and the coefficient of performance (COP) of the heat pump is

A. $C O P=\eta$
B. $O C P=1+\eta$
C. $C O P=\eta^{-1}$
D. $C O P=\eta^{-1}-1$

## Ans. C

Sol. For heat engine, $\eta=\frac{w}{Q_{1}}$
For Heat engine, $\operatorname{COP}=\frac{\mathrm{Q}_{3}}{\mathrm{~W}}$
Energy balance for Heat engine gives $Q_{2}=Q_{1}-$ W
And energy balance for Heat pump gives $Q_{3}=$ $\mathrm{Q}_{2}+\mathrm{W}=\mathrm{Q}_{1}-\mathrm{W}+\mathrm{W}=\mathrm{Q}_{1}$
Hence, $\operatorname{coP}=\frac{Q_{1}}{W}=\frac{1}{\eta}=\eta^{-1}$
48. Three sets of parallel plates $L M, N R$ and $P Q$ are given in Figures 1, 2 and 3. The view factor $\mathrm{Fu}^{\prime}$ is defined as the fraction of radiation leaving plate I that is intercepted by plate J. Assume that the values of $F_{L m}$ and $F_{N R}$ are 0.8 and 0.4 , respectively. The value of $\mathrm{F}_{\mathrm{PQ}}$ (round off to one decimal place) is $\qquad$ -


Ans. 0.6
Sol. $\mathrm{F}_{\mathrm{PQ}}=\mathrm{F}_{\mathrm{NR}}+\frac{\mathrm{F}_{\mathrm{LM}}-\mathrm{F}_{\mathrm{NR}}}{2}=0.4+\frac{0.8-0.4}{2}=0.6$
49. Hot and cold fluids enter a parallel flow double tube heat exchanger at $100^{\circ} \mathrm{C}$ and $15^{\circ} \mathrm{C}$, respectively. The heat capacity rates of hot and cold fluids are $C_{h}=2000 \mathrm{~W} / \mathrm{K}$ and $\mathrm{Cc}=1200$ W/K, respectively. If the outlet temperature of the cold fluid is $45^{\circ} \mathrm{C}$, the $\log$ mean temperature difference (LMTD) of the heat exchanger is $\qquad$ $K$ (round off to two decimal places).
Ans.
Sol. Given data
$\mathrm{T}_{\mathrm{h}_{\mathrm{i}}}=100^{\circ} \mathrm{C}, \mathrm{T}_{\mathrm{c}_{\mathrm{i}}}=15^{\circ} \mathrm{C}, \mathrm{T}_{\mathrm{c}_{\mathrm{e}}}=45^{\circ} \mathrm{C}$
$\mathrm{C}_{\mathrm{h}}=2000 \mathrm{~W} / \mathrm{K}, \mathrm{C}_{\mathrm{c}}=1200 \mathrm{~W} / \mathrm{K}$
Using the energy balance equation for the heat exchanger

$$
\begin{gathered}
\mathrm{C}_{\mathrm{h}}\left(\mathrm{~T}_{\mathrm{h}_{\mathrm{i}}}-\mathrm{T}_{\mathrm{h}_{\mathrm{e}}}\right)=\mathrm{C}_{\mathrm{c}}\left(\mathrm{~T}_{\mathrm{c}_{\mathrm{e}}}-\mathrm{T}_{\mathrm{c}_{\mathrm{i}}}\right) \\
\Rightarrow 2000\left(100-\mathrm{T}_{\mathrm{h}_{\mathrm{e}}}\right)=1200(45-15) \\
\Rightarrow \mathrm{T}_{\mathrm{h}_{\mathrm{e}}}=82^{\circ} \mathrm{C}
\end{gathered}
$$



$$
\begin{gathered}
\Delta \mathrm{T}_{\mathrm{i}}=\mathrm{T}_{\mathrm{h}_{\mathrm{i}}}-\mathrm{T}_{\mathrm{c}_{\mathrm{i}}}=100-15=85^{\circ} \mathrm{C} \\
\Delta \mathrm{~T}_{\mathrm{e}}=\mathrm{T}_{\mathrm{h}_{\mathrm{e}}}-\mathrm{T}_{\mathrm{c}_{\mathrm{e}}}=82-45=37^{\circ} \mathrm{C} \\
\text { LMTD }=\frac{\Delta \mathrm{T}_{\mathrm{i}}-\Delta \mathrm{T}_{\mathrm{e}}}{\ln \frac{\Delta \mathrm{~T}_{\mathrm{i}}}{\Delta \mathrm{~T}_{\mathrm{e}}}}=\frac{85-37}{\ln \frac{85}{37}}=57.71^{\circ} \mathrm{C}
\end{gathered}
$$

50. A differential equation is given as $x^{2} \frac{d^{2} y}{d x^{2}}-2 x \frac{d y}{d x}+2 y=4$.

The solution of the differential equation in terms of arbitrary constants $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ is
A. $\mathrm{y}=\frac{\mathrm{C}_{1}}{\mathrm{x}^{2}}+\mathrm{C}_{2} \mathrm{x}+4$
B. $y=C_{1} x^{2}+C_{2} x+4$
C. $y=C_{1} x^{2}+C_{2} x+2$
D. $y=\frac{C_{1}}{x^{2}}+C_{2} x+2$

Ans. D
Sol. Given differential equation

$$
(D(D-1)-2 D+2) y=4 \text { where } x=e^{z}
$$

Auxiliary equation,

$$
(D-2)(D-1)=0
$$

D $=1,2$
$C F=C_{1} \mathrm{e}^{1 \mathrm{x}}+\mathrm{C}_{2} \mathrm{e}^{2 \mathrm{x}}$
$\operatorname{PI}=\frac{4}{(D-2)(D-1)}=2$
$\mathrm{Y}=\mathrm{CF}+\mathrm{PI}$
$\therefore \mathrm{y}=\mathrm{C}_{1} \mathrm{e}^{1 \mathrm{x}}+\mathrm{C}_{2} \mathrm{e}^{2 \mathrm{x}}+2$
51. Water flowing at the rate of $1 \mathrm{~kg} / \mathrm{s}$ through a system is heated using an electric heater such that the specific enthalpy of the water increases by $2.50 \mathrm{~kJ} / \mathrm{kg}$ and the specific entropy increases by $0.007 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$. The power input to the electric heater is 2.50 kW . There is no other work or heat interaction between the system and the surroundings. Assuming an ambient temperature of 300 K , the irreversibility rate of the system is
$\qquad$ kW (round off to two decimal places).

Ans. 2.1

Sol. The entropy generation $\left(S_{g e n}\right)$ is given by rise in entropy rise of the system (Since no heat interaction is involved)
So, entropy generation ( $\mathrm{S}_{\mathrm{gen}}$ )
$=\dot{\mathrm{m}} \times$ specific entropy $=1 \times 0.007=0.007 \mathrm{~kW} / \mathrm{K}$
Now, according to Gouy Stodola theorem,
Irreversibility =

$$
\mathrm{T}_{\mathrm{o}} \times \mathrm{S}_{\text {gen }}=300 \times 0.007=2.1 \mathrm{~kW}
$$

52. In an orthogonal machining with a single point cutting tool of rake angle $10^{\circ}$, the uncut chip thickness and the chip thickness are 0.125 mm and 0.22 mm , respectively. Using Merchant's first solution for the condition of minimum cutting force, the coefficient of friction at the chip-tool interface is $\qquad$ (round off to two decimal places).
Ans. 0.74
Sol. uncut chip thickness
( t$)=0.125 \mathrm{~mm}$
Cut chip thickness $\left(\mathrm{t}_{\mathrm{c}}\right)=0.22 \mathrm{~mm}$
So, ship thickness ratio
$=\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{c}}}=\frac{0.125}{0.22}=0.568$
Rake angle $(\alpha)=10^{\circ}$
$\tan \phi=\frac{r \cos \alpha}{1-r \sin \alpha}=\frac{0.568 \times \cos 10}{1-0.568 \times \sin 10}=0.6206$
Or shear angle $(\phi)=31.82^{\circ}$
According to Merchant's theory, we have

$$
2 \phi+\beta-\alpha=\frac{\pi}{2}
$$

So, $\beta=90^{\circ}-63.64+10=36.36^{\circ}$
Now,

$$
\tan \beta=\mu \text { (coefficient of friction) }
$$

Therefore, coefficient of friction $=\tan 36.36^{\circ}=$ 0.74
53. An air standard Otto cycle has thermal efficiency of 0.5 and the mean effective pressure of the cycle is 1000 kPa . For air,
assume specific heat ratio $Y=1.4$ and specific gas constant $R=0.287 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$, If the pressure and temperature at the beginning of the compression stroke are 100 kPa and 300 K , respectively, then the specific net work output of the cycle is $\qquad$ $\mathrm{kJ} / \mathrm{kg}$ (round off to two decimal places).
Ans. 708.8
Sol.


Given data
$P_{1}=100 \mathrm{kPa}, \mathrm{T}_{1}=300 \mathrm{~K}$, mep
$=1000 \mathrm{kPa}, \gamma=1.4$, efficiency $(\eta)=0.5, \mathrm{R}=$ $0.287 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$

Using $\mathrm{P}_{1} \mathrm{~V}_{1}=\mathrm{RT}_{1}$
$\mathrm{V}_{1}=\frac{0.287 \times 300}{100}=0.861 \mathrm{~m}^{3} / \mathrm{kg}$
Efficiency $(\eta)=1-\frac{1}{(r)^{\gamma-1}}$ where, $r$ is the compression ratio $=\mathrm{V}_{1} / \mathrm{V}_{2}$

$$
\begin{aligned}
& 1-\frac{1}{(r)^{\gamma-1}}=0.5 \\
& 1-\frac{1}{(r)^{1.4-1}}=0.5
\end{aligned}
$$

Or, r $=5.657$
$\mathrm{V}_{1} / \mathrm{V}_{2}=5.657$
$\Rightarrow V_{2}=0.1522 \mathrm{~m}^{3} / \mathrm{kg}$
Swept volume $=\mathrm{V}_{1}-\mathrm{V}_{2}=0.861-0.1522=$ $0.7088 \mathrm{~m}^{3} / \mathrm{kg}$

Went $=$ Swept volume $\times$ mep
$=0.7088 \times 1000=708.8 \mathrm{~kJ} / \mathrm{kg}$
54. A through hole is drilled in an aluminum alloy plate of 15 mm thickness with a drill bit of diameter 10 mm , at a feed of $0.25 \mathrm{~mm} / \mathrm{rev}$ and a spindle speed of 1200 rpm. If the specific energy required for cutting this material is 0.7 N.m/mm ${ }^{3}$, the power required for drilling is
$\qquad$ W (round off to two decimal places).
Ans. 274.89
Sol. $\operatorname{MRR}=\frac{\pi \mathrm{d}^{2}}{4} \times$ feed $\times$ rev per sec $=\frac{\pi(10)^{2}}{4} \times 0.25 \times$
$\frac{1200}{60}=125 \pi \mathrm{~mm}^{3}$
Specific energy $=0.7 \mathrm{Nm} / \mathrm{mm}^{3}$
So, power required for drilling
$=0.7 \times 125 \pi=274.89 \mathrm{~W}$
55. A horizontal cantilever beam of circular crosssection, length 1.0 m
and flexural rigidity $\mathrm{EI}=200 \mathrm{~N} . \mathrm{m}^{2}$ is subjected to an applied moment $M_{A}=1.0 \mathrm{~N}-\mathrm{m}$ at the free end as shown in the figure. The magnitude of the vertical deflection of the free end is
$\qquad$ mm (round off to one decimal place).


Ans. 2.5
Sol. When the cantilever beam is subjected to moment at the free end, then the deflection at the free end is given by

$$
y_{(\text {free end) }}=\frac{M L^{2}}{2 E I}
$$

So,

$$
y_{(\text {free end })}=\frac{1 \times 1^{2}}{2 \times 200}=2.5 \times 10^{-3} \mathrm{~m}=2.5 \mathrm{~mm}
$$

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