GATE 2020

Set-2

Mechanical Engineering

Questions & Solutions

SECTION: GENERAL APTITUDE

1. While I agree______his proposal this time, I do not often agree_____him.

A. to, to B. with, with

C. with, to D. to, with

Ans. D

Sol. While I agree to his proposal this time, I do not often agree with him.

2. The recent measures to improve the output would____the level of production to our satisfaction.

A. speed B. decrease

C. increase D. equalise

Ans. C

Sol. The recent measures to improve the output would increase the level of production to our satisfaction.

3. Select the word that files the analogy:

White: Whitening:: Light: _______

A. Lighting B. Enlightening

C. Lightening D. Lightning

Ans. D

Sol. White: Whitening:: Light:

Lightning

4. In one of the greatest innings ever seen in 142 years of Test history, Ben Stokes upped the tempo in a five-and-a-half hour long stay of 219 balls including 11 fours and 8 sixes that saw him finish on a 135 not out as England squared the five-match series.

Based on their connotations in the given passage, which one of the following meanings does not match?

A. tempo = enthusiasm

B. squared = lost

C. saw = resulted in

D. upped = increased

Ans. B

Sol. Squared means drawn

(it was draw)

5. There are five levels {P,Q,R,S,T} in a linear supply chain before a product reaches customers, as shown in the figure.

P ⇨ Q ⇨ R ⇨ S ⇨ T ⇨ Customers

At each of the five levels, the price of the product is increased by 25%. If the product is produced at level P at the cost of Rs. 120 per unit, what is the price paid (in rupees) by the customers?

A. 234.38 B. 292.96

C. 187.50 D. 366.21

Ans. D

Sol. $P \to Q \to R \to S \to T \to$ Customers

Price paid by customer $= 120 \times (1.25)^5$

$= 366.21$

6. Climate change and resilience deal with two aspects – reduction of non-renewable energy resources and reducing vulnerability of climate change aspects. The terms 'mitigation' and 'adaption' are used to refer to these aspects, respectively.

Which of the following assertions is best supported by the above information?

A. Mitigation deals with

consequences of climate change.

B. Mitigation deals with actions taken to reduce the use of fossil fuels.

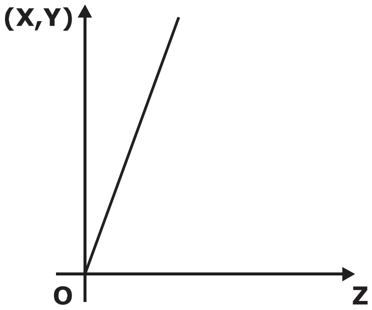
C. Adaptation deals with causes of climate change.

D. Adaptation deals with actions taken to combat green-house gas emissions.

Ans. B

Sol. Mitigation deals with actions taken to reduce the use of fossil fuels.

7. An engineer measures THREE quantities X, Y and Z in an experiment. She finds that they follow a relationship that is represented in the figure below: (the product of X and Y linearly varies with Z).



Then, which of the following statements is FALSE?

A. For fixed X; Z is proportional to Y

B. For fixed Z; X is proportional to Y

C. XY/Z is constant

D. For fixed Y; X is proportional to Z

Ans. B

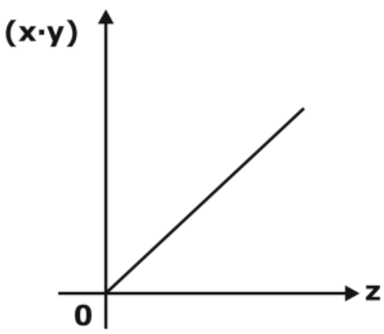
Sol. Line passes through origin

$x \cdot y = mZ$

Where m = slope of line.

For Constant Z

$x = \frac{1}{y}$



8. It was estimated that 52 men can complete a strip in a newly constructed highway connecting cities P and Q in 10 days. Due to an emergency, 12 men were sent to another project. How many number of days, more than the original estimate, will be required to complete the strip?

A. 3 days B. 10 days

C. 5 days D. 13 days

Ans. A

Sol. Since MD = Constant

Where M = No. of men

D = No. of days for work

Initially $M_1 = 52$

$D_1 = 10$ days.

Due to emergency, 12 men were sent. Thus, number of men remaining.

$M_2 = 52 - 12 = 40$

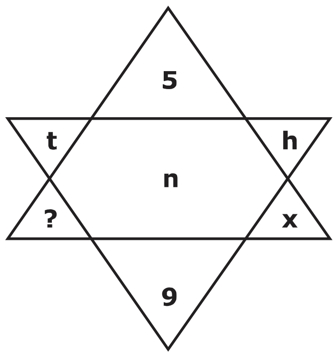
Now $52 \times 10 = 40 \times D_2$

$D_2 = 13$ days

No. of days, more than original estimate = 13-10

= 3 days

9. Find the missing element in the following figure.



A. y B. e

C. w D. d

Ans. D

Sol. Assume n = 4

5 + 4 = 9

Similarly, T = 20

x = t + n = 20 + 4

x = 24

similarly apply for the H = 8

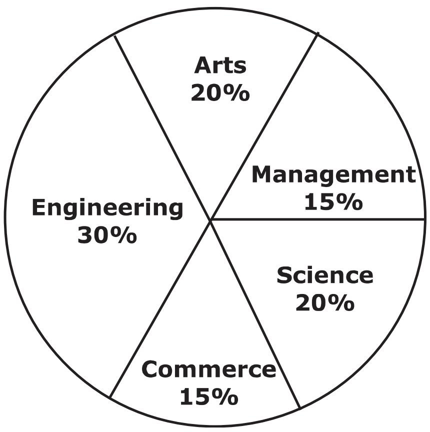
unknown + 4 = H

unknown + 4 = 8

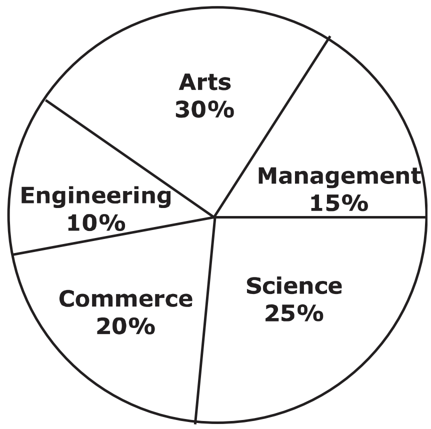
unknown = 4 = D

10. The two pie-charts given below show the data of total students and only girls registered in different streams in a university. If the total number of students registered in the university is 5000, and the total number of the registered girls is 1500; then the ratio of boys enrolled in Arts to the girls enrolled in Management is ____.

Percentage of students enrolled in different streams in a University



Percentage of girls enrolled in different streams



A. 11: 9 B. 9: 22

C. 2: 1 D. 22: 9

Ans. D

Sol. Total no. of students in the university = 5000

Total no. of girls = 1500

$(Boys)_{arts} = 0.2 \times 5000 - 0.3$

$\times 1500$

$= 550$

$(Girls)_{management} = 0.15 \times 1500$

$= 225$

$$\frac{(Boys)_{Arts}}{(Girls)_{management}} = \frac{550}{225} = \frac{22}{9}$$

= 22:9

MECHANICAL ENGINEERING

1. For an air-standard Diesel cycle,

A. heat addition is at constant pressure and heat rejection is at constant pressure

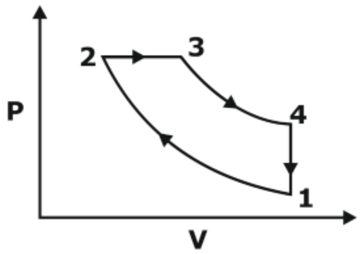
B. heat addition is at constant volume and heat rejection is at constant pressure

C. heat addition is at constant volume and heat rejection is at constant volume

D. heat addition is at constant pressure and heat rejection is at constant volume

Ans. D

Sol. Air standard diesel cycle



Process 1-2: Isentropic compression

Process 2-3: Constant pressure heat addition

Process 3-4: Isentropic expansion

Process 4-1: Constant volume heat rejection.

2. A machine member is subjected to fluctuating stress $\sigma = \sigma_0 \cos(8\pi t)$. The endurance limit of the material is 350 MPa. If the factor of safety used in the design is 3.5 then the maximum allowable value of σ_0 is ________ MPa (round off to 2 decimal places).

Ans. 100 MPa

Sol. Fluctuating stress (σ)

$=\sigma_0 \cos(8\pi t)$

Endurance strength (σ_e)

= 350 MPa.

FOS = 3.5

$\sigma_{max} = \sigma \times (1) = \sigma_0$

$\sigma_{min} = \sigma_0 \times (-1) = -\sigma_0$

Mean stress $(\sigma_m) = \dfrac{\sigma_{max} + \sigma_{min}}{2} = 0$

Amplitude stress $(\sigma_a) = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \sigma_0$

Now $\dfrac{\sigma_m}{S_{yt}} + \dfrac{\sigma_a}{S_e} = \dfrac{1}{Fos}$

$\dfrac{0}{S_{yt}} + \dfrac{\sigma_0}{350} = \dfrac{1}{3.5}$

$\sigma_0 = 100$ MPa

Thus, $\sigma_{max} = 100$ MPa

3. The values of enthalpies at the stator inlet and rotor outlet of a hydraulic turbo machine stage are h_1 and h_3 respectively. The enthalpy at the stator outlet (or, rotor inlet) is h_2. The condition $(h_2 - h_1) = (h_3 - h_2)$ indicates that the degree of reaction of this stage is

A. 100%

B. 75%

C. 50%

D. zero

Ans. C

Sol. Degree of reaction (R) $= \dfrac{\text{Enthalpy drop in rotor}}{\text{Total enthalpy drop}}$

$R = \dfrac{h_3 - h_2}{(h_3 - h_2) + (h_2 - h_1)}$

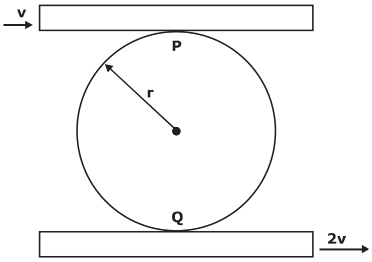
Given $h_2 - h_1 = h_3 - h_2$

Thus $R = \dfrac{(h_3 - h_2)}{2(h_3 - h_2)}$

R = 0.50

R = 50%

4. A circular disk of radius r is confirmed to roll without slipping at P and Q as shown in the figure.



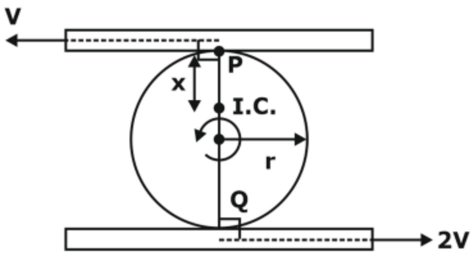
If the plates have velocities as shown, the magnitude of the angular velocity of the disk is

A. $\frac{v}{r}$ B. $\frac{v}{2r}$

C. $\frac{3v}{2r}$ D. $\frac{2v}{3r}$

Ans. C

Sol.



Instantaneous centre will be at intersection of perpendiculars of velocity vector.

Let IC be at a distance = x from point P.

Thus, $\frac{V}{x} = \frac{2V}{2r - x}$

$2Vr - Vx = 2Vx$

$2Vr = 3Vx$

$x = \frac{2r}{3}$

Thus, $V = x\omega$

$V = \frac{2r}{3}\omega$

$\omega = \frac{3V}{2r}$

5. Which one of the following statements about a phase diagram is INCORRECT?

A. It gives information on transformation rates

B. Solid solubility limits are depicted by it

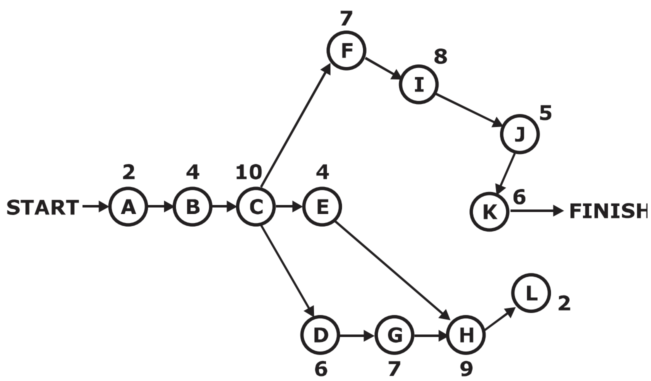
C. Relative amount of different phases can be found under given equilibrium conditions

D. It indicates the temperature at which different phases start to melt

Ans. A

Sol. Phase diagram: does not give any information on transformation rates. The transformation rates are observed on temperature, time and transformation rates.

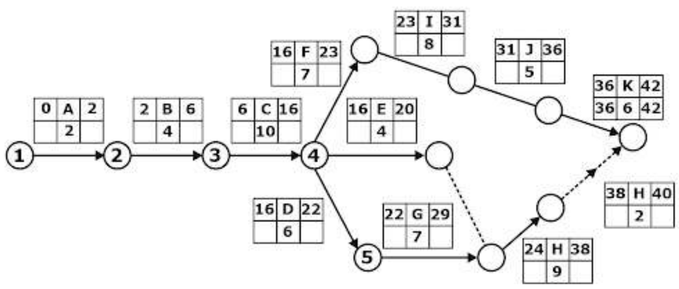
6. Consider the following network of activities, with each activity named A-L, illustrated in the nodes of the network.



The number of hours required for each activity is shown alongside the nodes. The slack on the activity L, is _______ hours.

Ans. 2

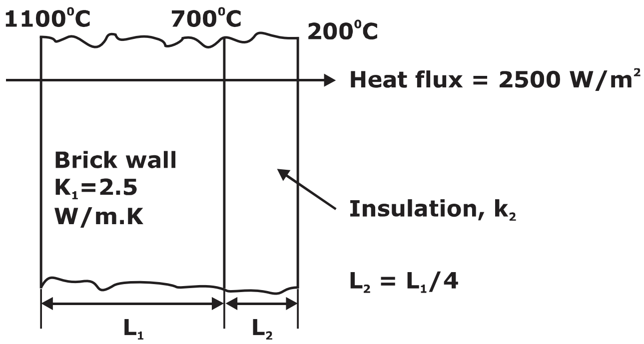
Sol.



Required Slack for activity

L = 2hours

7. In a furnace, the inner and outer sides of the brick wall (k_1 = 2.5 W/m.K) are maintained at 1100°C and 700°C, respectively as shown in figure.

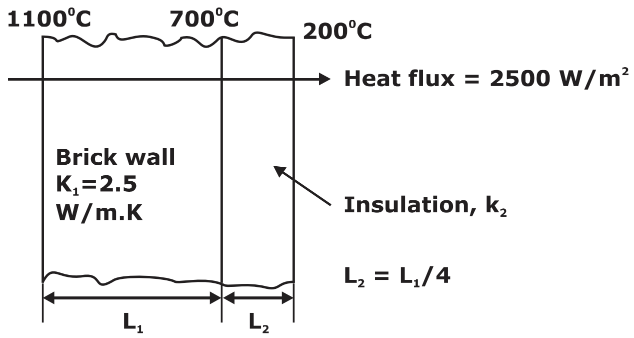


The brick wall is covered by an insulating material of thermal conductivity k_2. The thickness of the insulation is $1/4^{th}$ of the thickness of the brick wall. The outer surface of the insulation is at 200°C. The heat flux through the composite walls is 2500 W/m^2.

The value of k_2 is ________ W/m.K (round off to one decimal place).

Ans. 0.5

Sol.



Heat flux will be same through both the brick wall and insulation.

Thus, $Q = \frac{k_1(1100-700)}{L_1} = \frac{k_2(700-200)}{L_2}$

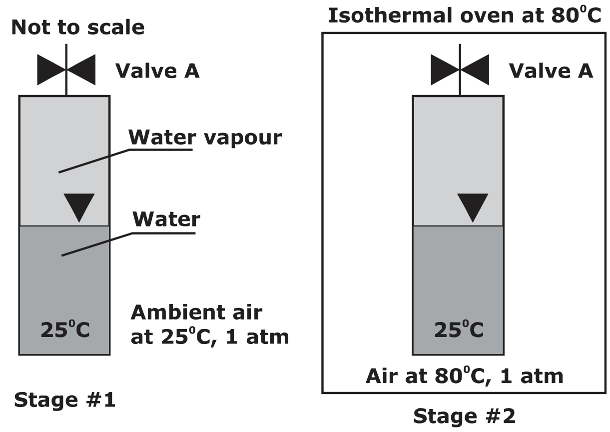
Given $L_2 = \frac{L_1}{4}$

$\frac{k_1 \times 400}{L_1} = \frac{k_2(500)\times 4}{L_1}$

$2.5 \times 400 = 2000 \times k_2$

$k_2 = \frac{1000}{2000} = 0.5\ W/m-k$

8. A closed vessel contains pure water, in thermal equilibrium with its vapour at 25°C (Stage #1), as shown.



The vessel in this stage is then kept inside an isothermal oven which is having an atmosphere of hot air maintained at 80°C. The vessel exchanges heat with the oven atmosphere and attains a new thermal equilibrium (Stage #2). If the Valve A is now opened inside the oven, what will happen immediately after opening the valve?

A. Hot air will go inside the vessel through Valve A

B. Water vapour inside the vessel will come out of the Valve A

C. All the vapour inside the vessel will immediately condense

D. Nothing will happen – the vessel will continue to remain in equilibrium

Ans. A

Sol. The vapour pressure will reach 1 atm when temperature is 100°C. Hence at 80°C also the pressure will be the than 1 atm 80 when valve is opened air will enter the valve.

The Valve A is now opened inside the oven Hot air will go inside the vessel through Valve A.

9. Let $I = \int_{x=0}^{1}\int_{y=0}^{x^2} xy^2\,dy\,dx$. Then, I may also be expressed as

A. $\int_{y=0}^{1}\int_{x=0}^{\sqrt{y}} yx^2\,dx\,dy$

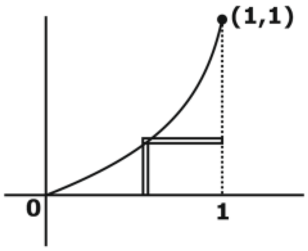
B. $\int_{y=0}^{1}\int_{x=0}^{\sqrt{y}} xy^2\,dx\,dy$

C. $\int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} xy^2\,dx\,dy$

D. $\int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} yx^2\,dx\,dy$

Ans. C

Sol.



$0 \le y \le x^2$ (This is represented by vertical stripe)

$0 \le x \le 1$

After change of order integration

$\sqrt{y} \le x \le 1$

$0 \le y \le 1$

$$I = \int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} xy^2\,dx\,dy$$

10. The number of qualitatively distinct kinematic inversions possible for a Grashof chain with four revolute pairs is

A. 2 B. 1

C. 3 D. 4

Ans. C

Sol. The number of qualitatively distinct kinematic inversions possible for a Grashof chain

1. Double crank mechanism
2. Crank-rocker mechanism
3. Double rocker mechanism

11. In Materials Requirement Planning, if the inventory holding cost is very high and the setup cost is zero, which one of the following lot sizing approaches should be used?

A. Economic Order Quantity

B. Lot-for-Lot

C. Fixed Period Quantity, for 2 periods

D. Base Stock Level

Ans. D

Sol. 1. MRP system, with very high inventory holding cost and the setup cost zero is base stock level system. A base stock policy is appropriate when the cost of placing orders is zero or Negligible.

2. Lot for Lot: There is never any inventory thus holding cost is zero.

3. Economic order quantity (EOQ) system and fixed period quantity, for 2 periods both have some ordering cost.

12. The sum of two normally distributed random variables X and Y is

A. normally distributed, only if X and Y have the same mean

B. normally distributed, only if X and Y are independent

C. always normally distributed

D. normally distributed, only if X and Y have the same standard deviation

Ans. C

Sol. The sum of two Normally distributed functions is always normally distributed.

$X = f_N(\mu_X, \sigma_x^2)$

$Y = f_N(\mu_Y, \sigma_y^2)$

Where subscript N stands for normally distribution.

M_X, μ_Y are means and σ_x^2, σ_y^2 are variances of function X and Y respectively.

$Z = X + Y = f_N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_y^2)$

13. Which of the following conditions is used to determine the stable equilibrium of all partially submerged floating bodies?

A. Metacentre must be at a higher level than the centre of gravity

B. Centre of buoyancy must be above the centre of gravity

C. Metacentre must be at a lower level than the centre of gravity

D. Centre of buoyancy must be below the centre of gravity

Ans. A

Sol. Condition of stability for partially submerged body, metacentre should always lie above centre of gravity.

14. The process, that uses a tapered horn to amplify and focus the mechanical energy for machining of glass, is

A. electrical discharge machining

B. ultrasonic machining

C. abrasive jet machining

D. electrochemical machining

Ans. B

Sol. The process that uses a tapered horn to amplify and focus the mechanical energy for machining glass is ultrasonic machining.

15. A bolt head has to be made at the end of a rod of diameter d = 12 mm by localized forging (upsetting) operation. The length of the unsupported portion of the rod is 40 mm. To avoid buckling of the rod, a closed forging operating has to be performed with a maximum die diameter of _______ mm.

Ans. 18mm

Sol. Rod diameter (d) = 12 mm.

Unsupported Length (l) = 40 mm

The diameter of upset made is not more than 1.5 times bar diameter. If this is kept more than 1.5d, the buckling will be excessive, and stock will fold in.

Thus, maximum die diameter (D) = 1.5 × 12 = 18 mm.

16. Two plates, each of 6 mm thickness, are to be butt-welded. Consider the following processes and select the correct sequence in increasing order of size of the heat affected zone.

1. Arc welding
2. MIG welding
3. Laser beam welding
4. Submerged arc welding

A. 4-3-2-1 B. 3-2-4-1

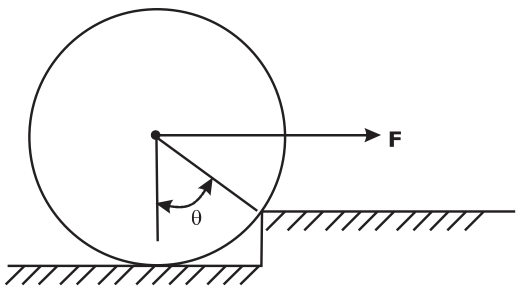
C. 1-4-2-3 D. 3-4-2-1

Ans. B

Sol. The correct sequence in increasing order of size of the heat affected zone:

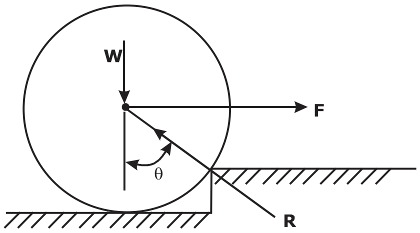
Laser beam welding < MIG welding < Submerged Arc welding < Arc welding.

17. An attempt is made to pull a roller of weight W over a curb (step) by applying a horizontal force F as shown in the figure.

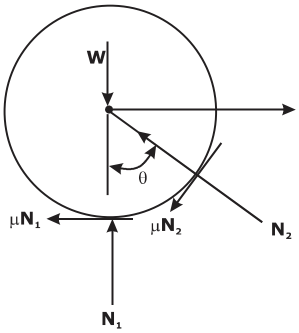


The coefficient of static friction between the roller and the ground (including the edge of the step) is μ. Identify the correct free body diagram (FBD) of the roller when the roller is just about to climb over the step.

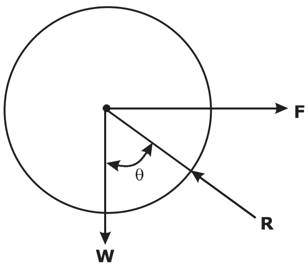
A.

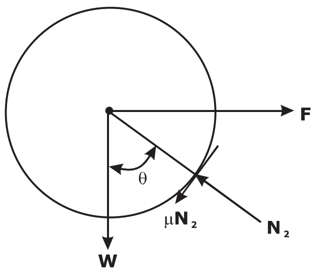


B.



C.

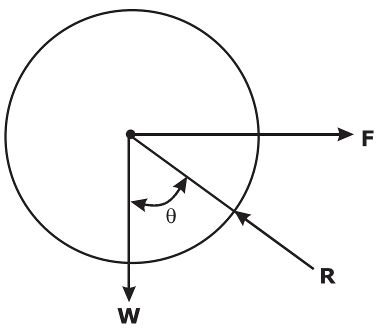




D.

Ans. C

Sol. Static friction at horizontal surface is zero because it is the moment of lifting the surface. At the point of tip of the surface whole roller is going to state of pure rolling. So only normal reaction is come.



18. A matrix P is decomposed into its symmetric part S and skew symmetric part V. If

$$S = \begin{pmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{pmatrix},\ V = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{pmatrix},$$

then matrix P is

A. $\begin{pmatrix} -4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

B. $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$

C. $\begin{pmatrix} -2 & 9/2 & -1 \\ -1 & 81/4 & 11 \\ -2 & 45/2 & 73/4 \end{pmatrix}$

D. $\begin{pmatrix} 4 & -6 & 1 \\ -2 & -3 & 0 \\ -5 & -7 & -2 \end{pmatrix}$

Ans. B

Sol. Matrix P is decomposed into its symmetric part S and skew symmetric part V if:-

$$S = \begin{bmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{bmatrix},\ V = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{bmatrix}$$

$$S = \frac{1}{2}\left(P + P^T\right)$$

$$V = \frac{1}{2}\left(P - P^T\right)$$

S + V = P

$$P = \begin{bmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{bmatrix}$$

19. A beam of negligible mass is hinged at support P and has a roller support Q as shown in the figure.

Ans. B

Sol. Matrix P is decomposed into its symmetric part S and skew symmetric part V if:-

$$S = \begin{bmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{bmatrix}, V = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{bmatrix}$$

$$S = \frac{1}{2}\left(P + P^T\right)$$

$$V = \frac{1}{2}\left(P - P^T\right)$$

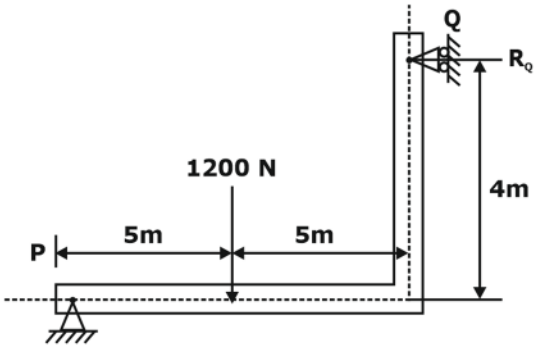
S + V = P

$$P = \begin{bmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{bmatrix}$$

A point load of 1200 N is applied at point R. The magnitude of the reaction force at support Q is ________ N.

Ans. 1500

Sol.



$$\frac{\Sigma M_p = 0}{1200 \times 5}$$

$R_Q = 0$

$R_Q = 300 \times 5$

$R_Q = 1500$ N

20. The solution of $\frac{d^2y}{dt^2} - y = 1$, which additionally satisfies

$y\big|_{t=0} = \frac{dy}{dt}\Big|_{t=0} = 0$ in the Laplace s-domain is

A. $\frac{1}{s-1}$

B. $\frac{1}{s(s-1)}$

C. $\frac{1}{s(s+1)}$

D. $\frac{1}{s(s+1)(s-1)}$

Ans. D

Sol. $\frac{d^2y}{dt^2} - y = 1,$

and $y\big|_{t=0} = \frac{dy}{dt}\Big|_{t=0} = 0$

$$L\left\{\frac{d^2y}{dt^2}\right\} - L(y) = \frac{1}{S}$$

$$S^2F(S) - Sf(0) - f'(0) - F(s) = \frac{1}{S}$$

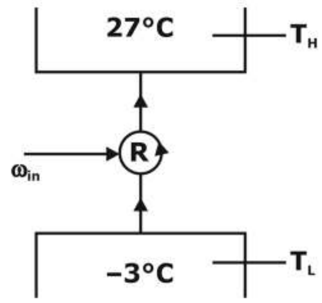
$$F(s)\{(S-1)(S+1)\} = \frac{1}{S}$$

$$F(S) = \frac{1}{S(S-1)(S+1)}$$

21. If a reversed Carnot cycle operates between the temperature limits of 27°C and −3°C, then the ratio of the COP of a refrigerator to that of a heat pump (COP of refrigerator/COP of heat pump) based on the cycle is _________ (round off to 2 decimal places).

Ans. 0.9

Sol.



Reversed correct cycle:

$$(COP)_{Ref} = \frac{T_L}{T_H - T_L}$$

$$= \frac{270}{300 - 270}$$

$(COP)_{Ref} = 9$

Since $(COP)_{Ref} + 1 = (COP)_{H.P}$

$(COP)_{H.P} = 9 + 1 = 10$

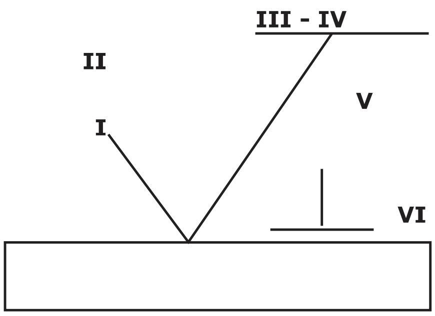
$$\frac{\text{COP of refrigerator}}{\text{COP of Heat pump}} = \frac{9}{10}$$

= 0.9

22. The figure below shows a symbolic representation of the surface texture in a perpendicular lay orientation with indicative values (I through VI) marking the various specifications whose definitions are listed below.

P: Maximum Waviness Height (mm); Q: Maximum Roughness Height (mm);

R: Minimum Roughness Height (mm); S: Maximum Waviness Width (mm); T: Maximum Roughness Width (mm); U: Roughness Width Cutoff (mm).



The correct match between the specifications and the symbols (I to VI) is

A. I-Q, II-U, III-R, IV-T, V-S, VI-P

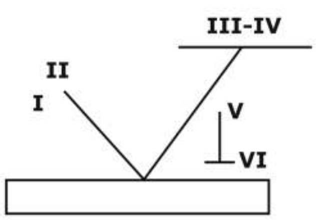
B. I-R, II-Q, III-P, IV-S, V-U, VI-T

C. I-R, II-P, III-U, IV-S, V-T, VI-Q

D. I-U, II-S, III-Q, IV-T, V-R, VI-P

Ans. B

Sol.



I = minimum roughness height (mm)

II = Maximum roughness height (mm)

III = Maximum waviness height (mm)

IV = Maximum waviness width (mm)

V = Roughness width cut off (mm)

VI = Maximum Roughness width (mm)

23. The equation of motion of a spring-mass-damper system is given by

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 9x = 10\sin(5t).$$

The damping factor for the system is

A. 0.5 B. 2

C. 0.25 D. 3

Ans. A

Sol. $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 9x = 10\sin(5t)$

Damping factor (ξ) =?

k = 9, c = 3, m = 1,

$c = 2\xi\sqrt{km}$

$3 = 2\xi\sqrt{1 \times 9}$

$\xi = \frac{3}{6} = 0.5$

24. In the space above the mercury column in a barometer tube, the gauge pressure of the vapour is

A. zero

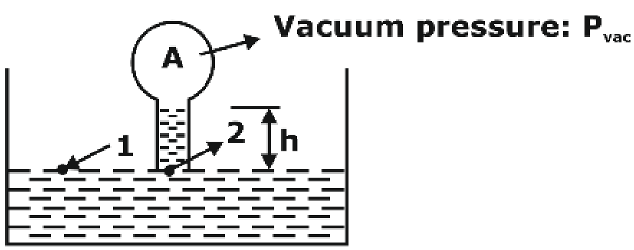
B. positive, but less than one atmosphere

C. positive, but more than one atmosphere

D. negative

Ans. D

Sol.



Absolute pressure at point

1 = absolute pressure of point 2

$(P_{abs})_1 = (P_{abs})_2$

$P_{atm} + 0 = P_{atm} + P_A + c P_A + \rho gh$

$P_A = -\rho gh$

So –ve gauge pressure.

25. Let I be a 100-dimensional identity matrix and E be the set of its distinct (no value appears more than once in E) real eigenvalues. The number of elements in E is _________.

Ans. [1]

Sol. $I = \begin{bmatrix} 1 & 0 & 0 & \cdots 0 \\ 0 & 1 & 0 & \cdots 0 \\ \vdots & & & \\ 0 & 0 & 0 & \cdots 1 \end{bmatrix}_{100\times100}$

Eigen values: 1

E= {0}

No. of different eigen values = 1

26. There are two identical shaping machines S_1 and S_2. In machine S_2, the width of the workpiece is increased by 10% and the feed is decreased by 10%, with respect to that of S_1. If all other conditions remain the same then the ratio of total time per pass in S_1 and S_2 will be ________ (round off to one decimal place).

Ans. 0.818

Sol. Two shaping machines S_1 and S_2:

Machine S_1 Machine S_2

Width = ω_1 with $(\omega_2) = 1.1\omega_1$

Feed = f_1 feed $(f_2) = 0.1f_1$

Let quick return ratio = R

$$(t_m)_1 = \frac{L}{V} \times \frac{\omega_1}{f_1}(1+R)$$

$$(t_m)_2 = \frac{L}{V} \times \frac{\omega_2}{f_2}(1+R)$$

$$\frac{(tm)_1}{(tm)_2} = \frac{\frac{LV}{V} \times \frac{\omega_1}{f_1}(1+R)}{\frac{L}{V} \times \frac{\omega_2}{f_2}(1+R)}$$

$$= \frac{f_2 \times \omega_1}{f_1 \times \omega_1}$$

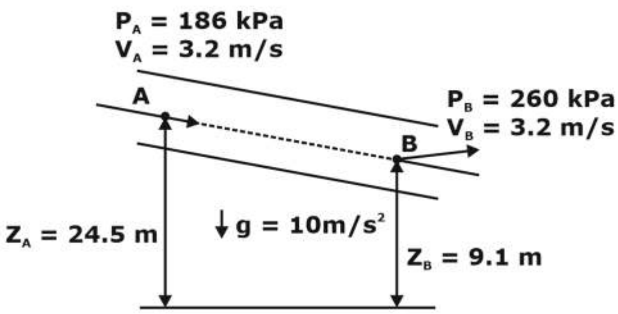
$$= \frac{0.9f_1 \times \omega_1}{f_1 \times 1.1\omega_1}$$

$$\frac{(tm)_1}{(tm)_2} = 0.818 = 0.82$$

27. Water (density 1000 kg/m^3) flows through an inclined pipe of uniform diameter. The velocity, pressure and elevation at section A are V_A = 3.2 m/s, p_A = 186 kPa and z_A = 24.5 m, respectively, and those at section B are V_B = 3.2 m/s, p_B = 260 kPa and z_B = 9.1 m, respectively. If acceleration due to gravity is 10 m/s^2 then the head lost due to friction is ________ m (round off to one decimal place).

Ans. 8

Sol.



Head loss (h_f) =?

Applying Bernoulli equation between A and B:

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_f$$

$$\frac{186\times10^3}{1000\times10} + \frac{(3.2)^2}{2\times10} + 24.5$$

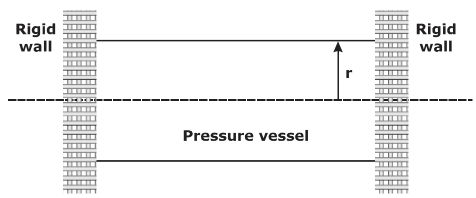
$$= \frac{260\times10^3}{1000\times10} + \frac{(3.2)^2}{2\times10} + 9.1 + h_f$$

$$h_f = (24.5 - 9.1) + \left(\frac{186-260}{10,000}\right)\times1000$$

= 15.4 – 7.4

h_f = 8 m

28. A thin-walled cylinder of radius r and thickness t is open at both ends, and fits snugly between two rigid walls under ambient conditions, as shown in the figure.



The material of the cylinder has Young's modulus E, Poisson's ratio v, and coefficient of thermal expansion α. What is the minimum rise in temperature ΔT of the cylinder (assume uniform cylinder temperature with no buckling of the cylinder) required to prevent gas leakage if the cylinder has to store the gas at an internal pressure of p above the atmosphere?

A. $\Delta T = \left(v - \frac{1}{4}\right)\frac{pr}{\alpha tE}$

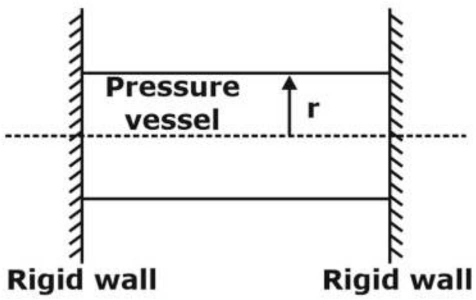
B. $\Delta T = \frac{3vpr}{2\alpha tE}$

C. $\Delta T = \left(v + \frac{1}{2}\right)\frac{pr}{\alpha tE}$

D. $\Delta T = \frac{vpr}{\alpha tE}$

Ans. D

Sol.



Strain along the longitudinal direction due to pressure

$$\varepsilon_l = \frac{\sigma_l}{E} - \nu\frac{\sigma_c}{E} = -\frac{\nu\sigma_c}{E}$$

$\left[\because \sigma_l = 0 \text{ (cylinder open at both ends}\right]$

Strain in longitudinal direction due to thermal expansion = $\alpha \Delta T$

Equate the magnitude

$$\frac{\nu\sigma_c}{E} = \alpha\Delta T$$

$$\frac{\nu Pr}{tE} = \alpha\Delta T$$

$$\Delta T = \frac{\nu Pr}{\alpha tE}$$

29. For the integral $\int_0^{\pi/2}(8 + 4\cos x)dx$, the absolute percentage error in numerical evaluation with the Trapezoidal rule, using only the end points, is ________ (round off to one decimal place).

Ans. 5.18%

Sol. $y = \int_0^{\pi/2}\left(8 + 4\cos x\right)dx$

Absolute percentage error in numerical evaluation with the trapezoidal rule, using only the end points:-

$$y_{actual} = \int_0^{\frac{\pi}{2}}\left(8 + 4\cos x\right)dx$$

$$= \left\{8x + 4\sin x\right\}_0^{\frac{\pi}{2}}$$

$$\left\{4\pi + 4\sin\frac{\pi}{2} - 8 \times 0 - 4 \times \sin 0\right\}$$

y_{actual} = 4π + 4

Using Trapezoidal Rule:

$$y_{trapezoidal} = \frac{\pi}{2 \times 2}\left[y_0 + y_{\frac{\pi}{2}}\right]$$

$$= \frac{\pi}{4}\left[\left(8 + 4\cos 0\right) + \left(8 + 4\cos\frac{\pi}{2}\right)\right]$$

$$= \frac{\pi}{4}\left[\left(8 + 4\right) + \left(8 + 0\right)\right]$$

=5π

Error in numerical evaluation

$$= \frac{Y_{actual} - Y_{trapezoidal}}{Y_{actual}} \times 100$$

$$= \frac{\left(4\pi + 4\right) - 5\pi}{\left(4\pi + 4\right)} \times 100$$

= 5.182%

30. A helical spring has spring constant k. If the wire diameter, spring diameter and the number of coils are all doubled then the spring constant of the new spring becomes

A. 8k
B. k
C. k/2
D. 16k

Ans. B

Sol. Given $K_3 = K$

d	D	n
2d	2D	2n

d → coil diameter

D → spring diameter

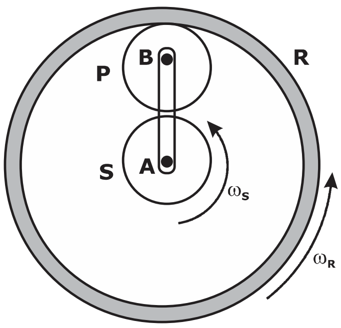
$$K = \frac{Gd^G}{8D^3n}$$

$$K \propto \frac{d^4}{D^3n} \Rightarrow \frac{K_2}{K_1} = \left(\frac{d_2}{d_1}\right)^4 \times \left(\frac{D_1}{D_2}\right)^3 \times \left(\frac{n_1}{n_x}\right)$$

$$\frac{K_2}{K_1} = (2)^4 \times \frac{1}{(2)^3} \times \frac{1}{2}$$

$$\boxed{K_2 = K_1}$$

31. The sun (S) and the planet (P) of an epicyclic gear train shown in the figure have identical number of teeth.



If the sun (S) and the outer ring (R) gears are rotated in the same direction with angular speed ω_S and ω_R, respectively, then angular speed of the arm AB is

A. $\frac{1}{4}\omega_R + \frac{3}{4}\omega_S$
B. $\frac{3}{4}\omega_R - \frac{1}{4}\omega_S$
C. $\frac{1}{2}\omega_R - \frac{1}{2}\omega_S$
D. $\frac{3}{4}\omega_R + \frac{1}{4}\omega_S$

Ans. D

Sol.

Arm	sum	Planet	ring
0	+x	−x	$-x\left(\frac{T_P}{T_R}\right)$
0	+x	−x	$-\frac{x}{3}$
Y	y + x	y − x	$y - \frac{x}{3}$

$y - x/3 = \omega_r \Rightarrow 3y - x = 3\omega_r$

$y + x = \omega_S$

$y + x = \omega_S$

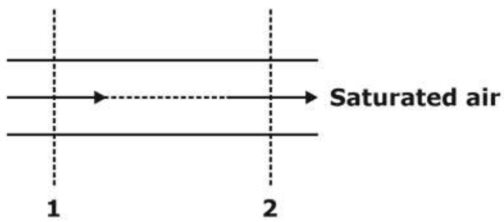
$4y = 3\omega_r + \omega_S$

$$\boxed{y = \frac{3}{4}\omega_r + \frac{1}{4}\omega_S}$$

32. Moist air at 105 kPa, 30°C and 80% relative humidity flows over a cooling coil in an insulated air-conditioning duct. Saturated air exits the duct at 100 kPa and 15°C. The saturation pressures of water at 30°C and 15°C are 4.24 kPa and 1.7 kPa respectively. Molecular weight of water is 18 g/mol and that of air is 28.94 g/mol. The mass of water condensing out from the duct is ______ g/kg of dry air (round off to the nearest integer).

Ans. 10.00

Sol.



m_{water} = 18 g/m.l

m_{air} = 28.94 g/mol

P_1 = 105 kPa $\quad p_2$ = 100 kPa

T1 = 30°C $\quad T_2$ = 15°C

φ = 80% $\quad \varphi_2$ = 100%

p_{vs1} = 4.24 kPa p_{vs2} = 1.7 kPa

At inlet:

At outlet

$$\phi_1 = \frac{p_{v1}}{p_{vs1}}$$

$$\phi_2 = \frac{p_{v2}}{p_{vs2}}$$

$$0.80 = \frac{p_{v1}}{p_{vs1}}$$

$$1.00 = \frac{p_{v_2}}{1.7}$$

0.80 × 4.24 = p_{v1}

p_{v2} = 1.7 kPa.

P_{v1} = 3.392 kPa.

$$\omega_1 = 0.622\frac{p_{v1}}{p_1 - p_{v1}} \times 1000 \text{ g / kg.d.a}$$

$$\omega_2 = 0.622 \times \frac{p_{v2}}{p_2 - p_{v2}} \times 1000$$

$$= 0.622 \times \frac{3.392}{105 - 3.392} \times 1000$$

$$= 0.622 \times \frac{1.7}{100 - 1.7} \times 1000$$

= 20.764 g/kg.da

ω_2 = 10.7568 g/kg.da

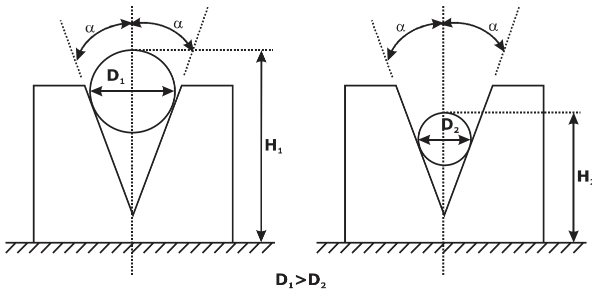
Mass of water condensing out from the duct

= $\omega_2 - \omega_1$

= 20.764 – 10.7568

= 10.00 g/kg.d.a.

33. Two rollers of diameters D_1 (in mm) and D_2 (in mm) are used to measure the internal taper angle in the V-groove of a machined component. The heights H_1 (in mm) and H_2 (in mm) are measured by using a height gauge after inserting the rollers into the same V-groove as shown in the figure.



Which one of the following is the correct relationship to evaluate the angle α as shown in the figure?

A. $\sin\alpha = \dfrac{(H_1 - H_2)}{(D_1 - D_2)}$

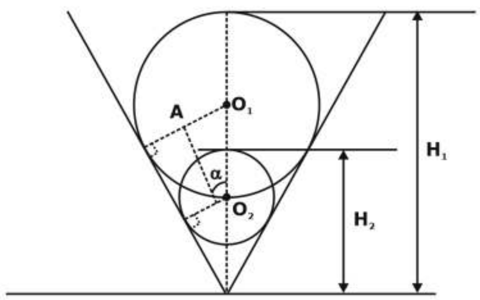
B. $\cos\alpha = \dfrac{(D_1 - D_2)}{2(H_1 - H_2) - 2(D_1 - D_2)}$

C. $\sin\alpha = \dfrac{(D_1 - D_2)}{2(H_1 - H_2) - (D_1 - D_2)}$

D. $\operatorname{cosec}\alpha = \dfrac{(H_1 - H_2) - (D_1 - D_2)}{2(D_1 - D_2)}$

Ans. C

Sol.



Upper roller having diameter

(D_1) =

Lower roller having diameter D_2

O_1O_2 = Distance between centre of roller

$$= \left(H_1 - \frac{D_1}{2}\right) - \left(H_2 - \frac{D_2}{2}\right)$$

$$O_1O_2 = (H_1 - H_2) - \left(\frac{D_1}{2} - \frac{D_2}{2}\right)$$

Difference in radius = $\left(\dfrac{D_1}{2} - \dfrac{D_2}{2}\right)$

By geometry in $\Delta\, AO_2O_1$

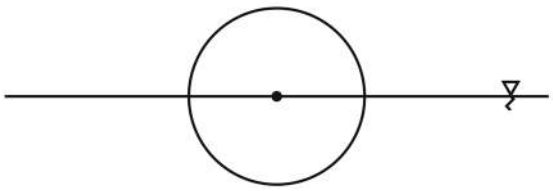
$$\sin\alpha = \frac{\left(\frac{D_1}{2}-\frac{D_2}{2}\right)}{\left(H_1-H_2\right)-\left(\frac{D_1}{2}-\frac{D_2}{2}\right)}$$

$$\sin\alpha = \frac{\left(D_1-D_2\right)}{2\left(H_1-H_2\right)-\left(D_1-D_2\right)}$$

34. A hollow spherical ball of radius 20 cm floats in still water, with half of its volume submerged. Taking the density of water as 1000 kg/m^3, and the acceleration due to gravity as 10 m/s^2, the natural frequency of small oscillations of the ball, normal to the water surface is _________ radians/s (round off to 2 decimal places).

Ans. 8.66

Sol.



For equilibrium

Buoyant force = weight of hollow sphere

$$\rho_\omega \times V_{disp} \times g = \omega_{sphere}$$

$$\rho_\omega \times \frac{V}{2} \times g = \rho_s \times Vg$$

$$\rho_s = \frac{\rho_\omega}{2}$$

Mass of hollow sphere = $\rho_s \times V$

$$= \frac{\rho_\omega}{2} \times V$$

$$= 500 \times \frac{4}{3}\pi \times (0.2)^3$$

= 16.7551 kg

Area cut by water surface in area of circle = πR^2

Now the sphere is displaced by small x so net disturbing force acting on the spherical ball for small oscillation = $\rho_\omega \pi R^2 gx$

Inertia force = $m\ddot{x}$

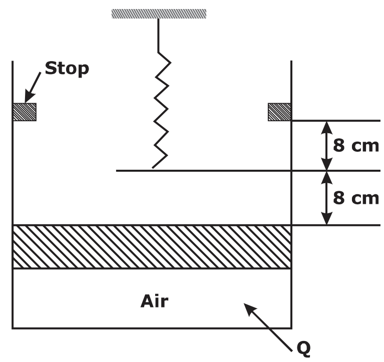
By D-Alembert's principle:

$$\omega_n = \sqrt{\frac{\rho_\omega \pi R^2 g}{m}}$$

$$= \sqrt{\frac{10^3 \times \pi \times (0.2)^2 \times 10}{16.7551}}$$

$$= \sqrt{75} = 8.66 \text{ rad/sec}$$

35. Air is contained in a frictionless piston-cylinder arrangement as shown in the figure.



The atmospheric pressure is 100 kPa and the initial pressure of air in the cylinder is 105 kPa. The area of piston is 300 cm^2. Heat is now added and the piston moves slowly from its initial position until it reaches the stops. The spring constant of the linear spring is 12.5 N/mm. Considering the air inside the cylinder as the system, the work interaction is _________ J (round off to the nearest integer).

Ans. 544

Sol. Considering Air inside the cylinder as the system.

Total work by gas = work done by the spring + work done by weight of piston + Work done by atmosphere

Work done by spring

$= \frac{1}{2}k\left(x_2^2 - x_1^2\right)$

$= \frac{1}{2} \times 12.5 \times 10^3 \times (0.08)^2$

= 40 J

Work done by atmosphere = P_{atm} × volume

= 100 × 10^3 × 0.03 × 0.16

= 480 J

Work done by weight of piston:

At initial state

$P_{initial}$ × Area = P_{atm} × Area + W_{piston}

$$105 \times 10^3 = 100 \times 10^3 + \frac{W_{piston}}{0.03}$$

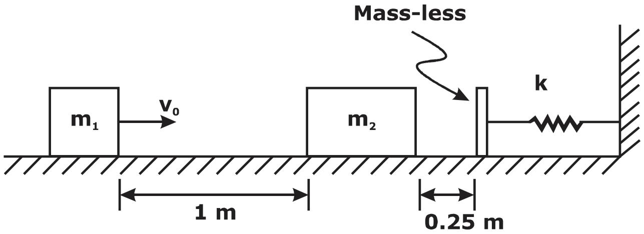
W_{piston} = 150 N

Work done by piston (W_{piston})= 150 × 0.16 = 24 Joule

So Total work done by gas = 40 + 480 + 24

= 544 Joule

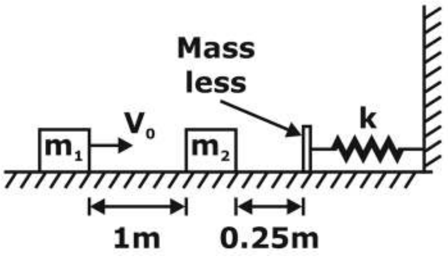
36. A rigid block of mass m_1 = 10 kg having velocity v_0 = 2 m/s strikes a stationary block of mass m_2 = 30 kg after travelling 1 m along a frictionless horizontal surface as shown in the figure.



The two masses stick together and jointly move by a distance of 0.25 m further along the same frictionless surface, before they touch the mass-less buffer that is connected to the rigid vertical wall by means of a linear spring having a spring constant k = 10^5 N/m. The maximum deflection of the spring is __________ cm (round off to 2 decimal places).

Ans. 1

Sol.



m_1 = 10 kg

v_0 = 2 m/s

m_2 = 30 kg

Since given surfaces are friction less surface. Thus, both energy and linear momentum will be conserve.

k = 10^5 N/m

After collision both mass sticks together.

Thus,

10 × 2 + 30 × 0 = (10 + 30)V_{common}

20 = 40 V_{common}

$$V_{common} = \frac{1}{2} \text{m/s}$$

$$\frac{1}{2}(m_1 + m_2)V_{common}^2 = \frac{1}{2}kx^2$$

$$40 \times \left(\frac{1}{2}\right)^2 = 10^5 \times x^2$$

10 × 10^{-5} = x^2

x = 10^{-2}m

x = 1 cm

37. Bars of 250 mm length and 25 mm diameter are to be turned on a lathe with a feed of 0.2 mm/rev. Each regrinding of the tool costs Rs. 20. The time required for each tool change is 1 min. Tool life equation is given as $VT^{0.2}$ = 24 (where cutting speed V is in m/min and tool life T is in min). The optimum tool cost per piece for maximum production rate is Rs. ________ (round off to 2 decimal places).

Ans. 26.98

Sol. Turning on Lath:

Length (L) = 250 mm

Diameter (D) = 25 mm

Feed (f) = 0.2 mm/rev

Regrinding tool cost = Rs 20/regrind

Tool change time (T_c) = 1 min.

Tool life equation $VT^{0.2} = 24$

Tool life for optimum production rate:

since $T = T_c\left(\frac{1}{n} - 1\right)$

$= 1\left(\frac{1}{0.2} - 1\right)$

Tool life (T) = 4 min

Now $VT^{0.2} = 24$

V = 18.188 m/min

Now $V = \frac{\pi DN}{1000}$

$18.188 = \frac{\pi \times 25 \times N}{1000}$

N = 23.158 rpm

Thus, Machining time (t_m)

$= \frac{L}{fN} = \frac{250\text{ mm}}{0.2\text{ mm / rev} \times 231.58}$

t_m = 5.3975 min

Thus, optimum tool cost $(C_T) = \frac{5.3975}{4} \times 20$

= 26.98 Rs/-

38. In a steam power plant, superheated steam at 10 MPa and 500°C, is expanded isentropically in a turbine until it becomes a saturated vapour. It is then reheated at constant pressure to 500°C. The steam is next expanded isentropically in another turbine until it reaches the condenser pressure of 20 kPa. Relevant properties of steam are given in the following two tables. The work done by both the turbines together is ________ kJ/kg (round off to the nearest integer).

Superheated Steam Table:

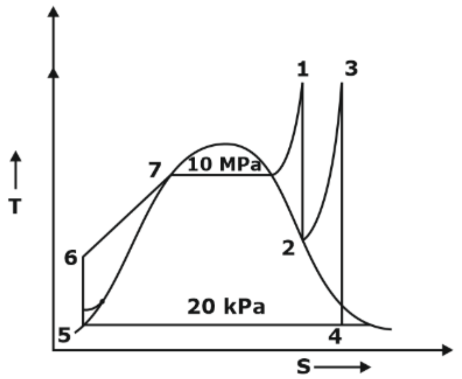
Pressure, p (MPa)	Temperature, T (°C)	Enthalpy, h (kJ/kg)	Entropy, s (kJ/kg.K)
10	500	3373.6	6.5965
1	500	3478.4	7.7621

Saturated Steam Table:

Pressure, p	Sat. Temp. T_{sat} (°C)	Enthalpy, h (kJ/kg)		Entropy, s (kJ/kg.K)	
		h_f	h_g	s_f	s_g
1 MPa	179.91	762.9	2778.1	2.1386	6.5965
20 kPa	60.06	251.38	2609.7	0.8319	7.9085

Ans. 1513.01

Sol.



Given

Work done by both turbines together:

$W_{turbines} = (h_1 - h_2) + (h_3 - h_4)$

From steam tables:

h_1 = 3373.6 kJ/kg

h_2 = 2778.1 kJ/kg

Since $S_3 = S_4$

S_3 = 7.7621 kJ/kg-k

7.7621 = 0.8319 + x

(7.9085 – 0.8319)

x = 0.9793

Thus, at exit of 2nd turbine will be in wet region with dryness fraction of 0.9793.

Thus, $h_4 = h_f + xh_{fg}$

= 251.38 + 0.9793

(2609.7 – 251.38)

= 2560.8827 kJ/kg.

$W_{Turbine}$ = (3373.6 – 2778.1) + (3478.4 – 2560.8827)

= 1513.01 kJ/kg

39. A cylindrical bar with 200 mm diameter is being turned with a tool having geometry 0° - 9° - 7° - 8° - 15° - 30° - 0.05 inch (Coordinate system, ASA) resulting in a cutting force F_{c1}. If the tool geometry is changed to 0° - 9° - 7° - 8° -15° - 0° - 0.05 inch (Coordinate system, ASA) and all other parameters remain unchanged, the cutting force changes to F_{c2}. Specific cutting energy (in J/mm³) is $U_c = U_0\,(t_1)^{-0.4}$, where U_0 is the specific energy coefficient, and t_1 is the uncut thickness in mm. The value of percentage change in cutting force F_{c2}, i.e. $\left(\frac{F_{c2}-F_{c1}}{F_{c1}}\right)\times 100$, is _______ (round off to one decimal place).

Ans. -5.59

Sol. Cylindrical bar:

Diameter (D) = 200 mm

In ASA system:

Tool Geometry (Tool 1) ⇒ 0° – 9° – 7° – 8° – 15° – 30° – 0.05 inch

Tool Geometry (Tool 2) ⇒ 0° – 9° – 7° – 8° – 15° – 0° 0.05 inch.

For Tool 1: Side cutting edge angle $(C_{s,1}) = 30°$

For tool 2: side cutting edge angle $(C_{s,2})$ = 0°

Since $C_s + \lambda$ =90°

Thus, λ_1 = 60°

λ_2 = 90°

Since specific energy consumption

$$(U_c) = \frac{F_c}{1000fd} = U_0(t_1)^{-0.4}$$

$F_c = U_0(t_1)^{-0.4} \times 1000fd$

Now $t_1 = f\sin\lambda$

$F_c = U_0(f\sin\lambda)^{-0.4} \times 1000fd$

$F_c \propto (\sin\lambda)^{-0.4}$

Thus,

$$\left(\frac{F_{c2}-F_{c1}}{F_{c1}}\right)\times 100$$

$$= \frac{(\sin\lambda_2)^{-0.4} - (\sin\lambda_1)^{-0.4}}{(\sin\lambda_1)^{-0.4}}\times 100$$

$$= \left\{\left(\frac{\sin\lambda_2}{\sin\lambda_1}\right)^{-0.4} - 1\right\}\times 100$$

$$= \left\{\left(\frac{\sin 90°}{\sin 60°}\right)^{-0.4} - 1\right\}\times 100$$

= –5.59%

40. A fair coin is tossed 20 times. The probability that 'head' will appear exactly 4 times in the first ten tosses, and 'tail' will appear exactly 4 times in the next ten tosses is __________ (round off to 3 decimal places).

Ans. 0.042

Sol. Required probability

$$= 10C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 \times 10C_4\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^6$$

= 0.042

41. The forecast for the monthly demand of a product is given in the table below.

Month	Forecast	Actual Sales
1	32.00	30.00
2	31.80	32.00
3	31.82	30.00

The forecast is made by using the exponential smoothing method. The exponential smoothing coefficient used in forecasting the demand is

A. 1.00 B. 0.10

C. 0.50 D. 0.40

Ans. B

Sol.

Month	Forecast	Actual sales
1	32.00	30.00
2	31.80	32.00
3	31.82	30.00

Since $f_t = f_{t-1} + \alpha (D_{t-1} - F_{t-1})$

$F_3 = F_2 + \alpha (D_2 - F_2)$

$31.82 = 31.80 + \alpha (32 - 31.80)$

$\alpha = 0.10$

42. A steel spur pinion has a module (m) of 1.25 mm, 20 teeth and 20° pressure angle. The pinion rotates at 1200 rpm and transmits power to a 60 teeth gear. The face width (F) is 50 mm, Lewis form factor Y = 0.322 and a dynamic factor K_v = 1.26. The bending stress (σ) induced in a tooth can be calculated by using the Lewis formula given below.

If the maximum bending stress experienced by the pinion is 400 MPa, the power transmitted is _______ kW (round off to one decimal place).

Lewis formula: $\sigma = \frac{k_v W^t}{FmY}$, where W^t is the tangential load acting on the pinion.

Ans. 10.03

Sol. A steel spur pinion:

Module (m) = 1.25 mm

Teeth (T_P) = 20 teeth

Pressure angle (φ) = 20°

N_P = 1200 rpm and Gear teeth (T_G) = 60

Face width (F) = 50 mm

Lewis form factor (Y) = 0.322

Dynamic factor (k_V) = 1.26

Maximum bending stress ($\sigma_{b,max}$) = 400 MPa

Power transmitted (P) ________?

Power transmitted $(P) = \frac{2\pi N_P M_{t,P}}{60}$

Where

$M_{t,p}$ = torque on pinion

$M_{t,P} = w^t \times r_p$

$m = \frac{d_p}{T_p} \Rightarrow d_p = 1.25 \times 20$

r_p = 12.5 mm

$M_{t,P} = w^t \times 0.0125$

$$\sigma_{b,max} = \frac{k_v w^t}{fmy}$$

$$\sigma_{b,max} = \frac{1.26 w^t}{50 \times 1.25 \times 0.322}$$

$$w^t = \frac{400 \times 50 \times 1.25 \times 0.322}{1.26} = 6388.88\text{ N}$$

Thus, Power(P)

$$= \frac{2\pi \times 1200 \times (6388.88 \times 0.0125)}{60}$$

Power (P) = 10035.62 Watt

P = 10.035 kW

43. A mould cavity of 1200 cm^3 volume has to be filled through a sprue of 10 cm length feeding a horizontal runner. Cross-sectional area at the base of the sprue is 2 cm^2. Consider acceleration due to gravity as 9.81 m/s^2. Neglecting frictional losses due to molten metal flow, the time taken to fill the mould cavity is __________ seconds (round off to 2 decimal places).

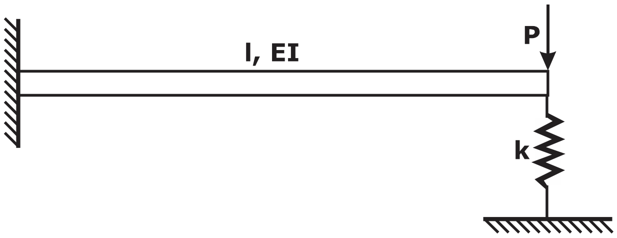
Ans. 4.28

Sol. time of filling $= \frac{V_m}{A_g\sqrt{2gh}}$

$$= \frac{1200}{2\sqrt{2 \times 981 \times 10}}$$

= 4.28 sec.

44. A cantilever of length l, and flexural rigidity EI, stiffened by a spring of stiffness k, is loaded by a transverse force P, as shown.



The transverse deflection under the load is

A. $\frac{Pl^3}{3EI}\left[\frac{3EI}{3EI+2kl^3}\right]$

B. $\frac{Pl^3}{3EI}\left[\frac{3EI-kl^3}{3EI}\right]$

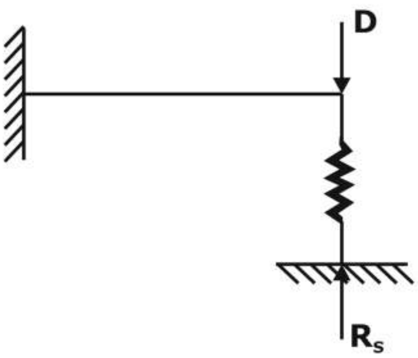
C. $\frac{Pl^3}{3EI}\left[\frac{3EI}{3EI+kl^3}\right]$

D. $\frac{Pl^3}{3EI}\left[\frac{6EI-kl^3}{6EI}\right]$

Ans. C

Sol. $-\frac{PL^3}{3EI}+\frac{R_S L^3}{3EI}=\frac{-R_S}{K}$

$$R_S\,\frac{1}{K}+\frac{L^3}{3EI}=\frac{PL^3}{3EI}$$

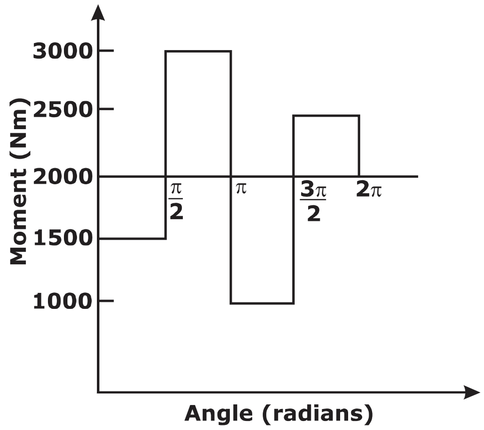


$$=\frac{-PL^3}{3EI}+\frac{R_S L^3}{3EI}=\frac{-PL^3}{3EL}$$

$$\frac{R_S}{1}\,\frac{PL^3}{3EI}\times\left(\frac{3EIK}{3EI+KL^3}\right)$$

$$\boxed{\delta=\frac{PL^3}{3EI+KL^3}}$$

45. The turning moment diagram of a flywheel fitted to a fictitious engine is shown in the figure.



The mean turning moment is 2000 Nm. The average engine speed is 1000 rpm. For fluctuation in the speed to be within ±2% of the average speed, the mass moment of inertia of the flywheel is _______ kg.m^2.

Ans. 3.58

Sol. Maximum function of energy

$$(\Delta E)_{max}=(3000-2000)\times\frac{\pi}{2}$$

$$=1000\times\frac{\pi}{2}$$

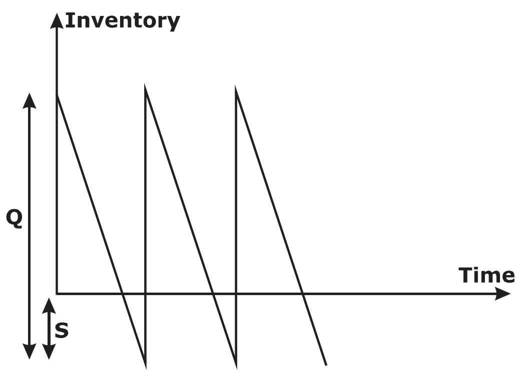
C_S = 0.04

$(\Delta E)_{max}$ = $I\omega^2 C_S$

$$1000\times\frac{\pi}{2}=I\times\left(\frac{22\times1000}{60}\right)^2\times0.04$$

I = 3.58 kg –m^2

46. For a single item inventory system, the demand is continuous, which is 10000 per year. The replenishment is instantaneous and backorders (S units) per cycle are allowed as shown in the figure.



As soon as the quantity (Q units) ordered from the supplier is received, the backordered quantity is issued to the customers. The ordering cost is Rs. 300 per order. The carrying cost is Rs. 4 per unit per year. The cost of backordering is Rs. 25 per unit per year. Based on the total cost minimization criteria, the maximum inventory reached in the system is _______ (round off to nearest integer).

Ans. 1137.15

Sol. In back order model,

$(Q^* - S^*)\,C_n = S^* \times C_b$

$$S^* = \left(\frac{Q^* C_h}{C_h + C_b}\right)$$

Maximum inventory level

$= (Q^* - S^*)$

$$= Q^* - \frac{Q^* C_h}{C_h + C_b}$$

$$= \left(\frac{Q^* C_b}{C_h + C_b}\right)$$

$$M^* = \sqrt{\frac{2DC_0}{C_h} \times \left(\frac{C_b \times C_h}{C_b}\right)} \times \left(\frac{C_b}{C_h + C_b}\right)$$

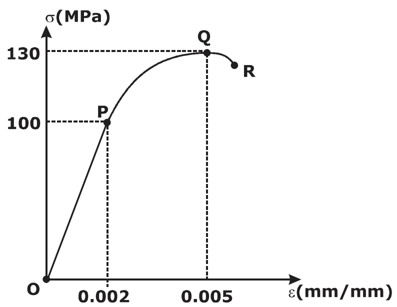
$$M^* = \sqrt{\frac{2DC_0}{C_h} \times \left(\frac{C_b}{C_b + C_h}\right)}$$

$$= \sqrt{\frac{2 \times 10000 \times 300}{4} \times \left(\frac{25}{29}\right)}$$

= 1137.147 unit

$\cong$ 1137 units

47. Uniaxial compression test data for a solid metal bar of length 1 m is shown in the figure.



The bar material has a linear elastic response from O to P followed by a nonlinear response. The point P represents the yield point of the material. The rod is pinned at both the ends. The minimum diameter of the bar so that it does not buckle under axial loading before reaching the yield point is ________ mm (round off to one decimal place).

Ans. 56.94

Sol. Critical load in case of bucking:

$$P_{cr} = \frac{\pi^2 EI}{l_e^2}$$

$$(\sigma_{max.} A) = \frac{\pi^2 EI}{l_e^2}$$

$$\text{young modules } E = \frac{\Delta\sigma}{E} = \frac{100}{2 \times 10^{-3}} = \frac{1}{2} \times 10^5$$

= 50,000 MPa

$$100 \times \frac{\pi}{4} d^2 = \frac{\pi^2 \times 5 \times 10^4}{10^6} \times \frac{\pi}{64} \times d^4$$

$16 \times 10^8 = \pi^2 \times 5 \times 10^4 \times d^2$

$$d_{min.} = \sqrt{\frac{16 \times 10^8}{\pi^2 \times 5 \times 10^4}}$$

$d_{min.}$ = 56.94 mm

48. Keeping all other parameters identical, the Compression Ratio (CR) of an air standard diesel cycle is increased from 15 to 21. Take ratio of specific heats = 1.3 and cut-off ratio of the cycle r_c = 2.

The difference between the new and the old efficiency values, in percentage, $(\eta_{new}|_{CR=21}) - (\eta_{old}|_{CR=15})$ = __________ % (round off to one decimal place).

Ans. 4.81

Sol. $$\eta_1 = 1 - \frac{1}{(r_1)^{\gamma-1}\gamma}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right]$$

$$= 1 - \frac{1}{(15)^{1.3-1} \times 1.3}\left[\frac{2^{1.3} - 1}{2 - 1}\right]$$

$= 1 - 0.341\,[1.4611] = 50.08\%$

$$\eta_2 = 1 - \frac{1}{(r_2)^{\gamma-1}\gamma}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right]$$

$$= 1 - \frac{1}{(21)^{1.3-1} \times 1.3}\left[\frac{2^{1.3} - 1}{2 - 1}\right]$$

$= 1 - 0.3085(1.4622)$

$= 54.89\%$

Difference in efficiencies $= \eta_2 - \eta_1$

$= 54.89 - 50.08$

$= 4.81\%$

49. The function f(z) of complex variable z = x + i y, where $i = \sqrt{-1}$, is given as $f(z) = (x^3 - 3xy^2) + i\,v(x, y)$. For this function to be analytic, v(x, y) should be

A. $(x^3 - 3x^2y)$ + constant

B. $(3x^2y^2 - y^3)$ + constant

C. $(3x^2y - y^3)$ + constant

D. $(3xy^2 - y^3)$ + constant

Ans. C

Sol. $f(z) = \left(x^3 - 3xy^2\right) + iv(x, y)$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}\ ;\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$3x^2 - 3y^2 = \frac{\partial v}{\partial y}$$

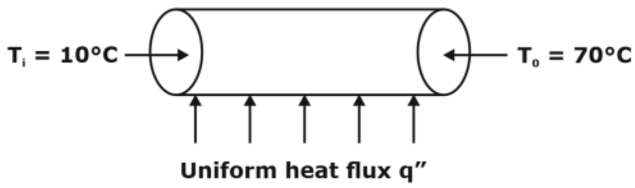
$$\int \partial v = \int \left(3x^2 - 3y^2\right)\partial y + \text{ constant}$$

$v = 3x^2y - y^3 +$ constant

50. Water flows through a tube of 3 cm internal diameter and length 20 m. The outside surface of the tube is heated electrically so that it is subjected to uniform heat flux circumferentially and axially. The mean inlet and exit temperatures of the water are 10°C and 70°C, respectively. The mass flow rate of the water is 720 kg/h. Disregard the thermal resistance of the tube wall. The internal heat transfer coefficient is 1697 $W/m^2 \cdot K$. Take specific heat C_p of water as 4.179 $kJ/kg \cdot K$. The inner surface temperature at the exit section of the tube is ____________ °C (round off to one decimal place).

Ans. 85.67

Sol. Dinner = 3 cm, L = 20 m



$$\text{Net heat flux} = \frac{\text{Total heat gain by water}}{\text{Total surface area}}$$

$$= \frac{\dot{m}C_p\left(T_0 - T_i\right)}{\pi D_i L}$$

$$= \frac{720 \times 4.179 \times 10^3 \times 60}{3600 \times \pi \times 0.03 \times 20}$$

$$= 26604.340\,\frac{W}{m^2}$$

Heat flux at the exist

$$= 26604.34\,\frac{w}{m^2}$$

$q'' = h\,[T_{exit} - T_0]$

$26604.34 = 1697 \times (T_{exit} - 70)$

The inner surface temperature at the exist section of the tube in $= \dfrac{26604.34}{1697} + 70$

$= 85.67°C$

51. One kg of air in a closed system undergoes an irreversible process from an initial state of p_1 = 1 bar (absolute) and T_1 = 27°C, to a final state of p_2 = 3 bar (absolute) and T_2 = 127°C. If the gas constant of air is 287 $J/kg \cdot K$ and the ratio of the specific heats $\gamma = 1.4$, then the change in the specific entropy (in J/kg.K) of the air in the process is

A. −26.3

B. 28.4

C. indeterminate, as the process is irreversible

D. 172.0

Ans. A

Sol. Given P_1 = 1 bar,

P_2 = 3 bar

T_1 = 300K

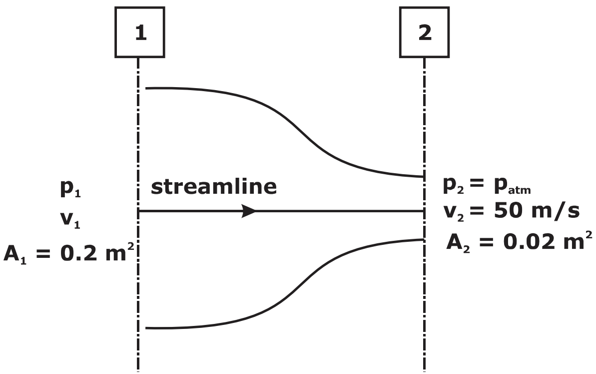
T_2 = 400K

$$\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

$$= 1005\ln\left(\frac{400}{300}\right) - 287\ln\left(\frac{3}{1}\right)$$

$$= -26.181\frac{J}{kgk}$$

52. Consider a flow through a nozzle, as shown in the figure below



The air flow is steady, incompressible and inviscid. The density of air is 1.23 kg/m^3. The pressure difference, (p_1 – p_{atm}) is ________ kPa (round off to 2 decimal places).

Ans. 1.52

Sol. Applying Bernoulli equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

(neglect loss of head due to friction)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_{atm}}{\rho g} + \frac{V_2^2}{2g}$$

$$(P_1 - P_{atm}) = \rho\frac{\left(V_2^2 - V_1^2\right)}{2}$$

$$= \frac{1}{2}\times 1.23\times\left(50^2 - 5^2\right)$$

= 1.522 kPa

53. The spectral distribution of radiation from a black body at T1 = 3000 K has a maximum at wavelength λ_{max}. The body cools down to a temperature T_2. If the wavelength corresponding to the maximum of the spectral distribution at T_2 is 1.2 times of the original wavelength λ_{max}, then the temperature T_2 is ____________ K (round off to the nearest integer).

Ans. 2500

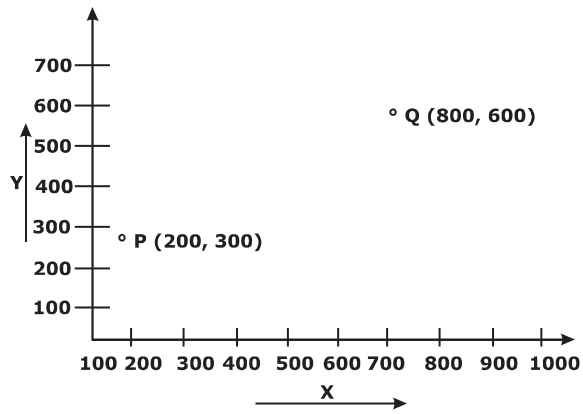
Sol. According to Wein's displacement law,

$\lambda_{max.}$ T = 2898 μm–K= const.

λ_{m1} × 3000 = 1.2 λ_{m2} × T_2

$$T_2 = \left(\frac{3000}{1.2}\right) = 2500\text{ K}$$

54. A point 'P' on a CNC controlled XY-stage is moved to another point 'Q' using the coordinate system shown in the figure below and rapid positioning command (G00).



A pair of stepping motors with maximum speed of 800 rpm, controlling both the X and Y motion of the stage, are directly coupled to a pair of lead screws, each with a uniform pitch of 0.5 mm. The time needed to position the point 'P' to the point 'Q' is ____________ minutes (round off to 2 decimal places).

Ans. 1.5

Sol. Since pair of steeping motors with controlling both the x and y motion so time needed in x-direction:

$$t_x = \frac{(x_2 - x_1)}{f_m} = \frac{(800-200)}{0.5 \times 800}$$

$$t_x = \frac{600}{400} = 1.5 \text{ min}$$

Time needed in y-direction

$$t_y = \frac{(y_2 - y_1)}{f_m} = \left(\frac{600-300}{400}\right)$$

= 0.75 min.

Since both are working simultaneously so the maximum time of both direction will be the machining time.

55. The directional derivative of f(x, y, z) = xyz at point (−1, 1, 3) in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ is

A. $\frac{7}{3}$

B. 7

C. $3\hat{i} - 3\hat{j} - \hat{k}$

D. $-\frac{7}{3}$

Ans. A

Sol. f(x, y, z) = xyz

$$\nabla f = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial_2}\right)(xyz)$$

$$= yz\hat{i} + xz\hat{j} + xy\hat{k}$$

$$(\nabla f)_{(-1,1,3)} = 3\hat{i} - 3\hat{j} - \hat{k}$$

$$\text{Normal vector} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1 + (-2)^2 + (2)^2}}$$

$$= \left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}\right)$$

So directional derivative of f in direction of $\vec{P}$

$$= \nabla f \cdot \frac{\vec{P}}{|\vec{P}|}$$

$$= (3\hat{i} - 3\hat{j} - k) \cdot \frac{(\hat{i} - 2\hat{j} + 2\hat{k})}{3} = \frac{7}{3}$$

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