## B BYJU'S

## GATE 2020

## Civil Engineering

## Shift-1

Questions \& Solutions

## SECTION: GENERAL APTITUDE

1. It is a common criticism that most of the academicians live in their $\qquad$ So, they are not aware of the real-life challenges.
A. Ivory towers
B. Homes
C. Glass palaces
D. Big flats

Ans. A
Sol. It is a common criticism that most of the academicians live in their ivory tower, so they are not aware of the real-life challenges.
2. His hunger for reading is insatiable. He reads indiscriminately. He is most certainly a/an
$\qquad$ reader.
A. all-round
B. voracious
C. wise
D. precocious

## Ans. B

Sol. His hunger for reading is insatiable, he reads indiscriminately. He is most certainly a/an voracious reader.
3. Select the word that fits the analogy Fuse : Fusion :: Use :
A. User
B. Uses
C. Usage
D. Usion

Ans. C
Sol. Fuse : Fusion :: use : Usage
4. If $0,1,2, \ldots \ldots . .7,8,9$ are coded as $O, P, Q, \ldots . . . .$. , $\mathrm{V}, \mathrm{W}, \mathrm{X}$, then 45 will be coded as
A. TS
B. SS
C. ST
D. SU

Ans. C
Sol.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O | P | Q | R | S | T | U | V | W | X |

So, $45 \rightarrow$ ST
5. The sum of two positive numbers is 100. After subtracting 5 from each number, the product of the resulting numbers is 0 . One of the original numbers is
A. 95
B. 85
C. 80
D. 90

Ans. A
Sol. Let one positive number be a other positive number $=100-$ a product of number after subtracting 5 from each number

$$
\begin{aligned}
& =(a-5) \times(95-a)=0 \\
& \Rightarrow 95 a-a^{2}-475+5 a=0 \\
& \Rightarrow a^{2}-100 a+475=0 \\
& \Rightarrow a^{2}-95 a-5 a+475=0 \\
& \Rightarrow a(a-95)-5(a-95)=0 \\
& \Rightarrow \text { either } a=5 \text { or } a=95
\end{aligned}
$$

From, the options given 95 is the answer.
6. The American psychologist Howard Gardner expounds that human intelligence can be sub categorised into multiple kinds, in such a way that individuals differ with respect to their relative competence in each kind. Based on this theory, modern educationists insist on prescribing multidimensional curriculum and evaluation parameters that enable development and assessment of multiple intelligences.
Which of the following statements can be inferred from the given text?
A. Howard Gardner insists that the teaching curriculum and evaluation needs to be multidimensional.
B. Modern educationists want to develop and assess the theory of multiple intelligences.
C. Modem educationists insist that the teaching curriculum and evaluation needs to be multidimensional.
D. Howard Gardner wants to develop and assess the theory of multiple intelligences.
Ans. C
7. Five friends $P, Q, R, S$ and $T$ went camping. At night, they had to sleep in a row inside the tent. $P, Q$ and $T$ refused to sleep next to $R$ since he snored loudly. $P$ and $S$ wanted to avoid $Q$ as he usually hugged people in sleep.
Assuming everyone was satisfied with the sleeping arrangements, what is the order in which they slept?
A. RSPTQ
B. SPRTQ
C. QTSPR
D. QRSPT

Ans. A
Sol. Option A is the only arrangement where given conditions are met.

In option $B \& C, R$ is sleeping next to $P$ In option $D, R$ is sleeping next to $Q$.
8. Insert seven numbers between 2 and 34, such that the resulting sequence including 2 and 34 is an arithmetic progression. The sum of these inserted seven numbers is
A. 124
B. 130
C. 120
D. 126

Ans. D
Sol. As per the given question, the sequence of given AP is, 2 _ $-{ }^{-}$_ 34
In this sequence first term (a) $=2$
Last term (tn) $=34$
So, as per the relation,

$$
t_{n}=a+(n-1) d
$$

$\mathrm{n} \rightarrow$ number of terms
d $\rightarrow$ common difference

$$
\begin{aligned}
& 34=2+(9-1) d \\
& d=4
\end{aligned}
$$

So, the A.P. becomes,

$$
2,6,10,14,18,22,26,30,34
$$

Sum of 7 terms between 2234 is 126
9. The unit's place in $26591749^{110016}$ is
A. 6
B. 1
C. 3
D. 9

Ans. B
Sol. The unit digit in the power of 9 can be found by,

$$
\begin{aligned}
& 9^{1}=9 \rightarrow \text { unit digit is } 9 \\
& 9^{2}=81 \rightarrow \text { so, unit digit is } 1 \\
& 9^{3}=729 \rightarrow \text { unit digit is } 9 \\
& 9^{4}=6561 \rightarrow \text { unit digit is } 1
\end{aligned}
$$

So, from the above sequence, if follows that 9 power, if even the unit digit will be 1 . and if 9 power, is odd unit digit will be 9 As per the question, $26591749^{110016}$

The answer of unit digit will be 1 .
10. The total expenditure of a family, on different activities in a month, is shown in the pie-chart. The extra money spent on education as compared to transport (in percent) is

A. 50
B. 100
C. 33.3
D. 55

Ans. A
Sol. Extra money spent on education as compared to transport

$$
\begin{aligned}
& =\left(\frac{15-10}{10}\right) \times 100 \\
& =50 \%
\end{aligned}
$$

## TECHNICAL

1. The probability that a 50 year flood may NOT occur at all during 25 years life of a project (round off to two decimal places), is $\qquad$ -.
Ans. 0.60
Sol. $\quad$ T = 50 years

$$
\begin{aligned}
& \qquad P=\frac{1}{T}=\frac{1}{50} \\
& q=1-P=\frac{49}{50} \\
& \text { Reliability }=q^{n}=\left(\frac{49}{50}\right)^{25}=0.6034=60.34 \%
\end{aligned}
$$

2. In a two-dimensional stress analysis, the state of stress at a point $P$ is

$$
[\sigma]=\left[\begin{array}{cc}
\sigma_{x x} & \tau_{x y} \\
\tau_{x y} & \sigma_{y y}
\end{array}\right]
$$

The necessary and sufficient condition for existence of the state of pure shear at the point $P$, is
A. $\left(\sigma_{x x}-\sigma_{y y}\right)^{2}+4 \tau_{x y}^{2}=0$
B. $\tau_{x y}=0$
C. $\sigma_{x x}+\sigma_{y y}=0$
D. $\sigma_{x x}+\sigma_{y y}-\tau_{x y}^{2}=0$

Ans. C

## Sol.



$$
\begin{aligned}
& \sigma_{1}=-\sigma_{2} \\
& \sigma_{1}=\frac{\sigma_{x x}+\sigma_{y y}}{2}+\text { Radius }
\end{aligned}
$$

$$
\begin{aligned}
& \sigma_{2}=\frac{\sigma_{x x}+\sigma_{y y}}{2}-\text { Radius } \\
& \frac{\sigma_{x x}+\sigma_{y y}}{2}+\text { Radius }=-\left[\left[\frac{\sigma_{x x}+\sigma_{y y}}{2}\right]-\text { Radius }\right] \\
& \sigma_{x x}+\sigma_{y y}=0
\end{aligned}
$$

3. The value of $\lim _{x \rightarrow \infty} \frac{x^{2}-5 x+4}{4 x^{2}+2 x}$ is
A. $\frac{1}{2}$
B. 1
C. $\frac{1}{4}$
D. 0

Ans. C
Sol. $\quad \lim _{x \rightarrow 0} \frac{x^{2}-5 x+4}{4 x^{2}+2 x}=\frac{4}{0}=0$
Applying L's Hospital rule

$$
\lim _{x \rightarrow 0} \frac{f^{1}(x)}{f^{1}(x)}=\frac{2 x-5}{8 x+2}=-v e
$$

So again applying L's Hospital rule

$$
\lim _{x \rightarrow 0} f^{\prime \prime}(x)=\frac{2}{8}=\frac{1}{4}
$$

4. A 4 m wide rectangular channel carries $6 \mathrm{~m}^{3} / \mathrm{s}$ of water. The Manning's ' $n$ ' of the open channel is 0.02 . Considering $\mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2}$, the critical velocity of flow (in $\mathrm{m} / \mathrm{s}$, round off to two decimal places) in the channel, is $\qquad$ -.
Ans. $2.44 \mathrm{~m} / \mathrm{sec}$
Sol. Velocity $=\frac{\text { Discharge }}{\text { Area }}=\frac{Q}{b . y}=\frac{q}{y}$

$$
\therefore V_{c}=\frac{q}{y_{c}}=\frac{q}{\left(\frac{q^{2}}{g}\right)^{1 / 3}}=q^{1 / 3} g^{1 / 3}
$$

$$
\mathrm{V}_{\mathrm{c}}{ }^{3}=\mathrm{qg}
$$

Also, $\mathrm{q}=\mathrm{V}_{\mathrm{c}} \times \mathrm{y}_{\mathrm{c}}$

$$
\mathrm{V}_{\mathrm{c}}^{3}=\mathrm{V}_{\mathrm{c}} \mathrm{y}_{\mathrm{c}} \mathrm{~g} \quad \text { or } \quad \mathrm{V}_{\mathrm{c}}^{2}=\mathrm{gyc}
$$

$$
\begin{gathered}
\therefore V_{c}=\sqrt{g y_{c}} \\
\text { Now, } y_{c}=\left(\frac{q^{2}}{g}\right)^{1 / 3}=\left(\frac{\left(\frac{6}{4}\right)^{2}}{9.81}\right)=0.612 \mathrm{~m} \\
\left\{\therefore q=\frac{Q}{B}\right\} \\
\therefore V_{c}=\sqrt{9.81 \times 0.612}=2.44 \mathrm{~m} / \mathrm{sec}
\end{gathered}
$$

5. A fully submerged infinite sandy slope has an inclination of $30^{\circ}$ with the horizontal. The saturated unit weight and effective angle of internal friction of sand are $18 \mathrm{kN} / \mathrm{m}^{3}$ and $38^{\circ}$, respectively. The unit weight of water is 10 $\mathrm{kN} / \mathrm{m}^{3}$. Assume that the seepage is parallel to the slope. Against shear failure of the slope, the factor of safety (round off to two decimal places) is
Ans. 0.6014
Sol. FOS $=\frac{\gamma_{\text {sub }}}{\gamma_{\text {sat }}} \frac{\tan \phi}{\tan \beta}$

$$
\mathrm{FOS}=\frac{(18-10)}{18} \times \frac{\tan 38^{\circ}}{\tan 30^{\circ}}=0.6014
$$

6. An amount of 35.67 mg HCl is added to distilled water and the total solution volume is made to one litre. The atomic weights of H and Cl are 1 and 35.5, respectively. Neglecting the dissociation of water, the pH of the solution, is
A. 350
B. 2.50
C. 2.01
D. 3.01

Ans. D
Sol. 1 mole of HCL given 1 mole $\mathrm{H}^{+}$ions 36.5 gm of HCl given 1 gm of $\mathrm{H}^{+}$ions Therefore,
35.67 mg of HCl gives,
$\frac{1}{36.5} \times 36.67 \mathrm{mg}$ of $\mathrm{H}^{+}$ions
$\therefore 35.67 \mathrm{mg}$ of $\mathrm{HCl}=0.977 \mathrm{mg}$ of $\mathrm{H}^{+}$

Or $\frac{0.977 \times 10^{-3}}{1}$

$$
=9.77 \times 10-4 \text { moles of } \mathrm{H}^{+} \text {ions }
$$

Now, we know that

$$
\begin{aligned}
& \mathrm{pH}=-\log _{10}\left[\mathrm{H}^{+}\right] \\
& \Rightarrow \mathrm{pH}=-\log _{10}\left(9.77 \times 10^{-4}\right) \\
& =-\log _{10} 9.77+\left(-\log _{10}\left(10^{-4}\right)\right) \\
& =-\log _{10} 9.77+4 \log _{10} 10 \\
& =-0.989+4 \\
& \Rightarrow \mathrm{pH}=3.01
\end{aligned}
$$

7. The Los Angeles test for stone aggregates is used to examine
A. Soundness
B. Abrasion resistance
C. Specific gravity
D. Crushing strength

Ans. B
Sol. Los Angels test in done to check the abrasion resistance of coarse aggregate as Per IS:2386 (Part IV)-1963.
Specific gravity of coarse aggregate $\qquad$ Using wire basket equipment, using IS:2386 Crushing strength deals with resistance to compressive action of aggregate Soundness is a test to Judge the durability of stone aggregate against weathering actions
8. A planar elastic structure is subjected to uniformly distributed load, as shown in the figure (not drawn to the scale)


Neglecting self-weight, the maximum bending moment generated in the structure (in kN.m, round off to the nearest integer), is $\qquad$ _.
Ans. 96

## Sol.



$$
\Sigma F_{x}=0 \Rightarrow H_{A}=0
$$

Neglecting self weight, the maximum bending moment. Taking " $A$ " as the origin and springing points " $A$ " and " $B$ " are at the same level.

Applying, $\Sigma M A=0$ to the FBD of entire arch.

$$
\begin{aligned}
& \Rightarrow 12 \times 8 \times\left(\frac{8}{\mathrm{~L}}\right)-\mathrm{V}_{\mathrm{B}}[8]=0 \\
& \Rightarrow V B=48 \mathrm{KN}
\end{aligned}
$$



$$
\begin{aligned}
& M_{x}+12 x\left(\frac{x}{2}\right)-V_{B}[x]=0 \\
& M_{x}=V_{B}[x]-6\left[x^{2}\right] \\
& \Rightarrow M_{x}=48(x)-6\left(x^{2}\right)
\end{aligned}
$$

For max B.M

$$
\begin{aligned}
& \frac{d M_{x}}{d x}=0 \\
& \Rightarrow 48-12 \times 20 \Rightarrow x=4 m \\
& (M x)_{\max }=48(4)-6(4)^{2}=96 \mathrm{KN}-\mathrm{m}
\end{aligned}
$$

9. A body floating in a liquid is in a stable state of equilibrium if its
A. Metacentre coincides with its Centre of gravity
B. Metacentre lies below its centre of gravity
C. Metacentre lies above its centre of gravity
D. Centre of gravity is below its centre of buoyancy

Ans. C
Sol. A measure of stability for floating bodies is the metacentric height GM, which is the distance between centre of gravity $G$ and the metacentre $M$.

A floating body is stable if point $M$ is above $G$, and thus GM is positive.


Equilibrium
Stable
10. A river has a flow of 1000 million litres per day (MLD). $\mathrm{BOD}_{5}$ of $5 \mathrm{mg} /$ litre and Dissolved Oxygen (DO) level of $8 \mathrm{mg} /$ litre before receiving the wastewater discharge at a location. For the existing environmental conditions, the saturation DO level is 10 $\mathrm{mg} /$ litre in the river. Wastewater discharge of 100 MLD with the $\mathrm{BOD}_{5}$ of $200 \mathrm{mg} /$ litre and DO level of $2 \mathrm{mg} /$ litre falls at that location. Assuming complete mixing of wastewater and river water, the immediate DO deficit (in $\mathrm{mg} /$ litre, round off to two decimal places), is
$\qquad$ —.

Ans. 2.545
Sol. For river,

$$
\begin{aligned}
& \text { flow rate }=\mathrm{Q}_{\mathrm{R}}=1000 \mathrm{MLD} \\
& \mathrm{BOD}_{5}=5 \mathrm{mg} / \mathrm{l} \\
& \text { Dissolved oxygen }=\mathrm{DO}_{\mathrm{R}}=8 \mathrm{mg} / \mathrm{l}
\end{aligned}
$$

For waste water,

$$
\text { Flow rate }=\mathrm{Q}_{\mathrm{w}}=100 \mathrm{MLD}
$$

$$
\mathrm{OBD}_{5}=200 \mathrm{mg} / \mathrm{l}
$$

$$
\text { Dissolved oxygen = DOw }=2 \mathrm{mg} / \mathrm{l}
$$

$$
\mathrm{DO}_{\text {mix }}=\frac{\mathrm{DO}_{R} \times \mathrm{Q}_{R}+\mathrm{DO}_{w} \times \mathrm{Q}_{\mathrm{w}}}{\mathrm{Q}_{R}+\mathrm{Q}_{\mathrm{w}}}
$$

$$
\Rightarrow \mathrm{DO}_{\text {mix }}=\frac{2 \times 100+8 \times 1000}{100+1000}
$$

$$
=7.45 \mathrm{mg} / \mathrm{l}
$$

Now, $\mathrm{DO}_{\text {def }}=\mathrm{DO}_{\text {sat }}-\mathrm{DO}_{\text {mix }}$
$=10-7.45=2.545 \mathrm{mg} / \mathrm{l}$
11. During the process of hydration of cement, due to increase in Dicalcium Silicate ( $\mathrm{C}_{2} \mathrm{~S}$ ) content in cement clinker, the heat of hydration
A. Does not change
B. Initially decreases and then increases
C. Decreases
D. Increases

Ans. B
Sol.


Contribution of Cement
Compounds to Strength of Cement


Due to increase in $\mathrm{C}_{2} \mathrm{~S}$ content, early age strength of cement is reduced indicating @ early ages like till 7 to 14 days,
The evolution of heat of hydration is less @ early ages. But increase in $\mathrm{C}_{2} \mathrm{~S}$ content contribute to later age strength i.e., rate of gain of strength in more @ later ages (after 7 to 14 days), indicating increase in heat of hydration at later ages.
12. The data for an agricultural field for a specific month are given below:
Pan Evaporatiom $=100 \mathrm{~mm}$
Effective Rainfall $=20 \mathrm{~mm}$ (after deducting losses due to runoff and deep percolation)
Crop Coefficient $=0.4$
Irrigation Efficiency $=0.5$
The amount of irrigation water (in mm ) to be applied to the field in that month, is
A. 10
B. 20
C. 80
D. 40

Ans. D
Sol. Effective rainfall $=P_{\text {eff }}=20 \mathrm{~mm}$
Water requirement $=$ pan evaporation $\times$ crop coefficient $=100 \times 0.4=40 \mathrm{~mm}$
$\therefore$ Total additional water requirement

$$
=40-20=20 \mathrm{~mm}
$$

Taking efficiency into consideration, we get the requirement $=\frac{20}{0.5}=40 \mathrm{~mm}$
13. The true value of $\ln (2)$ is 0.69 . If the value of $\ln (2)$ is obtained by linear interpolation between $\ln (1)$ and $\ln (6)$, the percentage of absolute error (round off
A. 84
B. 69
C. 48
D. 35

Ans. C
Sol.

$$
\begin{aligned}
& \mathrm{T} . \mathrm{V}=0.69 \\
& \mathrm{In}_{1}=0 \\
& \mathrm{In}_{6}=1.79
\end{aligned}
$$

from interpolating $(\ln 1 \& \ln 6)$ we get $=0.358$ (mv)

$$
\begin{aligned}
& \% \text { Error }=\frac{T \cdot v-m v}{T . v} \times 100 \\
& \% \text { Error }=48.11 \%
\end{aligned}
$$

14. The area of an ellipse represented by an equation $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ is
A. $\frac{\pi \mathrm{ab}}{4}$
B. $\frac{4 \pi \mathrm{ab}}{3}$
C. $\pi \mathrm{ab}$
D. $\frac{\pi \mathrm{ab}}{2}$

Ans. C
Sol. Equation of ellipse is $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$


$$
\begin{aligned}
& \text { Major axis }=2 a \\
& \text { Minor axis }=2 b
\end{aligned}
$$

Area of ellipse for standard equation is $\mathrm{m} a b$.
15. A reinforcing steel bar, partially embedded in concrete, is subjected to a tensile force $P$. The figure that appropriately represents the distribution of the magnitude of bond stress (represented as hatched region), along the embedded length of the bar, is
A.

B.

C.

D.


Ans. D
Sol. As per IS: 456-2000 bond strength varies parabolically, near the applied load is more and away from the applied load is less.
16. Which one of the following statements is NOT correct?
A. In case of a point load. Boussinesq's equation predicts higher value of vertical stress at a point directly beneath the load as compared to Westergaard's equation.
B. The cohesion of normally consolidated clay is zero when triaxial test is conducted under consolidated undrained condition.
C. The ultimate bearing capacity of a strip foundation supported on the surface of sandy soil increases in direct proportion to the width of footing.
D. A clay deposit with a liquidity index greater than unity is in a state of plastic consistency.
Ans. D
Sol. For IL $>1$ is in under liquid condition not plastic.
17. In a soil investigation work at a site, Standard Penetration Test (SPT) was conducted at every 1.5 m interval up to 30 m depth. At 3 m depth, the observed number of hammer blows for three successive 150 mm penetrations were 8 . 6 and 9 , respectively. The SPT $N$-value at 3 m depth is
A. 15
B. 23
C. 17
D. 14

Ans. A
Sol. Blows for first 150 mm is not noted. Thereafter for next 300 mm penetration number of blows are counted.
$\therefore$ SPT value $=9+6=15$
18. In an urban area, a median is provided to separate the opposing streams of traffic.
As per IRC: 86-1983, the desirable minimum width (in $m$, expressed as integer) of the median, is $\qquad$ .
Ans. 3-5
Sol. As per IRC, for urban area the width of median varies from 0.9 m to 5 m . whereas for rural areas, the width varies from 3 m to 5 m .
19. During chlorination process, aqueous (aq) chlorine reacts rapidly with water to form $\mathrm{Cl}^{-}$, HOCl , and $\mathrm{H}^{+}$as shown below

$$
\mathrm{Cl}_{2}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{HOCl}+\mathrm{Cl}^{-}+\mathrm{H}^{+}
$$

The most active disinfectant in the chlorination process from amongst the following, is
A. HOCl
B. $\mathrm{H}^{+}$
C. $\mathrm{H}_{2} \mathrm{O}$
D. $\mathrm{Cl}^{-}$

## Ans. A

Sol. HOCl is 80 times more active than $\mathrm{OCl}^{-}$as it is very unstable. The amount or sum of HOCl and OCl is known as free available chlorine.
20. In a drained triaxial compression test, a sample of sand fails at deviator stress of 150 kPa under confining pressure of 50 kPa . The angle of internal friction (in degree, round off to the nearest integer) of the sample is

Ans. $36.86^{\circ}$
Sol. $\quad \sigma_{3}=50$

$$
\sigma_{1}=200(50+150)
$$

We know that

$$
\sigma_{1}=\sigma_{3}\left(\frac{1+\sin \phi}{1-\sin \phi}\right)+2 C \sqrt{\frac{1+\sin \phi}{1-\sin \phi}}
$$

For sand $C=0$

$$
\begin{aligned}
& 200=50\left(\frac{1+\sin \phi}{1-\sin \phi}\right) \\
& 4=\frac{1+\sin \phi}{1-\sin \phi} \\
& 4-4 \sin \varphi=1+\sin \varphi \\
& 5 \sin \varphi=3 \\
& \phi=\sin ^{-1}\left(\frac{3}{5}\right) \\
& \phi=36.86^{\circ}
\end{aligned}
$$

21. Consider the planar truss shown in the figure (not drawn to the scale)


Neglecting self-weight of the members, the number of zero-force members in the truss under the action of the load $P$, is
A. 7
B. 9
C. 6
D. 8

Ans. A
Sol. Using Golden Rules -1 \& 2,


No. of zero force members $=8$
Idealized structure


No. of zero force members are 8.
Using golden rules 1 and 2 we can find 0 forces. No. of zero force members are " 8 "
As displacement between "A" \& "B" points in vertical direction is $=0$, i.e. $\Delta_{A B}=0$,
Resisting force $=F_{A B}=0$
22. Uniform flow with velocity $U$ makes an angle $\theta$ with the $y$-axis, as shown in the figure


The velocity potential $(\varphi)$, is
A. $\pm U(x \sin \theta-y \cos \theta)$
B. $\pm U(y \sin \theta+x \cos \theta)$
C. $\pm U(x \sin \theta+y \cos \theta)$
D. $\pm U(y \sin \theta-x \cos \theta)$

Ans. C
Sol. We know that, $u_{x}=\frac{-\partial \phi}{\partial x}$

$$
u_{y}=\frac{-\partial \phi}{\partial y}
$$

velocity in $x$-direction, $u_{x}=u \sin \theta$ velocity in $y$-direction, $u_{y}=u \cos \theta$ Now,

$$
\frac{-\partial \phi}{\partial x}=u_{x} \Rightarrow \frac{-\partial \phi}{\partial x}=u \sin \theta
$$

Integrating above equation, we get

$$
\varphi=-u \sin \theta x+f(y)+c \rightarrow(1)
$$

Similarly

$$
\varphi=-u \cos \theta y+f(x)+c \rightarrow(2)
$$

From equation (1) and (2), we get

$$
\varphi=-u(x \sin \theta+y \cos \theta)
$$

when, $\frac{\partial \phi}{\partial x}=u_{x}$ and $\frac{\partial \phi}{\partial y}=u_{y}$

$$
\begin{aligned}
& \varphi=\mathrm{u}(\mathrm{x} \sin \theta+\mathrm{y} \cos \theta) \\
& \therefore \varphi= \pm \mathrm{u}(\mathrm{x} \sin \theta+\mathrm{y} \cos \theta)
\end{aligned}
$$

23. Velocity of flow is proportional to the first power of hydraulic gradient in Darcy's law. This is law is applicable to
A. Turbulent flow in porous media
B. Transitional flow in porous media
C. Laminar flow in porous media
D. Laminar as well as turbulent flow in porous media

Ans. C
Sol. The simplest form of Darcy's law states that

$$
\mathrm{V}=\frac{\mathrm{Q}}{\mathrm{~A}}=-\mathrm{K} \frac{\partial \mathrm{~h}}{\partial \ell}
$$

Since the velocity in laminar flow is proportional to the first power of the hydraulic gradient (Poiseuille's law), it is reasonable to apply Darcy's law to laminar flow in porous media.
24. In the following partial differential equation, $\theta$ is a function of $t$ and $z$. and $D$ and $K$ are functions of $\theta$

$$
\mathrm{D}(\theta) \frac{\partial^{2} \theta}{\partial \mathbf{z}^{2}}+\frac{\partial \mathrm{K}(\theta)}{\partial \mathbf{z}}-\frac{\partial \theta}{\partial \mathrm{t}}=0
$$

The above equation is
A. A second order linear equation
B. A second-degree non-linear equation
C. A second order non-linear equation
D. A second-degree linear equation

Ans. C
25. A road in a hilly terrain is to be laid at a gradient of $4.5 \%$. A horizontal curve of radius 100 m is laid at a location on this road. Gradient needs to be eased due to combination of curved horizontal and vertical profiles of the road. As per IRC, the compensated gradient (in \%, round off to one decimal place), is $\qquad$ -.
Ans. 4

Sol. According to IRC

$$
\left.\begin{array}{l}
\text { Grade compensation }=\frac{30+R}{R} \\
\frac{75}{R}
\end{array}\right\}_{\text {minm. }} \text { Now, } \frac{30+100}{100}=1.3 \% \text {. }
$$

Compensated grade $=4.5-0.75=3.75 \%$
But, according to IRC the minimum value of compensated grade is 4\%. Therefore answer is 4\%.
26. Traffic volume count has been collected on a 2lane road section which needs upgradation due to severe traffic flow condition. Maximum service flow rate per lane is observed as 1280 veh/h at level of service "C". The Peak Hour Factor is reported as 0.78125 . Historical traffic volume count provides Annual Average Daily Traffic as 12270 veh/day. Directional split of the traffic flow is observed to be 60:40. Assuming that traffic stream consists of "All Cars' and all drivers are 'Regular Commuters', the number of extra lane(s) (round off to the next higher integer) to be provided, is $\qquad$ .

Ans. 6
Sol. Given,
Peak hour factor $=$ PHF $=0.78125$
Maximum service flow rate of LOS

$$
\begin{aligned}
& =\mathrm{MSF}=1280 \\
& \mathrm{~F}_{\mathrm{HV}}=1
\end{aligned}
$$

Road under familiarity adjustment ejector

$$
=f_{p}=1
$$

Now, we know that, number of lane (N)

$$
\mathrm{N}=\frac{\mathrm{DDHV}}{\mathrm{PHF} \times \mathrm{MSF} \times \mathrm{F}_{\mathrm{HV}} \times \mathrm{f}_{\mathrm{p}}}
$$

Here, DDHV = Directional distribution hour volume.

$$
\text { DDHV }=122270 \times 0.6=7362
$$

$\therefore \mathrm{N}=\frac{7362}{0.78125 \times 1280 \times 1 \times 1}$
$=7.362=8$ lanes
Number of extra lanes $=8-2=6$ lanes.
27. A rigid, uniform, weightless, horizontal bar is connected to three vertical members $\mathrm{P}, \mathrm{Q}$ and $R$ as shown in the figure (not drawn to the scale). All three members have identical axial stiffness of $10 \mathrm{kN} / \mathrm{mm}$. The lower ends of bar $P$ and $R$ rest on a rigid horizontal surface. When NO load is applied, a gap of 2 mm exists between the lower end of the bar Q and the rigid horizontal surface. When a vertical load W is placed on the horizontal bar in the downward direction, the bar still remains horizontal and gets displaced by 5 mm in the vertically downward direction.


The magnitude of the load W (in kN, round off to the nearest integer), is
Ans. 130

Sol. $\quad \frac{A E}{L}=10 \mathrm{kN} / \mathrm{mm}$


$$
\mathrm{w}_{1}+\mathrm{w}_{2}+\mathrm{w}_{3}=\mathrm{w}
$$

from symmetry $\Rightarrow \mathrm{w}_{1}=\mathrm{w}_{3}$

$$
\begin{aligned}
& \delta_{P}=5 \mathrm{~mm}=\frac{\mathrm{w}_{1} \mathrm{~L}}{\mathrm{AE}} \\
& \therefore \mathrm{~W}_{1}=50 \mathrm{kN}=\mathrm{W}_{3} \\
& \delta_{Q}=\frac{\mathrm{W}_{2} \mathrm{~L}}{\mathrm{AE}} \\
& \therefore \mathrm{~W}_{2}=30 \mathrm{kN} \\
& \therefore \mathrm{~W}=130 \mathrm{kN}
\end{aligned}
$$

28. In a homogeneous unconfined aquifer of area $3.00 \mathrm{~km}^{2}$, the water table was at an elevation of 102.00 m . After a natural recharge of volume 0.90 million cubic meter $\left(\mathrm{Mm}^{3}\right)$, the water table rose to 103.20 m . After this recharge, ground water pumping took place and the water table dropped down to 101.20 m . The volume of ground water pumped after the natural recharge, expressed (in $\mathrm{Mm}^{3}$ and round off to two
Ans. 1.5
Sol. Volume of water coming $=0.9 \times 10^{6} \mathrm{~m}^{3}$
Height of water $=\frac{0.9 \times 10^{6}}{3 \times 10^{6}}=0.3 \mathrm{~m}$

$$
\begin{aligned}
& \mathrm{n}=\text { Porosity }=\frac{\mathrm{V}_{\mathrm{V}}}{\mathrm{~V}_{\mathrm{T}}} \\
& \mathrm{n}=\frac{0.3}{1.2}=0.25
\end{aligned}
$$

Now pumping
Volume $=$ Area $\times$ Height

$$
=3 \times 10^{6} \times 2 \times 0.25=1.5 \times 10^{6} \mathrm{~m}^{3}
$$

29. The total stress paths corresponding to different loading conditions, for a soil specimen under the isotropically consolidated stress state ( O ), are shown below



| Stress Path | Loading Condition |
| :---: | :---: |
| OP | I-Compression loading <br> $\left(\sigma_{1}-\right.$ increasing; $\sigma_{3}-$ constant $)$ |
| OQ | II-Compression unloading <br> $\left(\sigma_{1}-\right.$ constant; $\sigma_{3}-$ decreasing $)$ |
| OR | III- Extension unloading <br> $\left(\sigma_{1}-\right.$ decreasing; $\sigma_{3}-$ constant $)$ |
| OS | IV - Extension loading <br> $\left(\sigma_{1}-\right.$ constant; $\sigma_{3}$-increasing $)$ |

The correct match between the stress paths and the listed loading conditions, is
A. OP - IV, OQ - III, OR-I, OS - II
B. OP - III, OQ - II, OR - I, OS - IV
C. OP - I, OQ - III, OR - II, OS - IV
D. OP - I, OQ - II, OR - IV, OS - III

Ans. A

Sol.

$\sigma_{1}$ const $\sigma_{3} \uparrow \rightarrow$ y coord $\downarrow \& \times \uparrow$

$\sigma_{1} \downarrow \& \sigma_{3}=$ const $\rightarrow$ yaxis $\downarrow \& \times \downarrow$

$\sigma_{1} \uparrow \& \sigma_{3}=$ const $=x a x i s \uparrow \& y \uparrow$

$\sigma_{1}$ const $\sigma_{3} \downarrow \rightarrow$ yaxis $\uparrow \& y \downarrow$

30. A continuous function/ft) is defined. If the third derivative at $x$, is to be computed by using the fourth order central finite-divided-difference scheme (with step length = h), the correct formula is

$$
f\left(x_{i+3}\right)-8 f\left(x_{i+2}\right)+13 f\left(x_{i+1}\right)
$$

A. $f^{\prime \prime \prime}\left(x_{i}\right)=\frac{+13 f\left(x_{i-1}\right)-8 f\left(x_{i-2}\right)-f\left(x_{i-3}\right)}{8 h^{3}}$

$$
-f\left(x_{i+3}\right)-8 f\left(x_{i+2}\right)-13 f\left(x_{i+1}\right)
$$

B. $f^{\prime \prime \prime}\left(x_{i}\right)=\frac{+13 f\left(x_{i-1}\right)+8 f\left(x_{i-2}\right)-f\left(x_{i-3}\right)}{8 h^{3}}$

$$
f\left(x_{i+3}\right)-8 f\left(x_{i+2}\right)-13 f\left(x_{i+1}\right)
$$

C. $f^{\prime \prime \prime}\left(x_{i}\right)=\frac{+13 f\left(x_{i-1}\right)+8 f\left(x_{i-2}\right)+f\left(x_{i-3}\right)}{8 h^{3}}$

$$
-f\left(x_{i+3}\right)+8 f\left(x_{i+2}\right)-13 f\left(x_{i+1}\right)
$$

D. $f^{\prime \prime \prime}\left(x_{i}\right)=\frac{+13 f\left(x_{i-1}\right)-8 f\left(x_{i-2}\right)+f\left(x_{i-3}\right)}{8 h^{3}}$

Ans. D
31. A gaseous chemical has a concentration of $41.6 \mu \mathrm{~mol} / \mathrm{m}^{3}$ in air at 1 atm pressure and temperature 293 K . The universal gas constant $R$ is $82.05 \times 10^{-6}\left(\mathrm{~m}^{3} \mathrm{~atm}\right) /(\mathrm{mol} \mathrm{K})$. Assuming that ideal gas law is valid, the concentration of the gaseous chemical (in ppm, round off to one decimal place), is $\qquad$ .

Ans. 1
Sol. We know that,

$$
\mathrm{PV}=\mathrm{Nrt}
$$

Where, $\mathrm{P}=$ Pressure
$\mathrm{V}=$ Volume
$\mathrm{n}=$ number of moles
R = Universal gas constant
T = Temperature
So, $V=\frac{n R T}{P}$

$$
\begin{aligned}
& =\frac{41.6 \times 10^{-6} \times 32.05 \times 10^{-6}}{1} \times 293 \\
& \Rightarrow V=10^{-6} \mathrm{~m}^{3} \\
& 1 \mathrm{ppm}=\frac{1 \text { part of gas }}{10^{6} \text { parts of air }}=\frac{1 \mathrm{~m}^{3} \text { of gas }}{10^{6} \mathrm{~m}^{3} \text { of air }} \\
& \Rightarrow 1 \mathrm{ppm}=\frac{41.6 \times 10^{6}-4 \text { moles }}{10^{6} \mathrm{~cm}^{3}}
\end{aligned}
$$

Since, 41.6-4 mole of gas volume of $10^{-6} \mathrm{~m}^{3}$ $\therefore 41.6-4 \mathrm{moles} / \mathrm{m}^{3}=1 \mathrm{ppm}$
32. Water flows in the upward direction in a tank through 2.5 m thick sand layer as shown in the figure. The void ratio and specific gravity of sand are 0.58 and 2.7 , respectively. The sand is fully saturated. Unit weight of water is 10 $k N / m^{3}$.


The effective stress (in kPa, round off to two decimal places) at point $A$, located 1 m above the base of tank, is
Ans. 8.925 kw/m ${ }^{3}$
Sol.


Taking 2 as datum

|  | PH | VH | DH | TH |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 4.7 | 0 | 0 | 4.7 |
| 1 | 0 | 0 | 3.5 | 3.5 |
| A | x | 0 | 1 | $4.7-0.48=4.22$ |

$$
\begin{aligned}
& x+0+1=4.22 \\
& x=3.22 \\
& i=\frac{h c}{c}=\frac{1.2}{2.5}=.48 \\
& \bar{C}_{A}^{\bar{A}}=\sigma-u \\
& { }_{A}^{\bar{\sigma}}=8.925 \mathrm{kw} / \mathrm{m}^{3}
\end{aligned}
$$

33. For the Ordinary Differential Equation $\frac{d^{2} x}{d t^{2}}-5 \frac{d x}{d t}+6 x=0 \quad, \quad$ with initial conditions $x(0)=0$ and $\frac{d x}{d t}(0)=10$, the solution is
A. $-10 e^{2 t}+10 e^{3 t}$
B. $10 \mathrm{e}^{2 \mathrm{t}}+10 \mathrm{e}^{3 \mathrm{t}}$
C. $5 e^{2 t}+6 e^{3 t}$
D. $-5 e^{2 t}+6 e^{3 t}$

Ans. A
Sol. $\quad \frac{d^{2} x}{{d t^{2}}^{2}}-\frac{5 d x}{d t}+6 x=0$

$$
\begin{aligned}
& D^{2}-5 D+6=0 \\
& D^{2}-3 D-2 D+6=0 \\
& D(D-3)-2(D-3)=0 \\
& D=2,3
\end{aligned}
$$

Solution will be, $x=C_{1} e^{3 t}+C_{2} e^{2 t}$
Applying boundary condition

$$
\begin{aligned}
& x(0)=0 \\
& C_{1}+C_{2}=0 \rightarrow(1) \\
& \frac{d x}{d t}=3 C_{1} e^{3 t}+2 c_{2} e^{2 t} \\
& \frac{d x}{d t}(0)=10 \\
& 10=3 c 1+2 c 2 \rightarrow(2)
\end{aligned}
$$

Form equation (1) and (2)

$$
C_{1}=10 C_{2}=-10
$$

So, final solution becomes, $x=10 e^{3 t}-10 e^{2 t}$
Or

$$
x=-10 e^{2 t}+10 e^{3 t}
$$

34. The soil profile at a site up to a depth of 10 m is shown in the figure (not drawn to the scale). The soil is preloaded with a uniform surcharge (q) of $70 \mathrm{kN} / \mathrm{m}^{2}$ at the ground level. The water table is at a depth of 3 m below ground level. The soil unit weight of the respective layers is shown in the figure. Consider unit weight of water as $9.81 \mathrm{kN} / \mathrm{m}^{3}$ and assume that the surcharge ( $q$ ) is applied instantaneously.


Boulder clay
Immediately after preloading, the effective stresses (in kPa ) at points P and Q . respectively, are
A. 54 and 95
B. 36 and 90
C. 36 and 126
D. 124 and 204

Ans. A
Sol. Surcharge is applied instantaneous
$\therefore$ Extra pore water pressure of 7 kPa is develated.

$$
\begin{aligned}
& \overline{\sigma_{p}}=\sigma_{p}-v_{p} \\
& \sigma_{p}=70+18 \times 3=124 \\
& u_{p}=0+70=70 \\
& \overline{\sigma_{p}}=124-70=54 \\
& \overline{\sigma_{q}}=\sigma_{q}-v_{q} \\
& \sigma_{q}=70+18 \times 3+20 \times 4=204 \\
& \mathrm{v}_{\mathrm{q}}=9.81 \times 4+70=109.24 \\
& \overline{\sigma_{p}}=204-109.24=94.76 \mathrm{kPa}
\end{aligned}
$$

35. A rigid weightless platform PQRS shown in the figure (not drawn to the scale) can slide freely in the vertical direction. The platform is held in position by the weightless member OJ and four weightless, frictionless rollers, Points $O$ and $J$ are pin connections. A block of 90 kN rests on the platform as shown in the figure


The magnitude of horizontal component of the reaction (in kN ) at pin O , is
A. 90
B. 120
C. 150
D. 180

Ans. B
Sol. Since the vertical load is resisted by bar OJ only


FBD of OJ

$$
\begin{aligned}
& \Sigma M J=0 \\
& H_{0} \times 3+90 \times 4=0 \\
& H_{0}=120 \mathrm{kN}
\end{aligned}
$$

36. A vertical retaining wall of 5 m height has to support soil having unit weight of $18 \mathrm{kN} / \mathrm{m}^{3}$, effective cohesion of $12 \mathrm{kN} / \mathrm{m}^{3}$, and effective friction angle of $30^{\circ}$. As per Rankine's carth pressure theory and assuming that a tension crack has occurred, the lateral active thrust on the wall per meter length (in $\mathrm{kN} / \mathrm{m}$, round off to two decimal places), is

Ans. $21.72 \mathrm{kw} / \mathrm{m}$
Sol.


$$
\mathrm{ka}=\frac{1-\sin 30}{1+\sin 30}=\frac{1-.5}{1+.5}=\frac{.5}{1.5}=1 / 3
$$

Depth of tension crack $\frac{2 c}{\gamma \sqrt{\text { ka }}}=\frac{2 \times 12}{18 \sqrt{\frac{1}{3}}}=2.309$

$$
\begin{aligned}
& \sigma_{H}=k a \sigma_{v}-2 c \sqrt{k a} \\
& \sigma_{H}=\frac{1}{3} \times 18 \times z-2 \times 12 \times \sqrt{1 / 3} \\
& \sigma_{H}=6 z-13.856 \\
& \sigma_{H} \text { AT TOP }(z=0)=-13.856 \\
& \sigma_{H} \text { AT BOTTOM }(z=5)=16.144 \\
& \text { Net thurst }=\left[\frac{0+16.144}{2}\right] \times 2.69 \times 1
\end{aligned}
$$

$$
=21.72 \mathrm{kw} / \mathrm{m}
$$

37. Surface Overflow Rate (SOR) of a primary settling tank (discrete settling) is 20000 litre/m² per day. Kinematic viscosity of water in the tank is $1.01 \times 10^{-2} \mathrm{~cm}^{2} / \mathrm{s}$. Specific gravity of the settling particles is 2.64 . Acceleration due to gravity is $9.81 \mathrm{~m} / \mathrm{s}^{2}$. The minimum diameter (in $\mu \mathrm{m}$, round off to one decimal place) of the particles that will be removed with $80 \%$ efficiency in the tank, is .

Ans. 14.46
Sol. According to stokes law,

$$
\begin{aligned}
& V_{S}=\frac{(4-1) \mathrm{gc}^{2}}{18 v} \\
& \text { Also, } \eta=80=\frac{u_{s}}{v_{s}} \times 100 \\
& \text { Here, } \mathrm{v}_{\mathrm{s}}=20000 \mathrm{I} / \mathrm{m}^{2} / \text { day } \\
& =\text { surface over flow rate } \\
& \Rightarrow u_{s}=\frac{0.8 \times 20000 \times 10^{-3}}{86400} \\
& =1.85 \times 10^{-4} \mathrm{~m} / \mathrm{sec} \\
& \therefore 1.85 \times 10^{-4}=\frac{(2.64-1) 9.81 \times \mathrm{d}^{2}}{18 \times 1.01 \times 10^{-2} \times 10^{-4}} \\
& \Rightarrow d=1.446 \times 10^{-5} \mathrm{~m} \\
& \Rightarrow d=14.46 \times 10^{-6} \mu \mathrm{~m}
\end{aligned}
$$

38. Distributed load (s) of $50 \mathrm{kN} / \mathrm{m}$ may occupy any position(s) (either continuously or in patches) on the girder PQRST as shown in the figure (not drawn to the scale)


The maximum negative (hogging) bending moment (in kN.m) that occurs at point $R$, is
A. 56.25
B. 150.00
C. 93.75
D. 22.50

Ans. A
Sol.


Maximum hogging moment at " $R$ " is possible if the UDL is loaded on over hang spans (PQ \& ST)
Magnitude of Hogging moment

$$
\begin{aligned}
& =50 \times\left\{-\frac{1}{2} \times 1.5 \times 0.6\right\}+50 \times\left\{-\frac{1}{2} \times 1.5 \times 0.9\right\} \\
& =-(0.45+0.675) \times 50 \\
& =-56.25 \mathrm{kN}-\mathrm{m}(\mathrm{Hog})
\end{aligned}
$$

39. A stream with a flow rate of $5 \mathrm{~m}^{3} / \mathrm{s}$ is having an ultimate BOD of $30 \mathrm{mg} /$ litre. A wastewater discharge of $0.20 \mathrm{~m} 3 / \mathrm{s}$ having $\mathrm{BOD}_{5}$ of 500 $\mathrm{mg} /$ litre joins the stream at a location and instantaneously gets mixed up completely. The cross-sectional area of the stream is $40 \mathrm{~m}^{2}$ which remains constant. BOD exertion rate constant is 0.3 per day (logarithm base to e). The BOD (in mg/litre, round off to two decimal places) remaining at 3 km downstream from the mixing location, is .

Ans. 49.57
Sol. We know that

$$
L_{1}=L o e^{-k \times t}
$$

$\mathrm{L}_{0}=$ Dissolved oxygen of $\mathrm{mi}=\mathrm{DO}_{\text {mix }}$

$$
\mathrm{DO}_{\text {mix }}=\frac{\mathrm{Q}_{\mathrm{R}} \mathrm{BOD}_{\mathrm{ur}_{\mathrm{R}}}+\mathrm{Q}_{\mathrm{S}} \cdot \mathrm{BOD}_{\mathrm{u}}}{\mathrm{Q}_{\mathrm{S}}+\mathrm{Q}_{\mathrm{R}}}
$$

Here, $\mathrm{Q}_{\mathrm{s}}=$ flow rate of sewage
$Q_{R}=$ flow rate of river.

For sewage

$$
\text { Also, } \begin{aligned}
& B O D_{s}=B O D_{u}\left(1-\mathrm{e}^{-5 \mathrm{k}}\right) \\
& \Rightarrow B O D_{o}=\frac{500}{1-\mathrm{e}^{-0.3 \times 5}} \\
&=643.66 \mathrm{mg} / \mathrm{l} \\
& D O_{\text {mix }}=\frac{5 \times 30+0.2 \times 643.66}{5+0.2} \\
& D O_{\text {mix }}=53.6 \mathrm{mg} / \mathrm{l} \\
& \mathrm{~L}_{x}=53.6 \mathrm{e}^{-0.3 \times 0.26} \\
& L_{x}=49.57 \mathrm{mg} / \mathrm{l}
\end{aligned}
$$

40. A water supply scheme transports 10 MLD (Million Litres per Day) water through a 450 mm diameter pipeline for a distance of 2.5 km . A chlorine dose of $3.50 \mathrm{mg} /$ litre is applied at the starting point of the pipeline to attain a certain level of disinfection at the downstream end. It is decided to increase the flow rate from 10 MLD to 13 MLD in the pipeline. Assume exponent for concentration, $\mathrm{n}=0.86$. With this increased flow, in order to attain the same level of disinfection, the chlorine dose
A. 3.95
B. 4.40
C. 5.55
D. 4.75

Ans. D
Sol. In the disinfection process we have the relationship

$$
\mathrm{tc}^{\mathrm{n}}=\mathrm{k}
$$

where, $\mathrm{t}=$ time required to kill all organism
$\mathrm{c}=$ concentration of disinfectant
$\mathrm{K}=$ constant

$$
\begin{aligned}
& \therefore \mathrm{t}_{1} \mathrm{C}_{1}^{\mathrm{n}}=\mathrm{t}_{2} \mathrm{c}_{2}^{\mathrm{n}} \\
& \Rightarrow \text { we know, } \mathrm{Q}=\frac{\mathrm{V}}{\mathrm{t}} \Rightarrow \mathrm{t}=\frac{\mathrm{V}}{\mathrm{Q}} \\
& \therefore \frac{\mathrm{~V}}{\mathrm{Q}_{1}} \mathrm{C}_{1}^{\mathrm{n}}=\frac{\mathrm{V}}{\mathrm{Q}_{2}} \mathrm{c}_{2}^{\mathrm{n}} \\
& \Rightarrow \frac{1}{10}(3.5)^{0.86}=\frac{1}{13} \times(\mathrm{c})^{0.86} \\
& \Rightarrow \mathrm{c}=4.75 \mathrm{mg} / \mathrm{l}
\end{aligned}
$$

41. If $C$ represents a line segment between ( 0,0 , $0)$ and (1, 1, 1) in Cartesian coordinate system, the value (expressed as integer) of the line integral
$\int_{C}[(y+z) d x+(x+z) d y+(x+y) d z]$ is $\qquad$ .

Ans. 3
Sol. Equation of a line segment

$$
\begin{aligned}
& \frac{x}{1}=\frac{y}{1}=\frac{z}{1}=k \\
& x=k, y=k, z=k \Rightarrow d x=d k \\
& 0 \leq x \leq 1 \\
& \therefore 0 \leq k \leq 1 \\
& \int_{0}^{1}(2 k) d k \cdot+\int_{0}^{1 / 2} k d k+\int_{0}^{1} 2 k d k \\
& =3\left[\mathrm{k}^{2}\right]_{0}^{1} \\
& =3
\end{aligned}
$$

42. The singly reinforced concrete beam section shown in the figure (not drawn to the scale) is made of M25 grade concrete and Fe500 grade reinforcing steel. The total cross-sectional area of the tension steel is $942 \mathrm{~mm}^{2}$.


As per Limit State design of IS 456-2000, the design moment capacity (in kN.m, round off to two decimal places) of the beam section, is

Ans. 158.277
Sol. $A_{s t}=942 \mathrm{~mm}^{2}, \mathrm{f}_{\mathrm{ck}}=25 \mathrm{~N} / \mathrm{mm}^{2}$
For Fe500 $\rightarrow \mathrm{X}_{\mathrm{u}, \max }=0.46 \mathrm{~d}$

$$
\begin{aligned}
& =0.46 \times(450) \\
& x_{u}, \max =207 \mathrm{~mm}
\end{aligned}
$$

Finding actual depth of neutral axis

$$
\begin{aligned}
& C=T \\
& 0.36 \mathrm{f}_{\mathrm{ck}} \cdot \mathrm{~b} \cdot \mathrm{x}_{\mathrm{u}}=0.87 \mathrm{fy} . \text { Ast } \\
& \rightarrow 0.36 \times 25 \times 300 \times \mathrm{xu} \\
& =0.87 \times 500 \times 942 \\
& x_{u}=151.766 \mathrm{~mm}
\end{aligned}
$$

$A_{s} X_{u}<X_{u, \max } \rightarrow$ The section is under reinforced section

$$
\begin{aligned}
& \quad M_{u}=M . R=0.36 f_{c k} . \text { b. } X_{u}\left(d-0.42 x_{u}\right) \\
&= 0.36 \times 25 \times 300 \times 151.766 \times(450-0.42 \\
& \times151.76)
\end{aligned}
$$

$$
M_{u}=158.277 \mathrm{kN}-\mathrm{m}
$$

43. A cantilever beam $P Q$ of uniform flexural rigidity (EI) is subjected to a concentrated moment $M$ at $R$ as shown in the figure


The deflection at the free end Q is
A. $\frac{3 \mathrm{ML}^{2}}{4 E I}$
B. $\frac{3 M L^{2}}{8 E I}$
C. $\frac{\mathrm{ML}^{2}}{4 \mathrm{EI}}$
D. $\frac{\mathrm{ML}^{2}}{6 \mathrm{EI}}$

Ans. B
Sol. $\quad \delta_{R}+\theta_{R} \cdot \frac{L}{2}$

$$
\begin{aligned}
& =\frac{M\left(\frac{L}{2}\right)^{2}}{2 E I}+\frac{M \frac{L}{2}}{E I} \times \frac{L}{2} \\
& =\frac{3}{8} \frac{M L^{2}}{E I}
\end{aligned}
$$

44. The flange and web plates of the doubly symmetric built-up section are connected by continuous 10 mm thick fillet welds as shown in the figure (not drawn to the scale). The moment of inertia of the section about its principal axis $X-X$ is $7.73 \times 10^{6} \mathrm{~mm}^{4}$. The permissible shear stress in the fillet welds is $100 \mathrm{~N} / \mathrm{mm}^{2}$. The design shear strength of the section is governed by the capacity of the fillet welds.


The maximum shear force (in $k N$, round off to one decimal place) that can be carried by the section, is $\qquad$ .

Ans. 393.5
Sol.


Permissible stress in weld $=\mathrm{q}=\frac{\mathrm{VA} \overline{\mathrm{y}}}{\mathrm{Ib}}$

$$
\begin{aligned}
& 100=\frac{(100 \times 10) \times(60-5)}{7.73 \times 10^{6} \times(4 \times 0.7 \times 10)} \\
& \mathrm{V}=393.527 \mathrm{kN} \\
& \mathrm{~V}=393.5 \mathrm{kN}
\end{aligned}
$$

45. A dowel bar is placed at a contraction joint. When contraction occurs, the concrete slab cracks at predetermined location (s). Identify the arrangement, which shows the correct placement of dowel bar and the place of occurrence of the contraction crack(s).
A.

B.

C.

D.


Ans. B
46. An open traverse PQRST is surveyed using theodolite and the consecutive coordinates obtained are given in the table

| Lin | Consecutive Coordinates |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Northin <br> $\mathbf{g ( m )}$ | Southin <br> $\mathbf{g ( m )}$ | Eastin <br> $\mathbf{g ( m )}$ | Westin <br> $\mathbf{g ( m )}$ |
| PQ | 110.2 | - | 45.5 | - |
| QR | 80.6 | - | - | 60.1 |
| RS | - | 90.7 | - | 70.8 |
| ST | - | 105.4 | 55.5 | - |

If the independent coordinates (Northing, Easting) of station P are ( 400 m, 200 m ), the independent coordinates (in m ) of station T , are
A. $194.7,370.1$
B. $405.3,229.9$
C. 205.3, 429.9
D. $394.7,170.1$

Ans. D
Sol.

$$
\begin{aligned}
& \Sigma L=190.8-196.1=-5.3 \\
& \Sigma D=101-130.9=-29.9 \\
& P(400 m, 200 m)
\end{aligned}
$$

Coordinate of T are

$$
\begin{aligned}
& =(400-5.3,200-29.9) \\
& =(394.7,170.1)
\end{aligned}
$$

47. Water flows at the rate of $12 \mathrm{~m}^{3} / \mathrm{s}$ in a 6 m wide rectangular channel. A hydraulic jump is formed in the channel at a point where the upstream depth is 30 cm (just before the jump). Considering acceleration due to gravity as $9.81 \mathrm{~m} / \mathrm{s}^{2}$ and density of water as 1000 $\mathrm{kg} / \mathrm{m}^{3}$, the energy loss in the jump is
A. 114.2 MW
B. 114.2 kW
C. $141.2 \mathrm{~J} / \mathrm{s}$
D. 141.2 h.p.

Ans. B
Sol. Head loss in a hydraulic jump

$$
=\mathrm{h}_{1}=\frac{\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right)^{3}}{4 \mathrm{y}_{1} \mathrm{y}_{2}}
$$

Here, $\mathrm{y}_{1}=0.3 \mathrm{~m}$
We know that,

$$
\begin{gathered}
Y_{2}=\frac{y_{1}}{2}\left(-1+\sqrt{1+8 \mathrm{~F}_{r_{1}}^{2}}\right) \\
\text { So, } \quad F_{r_{1}}=\left(\frac{\mathrm{q}^{2}}{\mathrm{qy}_{1}^{3}}\right)^{1 / 2}=\left(\frac{2^{2}}{\left(9.81 \times 0.3^{3}\right)}\right)^{1 / 2}=3.88 \\
\therefore \mathrm{y}_{2}=\frac{0.3}{2}\left[-1+\sqrt{1+8 \times 3.88^{2}}\right] \\
\Rightarrow \mathrm{y}_{2}=1.505 \mathrm{~m}
\end{gathered}
$$

$$
\text { Now, } \begin{aligned}
h_{2} & =\frac{(1.505-0.3)^{2}}{4 \times 1.505 \times 0.3} \\
& =0.968 \mathrm{~m}
\end{aligned}
$$

Power lost $=r_{w} Q_{2}=9.81 \times 12 \times 0.968 \mathrm{~kW}$

$$
=114.04 \mathrm{~kW}
$$

48. A circular water tank of 2 m diameter has a circular orifice of diameter 0.1 m at the bottom. Water enters the tank steadily at a flow rate of 20 litre/s and escapes through the orifice. The coefficient of discharge of the orifice is 0.8. Consider the acceleration due to gravity as $9.81 \mathrm{~m} / \mathrm{s}^{2}$ and neglect frictional losses. The height of the water level (in $m$, round off to two decimal places) in the tank at the steady state, is $\qquad$ .
Ans. 0.52
Sol. Assuming, H is the level of water in the circular tank


Discharge through orifice $=$ water enters in the tank

$$
\begin{aligned}
& \text { Cd. a } \sqrt{2 \mathrm{gH}}=20 \times 10^{-3} \\
& \Rightarrow 0.8 \times \frac{\pi}{4}(0.1)^{2} \times \sqrt{2 \times 9.81 \times \mathrm{H}} \\
& =20 \times 10^{-3} \\
& \Rightarrow \mathrm{H}=0.5164 \mathrm{~m}
\end{aligned}
$$

49. A simply supported prismatic concrete beam of rectangular cross-section, having a span of 8 m . is prestressed with an effective prestressing force of 600 kN . The eccentricity of the prestressing tendon is zero at supports and
varies linearly to a value of e at the mid-span. In order to balance an external concentrated load of 12 kN applied at the mid-span, the required value of e (in mm. round off to the nearest integer) of the tendon, is $\qquad$ _.

Ans. 40 mm
Sol.

50. Consider the system of equations

$$
\left[\begin{array}{rrr}
1 & 3 & 2 \\
2 & 2 & -3 \\
4 & 4 & -6 \\
2 & 5 & 2
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]=\left[\begin{array}{l}
1 \\
1 \\
2 \\
1
\end{array}\right]
$$

The value of $x_{3}$ (round off to the nearest integer) is $\qquad$ .
Ans. 3
Sol. Solving the above matrix by multiplication

$$
\begin{align*}
& x_{1}+3 x_{2}+2 x_{3}=1  \tag{1}\\
& 2 x_{1}+2 x_{2}-3 x_{3}=1  \tag{2}\\
& 4 x_{1}+4 x_{2}-6 x_{3}=2  \tag{3}\\
& 2 x_{1}+5 x_{2}+2 x_{3}=1 \tag{4}
\end{align*}
$$

Solving equation (1) and (2) eliminating $x_{1}$ we get,

$$
\begin{align*}
& 2 x_{1}+6 x_{2}+4 x_{3}=2 \\
& 2 x_{1}+2 x_{2}-3 x_{3}=1 \\
& -\quad-\quad+\quad  \tag{5}\\
& \hline 4 x_{2}+7 x_{3}=1
\end{align*}
$$

Solving equation (3) and (4) eliminating $x_{1}$, we get

$$
\begin{align*}
& 4 x_{1}+4 x_{2}-6 x_{3}=2 \\
& 4 x_{1}+10 x_{2}+4 x_{3}=2 \\
& -\quad-\quad-\quad-  \tag{6}\\
& \hline-6 x_{2}+10 x_{3}=0
\end{align*}
$$

Solving equation (5) and (6) we get

$$
\begin{aligned}
& 4 x_{2}+7 x_{3}=1 \\
& -6 x_{2}-10 x_{3}=0 \\
& 24 x / 2+42 x_{3}=6 \\
& \frac{-224 x_{2}-40 x_{3}=0}{2 x_{3}=6} \\
& x_{3}=3
\end{aligned}
$$

51. The relationship between traffic low rate (q) and density ( $D$ ) is shown in the figure


The shock wave condition is depicted by
A. Flow with respect to point $1\left(q_{1}=q_{\max }\right)$
B. Flow changing from point 2 to point $6\left(\mathrm{q}_{2}>\right.$ q6)
C. Flow with respect to point 4 and point 5 ( $\mathrm{q}_{4}$ = q5)
D. Flow changing from point 3 to point $7\left(\mathrm{q}_{3}<\right.$ q7)
Ans. B
52. Three reservoirs $P, Q$, and $R$ are interconnected by pipes as shown in the figure (not drawn to the scale). Piezomtric head at the junction S of
the pipes is 100 m . Assume acceleration due to gravity as $9.81 \mathrm{~m} / \mathrm{s}^{2}$ and density of water as $1000 \mathrm{~kg} / \mathrm{m}^{3}$. The length of the pipe from junction $S$ to the inlet of reservoir $R$ is 180 m .


Considering head loss only due to friction (with friction factor of 0.03 for all the pipes), the height of water level in the lowermost reservoir $R$ (in $m$, round off to one decimal place) with respect to the datum, is $\qquad$ .
Ans. 97.51
Sol.


We know that, $\mathrm{Q}_{\mathrm{s}}=\mathrm{Q}_{\mathrm{B}_{1}}+\mathrm{Q}_{\mathrm{B}_{2}}$
By continuity equation,

$$
\begin{aligned}
& \Rightarrow Q_{s}=A_{1} V_{1}+A_{2} V_{2} \\
& =Q=\frac{\pi}{4} \times 0.3^{2} \times 2.56+\frac{\pi}{4} \times 0.3^{2} \times 1.98 \\
& =0.3209 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

Apply energy equation at 1 and 2

$$
\therefore \mathrm{H}_{1}=\mathrm{H}_{2}+\mathrm{h}_{\mathrm{f}}
$$

$$
\begin{aligned}
& 100=z+\frac{8 Q_{3}^{2}}{z^{2} g} \times \frac{f L_{3}}{D_{3}^{5}} \\
& \Rightarrow 100=z+\frac{8 \times 0.3209^{2}}{z^{2} g} \times \frac{0.03 \times 180}{0.45^{5}} \\
& \Rightarrow z=97.51 \mathrm{~m}
\end{aligned}
$$

53. The appropriate design length of a clearway is calculated on the basis of 'Normal Take-off condition. Which one of the following options correctly depicts the length of the clearway? (Note: None of the options are drawn to scale)
A.

B.

C.


Ans. A
Sol. For standard case, of take-off, Clearway should not be more than

$$
\frac{1}{2}\binom{1.5 \times \text { take off distan ce }}{-1.15 \text { of left off distan ce }}
$$

Here, clearway

$$
\begin{aligned}
& =\frac{1}{2}(1.15 \times 1625-1.15 \times 875) \\
& =431.25 \mathrm{~m}
\end{aligned}
$$

So, clearway is less them 432 m
54. A 10 m thick clay layer is resting over a 3 m thick sand layer and is submerged. A fill of 2 m thick sand with unit weight of $20 \mathrm{kN} / \mathrm{m}^{3}$ is
placed above the clay layer to accelerate the rate of consolidation of the clay layer. Coefficient of consolidation of clay is $9 \times 10^{-2}$ $\mathrm{m}^{2} / y e a r$ and coefficient of volume compressibility of clay is $2.2 \times 10^{-4} \mathrm{~m}^{2} / \mathrm{kN}$. Assume Taylor's relation between time factor and average degree of consolidation.


The settlement (in mm. round off to two decimal places) of the clay layer, 10 years after the construction of the fill, is
Ans. 18.8
Sol.

$$
\begin{aligned}
& \Delta \bar{\sigma}=40 \mathrm{kw} / \mathrm{m}^{2} \\
& \Delta \mathrm{H}=\mathrm{m}_{\mathrm{v}} \times \mathrm{H}_{0} \times \Delta \bar{\sigma} \\
& =2.2 \times 10^{-4} \times 10 \times 40 \\
& =088 \mathrm{~m} \\
& =88 \mathrm{~mm} \rightarrow \text { Total consolidation } \\
& \mathrm{T}_{\mathrm{V}}=\frac{\mathrm{c}_{\mathrm{v}} \mathrm{t}}{\mathrm{~d}^{2}}=\frac{9 \times 10^{-2} \times 10}{5^{2}}=.036 \\
& \mathrm{Tv}=\pi / 4 \mathrm{u}^{2} \mathrm{~V}<60 \% \\
& \mathrm{U}=21.4 \%
\end{aligned}
$$

After layer statement $=.214 \times 88=18.8 \mathrm{~mm}$
55. The length and bearings of a traverse PQRS are

| Segment | Length(m) | Bearing |
| :---: | :---: | :---: |
| PQ | 40 | $80^{\circ}$ |
| QR | 50 | $10^{\circ}$ |
| RS | 30 | $210^{\circ}$ |

The length of line segment SP (in m, round off to two decimal places), is

Ans. 44.802
Sol.

| Segment | Length | Bearing | Lat.(Icos $\theta)$ | Dep. <br> (Isin $\theta)$ |
| :---: | :---: | :---: | :---: | :---: |
| PQ | 40 | 80 | 6.945 | 39.392 |
| QR | 50 | 10 | 49.250 | 8.682 |
| RS | 30 | 210 | -25.980 | -15 |
| SP | I | $\theta$ | $I \cos \theta$ | $I \sin \theta$ |

$\Sigma \mathrm{L}=\mathrm{I} \cos \theta+30.215=0$
$\Sigma \mathrm{D}=1 \sin \theta+33.081=0$
$I \cos \theta=-30.215$
$1 \sin \theta=-33.081$
$\mathrm{I}=\sqrt{(30.215)^{2}+(33.081)^{2}}$
$\mathrm{I}=44.802 \mathrm{~m}$

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