# GATE 2020 

Civil Engineering

## Shift-2

Questions \& Solutions

## SECTION: GENERAL APTITUDE

1. Rescue teams deployed $\qquad$ disaster hit areas combat $\qquad$ a lot of difficulties to save the people.
A. with, with
B. in, with
C. with, at
D. to, to

Ans. B
2. Select the most appropriate word that can replace the underlined word without changing the meaning of the sentence:
Now-a-days, most children have a tendency to belittle the legitimate concerns of their parents.
A. Disparage
B. Begrudge
C. Reduce
D. Applaud

Ans. A
Sol. Belittle: Disparage
3. Select the word that fits the analogy:

Partial : Impartial :: Popular : $\qquad$
A. Dispopular
B. Impopular
C. Unpopular
D. Mispopular

Ans. C
Sol. Partial : Impartial :: Popular: Unpopular
4. After the inauguration of the new building, the Head of the Department (HoD) collated faculty preferences for office space. $P$ wanted a room adjacent to the lab. Q wanted to be close to the lift. $R$ wanted a view of the playground and $S$ wanted a corner office.
Assuming the everyone was satisfied, which among the following shows a possible allocation?

B.

C.

D.


Ans. C
Sol. Option C is the only arrangement where all the conditions are successfully fulfilled.
5. If $f(x)=x^{2}$ for each $x \in(-\infty, \infty)$, then $\frac{f(f(f(x)))}{f(x)}$ is equal to $\qquad$ .
A. $(f(x))^{4}$
B.
$(f(x))^{2}$
C. $(f(x))^{3}$
D. $f(x)$

Ans. C
6. Nominal interest rate is defined as the amount paid by the borrower to the lender for using the borrowed amount for a specific period of time. Real interest rate calculated on the basis of actual value (inflation-adjusted), is approximately equal to the difference between nominal rate and expected rate of inflation in the economy.
Which of the following assertions is best supported by the above information?
A. Under low inflation, real interest rate is low and borrowers get benefited.
B. Under high inflation, real interest rate is low and lenders get benefited.
C. Under high inflation, real interest rate is low and borrowers get benefited.
D. Under low inflation, real interest rate is high and borrowers get benefited.
Ans. C
7. For the year 2019, which of the previous year's calendar can be used?
A. 2013
B. 2014
C. 2012
D. 2011

Ans. A
Sol. A year has 365 days, except a leap year, which has 366 days.

365 days $=52$ weeks +1 day .
366 days $=52$ weeks +2 days.
So, for a normal year, the next year's calendar will shift by one day and for a leap year, the next year's calendar will shift by two days.
Assuming 2019 starts on a Sunday.
2018: Saturday
2017: Friday
2016: Wednesday (Since 2016 is a leap year)
2015: Tuesday
2014: Monday
2013: Sunday
8. The ratio of 'the sum of the odd positive integers from 1 to 100 ' to 'the sum of the even positive integers from 150 to $200^{\prime}$ is $\qquad$ .
A. $45: 95$
B. $1: 2$
C. $50: 91$
D. $1: 1$

Ans. C
Sol. Sum of the odd positive integers from 1 to 100
$=1+3+5 \ldots \ldots+99$
No. of terms $=50$,
First term $=1$; Last term $=99$
Therefore, Sum $=50 \times(1+99) / 2=2500$
Sum of the even positive integers from 150 to 200
$=150+152+154+\ldots \ldots \ldots+198+200$
No. of terms $=26$
First term $=150$; Last term $=200$
Therefore, Sum $=26 \times(150+200) / 2=4550$
$\therefore$ ratio $=2500 / 4550=50 / 91$
9. In a school of 1000 students, 300 students play chess and 600 students play football. If 50 students play both chess and football, the number of students who play neither is $\qquad$ .
A. 150
B. 200
C. 100
D. 50

Ans. A
Sol. $N(S)=1000$
$N(C)=300$
$N(F)=600$
$N(C \& F)=50$
$N(S)=N(C)+N(F)-N(C \& F)+N($ none $)$
$\therefore \mathrm{N}$ (none) $=150$
10. The monthly distribution of 9 Watt LED bulbs sold by two firms $X$ and $Y$ from January to June 2018 is shown in the pie-chart and the corresponding table. If the total number of LED bulbs sold by two firms during April-June 2018 is 50000, then the number of LED bulbs sold by the firm $Y$ during April-June 2018 is $\qquad$ -.


| Month | Ratio of LED bulbs sold by two <br> firms (X:Y) |
| :---: | :---: |
| January | $7: 8$ |
| February | $2: 3$ |
| March | $2: 1$ |
| April | $3: 2$ |
| May | $1: 4$ |
| June | $9: 11$ |

A. 8750
B. 8250
C. 9750
D. 11250

Ans. C

## TECHNICAL

1. A triangular direct runoff hydrograph due to a storm has a time base of 90 hours. The peak flow of 60 $\mathrm{m}^{3} / \mathrm{s}$ occurs at 20 hours from the start of the storm. The area of catchment is $300 \mathrm{~km}^{2}$. The rainfall excess of the storm (in cm ), is
A. 3.24
B. 6.48
C. 5.40
D. 2.00

Ans. A
Sol.


Area of DRH $=$ Vol. of Rainfall Excess
$\frac{1}{2} \times 90 \times 60 \times 3600=300 \times 10^{6} \times x \times 0.01$
$x=3.24 \mathrm{~cm}$
2. Soil deposit formed due to transportation by wind is termed as
A. lacustrine deposit
B. aeolian deposit
C. estuarine deposit
D. alluvial deposit

Ans. B
3. For an axle load of 15 tonne on a road, the Vehicle Damage Factor (round off to two decimal places), in in terms of the standard axle load of 8 tonne, is
$\qquad$ .

Ans. 12.36
Sol. $\quad V D F=\left(\frac{\text { axle load }}{\text { standard load }}\right)^{4}=\left(\frac{15}{8}\right)^{4}=12.36$
4. The traffic starts discharging from an approach at an intersection with the signal turning green. The constant headway considered from the fourth or fifth headway position is referred to as
A. Discharge headway
B. Saturation headway
C. Intersection headway
D. Effective headway

Ans. B
Sol. Saturation headway
After initial some vehicle, Time headway remains constant.

5. A weightless cantilever beam of span $L$ is loaded as shown in the figure. For the entire span of the beam, the material properties are identical and the cross section is rectangular with constant width.


From the flexure-critical perspective, the most economical longitudinal profile of the beam to carry the given loads amongst the options given below, is
A.

B.


D.


Ans. D
Sol.


As bending moment at $A$ is zero and keeps on increasing linearly till point $B$, we need stronger section at $B$ as compared to what we need at $A$.


Suitable option (D)
6. As per IS 456:2000, the pH value of water for concrete mix shall NOT be less than
A. 6.0
B. 4.5
C. 5.0
D. 5.5

Ans. A
Sol. As per IS: 456-2000, Page no-14, clause 5.4.2, the $p^{H}$ value of water $\nless 6$ of the water used in construction should not be acidic in nature. If water used in construction is acidic, durability problem of concrete occurs.
7. The ordinary differential equation
$\frac{d^{2} u}{d x^{2}}-2 x^{2} u+\sin x=0$ is
A. linear and homogeneous
B. linear and nonhomogeneous
C. nonlinear and homogeneous
D. nonlinear and nonhomogeneous

Ans. B
Sol. $\frac{d^{2} x}{d x^{2}}-2 x^{2} u+\sin x=0$
The degree of all terms is different; hence it is non-homogeneous equation.

The given ordinary differential equation is linear because the coefficient of $y$ and $y^{\prime}, y^{\prime \prime}$ is the function of $x$.
8. A one-dimensional consolidation test is carried out on a standard 19 mm thick clay sample. The oedometer's deflection gauge indicates a reading of 2.1 mm, just before removal of the load, without allowing any swelling. The void ratio is 0.62 at this stage. The initial void ratio (round off to two decimal places) of the standard specimen is $\qquad$ _.

Ans. 0.8213
Sol. $\frac{\Delta e}{1+e_{f}}=\frac{\Delta H}{H_{f}}$
$\frac{\mathrm{e}_{\mathrm{i}}-.62}{\mathrm{I}+.62}=\frac{2.1}{16.9}$
$e_{i}=.8213$
9. The relationship between oxygen consumption and equivalent biodegradable organic removal (i.e., BOD) in a closed container with respect to time is shown in the figure


Assume that the rate of oxygen consumption is directly proportional to the amount of degradable organic matter and is expressed as $\frac{\mathrm{dL}_{\mathrm{t}}}{\mathrm{dt}}=-k L_{\mathrm{t}}$, where, $\mathrm{L}_{\mathrm{t}}$ (in $\mathrm{mg} /$ litre) is the oxygen equivalent of the organics remaining at time $t$ and $k\left(\right.$ in $\left.^{-1}\right)$ is the degradation rate constant. Lo is the oxygen equivalent of organic matter at time, $t=0$.
In the above context, the correct expression is
A. $\mathrm{BOD}_{\mathrm{t}}=\mathrm{L}_{0}-\mathrm{L}_{\mathrm{t}}$
B. $\mathrm{BOD}_{5}=\mathrm{L}_{5}$
C. $L_{t}=L_{0}\left(1-e^{-k t}\right)$
D. $\mathrm{Lo}=\mathrm{Lt}_{\mathrm{t}} \mathrm{e}^{-\mathrm{kt}}$

Ans. A
Sol. Here, $L_{o}=$ total oxygen equivalent of the organics at time, $\mathrm{t}=\mathrm{o}$
$L_{t}=$ total oxygen equivalent at $t=t$
Given, $\frac{d L_{t}}{d t}=-k L_{t}$
$\Rightarrow \quad \frac{d L_{t}}{L_{t}}=-k d t$
$\Rightarrow \quad \int_{L_{0}}^{L_{t}} \frac{d L_{t}}{L_{t}}=-k \int_{0}^{t} d t$
$\Rightarrow \quad \ln \left(\frac{L_{t}}{L_{o}}\right)=-k t$
$\Rightarrow \quad L_{t}=L_{o} e^{-k t}$
$\therefore$ BOD exerted $=B O D t=\left(L_{o}-L_{t}\right)=L_{o}-L_{o} e^{-k t}=$ Lo (1 - $\mathrm{e}^{-k t}$ )
10. The velocity components in the $x$ and $y$ directions for an incompressible flow are given as $u=(-5+$ $6 x)$ and $v=-(9+6 y)$, respectively. The equation of the streamline is
A. $\frac{-5+6 x}{9+6 y}=$ constant
B. $(-5+6 x)(9+6 y)=$ constant
C. $(-5+6 x)-(9+6 y)=$ constant
D. $\frac{9+6 y}{-5+6 x}=$ constant

Ans. B
Sol. Equation of stream line can be given as
$\frac{d x}{u}=\frac{d y}{v}$
Given: $u=(-5+6 x)$
$V=-(9+6 y)$

$$
\begin{aligned}
& \therefore \frac{d x}{(-5+6 x)}=\frac{d y}{-(9+6 y)} \\
& \Rightarrow \quad \ln (-5+6 x)+\ln (9+6 y)=\text { constant } \\
& \Rightarrow \quad(-5+6 x)(9+6 y)=\text { constant }
\end{aligned}
$$

11. Velocity distribution in a boundary layer is given by $\frac{\mathrm{u}}{\mathrm{U}_{\infty}}=\sin \left(\frac{\pi}{2} \frac{\mathrm{y}}{\delta}\right)$, where u is the velocity at vertical coordinate $y, U_{\infty}$ is the free stream velocity and $\delta$ is the boundary layer thickness. The values of $U_{\infty}$ and $\delta$ are $0.3 \mathrm{~m} / \mathrm{s}$ and 1.0 m , respectively. The velocity gradient $\left(\frac{\partial u}{\partial y}\right)$ (in $s^{-1}$, round off to two decimal places) at $y=0$, is $\qquad$ .

Ans. 0.4712
Sol. $\frac{U}{U_{\infty}}=\sin \left(\frac{\pi}{2} \cdot \frac{y}{\delta}\right)$
Velocity gradient $\left(\frac{d U}{\partial y}\right)_{y=0}$
$U=U_{\infty} \sin \left(\frac{\pi}{2} \frac{y}{\delta}\right)$
$\frac{\mathrm{dU}}{\partial \mathrm{y}}=\frac{\pi \mathrm{U}_{\infty}}{2 \rho} \cos \left(\frac{\pi}{2} \frac{\mathrm{y}}{\delta}\right)$

$$
\begin{aligned}
& \left(\frac{\partial y}{\partial y}\right)_{y=0}=\frac{\pi}{2} \frac{U_{\infty}}{\rho} \cos \left(\frac{\pi}{2} \times 0\right) \\
& =\frac{\pi}{2} \frac{U_{\infty}}{\rho} \cos 0 \quad \because \cos 0=1 \\
& =\frac{\pi}{2} \frac{U_{\infty}}{\rho}=\frac{0.3}{1}=0.3 \times \frac{\pi}{2} \\
& =0.4712
\end{aligned}
$$

12. The maximum applied load on a cylindrical concrete specimen of diameter 150 mm and length 300 mm tested as per the split tensile strength test guidelines of IS 5816:1999 is 157 kN . The split tensile strength (in MPa, round off to one decimal place) of the specimen is
Ans. $2.22 \mathrm{~N} / \mathrm{mm}^{2}$
Sol. Split tensile strength
$=\frac{2 \mathrm{P}}{\pi \mathrm{LD}}=\frac{\left(2 \times 157 \times 10^{3}\right)}{(\pi \times 300 \times 150)}=2.22 \mathrm{~N} / \mathrm{mm}^{2}$
13. The ratio of the plastic moment capacity of a beam section to its yield moment capacity is termed as
A. load factor
B. shape factor
C. slenderness ratio
D. aspect ratio

Ans. B
Sol. Shape factor is defined as the ratio of plastic moment of resistance of the section to yield moment of resistance of section.
14. A gas contains two types of suspended particles having average sizes of $2 \mu \mathrm{~m}$ and $50 \mu \mathrm{~m}$. Amongst the options given below, the most suitable pollution control strategy for removal of these particles is
A. Electrostatic precipitator followed by venture scrubber
B. Electrostatic precipitator followed by cyclonic separator
C. Settling chamber followed by bag filter
D. Bag filter followed by electrostatic precipitator

Ans. C
15. Two identically sized primary settling tanks receive water for Type-I settling (discrete particles in dilute suspension) under laminar flow conditions. The Surface Overflow Rate (SOR) maintained in the two tanks are $30 \mathrm{~m}^{3} / \mathrm{m}^{2}$.d and $15 \mathrm{~m}^{3} / \mathrm{m}^{2}$. d. The lowest diameters of the particles, which shall be settled out completely under SORs of $30 \mathrm{~m}^{3} / \mathrm{m}^{2}$.d and 15 $\mathrm{m}^{3} / \mathrm{m}^{2} / \mathrm{d}$ are designated as $\mathrm{d}_{30}$ and $d_{15}$, respectively. The ratio, $d_{30} / d_{15}$ (round off to two decimal places), is $\qquad$ .

Ans. 1.414
Sol. According to stokes law
$V_{\mathrm{s}}=\frac{(\mathrm{G}-1) \mathrm{gd}^{2}}{18 \mathrm{u}}$
Where, $\mathrm{V}_{\mathrm{s}}=$ settling velocity
$G=$ specific gravity $y$ the particle
$d=$ diameter of particle
$v=$ kinematic viscosity
$\therefore$ By above expression we can imply
$V_{s} \alpha d^{2}$
$\Rightarrow \quad \frac{V_{30}}{V_{15}}=\left(\frac{d_{30}}{d_{15}}\right)^{2}$
$\Rightarrow \quad \frac{d_{30}}{d_{15}}=\sqrt{\left(\frac{30}{15}\right)} \quad\left\{\begin{array}{r}\because V_{30}=30 \mathrm{~m} / \mathrm{d} \\ \mathrm{V}_{15}=15 \mathrm{~m} / \mathrm{d}\end{array}\right.$
$\Rightarrow \quad \frac{d_{30}}{d_{15}}=1.414$
16. Super passage is a canal cross-drainage structure in which
A. Natural stream water flows under pressure below a canal
B. Canal water flows under pressure below a natural stream
C. Natural stream water flows with free surface below a canal
D. Canal water flows with free surface below a natural stream

Ans. D
17. A soil has dry unit weight of $15.5 \mathrm{kN} / \mathrm{m}^{3}$, specific gravity of 2.65 and degree of saturation of $72 \%$. Considering the unit weight of water as $10 \mathrm{kN} / \mathrm{m}^{3}$, the water content of the soil (in \%, round off to two decimal places) is $\qquad$ —.
Ans. 19.27\%
Sol. $\mathrm{Y}_{\mathrm{d}}=15.5 \mathrm{kN} / \mathrm{m}^{3}$
$G=2.65$
$S=.72$
$\mathrm{Y}_{\mathrm{w}}=10 \mathrm{kN} / \mathrm{m}^{3}$
$\mathrm{w}=$ ?
$\gamma_{\mathrm{d}}=\frac{\mathrm{Gy}_{\mathrm{w}}}{1+\mathrm{e}}$
$15.5=\frac{2.65 \times 10}{1+e}$
$e=.7096$
$\mathrm{Se}=\mathrm{wG}$
$0.72 \times 0.7096=w \times 2.65$
$w=0.1927=19.27 \%$
18. The following partial differential equation is defined for u:u (x, y)
$\frac{\partial u}{\partial y}=\frac{\partial^{2} u}{\partial x^{2}} ; \quad y \geq 0 ; \quad x_{1} \leq x \leq x_{2}$
The set of auxiliary conditions necessary to solve the equation uniquely, is
A. Two initial conditions and one boundary condition
B. Three boundary conditions
C. One initial condition and two boundary conditions
D. Three initial condition

Ans. A
Sol. Let $u=x y$

$$
\begin{aligned}
& \frac{\partial u}{\partial y}=x y^{\prime} \\
& \frac{\partial^{2} u}{\partial x^{2}}=y x^{\prime \prime} \\
& \text { Given } \frac{\partial u}{\partial y}=\frac{\partial^{2} u}{\partial x^{2}} \\
& x y^{\prime}=y x^{\prime \prime} \\
& \Rightarrow \frac{y^{\prime}}{y}=\frac{x^{\prime \prime}}{x}=k
\end{aligned}
$$

$y^{\prime}-y k=0$
$x^{\prime \prime}-x k=0$
Take equation (1)
Auxiliary equation is
D $-k=0$
D $=k$
So the solution is
$y=C_{0} e^{k y}$
Take equation (2)
$x^{\prime \prime}-x k=0$
Auxiliary equation is
$D^{2}-k=0$
$D= \pm \sqrt{k}$
So the solution is
$X=C_{1} e^{\sqrt{k} x}+C_{2} e^{-\sqrt{k} x}$
But $u=x y$
$=\left(C_{1} E^{\sqrt{k} x}+C_{2} e^{-\sqrt{k} x}\right) C_{0} e^{k y}$
$=C_{1} C_{0} \mathrm{e}^{\sqrt{k} x k y}+C_{2} C_{0} \mathrm{e}^{-\sqrt{k} k x y}$
$=k_{1} e^{k^{3 / 2} x y}+k_{2} e^{-k^{3 / 2} x y}$
Hence option (A).
19. Muskingum method is used in
A. Hydrologic channel routing
B. hydrologic reservoir routing
C. Hydraulic channel routing
D. hydraulic reservoir routing

Ans. A
20. The integral

$$
\int_{0}^{1}\left(5 x^{3}+4 x^{2}+3 x+2\right) d x
$$

is estimated numerically using three alternative methods namely the rectangular, trapezoidal and Simpson's rules with a common step size. In this context, which one of the following statements is TRUE?
A. Simpson's rule as well as rectangular rule of estimation will give NON-zero error.
B. Simpson's rule, rectangular rule as well as trapezoidal rule of estimation will give NON-zero error.
C. Only Simpson's rule of estimation will give zero error.
D. Only the rectangular rule of estimation will give zero error.
Ans. C
Sol. Simpson uses a cubic function for interpolation and it does not give an error for a cubic functions.
21. The value of $\lim _{x \rightarrow \infty} \frac{\sqrt{9 x^{2}+2020}}{x+7}$ is
A. 3
B. indeterminable
C. $\frac{7}{9}$
D. 1

Ans. A
Sol. $\lim _{x \rightarrow \infty} \frac{\sqrt{9 x^{2}+2020}}{x+7}$
Dividing numerator and denominator by ' $x$ '
$\lim _{x \rightarrow \infty} \frac{\frac{\sqrt{9 x^{2}+2020}}{x}}{\frac{x+7}{x}}$
$\lim _{x \rightarrow \infty} \frac{\sqrt{\frac{9 x^{2}}{x^{2}}+\frac{2020}{x^{2}}}}{1+\frac{7}{x}}$
$\lim _{x \rightarrow \infty} \frac{\sqrt{\frac{9+2020}{x^{2}}}}{1+\frac{7}{x}}$
Putting $\mathrm{x} \rightarrow \infty \Rightarrow \frac{1}{\mathrm{x}^{2}} \rightarrow 0, \frac{1}{\mathrm{x}} \rightarrow 0$
$=\frac{\sqrt{9+0}}{1+0}=\frac{\sqrt{9}}{1}=3$
22. A sample of 500 g dry sand, when poured into a 2litre capacity cylinder which is partially filled with
water, displaces $188 \mathrm{~cm}^{3}$ of water. The density of water is $1 \mathrm{~g} / \mathrm{cm}^{3}$. The specific gravity of the sand is
A. 2.72
B. 2.66
C. 2.52
D. 2.55

Ans. B
Sol. Mass $=500 \mathrm{gm}$
Volume displaced $=188 \mathrm{~cm}^{3}$
$G=\frac{\rho_{\text {SOLID }}}{\rho_{\text {WATER }}}$
$\mathrm{G}=\frac{500 \mathrm{gm} / 188 \mathrm{cc}}{1 \mathrm{gm} / \mathrm{cc}}$
$G=2.659$
23. A fair (unbiased) coin is tossed 15 times. The probability of getting exactly 8 Heads (round off to three decimal places), is $\qquad$ —.

Ans. 0.196
Sol. $x=15$
$P(r=8)={ }^{n} C_{r} p^{r} q^{n-r}$
$={ }^{15} C_{8} p^{8} q^{15-8}$
$={ }^{15} \mathrm{C}_{8} \mathrm{p}^{8} q^{7}$
$P=($ probability of success in toss $)=\frac{1}{2}$
$\mathrm{q}=$ (probability of failure during toss) $=\frac{1}{2}$
$P(r=8)=15 C 8\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{7}$
$={ }^{15} C_{8}\left(\frac{1}{2}\right)^{15}=0.196$
24. The state of stress represented by Mohr's circle shown in the figure is

A. pure shear
B. biaxial tension of equal magnitude
C. hydrostatic stress
D. uniaxial tension

Ans. A
Sol. $\therefore \sigma_{1}=-\sigma_{2}$
$\therefore \mathrm{A}$
25. 24-h traffic count at a road section was observed to be 1000 vehicles on a Tuesday in the month of July. If daily adjustment factor for Tuesday is 1.121 and monthly adjustment factor for July is 0.913, the Annual Average Daily Traffic (in veh/day, round off to the nearest integer) is $\qquad$ .
Ans. 1023
Sol. 24 hr daily volume of Tuesday $=1000 \mathrm{vch}$
Daily adjustment factor $=1.121$
Monthly adjustment factor $=0.913$
Vol. d week $=1000 \times 1.21=1121$
AADT $=$ Monthly Adjustment factor $\times$ ADT $=0.913$
$\times 1121$
$=1023.473$ veh.per day
$=1023$ veh.per day
26. For the hottest month of the year at the proposed airport site, the monthly mean of the average daily temperature is $39^{\circ} \mathrm{C}$. The monthly mean of the maximum daily temperature is $48^{\circ} \mathrm{C}$ for the same month of the year. From the given information, the calculated Airport Reference Temperature (in ${ }^{\circ} \mathrm{C}$ ), is
A. 42
B. 48
C. 39
D. 36

Ans. A
Sol. Airport reference temperature is given by following expression.
$A R T=\left[T_{a}+\frac{T_{m}+T_{a}}{3}\right]$
where, $T_{a}=$ average daily temperature
$\mathrm{T}_{\mathrm{m}}=$ maximum daily temperature
$\therefore$ ART $=\left[39+\frac{48-39}{3}\right]$
$\therefore$ ART $=42^{\circ} \mathrm{C}$
27. The flow-density relationship of traffic on a highway is shown in the figure


The correct representation of speed-density relationship of the traffic on this highway is
A.

B.

C.

D.


Ans. A
28. Permeability tests were carried out on the samples collected from two different layers as shown in the figure (not drawn to the scale). The relevant horizontal ( $\mathrm{k}_{\mathrm{h}}$ ) and vertical ( $\mathrm{k}_{\mathrm{v}}$ ) coefficients of permeability are indicated for each layer.


The ratio of the equivalent horizontal to vertical coefficients of permeability, is
A. 37.29
B. 68.25
C. 80.20
D. 0.03

Ans. A
Sol. $\frac{\mathrm{keq}_{H}}{\mathrm{keq}_{v}}=\frac{\frac{4.4 \times 10^{-3} \times 3+6 \times 10^{-1} \times 4}{7}}{\frac{7}{\frac{3}{4 \times 10^{-3}}+\frac{4}{5.5 \times 10^{-1}}}}$

$$
=37.29
$$

29. A $4 \times 4$ matrix [P] is given below
$[P]=\left[\begin{array}{cccc}0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6\end{array}\right]$
The eigenvalues of [P] are
A. 1, 2, 3, 4
B. 1, 2, 5, 7
C. $3,4,5,7$
D. $0,3,6,6$

Ans. B
Sol. $|P|=-(-2)\left|\begin{array}{lll}1 & 3 & 0 \\ 0 & 6 & 1 \\ 0 & 1 & 6\end{array}\right|$
$=2\left|\begin{array}{lll}1 & 3 & 0 \\ 0 & 6 & 1 \\ 0 & 1 & 6\end{array}\right|$
$|P|=2\{1(36-1)-3(0-0)-0(0-0)\}$
$|P|=70$
As, the determinant of the given matrix is non-zero Hence, the matrix is singular.
The product of eigen values $=|A|$
$\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}=70$
sum of eigen values $=$ trace of matrix $P$
trace of matrix $P=0+3+6+6$
$=15$
$\lambda_{1}+\lambda_{2}+\lambda_{3}+\lambda_{4}=50$
Checking the above options, only (B) option is correct.
$\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}=1,2,5,7$
30. The diameter and height of a right circular cylinder are 3 cm and 4 cm , respectively. The absolute error in each of these two measurements is 0.2 cm . The absolute error in the computed volume (in $\mathrm{cm}^{3}$, round off to three decimal places), is $\qquad$ -.
Ans. $5.18 \mathrm{~cm}^{3}$
Sol.


$$
\begin{aligned}
& \text { diameter }=\mathrm{d}=3 \mathrm{~cm} \\
& \text { height }=\mathrm{h}=4 \mathrm{~cm} \\
& \mathrm{e}_{\mathrm{d}}=\mathrm{e}_{\mathrm{h}}= \pm 0.2 \mathrm{~cm} \\
& \mathrm{v}=\mathrm{f}(\mathrm{~d}, \mathrm{~h}) \\
& \mathrm{v}=\frac{\pi \mathrm{d}^{2}}{4} \mathrm{~h} \\
& \mathrm{dv}=\left(\frac{\partial \mathrm{v}}{\partial \mathrm{~d}}\right) \mathrm{dd}+\left(\frac{\partial \mathrm{v}}{\partial \mathrm{~h}}\right) \mathrm{dh} \\
& =\frac{\pi \mathrm{dh}}{2} \times \mathrm{e}_{\mathrm{d}}+\frac{\pi \mathrm{d}^{2}}{4} \times \mathrm{e}_{\mathrm{h}}
\end{aligned}
$$

$\therefore$ Absolute error $=5.18 \mathrm{~cm}^{3}$
31. Two steel plates are lap jointed in a workshop using 6 mm thick fillet weld as shown in the figure (not drawn to the scale). The ultimate strength of the weld is 410 MPa .


As per Limit State Design of IS 800:2007, the design capacity (in kN , round off to three decimal places) of the welded connection, is $\qquad$ _.
Ans. 413.586 kN
Sol. $\left(\frac{f_{u w}}{\sqrt{3} \times 1.25}\right) \times r \times\left(t_{t}\right)$
$=\left(\frac{410}{\sqrt{3} \times 1.25}\right) \times 520 \times(0.7 \times 6)=413.586 \mathrm{kN}$.
32. The ion product of water $\left(p K_{w}\right)$ is 14. If a rain water sample has a pH of 5.6 , the concentration of $\mathrm{OH}^{-}$in the sample (in $10^{-9} \mathrm{~mol} / \mathrm{litre}$, round off to one decimal place), is $\qquad$ —.
Ans. 3.981
Sol. $\mathrm{pH}+\mathrm{POH}=14$
$\Rightarrow 5.6+\mathrm{POH}=14$
$\Rightarrow \mathrm{POH}=14-5.6=8.4$
Also, $\mathrm{POH}=-\log \left[\mathrm{OH}^{-}\right]$
$\Rightarrow 8.4=-\log \left[\mathrm{OH}^{-}\right]$
$\Rightarrow 10^{-8.4}=\left[\mathrm{OH}^{-}\right]$
$\Rightarrow\left[\mathrm{OH}^{-}\right]=3.981 \times 10^{-9} \mathrm{M}$
33. A sample of water contains an organic compound $\mathrm{C}_{8} \mathrm{H}_{16} \mathrm{O}_{8}$ at a concentration of $10^{-3} \mathrm{~mol} /$ litre. Given that the atomic weight of $\mathrm{C}=12 \mathrm{~g} / \mathrm{mol}, \mathrm{H}=1 \mathrm{~g} / \mathrm{mol}$, and $\mathrm{O}=16 \mathrm{~g} / \mathrm{mol}$, the theoretical oxygen demand of water (in g of $\mathrm{O}_{2}$ per litre, round off to two decimal places), is $\qquad$ —.
Ans. $0.256 \mathrm{~g} / \mathrm{l}$

Sol. $\mathrm{C}_{8} \mathrm{H}_{16} \mathrm{O}_{8}+8 \mathrm{O}_{2} \rightarrow 8 \mathrm{CO}_{2}+8 \mathrm{H}_{2} \mathrm{O}$
1 mole of $\mathrm{C}_{8} \mathrm{H}_{16} \mathrm{O}_{8}$ require 8 mole of $\mathrm{O}_{2}$
$\therefore 10^{-3}$ mole/l require $=8 \times 10^{-3} \mathrm{~mole} / \mathrm{l}$
$8 \times 10^{-3}=\frac{x}{32} \Rightarrow x=0.256 \mathrm{~g} / \mathrm{l}$
34. A waste to energy plant burns dry solid waste of composition: Carbon $=35 \%$, Oxygen $=26 \%$, Hydrogen $=10 \%$, Sulphur $=6 \%$, Nitrogen $=3 \%$ and Inerts $=20 \%$. Burning rate is 1000 tonnes/d. Oxygen in air by weight is $23 \%$. Assume complete conversion of Carbon to $\mathrm{CO}_{2}$, Hydrogen to $\mathrm{H}_{2} \mathrm{O}$, Sulphur to $\mathrm{SO}_{2}$ and Nitrogen to $\mathrm{NO}_{2}$.
Given Atomic weights: $\mathrm{H}=1, \mathrm{C}=12, \mathrm{~N}=14, \mathrm{O}=$ $16, S=32$
The stoichiometric (theoretical) amount of air (in tones/d, round off to the nearest integer) required for complete burning of this waste, is $\qquad$ —.
Ans. 6965.2
Sol. Carbon to $\mathrm{CO}_{2}$ :


Oxygen required for $1 \mathrm{t} /$ day $=\frac{32}{12}=2.6667$ moles
Amount of air required for combustion of carbon $=$
$2.6667 \times 350=933.33$ tonnes/day
Similarly,

| for hydrogen | $\frac{32}{4} \times 100=800$ |
| :--- | :--- |
| for Sulphur | $\frac{32}{32} \times 60=60$ |
| for nitrogen | $\frac{32}{14} \times 30=68.57$ |

$\therefore$ total oxygen required $=933.34+800+60+$
68.57 ~ 1862 t/day

Available amount of oxygen $=260 \mathrm{t} /$ day
$\therefore$ required amount of oxygen $=1862-260=$
1602 t/day
$\therefore$ Amount of air required $=1602 / .23=6965.2$ tonnes/day
35. A constant-head permeability test was conducted on a soil specimen under a hydraulic gradient of 2.5 . The soil specimen has specific gravity of 2.65 and
saturated water content of $20 \%$. If the coefficient of permeability of the soil is $0.1 \mathrm{~cm} / \mathrm{s}$, the seepage velocity (in $\mathrm{cm} / \mathrm{s}$, round off to two decimal places) through the soil specimen is $\qquad$ —.

Ans. 0.7217
Sol. $\quad \mathrm{V}_{\text {discharge }}=\mathrm{Ki}$
$=.1 \times 2.5$
$=.25 \mathrm{~cm} / \mathrm{sec}$
$V_{\text {seapage }}=\frac{V_{\text {discharge }}}{n}$
$\mathrm{n}=\frac{\mathrm{e}}{1+\mathrm{e}}=\frac{.53}{1+.53}=.3464$
$\mathrm{Se}=\mathrm{WG}$
$1 \mathrm{e}=.2 \times 2.65$
$e=.53$
$V_{\text {seapage }}=\frac{.25}{.3464}$
$V_{\text {seapage }}=0.7217$
36. Crops are grown in a field having soil, which has field capacity of $30 \%$ and permanent wilting point of $13 \%$. The effective depth of root zone is 80 cm . Irrigation water is supplied when the average soil moisture drops to $20 \%$. Consider density of the soil as 1500 $\mathrm{kg} / \mathrm{m}^{3}$ and density of water as $1000 \mathrm{~kg} / \mathrm{m}^{3}$. If the daily consumptive use of water for the crops is 2 mm , the frequency of irrigating the crops (in days), is
A. 13
B. 10
C. 11
D. 7

Ans. A
Sol. Given: Final capacity $=30 \%$
Permanent wilting point $=13 \%$
Depth of root zone $=80 \mathrm{~cm}$
Density of soal $=1500 \mathrm{~kg} / \mathrm{m}^{3}$
Density of water $=1000 \mathrm{~kg} / \mathrm{m}^{3}$
Consumptive use $=\mathrm{Cu}=2 \mathrm{~mm}$

$$
\begin{aligned}
& \mathrm{d}=\frac{\delta \times \mathrm{d}_{\mathrm{rz}} \times(\mathrm{FC}-\mathrm{PWP}) \times \mathrm{D}}{\delta_{\mathrm{w}}} \\
& \mathrm{~d}=\frac{1500 \times 0.8 \times(0.3-0.13) \times 0.8}{1000}=163.2 \mathrm{~mm}
\end{aligned}
$$

$F_{01}=\frac{d}{C u}=\frac{164}{2}=82$ days
(No option matching)
37. The Fourier series to represent $x-x^{2}$ for $-\pi \leq x \leq \pi$ is given by
$x-x^{2}=\frac{a_{0}}{2}+\sum_{n=1}^{\infty} a_{n} \cos n x+\sum_{n=1}^{\infty} b_{n} \sin n x$
The value of $a_{0}$ (round off to two decimal places), is
$\qquad$ .
Ans. -6.58
Sol. Comparing the above equation with standard trigonometric farriers series representation,
$f(x)=\frac{a_{o}}{2}+\sum_{n=1}^{\infty} a_{n} \cos n x+\sum_{n=1}^{\infty} b_{n} \sin n x$
$\frac{a_{0}}{2}=\frac{1}{T} \int_{0}^{T} f(x) d x,-\pi<f(x)<\pi$
$\frac{a_{0}}{2}=\frac{1}{2 \pi} \int_{-\pi}^{\pi}\left(x-x^{2}\right) d x$
$\frac{a_{0}}{2}=\frac{1}{2 \pi}\left[\int_{-\pi}^{\pi} x d x-\int_{-\pi}^{\pi} x^{2} d x\right]$
$\frac{\mathrm{a}_{\mathrm{o}}}{2}=\frac{1}{2 \pi}\left[\left[\frac{\mathrm{x}^{2}}{2}\right]_{-\pi}^{\pi}-\left[\frac{\mathrm{x}^{3}}{3}\right]_{-\pi}^{\pi}\right]$
$\frac{a_{0}}{2}=\frac{1}{2 \pi}\left[\left(\frac{\pi^{2}}{2}-\frac{\pi^{2}}{2}\right)-\left(\frac{\pi^{3}}{3}-\frac{-\pi^{3}}{3}\right)\right]$
$\frac{a_{0}}{2}=\frac{1}{2 \pi}\left[\frac{-2 \pi 3}{3}\right]$
$\frac{a_{0}}{2}=\frac{-1}{2 \pi} \times \frac{2 \pi^{3}}{3}$
$\frac{a_{0}}{2}=\frac{-\pi^{2}}{3}$
$a_{0}=\frac{-2}{3} \pi^{2}=-6.58$
38. A prismatic linearly elastic bar of length $L$, crosssectional area A, and made up of a material with Young's modulus E , is subjected to axial tensile force as shown in the figures. When the bar is subjected
to axial tensile forces $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$, the strain energies stored in the bar are $\mathrm{U}_{1}$ and $\mathrm{U}_{2}$, respectively.


If $U$ is the strain energy stored in the same bar when subjected to an axial tensile force $\left(P_{1}+P_{2}\right)$, the correct relationship is
A. $U>U_{1}+U_{2}$
B. $U=U_{1}-U_{2}$
C. $\mathrm{U}<\mathrm{U}_{1}+\mathrm{U}_{2}$
D. $\mathrm{U}=\mathrm{U}_{1}+\mathrm{U}_{2}$

Ans. A
Sol.

$$
\begin{aligned}
& \lambda \\
& U_{1}=\frac{P_{1}^{2} \ell}{2 A E} \\
& \lambda \\
& U_{2}=\frac{P_{2}^{2} \ell}{2 A E} \\
& \lambda \\
& \lambda \\
& \lambda \\
& U=\frac{\left(P_{1}+P_{2}\right)^{2} \ell}{2 A E} \\
& U=\frac{P_{1}^{2} \ell}{2 A E}+\frac{P_{2}^{2} \ell}{2 A E}+\frac{P_{1} P_{2} \ell}{A E} \\
& \therefore U P P_{1}+P_{2} \\
& \therefore U+U_{2}
\end{aligned}
$$

39. Group-I gives a list of test methods for evaluating properties of aggregates. Group-II gives the list of properties to be evaluated.

| Group-I: Test methods | Group-II: Properties |
| :---: | :---: |
| P. Soundness test | 1. Strength |
| Q. Crushing test | 2. Resistance to <br> weathering |
| R. Los Angeles abrasion <br> test | 3. Adhesion |
| S. Stripping value test | 4. Hardness |

The correct match of test methods under Group-I to properties under Group-II, is
A. P-2; Q-1; R-4; S-3
B. P-2; Q-4; R-3; S-1
C. P-4; Q-1; R-2; S-3
D. P-3; Q-4; R-1; S-2

Sol. A

|  | (A) |  | (B) |
| :---: | :---: | :---: | :---: |
| P. | Soundness test | 2. | Weathering <br> resistance |
| Q. | Crushing test | 1. | Strength |
| R. | Los Angeles Abrasion <br> test | 4. | Hardness |
| S. | Stripping value test | 3. | Adhesion |

40. The plane truss has hinge supports at $P$ and $W$ and is subjected to the horizontal forces as shown in the figure (not drawn to the scale)


Representing the tensile force with ' + ' sign and the compressive force with '-' sign, the force in member XW (in kN, round off to the nearest integer), is
$\qquad$ -.

Ans. -30 kN
Sol. First find support reaction


Chose the section as shown above and take the FBD of structure above the section $\mathrm{X}-\mathrm{X}$


For the above FBD apply $\Sigma M_{Q}=0$
$(10 \times 4)+(10 \times 8)+F_{x w}[4]=0$
$\mathrm{F}_{\mathrm{xw}}=-30 \mathrm{kN}$ (Compression force)


Sign convention while writing equilibrium equation.
41. Joints $I, J, K, L, Q$ and $M$ of the frame shown in the figure (not drawn to the scale) are pins. Continuous members IQ and LJ are connected through a pin at N. Continuous members JM and KQ are connected through a pin at $P$. The frame has hinge supports at joints $R$ and $S$. The loads acting at joints $I, J$ and $K$ are along the negative $Y$ direction and the loads acting at joints $L$ and $M$ are along the positive $X$ direction.


The magnitude of the horizontal component of reaction (in kN ) at S , is
A. 5
B. 10
C. 20
D. 15

Ans. B
42. A 5 m high vertical wall has a saturated clay backfill. The saturated unit weight and cohesion of clay are $18 \mathrm{kN} / \mathrm{m}^{3}$ and 20 kPa , respectively. The angle of internal friction of clay is zero. In order to prevent development of tension zone, the height of the wall is required to be increased. Dry sand is used as backfill above the clay for the increased portion of the wall. The unit weight and angle of internal friction of sand are $16 \mathrm{kN} / \mathrm{m}^{3}$ and $30^{\circ}$, respectively. Assume that the back of the wall is smooth and top of the backfill is horizontal. To prevent the development of tension zone, the minimum height (in $m$, round off to one decimal place) by which the wall has to be raised, is
Ans. 2.5

## Sol.


$\sigma_{H}=k_{a} \sigma_{v}-2 c \sqrt{k_{a}}$
$\varphi=0$
$\therefore \mathrm{k}_{\mathrm{a}}=\frac{1-\sin \phi}{1+\sin \phi}=1$
for no tension crack
$\sigma_{H} \geq 0$
$\sigma_{H}=k_{a}\left(\sigma_{v}\right)-2 c \sqrt{k_{a}}=0$
$=1(4 d x)-2 c \sqrt{k_{a}}=0$
$=16 x-40=0$
$x=\frac{40}{16}=2.5 \mathrm{~m}$
43. A footing of size $2 \mathrm{~m} \times 2 \mathrm{~m}$ transferring a pressure of $200 \mathrm{kN} / \mathrm{m}^{2}$, is placed at a depth of 1.5 m below the ground as shown in the figure (not drawn to the scale). The clay stratum is normally consolidated. The clay has specific gravity of 2.65 and compression index of 0.3 .


Dense Sand
Considering 2:1 (vertical to horizontal) method of load distribution and $\gamma_{w}=10 \mathrm{kN} / \mathrm{m}^{3}$, the primary
consolidation settlement (in mm, round off to two decimal places) of the clay stratum is $\qquad$ -.
Ans. 74.27
Sol.


For CLAY
$\Rightarrow y_{S A T}=\left(\frac{G+e}{1+e}\right) y_{w}=\left(\frac{2.65+e}{1+e}\right) \times 10$
$=17 \mathrm{kw} / \mathrm{m}^{3}$
$\mathrm{eo}=1.357$
$\mathrm{Ho}=1.5 \mathrm{~m}$
$\sigma_{0} A T \times x=24 d+0.54$ sant +0.754 SAT $-1.25 y_{w}$
$=39.25 \mathrm{kN} / \mathrm{m}^{2}$
$\Delta \bar{\sigma}=\frac{q B^{2}}{(B+2 n 2)^{2}}=\frac{\text { force }}{\text { Area }}=\frac{200 \times 2 \times 2}{\left(2+2 \times \frac{1}{2} \times 1.75\right)^{2}} \Delta \bar{\sigma}=$
$56.88 \mathrm{kN} / \mathrm{m}^{2}$
$\Delta \mathrm{H}=\frac{\mathrm{C}_{\mathrm{C}} \mathrm{H}_{0}}{1+\mathrm{e}_{0}} \log \left(\frac{\overline{\sigma_{0}}+\overline{\Delta \sigma}}{\overline{\sigma_{0}}}\right)$
$=\frac{1.5 \times 0.3}{1+1.357} \log \left(\frac{39.25+56.88}{39.25}\right)$
$\Delta \mathrm{H}=74.27 \mathrm{~mm}$
44. A hydraulic jump occurs in a triangular (V-shaped) channel with side slopes 1:1 (vertical to horizontal). The sequent depths are 0.5 m and 1.5 m . The flow rate (in $\mathrm{m}^{3} / \mathrm{s}$, round off to two decimal places) in the channel is $\qquad$ —.

Ans. $1.728 \mathrm{~m}^{3} / \mathrm{sec}$
Sol. In case of $V$ shaped channel, flow rate can be given as

Flow rate $=Q=y_{1} y_{2} \sqrt{\frac{g}{3}\left(\frac{y_{2}^{3}-y_{1}^{3}}{y_{2}^{2}-y_{1}^{2}}\right)}$
$\Rightarrow \mathrm{Q}=0.5 \times 1.5 \sqrt{\frac{9.81}{3}\left(\frac{1.5^{3}-0.5^{3}}{1.5^{2}-0.5^{2}}\right)}$
$\mathrm{Q}=1.728 \mathrm{~m}^{3} / \mathrm{sec}$
45. The planar structure RST shown in the figure is roller-supported at $S$ and pin-supported at $R$. Members RS and ST have uniform flexural rigidity (EI) and $S$ is a rigid joint. Consider only bending deformation and neglect effects of self-weight and axial stiffening.


When the structure is subjected to a concentrated horizontal load $P$ at the end $T$, the magnitude of rotation at the support $R$, is
A. $\frac{\mathrm{PL}^{2}}{12 \mathrm{EI}}$
B. $\frac{\mathrm{PL}}{6 \mathrm{EI}}$
C. $\frac{\mathrm{PL}^{2}}{6 \mathrm{EI}}$
D. $\frac{\mathrm{PL}^{3}}{12 \mathrm{EI}}$

Ans. A
Sol.

(Finding support reactions at "R" \& "S")
Using unit load method
$\int_{0}^{L} \frac{M_{x} \cdot m_{x} \cdot d x}{E I}$; Take " $R$ " as the origin
$M_{x} \rightarrow$ B.M due to applied loads/moments in the idealised structure
$m_{x} \rightarrow B . M$ in the given structure due to unit load alone

$M_{x}=\left[\frac{-P}{2}\right] \cdot x ; 0 \leq x \leq L$
$m_{x}=\left\{\frac{x}{L}-1\right\} ; 0 \leq x \leq L$
$\theta_{R}=\frac{1}{\operatorname{EI}}\left[\left(\frac{-P}{2}\right) x\right] \cdot\left\{\frac{x}{L}-1\right\} d x$
$=\frac{P}{2}\left[x-\frac{x^{2}}{L}\right] d x=\frac{P}{2}\left[\frac{x^{2}}{2}-\frac{x^{3}}{3 L}\right]_{0}^{L}=\frac{P L^{2}}{12 E I}$
(Anti-Clock)
46. A concrete beam of span $15 \mathrm{~m}, 150 \mathrm{~mm}$ wide and 350 mm deep is prestressed with a parabolic cable as shown in the figure (not drawn to the scale). Coefficient of friction for the cable is 0.35, and coefficient of wave effect is 0.0015 per meter.


If the cable is tensioned from one end only, the percentage loss (round off to one decimal place) in the cable force due to friction, is $\qquad$ —.
Ans. 4.49

## Sol.


$M=0.35, K=0.0045 / m$
Jacking from one end
$\therefore \mathrm{x}=\mathrm{L}=15 \mathrm{M}$
Wobble coefficient (or) wave coefficient $k=$ 0.0015/m
$\alpha=\tan \alpha=\frac{8 \mathrm{~h}}{\mathrm{~L}}=\frac{8 \times(50+70)}{15,000}=0.064$
...... of
of......$=p o(M a+t x)$
$=\operatorname{Po}(0.35 \times 0.064+(0.0015) \times 15)$
$\ldots \ldots \ldots=\frac{\mathrm{Po}(\mathrm{M} \alpha+\mathrm{kx})}{\mathrm{Po}} \times 100$
$=[(0.35 \times 0.064)+(0.0015 \times 15)] \times 100$
$=4.49 \%$
47. A cast iron pipe of diameter 600 mm and length 400 m carries water from a tank and discharges freely into air at a point 4.5 m below the water surface in the tank. The friction factor of the pipe is 0.018 . Consider acceleration due to gravity as $9.81 \mathrm{~m} / \mathrm{s}^{2}$. The velocity of the flow in pipe (in $\mathrm{m} / \mathrm{s}$, round off to two decimal places) is $\qquad$ -.
Ans. $2.56 \mathrm{~m} / \mathrm{s}$
Sol. $\mathrm{h}_{\mathrm{f}}=4.5 \mathrm{~m}=$ frictional loss + entry loss + exit loss $h_{f}=\frac{f L V^{2}}{2 g D}+\frac{0.5 \mathrm{~V}^{2}}{2 g}+\frac{\mathrm{V}^{2}}{2 g}$
or $4.5=\frac{0.018 \times 400 \times \mathrm{V}^{2}}{2 \times 9.81 \times 0.6}+\frac{0.5 \mathrm{~V}^{2}}{2 \mathrm{~g}}+\frac{\mathrm{V}^{2}}{2 \mathrm{~g}} \Rightarrow \mathrm{~V}=2.56 \mathrm{~m} / \mathrm{s}$
48. An ordinary differential equation is given below
$6 \frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}-y=0$

The general solution of the above equation (with constants $C_{1}$ and $C_{2}$ ), is
A. $y(x)=C_{1} x e^{-\frac{x}{3}}+C_{2} e^{\frac{x}{2}}$
B. $y(x)=C_{1} e^{-\frac{x}{3}}+C_{2} x e^{\frac{x}{2}}$
C. $y(x)=C_{1} e^{-\frac{x}{3}}+C_{2} e^{\frac{x}{2}}$
D. $y(x)=C_{1} e^{\frac{x}{3}}+C_{2} e^{-\frac{x}{2}}$

Ans. D
Sol. $\frac{6 d^{2} y}{d x^{2}}+\frac{d y}{d x}-y=0$
$\left(6 m^{2}+m-1\right) y=0$
$\mathrm{m}=\frac{\mathrm{d}}{\mathrm{dx}}$
Auxiliary equation is
$6 m^{2}+m-1=0$
$6 m^{2}+3 m-2 m-1=0$
$3 m(2 m+1)-1(2 m+1)=0$
$(2 m+1)(3 m-1)=0$
$m=-\frac{1}{2}, \frac{1}{3}$
C.F. $=C_{1} e^{-\frac{1}{2} x}+C_{2} e^{\frac{1}{3} x}$

PI $=0$ as $f(x)=0$
General solution
$y=C_{1} e^{-x / 2}+C_{2} e^{x / 3}$
49. A theodolite is set up at station $A$. The $R L$ of instrument axis is 212.250 m . The angle of elevation to the top of a 4 m long staff, held vertical at station $B$, is $7^{\circ}$. The horizontal distance between stations $A$ and $B$ is 400 m . Neglecting the errors due to curvature of earth and refraction, the RL (in $m$, round off to three decimal places) of station $B$ is
$\qquad$ .

Ans. 257.363 m

## Sol.


$\frac{P Q}{400}=\tan 7^{\circ}$
$P Q=49.113$
$R L$ of $B=212.250+49.113-4$
$=257.363 \mathrm{~m}$
50. A 10 m high slope of dry clay soil (unit weight $=20$ $\mathrm{kN} / \mathrm{m}^{3}$ ), with a slope angle of $45^{\circ}$ and the circular slip surface, is shown in the figure (not drawn to the scale). The weight of the slip wedge is denoted by $W$. The undrained unit cohesion $\left(\mathrm{C}_{\mathrm{u}}\right)$ is 60 kPa .


The factor of safety of the slope against slip failure, is
A. 1.57
B. 1.84
C. 1.67
D. 0.58

Ans. B
Sol. Area of circular Arc soil
$=\frac{\mathrm{Q}}{360} \times \mathrm{Ar}^{2}-\frac{1}{2}$ base $\times h t$
$=\frac{90}{360} \times \pi \times 10^{2}-\frac{1}{2} \times 10 \times 10$
$=28.59 \mathrm{~m}^{2}$
Weight d wedge $=$ volume $\times \mathrm{y}=$ Area $\times 1 \times \mathrm{y}=$ $28.54 \times 1 \times 20=570.8 \mathrm{kN}$

FOS =
$\frac{\text { RestoringMoment }}{\text { OverterningMoment }}=\frac{60 \times 10 \times \pi / 4 \times 10}{570.8 \times 4.48}$
$=\frac{C \times(r Q) \times r}{w \times x}$
$\mathrm{FOS}=1.84$
51. A concrete dam holds 10 m of static water as shown in the figure (not drawn to the scale). The uplift is assumed to vary linearly from full hydrostatic head at the heel, to zero at the toe of dam. The coefficient of friction between the dam and foundation soil is 0.45 . Specific weights of concrete and water are 24 $\mathrm{kN} / \mathrm{m}^{3}$ and $9.81 \mathrm{kN} / \mathrm{m}^{3}$, respectively.


For NO sliding condition, the required minimum base width $B$ (in $m$, round off to two decimal places) is
$\qquad$ .

Ans. 15.873
Sol. For no sliding, minimum width of dam
$\mathrm{B}_{\text {min }}=\frac{\mathrm{H}}{\mu(\mathrm{G}-1)}$
Here $\mathrm{H}=10 \mathrm{~m}$
$\mu=0.45$
$G=2.4$
$\therefore \mathrm{B}_{\text {min }}=\frac{10}{0.45(2.4-1)}=15.873 \mathrm{~m}$
52. The design speed of a two-lane two-way road is 60 $\mathrm{km} / \mathrm{h}$ and the longitudinal coefficient of friction is 0.36 . The reaction time of a driver is 2.5 seconds. Consider acceleration due to gravity as $9.8 \mathrm{~m} / \mathrm{s}^{2}$. The intermediate sight distance (in $m$, round off to the nearest integer) required for the road is $\qquad$ ـ.

Ans. 162 m

Sol. Given: $V=60 \mathrm{kmph}=16.67 \mathrm{~m} / \mathrm{sec}$
$f=0.36$
$\mathrm{t}=2.5 \mathrm{sec}$
$\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$
As we know, 1SD = 2SSD (stopping sight distance)
$\Rightarrow S S D=v t+\frac{v^{2}}{2 g f}$
$=16.67 \times 2.5+\frac{16.67^{2}}{2 \times 9.81 \times 0.36}$
$=41.675+39.34=81.01 \mathrm{~m}$
$\therefore 1 \mathrm{SD}=2 \times 81.01=162 \mathrm{~m}$.
53. Alkalinity of water, in equivalent/litre (eq/litre), is given by
$\left\{\mathrm{HCO}_{3}^{-}\right\}+2\left\{\mathrm{CO}_{3}^{2-}\right\}+\left\{\mathrm{OH}^{-}\right\}-\left\{\mathrm{H}^{+}\right\}$
Where, $\{$ \} represents concentration in $\mathrm{mol} / \mathrm{litre}$. For a water sample, the concentrations of $\mathrm{HCO}_{3}{ }^{-}=$ $2 \times 10^{-3} \mathrm{~mol} / \mathrm{litre}, \mathrm{CO}_{3}{ }^{2-}=3.04 \times 10^{-4} \mathrm{~mol} /$ litre and the pH of water $=9.0$. The atomic weights are: Ca = 40; $\mathrm{C}=12$; and $\mathrm{O}=16$. If the concentration of $\mathrm{OH}^{-}$and $\mathrm{H}^{+}$are NEGLECTED, the alkalinity of the water sample (in $\mathrm{mg} /$ litre as $\mathrm{CaCO}_{3}$ ), is
A. 50.0
B. 130.4
C. 100.0
D. 65.2

Ans. B
Sol. We know
$\mathrm{HCO}_{3}^{-}=2 \times 10^{-3} \mathrm{~mol} / \ell$
$\mathrm{CO}_{3}^{2-}=3.04 \times 10^{-4} \mathrm{~mol} / \ell$
According to question neglecting $\mathrm{OH}^{-}$and $\mathrm{H}^{+}$,
Alkalinity of water $=\left(2 \times 10^{-3}+2 \times 3.04 \times 10-4\right.$
$=2.608 \times 10-3$ of eq/ $\ell$
In terms of $\mathrm{CaCO}_{3}$,
Alkalinity of water $=2.608 \times 10^{-3} \times 50 \mathrm{gm} / \ell$ as caco
$=130.4 \mathrm{mg} / \ell$ as $\mathrm{cacos}_{3}$.
54. The cross-section of the reinforced concrete beam having an effective depth of 500 mm is shown in the figure (not drawn to the scale). The grades of concrete and steel used are M35 and Fe550, respectively. The area of tension reinforcement is $400 \mathrm{~mm}^{2}$. It is given that corresponding to $0.2 \%$
proof stress, the material safety factor is 1.15 and the yield strain of Fe550 steel is 0.0044 .


As per IS 456:2000, the limiting depth (in mm, round off to the nearest integer) of the neutral axis measured from the extreme compression fiber, is
$\qquad$ .
Ans. 221.52 mm
Sol.


D $=500 \mathrm{~mm}=$ Effective depth
Strain block@ Balanced failure
$0.0044=\left[0.002+\frac{0.87 \mathrm{f}_{\mathrm{y}}}{\mathrm{E}_{\mathrm{S}}}\right]$
From similar triangles properties
$\frac{0.0035}{\left(x_{u}\right) \text { limit }}=\frac{0.0044}{d-\left(x_{u}\right) \text { limit }} ; d=500 \mathrm{~mm}$
$x_{u}$, limit $=221.52 \mathrm{~mm}$
55. A theodolite was set up at a station P. The angle of depression to a vane 2 m above the foot of a staff held at another station Q was $45^{\circ}$. The horizontal distance between stations $P$ and $Q$ is 20 m . The staff reading at a benchmark $S$ of $R L 433.050 \mathrm{~m}$ is 2.905 m . Neglecting the errors due to curvature and refraction, the RL of the station Q (in m ), is
A. 413.955
B. 413.050
C. 431.050
D. 435.955

Ans. A
Sol.


$$
\begin{aligned}
& \frac{x}{20}=\tan 45 \\
& x=20 \\
& R L ~ d Q=433.05+2.905-x-2 \\
& =413.955 \mathrm{~m}
\end{aligned}
$$

## Benefits of Online Classroom Program

1. GATE Learning Tablet
> Access high-quality classes at your convenience, anywhere and anytime with the tablet
2. Live Classroom Sessions
> Get Access to Live Classes By India's Leading GATE Faculty
3. Previous Year Question Books
> 20+ Years PYQs with Solutions
4. Workbooks
> Access to 3000+ Practice Questions with solutions
5. Regular Quizzes
, Sample Quizzes for daily practice and regular tests along with live class

## 6. Doubt Resolution

> Complete Doubt Resolution within 24 hours by Subject Experts

## Additional Offerings

> Test Series - Mock Tests based on GATE Exam pattern
> Preparation Guidance - Get a competitive advantage from our Experts
> Subject wise formula Notes - Comprehensive short notes for Revision
, Report Card - Regular performance analysis along with Live Class


