Electrical Engineering

GATE 2021

Questions with Solutions

1. Which one of the following numbers is exactly divisible by $(11^{13} + 1)$?

A. $11^{13} + 1$ B. $11^{26} + 1$

C. $11^{39} - 1$ D. $11^{52} - 1$

Ans. D

Sol. Using Algebraic property,

$(11^{52} - 1) = (11^{26} + 1)(11^{26} - 1) = (11^{26} + 1)(11^{13} - 1)(11^{13} + 1)$

Since $(11^{52} - 1)$ has factor of $(11^{13} + 1)$, hence $(11^{52} - 1)$ is completely divisible by $(11^{13} + 1)$.

2. For a regulator polygon having 10 sides, the interior angle between the sides of the polygon, in degrees, is:

A. 216 B. 396

C. 324 D. 144

Ans. D

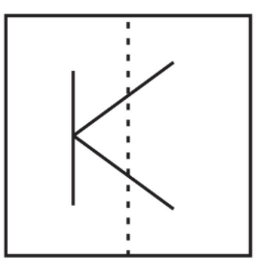
Sol. The interior angle between the sides of polygon $= \left(\frac{n-2}{n}\right) \times 180^\circ$

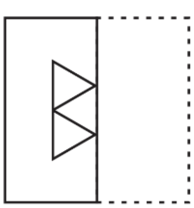
where n = Number of sides

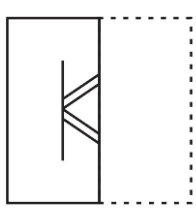
Hence interior angle

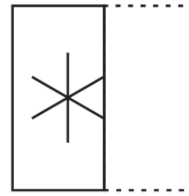
$= \left(\frac{10-2}{10}\right) \times 180^\circ = \left(\frac{8}{10}\right) \times 180^\circ = 144^\circ$

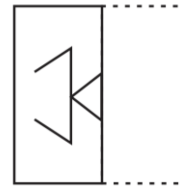
3. A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like ________.



A. 

B. 

C. 

D. 

Ans. A

Sol. Since half of the paper fold. Hence the structure will look like two triangles on a vertical strip. Hence option A is correct.

4. Let X be a continuous random variable denoting the temperature measured. The range of temperature is [0, 100] degree Celsius and let the probability density function X be f(x) = 0.01 for $0 \le X \le 100$. Then mean of X is ________.

A. 5.0 B. 50.0

C. 2.5 D. 25.0

Ans. B

Sol. PDF of X be f(x) = 0.01 for $0 \le x \le 100$

PDF f(x) = 0.01 for $0 \le x \le 100$

$$E(x) = \int_0^{100} xf(x)dx = \int_0^{100} x(0.01)dx$$

$$= \frac{1}{100}\left[\frac{x^2}{2}\right]_0^{100} = \frac{100}{2} = 50$$

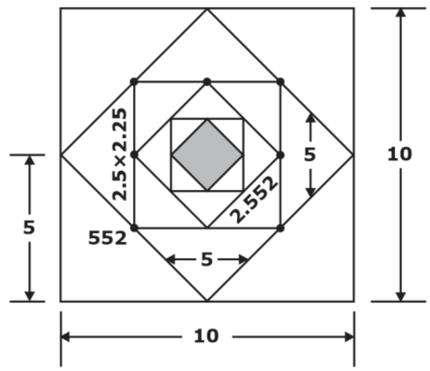
5. In the figure shown below, each inside square is formed by joining the midpoints of the slides of the next larger square. The area of the smallest square (shaded) as shown, in cm^2 is:

A. 1.5625 B. 12.50

C. 3.125 D. 6.25

Ans. C

Sol.



Second inner circle side,

$$a = \sqrt{\left(\frac{2.5\sqrt{2}}{2}\right)^2 + \left(\frac{2.5\sqrt{2}}{2}\right)^2}$$

$$a = \sqrt{\left(\frac{5\sqrt{2}}{4}\right)^2 + \left(\frac{5\sqrt{2}}{4}\right)^2}$$

$$a = \sqrt{\frac{2}{16}\left(5\sqrt{2}\right)^2} = \frac{1}{2\sqrt{2}}\left(5\sqrt{2}\right) \Rightarrow \frac{5}{2}$$

Side of shaded square,

$$\frac{a}{2}\sqrt{2} \Rightarrow \frac{(5/2)\sqrt{2}}{2} = \frac{5\sqrt{2}}{4} = \frac{5}{2\sqrt{2}}$$

Area of shaded region $= \left(\frac{a}{2}\sqrt{2}\right) \Rightarrow \frac{25}{8}$

$$\Rightarrow \frac{25}{8} = 3.125$$

6. The people _______ were at the demonstration were from all sections of society.

A. whom B. which
C. whose D. who

Ans. D

Sol. The people **who** were at demonstration were from all section of society.

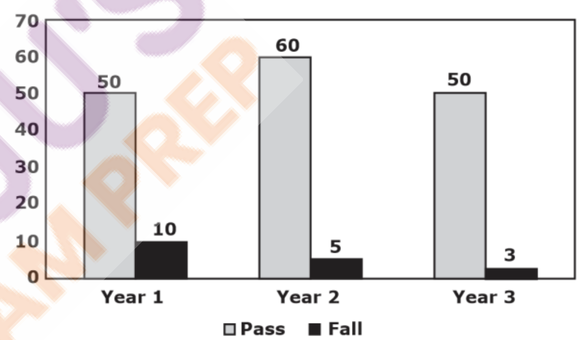
7. Oasis is to sand as island is to ________. Which one of the following options maintains a similar logical relation in the above sentence?

A. Water B. Mountain
C. Stone D. Land

Ans. A

Sol. Oasis is to sand as island is to Water.
So, Option A is correct.

8. The number of students passing or failing in an exam for a particular subject is presented in the bar chart above. Students who pass the exam cannot appear for the exam again. Students who fail the exam in the first attempt must appear for the exam in the following year. Students always pass the exam in their second attempt. The number of students who took the exam for the first time in the year 2 and the year 3 respectively, are ______.



A. 60 and 50 B. 55 and 48
C. 65 and 53 D. 55 and 53

Ans. B

Sol. Total students in year 2 who have given exam = 60 + 5.
But 10 students who have passed in year 2 are those.
Which were failed in year 1.
Hence, students in year 2 appearing for first time = 65 – 10 = 55.
Total students in year 3 who have given exam = 50 + 3 = 53.
But 5 students who have passed in year 2 are those which were failed in year 1.
Hence, students in year 3 appearing for time = 53 – 5 = 48.
Hence, option B is correct.

9. Seven cars P, Q, R, S, T, U and V are parked in a row not necessarily in that order. The cars T and U should be parked next to each other. The cars S and V also should be parked next to each other, whereas P and Q cannot be parked next to each other. Q and S must be parked next to each other. R is parked to the immediate right of V. T is parked to the left of U. Based on the above statements. the only **INCORRECT** option given below is:

A. Car P is parked at the extreme end.

B. Q and R are not parked together.

C. There are two cars parked in between Q and V.

D. V is the only car parked in between S and R.

Ans. C

Sol. From the given statement,

⇒ Cars S and V are parked together.

⇒ Cars Q and S are parked together.

First ⇒ QSV

Second ⇒ VSQ

Now, R is parked immediate right of V.

So, second arrangement is wrong as their S is parked at immediate right of V.

Possible arrangement = QSVR

As P and Q are not parked together and parked left to U.

So, final arrangement is **PTUQSVR** or **PUTQSVR.**

So, from the given options, option C is the only incorrect statement.

10. The importance of sleep is often overlooked by students when they are preparing for exams. Research has consistently shown that sleep deprivation greatly reduces the ability to recall the material learnt. Hence. cutting down on sleep to study longer hours can be counterproductive. Which one of the following statements is the **CORRECT** inference from the above passage?

A. Sleeping well alone is enough to prepare for an exam. Studying has lesser benefit.

B. Students are efficient and are not wrong in thinking that sleep is a waste of time.

C. To do well in an exam, adequate sleep must be part of the preparation.

D. If a student is extremely well prepared for an exam, he needs little or no sleep.

Ans. C

Sol. **Option A:** It is wrong as there is no information of benefits of studying an exam in given paragraph.

Option B: The paragraph does not give information about efficiency of student. Hence this option is also incorrect.

Option C: This option is correct as it explains about importance of sleep and adequate sleep is a part of preparation.

Option D: This option is incorrect as if a student is extremely well prepared for the exam, he needs little or no sleep, this information is not achieved from the given passage.

11. The causal signal with z-transform $z^2(z - a)^{-2}$ is: (u[n] is the unit step signal).

A. $n^{-1}a^n u[n]$ B. $(n + 1)a^n u[n]$

C. $n^2 a^n u[n]$ D. $a^{2n} u[n]$

Ans. B

Sol. $f(z) = \dfrac{z^2}{(z-a)^2}$

$$a^n u[n] \leftrightarrow \frac{z}{(z-a)}$$

$$n.a^n u[n] \leftrightarrow -z\left[\frac{d}{dz}\left(\frac{z}{z-a}\right)\right]$$

$$= -z\left[\frac{(z-a)1-z}{(z-a)^2}\right]$$

$$n.a^n u[n] \leftrightarrow \frac{az.}{(z-a)^2}$$

$$n.an^{-1}u[n] \leftrightarrow \frac{z}{(z-a)^2}$$

$$(n+1)a^n u[n] \leftrightarrow \frac{z^2}{(z-a)^2}$$

12. Suppose I_A, I_B and I_C are a set of unbalanced current phasors in a three-phase system. The phase B zero-sequence current $I_{B0} = 0.1\angle 0°$ p.u. If phase A current $I_A = 1.1\angle 0°$ p.u. and phase C current $I_C = (1\angle 120° + 0.1)$ p.u., then I_B in p.u. is

A. $1\angle -120° + 0.1\angle 0°$

B. $1.1\angle 240° - 0.1\angle 0°$

C. $1.1\angle -120° + 0.1\angle 0°$

D. $1\angle 240° - 0.1\angle 0°$

Ans. A

Sol. Facilitate per phase analysis of a complex 3-phase unbalanced system.

Given,

$I_{B0} = 0.1\ \angle 0°$ p.u. $= 0.1$

$I_A = 1.1\ \angle 0°$ p.u. $= 1.1$

$I_C = 1\ \angle 120° + 0.1$ p.u.

$I_B = ?$

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$I_{B0} = I_{A0} = \frac{1}{3}(I_A + I_B + I_C)$$

$$\alpha = 1\angle 120°,\ \alpha^2 = 1\angle 240°$$

$$I_{A0} = I_{C0}(\text{Cophesal})$$

$I_B = 0.1 + 1\angle 240°$ p.u.

$$I_{B0} = \frac{1}{3}(I_A + I_B + I_C)$$

$I_B = 3I_{B0} - I_A - I_C = 3 \times 0.1 - 1.1 - 1\angle 120° - 0.1 = 0.3 - 1.1 - 0.1 - 1\angle 120°$

$$= -0.9 - 1\angle 120° = -0.9 - \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$I_B = -0.9 + 0.5 - j\frac{\sqrt{3}}{2}$$

$$= -0.4 - j\frac{\sqrt{3}}{2} = 0.4 + 0.5 + \left[-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right]$$

$$= 0.1 + 1\angle 240° \text{ or } 0.1 + 1\angle -120°$$

13. If the input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \max(0, x(t))$, then the system is:

A. Linear and time-invariant

B. Non-linear and time-invariant

C. Linear and time-variant

D. Non-linear and time-variant

Ans. B

Sol. $y(t) = \max(0, x(t))$

$$y(t) = \begin{cases} 0 & x(t) < 0 \\ x(t) & x(t) \geq 0 \end{cases}$$

$y(t) = x(t).(x(t))$

For Linearity:

Additive + homogeneity

'Homogeneity'

$y(t) = x(t)\ (x(t))$

$K\ y(t) = K\ x(t)\ (x(t))$

'additive'

$y_1(t) = x_1(t)\ (x(t))$ --------(1)

$y_2(t)\ x_2(t)\ (x_2(t))$ --------(2)

$y_1(t) + y_2(t) - x_1(t)\ (x_1(t)) + x_2(t)\ (x_2(t))$ --(3)

$y_3(t) = [x_1(t) + x_2(t)]\,[x_1(t) + x_2(t)]$ ----(4)

$y_3(t) \neq y_1(t) + y_2(t)$

So, system is Non-linear.

For time variant or time invariant:

$y(t) = x(t)\,(x(t))$

$y(t, t_0) = x(t - t_o)\,(x(t - t_0))$ -------(1)

$y(t - t_0) = x(t - t_0)\,(x(t) - t_0)$ -------(2)

$y(t, t_0) = y(t - t_0)$

So, system is time Invariant.

14. Which one of the following vector functions represents a magnetic field $\vec{B}$? ($\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis, y-axis, and z-axis, respectively).

A. $10x\hat{x} + 20y\hat{y} - 30z\hat{z}$

B. $10x\hat{x} - 30z\hat{y} + 20y\hat{z}$

C. $10y\hat{x} - 20x\hat{y} - 10z\hat{z}$

D. $10z\hat{x} + 20y\hat{y} - 30x\hat{z}$

Ans. A

Sol. $\nabla \cdot \vec{B} = 0$ [Gauss law for magnetic fields]

Applying on options,

10 + 20 – 30 = 0

∴ Option ‘A‘ is the answer.

15. In a single-phase transformer, the total iron loss is 2500 W at nominal voltage of 440 V and frequency 50 Hz. The total iron loss is 850 W at 220 V and 25 Hz. Then, at nominal voltage and frequency, the hysteresis loss and eddy current loss respectively are:

A. 1600 W and 900 W

B. 900 W and 1600 W

C. 250 W and 600 W

D. 600 W and 250 W

Ans. B

Sol. $W_i = W_h + W_e$

$W_i = A_f + Bf^2$

$$\frac{V_2}{f_2} = \frac{V_1}{f_1} = \text{constant}$$

$B_{m2} = B_{m1}$

$$\frac{W_i}{f} = A + Bf$$

$$\frac{2500}{50} = A + B(50)$$

$$\frac{850}{250} = A + B(25)$$

$$\begin{array}{c} - \quad\quad - \quad - \\ \hline 16 = 25\,B \end{array}$$

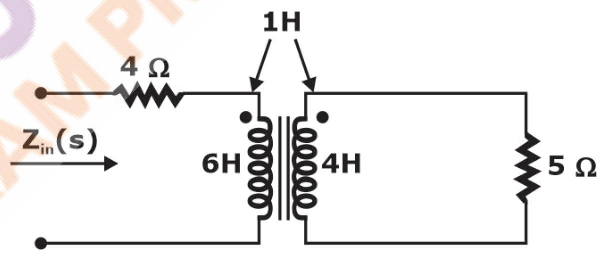
B = 0.64

A = 18

$W_h = A \times f = 18 \times 50 = 900$ W

$W_e = B \times f^2 = 0.64 \times 50^2 = 1600$ W

16. The input impedance, $Z_{in}(s)$, for the network shown is:



A. 6s + 4

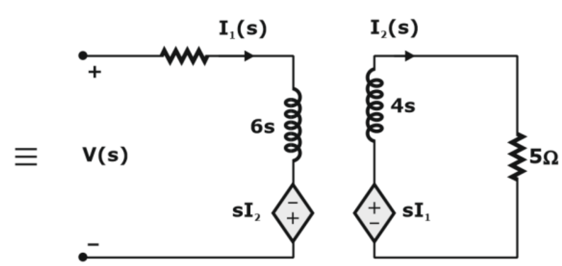
B. $\dfrac{23s^2 + 46s + 20}{4s + 5}$

C. 7s + 4

D. $\dfrac{25s^2 + 46s + 20}{4s + 5}$

Ans. B

Sol.



KVL at primary side,

$-V(s) + 4I_1(s) + 6sI_1(s) - sI_2(s) = 0$

$V(s) = (4s + 6s)\ I_1(s) - sI_2(s)$ ....(1)

KVL at secondary side,

$4s\ I_2(s) + 5I_2(s) - sI_1(s) = 0$

$$I_2(s) = \frac{s}{4s+5} I_1(s)$$

Put in (1)

$$V(s) = \left[(4+6s) - \frac{s^2}{4s+5}\right] I_1(s)$$

$$Z(s) = \frac{(4s+5)(4+6s) - s^2}{4s+5}$$

$$= \frac{16s + 24s^2 + 20 + 30s - s^2}{4s+5}$$

$$= \frac{23s^2 + 46s + 20}{4s+5}$$

17. Let f(t) be an even function, i.e. f(–t) = f(t) for all t. Let the Fourier transform of f(t) be defined as $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$. Suppose $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$ for all ω, and F(0) = 1. Then

A. f(0) < 1 B. f(0) > 1

C. f(0) = 0 D. f(0) = 1

Ans. A

Sol.

Given, $\frac{d}{d\omega} F(\omega) = -\omega F(\omega)$

$$\Rightarrow \frac{dF(\omega)}{F(\omega)} = -\omega\, d\omega$$

$$\int \frac{dF(\omega)}{F(\omega)} = -\int \omega\, d\omega$$

$$\Rightarrow \ln F(\omega) = -\frac{\omega^2}{2} + C$$

$$\because F(0) = 1 \Rightarrow \therefore C = 0$$

$$\ln F(\omega) = -\frac{\omega^2}{2} + 0$$

$$F(\omega) = e^{-\frac{\omega^2}{2}}$$

$$\because e^{-at^2} \xleftrightarrow{\text{Fourier Transform}} \sqrt{\frac{\pi}{a}}\, e^{-\frac{\omega^2}{4a}}$$

Put, $a = \frac{1}{2}$

$$e^{-\frac{t^2}{2}} \xleftrightarrow{\text{Fourier Transform}} \sqrt{2\pi}\, e^{-\frac{\omega^2}{2}}$$

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \xleftrightarrow{\text{Fourier Transform}} F(\omega) = e^{-\frac{\omega^2}{2}}$$

$$\therefore f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

$$f(0) = \frac{1}{\sqrt{2\pi}} e^0 = \frac{1}{\sqrt{2\pi}} = 0.4 < 1$$

$$\therefore f(0) < 1$$

18. Let $p(z) = z^3 + (1+j)z^2 (2+j)z + 3$, where z is complex number. Which one of the following is true?

A. The sum of the roots of p(z) = 0 is a real number.

B. The complex roots of the equation p(z) = 0 come in conjugate pairs.

C. Conjugate {p(z)}=p(conjugate{z})for all z.

D. All the roots cannot be real.

Ans. D

Sol. If $p(z) = z^3 + (1+j)z^2 (2+j)z + 3$, where z is complex number.

Since, coefficients are complex numbers, therefore, all the roots cannot be real. Hence, option D is correct.

Complex roots occur in conjugate pairs → false. Complex roots occur in conjugate pairs only if coefficients are all real nos.

Sum of the roots is real → false sum of all roots will be real number if complex roots occur in conjugate pairs. But their complex roots are not in conjugate pain.

$\overline{P(z)} = P(\bar{z})$ is false statement.

19. Consider a power system consisting of N number of buses. Buses in this power system are categorized into slack bus, PV buses and PQ buses for load flow study. The number of PQ buses is N_L. The balanced Newton-Raphson method is used to carry out load flow study in polar form. H, S, M, and R are sub-matrices of the Jacobian matrix J as shown below:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}; \text{ where } J = \begin{bmatrix} H & S \\ M & R \end{bmatrix}$$

The dimension of the sub-matrix M is:

A. $(N - 1) \times (N - 1 - N_L)$

B. $N_L \times (N - 1)$

C. $N_L \times (N - 1 + N_L)$

D. $(N - 1) \times (N - 1 + N_L)$

Ans. B

Sol. $$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{array}{cc} l & m \\ \left[\begin{matrix} H & S \\ M & R \end{matrix}\right. & \left.\right] \end{array} \begin{bmatrix} \Delta\delta \\ \Delta |V| \end{bmatrix}$$

$l \rightarrow$ Number of P specified

$m \rightarrow$ Number of Q specified

$M_{\underset{\uparrow\, m}{N_L} \times \underset{\uparrow\, l}{(N-1)}}$

Total buses = N

Slack bus = 1

Number of PQ buses = N_L

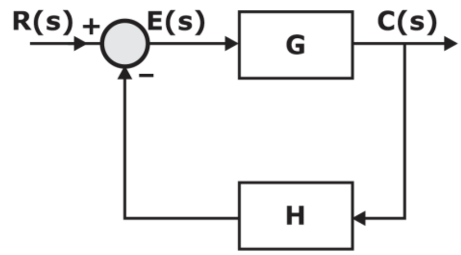
Number PV buses = $N - (1 + N_L)$

$l \rightarrow$ Number of P specified = $N/L + N - 1 - N/L$

$m \rightarrow$ Number of Q specified = N_L

size of M = $N_L \times (N - 1)$

20. For the closed-loop system shown, the transfer function $\dfrac{E(s)}{R(s)}$ is:



A. $\dfrac{G}{1+GH}$

B. $\dfrac{GH}{1+GH}$

C. $\dfrac{1}{1+G}$

D. $\dfrac{1}{1+GH}$

Ans. D

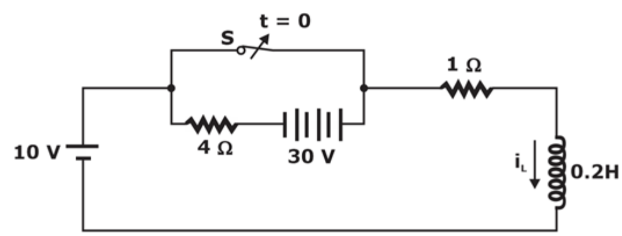
Sol. E = R − CH

E = R − (EG)H

E(1 + GH) = R

$\dfrac{E}{R} = \dfrac{1}{1+GH}$

21. In the circuit, switch 'S' is in the closed position for a very long time. If the switch is opened at time t = 0, then $i_L(t)$ in amperes, for t ≥ 0 is:



A. $8e^{-10t}$

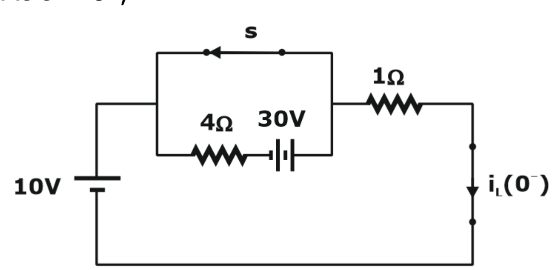
B. $8 + 2e^{-10t}$

C. 10

D. $10(1 - e^{-2t})$

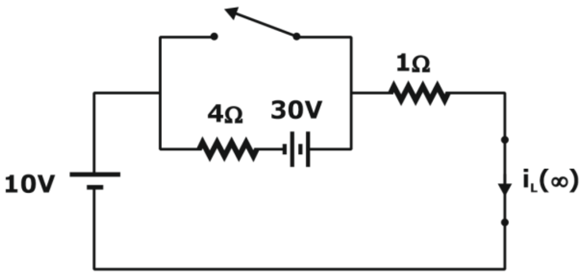
Ans. B

Sol. At $t = 0^-$;



$$i_L(0^-) = \frac{10}{1} = 10A$$

At t = ∞;



$$i_L(\infty) = \frac{40}{5} = 8\ A$$

Time constant

$$\tau = \frac{L}{R}$$

$$\tau = \frac{0.5}{5} = \frac{1}{10}\sec$$

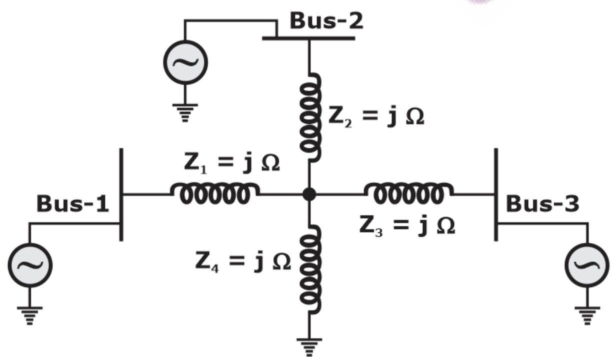
For Ist order R – L circuit,

$i_L(t) = i_L(\infty) - [i_L(\infty) - i_L(0^+)]e^{-t}/\tau$

$= 8 - [8 - 10]e^{-10t}$

$= 8 + 2e^{-10t}$ A

22. A 3-Bus network is shown. Consider generators as ideal voltage sources. If row 1, 2 and 3 of the Y_{BUS} matrix correspond to Bus 1, 2 and 3, respectively, then Y_{BUS} of the network is:



A. $\begin{bmatrix} -\frac{1}{2}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{1}{2}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{1}{2}j \end{bmatrix}$

B. $\begin{bmatrix} -4j & 2j & 2j \\ 2j & -4j & 2j \\ 2j & 2j & -4j \end{bmatrix}$

C. $\begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{3}{4}j \end{bmatrix}$

D. $\begin{bmatrix} -4j & j & j \\ j & -4j & j \\ j & j & -4j \end{bmatrix}$

Ans. C

Sol. $I_1 + I_3 + I_4 = I_2$ …………(1)

$I_1 = (V_1 - V_2)(-j1)$

$I_1 = (-j1)V_1 + (j1)V_2$ ………..(2)

$I_2 = V_2(-j1)$ …………(3)

$I_3 = (V_3 - V_2)(-j1)$

$I_3 = (-j1)V_2 + (j1)V_3$ ………..(4)

$I_4 = (V_4 - V_2)(-j1)$

$I_4 = (j1)V_2 - (j1)V_4$ ……………(5)

From equation (1) & (3)

$I_1 + I_3 + I_4 = V_2(-j1)$

$V_2 = (j1)[(-j1)V_1 + (j1)V_2 + (j1)V_2 + (-j1)V_3 + (j1)V_2 - (j1)V_4]$

$V_2 = V_1 - 3V_2 + V_3 + V_4$

$4V_2 = V_1 + V_3 + V_4$

$$V_2 = \frac{1}{4}V_1 + \frac{1}{4}V_3 + \frac{1}{4}V_4 \text{…………}(6)$$

Now,

$$I_1 = (1j1)V_1 + (j1)\left[\frac{1}{4}V_1 + \frac{1}{4}V_3 + \frac{1}{4}V_4\right]$$

$$I_1 = -j\frac{3}{4}V_1 + j\frac{1}{3}V_3 + j\frac{1}{4}V_4 \ldots\ldots(7)$$

$$I_3 = (-j1)\left[V_3 - \frac{1}{4}V_1 - \frac{1}{4}V_3 - \frac{1}{4}V_4\right]$$

$$I_3 = j\frac{1}{4}V_1 - j\frac{3}{4}V_3 + j\frac{1}{4}V_4 \ldots\ldots\ldots(8)$$

$$I_4 = (j1)\left[\frac{1}{4}V_1 + \frac{1}{4}V_3 + \frac{1}{4}V_4\right] - (j1)V_4$$

$$I_4 = j\frac{1}{4}V_1 + j\frac{1}{4}V_3 - J\frac{3}{4}V_4 \ldots\ldots\ldots\ldots(9)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -j\frac{3}{4} & j\frac{1}{4} & j\frac{1}{4} \\ j\frac{1}{4} & -j\frac{3}{4} & j\frac{1}{4} \\ j\frac{1}{4} & j\frac{1}{4} & -j\frac{3}{4} \end{bmatrix}}_{Y_{BUS}} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix}$$

23. Suppose the probability that a coin toss shows "head" is p, where $0 < p < 1$. The coin is tossed repeatedly until the first "head" appears. The expected number of tosses required is:

A. 1/p
B. p/(1–p)
C. $1/p^2$
D. (1 – p)/p

Ans. A

Sol.

Let X denotes the number of tosses required to get the first head,

X	1	2	3	4	--------
P(X)	p	pq	pq^2	pq^3	--------

$E(x) = \sum x.P(x)$

$E(x) = p + 2pq + 3pq^2 + 4pq^3 + \text{-------}$

$E(x) = p\,[1 + 2q + 3q^2 + 4q^3 + \text{------}]$

$\because 1 + 2A + 3A^2 + 4A^3 + \text{-------} = (1 - A)^{-2}$

$E(x) = p[1 - q]^{-2} \because p + q = 1$

$E(x) = p.p^{-2}\ p = 1 - q$

$E(x) = \frac{1}{p}$

24. Let f(x) be a real-valued function such that $f'(x_0) = 0$ for some $x_0 \in (0, 1)$, and $f''(x) > 0$ for all $x \in (0, 1)$. Then f(x) has

A. no local minimum in (0, 1)
B. one local maximum in (0, 1)
C. two distinct local minima in (0, 1)
D. exactly one local minimum in (0, 1)

Ans. D

Sol. (1). f(x) is real valued function.

(2). $f'(x_0) = 0;\ x_0 \in (0, 1)$.

$\Rightarrow x_0$ is the stationary point.

(3) $f''(x) > 0;\ \forall x \in (0,1)$

$\Rightarrow f''(x_0) > 0$

$\therefore$ f(x) is minimum at $x = x_0$

$\therefore$ exactly one local minimum occurs in (0, 1) and if occurs at 'x_0'

25. Two generators have cost functions F_1 and F_2. Their incremental-cost characteristics are:

$$\frac{dF_1}{dP_1} = 40 + 0.2P_1$$

$$\frac{dF_2}{dP_2} = 32 + 0.4P_2$$

They need to deliver a combined load of 260 MW. Ignoring the network loses, for economic operation, the generations P_1 and P_2 (in MW) are

A. $P_1 = 120,\ P_2 = 140$
B. $P_1 = P_2 = 130$
C. $P_1 = 160,\ P_2 = 100$
D. $P_1 = 140,\ P_2 = 120$

Ans. C

Sol. Coordination equation,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$40 + 0.2\, P_1 = 32 + 0.4\, P_2$ ....(1)

$P_1 + P_2 = 260$ MW ....(2)

$$P_1 - 2P_2 = -40 \ldots\ldots\ldots(1)$$

$$\Rightarrow \underline{-P_1 \pm P_2 = -260 \ldots\ldots\ldots(2)}$$

$$-3P_2 = -300$$

$P_2 = 100$ MW

$P_1 = 160$ MW

26. A counter is constructed with three D flip-flops. The input-output pairs are named (D_0, Q_0), (D_1, Q_1), and (D_2, Q_2), where the subscript 0 denotes the least significant bit. The output sequence is desired to be the Gray-code sequence 000, 001, 011, 010, 110, 111, 101, and 100, repeating periodically. Note that the bits are listed in the $Q_2\ Q_1\ Q_0$ format. The combinational logic expression for D_1 is:

A. $Q_2Q_0 + Q_1\bar{Q}_0$ B. $Q_2Q_1 + \bar{Q}_2\bar{Q}_1$

C. $Q_2Q_1Q_0$ D. $\bar{Q}_2Q_0 + Q_1\bar{Q}_0$

Ans. D

Sol.

$Q_2\ Q_1\ Q_0$	$Q_2^+\ Q_1^+\ Q_0^+$	$D_2\ D_1\ D_0$
0 0 0	0 0 1	0 0 1
0 0 1	0 1 1	0 1 1
0 1 0	1 1 0	1 1 0
0 1 1	0 1 0	0 1 0
1 0 0	0 0 0	0 0 0
1 0 1	1 0 0	1 0 0
1 1 0	1 1 1	1 1 1
1 1 1	1 0 1	1 0 1

For D_1:

Q_2 \ Q_1Q_0	00	01	11	10
0		1	1	1
1				1

$$D_1 = \overline{Q_2}Q_0 + Q_1\overline{Q_0}$$

27. In the open interval (0, 1), the polynomial $p(x) = x^4 - 4x^3 + 2$ has

A. two real roots B. one real root

C. three real root D. no real roots

Ans. B

Sol. $p(x) = x^4 - 4x^3 + 2 \Rightarrow P(-x) = x^4 + 4x^3 + 2$

Number of real (+ve) roots ≤ Number of sign changes in p(x) = 2

Number of real (−ve) rots ≤ Number of sign changes in p(−x) = 0

So, p(x) = 0 has no (−ve) real roots and a maximum of 2 (+ve) real roots.

∴ Number of real roots of 'P(x) = 0' ≤ 2

$$\left.\begin{array}{l} P(0) = 2 \\ P(1) = -1 \end{array}\right\} \text{One real root lies between 0 \& 1}$$

$$\left.\begin{array}{l} P(3) = 81 - 108 + 2 < 0 \\ P(4) = 256 - 256 + 2 > 0 \end{array}\right\} \text{One real root lies between 3 \& 4}$$

P(x) has only two real roots.

One real root lies in (0, 1) and One real root lies is (3, 4)

28. Let p and q be real numbers such that $p^2 + q^2 = 1$. The eigenvalues of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are:

A. j and −j B. pq and −pq

C. 1 and 1 D. 1 and −1

Ans. D

Sol. Let $A = \begin{bmatrix} p & q \\ q & -p \end{bmatrix}$

The characteristic equation

$|A - \lambda I| = 0$

$$\begin{vmatrix} p-\lambda & q \\ q & -p-\lambda \end{vmatrix} = 0$$

$\Rightarrow (p^2 - \lambda^2) - q^2 = 0$

$\Rightarrow p^2 + q^2 - \lambda^2 = 0$

$\Rightarrow 1 - \lambda^2 = 0$

$\lambda^2 = 1$

$\lambda = \pm 1$

29. Consider the table given:

Constructional feature	**Machine type**	**Mitigation**
(P) Damper bars	(S) Induction motor	(X) Hunting
(Q) Skewed rotor slots	(T) Transformer	(Y) Magnetic locking
(R) Compensating winding	(U) Synchronous machine	(Z) Armature reaction
	(V) DC machine	

The correct combination that relates the constructional feature. machine type and mitigation is:

A. P-V-X, Q-U-Z, R-T-Y

B. P-T-Y, Q-V-Z, R-S-X

C. P-U-X, Q-S-Y, R-V-Z

D. P-U-X, Q-V-Y, R-T-Z

Ans. C

Sol. PUX – QSY – RVZ

30. Inductance is measured by:

A. Schering bridge B. Kelvin bridge

C. Maxwell bridge D. Wien bridge

Ans. C

Sol. For measurement of inductance maxwell bridge is use.

31. Two discrete-time linear time-invariant systems with impulse responses $h_1[n] = \delta[n-1] + \delta[n+1]$ and $h_2[n] = \delta[n] + \delta[n-1]$ are connected in cascade, where $\delta[n]$ is the Kronecker delta. The impulse response of the cascaded system is:

A. $\delta[n]\delta[n-1] + \delta[n-2]\delta[n+1]$

B. $\delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$

C. $\delta[n-1]\delta[n] + \delta[n+1]\delta[n-1]$

D. $\delta[n-2] + \delta[n+1]$

Ans. B

Sol. $h_1[n] = \delta[n-1] + \delta[n+1]$

$h_2[n] = \delta[n] + \delta[n-1]$

$h_1[n] * h_2[n] = \delta[n-1] + \delta[n-2] + \delta[n+1] + \delta[n]$

32. Let $(-1 - j)$, $(3 - j)$, $(3 + j)$ and $(-1 + j)$ be the vertices of a rectangle C in the complex plane. Assuming that C is traversed in counter-clockwise direction, the value of the contour integral $\oint_C \frac{dz}{z^2(z-4)}$ is:

A. $j\pi/16$ B. 0

C. $-j\pi/8$ D. $j\pi/2$

Ans. C

Sol. Vertices of rectangle are $(-1, -1)$, $(3, -1)$, $(3, 1)$ and $(-1, 1)$

Let $f(z) = \frac{1}{z^2(z-4)}$

Poles of f(z) are z = 4

z = 0 of index 'z'

∴ z = 0 lies inside 'c'

∴ 'z = 0' is a singular point of index

(n + 1) = z

Residue of f(z) 'at z = a' $= \frac{1}{n!}\lim_{z\to a}\frac{d^n}{dz^n} z^{n+1} f(z)$

n = 1, a = 0

$$R = \frac{1}{1!}\lim_{z\to 0}\frac{d}{dz}z^2\frac{1}{(z-4)^2}$$

$$= \lim_{z\to 0}\frac{(-1)}{(z-4)^2} = \frac{-1}{16}$$

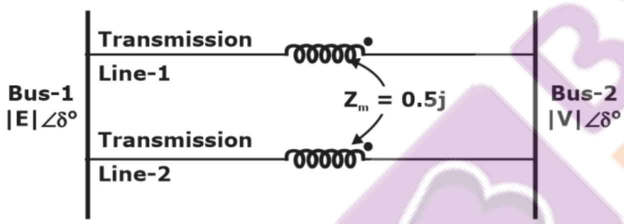
By Cauchy's Residue theorem

$$\oint_C f(z)dz = 2\pi j[\text{sum of residues}]$$

$$\oint_C \frac{dz}{z^2(z-4)} = 2\pi j\ R = 2\pi j\left(\frac{-1}{16}\right)$$

$$= \frac{-j\pi}{8}$$

33. In the figure shown, self-impedances of the two transmission lines are 1.5j p.u. each, and Z_m = 0.5j p.u. is the mutual impedance. Bus voltages shown in the figure are in p.u. Given that δ > 0, the maximum steady-state real power that can be transferred in p.u from Bus-1 to Bus-2 is:

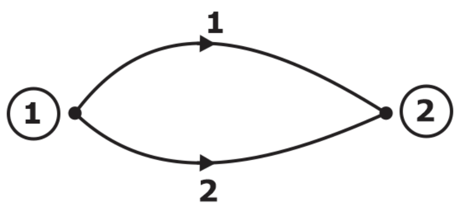


A. $\frac{3|E||V|}{2}$ B. $|E||V|$

C. $\frac{|E||V|}{2}$ D. $2|E||V|$

Ans. B

Sol. $P_{max} = \frac{|E||V|}{X_{Th}}$



Singular Tf

$$A = \frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$$

$$y_{prim} = Z_{prim}^{-1}$$

$$Z_{prim} = \begin{bmatrix}1.5 & 0.5\\ 0.5 & 1.5\end{bmatrix}$$

1.5 × 1.5 – 0.25

2.25 – 0.25 = 2

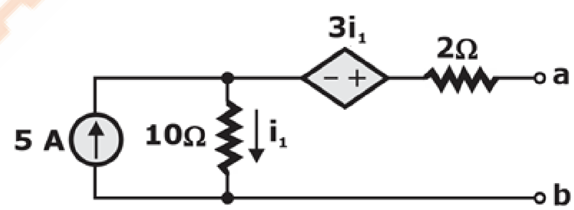
$$y_{prim} = \frac{1}{2}\begin{bmatrix}1.5 & -0.5\\ -0.5 & 1.5\end{bmatrix}$$

$$y_{Bus} = A^T[y_{prim}]A$$

$$= \frac{1}{2}\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}1.5 & -0.5\\ -0.5 & 1.5\end{bmatrix} = \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$Z_{Bus} = y_{Bus}^{-1} = \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

34. For the network shown, the equivalent Thevenin voltage and Thevenin impedance as seen across terminals 'ab' is:



A. 50 V in series with 2 Ω

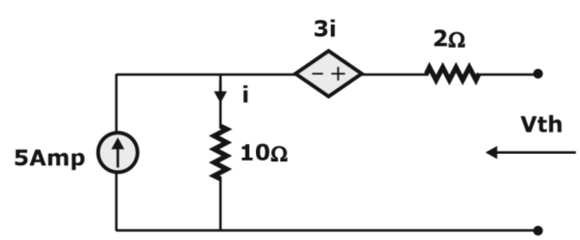
B. 65 V in series with 15 Ω

C. 35 V in series with 2 Ω

D. 10 V in series with 12 Ω

Ans. B

Sol.



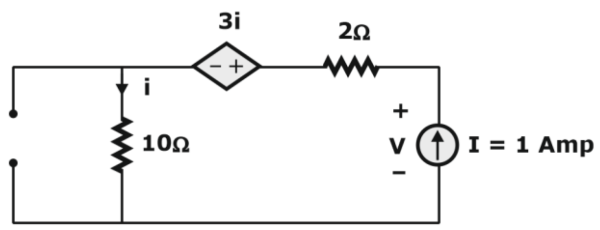
$V_{th} = V_{ab} = 2 \times 3i + 10i$

$= 13i$ volt

But $i = 5$ A

$V_{th} = V_{ab} = 13 \times 5 = 65$ volt

For R_{th}:



$-V + 2 \times 1 + 3i + 10i = 0$

$V = 15$ volts

$$R_{Th} = \frac{V}{1} = 15\Omega$$

35. One coulomb of point charge moving with a uniform velocity $10\hat{x}$ m/s enters the region $x \geq 0$ having a magnetic flux density $\vec{B} = (10y\hat{x} + 10x\hat{y} + 10\hat{z})$ T. The magnitude of force on the charge at $x = 0^+$ is ________ N. ($\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis, y-axis, and z-axis, respectively).

Ans. 100

Sol. $f = Q(v \times B) = 10[a_x \times (10ya_x + 10xa_y + 10a_z)]$

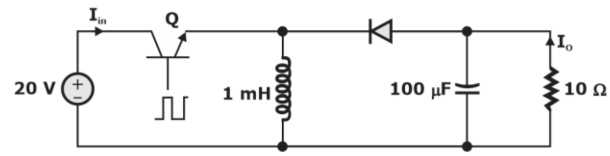
$F = [10xa_z - 10a_y]10$

At $x = 0$

$F = [10a_y]10$

$|F| = 100$ N

36. Consider the buck-boost converter shown. Switch Q is operating at 25 kHz and 0.75 duty-cycle. Assume diode and switch to be ideal. Under steady-state condition, the average current flowing through the inductor ________ A.



Ans. (24 – 24)

Sol.

$$I_L = \frac{\alpha V_{dc}}{R(1-\alpha)^2} = \frac{0.75 \times 20}{10(1-0.75)^2} = 24 \text{ A}$$

37. Let A be a 10 × 10 matrix such that A^5 is a null matrix, and let I be the 10 × 10 identity matrix. The determinant of A + I is ________.

Ans. (1 – 1)

Sol.

Given, $A^5 = 0$

∴ A is a nilpotent matrix of index '5'.

∴ eigen values of A are all 0.

$|A|$ = product of eigen values = 0

$|A| = 0$

Now, $|A + I| = |A| + |I|$

$= 0 + 1 = 1$

38. A 16-bit synchronous binary up-counter is clocked with a frequency f_{CLK}. The two most significant bits are OR-ed together to form an output Y. Measurements show that Y is periodic, and the duration for which Y remains high in each period is 24 msec. The clock frequency f_{CLK} is ________ MHz. (Round off to 2 decimal places).

Ans. 2.048

Sol.

For a 16-bit synchronous binary counter:

Total combination (no. of states) = 216

In 216 combination, half of it having MSB "1" and in remaining half again having D_{14} is high.

Output will become high $= 2^{15} + \frac{2^{15}}{2} = 3 \times 2^{14}$

$3 \times 2^{14} \times T_{CLK} = 24 \times 10^{-3}$ sec

$T_{CLK} = \frac{8 \times 10^{-3}}{2^{14}}$ sec

$T_{CLK} = \frac{1}{2^{11}}$ msec

$f_{CLK} = 2^{11}$ kHz

$f_{CLK} = 2.048$ MHz

39. Consider a large parallel plate capacitor. The gap d between the two plates is filled entirely with a dielectric slab of relative permittivity 5. The plates are initially charged to a potential difference of V volts and then disconnected from the source. If the dielectric slab is pulled out completely, then the ratio of the new electric field E_2 in the gap to the original electric field E_1 is ________.

Ans. 5

Sol.

$E \propto \frac{1}{\varepsilon}$

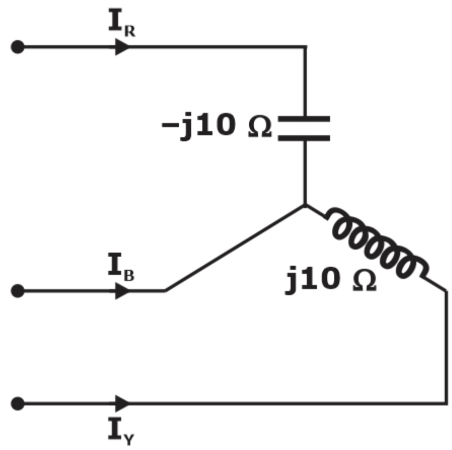
$E_2 \propto \frac{1}{\varepsilon_2} \propto \frac{1}{\varepsilon_0 \varepsilon_1}$

$E_1 \propto \frac{1}{\varepsilon_1} \propto \frac{1}{\varepsilon_0}$

$$\frac{E_2}{E_1} = \frac{\frac{1}{\varepsilon_0 \varepsilon_r}}{\frac{1}{\varepsilon_0}} = \frac{1}{\varepsilon_r} = \frac{1}{5}$$

$$\boxed{\frac{E_2}{E_1} = \frac{1}{5} \Rightarrow \therefore \frac{E_1}{E_2} = 5:1}$$

40. A three-phase balanced voltage is applied to the load shown. The phase sequence is RYB. The ratio $\frac{|I_B|}{|I_R|}$ is ________.



Ans. 1.732

Sol.

Phase sequence is RYB,

$V_R = V\angle 0°$, $V_Y = V\angle -120°$, $V_B = V\angle -240°$

$I_R = \frac{V_R}{z} = \frac{V\angle 0°}{j10} = \frac{V}{10}\angle 90°$ A

$I_Y = \frac{V\angle -120°}{j10} = \frac{V}{10}\angle -210° A$

$I_Y = \frac{V\angle -120°}{j10} = \frac{V}{10}\angle -210° A$

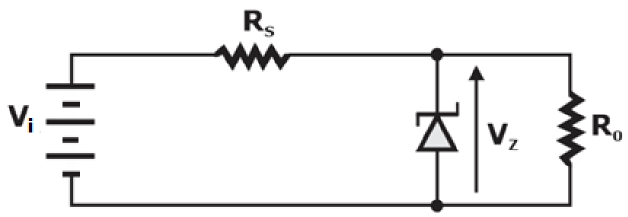
$\vec{I}_B = -\left(\hat{I}_R + \hat{I}_Y\right)$

$= -\left(\frac{V}{10}\angle 90° + \frac{V}{10}\angle -210°\right)$

$= -\frac{V}{10}(1\angle 90° + 1\angle -210°) = \frac{-V\sqrt{3}}{10}\angle +120°$

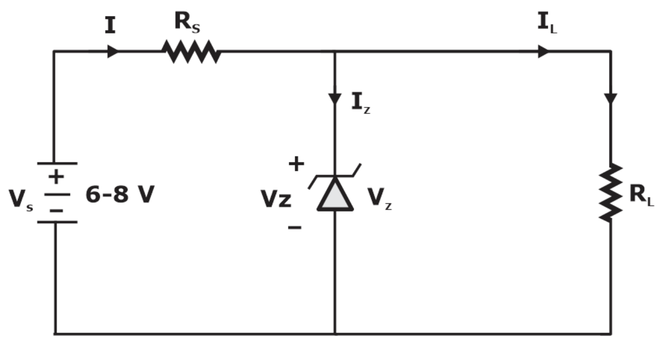
$\frac{|\vec{I}_B|}{|\vec{I}_R|} = \sqrt{3} = 1.732$

41. In the circuit shown, a 5 V Zener diode is used to regulate the voltage across load R_0. The input is an unregulated DC voltage with a minimum value of 6 V and a maximum value of 8 V. The value of R_S is 6 Ω. The Zener diode has a maximum rated power dissipation of 2.5 W. Assuming the Zener diode to be ideal, the minimum value of R_0 is _________ Ω.



Ans. 30

Sol.



$$I_{z\ \text{Mininum}} = I_{\text{Mininum}} - I_{L\ \text{Maximum}} \ge I_{zk}$$

I_{zk} = Knee current

$$I_{z\ \text{Minimum}} = \frac{V_{i\ \text{Minimum}} - V_z}{R_s} - \frac{V_z}{R_{L\ \text{Minimum}}} \ge 0$$

$$= \frac{6-5}{6} - \frac{5}{R_{L\ \text{Minimum}}} \ge 0$$

$$= \frac{1}{6} \ge \frac{5}{R_{L\ \text{Minimum}}}$$

$$R_{L\ \text{Minimum}} \ge 6 \times 5 = 30\,\Omega$$

42. Suppose the circles $x^2 + y^2 = 1$ and $(x - 1)^2 + (y - 1)^2 = r^2$ intersect each other orthogonally at the point (u, v). Then u + v = ________.

Ans. 1

Sol. $x^2 + y^2 = 1$ ...(1)

$(x - 1)^2 + (y - 1)^2 = r^2$ ...(2)

If (u, v) lies on both the circles (1) & (2),

(1) $\Rightarrow u^2 + v^2 = 1$ ...(3)

$$\vec{N}_1 = \bar{\nabla}\phi_1 = \hat{i}(2x) + \hat{j}(2y) = 2u\,\hat{i} + 2v\hat{j}$$

$$\vec{N}_2 = \bar{\nabla}\phi_2 = 2(x-1)i + 2(y-1)\hat{j} = 2(u-1)\hat{i} + 2(v-1)\hat{j}$$

Now, $\vec{N}_1 \cdot \vec{N}_2 = 0$

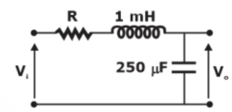
$2u.\ 2(u - 1) + 2v.\ 2(v - 1) = 0$

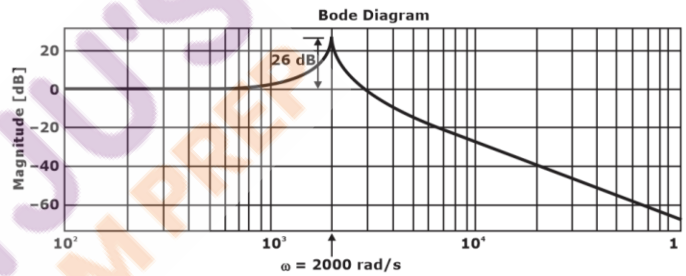
$4[u^2 + v^2 - u - v] = 0$

$4[1 - u - v] = 0 \Rightarrow -u - v + 1 = 0$

$u + v = 1$

43. The Bode magnitude plot for the transfer function $\frac{V_o(s)}{V_i(s)}$ of the circuit is as shown. The value of R is ______ Ω. (Round off to 2 decimal places.)





Ans. 0.1

Sol. Given M_r dB = 26 dB

$20 \log M_R = 26$

$M_r = 20$

$$M_r = \frac{1}{\sin 2\phi} = 20$$

$$\sin\phi = \frac{1}{\sin 2\phi} = 20$$

$2\varphi = 2.86, 177.14$

$\varphi = 88.57$ (> 450°)

$s = \cos\varphi = 0.025$

Now, T.F. of RLC series circuit is

$$\frac{V_o}{V_i} = \frac{1}{s^2LC + sRC + 1}$$

$$= \frac{1}{s^2(10^{-3} \times 250 \times 10^{-6})s(R \times 250 \times 10^{-6}) + 1}$$

$$= \frac{4\times10^6}{s^2+1500\,Rs+4\times10^6}$$

$$\omega_n^2 = 4\times10^6$$

$$\omega_n = 2\times10^3$$

$25\omega_n = 1000$ R

$2(0.25)\,(2 \times 10^3) = 1000R$

R = 0.1 Ω

44. An 8-pole, 50 Hz, three-phase, slip-ring induction motor has an effective rotor resistance of 0.08 Ω per phase. Its speed at maximum torque is 650 RPM. The additional resistance per phase that must be inserted in the rotor to achieve maximum torque at start is __________ Ω. (Round off to 2 decimal places.) Neglect magnetizing current and stator leakage impedance. Consider equivalent circuit parameters referred to stator.

Ans. 0.52

Sol.

Synchronous speed,

$$N_S = \frac{120f}{P} = \frac{15\times50}{8} = 750 \text{ rpm.}$$

Slip at maximum torque,

$$s_{T_{max}} = \frac{N_S - N_{T_{max}}}{N_S} = \frac{750-650}{750} = \frac{100}{750} = 0.1333$$

Also, we known that,

$$s_{T_{max}} = \frac{R_2'}{X_2'} \Rightarrow X_2' = \frac{R_2'}{s_{T_{max}}} = \frac{0.08}{0.1333} = 0.6\ \Omega$$

For producing maximum torque at starting,

$$s_{T_{max}} = 1 \Rightarrow \frac{R'_{2(New)}}{X_2'} = 1 \Rightarrow R'_{2(New)} = X_2' = 0.6$$

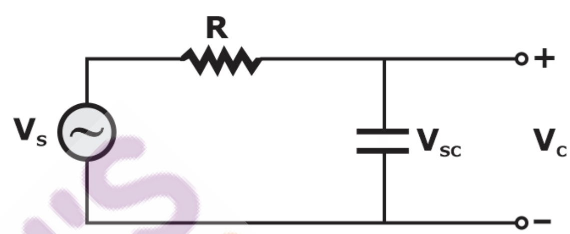
$$\Rightarrow R_2' + R_{ext} = 0.6$$

$$R_{ext} = 0.6 - R_2' = 0.6 - 0.08 = 0.52\ \Omega$$

45. A signal generator having a source resistance of 50 Ω is set to generate a 1 kHz sinewave. Open circuit terminal voltage is 10 V peak-to-peak. Connecting a capacitor across the terminals reduces the voltage to 8 V peak-to-peak. The value of this capacitor is _________ μF. (Round off to 2 decimal places.)

Ans. 2.38

Sol.



$$V_o = \frac{V_{SC}}{R+V_{SC}}V_S$$

$$V_o / V_s = A = \frac{1}{1+SCR} = \frac{1}{1+j\omega RC}$$

$$\frac{V_{OP-P}}{V_{SP-P}} = \frac{1}{|1+j2\pi fRC|}$$

$$\frac{8}{10} = \frac{1}{\sqrt{1+(2\pi\times10^3\times50C)^2}}$$

$1 + (2п\ 02 \times 10^3 \times 50C)^2 = (10/8)^2 = 1.5625$

$$2\pi\times10^3\times50C = \sqrt{0.5625}$$

$C = 2.38 \times 10^{-6}$ F

$$\boxed{C = 2.38\ \mu F}$$

46. A belt-driven DC shunt generator running at 300 RPM delivers 100 kW to a 200 V DC grid. It continues to run as a motor when the belt breaks, taking 10 kW from the DC grid. The armature resistance is 0.025 Ω, field resistance is 50 Ω, and brush drop is 2 V. Ignoring armature reaction, the speed of the motor is _______RPM. (Round off to 2 decimal places.)

Ans. 275.18

Sol. $P_0 = 100 \times 10^3$

$$I_L = \frac{100 \times 10^3}{200}$$

$I_L = 500$ A

$$\rightarrow I_{sh} = \frac{200}{50} = 4A$$

$I_a = 500 + 4 = 504$ A

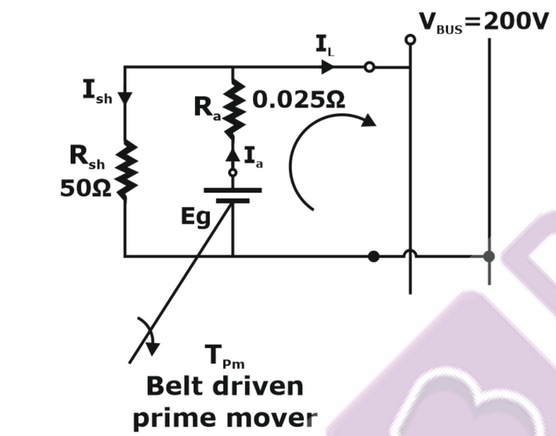
By KVL

$E_g - I_aR_a - V_B = V_{Bus}$

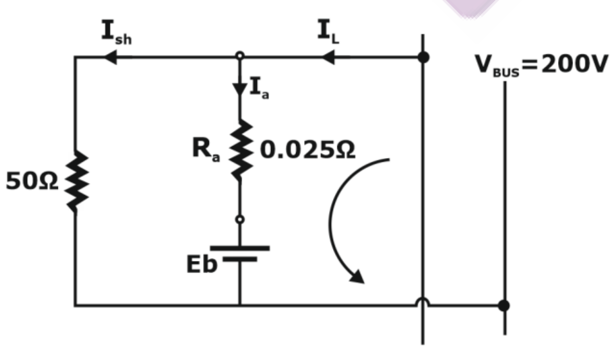
$E_g - V_{BUS} + I_aR_a + B.D.$

$= 200 + 504 \times 0.025 + 2$

$E_g = 214.6$ V



The machine starts consuming 10 kW Power due to absence of mechanical input.



$P = 10 \times 10^3$

$$I_L = \frac{10 \times 10^3}{200} = 50\,A$$

$$\rightarrow I_{sh} = \frac{200}{50} = 4A$$

$I_a = I_L - I_{sh} = 50 - 4 = 46$ A

By KVL

$V - I_a R_a - B.D = E_b$

$200 - 46 \times 0.025 - 2 = E_b$

$E_m = E_b = 196.85$ Volts

$$N \propto \frac{E_b}{\phi}$$

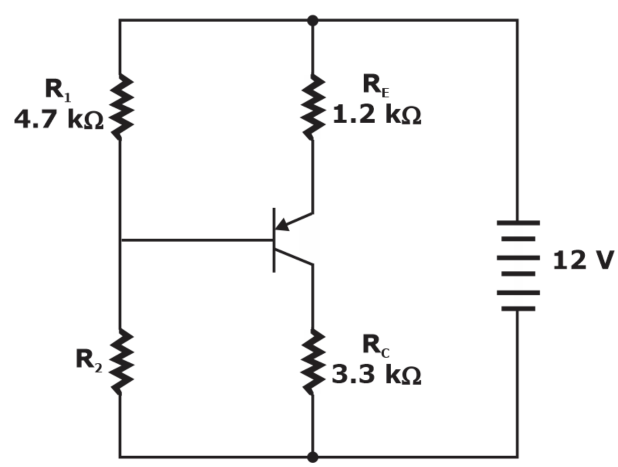
'φ' is constant since current through shunt field winding is constant.

$$\therefore \frac{N_m}{N_g} = \frac{E_m}{E_g}$$

$$N_m = \frac{196.85}{214.6} \times 300$$

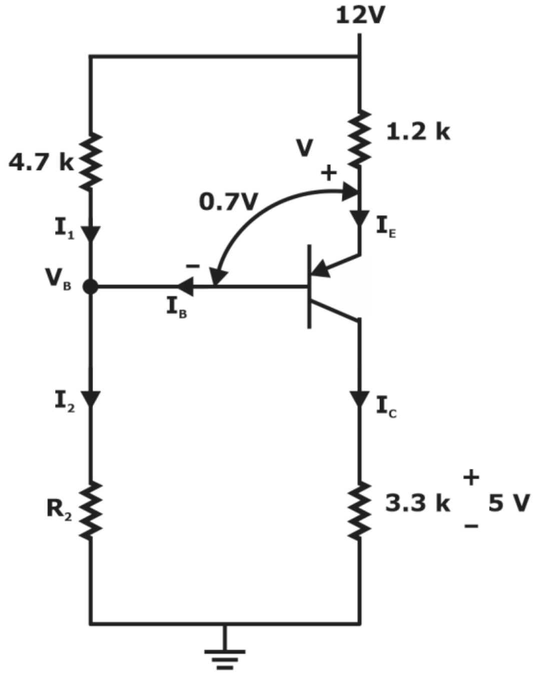
$N_m = 275.18$ rpm

47. In the BJT circuit shown, beta of the PNP transistor is 100. Assume $V_{BE} = -0.7$ V. The voltage across R_C will be 5 V when R_2 is ________ kΩ. (Round off to 2 decimal places).



Ans. 17.02

Sol.



$$R_2 = \frac{V_B}{I_2}$$

$$I_c = \frac{5}{3.3} = 1.515\text{ mA}$$

$$I_B = \frac{I_C}{B} = \frac{1.515}{100}\text{mA} = 0.0151\text{ mA}$$

$$I_E = I_c + I_B = 1.530\text{ mA}$$

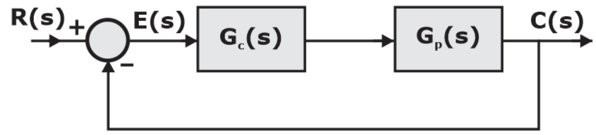
$$V_B = 12 - 1.2\ I_E - 0.7$$

$$V_B = 9.46\text{V}$$

$$I_2 = I_1 + I_B = \frac{12-9.46}{4.7} + \frac{1.515}{100} = 0.55\text{ mA}$$

$$R_2 = \frac{9.46}{0.550} = 17.02\text{ k}\Omega$$

48. Consider a closed-loop system as shown. $G_p(s) = \dfrac{14.4}{s(100.1s)}$ is the plant transfer function and $G_c(s) = 1$ is the compensator. For a unit-step input, the output response has damped oscillations. The damped natural frequency is _________ rad/s. (Round off to 2 decimal places.)



Ans. 10.91

Sol. $1 + G_c(a)\ G_p(s)\ H(s) = 1$

$$1 + 1 \times \frac{14.4}{s(1+0.1s)} = 0$$

$$0.1s^2 + s + 14.4 = 0$$

$$s^2 + 10s + 144 = 0$$

$$2\xi\omega_n = 10$$

$$\omega_n^2 = 144$$

$$2\xi \times 12 = 10$$

$$\xi = \frac{10}{24} = \frac{5}{12}$$

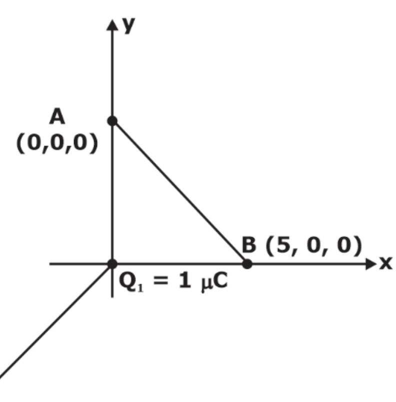
So, $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$= 12\sqrt{1 - \frac{25}{144}} = \sqrt{119} = 10.91\text{ rad/sec}$$

49. A 1 μC point charge is held at the origin of a cartesian coordinate system. If a second point charge of 10 μC is moved from (0, 10, 0) to (5, 5, 5) and subsequently to (5, 0, 0), then the total work done is _________ mJ. (Round off to 2 decimal places). Take $\dfrac{1}{4\pi\varepsilon_o} = 9 \times 10^9$ in SI units. All coordinates are in meters.

Ans. 9

Sol.



We know, E is conservative in nature, i.e., $\vec{\nabla}\times\vec{E}=0$ & VAB is independent of path taken.

$$V_{AB}=-\int_A^B \vec{E}\cdot d\vec{l}$$

$$=-\int_A^B \frac{Q_1}{4\pi\varepsilon_o r^2}\hat{a}_r\cdot dr\,\hat{a}_r$$

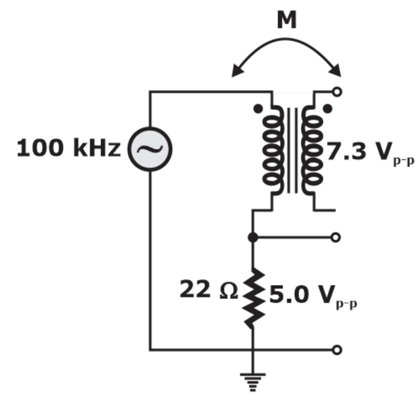
$$=\frac{Q_1}{4\pi\varepsilon_o}\left[\frac{1}{r}\right]_{r=0}^{r=5}=9\times10^9\times1\times10^{-6}\left[\frac{1}{5}-\frac{1}{10}\right]$$

So, $W=Q_2V_{AB}$

$$W=\left(10\times10^{-6}\right)\times9\times10^9\times1\times10^{-6}\left[\frac{1}{5}-\frac{1}{10}\right]$$

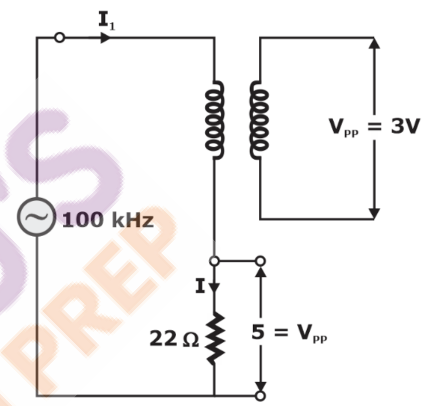
$$W=9\times10^{-2}\times\frac{1}{10}=9\times10^{-3}=9\text{ mJ}$$

50. An air-core radio-frequency transformer as shown has a primary winding and a secondary winding. The mutual inductance M between the windings of the transformer is _______μH. (Round off to 2 decimal places.)



Ans. 51.12

Sol.

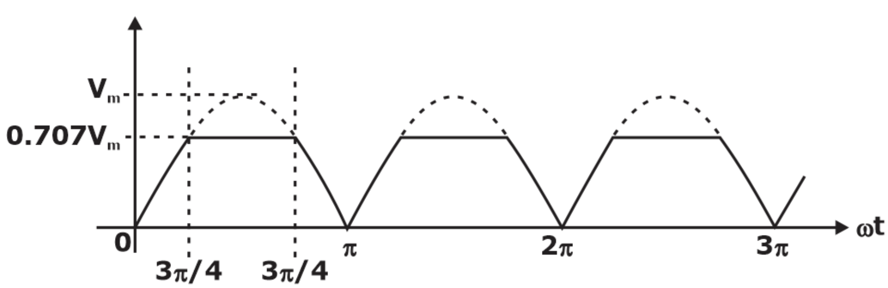


$$V=\omega MI$$

$$7.3=2\pi\times100\times10^3\times M\times\frac{5}{22}$$

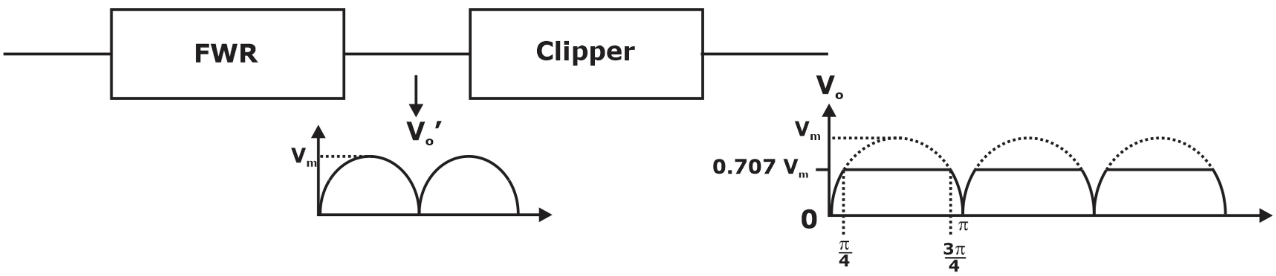
$$M=\frac{22\times7.3}{\pi\times10^6}=51.12\ \mu H$$

51. The waveform shown in solid line is obtained by clipping a full-wave rectified sinusoid (shown dashed). The ratio of the RMS value of the full-wave rectified waveform to the RMS value of the clipped waveform is _________. (Round off to 2 decimal places.)



Ans. 1.219

Sol.



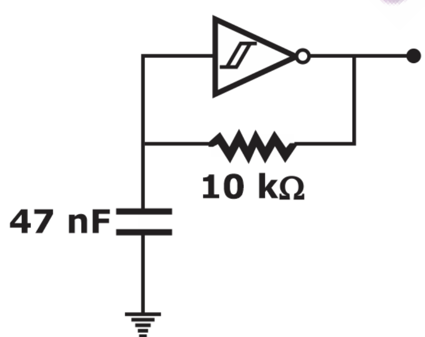
$$V_{rms\ Clipped} = \left[\frac{1}{\pi}\int_0^{\frac{\pi}{4}} V_m^2 \sin^2\theta \cdot d\theta + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left(\frac{V_m}{\sqrt{2}}\right)^2 \cdot d\theta + \int_{\frac{3\pi}{4}}^{\pi} V_m^2 \sin^2\theta \cdot d\theta\right]^{\frac{1}{2}}$$

$V_{rms\ clipped} = 0.58V_m$

$V_{rms\ Unclipped} = V_{rms}$ for FWR $= \dfrac{V_m}{\sqrt{2}}$

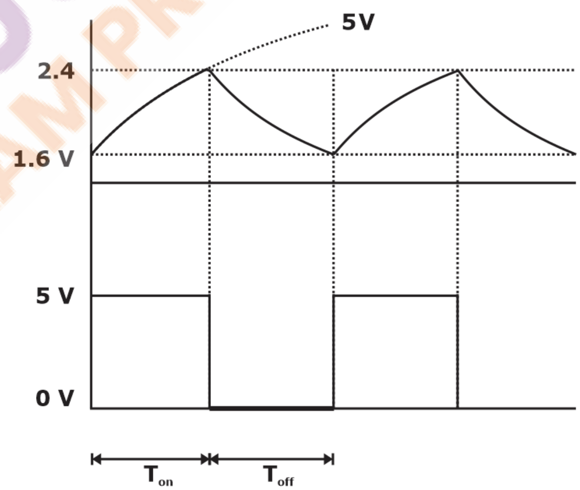
$$\frac{V_{rms\ Unclipped}}{V_{rms\ Clipped}} = \frac{\frac{V_m}{\sqrt{2}}}{0.58\,V_m} = 1.219$$

52. A CMOS Schmitt-trigger inverter has a low output level of 0 V and a high output level of 5 V. It has input thresholds of 1.6 V and 2.4 V. The input capacitance and output resistance of the Schmitt-trigger are negligible. The frequency of the oscillator shown is _________ Hz. (Round off to 2 decimal places.)



Ans. 3158

Sol.



During T_{on}

$$V_c = V(\infty) - \left[V(\infty) - V(0)\right]e^{\frac{-t}{\tau}}$$

$$V_c = 5 - \left[5 - 1.6\right]e^{\frac{-t}{RC}}$$

$$V_c = 5 - 3.4\,e^{\frac{-t}{RC}}$$

At $t = T_{on}$, $V_c = 2.4V$

$$\therefore\ 2.4 = 5 - 3.4\,e^{\frac{-T_{on}}{RC}}$$

T_{on} = 126 × 10^{-6} sec

= 126 μsec

During T_{off}

$$V_c = 0 - [0 - 2.4]e^{\frac{-t}{\tau}}$$

$$V_c = 2.4\,e^{\frac{-t}{RC}}$$

At t = T_{off}, V_c = 1.6 V

$$1.6 = 2.4\,e^{\frac{-T_{off}}{RC}}$$

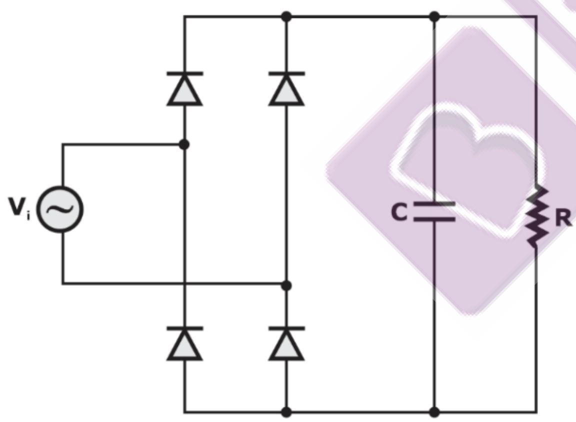
T_{off} = 190.56 μsec

T = T1 + T2 = 126 + 190.56

= 316.56 μsec

$$f = \frac{1}{T} = 3158 \text{ Hz}$$

53. In the circuit shown, the input V_i is a sinusoidal AC voltage having an RMS value of 230 V ± 20% . The worst-case peak-inverse voltage seen across any diode is _________ V. (Round off to 2 decimal places.)



Ans. 390.26

Sol.

$$V_{rms} = 230 \pm 20\%$$

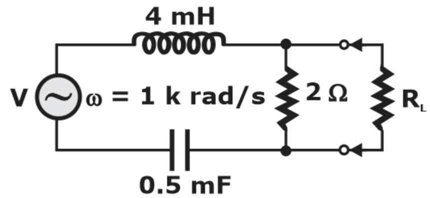
$$V_{rms} = (230 + 20\%)$$

$$V_{rms} = 230 + 46 = 276$$

PIV for the given circuit is to be $V_m = V_{rms} \times \sqrt{2}$

$$= 276 \times \sqrt{2} = 390.26 \text{ V}$$

54. In the given circuit, for maximum power to be delivered to R_L, its value should be ___________ Ω. (Round off to 2 decimal places.)

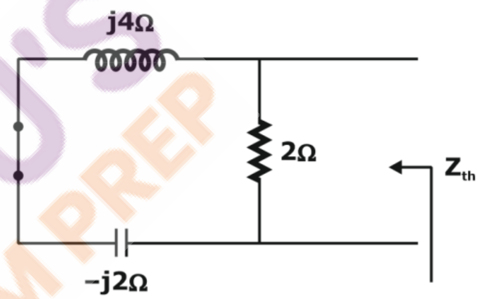


Ans. 1.414

Sol. For purely resistive load,

$R_L = |Z_{Th}|$

For Z_{Th}:



$$Z_{Th} = 2 \,||\, (j4 - j2) = 2 \,||\, (j2) = \frac{2 \times j2}{2 + j2} = \frac{j2}{1 + j1}$$

$$\frac{2\angle 90^\circ}{\sqrt{2}\angle 45^\circ} = \sqrt{2}\;\angle 45^\circ$$

$$|Z_{Th}| = \sqrt{2}\;\Omega$$

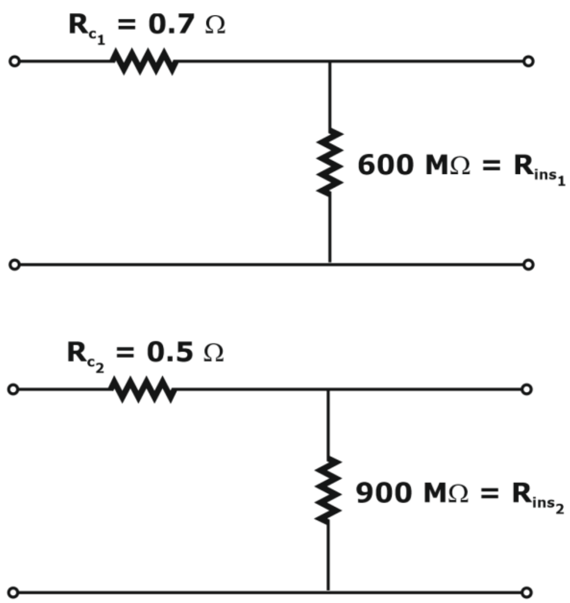
$$R_L = |Z_{Th}| = \sqrt{2}\,\Omega$$

R_L = 1.414 Ω

55. Two single-core power cables have total conductor resistances of 0.7 Ω and 0.5 Ω, respectively. and their insulation resistances (between core and sheath) are 600 MΩ and 900 MΩ, respectively. When the two cables are joined in series, the ratio of insulation resistance to conductor resistance is _________× 10^6.

Ans. 300

Sol.



$R_{C_{Total}} = R_{C_1} + R_{C_2}$ (series)

0.7 + 0.5 = 1.2

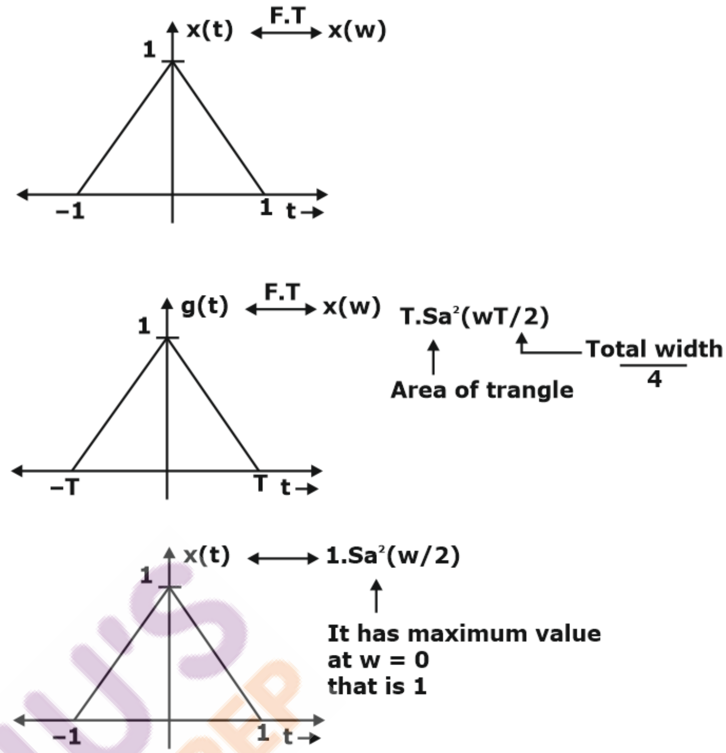
$$R_{ins_{Total}} = \frac{600 \times 900}{1500} = 360 \times 10^6$$

$$\frac{R_{ins_{Total}}}{R_{C_{Total}}} = \frac{360 \times 10^6}{1.2} = 300 \times 10^6$$

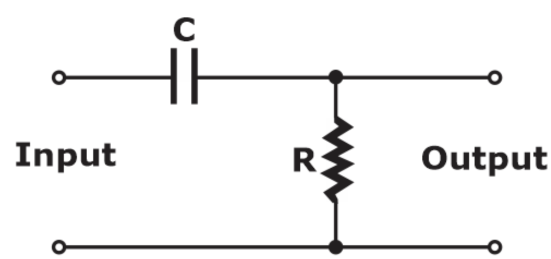
56. Consider a continuous-time signal x(t) defined by x(t) = 0 for |t| > 1, and x(t) = 1 – |t| for |t| ≤ 1. Let the Fourier transform of x(t) be defined as $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$. The maximum magnitude of $X(\omega)$ is ________.

Ans. 1

Sol.

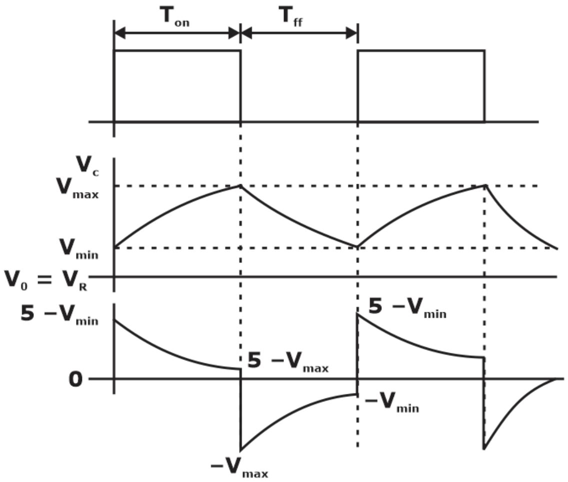


57. A 100 Hz square wave, switching between 0 V and 5 V, is applied to a CR high-pass filter circuit as shown. The output voltage waveform across the resistor is 6.2 V peak-to-peak. If the resistance R is 820 Ω , then the value C is ________μF. (Round off to 2 decimal places.)



Ans. 12.45

Sol.



During T_{on}:

V_i = 5 V 'C' will charge.

$V_o = V_i - V_c = 5 - V_c$

when V_c increases $V_R = V_o$ decreases exponentially.

$V_c = V(\infty) - [V(\infty) - V(0)]e^{-t/\tau}$ ...(1)

$V_c = 5 - (5 - V_{min})\, e^{-t/\tau}$ ...(2)

at $t = T_{on} = T/2$ $V_c = V_{max}$.

$V_{max} = 5 - [5 - V_{min}]e^{-T/2\tau}$ ...(3)

Solving of o/p:

$V_{p-p} = 5 - V_{min} - (-V_{max}) = 6.2$

$(V_{max} - V_{min}) = 1.2$ V ...(4)

During T_{off} 'C' will discharge towards zero from eq(1)

$V_c = 0 - [0 - V_{max}]e^{-t/\tau}$

$V_c = V_{max}\, e^{-t/\tau}$ ...(5)

At $t = T_{off} = T/2$ $V_c = V_{min}$

$V_{min} = V_{max}\, e^{-T/2\tau}$ ...(6)

We have to find $\tau = RC \Rightarrow C = \tau/R$ ...(7)

Putting the value of eq. 5 in eq. 3,

$$V_{max} = 5 - \left[5 - V_{max}e^{-T/2\tau}\right]e^{-T/2\tau}$$

$$V_{max}\left[1 - e^{-T/\tau}\right] = 5\left[1 - e^{-T/2\tau}\right]$$

$$V_{max} = \frac{5\left[1 - e^{-T/2\tau}\right]}{\left[1 - e^{-T/\tau}\right]} = \frac{5\left[1 - e^{-T/2\tau}\right]}{\left[1 - e^{-T/2\tau}\right]\left[1 + e^{-T/2\tau}\right]}$$

$$\left[\because a^2 - b^2 = (a+b)(a-b)\right]$$

$$V_{max} = \frac{5}{\left[1 + e^{-T/2\tau}\right]}$$

Putting the value of eq. 6 in eq. 4,

$$V_{max} - V_{max}e^{-T/2\tau} = 1.2$$

$$V_{max}\left[1 - e^{-T/2\tau}\right] = 1.2$$

Putting eq. 8 in above equation,

$$\left[\frac{5\left[1 - e^{-T/2\tau}\right]}{\left[1 + e^{-T/2\tau}\right]}\right] = 1.2$$

$$5 - 5e^{-T/2\tau} = 1.2 + 1.2e^{-T/2\tau}$$

$$3.8 = 6.2e^{-T/2\tau}$$

$$e^{T/2\tau} = \frac{6.2}{3.8}$$

$$\frac{T}{2\tau} = \ln\left(\frac{6.2}{3.8}\right) = 0.489$$

$$\tau = \frac{T}{2 \times 0.489} = 0.0102 \text{ sec}$$

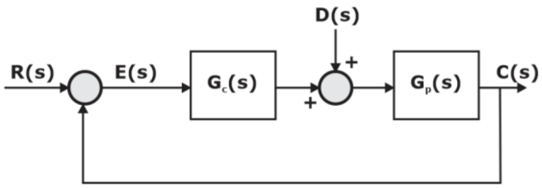
$$\tau = RC$$

$$C = \frac{\tau}{R} = \frac{0.0102}{820}$$

$$C = 1.245 \times 10^{-5} = 12.45\ \mu F$$

58. In the given figure, plant $G_p(s) = \dfrac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$ and compensator $G_c(s) = K\left(\dfrac{1+T_1 s}{1+T_2 s}\right)$. The external disturbance input is D(s). It is desired that when the disturbance is a unit step, the steady-state error should not exceed 0.1 unit.

The minimum value of K is __________. (Round off to 2 decimal places.)



Ans. 9.45

Sol. E(s) = R(s) – C(s)

$E(s) = R(s) - G_p(s)[D(s) + E(s)G_c(s)]$

$E = R - G_p D - G_p G_c E$

$E(1 + G_p G_c) = R - G_p D$

Put R(s) = 0

$$E = \frac{-G_p D}{1 + G_p G_c}$$

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$e_{ss} = \frac{\lim\limits_{s \to 0} s \times \frac{-1}{s} \times \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}}{1 + \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)} \times k\left(\frac{1+T_1 s}{1+T_2 s}\right)}$$

$$= \frac{-2.2}{1 + 2.2k}$$

If, $e_{ss} \le 0.1$

$K \ge -10.45$

$K_{min} = -10.45$

but 'k' for compensator cannot be negative.

so, let us take $|e_{ss}| \le 0.1$.

$$\frac{2.2}{1 + 20.2k} \le 0.1$$

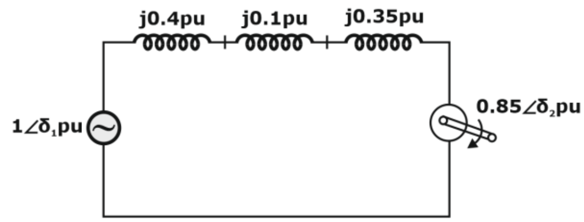
$K \ge 9.45$

$K_{min} = 9.45$

59. An alternator with internal voltage of $1\angle\delta_1$ p.u. and synchronous reactance of 0.4 p.u. is connected by a transmission line of reactance 0.1 p.u. to a synchronous motor having synchronous reactance 0.35 p.u. and internal voltage of $0.85\angle\delta_2$ p.u. If the real power supplied by the alternator is 0.866 p.u., then $(\delta_1 - \delta_2)$ is _________ degrees. (Round off to 2 decimal places.) (Machines are of non-salient type. Neglect resistances.)

Ans. 60

Sol. Total power transferred,



$$P = \frac{E_f V}{X_{eq}} \sin(\delta_1 - \delta_2)$$

$$\Rightarrow 0.866 = \frac{1 \times 0.85}{0.85} \sin(\delta_1 - \delta_2)$$

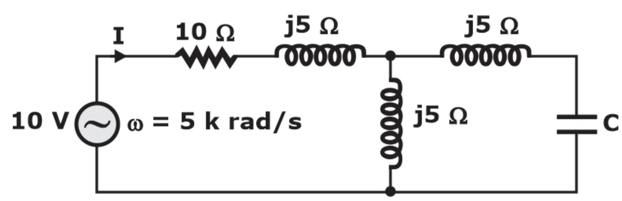
Total reactance of the system,

X_{eq} = j0.4 + j0.1 + j0.35 = j0.85 p.u.

$\sin(\delta_1 - \delta_2) = 0.866$

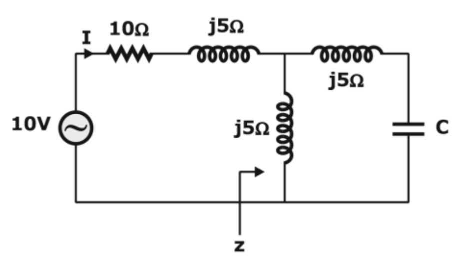
$(\delta_1 - \delta_2) = 60°$

60. In the given circuit, the value of capacitor C that makes current I = 0 is ________ μF.



Ans. 20

Sol.



To make I = 0,

Z = ∞

$[j5 \,||\, (j5 - jX_C)] = \infty$

So, $j10 - jX_C = 0$

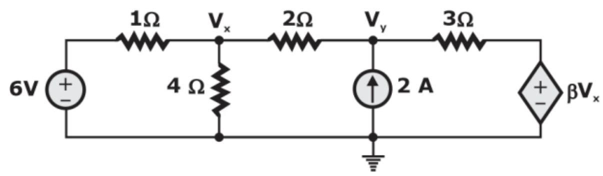
$X_C = 10$

$$\frac{1}{\omega C} = 10$$

$$C = \frac{1}{10 \times 5 \times 1000}$$

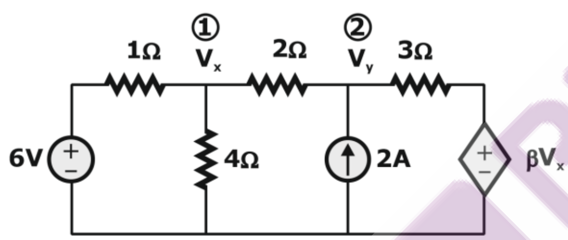
$C = 20\mu F$

61. In the given circuit, for voltage V_y to be zero, the value of β should be ________. (Round off to 2 decimal places).



Ans. –3.25

Sol.



Given, $V_y = 0$

Apply, KCL at node (1),

$$\frac{V_x - 6}{1} + \frac{V_x}{4} + \frac{V_x - 0}{2} = 0$$

$4V_x - 24 + V_x + 2V_x = 0$

$$V_x = \frac{24}{7}\ V$$

Apply, KCL at node (2),

$$\frac{V_x}{2} + 2 = \frac{0 - \beta V_x}{3}$$

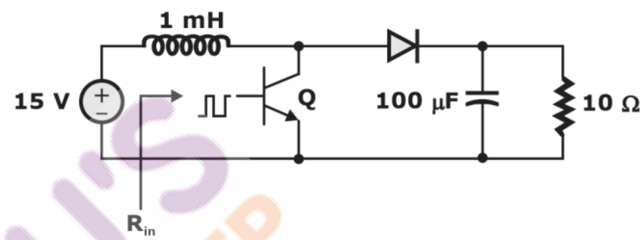
$$\frac{3V_x}{2} + 6 = -\beta V_x$$

$$\frac{36}{7} + 6 = -\frac{24}{7}\beta$$

$$\beta = -\frac{78}{24}$$

$\beta = -3.25$

62. Consider the boost converter shown. Switch Q is operating at 25 kHz with a duty cycle of 0.6. Assume the diode and switch to be ideal. Under steady-state condition, the average resistance R_{in} as seen by the source is ______ Ω. (Round off to 2 decimal places.)



Ans. 1.6

Sol.

$$V_o = \frac{V_s}{1 - \alpha}$$

$$V_o = (1 - \alpha) I_s$$

$$R_o = \frac{R_{in}}{(1 - \alpha)^2}$$

$$R_{in} = (1 - \alpha)^2 R_o$$

$$R_{in} = (1 - 0.6)^2 \times 10$$

$$= (1 - 0.6)^2 \times 10$$

$$= 1.6\,\Omega$$

63. The power input to a 500 V, 50 Hz, 6-pole, 3-phase induction motor running at 975 RPM is 40 kW. The total stator losses are 1 kW. If the total friction and windage losses are 2.025 kW, then the efficiency is ________%.

Ans. 90

Sol.

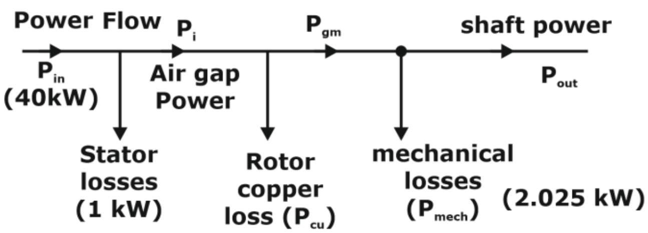
$$s = \frac{N_s - N_r}{N_s} = \frac{1000 - 975}{1000}$$

s = 0.025

$P_{gm} = (1 - s)\, p_i = (1 - 0.025)\,(P_{in} - \text{stator loss})$

$= (0.975)\,(40 - 1)$

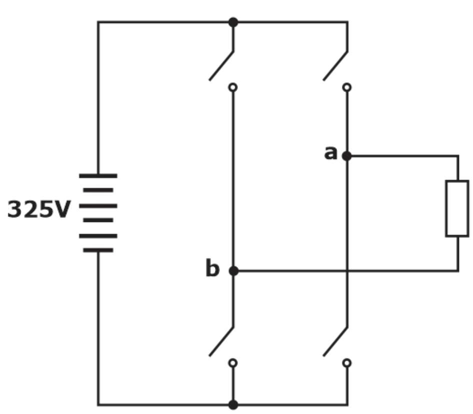
$P_{gm} = 38.025$ kW

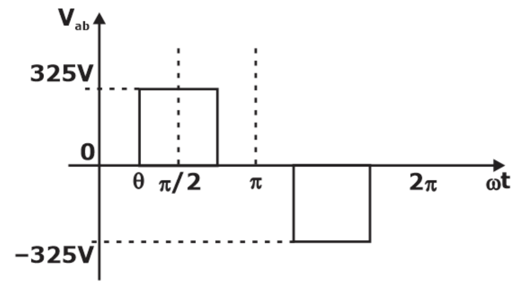


$$\eta_m = \frac{P_{out}}{P_{in}} = \frac{P_{gm} - P_{mech}}{40\text{ kW}} = \frac{38.025 - 2.025}{40}$$

$\eta_m = 0.9$ i.e. 90%

64. A single-phase full-bridge inverter fed by a 325 V DC produces a symmetric quasi-square waveform across 'ab' as shown. To achieve a modulation index of 0.8, the angle θ expressed in degrees should be ________ (Round off to 2 decimal places.) (Modulation index is defined as the ratio of the peak of the fundamental component of V_{ab} to the applied DC value.)





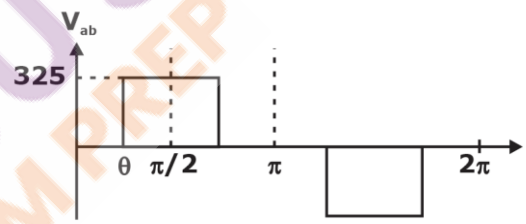
Ans. 51.06

Sol. Supply voltage = 325 V

Amplitude of fundamental component of output voltage can be expressed as:

$V_{01} = m_A \times V_s = 0.8 \times 325 = 260$ V ...(1)

From the given waveform,



Pulse width of output voltage,

$$2d = \pi - \phi \Rightarrow d = \frac{\pi}{2} - \phi$$

Fourier series expansion of square wave will be:

$$V_0 = \sum_{n=1,\,3,\,5}^{\infty} \frac{4V_s}{n\pi} \sin nd \sin n\omega t$$

Fundamental voltage, $V_{01} = \frac{4V_s}{\pi} \sin d$

$$V_{01} = \frac{4V_s}{n\pi} \sin\left(\frac{\pi}{2} - \theta\right) = \frac{4V_s}{\pi} \cos\theta \quad ...(2)$$

Equating equation (1) and (2),

$$\frac{4 \times 325}{\pi} \cos\theta = 260 \Rightarrow \cos\theta = 0.628$$

$\theta = 51.073°$

65. The state space representation of a first-order system is given as:

$$\dot{x} = -x + u$$

$$y = x$$

where, x is the state variable, u is the control input and y is the controlled output. Let u = –Kx be the control law, where K is the controller gain. To place a closed-loop pole at –2, the value of K is ________.

Ans. 1

Sol. $\dot{x} = -x + u = -x - Kx$

$\dot{x} = (-1 - K)x$

Hence, A = –1 – K

Characteristic of CLTF = |sI – A| = s – (–1 – K)

= s + 1 + K …eq.(1)

It is given CLTF at s = –2, Hence s + 2 =0 …eq.(2)

Comparing (1) & (2), we get,

s + 1 + K = s + 2

K = 1

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