GATE 2021

Set-1

Mechanical Engineering

Questions & Solutions

SECTION: GENERAL APTITUDE

1. Consider the following sentences:

(i) After his surgery. Raja hardly could walk.

(ii) After his surgery. Raja could barely walk.

(iii) After his surgery. Raja barely could walk.

(iv) After his surgery. Raja could hardly walk.

Which of the above sentences are grammatically CORRECT?

A. (iii) and (iv) B. (i) and (ii)

C. (i) and (iii) D. (ii) and (iv)

Ans. D

Sol.

2. Ms. X came out of building through its front dor to find her shadow due to the morning sun falling to her right side with the building to her back. From this, it can be inferred that building is facing____

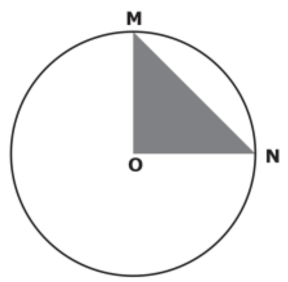
A. East B. West

C. South D. North

Ans. C

Sol. Since its shadow is falling to her right in the morning means the sun will be in his left i.e. east will be in his left hand thus building must be south facing.

3.



In the above figure. O is the center of the circle and, M and N lie on the circle.

The area of the right triangle MON is 50 cm^2.

What is the area of the circle in cm^2?

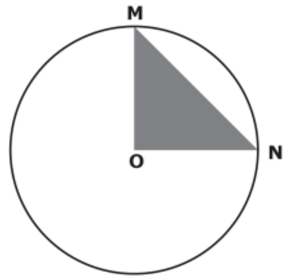
A. 100π B. 75π

C. 50π D. 2π

Ans. A

Sol. Given,

Area of the right triangle MON = 50 cm^2,



Area of ΔMON = 50 cm^2

$\frac{1}{2}(r)\times(r) = 50$

$r^2 = 100 \Rightarrow r = 10$ cm

So, Area of circle = πr^2 = 100 π cm^2

4. If $\begin{cases} "\oplus" \text{ means } "-", \\ "\otimes" \text{ means } "\div", \\ "\Delta" \text{ means } "+", \\ "\nabla" \text{ means } "\times", \end{cases}$

then, the value of the expression $\Delta\, 2 \oplus 3\, \Delta\, ((4 \otimes 2)\ \nabla 4) =$

A. 7

B. 6

C. −0.5

D. −1

Ans. A

Sol. $\Delta 2 \oplus 3 \Delta ((4 \otimes 2) \nabla 4)$

$+2 - 3 + ((4 \div 2) \times 4)$

$+2 - 3 + (2 \times 4) = 7$

5. The increased consumption of leafy vegetables in the recent months is a clear indication that the people in the state have begun to lead a healthy lifestyle"

Which of the following can be logically inferred from the information presented in the above statement?

A. Consumption of leafy vegetables may not be the only indicator of healthy lifestyle.

B. Leading a healthy lifestyle is related to a diet with leafy vegetables.

C. The people in the state did not consume leafy vegetables earlier.

D. The people in the state have increased awareness of healthy hazards causing by consumption of junk foods.

Ans. B

Sol.

6. Oxpeckers and rhinos manifest a symbiotic relationship in the wild. The oxpeckers warn the rhinos about approaching poachers, thus possibly saving the lives of the rhinos. Oxpeckers also feed on the parasitic ticks found on rhinos.

In the symbiotic relationship described above, the primary benefits for oxpeckers and rhinos respectively are.

A. Oxpeckers get a food source, rhinos have no benefit.

B. Oxpeckers get a food source, rhinos may be saved from the poachers.

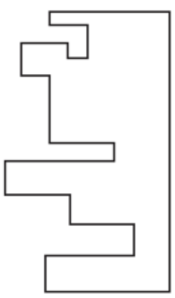
C. oxpeckers save the lives of poachers, rhinos save their own lives.

D. Oxpeckers save their habitat from poachers while the rhinos have no benefit.

Ans. B

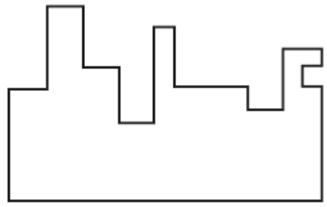
Sol.

7.

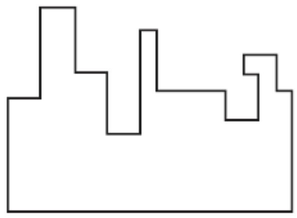


A jigsaw puzzle has 2 pieces. One of the pieces is shown above. Which one of the given options of for the missing piece when assembled will form a rectangle? The piece can be moved, rotated or flipped to assemble with the above piece.

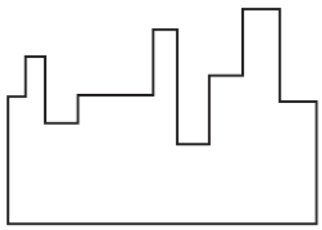
A.



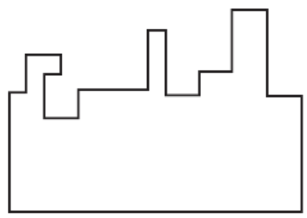
B.



C.



D.



Ans. B

Sol.

8. The number of hens, ducks and goats in farm P are 65, 91 and 169, respectively. The total number of hens, ducks and goats in a nearby farm Q is 416. The ratio of hens: ducks: goats in farm Q is 5:14:13. All ducks and goats are sent from farm Q to farm P.

The new ratio of hens: ducks: goats in farm P is ______

A. 21:10:26 B. 5:7:13

C. 10:21:26 D. 5:14:13

Ans. C

Sol. Given,

Farm P:

Hens = 65

Duck = 91

Goat = 169

Farm Q:

Total number of hens, ducks, and goats = 416,

Hens: Duck: Goat = 5 : 14 : 13,

So, 5K + 14K + 13K = 416,

32K = 416

K = 13,

So, Hens = 5K = 65; Duck = 14K = 182; Goats = 13K = 169

After Merging, Total numbers are

Hens = 65 + 65 = 130

Duck = 91 + 182 = 273

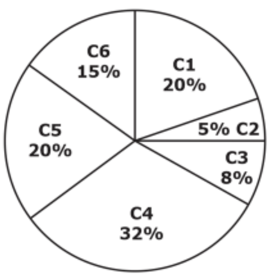
Goat = 169 + 169 = 338

Ratio: Hens : Duck : Goats = 130 : 273 : 338

= 13 × 10 : 13 × 21 : 13 × 26

Ratio: Hens : Duck : Goats = 10 : 21 : 26

9.



Company	Ratio
C1	3:2
C2	1:4
C3	5:3
C4	2:3
C5	9:1
C6	3:4

The distribution of employees at the rank of executives. Across different companies C1, C2, …, C6 is presented in the chart given above. The ration of executives with a management degree to those without a management degree in each of these companies is provided in the table above. The total number of executives across all companies is 10.000.

The total number of management degree holders among the executives in companies C2 and C5 together is______.

A. 1900 B. 2500

C. 225 D. 600

Ans. A

Sol. Given,

Total number of executives

= 10,000

Companies	Percentage distribution	Executives	Ratio with management to without management degree	With management	Without management
C_1	20	2000	3:2	1200	800
C_2	5	500	1:4	100	400
C_3	8	800	5:3	500	300
C_4	32	3200	2:3	1280	1920
C_5	20	2000	9:1	1800	200
C_6	15	1500	3:4	642.85	857.12

Total executives in C_2 & C_5 with management degree = 100 + 1800 = 1900

10. Five persons P, Q, R, S and T are sitting in a row not necessarily in the same order. Q and R are separated by one person, and S should be seated adjacent to Q.

The number of distinct seating arrangements possible is:

A. 8 B. 4

C. 10 D. 16

Ans. D

Sol. Possible way of sitting arrangement starting from left end

Q P R S T
Q T R S P
Q T R P S
Q P R T S
P Q T R S
T Q P R S
S P Q T R
S T Q P R

Similar way can be done from right side(Mirror image)

MECHANICAL ENGINEERING

1. If y(x) satisfies the differential equal $(\sin x)\frac{dy}{dx} + y\cos x = 1$, subject to the condition $y(\pi/2) = \pi/2$, then $y(\pi/6)$ is

A. $\frac{\pi}{2}$ B. 0

C. $\frac{\pi}{3}$ D. $\frac{\pi}{6}$

Ans. C

Sol. Given,

$$\sin x\frac{dy}{dx} + (\cos x)y = 1$$

Divide by sin x

$$\frac{dy}{dx} + (\cot x)y = \operatorname{cosec} x$$

Form: $\frac{dy}{dx} + P_y = Q$

⇒ Linear DE of 1st order

$$IF = e^{\int P dx} = e^{\int \cot x\, dx} = e^{\ln(\sin x)} = \sin x$$

$$y(IF) = \int Q(IF)dx + C$$

$$y(\sin x) = \int(\operatorname{cosec} x)(\sin)\,dx + C$$

$$y \sin x = \int dx + C$$

$$y \sin x = x + c$$

Given

$$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + C$$

⇒ C =0

$$y\left(\frac{\pi}{6}\right) = ?$$

$$y \sin\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + C$$

$$y \times \frac{1}{2} = \frac{\pi}{6} \Rightarrow y = \frac{\pi}{3} \approx 1.05$$

2. The value of $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ is

A. $\frac{1}{2}$ B. $\frac{1}{4}$

C. 1 D. $\frac{1}{3}$

Ans. A

Sol. Given,

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{0}{0} \text{ form}$$

Apply Hospital Rule,

$$= \lim_{x\to 0}\frac{\sin x}{2x} = \frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x} = \frac{1}{2}$$

3. The Dirac-delta function $\delta(t - t_0)$ for t, $t_0 \in R$, has the following property

$$\int_a^b \phi(t)\delta(t-t_0)dt = \begin{cases}\phi(t_0) & a < t_0 < b \\ 0 & \text{otherwise}\end{cases}$$

The Laplace transform of the Dirac-delta function δ(t − a) for a > 0; L(δ(t − a)) = F is

A. 0

B. ∞

C. e^{-sa}

D. e^{sa}

Ans. C

Sol. Given,

$$\int_a^b \phi(t)\,\delta(t-t_0)\,dt = \begin{cases}\phi(t_o) & a < t_o < b \\ 0 & \text{otherwise}\end{cases}$$

We know Laplace Transform of Dirac Delta function δ(t) is 1

L{δ(t)} = 1

Apply time shift property.

L{δ(t-a)} = 1 × e^{-as} = e^{-as}

4. The ordinary differential equation $\frac{dy}{dt} = -\pi y$ subject to an initial condition y(0) = 1 is solved numerically using the following scheme :

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

Where h is the time step, t_n = nh, and n = 0, 1, 2, … This numerical scheme is stable for all values of h in the interval _______ .

A. $0 < h < \frac{\pi}{2}$ B. for all h > 0

C. $0 < h < \frac{2}{\pi}$ D. 0 < h < 1

Ans. C

Sol. $$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

$$y_{n+1} = -\pi h y_n + y_n = (-\pi h + 1) y_n$$

it is recursion solution between y_{n+1} & y_n.

thus, Solution will be stable if

$$|-\pi h + 1| < 1$$

$$-1 < -\pi h + 1 < 1$$

$$-2 < -\pi h < 0$$

$$0 < \pi h < 2$$

$$0 < h < \frac{2}{\pi}$$

5. Consider a binomial random variable X. If X_1, X_2,…., X_n are independent and identically distributed samples from the distribution of X with sum $Y = \Sigma_{i=1}^{n} X_i$, then the distribution of Y as $n \to \infty$ can be approximated as

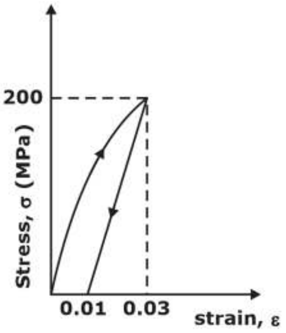
A. Binomial B. Normal

C. Exponential D. Bernoulli

Ans. B

Sol. When sample size of random variable tends to ∞ then the distribution will become Normal distribution.

6. The loading and unloading response of a metal is shown in the figure. The elastic and plastic strains corresponding to 200 MPa stress, respectively, are



A. 0.01 and 0.02 B. 0.02 and 0.02

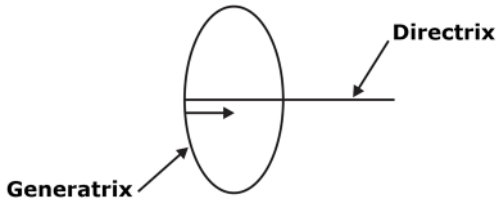
C. 0.02 and 0.01 D. 0.01 and 0.01

Ans. C

Sol. Elastic strains = 0.03 -0.01 = 0.02

Plastic strains = 0.01

7. In a machining operation, if a cutting tool traces the workpiece such that the directrix is perpendicular to the plane of the generatrix as shown in figure, the surface generated is



A. cylindrical

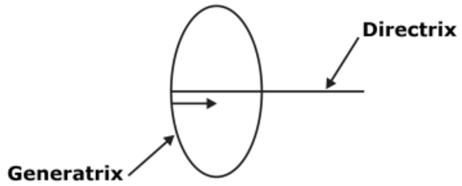
B. spherical

C. a surface of revolution

D. plane

Ans. A

Sol. When the directrix is perpendicular to the plane of the generatrix as shown in figure, the surface generated is Cylindrical.



8. The correct sequence of machining operations to be performed to finish a large diameter through hole is

A. boring, reaming, drilling

B. drilling, boring, reaming

C. boring, drilling, reaming

D. drilling, reaming, boring

Ans. B

Sol. The correct sequence of machining operations to be performed to finish a large diameter through hole is drilling, boring, reaming.

9. In modern CNC machine tools, the backlash has been eliminated by

A. rack and pinion

B. rather and pinion

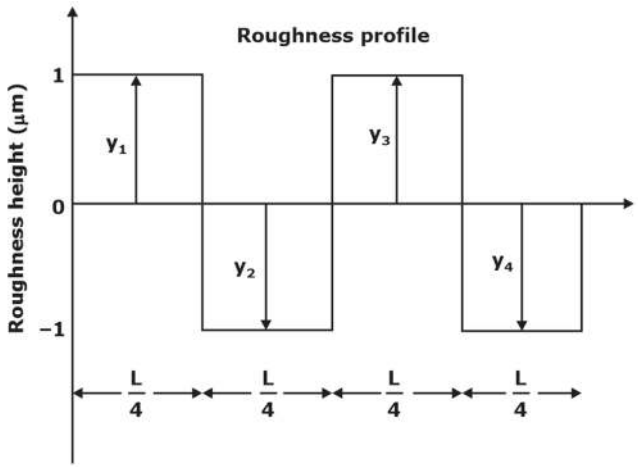
C. slider crank mechanism

D. preloaded ballscrews

Ans. D

Sol. In modern CNC machine tools, the backlash has been eliminated by preloaded ballscrews.

10. Consider the surface roughness profile as shown in the figure.



The center line average roughness (R_a, in μm) of the measured length (L) is

A. 1 B. 0

C. 4 D. 2

Ans. A

Sol. Given,

Surface roughness profile;

Centre line average $= R_a = \frac{\Sigma y}{n}$

$$R_a = \frac{|y_1| + |y_2| + |y_3| + |y_4|}{4} = \frac{4}{4} = 1\text{Micron}$$

11. In which of the following pairs of cycles, both cycles have at least one isothermal process?

A. Bell-Coleman cycle and Vapour compression refrigeration cycle

B. Diesel cycle and Otto cycle

C. Carnot cycle and Stirling cycle

D. Brayton cycle and Rankine cycle

Ans. C

Sol. Carnot cycle consist of two isothermal and two isentropic process and Stirling cycle contains two isothermal and two constant volume process.

12. Superheated steam at 1500 kPa, has a specific volume of 2.75 m3/kmol and compressibility factor (Z) of 0.95. The temperature of steam is _______ °C (round off to the nearest integer).

A. 198 B. 249

C. 471 D. 522

Ans. B

Sol. Given,

Superheated steam,

Pressure (P) = 1500 kPa

Specific volume ($\bar{v}$)

= 2.75 m^3/kmole

$P\bar{v} = ZRT$

$$T = \frac{1500 \times 2.75}{8.314 \times 0.95} = 522K$$

T= 522 – 273 = 249°C

13. A hot steel spherical ball is suddenly dipped into a low temperature oil bath. Which of the following dimensionless parameters are

required to determine instantaneous center temperature of the ball using a Heisler chart?

A. Reynolds number and Prandtl number

B. Biot number and Fourier number

C. Biot number and Froude number

D. Nusselt number and Grashoff number

Ans. B

Sol. Biot number and Fourier number are required to determine instantaneous center temperature of the ball using a Heisler chart.

14. An infinitely long pin fin, attached to an isothermal hot surface, transfer hear at steady rate of Q_1 to the ambient air. If the thermal conductivity of the fin metrical is doubled, while keeping everything else constant, the rate of steady-state heat transfer from the fin becomes Q_2. The ration Q_2/Q_1 is

A. $\frac{1}{\sqrt{2}}$ B. $\sqrt{2}$

C. $\frac{1}{2}$ D. 2

Ans. B

Sol. Given,

An infinite long pin fin,

$$\dot{Q}_1 = \sqrt{hpKA_{cs}}(T_0 - T_\infty)$$

So, heat transfer $\dot{Q} \propto \sqrt{K}$

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \sqrt{\frac{K_2}{K_1}} = \sqrt{2}$$

15. The relative humidity of ambient air at 300 K is 50% with a partial pressure of water vapour equal to p_v. The saturation pressure of water at 300 K is p_{sat}. The correct relation for the air-water mixture is

A. $p_v = p_{sat}$

B. $p_v = 0.5\ p_{sat}$

C. $p_v = 0.622\ p_{sat}$

D. $p_v = 2\ p_{sat}$

Ans. B

Sol. Relative humidity, $\phi = \frac{p_v}{p_{sat}}$

$$0.5 = \frac{p_v}{p_{sat}}$$

$p_v = 0.5\ p_{sat}$

16. Consider a reciprocating engine with radius R and connecting rod of length L. The secondary unbalance force for this case is equivalent to primary unbalance force due to a virtual crank of ___________

A. radius $\frac{R}{4}$ rotating al half the engine speed

B. radius $\frac{L}{2}$ rotating at twice the engine speed

C. radius $\frac{L^2}{4R}$ rotating at half the engine speed

D. radius $\frac{R^2}{4L}$ rotating at twice the engine speed

Ans. D

Sol. Given,

Crank radius = R,

Connecting rod of length = L,

Secondary unbalance Force;

$$F_{sun} = m\left(\frac{r}{4m}\right)(2\omega)^2 \cos 2\theta$$

As we know, $\left[n = \frac{l}{r}\right]$

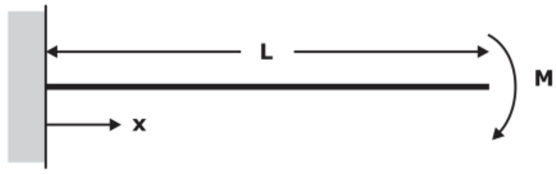
$$\text{Radius} = \frac{r}{4m}$$

$$n = \frac{L}{m}$$

$$\Rightarrow \text{radius} = \frac{R^2}{4L}$$

Angular velocity = twice speed

17. A cantilever beam of length. L, and flexural rigidity. EI, is subjected to an end moment. M, as shown in the figure. The deflection of the beam at x= $\frac{L}{Z}$ is

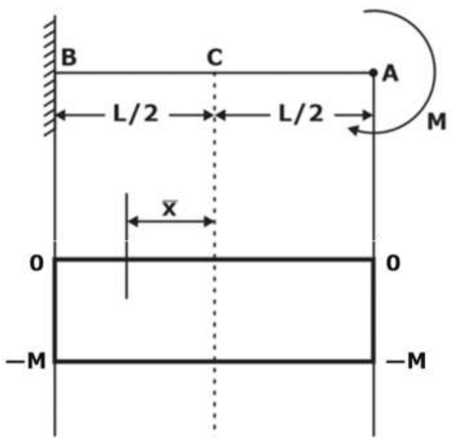


A. $\frac{ML^2}{4EI}$ B. $\frac{ML^2}{8EI}$

C. $\frac{ML^2}{2EI}$ D. $\frac{ML^2}{16EI}$

Ans. B

Sol.



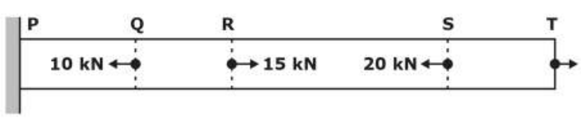
Simplified theorem (Moment Area Method)

$$y_c - y_b = \frac{[A\bar{x}]}{EI}$$

$$y_c - 0 = \frac{1}{EI}\left[M \times \frac{L}{2} \times \frac{L}{4}\right]$$

$$y_c = \frac{ML^2}{8EI}$$

18. A prismatic bar PQRST is subjected to axial loads as shown in the figure. The segments having maximum and minimum axial stresses, respectively, are



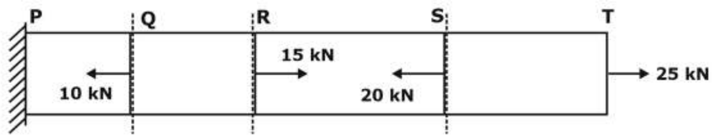
A. ST and PQ B. QR and PQ

C. QR and RS D. ST and RS

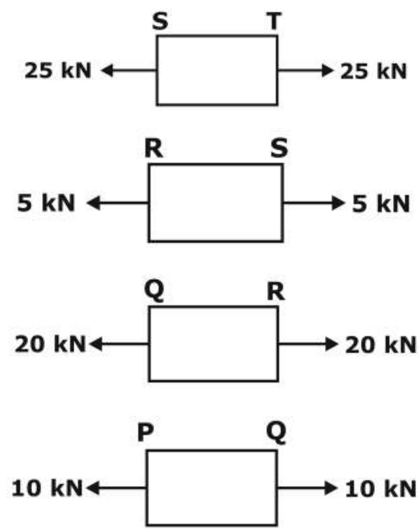
Ans. D

Sol. Given,

Prismatic bar PQRST,



By taking sections at points 'S', 'R', 'Q'



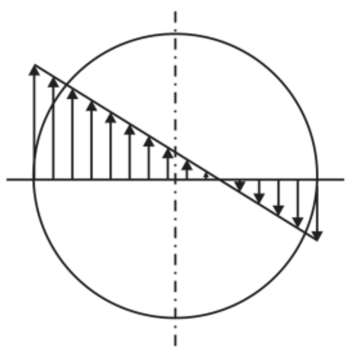
The max load of 25 kN is on 'ST' Portion, Hence

σ_{ST} = Max stress

The min load of 5 kN is on 'RS' portion, Hence

σ_{RS} = Min stress

19. Shear stress distribution one the cross-section of the coil wire in a helical compression spring is shown in the figure. This shear stress distribution represents.



A. torsional shear stress in the coil wire cross-section

B. combined direct shear and

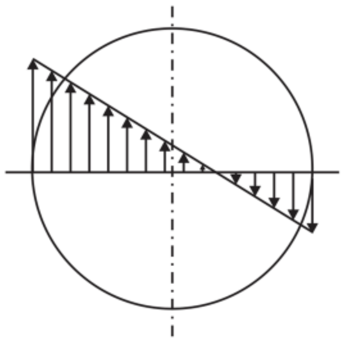
torsional shear stress in the coil wire cross-section

C. direct shear stress in the coil wire cross-section

D. combined direct shear and torsional shear stress along with the effect of stress concentration at inside edge of the coil wire cross-section.

Ans. B

Sol.



The shear stress distribution given represents Combined direct shear stress in the coil wire Cross-section.

20. Robot Ltd. Wishes to maintain enough safety stock during the lead time period between starting a new production run and its completion such that the probability of satisfying the customer demand during the lead time period is 95%. The lead time period is 5 days and daily customer demand can be assumed to follow the Gaussian (normal) distribution with mean 50 units and a standard deviation of 10 units. Using $\phi^{-1}(0.95) = 1.64$, where ϕ represents the cumulative distribution function of the standard normal random variable, the amount of safety stock that must be maintained by Robot Ltd, to achieve this demand fulfillment probability for the lead time period is _____ units (round off to two decimal places).

Ans. 332

Sol. Lead Time (LT) = 5 days

Mean of the demand: μ_d = 50

Variance of demand: σ_d = 10

Normal derivative for 95%

confidence level $Z_{0.95} = \varphi^{-1}(0.95)$

= 1.64

safety stock = Lead time × daily demand = (LT) x d

Daily demand for 95% confidence level

$$Z_{0.95} = \frac{x_d - \mu_d}{\sigma_d}$$

$$1.64 = \frac{x_d - 50}{10}$$

∴ x_d = 66.4

∴ Safety stock = (L.T.) x d = 5 ×66.4 = 332 units

21. A pressure measurement device fitted on the surface of a submarine, located at a depth H below the surface of an ocean, reads an absolute pressure of 4.2 MPa. The density of sea water is 1050 kg/m^3, the atmospheric pressure is 101 kPa, and the acceleration due to the gravity is 9.8 m/s^2. The depth H is _________ m (round off to the nearest integer).

Ans. 398.34

Sol. Given,

Absolute pressure at submarine

= 4.2 MPa

Atmospheric pressure: 101 kPa

Density of sea water = 1050 kg/m^3

$P_{atm} + \rho g H = P_{abs}$

$$H = \frac{P_{abs} - P_{atm}}{\rho g} = \frac{\left(4.2 \times 10^6\right) - \left(101 \times 10^3\right)}{1050 \times 9.8}$$

H = 398.34 m

22. Consider fully developed, steady state incompressible laminar flow of viscous fluid between two large parallel horizontal plates. The bottom plate is fixed and the top plate moves with a constant velocity of U = 4 m/s. Separation between the plates is 5 mm. There is no pressure gradient in the direction of flow. The density of fluid is 800 kg/m^3, and the kinematic viscosity is 1.25 × 10^{-4} m^2/s. The average shear stress in the fluid is _______ (round off to the nearest integer).

Ans. 80

Sol. Given,

Top plate moves with velocity(U)

= 4 m/s,

Separation between the plate = 5 mm,

Given Zero pressure gradient in the direction of flow implies linear velocity profile

$$\tau = \frac{\mu V}{h} = \frac{\rho \nu V}{h} = \frac{800 \times 1.25 \times 10^{-4} \times 4}{5 \times 10^{-3}} = 80\text{Pa}$$

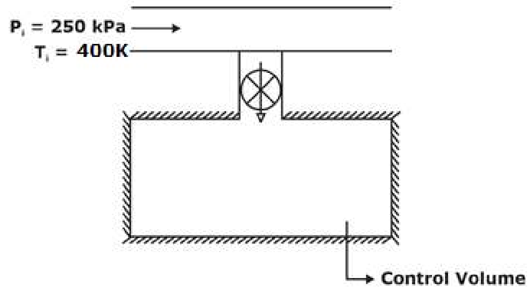
23. A rigid insulated tank is initially evacuated. It is connected through a valve to a supply line that carries air at a constant pressure and temperature of 250 kPa and 400 K respectively. Now the valve is opened and air is allowed to flow into the tank until the pressure inside the tank reaches to 250 kPa at which point the valve is closed. Assume that the air behaves as a perfect gas with constant properties (c_p = 1.005 kJ/kg.K, c_v = 0.718 kJ/kg.K, R = 0.287 kJ/kg.K). Final temperature of the air inside the tank is _______ K (round off to one decimal place).

Ans. 559.88

Sol. Given,

Rigid insulated tank,

Supply line condition 250kPa and 400K



Conservation of mass,

$$\frac{(dm)_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

As no mass is leaving $\dot{m}_e = 0$

$$\frac{(dm)_{cv}}{dt} = \dot{m}_i$$

Also, $\frac{(du)}{dt}_{cv} = \dot{m}_i h_i + \dot{Q}_i - \dot{m}_e h_e - \dot{W}_{cv}$

Since, the tank insulated $\dot{Q}_i = 0$

Also, $W_{cv} = 0$

$$\Rightarrow \frac{(du)_{cv}}{dt} = h_i \times \frac{(dm)}{dt}$$

$\Rightarrow U_2 - U_1 = (m_2 - m_1)\, h_i$

$\Rightarrow m_2 u_2 - m_1 u_1 = m_2 h_i - m_1 h_i$ [Since, m_1 = 0]

$\Rightarrow m_2 u_2 = m_2 h_i$

$\Rightarrow u_2 = h_i$

<u>For ideal gas</u>

$U = mC_vT \Rightarrow u = C_vT$ and $h = C_pT$

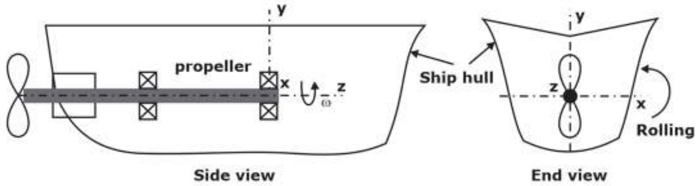
$C_vT_2 = C_pT_i$

$$\Rightarrow u_2\ T_2 = \frac{C_p}{C_v} T_i$$

$\Rightarrow T_2 = \gamma\, T_i$

$$T_2 = \frac{1.005}{0.718} \times 400 = 559.88\text{K}$$

24. The figure shows an arrangement of a heavy propeller shaft in a ship. The combined polar mass moment of inertia of the propeller and the shaft is 100 kg.m^2. The propeller rotates at ω =12 rad/s. The waves acting on the ship hull induces a rolling motion as shown in the figure with an angular velocity of 5 rad/s. The gyroscopic moment generated on the shaft due to motion described is ___________ N.m (round off to the nearest integer).

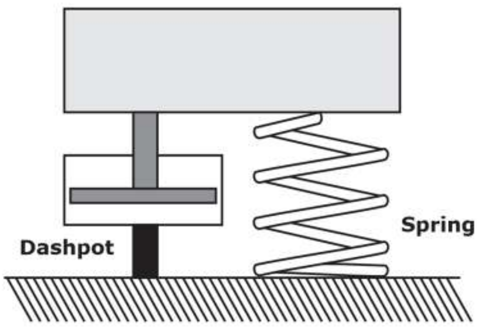


Ans. 0

Sol. In case of rolling of a ship the axis of precession is always parallel to the axis of spin for all positions. Hence there is no effect of the gyroscopic couple acting on the body of the ship during rolling.

Rolling, w and w_p have same axis $C = I\omega\omega_p = 0$

25. Consider a single degree of freedom system comprising a mass M. Supported on a spring and a dashpot as shown in the figure.



If the amplitude of the free vibration response reduces from 8 mm to 1.5 mm in 3 cycles, the damping ratio of the system is _______ (round off to three decimal places).

Ans. 0.088

Sol. Given,

free vibration response reduces from 8 mm to 1.5 mm in 3 cycles,

Decrement Ratio;

$$\frac{x_o}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} = e^{\delta}$$

$$\frac{x_o}{x_1} \times \frac{x_1}{x_2} \times \frac{x_2}{x_3} = e^{3\delta}$$

$$\frac{8}{1.5} = e^{3\delta}$$

$$\delta = 0.558$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$0.558 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

So damping ratio = 0.088

26. Consider a vector p in 2-dimensional space. Let its direction (counter-clockwise angle with the positive x-axis) be θ. Let p be an eigenvector of a 2 × 2 matrix A with corresponding eigenvalue λ, $\lambda > 0$. If we denote the magnitude of a vector v by ||v||, identify the **VALID** statement regarding p′, where p′ = Ap.

A. Direction of $p' = \lambda\theta$, $\| p' \| = \| p \|$

B. Direction of $p' = \lambda\theta$, $\| p' \| = \lambda \| p \|$

C. Direction of p′ = θ, $\| p' \| = \| p \| / \lambda$

D. Direction of p′ = θ, $\| p' \| = \lambda \| p \|$

Ans. D

Sol. A is 2×2 matrix and P is the eigen vector of matrix A with corresponding eigen value of λ.

Given, P′ = P & AP = λP

Hence, P′ = λP

$\| p' \| = \| \lambda P \| = \lambda \| p \|$

And direction of vector P will be same as that of Vector p′.

27. Let C represent the unit circle centered at origin in the complex plane, and complex variable, z = x + iy. The value of the contour integral $\oint_C \frac{\cosh 3z}{2z} dz$ (where integration is taken counter clockwise) is

A. $2\pi i$ B. 2

C. 0 D. πi

Ans. D

Sol. Given,

Complex variable, Z = x +i y,

Contour integral $=\oint_c \frac{\text{Cosh}3z}{2z}dz$

By Cauchy's Integral formula,

$\oint_c F(z)dz = 2\pi i \times$

(Sum of Residue of all poles inside C)

Residue at Z=0,

$\text{Cosh}z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + ...$

So $= \lim z \to 0 \frac{1 + \frac{z^2}{2!} + \frac{z^4}{4!} + ...}{2z}$,

So, Residue at z=0 is equal to 0.5,

$\oint_c F(z)dz = 2\pi i \times (0.5) = \pi i$

28. A set of jobs A, B, C, D, E, F, G, H arrive at time t = 0 for processing on turning and grinding machines. Each job needs to be processed in sequence – first on the turning machine and second on the grinding machine, and the grinding must occur immediately after turning. The processing times of the jobs are given below.

Job	A	B	C	D	E	F	G	H
Turning (minutes)	2	4	8	9	7	6	5	10
Grinding (minutes)	6	1	3	7	9	5	2	4

If the make span is to be minimized, then optimal sequence in which these jobs must be processed on the turning and grinding machine is

A. G-E-D-F-H-C-A-B

B. A-E-D-F-H-C-G-B

C. A-D-E-F-H-C-G-B

D. B-G-C-H-F-D-E-A

Ans. B

Sol. For minimum make spam Johnson's rule can be applied.

	2	1	4	8	7	6	3	5
Job	A	B	C	D	E	F	G	H
Tunning (min)	2	4	8	9	7	6	5	10
Grainding (min)	6	1	3	7	9	5	2	4
Preference	H	L	L	L	H	L	L	L

According to the Johnson's Rule the sequencing is,

A	E	D	F	H	C	G	B

29. The fundamental thermodynamic relation for a rubber band is given by dU = TdS + τdL, where T is the absolute temperature, S is the entropy, τ is the tension in the rubber band, and L is the length of the rubber band.

Which one of the following relations is CORRECT.

A. $\left(\frac{\partial T}{\partial S}\right)_L = \left(\frac{\partial \tau}{\partial L}\right)_S$

B. $\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial \tau}{\partial S}\right)_L$

C. $T = \left(\frac{\partial U}{\partial S}\right)_\tau$

D. $\tau = \left(\frac{\partial U}{\partial S}\right)_L$

Ans. B

Sol. By exact differential equation theorem,

If dz = M dx + N dy

So, if dz is to be exact differential then,

$\left(\frac{\partial M}{dy}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$

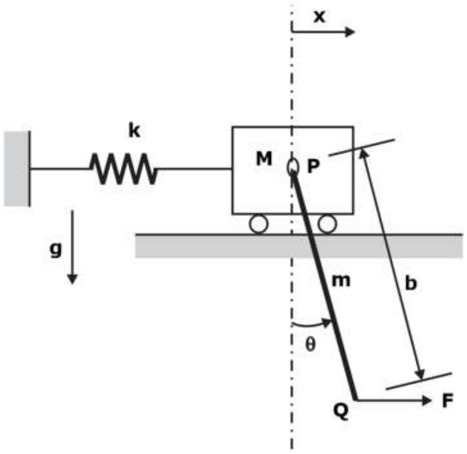
So, by using given relation,

$du = Tds + \tau dL$

If du is to be exact differential

$\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial \tau}{\partial s}\right)_L$

30. Consider a two degree of freedom system as shown in the figure, where PQ is a rigid uniform rod of length, b and mass, m.



Assume that the spring deflects only horizontally and force F is applied horizontally at Q. For this system, the Lagrangian, L is

A. $\frac{1}{2}(M+m)\dot{x}^2+\frac{1}{6}mb^2\dot{\theta}^2-\frac{1}{2}kx^2+mg\frac{b}{2}\cos\theta$

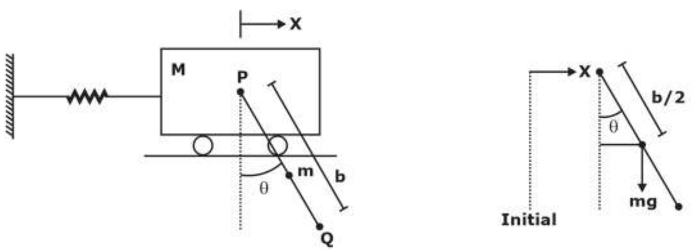
B. $\frac{1}{2}(M+m)\dot{x}^2+\frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta+\frac{1}{6}mb^2\dot{\theta}^2-\frac{1}{2}kx^2+mg\frac{b}{2}\cos\theta$

C. $\frac{1}{2}M\dot{x}^2+\frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta+\frac{1}{6}mb^2\dot{\theta}^2-\frac{1}{2}kx^2+mg\frac{b}{2}\cos\theta+Fb\sin\theta$

D. $\frac{1}{2}M\dot{x}^2+\frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta+\frac{1}{6}mb^2\dot{\theta}^2-\frac{1}{2}kx^2$

Ans. B

Sol.



Lagrange L = KE – PE

$$KE=\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\left(\frac{d}{dt}\left(X+\frac{1}{2}\sin\theta\right)\right)^2+\frac{1}{2}Iw^2$$

$$=\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\left[\left(\dot{X}+\frac{b}{2}\cos\theta\cdot\dot{\theta}\right)^2\right]+\frac{1}{2}\frac{m\left(\frac{b}{2}\right)^2}{3}\cdot\dot{\theta}^2$$

$$=\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\left(\dot{X}^2+\frac{b^2}{4}\cos^2\theta\cdot\dot{\theta}^2+2\dot{X}\left(\frac{b}{2}\right)\cos\theta\cdot\dot{\theta}\right)$$

$$+\frac{1}{2}m\left(\frac{b^2}{12}\right)\dot{\theta}^2$$

$$=\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\dot{X}^2+\frac{1}{2}m\left(\frac{b^2}{4}+\frac{b^2}{12}\right)$$

$$\dot{\theta}^2+\frac{1}{2}m\left[b\dot{X}\dot{\theta}\cos\theta\right]$$

(Since θ is small so $\cos\theta = 1$)

$$KE=\frac{1}{2}(M+m)\dot{X}^2+\frac{1}{6}mb^2\dot{\theta}^2+m\left(\dot{X}\dot{\theta}\right)\frac{b}{2}\cos\theta$$

$KE=\frac{1}{2}KX^2$ (Spring) PE_{PQ}

$=mg\left(-\frac{b}{2}\cos\theta\right)$ (below ref. line)

Lagrange L = KE – PE

$$=\frac{1}{2}(M+m)\dot{X}^2+\frac{1}{6}mb^2\dot{\theta}^2+$$

$$\frac{1}{2}mb\dot{\theta}\dot{X}\cos\theta-\frac{1}{2}KX^2+mg\frac{b}{2}\cos\theta$$

31. A right solid circular cone standing on its base on a horizontal surface is of height H and base radius R. The cone is made of a material with specific weight w and elastic modulus E. The vertical deflection at the mid-height of the cone due to self-weight is given by

A. $\frac{wRH}{8E}$

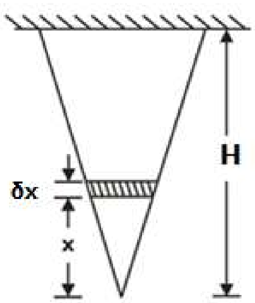
B. $\frac{wRH}{6E}$

C. $\frac{wH^2}{6E}$

D. $\frac{wH^2}{8E}$

Ans. D

Sol. Given,



Consider an elementary section of length δx, at a distance x from the free end.

Let A_x be the area of cross section of the elementary section.

The extension of this elementary section is given by,

$$\delta\Delta = \frac{W_x \cdot \delta x}{A_x \cdot E}$$

where W_x = weight of the conical portion

below the section = $\frac{A_x \cdot x \cdot w}{3}$

(where w is the specific weight or unit weight of the material)

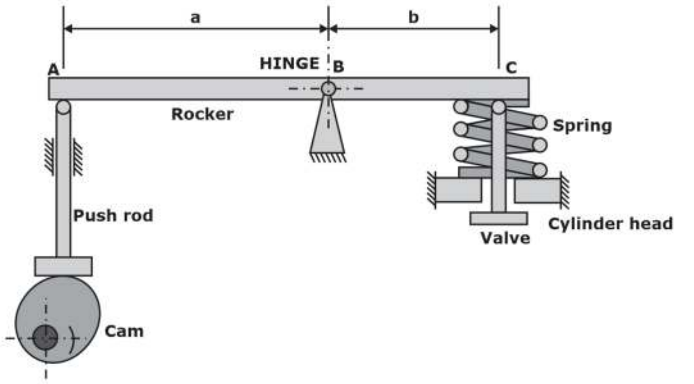
$$\therefore \quad \delta\Delta = \frac{1}{3}\frac{A_x \cdot x \cdot w\,\delta x}{A_x \cdot E} = \frac{x\,w\,\delta x}{3E}$$

Hence total extension of $\Delta L = \sum_{x=\frac{H}{2}}^{x=H} \delta\Delta = \int_{\frac{H}{2}}^{H} \frac{x\,w\,\delta x}{3E}$

$$\Delta L = \frac{w}{3E}\left(H^2 - \frac{H^2}{4}\right)$$

$$\Delta L = \frac{wH^2}{8E}$$

32. A tappet valve mechanism in an IC engine comprises a rocker arm ABC that is hinged at B as shown in the figure. The rocker is assumed rigid and it oscillates about the hinge B. The mass moment of inertia of the rocker about B is 10^{-4} kg.m². The rocker arm dimensions are a = 3.5 cm and b = 2.5 cm. A pushrod pushes the rocker at location A, when moved vertically by a cam that rotates at N rpm. The pushrod is assumed massless and has stiffness of 15 N/mm. At other end C, the rocker pushes a valve against spring of stiffness 10 N/mm. The valve is assumed massless and rigid.



Resonance in the rocker system occurs when the cam shaft runs at a speed of rpm (round off to the nearest integer).

A. 4739
B. 2369
C. 496
D. 790

Ans. A

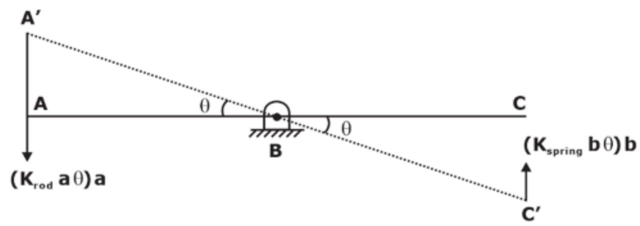
Sol. Given,

mass moment of inertia of the rocker about B = 10^{-4} kg.m²,

The rocker arm dimensions are a = 3.5 cm and b = 2.5 cm,

pushrod has stiffness = 15 N/mm,

spring of stiffness 10 N/mm,



Apply D'Alembert Principle,

$$I\ddot{\theta} + K_{rod}a^2\theta + K_{spring}b^2\theta = 0$$

$$I\ddot{\theta} + \left(K_{rod}a^2 + K_{spring}b^2\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{\left(K_{rod}a^2 + K_{spring}b^2\right)}{I}}$$

$$\omega_n = \sqrt{\frac{\left(15\times10^3\times(0.035)^2 + 10\times10^3\times(0.025)^2\right)}{10^{-4}}}$$

$$\frac{2\pi N_n}{60} = 496.235$$

$$N_n = 4738.703 \cong 4739\text{rpm}$$

33. Customers arrive at a shop according to the Poisson distribution with a mean of 10 customers/hour. The manager notes that no customer arrives for the first 3 minutes after the shop opens. The probability that a customer arrives within the next 3 minutes is

A. 0.61 B. 0.50

C. 0.86 D. 0.39

Ans. D

Sol. $\lambda = \frac{10 \text{ customers}}{\text{hour}} = \frac{1}{6} \text{ customers / min.}$

Arrival rate, $P_m(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$ (Poisson distribution)

Prob. of n-arrivals in time 't'

For n = 0

$P_0(t) = e^{-\lambda t}$

Prob. of no. arrivals in time 't'

P (inter arrival time (T) > t)

So, P (inter arrival time (T) ≤ t) = $1 - e^{-\lambda t}$

⇒ P (inter arrival time (T) ≤ t) = $1 - e^{-\frac{1}{6}\times 3}$

⇒ = 0.39

34. Let f(x) = $x^2 - 2x + 2$ be a continuous function defined on x ∈ [1, 3].The point x at which the tangent of f(x) becomes parallel to the straight line joining f(1) and f(3) is

A. 2 B. 3

C. 0 D. 1

Ans. A

Sol. Given,

F(x) = $x^2 - 2x + 2$

F (1) = 1 – 2 + 2 = 1

F (3) = 9 – 6 + 2 = 5

So, point are A (1,1) and B(3, 5)

Slope of line AB $= \frac{5-1}{3-1} = 2$

We need 'x' for which F′(x) = slope of AB

2x – 2 = 2

x = 2

35. Activities A, B, C and D from the critical path for a project with a PERT network. The means and variances of the activity duration for each activity are given below. All activity durations follow the Gaussian (normal) distribution, and are independent of each other.

Activity	A	B	C	D
Mean (days)	6	11	8	15
Variance ($days^2$)	4	9	4	9

The probability that the project will be completed within 40 days is ________.

(round off to two decimal places).

(Note : Probability is a number between 0 and 1)

Ans. 0.50

Sol. Given,

PERT Network,

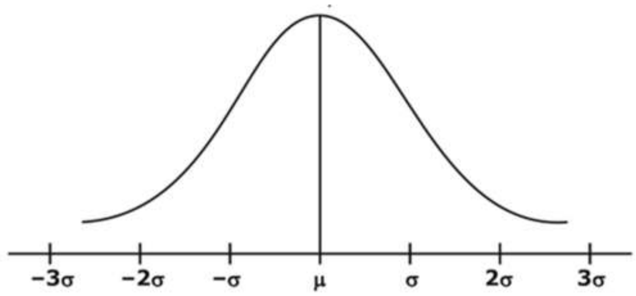
Estimated completion time at Project,

T_e = 6 + 11 + 8 + 15 = 40

Variance of project σ^2 = 26

Probability of project completion in 40 days

P(T_e=40) = 0.50 =50%



T_e days =	(< 40days)	40	(>40 days)
Probability =	< 50%	50%	>50%

36. A true centrifugal casting operation needs to be performed horizontally to make copper tube sections with outer diameter of 250 mm and inner diameter of 230 mm. The value of acceleration due to gravity, $g = 10\ m/s^2$. If a G-factor (ratio of centrifugal force to weight) of 60 is used for casting the tube, rotational speed required is ________ rpm (round off to the nearest integer).

Ans. 655.28

Sol. Largest centrifugal force is on outer diameter of tube

∴ Angular acceleration = $\alpha = r\omega^2$

$$\omega = \sqrt{\frac{\alpha}{r}}$$

$$\omega = \sqrt{\frac{60 \times 9}{r_{outer}}}$$

∴ $d_0 = 250$ mm

∴ $r_0 = 125$ mm = 0.125 m

$$\omega = \sqrt{\frac{60 \times 9.81}{0.125}} = 2\pi N$$

N = 10.92 rps

N = 655.28 rpm

37. The resistance spot welding of two 1.55 mm thick metal sheets is performed using welding current of 10000 A for 0.25 s. The contact resistance at the interface of the metal sheets is 0.0001 Ω. The volume of weld nugget formed after welding is 70 mm^3, the thermal efficiency of the welding process is __________ % (round off to one decimal place).

Ans. 33.60

Sol. Given,

Thickness of the sheet: t = 1.5 mm

Welding Current: I = 10000 A

Welding Time: t = 0.25 s

Contact resistance: R = 0.0001 Ω

Volume of the nugget formed: V = 70 mm^3

Heat required to melt: Δ = 12 J/mm^3

∴ H.F. = I^2Rt

= $(10,000)^2 \times 0.0001 \times 0.25$

Heat Generated = 2500 J

Heat required to melt the nugget = V × Δ

= 70 × 12 = 840 J

thermal efficiency,

$$\therefore \eta_H = \frac{\text{heat Required}}{\text{heat generated}} = \left(\frac{840}{2500}\right) = 0.336$$

η_H = 33.6%

38. An orthogonal cutting operation is performed using a single point cutting tool with a rake angle of 12° on a lathe. During turning, the cutting force and the friction force are 1000 N and 600 N, respectively. If the chip thickness and the uncut chip thickness during turning are 1.5 mm and 0.75 mm, respectively, then the shear force is _________ N (round off to two decimal places).

Ans. 685.81

Sol. Given,

Rake angle: α = 12º,

Cutting Force: F_c = 1000N,

Friction Force: F = 600N,

Uncut chip thickness: t_1 = 0.75 mm,

Chip thickness: t_2 = 1.5 mm

$$\therefore r = \frac{t_1}{t_2} = \frac{0.75}{1.5} = 0.5$$

$$\phi = \tan^{-1}\left[\frac{r\cos\alpha}{1 - r\sin\alpha}\right] = \tan^{-1}\left[\frac{0.5\cos 12}{1 - 0.5\sin 12}\right]$$

Shear angle: φ = 28.62º

and F = F_c sinα + F_t cos α

∴600 = 1000 sin 12 + F_t cos 12

F_t = 400.84N

F_s = F_c cosφ − F_t sinφ

28.62

F_s = 685.81 N

39. In a grinding operation of a metal, specific energy consumption is 15 J/mm^3. If a grinding wheel with diameter of 200 mm is rotating at 3000 rpm to obtain a material removal rate of 6000 mm^3/min, then the tangential force on the wheel is _______ N (round off to two decimal place).

Ans. 47.746

Sol. Given,

Specific energy consumption

=15 J/mm^3,

Grinding wheel diameter (D)

= 200mm,

Material removal rate = 6000 mm^3/min.

Specific energy

$$= \frac{\text{Work done}}{\text{MRR}} = \frac{F_C V_C}{Q}$$

Cutting speed, $V_C = \frac{\pi DN}{1000} = \frac{\pi \times 200 \times 3000}{1000}$

$V_C = 1884.95$ m/min.m/min

So $F_C = \frac{15 \times 6000}{1884.95}$ = 47.746 N

40. A 200 mm wide plate having a thickness of 20 mm is fed through a rolling mill with two rolls. The radius of each roll is 300 mm. The plate thickness is to be reduced to 18 mm in one pass using a roll speed of 50 rpm. The strength coefficient (K) of the work material flow curve is 300 MPa and the strain hardening exponent, n is 0.2. The coefficient of friction between the rolls and the plate is 0.1. If the friction is sufficient to permit the rolling operation then the roll force will be _________ kN (round off to the nearest integer).

Ans. 955.9

Sol. Given,

Width of plate b = 200 m

h_0 = 20 mm, h_f = 18 mm

N = 50 rpm, K = 300 MPa

n = 0.2, μ = 0.1

$$h_{avg} = \frac{h_0 + h_f}{2} = \frac{20+18}{2} = 19\text{mm}$$

$$L = \sqrt{R\Delta h} = \sqrt{300(20-18)} = 24.49\text{mm}$$

$$\Delta = \frac{h_{avg}}{L} = \frac{19}{24.49} = 0.77 < 1$$

∴ Therefore friction is important

Now,

$$\varepsilon_f = \ln\left(\frac{h_f}{h_0}\right) = \ln\left(\frac{18}{20}\right) = -0.1054$$

$$\bar{\sigma}_0 = \frac{K\varepsilon_f^n}{1+n} = \frac{300 \times (0.1054)^{0.1054}}{1+0.2}$$

$$F = 1.15\sqrt{R\Delta h} \text{ x } b \text{ x } \sigma_0 \text{ x} \left(1 + \frac{\mu\sqrt{R\Delta h}}{h_0 + h_f}\right)$$

$$F = 1.15\sqrt{300 \times 2} \times 200 \times 159.4 \times \left(1 + \frac{0.1 \times \sqrt{300 \times 2}}{20+18}\right)$$

F = 955.9 kN

41. The XY table of a NC machine tool is to move from P(1.1) to Q(51.1), all coordinates are in mm. The pitch of the NC drive leadscrew is 1 mm. If the backlash between the leadscrew and the nut is 1.8^0. then the total backlash of the table on moving from P to Q is mm _______ (round off to two decimal places).

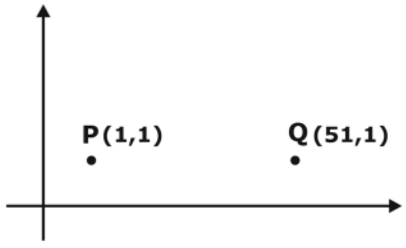
Ans. 0.25

Sol. Given,

Tool move from P(1,1) to Q(51,1)

Total Angular backlash = $\frac{1.8°}{360°}$

= 0.005



As motion between point P and Q is (51 – 1) = 50 mm

So, Total backlash = Angular backlash × Total distance between P and Q

Total backlash = 0.005 × 50 = 0.25 mm

42. Consider a single machine workstation to which jobs arrive according to a Poisson distribution with a mean arrival rate of 12 jobs/hour. The process time of the workstation is exponentially distributed with a mean of 4 minutes. The expected number of jobs at the workstation at any given point of time is ________ (round off to the nearest integer).

Ans. 4

Sol. Given,

Arrival rate λ = 12 Job/hr

Service time $1/\mu$ = 4 min/Job

Length of system L_S = ?

Service rate

$$\mu = \frac{1}{4}\frac{\text{Job}}{\text{Min}} \times \frac{60}{60} = 15\ \text{Job / hr}$$

No of Jobs (n)at workstation = Length of the system

$$= \frac{\lambda}{\mu - \lambda} - = \frac{12}{15-12} = \frac{12}{3} = 4\ \text{Jobs}$$

43. An uninsulated cylindrical wire of radius 1.0 mm produces electric heating at the rate of 5.0 W/m. The temperature of the surface of the wire is 75°C when placed in air at 25°C. When the wire is coated with PVC of thickness 1.0 mm, the temperature of the surface of the wire reduces to 55°C. Assume that the heat generation rate from the wire and the convective heat transfer coefficient are same for both uninsulated wire and the coated wire. The thermal conductivity of PVC is _______ W/m.K (round off two decimal places).

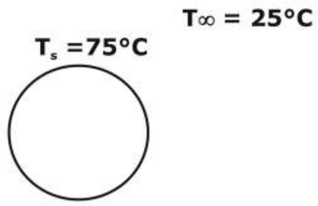
Ans. 0.11

Sol. Given,

Wire radius= 1mm,

Electric heating at the rate = 5.0 W/m,

Case -I:



Net initial heat transfer,

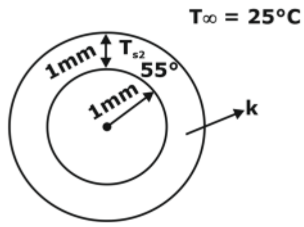
$$\dot{Q} = h \times 2\pi r_1 \times L(75-25)$$

5 = h × 2п × 10^{-3} × 1× 50

h = 15.92 W/m^2k

Case II:

Given assume same for both case;



$$\dot{Q} = \frac{(T_{s2} - T_\infty)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} + \frac{1}{h 2\pi r_2 L}}$$

r_2 = 2 mm, r_1 = 1mm

L = 1 m

$$\dot{Q} = \frac{(55-25)}{\frac{\ln\left(\frac{2}{1}\right)}{2\pi k} + \frac{1}{h \times 2\pi r_2}}$$

$$5 = \frac{30}{\frac{\ln 2}{2\pi k} + \frac{1}{15.92 \times 2\pi \times 2 \times 10^{-3}}}$$

k = 0.11 W/mK

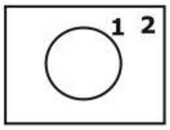
44. A solid sphere of radius 10 mm is placed at the centroid of a hollow cubical enclosure of side length 30 mm. The outer surface of the sphere is denoted by 1 and the inner surface of the cube is denoted by 2. The view factor F_{22} for radiation heat transfer is ________ (rounded off two decimal places).

Ans. 0.768

Sol. Given,

Solid sphere of radius = 10mm,

Hollow cubical enclosure of side length = 30mm



$F_{12} = 1$

$$F_{21} = \frac{A_1}{A_2} = \frac{4\pi r^2}{6a^2} = \frac{4\pi \times 10^2}{6 \times 30^2} = 0.232$$

$F_{22} = 1 - F_{21} = 1 - 0.232 = 0.768$

45. Consider a steam power plant operating on an ideal reheat Rankine cycle. The work input to the pump is 20 kJ/kg. The work output from the high pressure turbine is 750 kJ/kg. The work output from the low pressure turbine is 1500 kJ/kg. The thermal efficiency of the cycle is 50%. The enthalpy of saturated liquid and saturated vapour at condenser pressure are 200 kJ/kg and 2600 kJ/kg. respectively. The quality of steam at the exit of the low pressure turbine is _________ % (round off to the nearest integer).

Ans. 92.90

Sol. Given,

$W_p = 20$ kJ/kg

$W_{T_H} = 750 \text{ kJ / kg}$

$W_{T_L} = 1500 \text{ kJ/kg}$

At condenser pressure

$h_f = 200$ kJ/kg

$h_g = 2600$ kJ/kg

$$\eta = \frac{W_{net}}{Q_s} = \frac{W_{net}}{W_{net} + Q_R}$$

$$\Rightarrow 0.5 = \frac{2230}{2230 + Q_R}$$

$Q_R = 2230$ ks/kg

$Q_R = h_4 - h_5$

$h_4 = 2430$ kJ/kg

$h_4 = h_f + x\,(h_g - h_f)$

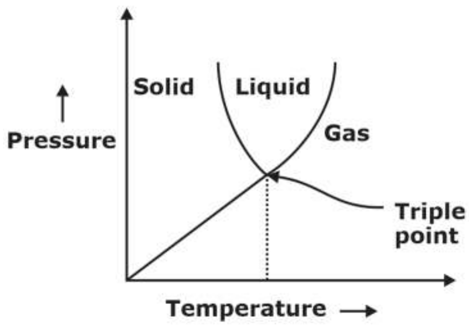
$2430 = 200 + x\,(2600 - 200)$

$x = 0.929 = 92.9\%$

46. In the vicinity of the triple point, the equation of liquid-vapour boundary in the P – T phase diagram for ammonia is ln P = 24.38 – 3063/T, where P is pressure (in Pa) and T is temperature (in K). Similarly, the solid-vapour boundary is given by ln P = 27.92 – 3754/T. The temperature at the triple point is ___________K (around off to one decimal place).

Ans. 195.197

Sol. Given,



At triple point liquid vapour line and solid vapour line will coincide

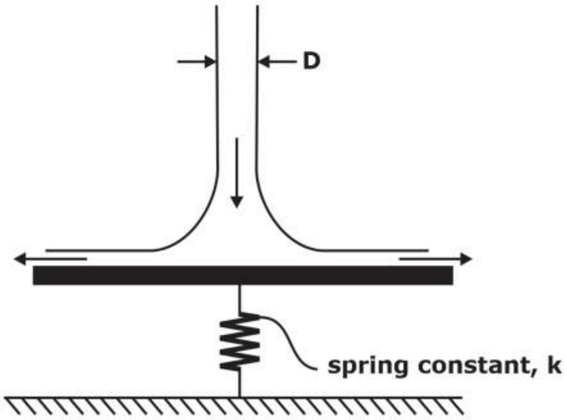
$$\therefore\ 24.38 - \frac{3063}{T} = 27.92 - \frac{3754}{T}$$

$$\frac{3754}{T} - \frac{3063}{T} = 27.92 - 24.38$$

$$\frac{691}{T} = 3.54$$

$T = 195.197\ ^{o}C$

47. A cylindrical jet of water (density = 1000 kg/m^3) impinges at the center of a flat, circular plate and spreads radially outwards, as shown in the figure. The plate is resting on a linear spring with a spring constant k = 1 kN/m. The incoming jet diameter is D = 1 cm.



If the spring shows a steady deflection of 1 cm upon impingement of jet, then the velocity of the incoming jet is _______ m/s (round off to one decimal place).

Ans. 11.28

Sol. From momentum equation in normal direction

$F = \rho A V^2 = Kx$

$$V = \sqrt{\frac{Kx}{\rho A}} = \sqrt{\frac{4Kx}{\rho \pi D^2}}$$

$$= \sqrt{\frac{4 \times 1 \times 10^3 \times 0.01}{1000 \times \pi \times 0.01^2}}$$

$$= 11.28 \text{ m/s}$$

48. A single jet Pelton wheel operates at 300 rpm. The mean diameter of the wheel is 2 m. Operating head and dimensions of jet are suck that water comes out of the jet with a velocity of 40 m/s and flow rate of 5 m^3/s. The jet is deflected by the bucket at an angle of 165^0. Neglecting all losses, the power developed by the Pelton wheel is _____________ MW (round off to two decimal places).

Ans. 2.65

Sol. Given,

Speed of Pelton wheel (N)

= 300 rpm,

Mean diameter of wheel (D_m)= 2 m,

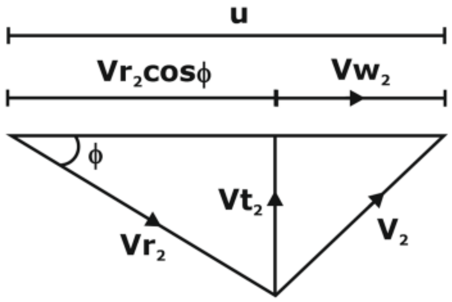
Velocity of jet ($V_1 = V_{w1}$) = 40 m/s,

Flow rate (Q) = 5 m^3/s,

Jet is deflected by (δ) = 165°,

So φ = 15°,

Wheel speed $u = \frac{\pi DN}{60} = 31.416$ m/s



$V_{r1} = V_1 - u = 8.58$ m/s $= V_{r2}$,

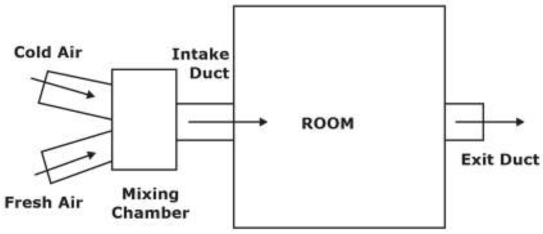
$V_{w2} = u - V_{r2} \cos \varphi = 23.12$ m/s,

$$P = \rho Q\ [V_{w1} + V_{w2}] \cdot u$$

$$= \frac{1000 \times 5[40 - 23.12] \times 31.41}{1000}$$

$$= 2.65 \text{ MW}$$

49. An air-conditioning system provides a continuous flow of air to a room using an intake duct and an exit duct, as shown in the figure. The maintain the quality of the indoor air, the intake duct supplies a mixture of fresh air with a cold air stream. The two streams are mixed in an insulated mixing chamber located upstream of the intake duct. Cold air enters the mixing chamber at 5°C, 105 kPa with a volume flow rate of 1.25 m^3/s during steady state operation. Fresh air enters the mixing chamber at 34°C and 105 kPa. The mass flow rate of the fresh air is 1.6 times of the cold air stream. Air leaves the room through the exit duct at 24°C.



Assuming the air behaves as an ideal gas with c_p = 1.005 kJ/kg.K and R = 0.287 kJ/kg.K, the rate of heat gain by the air from is ________ kW (round off to two decimal places).

Ans. 4.96

Sol. Given,

Cold air temperature (T_1) = 5°C

Cold Air pressure: P_1 = 105 kPa

Fresh Air temperature: T_2 = 34°C

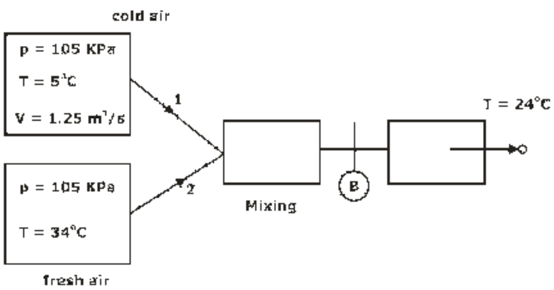
Fresh Air pressure: P_2 = 105 kPa

Volume flow rate of cold Air: V_1 = 1.25m^3/s

Temperature of air exiting room: T_o = 24°C

Assuming the air behaves as an ideal gas,

C_p = 1.005 kJ/kg.K and R = 0.287 kJ/kg.K,



Assuming the air behaves as an ideal gas

$$p_1\dot{v}_1 = \dot{m}_1 R T_1$$

$$105 \times 1.25 = \dot{m}_1 \times 0.287 \times 278$$

$$\dot{m}_2 = 1.645 \text{ kg/s} \left(\dot{m}_1 \Rightarrow \dot{m} \text{ cold air}\right)$$

Given, $\dot{m}_2 = 1.6\,\dot{m}_1 = 2.632$ kg/s

$$\dot{m}_3 = \dot{m}_2 + \dot{m}_1 = 4.277 \text{ kg/s}$$

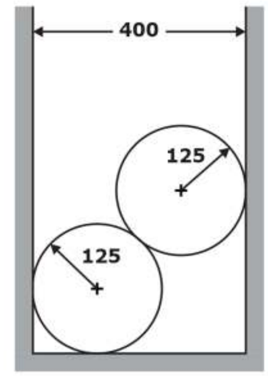
Applying energy principle ,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\dot{m}_1 c_p T_1 + \dot{m}_2 c_p T_2 = \dot{m}_3 c_p T_3$$

$$T_3 = 22.846^oC$$

50. Two smooth identical spheres each of radius 125 mm ad weight 100 N rest a horizontal channel having vertical walls. The distance between vertical walls of the channel is 400 mm.



All dimensions are in mm

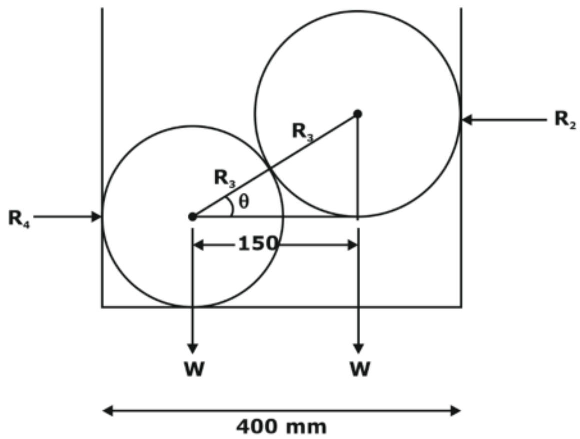
The reaction at the point of contact between two spheres is _______ N(round off to one decimal places).

Ans. 125

Sol. Given,

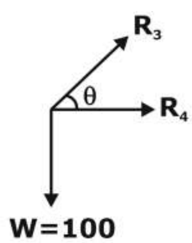
Radius of spheres ball(R) **=** 125mm,

Weight of ball (W) = 100 N,



$$Cos\,\theta = \frac{150}{250} = 0.6$$

θ= 53.13°

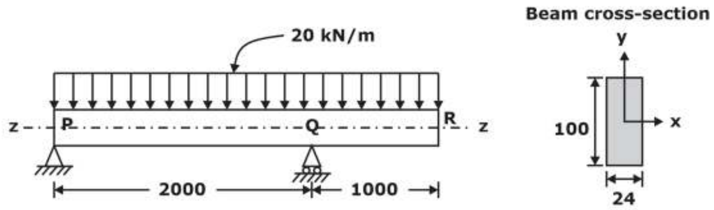


$R_3 \sin\theta = 100$

$$R_3 = \frac{100}{\sin\theta}$$

$R_3 = 125$ N

51. An overhanging beam PQR is subjected to uniformly distributed load 20 kN/m as shown in the figure.

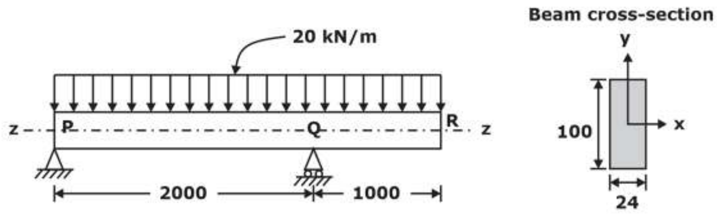


All dimensions are in mm

The maximum bending stress developed in the beam is __________ MPa (round off to one decimal place).

Ans. 250

Sol. Given,



All dimensions are in mm

Uniform distributed load = 20 kN

Width: b = 24 mm

Depth: d = 100 mm

$R_p + R_Q = 20 \times 3 = 60$ kN

Taking moment about Q

$R_P \times 2 = 60 \times 0.5$

$R_P = 15$ kN

$R_Q = 45$ kN

Taking bending moment at x-distance from P, between P and Q

$$M_{x-x} = (R_P \times x) - (20 \times x \times \frac{x}{2})$$

$$M_{x-x} = 15x - 10x^2$$

For maximum bending moment,

$$\frac{dM_{x-x}}{dx} = 0$$

$$15 - 20x = 0 \Rightarrow x = \frac{3}{4}$$

Maximum bending moment between P and Q

$$(M_{max})_{P-Q} = 15 \times \frac{3}{4} - 10 \times \left(\frac{3}{4}\right)^2$$

$$= 5.625\,kN$$

Bending moment at Q

$$M_Q = 20 \times 1 \times 0.5 = 10\,kN$$

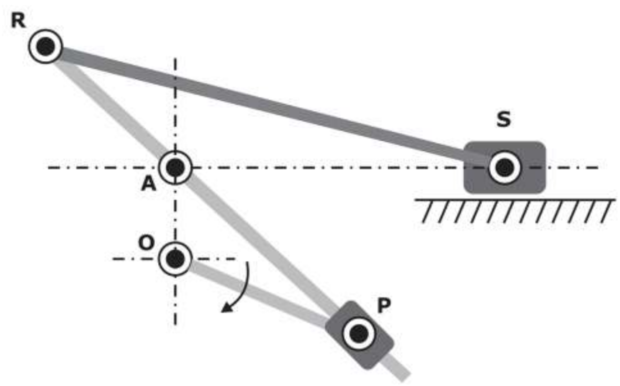
Maximum bending moment over entire beam

(M_{max}) = 10 kN

$$\text{Bending stress}(\sigma_b) = \frac{6 \times M_{max}}{bd^2}$$

$$= \frac{6 \times 10 \times 10^3}{0.024 \times 0.1^2} = 250\,MPa$$

52. The Whitworth quick return mechanism is shown in the figure with link lengths as follows: OP = 300 mm, OA = 150 mm, AR = RS = 450 mm.



The quick return ratio for the mechanism is _______ (round off to one decimal place).

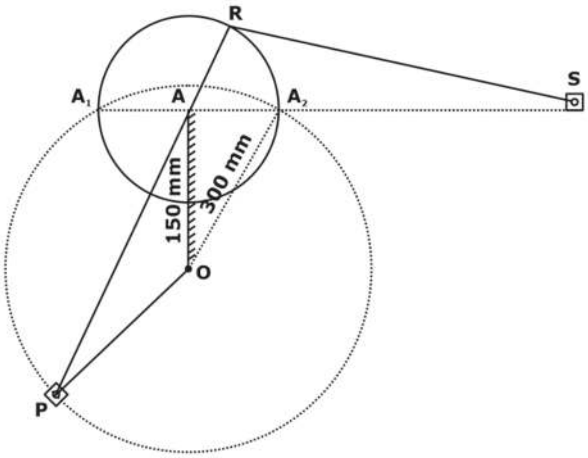
Ans. 2

Sol. Given,

Driving crank (OP) = 300 mm,

Fixed crank length (OA) = 150 mm,

AR = RS = 450 mm,



In triangle OAA_2,

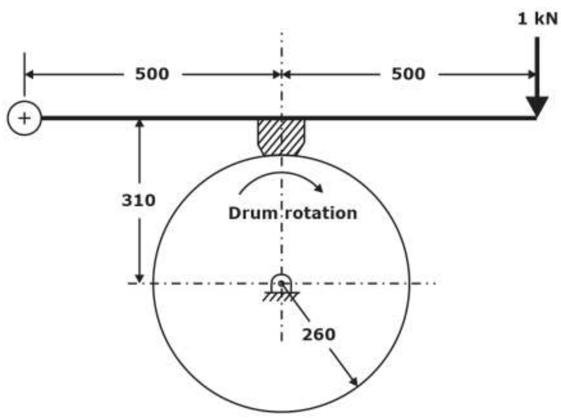
$$\cos\alpha = \frac{150}{300} = \frac{1}{2}$$

$$\alpha = 60^{o}$$

$$QRR = \frac{\text{Cutting time}}{\text{Return time}}$$

$$= \frac{(360^{o} - 2\alpha)}{2\alpha} = \frac{240}{120} = 2$$

53. A short shoe drum (radius 260 mm) brake is shown in the figure. A force of 1 kN is applied to the lever. The coefficient of friction is 0.4.



The magnitude of the torque applied by the brake is _________ Nm(round off to one decimal place).

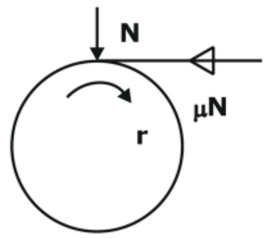
Ans. 200

Sol. Given,

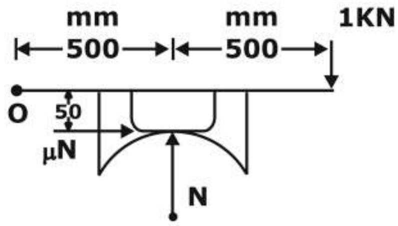
Radius of the drum = 260 mm

Coefficient of friction = 0.4

FBD of Drum



FBD of Lever



$1000 \times (1000) - N \times 500 - (0.4)\, N \times 50 = 0$

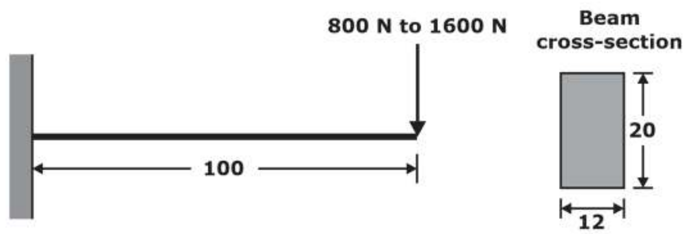
$N = 1923.07$ Newtons

Torque: $T_b = \mu NR$

$T = (0.4)\,(1923.07)\,(0.260)$

$T = 200$ Nm

54. A machine part in the form of cantilever beam is subjected to fluctuating load as shown in the figure. The load varies from 800 N to 1600 N. The modified endurance. Yield and ultimate strengths of the metrical are 200 MPa, 500 MPa and 600 MPa, respectively.



All dimensions are in mm

The factor of safety of the beam using modified Goodman criterion is _______ (round off to one decimal places).

Ans. 2

Sol. Given,

Load varies from 800N to 1600N,

Maximum load (P_{max}) = 1600,

Minimum load ($P_{min.}$) =800N,

$$(\sigma_b)_{max.} = \frac{M_{max.} \times Y}{I}$$

$$= \frac{1600 \times 100 \times (10) \times 12}{12 \times 20^3} = \frac{1600}{8} = 200\text{MPa}$$

$$(\sigma_b)_{min.} = \frac{M_{min.} \times Y}{I}$$

$$= \frac{800 \times 100 \times (10) \times 12}{12 \times 20^3} = \frac{800}{8} = 100\text{MPa}$$

$$\sigma_{mean} = \frac{200 + 100}{2} = 150\text{MPa}$$

$$\sigma_{amp} = \frac{100}{2} = 50\text{MPa}$$

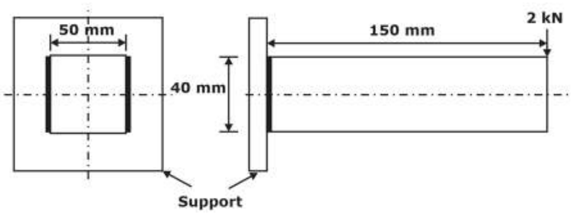
Modified Goodman Criteria,

$$\frac{\sigma_m}{\sigma_{ut}} + \frac{\sigma_a}{\sigma_e} = \frac{1}{FOS}$$

$$\frac{150}{600} + \frac{50}{200} = \frac{1}{FOS}$$

FOS = 2

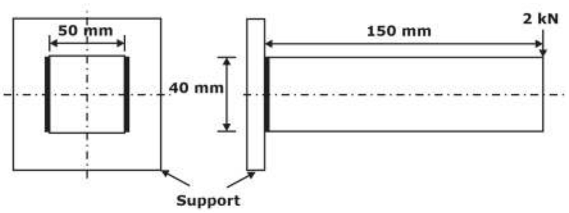
55. A cantilever beam of rectangular cross-section is welded to a support by means of two fillet welds as shown in figure. A vertical load of 2 kN acts at free ends of the beam.



Considering that the allowable shear stress in weld is 60 N/mm^2. The minimum size (leg) of the weld required is _______ mm (round off to one decimal place).

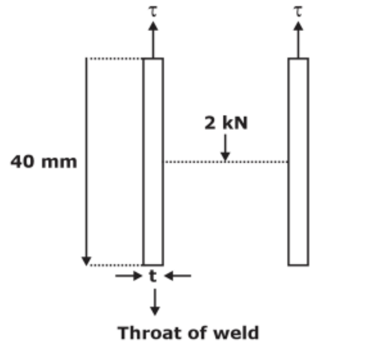
Ans. 6.656

Sol.



The applied force 2kN causes both shearing and bending.

(1) Shear Stress

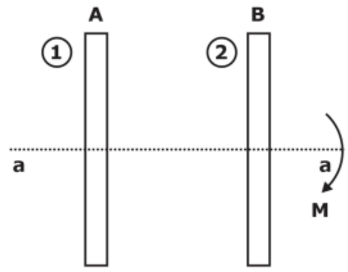


Shear area = (40)t + (40t) = 80t

$$\text{Shear stress } \tau = \frac{F}{SA} = \frac{2 \times 10^3}{80t}$$

$$= \frac{25}{t} N/mm^2 \qquad ......(i)$$

(2) Bending Stress



Bending Moment: M = 2 × 10^3 ×150 = 3 × 10^5 mm

$$I = (I_{xx})_1 + (I_{xx})_2$$

$$= \frac{t(40)^3}{12} + \frac{t(40)^3}{12} = \frac{t(40)^3}{6}$$

$$\sigma_b = \frac{3 \times 10^5}{\frac{t \times (40)^3}{6}} \times 20 = \frac{562.5}{t} \quad ..... (ii)$$

Maximum shear stress

$$\tau_{Max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$60 = \frac{1}{t}\sqrt{\left(\frac{562.5}{2}\right)^2 + (25)^2}$$

$$60 = \frac{1}{t}\sqrt{79726.5}$$

$$\Rightarrow \ t = \frac{282.36}{60}$$

$$= 4.706\,\text{mm}$$

$$\text{Leg of weld } b = \frac{t}{0.707} = 6.656 \text{ mm}$$

****