BYJU'S EXAM PREP

Electrical Engineering

GATE 2018

Questions with Solutions

1. "A common misconception among writers is that sentence structure mirrors thought; the more _____ the structure, the ted the ideas".

A. detailed B. simple

C. clear D. convoluted

Ans. D

Sol. Because the second half of the sentence illustrates the idea that "structure mirrors thought," any word that fills the blank must be similar in meaning to "convoluted." The two words that are similar to "convoluted" are "complicated" and "involved", which produce sentences alike in meaning. "Fanciful," while somewhat similar in meaning to "convoluted," is not as similar to either "complicated" or "involved" as those words are to each other. The other answer choices are not similar in meaning to "convoluted," and thus do not produce coherent sentences.

Thus the correct answer is complicated.

2. Functions, F(a, b) and G(a, b) are defined as follows:

$F(a, b) = (a - b)^2$ and $G(a, b) = |a - b|$, where $|x|$ represents the absolute value of x. What would be the value of G(F(1, 3), G(1, 3))?

A. 2 B. 4

C. 6 D. 36

Ans. A

Sol. $F(a, b) = (a - b)^2$

$G(a, b) = |a - b|$

$G(F(1, 3), G(1, 3)) = G((1 - 3)^2, |1 - 3|)$

$= G(4, 2)$

$= |4 - 2| = 2$

3. The three roots of the equation f(x) = 0 are x = {–2, 0, 3}.

What are the three values for which f(x – 3) = 0 ?

A. -5, -3, 0 B. -2, 0, 3

C. 0, 6, 8 D. 1, 3, 6

Ans. D

Sol. f(x) = 0 for x = {–2, 0, 3}

So, f(x) = (x + 2)x(x – 3) = 0

f(x – 3) = (x – 3 + 2) (x – 3) (x – 3 – 3) = 0

x = 1, 3, 6

4. "Since you have gone off the _____, the _____ sand is likely to damage the car". The words that best fill the blanks in the above sentence are

A. course, coarse B. course, course

C. coarse, course D. coarse, coarse

Ans. A

Sol. Going off the course – not following the intended route.

Coarse sand – harsh in texture

5. For what values of k given below is $\frac{(k+2)^2}{k-3}$ an integer?

A. 4, 8, 18 B. 4, 10, 16

C. 4, 8, 28 D. 8, 26, 28

Ans. C

Sol. Since $\frac{(k+2)^2}{k-3}$ must be an integer, we can verify the options.

$\frac{(4+2)^3}{4-3} = 36$; $\frac{(8+2)^2}{8-3} = 20$

; $\frac{(28+2)^2}{28-3} = 36$

So, option (c) ⇒ 4, 8, 28

6. A class of twelve children has two more boys than girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than boys?

A. 0 B. $\frac{325}{864}$

C. $\frac{525}{864}$ D. None of the above.

Ans. D

Sol. None of the options are correct. (Marks to all by IIT)

There are 7B and 5G

Through as question is stating 3 students are taken at random.

This can be a possible way to approach it.

G B

2 1

3 0

$$= \frac{{}^5C_2 + {}^7C_1 + {}^5C_3 + {}^7C_0}{{}^{12}C_3}$$

$$= \frac{10\times 7 + 10\times 1}{\frac{12\times 10\times 11}{6}} = \frac{80}{220} = \frac{4}{11} \approx 0.3636$$

7. P, Q, R and S crossed a lake in a boat that can hold a maximum of two persons, with only one set of oars. The following additional facts are available

i. The boat held two persons on each of the three forward trips across lake and one person on each of the two return trips.

ii. P is unable to row when someone else is in the boat.

iii. Q is unable to row with anyone else except R.

iv. Each person rowed for at least one trip.

v. Only one person can row during a trip.

Who rowed twice?

A. P B. Q

C. R D. S

Ans. C

Sol. Since, P cannot row with anyone else so P must row while on return journey.

So, on first trip P & S leave in which S rows.

On the return trip P will row

On next trip P will leave with R and R will row & P will get down on other end.

Now R will return & row to pick Q up.

On the last trip Q & R will go in which Q will row the boat.

Hence, R rowed the boat twice

8. A designer uses marbles of four different colours of his designs. The cost of each marble is the same, irrespective of the colour. The table below shows the percentage of marbles of each colour used in the current design. The cost of each marble increased by 25%. Therefore, the designer decided to reduce equal numbers of marbles of each colour to keep the total cost unchanged. What is the percentage of blue marbles in the new design?

Blue	Black	Red	Yellow
40%	25%	20%	15%

A. 35.75 B. 40.25

C. 43.75 D. 46.25

Ans. C

Sol. Assume total number of marbles = 100

Blue marble = 40

Black marble = 25

Red marble = 20

Yellow marble = 15

Cost of 1 marble be Rs. 1

Cost of marbles increased = 25%

New cost of 1 marble = Rs. 1.25

Let x marbles are reduced from each type but the total cost must remain same i.e. Rs 100

Now,

1.25 [(40 – x) + (25 – x) + (20 – x) + (15 – x)] = 100

125 – 5x = 100

25 = 5x

x = 5

So, now the numbers of different marbles are

Blue marble = 35

Black marble = 20

Red marble = 15

Yellow marble = 10

% blue marbles in new design $= \frac{35}{80} \times 100 = 43.75\%$

9. An e-mail password must contain three characters. The password has to contain one numeral from 0 to 9, one upper case and one lower case character from the English alphabet. How many distinct passwords are possible?

A. 6,760 B. 13,520

C. 40,560 D. 1,05,456

Ans. C

Sol. Since, there are three positions which can be filled either by upper case letter, lower case letter or a number from 0 9 –

Hence, total choices are = 26 × 26 × 10 = 6760

3 characters can be arranged in 3! ways & hence total no of password can be = 3! × 6760 = 40560

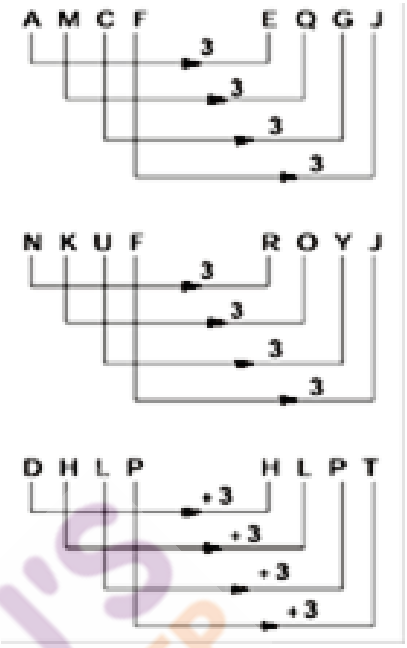
10. In a certain code AMCF is written as EQGJ and NKUF is written as ROYJ. How will DHLP be written in the code?

A. RSTN B. TLPH

C. HLPT D. XSVR

Ans. C

Sol.



11. A single-phase 100 kVA, 1000 V/100 V, 50 Hz transformer has a voltage drop of 5% across its series impedance at full load. Of this, 3% is due to resistance. The percentage regulation of the transformer at full load with 0.8 lagging power factor is

A. 4.8 B. 6.8

C. 8.8 D. 10.8

Ans. A

Sol. Percent voltage regulation

$$= \left(\frac{I_0 R_{02}}{E_2} \times 100\right)\cos\phi_2 \pm \left(\frac{I_2 X_{02}}{E_2}\right)$$

$\%\ \text{V.R.} = V_r \cos\phi_2 \pm \sin\phi_2$

(where, '+' lag p.f. and '-' lead p.f.)

At full load:

Given, V_r = 3%

Impedance drop, V_z = 5%

Reactance drop, $V_x = \sqrt{5^2 - 3^2} = 4\%$

Voltage regulation at full load at 0.8 p.f. lagging

V.R. = 3(0.8) + 4(0.6)

= 2.4 + 2.4 = 4.8%

12. Let be a real-valued function of a real variable defined as $f(x) = x^2$ for $x \geq 0$, and $f(x) = -x^2$ for $x < 0$. Which one of the following statements is true?

A. f(x) is discontinuous at x = 0

B. f(x) is continuous but not differentiable at x = 0

C. f(x) is differentiable but its first derivative is not continuous at x = 0

D. f(x) is differentiable but its first derivative is not differentiable at x = 0

Ans. D

Sol.

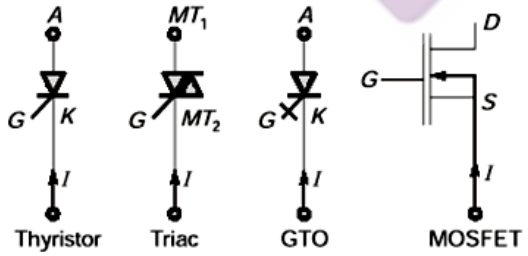
$$f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x, & x \geq 0 \\ -2x, & x < 0 \end{cases}$$

$$f''(x) = \begin{cases} 2, & x \geq 0 \\ -2, & x < 0 \end{cases}$$

The function is differentiable but its first derivative is not differentiable at x = 0.

13. Four power semiconductor devices are shown in the figure along with their relevant terminals. The device(s) that can carry dc current continuously in the direction shown when gated appropriately is (are)



A. Triac only

B. Triac and MOSFET

C. Triac and GTO

D. Thyristor and Triac

Ans. A

Sol. Since triac is bi-directional switch it can carry current in reverse direction.

GTO are unidirectional switched so they cannot carry current in reverse direction.

14. The value of the integral $\oint \frac{z+1}{z^2-4}\,dz$ in counter clockwise direction around a circle C of radius 1 with center at the point z = −2 is

A. $\frac{\pi i}{2}$ B. $2\pi i$

C. $-\frac{\pi i}{2}$ D. $-2\pi i$

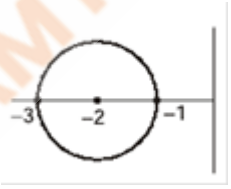
Ans. A

Sol. $\int \frac{z+1}{z^2-4}\,dz$

$$\int \frac{z+1}{(z-2)(z+2)}dz$$

$$\int \frac{\left(\frac{z+1}{z-2}\right)}{(z+2)}dz$$

where, $f(z) = \frac{z+1}{z-2}$

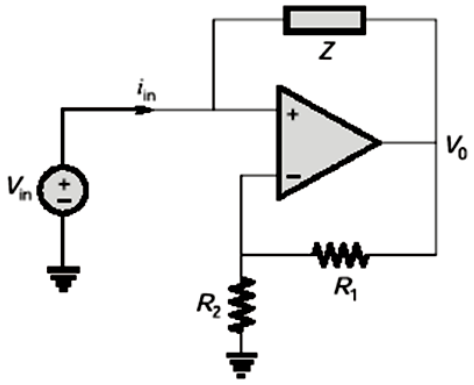


$$= 2\pi i\, f(-2)$$

$$= 2i\left(\frac{-2+1}{-2-2}\right)$$

$$= 2\pi i\left(\frac{-1}{-4}\right) = \frac{\pi i}{2}$$

15. The op-amp shown in the figure is ideal. The input impedance is given by



A. $-Z\frac{R_2}{R_1}$ B. $-Z\frac{R_1}{R_2}$

C. Z D. $-Z\frac{R_1}{R_1+R_2}$

Ans. A

Sol. According to virtual ground

$V_+ = V_- = V_{in}$

By KVL

$V_0 = V_{in} - I_{in}Z$

$I_{in} = \frac{V_{in} - V_0}{Z}$ .....(i)

Also by voltage division

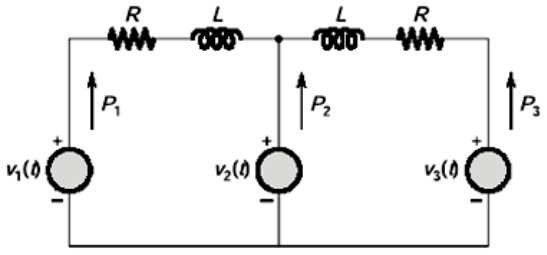
$V_0 = V_{in}\left(\frac{R_1}{R_2} + 1\right)$ .....(ii)

Equation (ii) in equation (i)

$I_{in} = -\frac{V_{in}R_1}{R_2Z}$

$Z_{in} = \frac{V_{in}}{I_{in}} = -Z\frac{R_2}{R_1}$

16. In the figure, the voltages are $v_1(t) = 100\cos(\omega t), v_2 t = 100\cos\left(\omega t + \frac{\pi}{18}\right)$ and $V_3(t) = 100\cos\left(\omega t + \frac{\pi}{36}\right)$. The circuit is in sinusoidal steady-state, and $R \ll \omega L$. P_1, P_2 and P_3 are the average power outputs. Which one of the following statements is true?



A. $P_1 = P_2 = P_3 = 0$

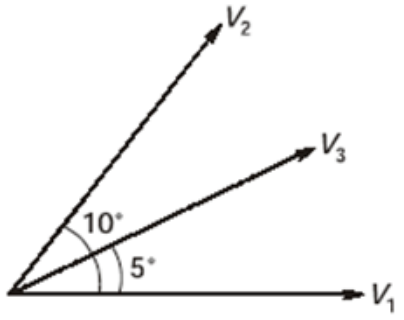
B. $P_1 < 0, P_2 > 0, P_3 > 0$

C. $P_1 < 0, P_2 > 0, P_3 < 0$

D. $P_1 > 0, P_2 < 0, P_3 > 0$

Ans. C

Sol.



$V_2 : \frac{\pi}{18} = \frac{180°}{18} = 10°$

$V_3 = \frac{\pi}{36} = \frac{180°}{36} = 5°$

V_2 leads V_1 and V_3

So, current will flow from V_2 towards V_1 and V_3.

Hence, $P_1 < 0$, $P_2 > 0$, $P_3 < 0$

17. A positive charge of 1 nC is placed at (0, 0, 0.2) where all dimensions are in meters. Consider the x-y plane to be a conducting ground plane. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. The z component of the E field at (0, 0, 0.1) is closed to

A. 899.18 V/m

B. -899.18 V/m

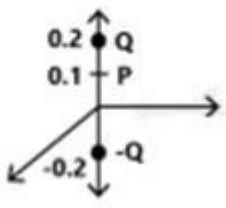
C. 999.09 V/m

D. -999.09 V/m

Ans. D

Sol. Due to infinite conducting plane, we have to consider an image charge symmetrically below plane. Since one charge is placed at (0, 0, 0.2),

the image charge will be at



(0, 0, -0.2)

Electric field at P will be sum of Field due to Q & -Q

Net electric field at point due to a charge Q is given by

$$E = \frac{Q}{\pi\varepsilon_0 r^2}$$

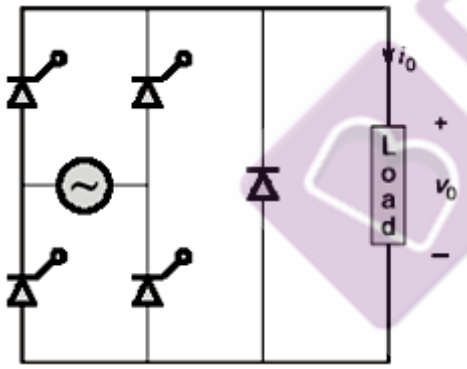
So, electric field a point P is

$$E_P = \frac{Q}{4\pi\varepsilon_0(0.1)^2} + \frac{Q}{4\pi\varepsilon_0(0.3)^2}$$

$$E_P = \frac{10^{-9}}{4\pi \times 8.85 \times 10^{-12}}\left(\frac{1}{0.01} + \frac{1}{0.09}\right) = 999.09 \text{ V / m}$$

Since, it is in downward direction so, $E_P = -999.09$ V/m

18. A single-phase fully controlled rectifier is supplying a load with an anti-parallel diode as shown in the figure. All switches and diodes are ideal. Which one of the following is true for instantaneous load voltage and current?



A. $v_0 \geq 0$ and $i_0 < 0$

B. $v_0 < 0$ and $i_0 < 0$

C. $v_0 \geq 0$ and $i_0 \geq 0$

D. $v_0 < 0$ and $i_0 \geq 0$

Ans. C

Sol. Since, this converter has freewheeling diode, the output voltage cannot go negative $v_0 \geq 0$.

Since, diodes and SCR are unidirectional current devices so current cannot reverse $i_0 \geq 0$.

19. Match the transfer functions of the second-order systems with the nature of the system given below.

Transfer functions Nature of system

P. $\frac{15}{s^2 + 5s + 15}$ I. Overdamped

Q. $\frac{25}{s^2 + 10s + 25}$ II. Critically damped

R. $\frac{35}{s^2 + 18s + 35}$ III. Underdamped

A. P-I, Q-II, R-III

B. P-II, Q-I, R-III

C. P-III, Q-II, R-I

D. P-III, Q-I, R-II

Ans. C

Sol. $P = \frac{15}{s^2 + 5s + 15}$

$\omega_n = \sqrt{15} = 3.872$ rad/s

$2\xi\omega_n = 5$

$\xi = 0.645$

So, P is underdamped.

$$Q = \frac{25}{s^2 + 10s + 25}$$

$\omega_n = \sqrt{25} = 5$ rad/s

$2\xi\omega_n = 10$

$\xi = 1$

So, Q is critically damped.

Observing all the options, option (c) is correct.

20. In a salient pole synchronous motor, the developed reluctance torque attains the maximum value when the load angle in electrical degree is

A. 0 B. 45°

C. 60° D. 90°

Ans. B

Sol. Reluctance power is given by

$$P = \frac{V_t^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right)\sin 2\delta$$ (V_t is the line voltage)

So, reluctance torque is

$$T = \frac{P}{\omega_s} = \frac{V_t^2}{2\omega_s}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

So, torque is maximum when $2\delta = 90°$

$\delta = 45°$

21. The graph of a network has 8 nodes and 5 independent loops. The number of branches of the graph is

A. 11 B. 12

C. 13 D. 14

Ans. B

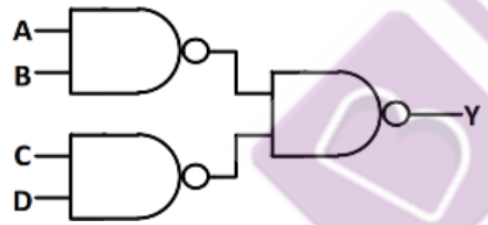
Sol. loops $= b - (N - 1)$

$5 = b - (8 - 1)$

$5 = b - 7$

$b = 12$

22. In the logic circuit shown in the figure, Y is given by,



A. Y = ABCD

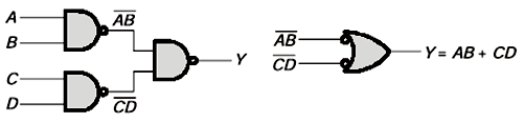
B. Y = (A + B)(C + D)

C. Y = A + B + C + D

D. Y = AB + CD

Ans. D

Sol.



As NAND gate can also be represented as bubbled OR. So, Y comes out be AB+CD as shown above.

23. A continuous-time input signal x(t) is an eigen function of an LTI system, if the output is

A. kx(t) where is an eigenvalue.

B. $ke^{j\omega t}$ x(t), where is an eigenvalue and is a complex exponential signal.

C. $x(t)e^{j\omega t}$, where $e^{j\omega t}$ is a complex exponential signal.

D. kH(ω), where k is an eigenvalue and H(ω) is a frequency response of the system.

Ans. A

Sol. If the output signal is a scalar multiple of input signal, the signal is refereed as an eigen function (or characteristic function) and the multiplier is referred as an eigen value (or characteristic value).

If x(t) is the eigen function and k is the eigen value, then output, y(t) = kx(t).

Hence, the correct option is (A).

24. Two wattmeter method is used for measurement of power in a balanced three-phase load supplied from a balanced three-phase system. If one of the wattmeter reads half of the other (both positive), then the power factor of the load is

A. 0.532 B. 0.632

C. 0.707 D. 0.866

Ans. D

Sol. In two Wattmeter method

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

Given : $W_2 = \frac{W_1}{2}$

$$\tan\phi = \frac{\sqrt{3}\left(W_1 - \frac{W_1}{2}\right)}{\left(W_1 + \frac{W_1}{2}\right)}$$

$\varphi = 30°$

$\cos\varphi = \cos 30° = 0.866$

25. Consider a lossy transmission line with V_1 and V_2 as the sending and receiving end voltages, respectively. Z and X are the series impedance and reactance of the line, respectively. The steady-state stability limit for the transmission line will be

A. greater than $\left|\frac{V_1V_2}{X}\right|$

B. less than $\left|\frac{V_1V_2}{X}\right|$

C. equal to $\left|\frac{V_1V_2}{X}\right|$

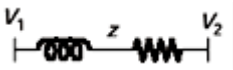
D. equal to $\left|\frac{V_1V_2}{Z}\right|$

Ans. B

Sol. With only x :

$$P_{max} = \left|\frac{V_1V_2}{X}\right|$$

With Lossy Tr. Line



$$P = \left|\frac{V_1V_2}{Z}\right|\cos(\beta-\delta) - \left|\frac{AV_2^2}{Z}\right|\cos(\beta-\alpha)$$

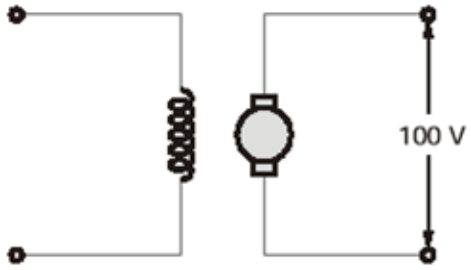
$\therefore$ With Lossy Line $P_{max} < \left|\frac{V_1V_2}{X}\right|$

26. A separately excited dc motor has an armature resistance R_a = 0.05 Ω. The field excitation is kept constant. At an armature voltage of 100 V, the motor produces a torque of 500 Nm at zero speed. Neglecting all mechanical losses, the no-load speed of the motor (in radian/s) for an armature voltage of 150 V is _____ (upto 2 decimal places).

Ans. 600

Sol. Given, separately initiated DC motor.

Field excitation is constant.



Producing a torque of 500 N-m

At zero speed,

N = 0 and E_b = 0

and $E_b = V_t - I_ar_a$

$$I_a = \frac{V_t}{r_a} = \frac{100}{0.05} = 2000A$$

Since $T = K_a\phi I_a$

$$K_a\phi = \frac{500}{2000} = \frac{1}{4}$$

When motor runs on no-load given all mechanical losses neglected. No-load current is negligible and the voltage drop at no-load can be negligible.

So, $E_b = V_t$ = 150 V

$$E_b = K_a\phi\omega_m$$

$$\omega_m = \frac{150}{\frac{1}{4}}$$

ω_m = 150 × 4 = 600 rad/s

27. The positive, negative and zero sequence impedances of a 125 MVA, three-phase, 15.5 kV, star-grounded, 50 Hz generator are j0.1 pu, j0.05 pu and j0.01 pu respectively on the machine rating base. The machine is unloaded and working at the rated terminal voltage. If the grounding impedance of the generator is j0.01 pu then the magnitude of fault current for a b-phase to ground fault (in kA) is _____ (upto 2 decimal places).

Ans. 73.52

Sol. For LG fault,

$$I_{f_{pu}} = \frac{3.V_{Th}}{Z_1 + Z_2 + Z_0 + 3Z_n}$$

$$= \frac{3 \times 1}{0.1 + 0.05 + 0.01 + 3(0.01)}$$

$$I_{base} = \frac{P}{\sqrt{3}.V_L} = \frac{125 \times 10^6}{\sqrt{3} \times 15.5 \times 10^3} = 4656.05A$$

$$I_f = I_{f_{pu}} \times I_{base} = 15.789 \times 4656.05$$

= 73516.538 = 73.52 kA

28. A 1000 × 1000 bus admittance matrix for an electric power system has 8000 non-zero elements. The minimum number of branches (transmission lines and transformers) in this system are _____ (upto 2 decimal places).

Ans. 3500

Sol. Number of buses = Number of diagonal elements = 1000

No. of non-zero elements = 8000

No. of non-zero off diagonal elements = 7000

No. of transmission lines = $\frac{\text{No. of non} - \text{zero off diagonal elements}}{2}$

$$= \frac{7000}{2} = 3500$$

29. Consider a non-singular 2 x 2 square matrix A. If trace (A)=4 and trace (A^2)=5, the determinant of the matrix A is _____

Ans. 5.5

Sol. Eigen value of matrix A = λ_1, λ_2

Eigen value of matrix A^2 = λ_1^2, λ_2^2

We know that,

Sum of Eigen value = Trace of matrix

Product of Eigen value = Determinant of matrix

Hence,

$\lambda_1 + \lambda_2 = 4$

$\lambda_1^2 + \lambda_2^2 = 5$

Now,

$(\lambda_1 + \lambda_2)^2 = \lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2$

$4^2 = 5 + 2\lambda_1\lambda_2$

$2\lambda_1\lambda_2 = 11$

$\lambda_1\lambda_2 = 5.5$

30. Consider a unity feedback system with forward transfer function given by

$$G(s) = \frac{1}{(s+1)(s+2)}$$

The steady-state error in the output of the system for a unit-step input is _____ (upto 2 decimal places).

Ans. 0.66

Sol. Steady-state error for type-0 and step input,

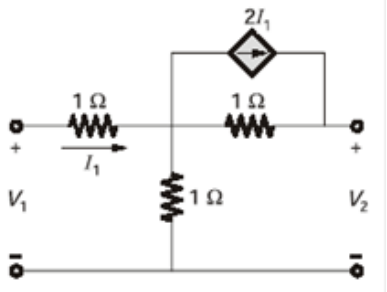
$$e_{ss} = \frac{1}{1 + k_p}$$

$$k_p = \lim_{s \to 0} \frac{1}{(s+1)(s+2)} = \frac{1}{2}$$

$$e_{ss} = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3} = 0.66 \text{ units}$$

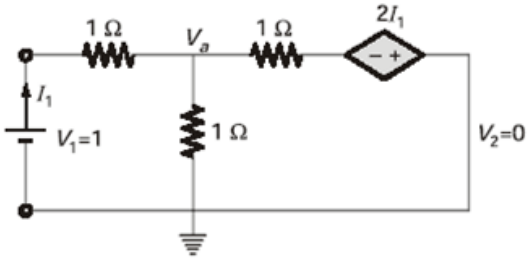
31. In the two-port network shown, the h_{11} parameter

$\left(\text{where, } h_{11} = \frac{V_1}{I_1}, \text{ when } V_2 = 0\right)$ in ohms is _____ (up to 2 decimal places).



Ans. 05

Sol. To find $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ the equivalent circuit is given below.



By KCL,

$$\frac{V_a - 1}{1} + \frac{V_a}{1} + \frac{V_a + 2I_1}{1} = 0$$

$$3V_a + 2I_1 = 1 \quad ....(i)$$

$$I_1 = \frac{1 - V_a}{1} \quad ...(ii)$$

Substitute equation (ii) in equation (i),

$$V_a = -1$$

$$I_1 = \frac{1 - V_a}{1} = \frac{1 - (-1)}{1} = 2$$

$$h_{11} = \frac{V_1}{I_1} = \frac{1}{2} = 0.5\,\Omega$$

32. The series impedance matrix of a short three-phase transmission line in phase coordinates is $\begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}$. If the positive sequence impedance is (1 + j10)Ω, and the zero sequence is (4 + j31)Ω, then the imaginary part of Z_m (in Ω) is _____ (up to 2 decimal places).

Ans. 7

Sol. We know,

$$Z_1 = (1 + j10)\,\Omega$$

$$Z_0 = (4 + j31)\,\Omega$$

$$Z_1 = Z_s - Z_m$$

$$Z_0 = Z_s + 2Z_m$$

$$---------$$

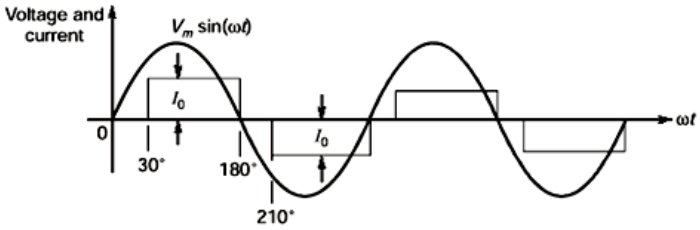
$$Z_1 - Z_0 = -3Z_m$$

$$Z_m = \frac{Z_0 - Z_1}{3}$$

$$= \frac{4 + j31 - 1 - j10}{3} = \frac{3 + j21}{3} = (1 + j7)$$

The imaginary part of Z_m is 7

33. The waveform of the current drawn by a semi-converter from a sinusoidal AC voltage source is shown in the figure. If I_0 = 20A, the rms value of fundamental component of the current is _____ A (up to 2 decimal places).



Ans. 17.39

Sol. $i_{s1} = \frac{4I_0}{\pi}\cos\frac{\alpha}{2}$

$$I_{s1}(\text{rms}) = \frac{2\sqrt{2}}{\pi} I_0 \cdot \cos\frac{\alpha}{2}$$

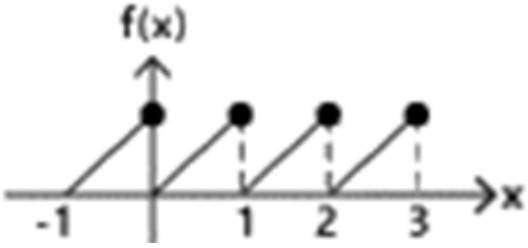
$$= \frac{2\sqrt{2}}{\pi} \times 20 \times \cos\left(\frac{30°}{2}\right) = 17.39A$$

34. Let f be a real valued function of a real variable defined as f(x) = x – [x], where [x] denotes the largest integer less than or equal to x. The value of $\int_{0.25}^{1.25} f(x)\,dx$ is _____ (up to 2 decimal places).

Ans. 0.5

Sol. f(x) = x – [x] = {x} = fractional part of x

Graph of fractional part of x is given below.



It is periodic with period 1.

$$\int_{0.25}^{1.25} f(x)\,dx = \int_{0}^{1} f(x)\,dx = \frac{1}{2} \times 1 \times 1 = 0.5$$

35. The value of the directional derivative of the function φ (x, y, z) = $xy^2 + yz^2 + zx^2$ at the point (2, -1, 1) in the direction of the vector p = i + 2j + 2k is

Ans. 1

Sol. $\varphi = xy^2 + yz^2 + zx^2$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= (y^2 + 2xz)\hat{i} + (2xy + z^2)\hat{j} + (x^2 + 2yz)\hat{k}$$

$$\nabla\phi_{(2,\,-1,1)} = 5\hat{i} - 3\hat{j} + 2\hat{k}$$

The directional derivative of $\varphi(x, y, z)$ at in (2, -1, 1) the direction of $\bar{P}$ is $\nabla\phi_{at.P''} \cdot \frac{\bar{P}}{|\bar{P}|}$

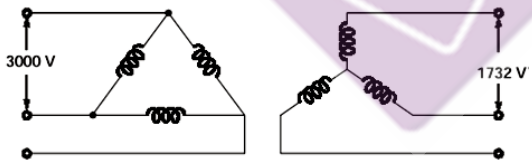
$$= \left(5\bar{i} - 3\bar{j} + 2\bar{k}\right).\left(\frac{\bar{i} + 2\bar{j} + 2\bar{k}}{3}\right)$$

$$= \frac{5 - 6 + 4}{3} = 1$$

36. A three-phase, 900 kVA, $3\text{kV} / \sqrt{3}\,\text{kV}\,(\Delta / Y)$ 50 Hz transformer has primary (high voltage side) resistance per phase of 0.3Ω and secondary (low voltage side) resistance per phase of 0.02Ω. Iron loss of the transformer is 10 kW. The full load % efficiency of the transformer operated at unity power factor is _____ (upto 2 decimal places).

Ans. 97.36

Sol. 900 kVA, Δ/Y, 3-phase transformer,



Given,

$R_1 = 0.3$

$R_2 = 0.02$ Ω/ph

Iron loss = 10 kW

Phase voltage on delta (HV) side = 3kV

Phase voltage on star (LV) side = $\frac{\sqrt{3}}{\sqrt{3}} = 1\,\text{kV}$

Primary resistance referred to secondary

$$= \left(\frac{1}{3}\right)^2 \times 0.3 = 0.033$$

Total resistance on secondary side = 0.033+0.02 = 0.0533

Full load phase current on secondary side I

$$= \frac{900 \times 10^3}{\sqrt{3} \times \sqrt{3} \times 10^3} = 300\text{A}$$

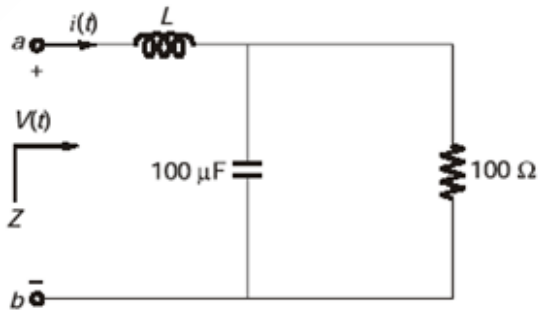
Total copper loss or ohmic loss

$= 3I^2R = 3 \times 300^2 \times 0.0533 = 14400\text{W} = 14.4\text{ kW}$

Efficiency $\eta = \frac{\text{output}}{\text{output} + \text{losses}}$

$$\eta = \frac{900 \times 1}{900 \times 1 + 14.4 + 10} = 0.9736 = 97.36\%$$

37. The voltage v(t) across the terminals a and b shown in the figure, is a sinusoidal voltage having a frequency ω = 100 rad/s. When the inductor current i(t) is in phase with the voltage v(t), the magnitude of the impedance Z seen between the terminals a and b is _____Ω. (up to 2 decimal places).



Ans. 50

Sol. Capacitive reactance

$$X_C = \frac{1}{j\omega C} = \frac{-j}{100 \times 100 \times 10^6} = -j100$$

$$Z = j\omega L + \frac{100 \times -j100}{100 - j100} = j\omega L + 50 - j50$$

When the inductor current i(t) is in phase with the voltage v(t), the circuit is in resonance. So the impedance will only be resistive at resonance. Hence, Z = 50Ω

38. The Fourier transform of a continuous time signal x(t) is given by $X(\omega) = \frac{1}{(10 + j\omega)^2}$. $-\infty < \omega < \infty$, where $j = \sqrt{-1}$ and ω denotes frequency. Then the value of |ln x(t)| at t = 1 is _____(upto 1 decimal place). (In denotes the logarithm to base e).

Ans. 10

Sol. $X(\omega) = \frac{1}{(10 + j\omega)^2}$

By taking inverse Fourier transform,

$x(t) = te^{-10t}\, u(t)$

Now, $x(t)_{t=1} = 1 \times e^{-10} \times 1 = e^{-10}$

Thus, $|\ln x(t)| = |\ln e^{-10}|$

$= |-10| = 10$

39. Let $f(x) = 3x^3 - 7x^2 + 5x + 6$. The maximum value of f(x) over the interval [0, 2] is _____ (upto 1 decimal place).

Ans. 12

Sol. $f(x) = 3x^3 - 7x^2 + 5x + 6$

$f'(x) = 9x^2 - 14x + 5$

$f''(x) = 18x - 14$

$f'(x) = 0$

$x^2 - 14x + 5 = 0$

$x = 1, 0.55$

$x = 1$

$f''(1) = 18 - 14 = 4 > 0$, minima

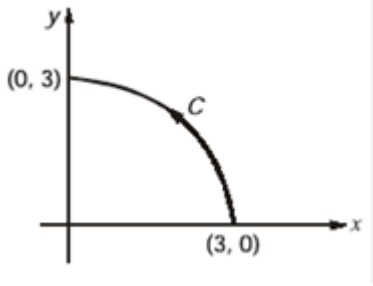
$x = 0.55$

$f''(0.55) = -4.1 <$ maxima

maximum {f(0), f(0.55), f(2)}

maximum {6, 7.13, 12} = 12

40. As shown in the figure, C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral $\int_C (y^2 + 2yx)dx + (2xy + x^2)\,dy$ is _____ (upto 2 decimal places).



Ans. 0

Sol. Consider a closed curve C and the coordinate axes.

Applying Green's Theorem

$$\int_C (y^2 + 2yx)dx + (2xy + x^2)\,dy$$

$$= \iint \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

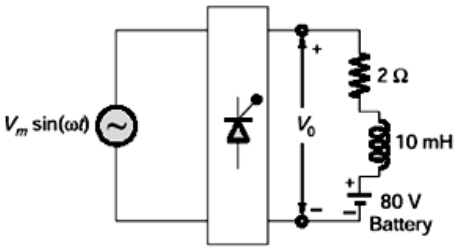
$$\frac{\partial N}{\partial x} = \frac{\partial (2xy + x^2)}{\partial x} = 2x + 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial (y^2 + 2xy)}{\partial y} = 2y + 2x$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0$$

So, $\int_C (y^2 + 2yx)dx + (2xy + x^2)\,dy = 0$

41. A phase controlled single-phase rectifier, supplied by an AC source, feeds power to an R-L-E load as shown in the figure. The rectifier output voltage has an average value given by $V_0 = \frac{V_m}{2\pi}(3 + \cos\alpha)$, where $V_m = 80\pi$ volts and α is the firing angle. If the power delivered to the lossless batter is 1600 W. α in degree is _____ (upto 2 decimal places).



Ans. 90

Sol. Power delivered to battery = 1600W

$$\text{Current} = \frac{P}{E} = \frac{1600}{80} = 20A$$

$V_0 = E + I_0R = 80 + 20 \times 2 = 120$ V

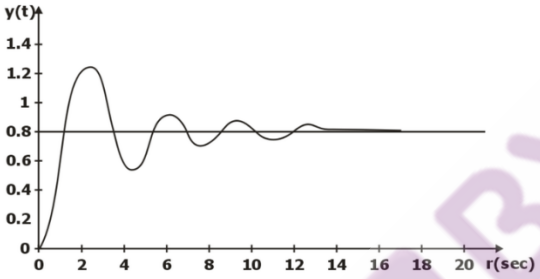
$$V_0 = \frac{V_m}{2\pi}(3 + \cos\alpha)$$

$$V_0 = \frac{80\pi}{2\pi}(3 + \cos\alpha)$$

$3 + \cos\alpha = 3$

$\alpha = 90°$

42. The unit step response y(t) of a unity feedback system with open-loop transfer function $G(s)H(s) = \frac{K}{(s+1)^2(s+2)}$ is shown in the figure. The value of K is _____ (upto 2 decimal places).



Ans. 8

Sol. From the response steady state value of the function is 0.8. So finding the steady state value.

Closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{(s+1)^2(s+2)}}{1 + \frac{K}{(s+1)^2(s+2)}}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)^2(s+2) + K}$$

Given, $R(s) = \frac{1}{s}$

$$C(s) = \frac{K}{s\left[(s+1)^2(s+2) + K\right]}$$

$$\lim_{s\to 0} sC(s) = 0.8 \quad \text{(given)}$$

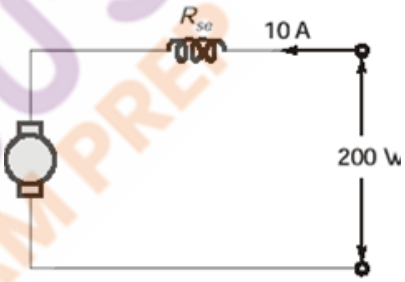
$$\frac{K}{2+K} = 0.8$$

$\Rightarrow K = 8$

43. A 200 V DC series motor, when operating from rated voltage while driving a certain load, draws 10 A current and runs at 1000 rpm. The total series resistance is 1Ω. The magnetic circuit is assumed to be linear. At the same supply voltage, the load torque is increased by 44%. The speed of the motor in rpm (rounded to the nearest integer) is _____.

Ans. 824.56

Sol.



Given, $I_{a1} = 10A$

$N_1 = 1000$ rpm

$(R_a + R_{se}) = 1\Omega$

$T_2 = 1.44\ T_1$

In series motor,

$$T \propto I_a^2$$

$$\frac{T_1}{T_2} = \frac{I_{a_1}^2}{I_{a_2}^2}$$

$$\frac{T_1}{1.44T_1} = \frac{10^2}{I_{a_2}^2}$$

$$I_{a_2} = 12A$$

Also,

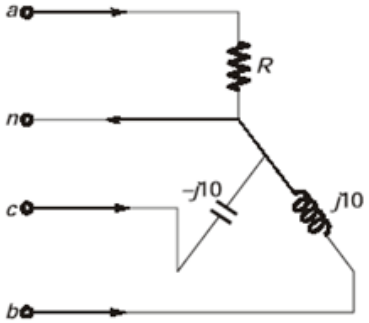
$$E_b \propto I_aN$$

$$\frac{E_{b_1}}{E_{b_2}} = \frac{I_{a_1}N_1}{I_{a_2}N_2}$$

$$\frac{200-10(1)}{200-12(1)}=\frac{10\times1000}{12\times N}$$

N = 824.56 rpm

44. A three-phase load is connected to a three-phase balanced supply as shown in the figure. If V_{an} = 100∠0°V, V_{bn} = 100 ∠−120°V and V_{cn} = 100 ∠−240° (angles are considered positive in the anti-clockwise direction), the value of R for zero current in the neutral wire is _____ Ω (upto 2 decimal places).



Ans. 5.77

Sol. From the given voltages,

$$I_R=\frac{V_{RN}}{R}=\frac{100\angle0^\circ}{R}$$

$$I_R=\frac{V_{RN}}{R}=\frac{100\angle0^\circ}{R}$$

$$I_Y=\frac{V_{YN}}{jX_L}=\frac{100\angle120^\circ}{j10}=10\angle-210^\circ$$

$$I_B=\frac{V_{BN}}{-jX_C}=\frac{100\angle120^\circ}{j10}=10\angle-150^\circ$$

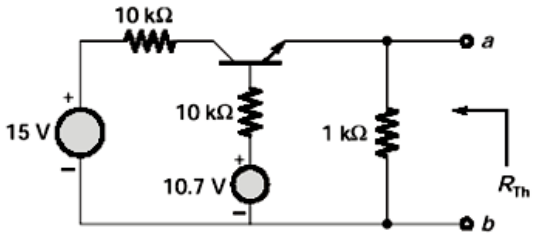
For I_N = 0

$$I_R+I_Y+I_B=0$$

$$\frac{100}{R}+10\angle-210^\circ+10\angle-150^\circ=0$$

$$R=5.77\Omega$$

45. In the circuit shown in the figure, the bipolar junction transistor (BJT) has a current gain β = 100. The base-emitter voltage drop is a on stant, V_{BE} = 0.7 V. The value of the Thevenin equivalent resistance R_{Th} (in Ω) as shown in the figure is _____ (upto 2 deimal places).



Ans. 90.09

Sol. KVL @ input loop,

$10.7 - 10\,ki_b - 0.7 - 1\,ki_e = 0$

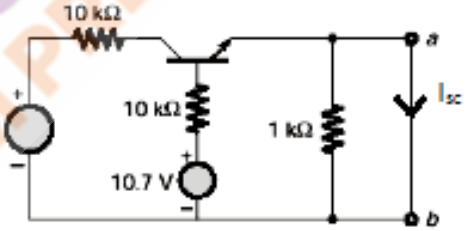
$10.7 - 10\,ki_b - 0.7 - 1k(1+\beta)i_b = 0$

$10 = 111i_b$

$$i_b=\frac{10}{100}\text{ mA}$$

$$V_{ab}=1k\times i_e=k(1+\beta)i_b=\frac{1010}{111}\text{V}$$

For short circuit condition



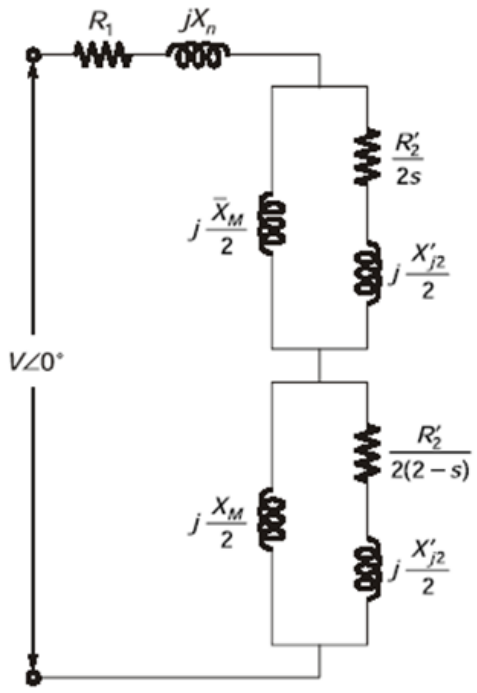
$10.7 - 10\,ki_b - 0.7 = 0$

i_b = 1mA

$I_{SC} = i_e = (1+\beta)i_b = 101 \times 1 = 101$ mA

$$R_{Th}=\frac{V_{ab}}{I_{SC}}=\frac{1010}{111\times101\text{m}}=\frac{1010\times1000}{111\times101}=90.09\,\Omega$$

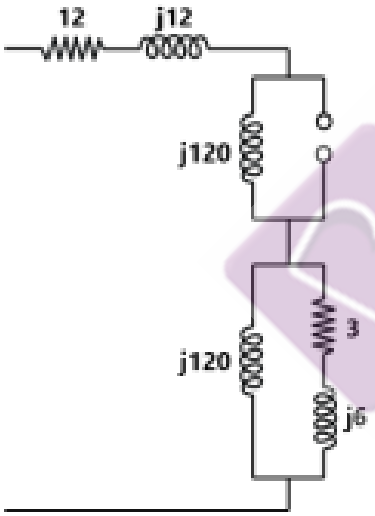
46. The equivalent circuit of a single-phase induction motor is shown in the figure, where the parameters are R_1 = R_2 = X_{l1} = X_{l2} = 12 Ω, X_M = 240 Ω and s is the slip. At no-load, the motor speed can be approximated to be the synchronous speed. The no-load lagging power factor of the motor is _____ (upto 3 decimal places).



Ans. 0.1

Sol. At synchronous speed s=0

So, the equivalent circuit will be



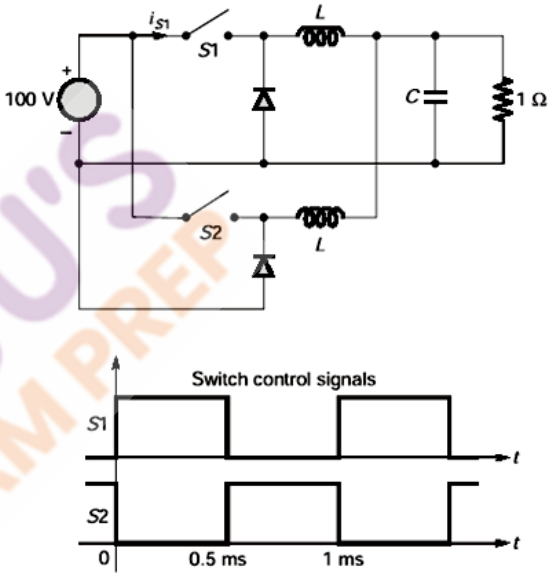
$$Z_{eq} = \frac{(3+j6)(j120)}{(3+j126)} + (12+j132)$$

= (14.72 + 137.78) Ω = 138.86∠ 83.9°Ω

Impedance angle will be p.f. angle

∴ No-load lagging p.f. of the motor is cos 83.9° =0.1

47. The figure shows two buck converters connected in parallel. The common input dc voltage for the converters has a value of 100 V. The converters have inductors of identical value. The load resistance is 1Ω. The capacitor voltage has negligible ripple. Both converters operate in the continuous conduction mode. The switching frequency is 1 kHz, and the switch control signals are as shown. The circuit operates in the steady-state. Assuming that the converters share the load equally, the average value of i_{s1}, the current of switch S1 (in ampere) is _____ (upto 2 decimal places).



Ans. 12.5

Sol. $V_0 = DV_s = 0.5 \times 100 = 50V$

$$I_0 = \frac{V_0}{R}$$

$P_0 = V_0I_0 = 2500W$

By power conversion $V_SI_S(avg) = V_0I_0$

$$I_S(avg) = \frac{2500}{100} = 25A$$

Since both the converter share equal current

I_S (avg) = 12.5 A

48. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3 × 3 identity matrix. The determinant of *B* is _____ (upto 1 decimal place).

Ans. 1

Sol.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$(1 - \lambda)((2 - \lambda)(-2 - \lambda)) - 1(0 - 0) = 0$

$\lambda = 1, 2, -2$

Eigen values of A are 1,2, –2

Eigne values of A^3 are 1, 8, –8

A^2 are 1, 4, 4

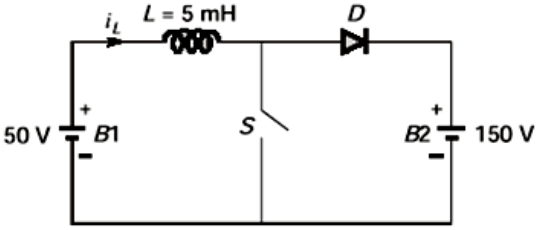
4A are 4, 8, –8

5I are 5, 5, 5

$A^3 - A^2 - 4A + 5I$ are 1, 1, 1

$\therefore |B| = (1)(1)(1) = 1$

49. A dc to dc converter shown in the figure is charging a battery bank, B2 whose voltage is constant at 150 V. B1 is another batter bank whose voltage is constant at 50 V. The value of the inductor. Is 5 mH and the ideal switch, is operated with a switching frequency of 5 kHz with a duty ratio of 0.Once the circuit has attained steady state and assuming the diode to be ideal, the power transferred from to (in Watt) is _____ (upto 2 decimal places).

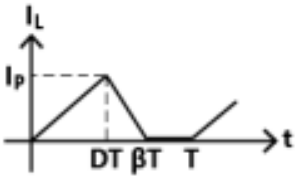


Ans. 12

Sol. For continuous conduction:

$$V_0 = \frac{V_s}{1-D} = \frac{50}{1-0.4} = \frac{50}{0.6} = 83.33V$$

Since $V_0 < 150$, it is discontinuous mode



When S ON

$$V_s = L\frac{dI_L}{dt} \Rightarrow \frac{V_s}{L} = \frac{dI_L}{dt}$$

$$I_P = \frac{V_s}{L} \times DT \quad \text{..... (i)}$$

When S OFF

$$V_s - V_0 = L\frac{dI_L}{dt} \Rightarrow \frac{V_s - V_0}{L} = \frac{dI_L}{dt}$$

$$I_P = \frac{V_s - V_0}{L} \times (D - \beta)T \quad \text{..... (ii)}$$

From (i) and (ii)

$$\frac{V_s}{L} \times DT = \frac{V_s - V_0}{L} \times (D - \beta)T$$

Solving, we get

$$V_0 = \frac{\beta}{\beta - D}$$

$$150 = \frac{\beta}{\beta - 0.4} \times 50$$

$\beta = 0.6$

The power is transferred to 150V during DT to βT

From (i) $I_P = \frac{50}{5 \times 10^{-3}} \times 0.4 \times \frac{1}{5 \times 10^3} = 0.8A$

Power transferred to 150V source

$$P = \frac{1}{T} \times \frac{1}{2} \times I_P \times V_0 \times (\beta - D)T$$

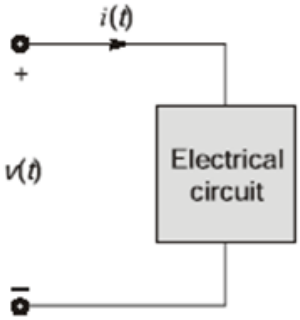
$$= \frac{1}{2} \times 0.8 \times 150 \times (0.6 - 0.4) = 12W$$

50. The voltage across the circuit in the figure and the current through it, are given by the following expressions:

$$v(t) = 5 - 10\cos(\omega t + 60°)V$$

$$i(t) = 5 + X\cos(\omega t)A$$

where, $\omega = 100\pi$ radian/s. If the average power delivered to the circuit is zero, then the value of X(in ampere) is _____ (upto 2 decimal places).



Ans. 10

Sol. For power calculation same frequency terms must be used in both voltage and current.

$$P = V_0 I_0 + \frac{1}{2} V_1 I_1 \cos\left(\theta_v - \theta_i\right)$$

$$V_1 = -10\cos\left(\omega t + 60^\circ\right) = 10\cos\left(\omega t - 120^\circ\right)$$

$$I_1 = X\cos\left(\omega t\right)$$

$$P = 5\times 5 + \frac{1}{2}\times 10 \times X\cos\left(120\right) = 0$$

$$25 + 5X\cos\left(120\right) = 0$$

$$X = 10$$

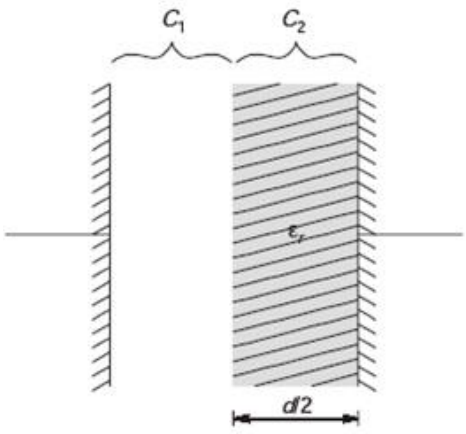
51. The capacitance of an air-filled parallel-plate capacitor is 60 pF. When a dielectric slab whose thickness is half the distance between the plates, is placed on one of the plates covering it entirely, the capacitance becomes 86 pF. Neglecting the fringing effects, the relative permittivity of the dielectric is _____ (upto 2 decimal places).

Ans. 2.53

Sol. Given :

$$C = \frac{\varepsilon_0 A}{d} = 60\text{pF}$$

In second case:



Capacitance, $C_1 = \dfrac{\varepsilon_0 A}{d/2}$

$$= \frac{2\varepsilon_0 A}{d} 2\times(60\text{pF}) = 120\text{pF}$$

and $C_2 = \dfrac{2\varepsilon_0\varepsilon_r A}{d} = (2\times 60)\varepsilon_r\text{pF} = 120\varepsilon_r\text{pF}$

Now,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{120\times 120\varepsilon_r}{(120 + 120\varepsilon_r)}\text{pF} = 86\text{pF}$$ (given)

or,

$$\frac{86}{120} = \frac{\varepsilon_r}{1+\varepsilon_r}$$

or, $\varepsilon_r = \dfrac{86}{34} = 2.53$

52. The signal energy of the continuous-time signal

$$x(t) = \left[(t-1)u(t-1)\right] - \left[(t-2)u(t-2)\right] - \left[(t-3)u(t-3)\right] + \left[(t-4)u(t-4)\right]$$

is

A. $\frac{11}{3}$ B. $\frac{7}{3}$

C. $\frac{1}{3}$ D. $\frac{5}{3}$

Ans. D

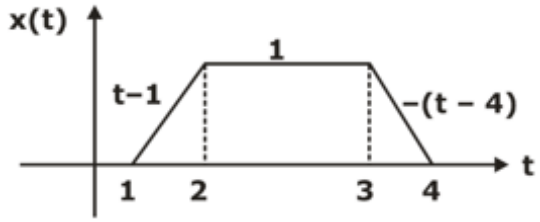
Sol.

$$x(t) = r(t-1) - r(t-2) - r(t-3) + r(t-4)$$

Graph of x(t) is shown below,

Energy of signal x(t)

$$E_x(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$$



$$= \int_1^2 (t-1)^2 dt + \int_2^3 t^2 dt + \int_3^4 \{-(t-4)\}^2 dt$$

$$\left[\frac{(t-1)^3}{3}\right]_1^2 + [t]_2^3 + \left[\frac{(t-4)^3}{3}\right]_3^4$$

$$= \frac{1}{3} + 1 + \frac{1}{3} = \frac{2}{3} + 1 = \frac{5}{3}$$

53. Consider the two continuous-time signals defined below:

$$x_1(t) = \begin{cases} |t|, -1 \le t \le 1 \\ 0, \text{ otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} 1-, |t|, -1 \le t \le 1 \\ 0, \text{ otherwise} \end{cases}$$

These signals are sampled with a sampling period of T = 0.25 seconds to obtain discrete-time signals $X_1[n]$ and $x_2[n]$, respectively. Which one of the following statements is true?

A. The energy of $x_1[n]$ is greater than the energy of $x_2[n]$.

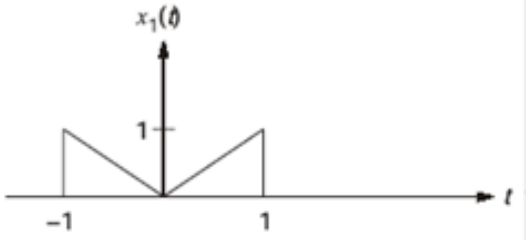
B. The energy of $x_2[n]$ is greater than the energy of $x_1[n]$.

C. $x_1[n]$ and $x_2[n]$ have equal energies.

D. Neither $x_1[n]$ nor $x_2[n]$ is a finite energy signal.

Ans. A

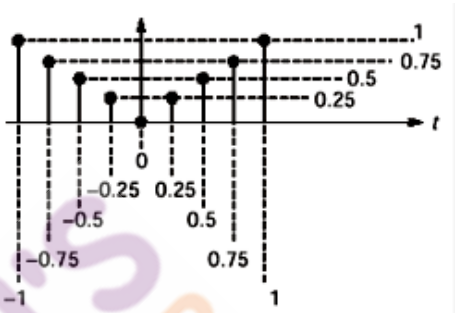
Sol. $x_1(t) = \begin{cases} |t|, -1 \le t \le 1 \\ 0, \text{ otherwise} \end{cases}$



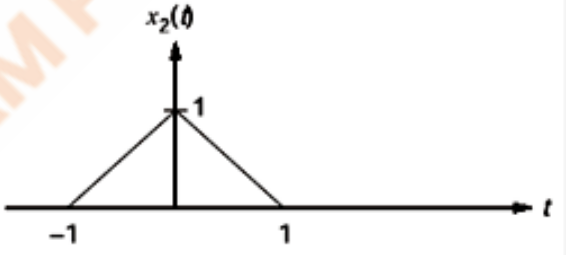
T_S = sampling time-period

= 0.25 sec

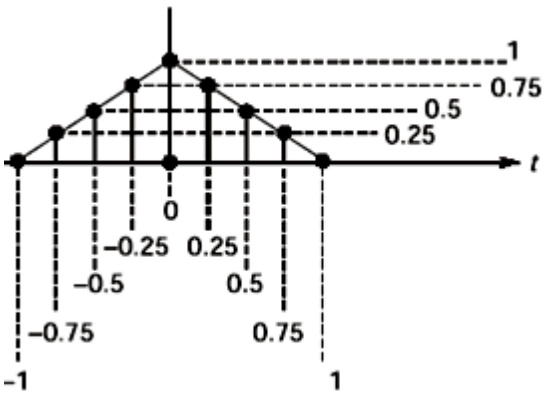
$X_1(n)$ = {1, 0.75, 0.5, 0.25, 0.25, 0.5, 0.75, 1}



Now, $x_2(t) = \begin{cases} 1-|t|, -1 \le t \le 1 \\ 0, \text{ otherwise} \end{cases}$

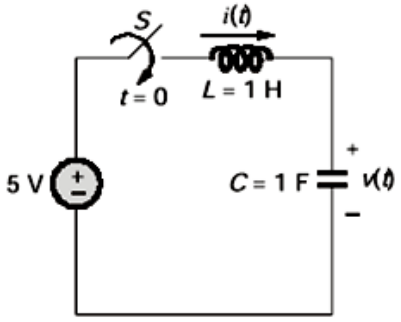


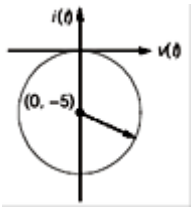
$X_2(n)$ = {0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0}

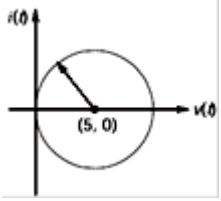


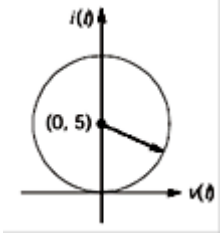
Since $X_1(n)$ is having one more non-zero sample of amplitude '1' as compared to $X_2(n)$. Therefore, energy of $X_1(n)$ is greater than energy of $X_2(n)$.

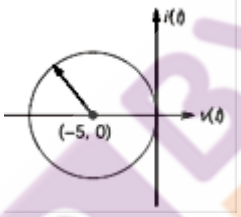
54. A DC voltage source is connected to a series L-C circuit by turning on the switch S at time t = 0 as shown in the figure. Assume i(0) = 0, v(0) = . Which one of the following circular loci represents the plot of i(t) versus v(t)?



A. 

B. 

C. 

D. 

Ans. B

Sol. $i = V_s\sqrt{\frac{C}{L}}\sin\omega t$ where, $\omega = \frac{1}{\sqrt{LC}}$

So, I = 5sint

$i = C\frac{dV}{dt}$

$v = \frac{1}{C}\int_0^t i\,dt = \int_0^t 5\sin t\,dt = 5(1-\cos t)$

= 5 sin t

and $v = 5 - 5\cos t \Rightarrow (5-v) = 5\cos t$

Squaring and adding

$(5-v)^2 + i^2 = 25$

Or, $(v-5)^2 + i^2 = 25$

This is the equation of circle with center (5,0)

55. A 0-1 ampere moving iron ammeter has an internal resistance of 50 mW and inductance of 0.1 mH. A shunt coil is connected to extend its range to 0-10 Ampere for all operating frequencies. The time constant (in milliseconds) and resistance in (mW)of the shunt coil respectively are

A. 2 & 5.55 B. 2 & 1

C. 2.18 & 0.55 D. 11.1 & 2

Ans. A

Sol. For all frequencies time constants of the shunt and meter arm should be equal.

i.e., $\frac{\omega L_m}{R_m} = \frac{\omega L_{sh}}{R_m}$

or, $\frac{L_m}{R_m} = \frac{L_{sh}}{R_{sh}}$

or, $\frac{L_m}{R_m} = \frac{0.1\times10^{-3}}{50\times10^{-3}} = 0.002$

and given, I_m = 1A, Rm = 50 mW

L_m = 0.1 mH, I = 10 A

We know, $R_{sh} = \frac{R_m}{(m-1)}; m = \frac{I}{I_m}$

Here, $\frac{10}{1} = 10$

$R_{sh} = \frac{50\times10^{-3}}{10-1} = \frac{50\times10^{-3}}{9} = 5.55\,m\Omega$

∴ Option (A) is correct.

56. The per unit power output of a salient pole generator which is connected to an infinite bus, is given by the expression, p =1.4 sind + 0.15 sin 2d where is the load angle. Newton-Raphson method is used to calculate the value of d for P = 0.8pu. If the initial guess is 30°, then its value (in degree) at the end of the first iteration is

A. 15° B. 28.48°

C. 28.74° D. 31.20°

Ans. C

Sol.

$$P(\delta) = 1.4\sin\delta + 0.15\sin 2\delta = 0.8$$

$$= 1.4\sin\delta + 0.15\sin 2\delta - 0.8 = 0$$

$$P'(\delta) = \frac{d}{d\delta}(P(\delta))$$

$$= 1.4\cos\delta + 0.30\cos 2\delta$$

Given, $\delta_0 = 30°$

By using Newton Raphson method

$$\Delta\delta = \delta_1 - \delta_0 = \frac{f(\delta_0)}{f'(\delta_0)} = -\frac{P(\delta_0)}{P'(\delta_0)}$$

$$\delta_1 - 30° = \frac{1.4\sin 30° + 0.15\sin 60° - 0.8}{1.4\cos 30° + 0.3\cos 60°}$$

$$\delta_1 - 30° = -0.0219\,\text{rad} = -1.26°$$

$$\delta_1 = 28.74°$$

57. If C is a circle |z| = 4 and $f(z) = \frac{z^2}{(z^2 - 3z + 2)^2}$, then $\oint_C f(z)\,dz$ is

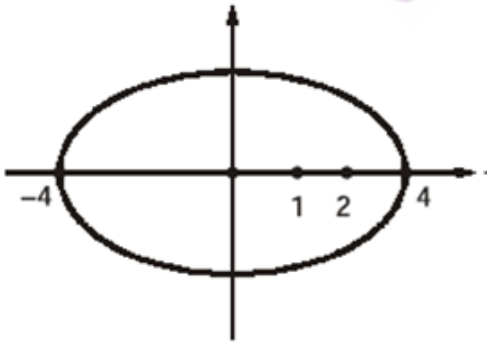
A. 1 B. 0

C. -1 D. -2

Ans. B

Sol. $\int \frac{z^2}{(z^2 - 3z + 2)}\,dz$

$$\int \frac{z^2}{(z-1)(z-2)^2}\,dz$$



$$\underset{z-1}{Res}\,f(z) = \underset{z\to 1}{Lt}\frac{1}{1!}\frac{d}{dz}\left((z-1)^2 \cdot \frac{z^2}{(z-1)^2(z-2)^2}\right)$$

$$= \underset{z\to 1}{Lt}\left(\frac{2z(z-2)^2 - 2z(z-2)}{(z-1)^4}\right)$$

$$= \underset{z\to 1}{Lt}\left(\frac{2z(z-2) - 2z^2}{(z-2)^3}\right)$$

$$= \frac{-4}{-1} = 4$$

$$\underset{z-2}{Res}\,f(z) = \underset{z\to 2}{Lt}\frac{1}{1!}\frac{d}{dz}\left((z-2)^2 \cdot \frac{z^2}{(z-1)^2(z-2)^2}\right)$$

$$= \underset{z\to 2}{Lt}\left(\frac{(z-1)^2 \cdot 2z - z^2(z-1)}{(z-1)^4}\right)$$

$$= \underset{z\to 2}{Lt}\left(\frac{2z(z-1) - 2z^2}{(z-1)^3}\right)$$

$$= \frac{4-8}{1} = -4$$

By residue theorem, $I = 2\pi i(4-4) = 0$

58. The positive, negative and zero sequence impedances of a three-phase generator are Z_1,Z_2 and Z_0 respectively. For a line-to-line fault with fault impedance Z_f, the fault current is $I_{f1} = K_{If}$, where I_f is the fault current with zero fault impedance. The relation between Z_f and k is

A. $Z_f = \frac{(Z_1 + Z_2)(1-k)}{k}$

B. $Z_f = \frac{(Z_1 + Z_2)(1+k)}{k}$

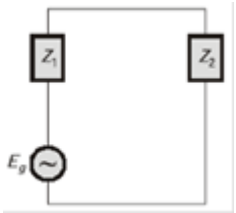
C. $Z_f = \frac{(Z_1 + Z_2)k}{1-k}$

D. $Z_f = \frac{(Z_1 + Z_2)k}{1+k}$

Ans. A

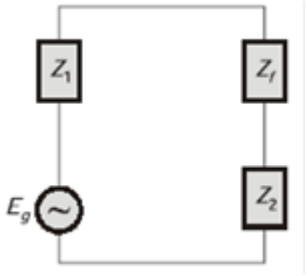
Sol. For LL fault:

Without Z_f,



$$I_f = \frac{\sqrt{3}E_0}{Z_1 + Z_2}$$

With Z_f,



$$I_{f1} = \frac{\sqrt{3}E_0}{Z_1 + Z_2 + Z_f}$$

Given, $I_{f1} = k \cdot I_f$

$$\frac{\sqrt{3} \cdot E_0}{Z_1 + Z_2 + Z_f} = \left(\frac{\sqrt{3}E_0}{Z_1 + Z_2}\right)k$$

$Z_1 + Z_2 = k(Z_1 + Z_2 + Z_f)$

$$Z_f = \frac{(Z_1 + Z_2)(1-k)}{k}$$

59. Consider the two bus power system network with given loads as shown in the figure. All the values shown in the figure are in per unit. The reactive power supplied by generator and are and respectively. The per unit values of and line reactive power loss respectively are

A. 5.00, 12.68, 2.68

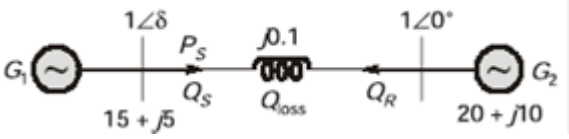
B. 6.34, 10.00, 1.34

C. 6.34, 11.24, 2.68

D. 5.00, 11.34, 1.34

Ans. C

Sol.



At G_2 load demand is 20 pu, G_2 supplies only 15. Remaining supplied by G_1 through transmission line.

$$P_s = \left|\frac{V_S V_R}{X_L}\right| \sin\delta$$

$$5 = \frac{1 \times 1}{0.1}\sin\delta$$

$$\delta = 30°$$

$$Q_S = \frac{V_S^2}{X_L} - \frac{V_S V_R}{X_L}\cos\delta$$

$$= \frac{1^2}{0.1} - \frac{1 \times 1}{0.1}\cos 30° = 1.34\,\text{p.u.}$$

$$Q_R = \left|\frac{V_S V_R}{X_L}\right|\cos\delta - \left|\frac{V_R^2}{X_L}\right|$$

$$= \left|\frac{1 \times 1}{0.1}\right|\cos 30° - \frac{1^2}{0.1} = -1.34\,\text{p.u.}$$

$$Q_{loss} = Q_S - Q_R$$

$$= 1.34 - (-1.34) = 2.68\,\text{p.u.}$$

At G_1:

$Q_{G1} = Q_{loss} + Q_S$

$= 5 + 1.34 = 6.34$ p.u.

At G_2:

$Q_{G2} = Q_{loss} + (-Q_R)$

$= 10 - (-1.34) = 11.34$ p.u.

60. Consider a system governed by the following equations:

$$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

The initial conditions are such that $x_1(0) < x_2(0) < \infty$. Let $x_{1f} = \lim_{t\to\infty} x_1(t)$ and $x_{2f} = \lim_{t\to\infty} x_2(t)$. Which one of the following is true?

A. $x_{1f} < x_{2f} < \infty$ B. $x_{2f} < x_{1f} < \infty$

C. $x_{2f} = x_{1f} < \infty$ D. $x_{2f} = x_{1f} = \infty$

Ans. C

Sol. Converting to state model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$A=\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$$

$$(SI-A)=\begin{bmatrix}S+1 & -1\\ 1 & S+1\end{bmatrix}$$

$$(SI-A)^{-1}=\frac{1}{S^2+2S}\begin{bmatrix}S+1 & -1\\ 1 & S+1\end{bmatrix}$$

$$(SI-A)^{-1}=\begin{bmatrix}\frac{S+1}{S(S+2)} & \frac{1}{S(S+2)}\\ \frac{1}{S(S+2)} & \frac{S+1}{S(S+2)}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2S}+\frac{1}{2(S+2)} & \frac{1}{2S}-\frac{1}{2(S+2)}\\ \frac{1}{2S}-\frac{1}{2(S+2)} & \frac{1}{2S}-\frac{1}{2(S+2)}\end{bmatrix}$$

$$\phi(t)=\frac{1}{2}\begin{bmatrix}1+e^{-1t} & 1-e^{-2t}\\ 1-e^{-2t} & 1+e^{-2t}\end{bmatrix}$$

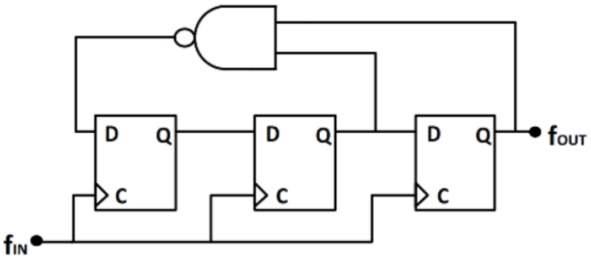
$$x(t)=\phi(t)x(0)=\frac{1}{2}\begin{bmatrix}1+e^{-2t} & 1-e^{-2t}\\ 1-e^{-2t} & 1+e^{-2t}\end{bmatrix}\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}x_1(0)+x_1(0)e^{-2t}+x_2(0)-x_2(0)e^{-2t}\\ x_1(0)-x_1(0)e^{-2t}+x_2(0)+x_2(0)e^{-2t}\end{bmatrix}$$

$$x_f=\lim_{t\to\infty}x(t)=\frac{1}{2}\begin{bmatrix}x_1(0)+x_2(0)\\ x_1(0)+x_2(0)\end{bmatrix}$$

Hence, $x_{1f}=x2f<\infty$

61. Which one of the following statements is true about the digital circuit shown in the figure?



A. It can be used for dividing the input frequency by 3.

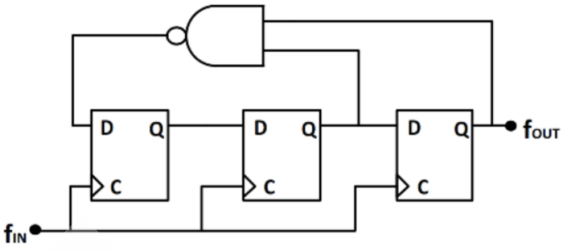
B. It can be used for dividing input frequency by 5.

C. It can be used for dividing the input frequency by 7.

D. It cannot be reliably used as a frequency divider due to disjoint internal cycles.

Ans. B

Sol.



$Q_B Q_C$	Q_A	Q_B			
D_A	D_B	D_C	Q_A	Q_B	Q_C
			0	0	0
1	0	0	1	0	0
1	1	0	1	1	0
1	1	1	1	1	1
0	1	1	0	1	1
0	0	1	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0

Cycle

Repeated

mod 5 counter

So, the frequency will be divided by 5.

62. Digital input signals A,B,C with A as the MSB and C as the LSB are used to realize the Boolean function f=m0+m2+m3+m5+m7 where m_i denotes the i^{th} minterm. In addition, F has a don't care for m_i .The simplified expression for F is given by,

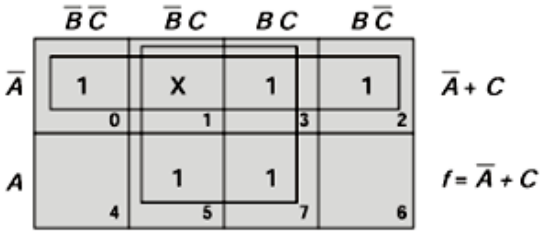
A. $\bar{A}\bar{C}+BC+AC$ B. $\bar{A}+C$

C. $\bar{C}+A$ D. $\bar{A}C+BC+A\bar{C}$

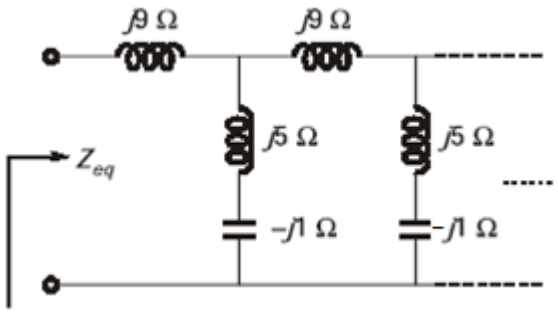
Ans. B

Sol. Given, f=m0+m2+m3+m5+m7

and m_1=don't care condition



63. The equivalent impedance Z_{eq} for the infinite ladder circuit shown in the figure is

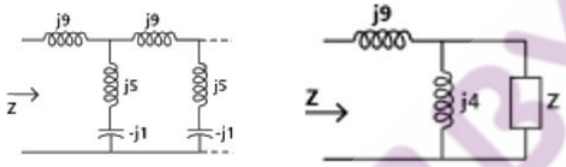


A. j12W B. –j12W

C. j13W D. 13W

Ans. A

Sol.



Assume equivalent impedance = Z

$$Z = j9 + \frac{Z(j4)}{Z + j4}$$

$$Z = \frac{j9(Z + j4) + j4Z}{Z + j4}$$

$Z^2 + j4Z = j9Z - 36 + j4Z$

$Z^2 - j9Z + 36 = 0$

$(Z - j12)(Z + j3) = 0$

$Z = j12$

64. The number of roots of polynomial, $S^7 + S^6 + 7S^5 + 14S^4 + 31S^3 + 73S^2 + 25S + 200$, in the open left half of the complex plane is

A. 3 B. 4

C. 5 D. 6

Ans. A

Sol. Characteristic equation, $S^7 + S^6 + 7S^5 + 14S^4 + 31S^3 + 73S^2 + 25S + 200$,

s^7	1	7	31	25
s^6	1	14	73	200
s^5	–7	–42	–175	0
s^4	8	48	200	0
s^3	32	96	0	0
s^2	24	200	0	0
s^1	–170	0	0	0
s^0	200	0	0	0

As all elements of the fifth row is zero.

Auxiliary equation, $A(S) = 8S^4 + 48S^2 + 200$

$$\frac{d}{ds}A(S) = 32S^3 + 96S$$

Roots of auxiliary equation are = -3 + j4, -3 - j4

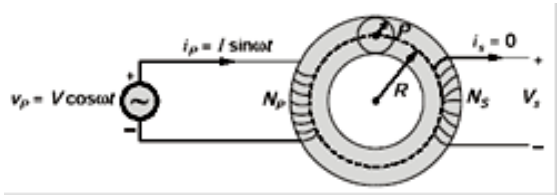
Total no of poles = 7

Two sign change above auxiliary equation = 2 poles in RHS

Two sign changes below auxiliary equation implies out of 4 symmetric roots about origin two poles are in LHS and two poles are in RHS.

∴ 3 poles in LHS and 4 poles in RHS.

65. A transformer with tororidal core of permeability μ is shown in the figure. Assuming uniform flux density across the circular core cross-section of radius r<<R, and neglecting any leakage flux, the best estimate for the mean radius R is

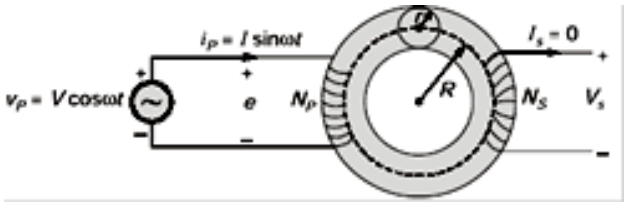


A. $\frac{\mu V r^2 N_p^2 \omega}{I}$ B. $\frac{\mu I r^2 N_p N_S \omega}{V}$

C. $\frac{\mu I r^2 N_p^2 \omega}{2I}$ D. $\frac{\mu I r^2 N_p^2 \omega}{2V}$

Ans. D

Sol.



$$v_p = N_P \frac{d\phi}{dt} = N_P \frac{d}{dt}\left(\frac{N_p i_p}{R_l}\right) R_l = \text{reluctance}$$

$$v_P = \frac{N_p^2 d\left(I\sin\omega t\right)}{dt} = \frac{N_P^2}{R_l}\omega I\cos\omega t = V\cos\omega t$$

$$V = \frac{N_p^2\omega I}{R_l} = \frac{N_p^2\omega I}{l} = \frac{\mu N_p^2\omega IA}{l} = \frac{\mu N_p^2\omega I\pi r^2}{2\pi R}$$

$$\text{Hence, } R = \frac{\mu I r^2 N_p^2\omega}{2V}$$

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