Electrical Engineering

GATE 2019

Questions with Solutions
1. Newspapers are a constant source of delight and recreation for me. The ________ trouble is that I read _______ many of them
A. even, too   B. even, quite   C. only, quite   D. only, too
Ans. D
Sol. Newspapers are a constant source of delight and recreation for me. The only (what bother’s) trouble is that I read too (a lot/large) many of them.

2. The missing number in the given sequence 343, 1331, ______, 4913 is
A. 2744   B. 2197   C. 4096   D. 3375
Ans. B
Sol. 343 = 7³
1331 = 11³
4913 = 17³
All numbers given are cube of prime numbers so 1³ = 2917 satisfy the missing number.

3. The passengers were angry _______ the airline staff about the delay.
A. towards   B. On   C. with   D. about
Ans. C
Sol. The passengers were angry with the airline staff about the delay.

4. It takes two hours for a person X to mow the lawn. Y can move the same lawn in four hours. How long (in minutes) will it take X and Y, if they work together to move the lawn?
A. 120   B. 80   C. 60   D. 90
Ans. B
Sol. Time taken by X to now the lawn = 2 hrs.
∴ Work done by X in 1 hr = \( \frac{1}{2} \)
Similarly,

Work done by 4 in hr = \( \frac{1}{4} \)
Work done by x + 4 in 1 hr = \( \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \)
∴ Total time taken by X & 4 together = \( \frac{4}{3} \) hours
= \( \frac{4}{3} \times 60 \) minutes
= 80 Minutes

5. I am not sure if the bus that has been booked will be able to ____ all the students.
A. sit   B. deteriorate   C. accommodate   D. fill
Ans. C
Sol. I am not sure if the bus that has been booked will be able to accommodate (occupy) all the students.

6. Given two sets X = \{1, 2, 3\} and Y = \{2, 3, 4\}, we construct a set Z of all possible fractions where the numerators belong to set X and the denominators belong to set Y. The product of element having minimum and maximum values in the set Z is __________.
A. 3/8   B. 1/12   C. 1/8   D. 1/6
Ans. A
Sol. Given that X = \{1, 2, 3\}
4 = \{2, 3, 4\}
\[ Z = \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{3}{3}, \frac{3}{3}, \frac{3}{3}, \frac{3}{3}, \frac{2}{4}, \frac{4}{4}, \frac{2}{3}, \frac{4}{4} \right\} \]
Minimum value in z = \( \frac{1}{4} \)
Maximum value in z = \( \frac{3}{2} \)
Product = \( \frac{3}{8} \)
7. The ratio of the number of boys and girls who participated in an examination is 4:3. The total percentage of candidates who passed the examination is 80 and the percentage of girls who passed is 90. The percentage of boys who passed is __________ 
A. 80.50 
B. 55.50 
C. 72.50 
D. 90.00 
Ans. C 
Sol. The correct statement can be concluded from Venn diagram or using the Sylllogism.

9. How many integers are there between 100 and 1000 all of whose digits are even? 
A. 100 
B. 60 
C. 90 
D. 80 
Ans. A 
Sol. For all digits of a number which lie between 100 and 1000 are even, 
Unit and tens digits can be filled from the set {0, 2, 4, 6, 8} 
But hundred’s digit does not include 0 as it will not remain a number which lie between 100 and 1000 
∴ Hundreds digit set is {2, 4, 6, 8} 
Total integer be = 5 \ \times 5 \ \times 4 
Total integer = 100 numbers 

10. Consider five people – Mita, Ganga, Rekha, Lakshmi and Sana. Ganga is taller than both Rekha and Lakshmi. Lakshmi is taller than Sana. Mita is taller than Ganga. 
Which of the following conclusions are true? 
1. Lakshmi is taller than Rekha 
2. Rekha is shorter than Mita 
3. Rekha is taller than Sana 
4. Sana is shorter than Ganga 
A. 1 and 3 
B. 1 only 
C. 2 and 4 
D. 3 only 
Ans. C 
Sol. Given that 
Ganga > Rekha, Lakshmi 
Lakshmi > Sana 
Mita > Ganga 
∴ Mita > Ganga > Rekha, Lakshmi > Sana 
∴ 2 and statement 4 are correct
1. The mean-square of a zero-mean random process is \( \frac{kT}{C} \), where \( k \) is Boltzmann’s constant, \( T \) is the absolute temperature, and \( C \) is capacitance. The standard deviation of the random process is
   A. \( \sqrt{\frac{kT}{C}} \)  
   B. \( \frac{kT}{C} \)  
   C. \( \frac{1}{\sqrt{C}} \)  
   D. \( \frac{c}{kT} \)

**Ans. C**

**Sol.** Given that

Mean square of random process \( = E(x^2) = \frac{kt}{C} \)

Mean is given zero \( \Rightarrow E(x) = 0 \)

We know that \( E(x^2) - [E(x)]^2 = \text{variance} \)

Variance \( = \frac{kT}{C} \)

Standard deviation \( = \sqrt{\text{variance}} = \sqrt{\frac{kT}{C}} \)

2. The characteristic equation of a linear time-invariant (LTI) system is given by
   \( \Delta(s) = s^4 + 3s^3 + 3s^2 + s + k = 0 \).
   The system is BIBO stable if
   A. \( 0 < k < \frac{12}{9} \)  
   B. \( 0 < k < \frac{8}{9} \)  
   C. \( k > 6 \)  
   D. \( k > 3 \)

**Ans. B**

**Sol.** Applying R.H criteria for stability

\[ \Delta(S) = S^4 + 3S^3 + 3S^2 + S + K = 0 \]

For stability, first column should be greater than zero

\[ \frac{8}{3} - \frac{3K}{8/3} > 0 \quad \text{and} \quad k > 0 \]

\( \therefore 0 < K < \frac{8}{9} \)

3. The inverse Laplace transform of \( H(s) = \frac{s + 3}{s^2 + 2s + 1} \) for \( t \geq 0 \) is
   A. \( 2te^{-t} + e^{-t} \)  
   B. \( 3te^{-t} + e^{-t} \)  
   C. \( 3e^{-t} \)  
   D. \( 4te^{-t} + e^{-t} \)

**Ans. A**

**Sol.**

\[ H(s) = \frac{s + 3}{s^2 + 2s + 1} \]

\[ H(t) = L^{-1} \left[ H(S) \right] \]

\[ = L^{-1} \left[ \frac{s + 3}{(s + 1)^2} \right] = L^{-1} \left[ \frac{s + 3}{(s + 1)^2} \right] = L^{-1} \left[ \frac{1}{s + 1} \right] + L^{-1} \left[ \frac{2}{(s + 1)^2} \right] \]

\[ H(t) = e^{-t} + 2te^{-t} \]

4. A 5 kVA, 50 V/100 V, single-phase transformer has a secondary terminal voltage of 95 V when loaded. The regulation of the transformer is
   A. 9\%  
   B. 4.5\%  
   C. 1\%  
   D. 5\%

**Ans. D**

**Sol.** We know that

\[ \text{Voltage Regulation} = \frac{V_{\text{NL}} - V_{\text{FL}}}{V_{\text{NL}}} \times 100 \]

Given that \( V_{\text{FL}} = 95 \text{V} \)

\( V_{\text{NL}} = 100 \text{V} \)

\[ \% \text{VR} = \frac{100 - 95}{100} \times 100 = 5\% \]
5. A three-phase synchronous motor draws 200 A from the line at unity power factor at rated load. Considering the same line voltage and load, the line current at power factor of 0.5 leading is
A. 400 A  B. 300 A  C. 200 A  D. 100 A
Ans. A
Sol. We know that \( P = VI \cos \phi \), as load and voltage are same
\[ \therefore I \cos \phi = \text{constant} \]
\[ I_1 \cos \phi_1 = I_2 \cos \phi_2 \]
\[ I_1 = 200A \]
\[ \cos \phi_1 = 1 \]
\[ \cos \phi_2 = 0.5 \]
\[ I_2 = \frac{I_1 \cos \phi_1}{\cos \phi_2} = 400A \]

6. A cv-axial cylindrical capacitor shown in Figure (i) has dielectric with relative permittivity \( \varepsilon_r_1 = 2 \). When one-fourth portion of the dielectric is replaced with another dielectric of relative permittivity \( \varepsilon_r_2 \), as shown to Figure (ii), the capacitance is doubled. The value of \( \varepsilon_r_2 \) is _________.

\[ C_1 = \frac{4\pi \varepsilon_0}{\ln \left( \frac{R}{r} \right)} \]

Total portion covers \[ \frac{2\pi}{4} = \frac{\pi}{2} \]
\[ \frac{\pi}{2} \text{ length for } \varepsilon_r_1 \]
and \[ \frac{3\pi}{2} \text{ length for } \varepsilon_r_1 \]
Both are connected in parallel

\[ C_2 = C_{r1} + C_{r2} \]
\[ = \frac{2\pi(2 \varepsilon_0)}{\ln \left( \frac{R}{r} \right)} \times \frac{3\pi}{2} + \frac{2\pi(\varepsilon_{p2} \varepsilon_0)}{\ln \left( \frac{R}{r} \right)} \times \frac{\pi}{2} \]
\[ = \frac{\pi \varepsilon_0}{\ln \left( \frac{R}{r} \right)} \left[ 3 + \frac{\varepsilon_{p2}}{2} \right] \]

Given \( C_2 = 2C_1 \)
\[ \frac{\pi \varepsilon_0}{\ln \left( \frac{R}{r} \right)} \left[ 3 + \frac{\varepsilon_{p2}}{2} \right] = 2 \left( \frac{4\pi E_o}{\ln \left( \frac{R}{r} \right)} \right) \]
\[ \varepsilon_{p2} = 10 \]
7. The parameter of an equivalent circuit of a three-phase induction motor affected by reducing the rms value of the supply voltage at the rated frequency is
A. rotor leakage reactance
B. stator resistance
C. rotor resistance
D. magnetizing reactance

Ans. D
Sol.

By reducing the rms value of supply voltage at rated frequency, magnetizing current changes which changes the magnetizing reactance

8. The output response of a system is denoted as \( y(t) \), and its Laplace transform is given by

\[
Y(s) = \frac{10}{s(s^2 + s + 100\sqrt{2})}. 
\]

The steady state value of \( y(t) \) is
A. \( 10\sqrt{2} \)
B. \( 100\sqrt{2} \)
C. \( \frac{1}{10\sqrt{2}} \)
D. \( \frac{1}{100\sqrt{2}} \)

Ans. C
Sol.

For finding steady state value, we will apply final value theorem

\[
\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)
\]

\[
y(\infty) = \lim_{s \to 0} \frac{10}{s^2 + s + 100\sqrt{2}}
\]

\[
y(\infty) = \frac{1}{10\sqrt{2}}
\]

9. The open loop transfer function of a unity feedback system is given by

\[
G(s) = \frac{\pi e^{-0.25s}}{s}.
\]

In \( G(s) \) plane, the Nyquist plot of \( G(s) \) passes through the negative real axis at the point
A. \((-0.5, j0)\)
B. \((-1.5, j0)\)
C. \((-1.25, j0)\)
D. \((-0.75, j0)\)

Ans. A
Sol.

Nyquist plot cut the negative real Axis at \( \omega = \) phase cross over frequency

\[
G(j\omega) = \frac{\pi e^{-0.25j\omega}}{j\omega}
\]

\[
= -90^\circ - 0.25\omega \times \frac{180^\circ}{\pi}
\]

\[
\angle G(j\omega) \mid_{\omega=\omega_{pc}} = -180^\circ
\]

\[
\omega_{pc} = -90^\circ - 0.25\omega_{pc} \times \frac{180^\circ}{\pi} = -180^\circ
\]

\[
90^\circ = \omega_{pc} \left( \frac{45^\circ}{\pi} \right)
\]

\[
\omega_{pc} = 2\pi
\]

Magnitude at cutting point

\[
X = |G(j\omega)|_{\omega_{pc}}
\]

\[
= \frac{\pi}{\omega_{pc}} = \frac{\pi}{2\pi} = \frac{1}{2}
\]

Then, the co-ordinates becomes \((-0.5, j0)\).
10. A current controlled current source (CCCS) has an input impedance of 10 Ω and output impedance of 100 KΩ. When this CCCS is used in a negative feedback closed loop with a loop gain of 9, the closed loop output impedance is
A. 100 KΩ  
B. 1000 KΩ  
C. 100 Ω  
D. 10 Ω

Ans. B

Sol. Given $Z_{in} = 10\,\Omega$, $Z_{o/p} = 100\,\Omega$
For CCCS

Series connection is output
$Z_{o/p} = Z_{o/p} (1 + A\beta) = 100 (1 + 9)$
$= 100\,\text{KΩ}$

11. A six-pulse thyristor bridge rectifier is connected to a balanced three-phase, 50 Hz AC source. Assuming that the DC output current of the rectifier is constant, the lowest harmonic component in the AC input current is
A. 100 Hz  
B. 150 Hz  
C. 300 Hz  
D. 250 Hz

Ans. D

Sol. We know that,
For 6-pulse converter harmonic present in AC current are $6K \pm 1$
General expression $NK \pm 1$  
[k = 0, 1, 2, 3]
For 6 pulse $n = 6$
Lowest order harmonic = 5
Lower harmonic frequency = $5 \times 50 = 250\,\text{Hz}$

12. The current $I$ flowing in the circuit shown below in amperes (round off to one decimal place) is ___________.

![Circuit Diagram]

Ans. 1.4

Sol.

Applying nodal analysis at point 1 whose voltage is assumed as $V_1$.
$\frac{V_1 - 20}{2} - 2 + \frac{V_1 - 5I}{3} = 0$  
(1)
$I = \frac{20 - V_1}{2}$  
(2)
Solving (1) and (2)
$-I - 2 + \frac{V_1 - 5I}{3} = 0$
$8I = V_1 - 6$
$8I = 20 - 2I - 6$
$10I = 14$
$I = 1.4\,\text{A}$

13. The partial differential equation

\[ \frac{\partial^2 u}{\partial t^2} - c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0; \text{ Where } c \neq 0 \]

is known as
A. Poisson’s equation  
B. wave equation  
C. Laplace equation  
D. heat equation

Ans. B
**14.** Which one of the following function is analytic in the region |z| ≤ 1?

A. \[ \frac{z^2 - 1}{z + 2} \]

B. \[ \frac{z^2 - 1}{z} \]

C. \[ \frac{z^2 - 1}{z - 0.5} \]

D. \[ \frac{z^2 - 1}{z - j0.5} \]

**Ans.** A

**Sol.** For \( \frac{z^2 - 1}{z + 2} \), the singularity \( z = -2 \) lies outside the \( |Z| \leq 1 \).

\( \therefore \) By Cauchy’s integral theorem

\[ \int \frac{z^2 - 1}{z + 2} \, dz = 0 \text{ for } |z| \leq 1 \]

**15.** If \( f = 2x^3 + 3y^2 + 4z \), the value of line integral \( \int_C \text{grad } f \cdot d\mathbf{r} \) evaluated over contour \( C \) formed by the segments (-3, -3, 2) \( \rightarrow \) (2, -3, 2) \( \rightarrow \) (2, 6, 2) \( \rightarrow \) (1, 6, -1) is ____________.

**Ans.** 139

**Sol.** Given that

\[ y = 2x^3 + 3y^2 + 4z \]

\[ \int_C \text{grad } f \cdot d\mathbf{r} = ? \]

\[ d\mathbf{r} = dx \, \hat{i} + dy \, \hat{j} + dz \, \hat{k} \]

\[ \text{grad } f = \frac{df}{dx} \, \hat{i} + \frac{df}{dy} \, \hat{j} + \frac{df}{dz} \, \hat{k} \]

\[ = 6x^2 \hat{i} + 6y \hat{j} + 4 \hat{k} \]

\[ \int_C \text{grad } f \cdot d\mathbf{r} = \int 6x^2 \, dx + \int 6y \, dy + \int 4 \, dz \]

Applying the limits

\[ \left[ \int \text{grad } f \cdot d\mathbf{r} = \left[ \frac{1}{2} \right] \begin{array}{c} 6x^2 \, dx \end{array} + \left[ \frac{1}{2} \right] \begin{array}{c} 6y \, dy \end{array} + \left[ \frac{1}{2} \right] \begin{array}{c} 4 \, dz \end{array} \right] \]

\[ = \left[ \frac{1}{2} \right] \begin{array}{c} 6x^2 \, dx \end{array} + \left[ \frac{1}{2} \right] \begin{array}{c} 6y \, dy \end{array} + \left[ \frac{1}{2} \right] \begin{array}{c} 4 \, dz \end{array} \]

\[ = \left[ 2x^3 \right]^2_3 + \left[ 3y^2 \right]^2_3 + \left[ 4z \right]^1_2 \]

\[ = 70 + 81 - 12 = 139 \]

**16.** Five alternators each rated 5 MVA, 13.2 kV with 25% of reactance on its own base are connected in parallel to a busbar. The short-circuit level in MVA at the busbar is ____________.

**Ans.** 100

**Sol.**

Net reactance of generator

\[ X = \frac{0.25}{5} = 0.05 \text{p.u.} \]

\[ I_{SC} = \frac{P}{e - \text{fault voltage}} \times X = \frac{1}{0.05} = 20 \text{p.u.} \]

Short Circuit MVA = \( I_{SC} \times \text{Base MVA} = 20 \times 5 = 100 \text{ MVA} \)

**17.** Given \( V_{gs} \) is the gate-source voltage, \( V_{ds} \) is the drain voltage, and \( V_{th} \) is threshold voltage of an enhancement type NMOS transistor, the conditions for transistor to be biased in saturation are

A. \( V_{gs} > V_{th}; V_{ds} \leq V_{gs} - V_{th} \)

B. \( V_{gs} < V_{th}; V_{ds} \geq V_{gs} - V_{th} \)

C. \( V_{gs} > V_{th}; V_{ds} \geq V_{gs} - V_{th} \)

D. \( V_{gs} < V_{th}; V_{ds} \leq V_{gs} - V_{th} \)

**Ans.** C
18. For NMOS transistor to be in saturation the condition will be
\[ V_{GS} > V_{th} \]
And \[ V_{DS} > V_{GS} - V_{th} \]

19. The total impedance of the secondary winding, leads, and burden of a 5 A CT is 0.01 \( \Omega \). If the fault current is 20 times the rated primary current of the CT, the VA output of the CT is __________.

Ans. 100

Sol.
\[ I_{sec} = 5 \times 20 = 100 \text{ A} \]
\[ V = I_{sec} R = 100 \times 0.01 = 1 \text{V} \]

VA output of CT = \( V I_{sec} = 100 \times 1 \) 100 VA

20. The \( Y_{bus} \) matrix of a two-bus power system having two identical parallel lines connected between them in p.u. is given as
\[
Y_{bus} = \begin{bmatrix} -j8 & j20 \\ j20 & -j8 \end{bmatrix}
\]

The magnitude of the series reactance of each line in p.u. (round off up to one decimal place) is ___.

Ans. 0.1

Sol.
\[ Y_{12} = -(y_{12}) = -j20 \]

Series admittance of each line =
\[
\frac{Y_{12}}{2} = \frac{-j20}{2} = -j10
\]

Series reactance of each line = \( \frac{1}{-j10} = j0.1 \text{pu} \)

21. The symbols, a and T, represent positive quantities, and \( u(t) \) is the unit step function. Which one of the following impulse responses is NOT the output of a causal linear time-invariant system?

A. \( e^{-a(t-T)}u(t) \)
B. \( e^{aT}u(t) \)
C. \( 1 + e^{-aT}u(t) \)
D. \( e^{-a(t+T)}u(t) \)

Ans. C

Sol.
\[ H(t) = 1 + e^{-at}u(t) \]

‘1’ is a constant and two sided so the impulse response cannot be causal as for causal it should satisfy
\[ h(t) = 0 \quad t < 0 \]
\[ \neq 0 \quad t > 0 \]
Which it is not satisfying due to presence of constant
\[ \therefore \text{It is not causal} \]

22. A system transfer function is
\[ H(s) = \frac{a_1s^2 + b_1s + c_1}{a_2s^2 + b_2s + c_2} \]
If \( a_1 = b_1 = 0 \), and all other coefficients are positive, the transfer function represents a

A. high pass filter  
B. band pass filter  
C. notch filter  
D. low pass filter

Ans. D
23. The output voltage of a single-phase full bridge voltage source inverter is controlled by unipolar PWM with one pulse per half cycle. For the fundamental rms component of output voltage to be 75% of DC voltage, the required pulse width in degrees (round off up to one decimal place) is ________.

Ans. 112.88

Sol. Waveform for output voltage of single-phase full bridge PWM inverter

\[ V_{o1} = \sum_{n=6k+1} \frac{4V_{dc}}{n\pi} \sin nd \sin \frac{n\pi}{2} \overline{\omega t} \]

\[ V_{o1\text{rms}} = \text{fundamental rms output voltage} \]

\[ V_{o1} = \frac{2\sqrt{2}}{\pi} V_{dc} \sin d \sin \frac{\pi}{2} \]

Given, \[ V_{o1} = 0.754 V_{dc} \]

\[ 0.75 \ V_{dc} = \frac{2\sqrt{2}}{\pi} V_{dc} \sin d \]

\[ d = \sin^{-1} \left[ \frac{0.75}{0.9} \right] = 56.44 \]

Pulse width = \[ 2d = 112.88 \]

24. In the circuit shown below, the switch is closed at \( t = 0 \). The value of \( \theta \) in degrees which will give the maximum value of DC offset of the current at the time of switching is ________.

A. 60
B. 90
C. -30
D. -45

Ans. D

Sol. For series R – L circuit, \( I(t) \) expression is

\[ i(t) = -\frac{V_m}{\sqrt{R^2 + X_L^2}} \sin(\theta - \phi) e^{-t/\tau} + \frac{V_m}{\sqrt{R^2 + X_L^2}} \sin(\omega t - \phi) \]

DC offset = \( A = \frac{-V_m}{Z} \sin(\omega t - \phi) \)

For Maximum value of DC offset \( A \)

\[ \theta - \varphi = -90 \]

\[ \theta - \tan^{-1} \left[ \frac{\omega L}{R} \right] = -90 \]

\[ \theta - \tan^{-1} \left[ \frac{377 \times 10 \times 10^{-3}}{3.77} \right] = -90 \]

\[ \theta - 45^\circ = -90^\circ \]

\[ \theta = -45^\circ \]
25. M is a 2 × 2 matrix with eigenvalues 4 and 9.
The eigenvalues of \( M^2 \) are
A. 2 and 3  
B. -2 and 3  
C. 16 and 81  
D. 4 and 9

Ans. C

Sol. M is a 2 × 2 Matrix with Eigen value 4 and 9
If has \( \lambda_1, \lambda_2 \) Eigen values
\( M^n \rightarrow \lambda_1^n, \lambda_2^n \) Eigen values
\( M^2 \rightarrow 4^2, 9^2 \)
∴ \( M^2 \) has Eigen values as 16 and 81

26. A three-phase 50 Hz, 400 kV transmission line is 300 km long. The line inductance is 1 mH/km per phase, and the capacitance is 0.01 μF/km per phase. The line is under open circuit condition at the receiving end and energized with 400 kV at the sending end, the receiving end and energized with 400 kV at the sending end, the receiving end line voltage end line voltage in kV (round off to two decimal places) will be ____________.

Ans. 418.85

Sol. \( V_S = 400 \) kV
\( I = 300 \) km
\( L_1 = 1 \) mH / km / phase
\( C_1 = 0.01 \) μF / km / phase
\( v = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{1 \times 10^{-3} \times 0.01 \times 10^{-6}}} = 3.16 \times 10^5 \) km/s

\( \beta' = \frac{2\pi f l}{v} = \frac{2\pi \times 50 \times 300}{3.16 \times 10^5} = 0.29 \)

\( A = 1 - \frac{\beta^2}{2} = 1 - \frac{(0.29)^2}{2} = 0.955 \)

\( V_R = \frac{V_S}{A} = \frac{400}{0.955} = 418.85 \) kV

27. The current I flowing in the circuit shown below in amperes is ____________.

Ans. 0

Sol. According to Mill man’s Theorem, the equivalent circuit of the given circuit is

\[ E_{eq} = \frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3} + \frac{E_4}{R_4} \]

\[ = \frac{200}{5} + \frac{160}{40} + \frac{-100}{25} + \frac{-80}{20} \]

\[ = \frac{1}{50} + \frac{1}{40} + \frac{1}{25} + \frac{1}{20} \]

\[ V_{eq} = 0V \]
So, the current I flowing is 0 A

28. A 220 V (line), three-phase, Y-connected, synchronous motor has a synchronous impedance of (0.25 + j2.5) Ω/phase. The motor draws the rated current of 10 A at 0.8 pf leading. The rms value of line-to-line internal voltage in volts (round off to two decimal places) is ____________.

Ans. 245.36

Sol. For synchronous motor
29. The closed loop line integral
\[ \int_{|z|=5} \frac{z^3 + z^2 + 8}{z + 2} \, dz \]
Evaluated counter-clockwise, is
A. +4j \pi  
B. -4j \pi  
C. +8j \pi  
D. -8j \pi  

Ans. C

Sol.
\[ \int_{|z|=5} \frac{z^3 + z^2 + 8}{z + 2} \, dz = 2\pi j \text{ (sum of residues)} \]
\[ = 2\pi j \times \lim_{z \to 2} \left( \frac{z^3 + z^2 + 8}{z + 2} \right) \]
\[ = 2\pi j \left( \frac{-8 + 4 + 8}{1} \right) = 8\pi j \]

30. The voltage across and the current through a load are expressed as follows
\[ v(t) = -170 \sin \left(377t - \frac{\pi}{3}\right) \, V \]
\[ i(t) = 8 \cos \left(377t + \frac{\pi}{6}\right) \, A \]

The average power in watts (round off to one decimal place) consumed by the load is ____________.

Ans. 588.89

Sol. \[ v(t) = -170 \sin \left(377t - \frac{\pi}{3}\right) \, V \]

31. A delta-connected, 3.7 kW, 400 V(line), three-phase, 4-pole, 50-Hz squirrel-cage induction motor has the following equivalent circuit parameters per phase referred to the stator: \( R_1 = 5.39 \, \Omega \), \( R_2 = 5.72 \, \Omega \), \( X_1 = X_2 = 8.22 \, \Omega \). Neglect shunt branch in the equivalent circuit. The starting line current in amperes (round off to two decimal places) when it is connected to a 100 V (line), 10 Hz, three-phase AC source is ____________.

Ans. 14.95

Sol. Given \( R_1 = 5.39 \, \Omega \), \( R_2 = 5.72 \, \Omega \), \( X_1 = X_2 = 8.22 \, \Omega \)
for frequency \( \rightarrow 10 \, Hz \)
\[ X_1 = X_2 = 8.22 \times \frac{10}{50} = 1.644 \, \Omega \]

Starting phase current at 10 Hz
\[ I_{pn} = \frac{V_{pn}}{\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}} \]
\[ = \frac{100}{\sqrt{(5.39 + 5.72)^2(1.644 + 1.644)^2}} \]
\[ I_{pn} = 8.63A \]

Starting line current = \( I_L = \sqrt{3}I_{pn} \)
\[ I_L = \sqrt{3} \times 8.63 \]
\[ I_L = 14.95A \]

32. In a 132 kV system, the series inductance up to the point of circuit breaker location is 50 mH. The shunt capacitance at the circuit breaker terminal is 0.05 μF. The critical value of resistance in ohms required to be connected across the circuit breaker contacts which will give no transient oscillation is \[ \text{Ans.} 500 \]

Sol. Given data \( L = 50\text{mH}, C = 0.05 \text{ μF} \)

Critical resistance to avoid current shopping will be given as
\[ R = \frac{1}{2} \sqrt{L} = \frac{1}{2} \sqrt{50 \times 10^{-3}} \]
\[ R = 500Ω \]

33. In the single machine infinite bus system shown below, the generator is delivering the real power of 0.8 p.u. at 0.8 power factor lagging to the infinite bus. The power angle of the generator in degrees (round off to one decimal place) is \[ \text{Ans.} 20.51 \]

Sol.
\[ X_{eq} = 0.25 + 0.2 + \frac{0.4}{2} \]
\[ X_{eq} = 0.65 \text{ p.u.} \]
\[ P = V_{PU} I_{PV} \cos \phi \]
\[ 0.8 = 1 \times I_{PV} \times 0.8 \]
\[ I_{PV} = 1 \text{ p.u.} \]
\[ I = i_{PV} - 36.86 \] \[ \text{[as 0.8 pf lagging]} \]
\[ \tilde{E} = \tilde{V} + j\tilde{I}X_{eq} \]
\[ \tilde{E} = 1 + 1 - 36.86 \times j0.65 = 1.484 \angle 20.51 \text{ Pu} \]
\[ \delta = 20.51 \text{ degrees} \]

34. In the circuit below, the operational amplifier is ideal. If \( V_1 = 10 \text{ mV} \) and \( V_2 = 50 \text{ mV} \), the output voltage \( (V_{out}) \) is

\[ \text{A. 600 mV} \quad \text{B. 500 mV} \quad \text{C. 400 mV} \quad \text{D. 100 mV} \]

\[ \text{Ans. C} \]

Sol.
\[ V_x = V_2 \frac{R_2}{R_1 + R_2} \quad \text{[Voltage division Rule]} \]

\[ V_{out} = V_x \left[ 1 + \frac{R_2}{R_1} \right] - V_1 \frac{R_2}{R_1} \]

\[ V_{out} = V_2 \frac{R_2}{R_1 + R_2} \left[ 1 + \frac{R_2}{R_1} \right] - V_1 \frac{R_2}{R_1} \]

\[ V_{out} = V_2 \frac{R_2}{R_1} - V_1 \frac{R_2}{R_1} = \frac{R_2}{R_1} (V_2 - V_1) \]

\[ V_{out} = \frac{100}{10} (50 - 10) \]

\[ V_{out} = 400 \text{mV} \]

35. In the circuit shown below, X and Y are digital inputs, and Z is a digital output. The equivalent circuit is a

A. XOR gate  
B. NAND gate  
C. XNOR gate  
D. NOR gate

**Ans. A**

**Sol.**

Output = \( \overline{X}Y + XY \)

= \( X \oplus Y \)

The above expression is for XOR gate

36. A 0.1 \( \mu \)F capacitor charged to 100 V is discharged through a 1 k\( \Omega \) resistor. The time in ms (round off to two decimal places) required for the voltage across the capacitor to drop to 1 V is __________.

**Ans. 0.46**

**Sol.** Discharging of capacitor equation

\[ V_C (t) = V_0 e^{-\frac{t}{\tau}} \]

Where \( \tau = RC = (10^3 \times 10^{-7}) = 10^{-4} \) sec

\[ V_0 = 100 \text{V} \]

\[ V_C(t) = 100 e^{-104t} \]

\[ V_C(t) = 1 \]

\[ 1 = 100 e^{-104t} \]

\[ T = 0.46 \text{ msec} \]

37. A periodic function \( f(t) \), with a period of 2\( \pi \), is represented as its Fourier series,

\[ f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt. \]

If

\[ f(t) = \begin{cases} A \sin t, & 0 \leq t \leq \pi \\ 0, & \pi < t < 2\pi \end{cases} \]

The Fourier series coefficients \( a_1 \) and \( b_1 \) of \( f(t) \) are

A. \( a_1 = 0; b_1 = A/\pi \)

B. \( a_1 = 0; b_1 = A/2 \)

C. \( a_1 = A/\pi ; b_1 = 0 \)

D. \( a_1 = A/2; b_1 = 0 \)

**Ans. B**

**Sol.**

\[ a_n = \frac{2}{T} \int_{0}^{T} x(t) \cos n\omega t \, d(\omega t) \]

\[ a_1 \bigg|_{n=1} = \frac{2}{2\pi} \int_{0}^{2\pi} A \sin t \cos t \, dt \]

\[ = \frac{A}{\pi} \int_{0}^{\pi} \sin t \cos t \, dt \]

\[ a_1 = \frac{A}{\pi} \int_{0}^{\pi} \frac{\sin 2t}{2} \, dt = \frac{A}{2\pi} \left[ \frac{-\cos 2t}{2} \right]_{0}^{\pi} \]
\[
a_1 = 0
\]
\[
b_n = \frac{2}{T} \int_0^T x(t) \sin n\omega t \, dt
\]
\[
b_1 = \frac{2}{2\pi} \int_0^\pi A \sin t \sin t \, dt
\]
\[
b_1 = \frac{A}{\pi} \int_0^\pi \sin^2 t \, dt
\]
\[
b_1 = \frac{A}{2}
\]

38. If \( A = 2x\hat{i} + 3y\hat{j} + 4z\hat{k} \) and \( u = x^2 + y^2 + z^2 \), then \( \text{div} (uA) \) at \((1, 1, 1)\) is \__________.

**Ans. 45**

**Sol.**
\[
A = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}, \quad U = x^2 + y^2 + z^2
\]
\[
UA = (2x^3 + 2xy^2 + 2xz^2)\hat{i} + (3x^2y + 3y^3 + 3yz^2)\hat{j} + (4x^2z + 4y^2z + 4z^3)\hat{k}
\]
\[
\text{div(UA)} = \frac{d}{dx}(2x^3 + 2xy^2 + 2xz^2) + \frac{d}{dy}(3x^2y + 3y^3 + 3yz^2) + \frac{d}{dz}(4x^2z + 4y^2z + 4z^3)
\]
\[
\text{div(UA)} = (6x^2 + 2y^2 + 2z^2) + (3x^2 + 9y^2 + 3z^2) + (4x^2 + 4y^2 + 12z^2)
\]
\[
at (1, 1, 1) \Rightarrow x = 1, \ y = 1, \ z = 1
\]
\[
\text{div(UA)} = 45
\]

39. A moving coil instrument having a resistance of 10 \( \Omega \), gives a full-scale deflection when the current is 10 mA. What should be the value of the series resistance, so that it can be used as a voltmeter for measuring potential difference up to 100 V?

A. 9990 \( \Omega \) \quad B. 990 \( \Omega \) \quad C. 99 \( \Omega \) \quad D. 9 \( \Omega \)

**Ans.** D

**Sol.** We know
\[
\dot{X} = AX + Bu
\]
\[
Y = CX + Du
\]
Comparing the above equation with the given problem
41. The enhancement type MOSFET in the circuit below operates according to the square law. \( \mu_n C_{ox} = 100 \mu A/V^2 \), the threshold voltage \( V_T \) is 500 mV. Ignore channel length modulation. The output voltage \( V_{out} \) is

\[
\begin{align*}
A &= \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \alpha \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 0 \\ -1 & \alpha & 2\beta \end{bmatrix}
\end{align*}
\]

Characteristic equation is

\[
|SI - A| = 0
\]

\[
\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix} = 0
\]

\[
S - 1 = \alpha \quad S + 2\beta \]

\[
s^2 + 2S\beta + \alpha = 0 \quad (1)
\]

\[
s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (2)
\]

Comparing (1) and (2)

\[
\omega_n^2 = \alpha
\]

\[
\omega_n = \sqrt{\alpha}
\]

\[
2\xi\omega_n = 2\beta
\]

\[
\xi = \frac{\beta}{\omega_n} = \frac{\beta}{\sqrt{\alpha}}
\]

The enhancement type MOSFET in the circuit below operates according to the square law. \( \mu_n C_{ox} = 100 \mu A/V^2 \), the threshold voltage \( V_T \) is 500 mV. Ignore channel length modulation. The output voltage \( V_{out} \) is

\[
I_D = \frac{1}{2} (\mu_n C_{ox}) \left( \frac{W}{L} \right) (V_{gs} - V_T)^2
\]

\[
5 \times 10^{-6} = \frac{1}{2} (100 \times 10^{-6}) \times (10) \times (V_{out} - 0.5)^2
\]

\[
(V_{out} - 0.5)^2 = 0.01
\]

\[
V_{out} = 0.6V = 600mV
\]

42. The asymptotic Bode magnitude plot of a minimum phase transfer function \( G(s) \) is shown below.

Consider the following two statements.

Statement I: Transfer function \( G(s) \) has three poles and one zero.

Statement II: At very high frequency \( (\omega \to \infty) \), the phase angle \( \angle G(j\omega) = -\frac{3\pi}{2} \).

Which one of the following options is correct?

A. Both the statements are true.
B. Both the statements are false.
C. Statement I is false, and statement II is true.
D. Statement I is true, and statement II is false.

Ans. C

Sol. From the given Bode plot,

\[
T(S) = \text{Transfer function} = \frac{K}{s \left( 1 + \frac{S}{1} \right) \left( 1 + \frac{S}{20} \right)}
\]

It has three poles and no zero

So, statement 1 is false
∠T(s) = \angle -90 - \tan^{-1} \frac{w}{20}

∠T(jω) \mid w \rightarrow \infty = \angle -270°

So, statement 2 is true

43. A single-phase transformer of rating 25 kVA, supplies a 12 kW load at power factor of 0.6 lagging. The additional load at unity power factor in kW (round off to two decimal places) that may be added before this transformer exceeds its rated kVA is _____________.

Ans. 7.20

Sol. Load supplied previously before adding extra load
12 KW at pf of 0.6

\[ S_{\text{load}} = 12 + j16 \]

Now, Let P be extra load added (\( Q_{\text{extra}} \) = as unity p.f)

\[ S_{\text{load}} = 12 + P + j16 \]

\[ |S_{\text{load}}| = \sqrt{(12 + P)^2 + 16^2} \]

Rated KVA \( |S_{\text{rated}}| = 25 \)

\[ 25 = \sqrt{(12 + P)^2 + 16^2} \]

\[ 25^2 = (12 + P)^2 + 16^2 \]

\[ P = 7.5, -31.2 \]

So, 7.20 KW is extra load which is added

44. Consider a 2 x 2 matrix \( M = [v_1 v_2] \), where. \( v_1 \) and \( v_2 \) are the column vectors. Suppose \( M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix} \), where \( u_1^T \) and \( u_2^T \) are the row vectors. Consider the following statements:

Statement 1: \( u_1^T v_1 = 1 \) and \( u_2^T v_2 = 1 \)

Statement 2: \( u_1^T v_1 = 0 \) and \( u_2^T v_1 = 0 \)

Which of the following options is correct?

A. Both the statements are false
B. Statement 2 is true and statement 1 is false
C. Statement 1 is true and statement 2 is false
D. Both the statements are true

Ans. D

Sol. \( M^{-1} M = I \)

\[ \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix} [V_1 V_2] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ \begin{bmatrix} U_1 T_{V_1} \\ U_2 T_{V_1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ U_1^T V_1 = 1 \quad U_1^T V_2 = 0 \]

\[ U_2^T V_1 = 0 \quad U_2^T V_2 = 1 \]

Statement 1 and 2 are both correct

45. A single-phase fully-controlled thyristor converter is used to obtain an average voltage of 180 V with 10 A constant current to feed a DC load. It is fed from single-phase AC supply of 230 V, 50 Hz. Neglect the source impedance. The power factor (round off to two decimal places) of AC mains is _____________.

Ans. 0.78

Sol. \( V_s I_{sr} \cos \phi = V_o I_o \)

For single phase fully - controlled converter

\[ I_o = I_{sr} = 10 A \]

\[ \cos \phi = \frac{V_o}{V_{sr}} = \frac{180}{230} = 0.78 \]

46. A DC-DC buck converter operates in continuous conduction mode. It has 48 V input voltage, and it feeds a resistive load of 24 Ω. The switching frequency of the converter is 250 Hz. If switch-on duration is 1 msec, the load power is

A. 12 W \hspace{1cm} B. 6 W
C. 24 W \hspace{1cm} D. 48 W

Ans. B
**Sol.** Given that

Switch frequency, $f_s = 250\text{Hz}$
Load resistance $R_L = 24\Omega$
Supply voltage $V_s = 48V$
$T_{ON} = 1\text{msec}$

$T = \frac{1}{f_s} = 4\text{ms}$

$\alpha = \frac{T_{ON}}{T} = 0.25$

Load power $P = \frac{V_o^2}{R} = \frac{(\alpha V_s)^2}{R} = \frac{(0.25 \times 48)^2}{24}$

$P = 6\text{watts}$

47. In a DC-DC boost converter, the duty ratio is controlled to regulate the output voltage at 48 V. The input DC voltage is 24 V. The output power is 120 W. The switching frequency is 50 kHz. Assume ideal components and a very large output filter capacitor. The converter operates at the boundary between continuous and discontinuous conduction modes. The value of the boost inductor (in $\mu\text{H}$) is ________.

**Ans.** 24

**Sol.** $P_o = 120\text{w}$, $V_s = 24\text{V}$, $V_o = 48\text{V}$

$V_o = \frac{V_s}{1 - \alpha}$

$1 - \alpha = \frac{24}{48}$

$\alpha = 0.5$ [Duty cycle]

$P_o = V_o I_o = 120$

$I_o = \frac{120}{48} = 2.54\text{A}$

$V_s I_s = V_o I_o$

$I_s = \frac{120}{24} = 5\text{A}$

At boundary of continuous & discontinuous

$\text{I}_L = \text{I}_s = \frac{\Delta \text{I}_L}{2}$

$\Delta \text{I}_L = \frac{\alpha V_s}{f L c} = 2 \times 5$

$L_c = \frac{0.5 \times 24}{50 \times 10^3 \times 10} = 24\mu\text{H}$

48. A 220 V DC shunt motor takes 3 A at no-load. It draws 25 A when running at full-load at 1500 rpm. The armature and shunt resistances are 0.5 Ω and 220 Ω, respectively.

The no-load speed in rpm (round off to two decimal places) is ________.

**Ans.** 1579.33

**Sol.**

**No load**

$I_{NL} = 3\text{A}$

$I_c = \frac{220}{Rf} = \frac{220}{220} = 1\text{A}$

$I_s = I_L - I_r = 2\text{A}$

Back cmf = $E_b N = V - I_s R_a$

= $220 - 2 \times 0.5 = 219\text{V}$

**Full load**

$I_{FL} = 25\text{A}$

$N_r = 1500\text{rpm}$

$I_r = 1\text{A}$

$I_a = I_{FL} - I_r = 24\text{A}$

$E_b F = V - I_a R_a = 220 - 24 \times 0.5 = 208\text{V}$

We know $E \propto \text{speed (N)}$

$E_{bF} = \frac{N_f}{N_N}$

$(N_N = \text{speed at no load})$

$208 = \frac{1500}{N_N}$

$N_N = 1579.33\text{rpm}$
49. A fully-controlled three-phase bridge converter is working from a 415 V, 50 Hz AC supply. It is supplying constant current of 100 A at 400 V to a DC load. Assume large inductive smoothing and neglect overlap. The rms value of the AC line current in amperes (round off to two decimal places) is ________.

**Ans.** 81.65

**Sol.** Ac line current rms = \( I_{\text{rms}} \)
\[
= I_0 \sqrt{3} = 100 \sqrt{3} = 81.65\text{A}
\]

50. The output expression for the Karnaugh map shown below is

A. \( QR + S \)  
B. \( QR + \bar{S} \)  
C. \( \bar{Q}R + \bar{S} \)  
D. \( Q \bar{R} + S \)

**Ans.** D

**Sol.**
\[
F(P, Q, R, S) = S + Q\bar{R}
\]

51. The probability of a resistor being defective is 0.02. There are 50 such resistors in a circuit. The probability of two or more defective resistors in the circuit (round off to two decimal places) is ________.

**Ans.** 0.26

**Sol.**
\[
P = 0.02
n = 50
\lambda = np = 50 (0.02) = 1
P(x \geq 2) = 1 - P(x < 2)
= 1 - [P(x = 0) + P(x = 1)]
= 1 - \left[ e^{-\lambda} \frac{\lambda^0}{0!} + e^{-\lambda} \frac{\lambda^1}{1!} \right]
= 1 - e^{-1} (1 + 1) = 0.26
\]

52. The magnetic circuit shown below has uniform cross-sectional area and air gap of 0.2 cm. The mean path length of the core is 40 cm. Assume that leakage and fringing fluxes are negligible. When the core relative permeability is assumed to be infinite, the magnetic flux density computed in the air gap is 1 tesla. With same Ampere-turns, if the core relative permeability is assumed to be 1000 (linear), the flux density in tesla (round off to three decimal places) calculated in the air gap is __________.

**Ans.** 0.26

**Sol.**
\[
F (P, Q, R, S) = S + Q\bar{R}
\]

[Diagram of magnetic circuit with dimensions]
53. A 30 kV, 50 Hz, 50 MVA generator has the positive, negative, and zero sequence reactance's of 0.25 p.u., 0.15 p.u., and 0.05 p.u., respectively. The neutral of the generator is grounded with a reactance so that the fault current for a bolted LG fault and that of a bolted three-phase fault at the generator terminal are equal. The value of grounding reactance in ohms (round off to one decimal place) is ___________.

Ans. 1.8

Sol. Fault current for SLG fault

\[ I_{\text{SLG}} = \frac{3V}{X_1 + X_2 + X_0 + 3X_n} \]

Fault current for 3ϕ fault

\[ I_3 = \frac{V}{X_1} \]

\[ V \]

\[ X_1 = \frac{3V}{X_1 + X_2 + X_0 + 3X_n} = \frac{V}{X_1} \]

\[ X_n = \frac{2X_1 - X_2 - X_0}{3} \]

\[ X_n = 0.1 \text{ Pu} \]

\[ X_n \text{ (in} \Omega) = 0.1 \times \frac{30^2}{50} \]

\[ \frac{Z_{\text{bass}} \times \text{MVA}}{\text{KVL}} \]

\[ X_n \text{ (in} \Omega) = 1.8 \Omega \]

54. The line currents of a three-phase four wire system are square waves with amplitude of 100 A. These three currents are phase shifted by 120° with respect to each other. The rms value of neutral current is

A. 0 A
B. 300 A
C. 100 A
D. \[ \frac{100}{\sqrt{3}} \] A

Ans. C

Sol.
55. The transfer function of a phase lead compensator is given by

\[ D(s) = \frac{3(s + \frac{1}{3T})}{(s + \frac{1}{T})}. \]

The frequency (in rad/sec), at which \( \angle T(j\omega) \) is maximum, is

A. \( \sqrt{3T} \)  
B. \( \sqrt{3T^2} \)  
C. \( \frac{1}{\sqrt{3T^2}} \)  
D. \( \frac{1}{\sqrt{T^2}} \)

Ans. C

Sol. \( T(s) = \frac{1 + 3Ts}{1 + Ts} \)

Frequency at which \( \angle T(j\omega) \) is maximum

(i)

\[ W_m = \frac{1}{T\sqrt{\alpha}} \]

\[ T(S) = \frac{1 + \alpha TS}{1 + TS} \text{ is The general phase lead compensator} \]

\[ \therefore \alpha = 3 \]

\[ W_m = \frac{1}{T\sqrt{3}} = \frac{1}{\sqrt{3T^2}} \]
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