1. If the sum of the first 40 terms of the series,

    $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \ldots$ is $(102)m$, then m is equal to :

    A. 10

    B. 25

    C. 5

    D. 20

2. In an increasing geometric series, the sum of the second and the sixth term

    is $\dfrac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of

    $4^{\text{th}}, 6^{\text{th}}$ and 8th terms is equal to :

    A. 35

    B. 30

    C. 26

    D. 32

3. If $0 < \theta, \phi < \dfrac{\pi}{2}$, $x = \sum\limits_{n=0}^{\infty} \cos^{2n} \theta$, $y = \sum\limits_{n=0}^{\infty} \sin^{2n} \phi$ and $z = \sum\limits_{n=0}^{\infty} \cos^{2n} \theta \cdot \sin^{2n} \phi$

    then :

    A. $xyz = 4$

    B. $xy - z = (x + y)z$

    C. $xy + xy + zx = z$

    D. $xy + z = (x + y)z$

4. The sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :

A. $\dfrac{41}{8}e + \dfrac{19}{8}e^{-1} - 10$

B. $-\dfrac{41}{8}e + \dfrac{19}{8}e^{-1} - 10$

C. $\dfrac{41}{8}e - \dfrac{19}{8}e^{-1} - 10$

D. $\dfrac{41}{8}e + \dfrac{19}{8}e^{-1} + 10$

5. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\dfrac{1}{2}$, then the greatest number amongst them is

A. 16

B. 27

C. 7

D. $\dfrac{21}{2}$

6. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in \mathbb{R}$ and $a > 0$, is equal to:

A. $a + \dfrac{1}{a}$

B. $a + 1$

C. $2a$

D. $2\sqrt{a}$

7. $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \cdots \infty) \log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\dfrac{2 \sin x}{\sin x + \sqrt{3} \cos x}, \left(0 < x < \dfrac{\pi}{2}\right)$ is :

    A. $\dfrac{3}{2}$

    B. $2\sqrt{3}$

    C. $\dfrac{1}{2}$

    D. $\sqrt{3}$

8. Let S_1 be the sum of first $2n$ terms of an arithemetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000 then sum of the first $6n$ terms of the arithmetic progression is equal to :

    A. 3000

    B. 7000

    C. 5000

    D. 1000

9. The sum of 10 terms of the series $\dfrac{3}{1^2 \times 2^2} + \dfrac{5}{2^2 \times 3^2} + \dfrac{7}{3^2 \times 4^2} + \cdots$ is

    A. $\dfrac{143}{144}$

    B. $\dfrac{99}{100}$

    C. $\dfrac{120}{121}$

    D. 1

10. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \cdots + (n-1) \cdot 1$, $n \geq 4$. The sum $\displaystyle\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to

    A. $\dfrac{e-1}{3}$

    B. $\dfrac{e}{3}$

    C. $\dfrac{e}{6}$

    D. $\dfrac{e-2}{6}$

11. If $0 < x < 1$ and $y = \dfrac{1}{2}x^2 + \dfrac{2}{3}x^3 + \dfrac{3}{4}x^4 + \cdots$, then the value of e^{1+y} at $x = \dfrac{1}{2}$ is

    A. $2e$

    B. $\dfrac{1}{2}e^2$

    C. $2e^2$

    D. $\dfrac{1}{2}\sqrt{e}$

12. If sum of the first 21 terms of the series $\log_{9^{\frac{1}{2}}} x + \log_{9^{\frac{1}{3}}} x + \log_{9^{\frac{1}{4}}} x + \cdots,$ where $x > 0$ is 504, then x is equal to:

    A. 81

    B. 243

    C. 9

    D. 7

13. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to:

   A. 1842

   B. 1852

   C. 1862

   D. 1872

14. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\dfrac{S_{4n}}{S_{2n}}$ is

   A. 4

   B. 2

   C. 6

   D. 8

15. If $[x]$ be the greatest integer less than or equal to x, then $\displaystyle\sum_{n=8}^{100} \left[\dfrac{(-1)^n n}{2} \right]$ is equal to

   A. 2

   B. -2

   C. 0

   D. 4

16. If $\tan\left(\dfrac{\pi}{9}\right)$, x, $\tan\left(\dfrac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\dfrac{\pi}{9}\right)$, y, $\tan\left(\dfrac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to

    A.   0

    B.   1

    C.   3

    D.   4

17. If for $x, y \in \mathbb{R}$, $x > 0$, $y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} + \ldots$ upto ∞ terms and $\dfrac{2 + 4 + 6 + \ldots + 2y}{3 + 6 + 9 + \ldots + 3y} = \dfrac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to:

    A. $(10^6, 9)$

    B. $(10^6, 6)$

    C. $(10^4, 6)$

    D. $(10^2, 3)$

18. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\dfrac{a_1 + a_2 + \cdots + a_{10}}{a_1 + a_2 + \cdots + a_p} = \dfrac{100}{p^2}$, $p \neq 10$, then $\dfrac{a_{11}}{a_{10}}$ is equal to

    A. $\dfrac{19}{21}$

    B. $\dfrac{100}{121}$

    C. $\dfrac{21}{19}$

    D. $\dfrac{121}{100}$

19. Let a, b and c be the $7^{\text{th}}, 11^{\text{th}}$ and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\dfrac{a}{c}$ is equal to :

    A. 4

    B. 2

    C. $\dfrac{7}{13}$

    D. $\dfrac{1}{2}$

20. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, then the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is:

    A. 36

    B. 32

    C. 28

    D. 24

1. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these seris are the maximum possible four digit numbers, then the number of common terms in these two series is equal to

2. Let
    $$S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \cdots \text{ up to } n\text{-terms},$$
    where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to

3. The sum of all $3-$digit numbers less than or equal to 500, that are formed without using the digit $"1"$ and they all are multiple of 11, is

4. If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \dfrac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

5. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\displaystyle\sum_{n=1}^{\infty} \dfrac{a_n}{2^{3n}}$ is equal to

1. The sum $\sum\limits_{r=1}^{10} (r^2 + 1) \times (r!)$ is equal to:

    A. $10 \times (11!)$

    B. $11 \times (11!)$

    C. $(11!)$

    D. $101 \times (10!)$

2. If $n \geq 2$ is a positive integer, then the sum of the series
    $^{n+1}C_2 + 2\left(^2C_2 + {}^3C_2 + {}^4C_2 + \cdots + {}^nC_2\right)$ is

    A. $\dfrac{n(n+1)^2(n+2)}{12}$

    B. $\dfrac{n(n-1)(2n+1)}{6}$

    C. $\dfrac{n(n+1)(2n+1)}{6}$

    D. $\dfrac{n(2n+1)(3n+1)}{6}$

3. The value of $-{}^{15}C_1 + 2 \times {}^{15}C_2 - 3 \times {}^{15}C_3 + \ldots - 15 \times {}^{15}C_{15}$ is :
    $+ {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$

    A. 2^{14}

    B. $2^{13} - 13$

    C. $2^{16} - 1$

    D. $2^{13} - 14$

4. The value of $\sum\limits_{r=0}^{6} \left({}^6C_r \cdot {}^6C_{6-r} \right)$ is equal to:

   A. 1124

   B. 924

   C. 1324

   D. 1024

5. Let $(1 + x + 2x^2)^{20} = a_0 + a_1 x + a_2 x^2 + \cdots + a_{40} x^{40}$. Then, $a_1 + a_3 + a_5 + \cdots + a_{37}$ is equal to:

   A. $2^{20}(2^{20} + 21)$

   B. $2^{19}(2^{20} + 21)$

   C. $2^{20}(2^{20} - 21)$

   D. $2^{19}(2^{20} - 21)$

6. For the natural numbers m, n, if $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \ldots + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m + n)$ is equal to:

   A. 64

   B. 80

   C. 88

   D. 100

7. The maximum value of the term independent of t in the expansion of
    $\left(tx^{1/5} + \dfrac{(1-x)^{1/10}}{t} \right)^{10}$ where $x \in (0,1)$ is :

    A. $\dfrac{10!}{\sqrt{3}(5!)^2}$

    B. $\dfrac{2 \times 10!}{3(5!)^2}$

    C. $\dfrac{10!}{3(5!)^2}$

    D. $\dfrac{2 \times 10!}{3\sqrt{3}(5!)^2}$

8. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}} + 5^{\frac{1}{8}} \right)^{60}$, then $(n-1)$ is divisible by:

    A. 8

    B. 26

    C. 7

    D. 30

9. If the fourth term in the expansion of $\left(x + x^{\log_2 x} \right)^7$ is 4480, then the value of x where $x \in N$ is equal to:

    A. 4

    B. 3

    C. 2

    D. 1

10. A possible value of x, for which the ninth term in the expansion of
$$\left\{ 3^{\log_3 \sqrt{25^{x-1}+7}} + 3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)} \right\}^{10}$$ in the increasing powers of
$3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is

    A. -1

    B. 0

    C. 1

    D. 2

11. If the greatest value of the term independent of x in the expansion of
$\left(x\sin\alpha + a\dfrac{\cos\alpha}{x} \right)^{10}$ is $\dfrac{10!}{(5!)^2}$, then the value of a is equal to

    A. -1

    B. -2

    C. 2

    D. 1

12. If the coefficients of x^7 in $\left(x^2 + \dfrac{1}{bx} \right)^{11}$ and x^{-7} in $\left(x - \dfrac{1}{bx^2} \right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to

    A. 1

    B. -2

    C. -1

    D. 2

13. The coefficient of x^7 in the expression
$(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \ldots + x^{10}$ is :

    A.   420

    B.   330

    C.   210

    D.   120

14. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $(x + \sqrt{x^2 - 1})^6 + (x - \sqrt{x^2 - 1})^6$, then :

    A. $\alpha + \beta = -30$

    B. $\alpha - \beta = -132$

    C. $\alpha + \beta = 60$

    D. $\alpha - \beta = 60$

15. The value of r for which
$^{20}C_r \, ^{20}C_0 + \, ^{20}C_{r-1} \, ^{20}C_1 + \, ^{20}C_{r-2} \, ^{20}C_2 + \ldots + \, ^{20}C_0 \, ^{20}C_r$ is maximum, is :

    A.   11

    B.   15

    C.   10

    D.   20

16. If the constant term in the binomal expansion of $\left(\sqrt{x} - \dfrac{k}{x^2} \right)^{10}$ is 405, then $|k|$ equals:

    A. 1

    B. 9

    C. 2

    D. 3

1. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is

2. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is

3. Let ^nC_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$. If
$$\sum_{k=0}^{10} \left(2^2 + 3k\right){}^nC_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}, \; \alpha, \; \beta \in \mathbb{R},$$ then $\alpha + \beta$ is equal to

4. The number of elements in the set
$$\{n \in \{1, 2, 3, \ldots, 100\} \,|\, (11)^n > (10)^n + (9)^n\} \text{ is}$$

5. Let n be a positive integer. Let
$$A = \sum_{k=0}^{n} (-1)^k \; {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]. \text{ If}$$
$63A = 1 - \dfrac{1}{2^{30}}$, then n is equal to

6. For a positive integer n the binomial expression $\left(1 + \dfrac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, $2 : 5 : 12$, then n is equal to

7. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \dfrac{1}{x^2}\right)^{22}$ is 1540, is

8. If $\left(\dfrac{3^6}{4^4}\right) k$ is the term, independent of x, in the binomial expansion of $\left(\dfrac{x}{4} - \dfrac{12}{x^2}\right)^{12}$, then k is equal to

9. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder

1. The total number of positive integral solutions (x, y, z) such that $xyz = 24$ is :

   A. 36

   B. 45

   C. 24

   D. 30

2. Consider a rectangle $ABCD$ having $5, 7, 6, 9$ points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

   A. 1890

   B. 795

   C. 717

   D. 1173

3. If the sides AB, BC and CA of a triangle ABC have $3, 5$ and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices is equal to:

   A. 360

   B. 240

   C. 333

   D. 364

4. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \dfrac{5}{6}, y > z$. Then the number of odd divisors of n, including 1, is:

    A. 11

    B. $6x$

    C. 12

    D. 6

5. If the number of five digit numbers with distinct digits and 2 at the 10th place is $336k$, then k is equal to :

    A. 8

    B. 7

    C. 4

    D. 6

6. Total number of six-digit numbers in which only and all the five digits $1, 3, 5, 7$ and 9 appear, is

    A. 5^6

    B. $\dfrac{1}{2}(6!)$

    C. $6!$

    D. $\dfrac{5}{2}(6!)$

7. The value of $\left(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \cdots \cdot \text{up to } 51^{\text{th}} \text{ term}\right)$ is equal

$+ \left(1! - 2! + 3! - 4! + \cdots \cdot \text{up to } 51^{\text{th}} \text{ term}\right)$

to

A. $1 - 51(51)!$

B. $1 + (52)!$

C. 1

D. $1 + (51)!$

8. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

A. $2!3!4!$

B. $(3!)^3 \cdot (4!)$

C. $3!(4!)^3$

D. $(3!)^2 \cdot (4!)$

9. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

A. 560

B. 1050

C. 1625

D. 575

10. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to:

    A.   5

    B.   6

    C.   2

    D.   4

11. If $^nP_r = {}^nP_{r+1}$ and $^nC_r = {}^nC_{r-1}$, then the value of r is equal to

    A.   4

    B.   3

    C.   2

    D.   1

12. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :

    A.   4

    B.   6

    C.   2

    D.   3

13. If $\sum_{i=1}^{20} \left(\dfrac{^{20}C_{i-1}}{^{20}C_i + {}^{20}C_{i-1}} \right)^3 = \dfrac{k}{21}$, then k equals :

    A.   50

    B.   100

    C.   200

    D.   400

14. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be a member of same team, is :

    A.   200

    B.   300

    C.   350

    D.   500

15. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:

    A.   2250

    B.   2255

    C.   1500

    D.   3000

1. The number of three-digit even numbers, formed by the digits $0, 1, 3, 4, 6, 7$ if the repetition of digits is not allowed, is

2. Let n be a non-negative integer. Then the number of divisors of the form $4n + 1$ of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to

3. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to

4. The number of six letter words (with or without meaning), formed using all the letters of the word $'VOWELS'$, so that all the consonants never come together, is

5. If the digits are not allowed to repeat in any number formed by using the digits $0, 2, 4, 6, 8$, then the number of all numbers greater than $10,000$ is equal to

6. The numbers of times the digit 3 will be written when listing the integers from 1 to 1000 is

7. If $^{1}P_{1} + 2 \cdot {}^{2}P_{2} + 3 \cdot {}^{3}P_{3} + \cdots + 15 \cdot {}^{15}P_{15} = {}^{q}P_{r} - s,\ 0 \le s \le 1$, then $^{q+s}C_{r-s}$ is equal to

8. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is

9. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is

10. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is $100k$, then k is equal to