UNITS, DIMENSION, MEASUREMENTS AND PRACTICAL PHYSICS

- **Fundamental or base quantities :** The quantities which do not depend upon other quantities for their complete definition are known as fundamental or base quantities. e.g. length, mass, time, etc.

- **Derived quantities :** The quantities which can be expressed in terms of the fundamental quantities are known as derived quantities . e.g. Speed, volume, accelaration, force, pressure, etc.

- **Units of physical quantities:** The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the unit of that quantity.
  Physical Quantity = Numerical Value × Unit

Systems of Units

Sr. No.	MKS	CGS	FPS	MKSQ	MKSA
(i)	Length (m)	Length (cm)	Length (ft)	Length (m)	Length (m)
(ii)	Mass (kg)	Mass (g)	Mass (pound)	Mass (kg)	Mass (kg)
(iii)	Time (s)	Time (s)	Time (s)	Time (s)	Time (s)

Fundamental Quantities in S.I. System and their units

Sr. No.	Physical Quantity	Name of Unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	electric Current	ampere	A
6.	Luminous Intensity	candela	cd
7.	Amount of Substance	mole	mol

Base Quantity	SI Units		
	Name	Symbol	Definition
Length	Meter	m	The meter is the length of the path travelled by light in vacuum during a time interval of 1/(299, 792, 458) of a second (1983)
Mass	Kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures, at Serves, near Pairs, France. (1889)
Time	Second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom (1967)
Electric Current	ampere	A	the ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, off negligible circular cross-section, and placed 1 meter a pat in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	Kelvin	k	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967).
Amount of Substance	Mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	Candela	Cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 × 102 hertz and that has a radiant intensity in that direction of 1/683 watt persteradian (1979).

- **Supplementary Units:**

  Radian (rad) - for measurement of plane angle

  Steradian (sr) - for measurement of solid angle

- **Dimensional Formula :**

  Relation which express physical quantities in terms of appropriate powers of fundamental units.

- **Use of dimensinal analysis :**

→ To check the dimensional correctness of a given physical relation

→ To derive realtionship between different physical quantities

→ To convert units of a physical quantity from one system to another

$$n_1u_1 = n_2u_2 \Rightarrow n_2 = n_1\frac{u_1}{u_2} = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c \text{ where } u = M^aL^bT^c$$

- **Limitations of this method :**

→ In mechanics the formula for a physical quantity depending on more than three other physical quantities cannot be derived. It can only be checked.

→ This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trignometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + 1/2at^2$ also can't be derived.

→ The relation derived from this method gives no information about the dimensionless constants.

→ If dimensions are given, physical qunatity may not be unique as many physical quantities have the same dimensions.

→ It gives no information whether a physical quantity is a scalar or a vector.

SI PREFIXES

The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain powers of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

UNITS, DIMENSION, MEASUREMENTS AND PRACTICAL PHYSICS

Units of important Physical Quantities

Physical Quantity	Unit	Physical Quantity	Unit
Angular Acceleration	Rad-s^{-2}	Frequency	Hertz (H_z)
Moment of inertia	Kg – m^2	Resistance	Kgm^2 A^{-2} s^{-3}
Self-inductance	Henry (H)	Surface tension	Newton/m
Magnetic flux	Weber (Wb)	Universal gas Constant	Joule K^{-1} mol^{-1}
Pole strength	A-m	Dipole moment	Coulomb-meter
Viscosity	Poise	Stefan constant	Watt m^{-2} K^{-4}
Reactance	Ohm	Permittivity of Free space (ε_0)	Colulomb^2/N-m^2
Specific heat	J/kg°C	Permeability of Free space (μ_0)	Weber/A-m
Strength of Magnetic field	Newton A^{-1} m^{-1}	Planck's constant	Joule-sec
Astronomical Distance	Parsec	Entropy	J/K

Dimensions of important Physical Quantities

Physical Quantity	Dimensions	Physical Quantity	Dimensions
Momentum	$M^1 L^1 T^{-1}$	Capacitance	$M^{-1} L^{-2} T^4 A^2$
Calorie	$M^1 L^2 T^{-2}$	Modulus of Rigidity	$M^1 L^{-1} T^{-2}$
Latent heat Capacity	$M^0 L^2 T^{-2}$	Magnetic Permeability	$M^1 L^1 T^{-2} A^{-2}$
Self inductance	$M^1 L^2 T^{-2} A^{-2}$	Pressure	$M^1 L^{-1} T^{-2}$
Coefficient of Thermal Conductivity	$M^1 L^1 T^{-3} K^{-1}$	Planck's constant	$M^1 L^2 T^{-1}$
Power	$M^1 L^2 T^{-3}$	Solar constant	$M^1 L^0 T^{-3}$
Impulse	$M^1 L^1 T^{-1}$	Magnetic flux	$M^1 L^2 T^{-2} A^{-1}$
Hole mobility in a Semi conductor	$M^{-1} L^0 A^1 T^2$	Current density	$M^0 L^{-2} T^0 A^1$
Bulk modulus of elasticity	$M^1 L^{-1} T^{-2}$	Young modulus	$M^1 L^{?1} T^{?2}$
Potential energy	$M^1 L^2 T^{-2}$	Magnetic field intensity	$M^0 L^{-1} T^0 A^1$
Gravitational constant	$M^{-1} L^3 T^{-2}$	Magnetic Induction	$M^1 T^{-2} A^{-1}$
Light year	$M^0 L^1 T^0$	Permittivity	$M^{-1} L^{-3} T^4 A^2$
Thermal resistance	$M^{-1} L^{-2} T^3 K$	Electric field	$M^1 L^1 T^{-3} A^{-1}$
Coefficient of viscosity	$M^1 L^{-1} T^{-1}$	Resistance	$M L^2 T^{-3} A^{-2}$

Sets of Quantities having same dimensions

Sr. No.	Quantities	Dimensions
1.	Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility, etc.	$[M^0 L^0 T^0]$
2.	Mass and inertia	$[M^1 L^0 T^0]$
3.	Momentum and impulse.	$[M^1 L^1 T^{-1}]$
4.	Thrust, force, weight, tension, energy gradient.	$[M^1 L^1 T^{-2}]$
5.	Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density.	$[M^1 L^{-1} T^{-1}]$
6.	Angular momentum and Planck's constant (h).	$[M^1 L^2 T^{-1}]$
7.	Acceleration, g and gravitational field intensity.	$[M^0 L^1 T^{-2}]$
8.	Surface tension, free surface energy (energy per unit area), force gradient, spring constant.	$[M^1 L^0 T^{-2}]$
9.	Latent heat capacity and gravitational potential.	$[M^0 L^2 T^{-2}]$
10.	Thermal capacity, Boltzmann constant, entropy.	$[ML^2 T^{-2} K^{-1}]$
11.	Work, torque, internal energy, potential energy, kinetic energy, moment of force, (q^2/C), (LI^2), (qV), (V^2C), (I^2Rt), $\frac{V^2}{R}t$, (VIt), (PV), (RT), (mL), $(mc\,\Delta T)$	$[M^1 L^2 T^{-2}]$
12.	Frequency, angular frequency, angular velocity, velocity gradient, radioactivity $\frac{R}{L}, \frac{1}{RC}, \frac{1}{\sqrt{LC}}$,	$[M^0 L^0 T^{-1}]$
13.	$\left(\frac{1}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{L}{R}\right), (RC), \left(\sqrt{LC}\right)$, time	$[M^0 L^0 T^1]$
14.	(VI), (I^2R), (V^2/R), Power	$[M L^2 T^{-3}]$

Some Fundamental Constants

Gravitational constant(G)	6.67×10^{-11} N m^2 kg^{-2}
Speed of light in vacuum (c)	3×10^8 ms^{-1}
Permeability of vacuum (μ_0)	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of vacuum (ε_0)	8.85×10^{-12} F m^{-1}
Planck constant (h)	6.63×10^{-34} Js
Atomic mass unit (amu)	1.66×10^{-27} kg
Energy equivalent of 1 amu	931.5 MeV
Electron rest mass (m_e)	9.1×10^{-31} kg $\equiv$ 0.511 MeV
Avogadro constant (N_a)	6.02×10^{23} mol^{-1}
Faraday constant (F)	9.648×10^4 C mol^{-1}
Stefan-Boltzmann constant (σ)	5.67×10^{-8} W m^{-2} K^{-4}
Wien constant (b)	2.89×10^{-3}mK
Rydberg constant (R_∞)	1.097×10^7 m^{-1}
Triple point for water	273.16 K (0.01°C)
Molar volume of ideal gas (NTP)	22.4L = 22.4×10^{-3} m^3 mol^{-1}

Key Points

- Trigonometric functions $\sin\theta$, $\cos\theta$, $\tan\theta$ etc and their arrangements are dimensionless.
- Dimensions of differential coefficients $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$
- Dimensions of integrals $\left[\int y\,dx\right] = [yx]$
- We can't add or subtract two physical quantities of different dimensions.
- Independent quantities may be taken as fundamental quantities in a new system of units.

PRACTICAL PHYSICS

Rules for counting significant Figures

For a number greater than 1

- All non-zero digits are significant.
- All zeros between two non-zero digits are significant. Location of decimal does not matter.
- If the number is without decimal part, then the terminal or trailing zeros are not significant.
- Trailing zeros in the decimal part are significant.

For a number less than 1

Any zero to the right of a non-zero digit is significant. All zeros between decimal point and first non-zero digit are not significant.

Ex. $0.108 \rightarrow 3SF$, $40.000 \rightarrow 5SF$,
$1.23 \times 10^{-19} \rightarrow 3SF$, $0.0018 \rightarrow 2SF$

Rounding off

Rounding Rules for Whole Numbers

Rounding rules for whole numbers is as follows:

- To get an accurate final result, always choose the smaller place value.
- Look for the next smaller place which is towards the right of the number that is being rounded off to. For example, if you are rounding off a digit from tens place, look for a digit in the ones place.
- If the digit in the smallest place is less than 5, then the digit is left untouched. Any number of digits after that number becomes zero and this is known as **rounding down**.
- If the digit in the smallest place is greater than or equal to 5, then the digit is added with +1. Any digits after that number become zero and this is known as **rounding up**.

Rounding Rules for Decimal Numbers

Rounding rules for decimal numbers are as follows:

- Determine the rounding digit and look at its righthand side.
- If the digits at the righthand side are less than 5, consider them as equal to zero.

- If the digits at the righthand side are greater than or equal to 5, then add to that digit and consider all other digits as zero.

Example:

$6.87 \to 6.9$,	$6.84 \to 6.8$,	$6.85 \to 6.8$,
$6.75 \to 6.8$,	$6.65 \to 6.6$,	$6.95 \to 7.0$

Order of magnitude :

Power of 10 required to represent a quantity
$49 = 4.9 \times 10^1 \approx 10^1 \Rightarrow$ order of magnitude = 1
$51 = 0.51 \times 10^2 \approx 10^2 \Rightarrow$ order of magnitude = 2
$0.051 = 5.1 \times 10^{-2} \approx 10^{-2} \Rightarrow$ order of magnitude = -2

Propagation of combination of errors

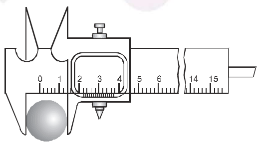
- **Error in Summation and Difference :** $x = a + b$ then $\Delta x = \pm (\Delta a + \Delta b)$
- **Error in Product and Division :** A physical quantity X depend upon Y & Z as $X = Y^a Z^b$ then maximum possible fractional error in X.

$$\frac{\Delta X}{X} = |a|\frac{\Delta Y}{Y} + |b|\frac{\Delta Z}{Z}$$

- **Error in Power of a Quantity :** $x = \frac{a^m}{b^n}$ then $\frac{\Delta x}{x} = \pm\left[m\left(\frac{\Delta a}{a}\right) + n\left(\frac{\Delta b}{b}\right)\right]$

- **Least count :** The smallest value of a physical quntity which can be measured accurately with an instrument is called the least count of the measuring instrument.

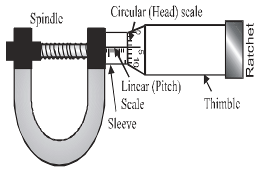
- **Vernier Callipers:** Least count = 1MSD – 1VSD
  (MSD $\to$ main scale division, VSD $\to$ Vernier scale division)



Ex. A vernier scale has 10 parts, which are equal to 9 parts of main scale having each parts equal to 1 mm then least count = 1 mm – $\frac{9}{10}$ mm = 0.1 mm [$\because$ 9 MSD = 10 VSD]

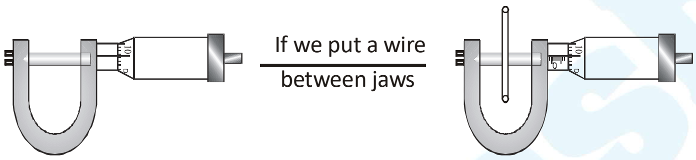
• Screw Gauge :

$$\text{Least Count} = \frac{\text{Pitch}}{\text{total no. of divisions on circular scale}}$$



Zero Error :

If there is no object between the jaws (i.e. jaws are in contact), the screwgauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This exces reading is called zero error.



Excess reading = 0.07 mm
(zero error)

It gives 3.07 mm
in which there is 0.07 mm excess reading which has to be substracted so actual thickness
$= \underset{\text{Observed (reading)}}{3.07} - \underset{\text{excess reading (zero error)}}{0.07} = 3.00$ mm

$$\boxed{\text{Actual reading} = \underset{\text{reading}}{\text{observed}} - \underset{\text{(zero error)}}{\text{excess reading}}}$$

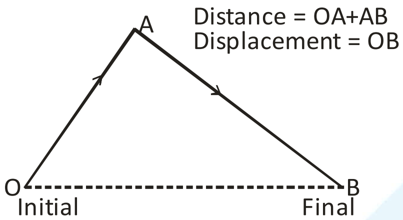
Excess reading = 0.07 mm (zero error)

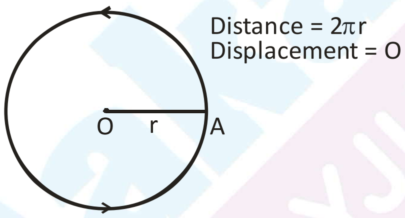
Ex. The distance moved by spindle of a screw gauge for each turn of head is 1mm. The edge of the humble is provided with an angular scale carrying 100 equal divisions. The least count $= \frac{1\text{mm}}{100} = 0.01$ mm

MOTION IN 1-D

1. Distance and Displacement

Distance	Displacement
(i) Length of actual path covered between the initial and final positions/points (ii) Scalar quantity (iii) Can have only +ve values	(i) Length of the shortest path between initial and final points (ii) Vector quantity (iii) Can have –ve, 0, +ve values.





- Distance $\geq$ |displacement| and average speed $\geq$ |average velocity|
  - If distance>|displacement| this implies
    (a) atleast at one point in path, velocity is zero.
    (b) the body must have retarded during the motion.

2. Displacement in terms of position vector : Let a body is displaced from

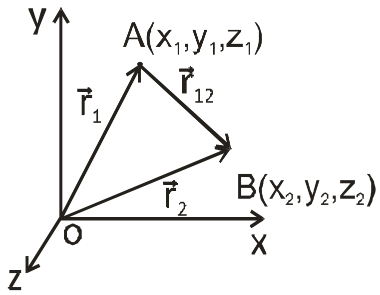
$A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ then its displacement is given by vector $\overrightarrow{AB}$

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

From ΔOAB $\quad \vec{r}_1 + \vec{r}_{12} = \vec{r}_2 \Rightarrow \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

$$\vec{r}_{12} = (x_2 - x_1)\hat{i} + (y_2 - y_2)\hat{j} + (z_2 - z_1)\hat{k}$$
$$|\vec{r}_{12}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



3. Velocity & Acceleration :

- **Velocity** $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$
- **For uniform motion**

  Average speed = |average velocity| = |instantaneous velocity|

- **Average velocity** $= \frac{\text{Displacement}}{\text{time interval}} = \vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$
- **Average speed =** $\frac{\text{Distance Travelled}}{\text{time interval}}$
- **Instantaneous velocity =** $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ if a body moving along a straight line travels distance S_1 in time t_1, S_2 in time t_2, ...S_n in time t_n then its average speed or average velocity is $<V> = \frac{S_1 + S_2 + ... + S_n}{t_1 + t_2 + ... + t_n}$
- If a body moving along a straight line travels distance S_1 with velocity V_1, distance S_2 with velocity V_2, distance S_3 with V_3, ...then

  average velocity $<V> = \frac{S_1 + S_2 + ...}{\frac{S_1}{V_1} + \frac{S_2}{V_2} + ...}$

  here if $S_1 = S_2 = S_3 = ... = S_n$

  then $<V> = \frac{1 + 1 + ...}{\frac{1}{V_1} + \frac{1}{V_1} + ...}$

- A particle moving along a straight line covers the first half distance with velocity V_1 and the remaining distance with velocity V_2 then its average velocity $<V>=\frac{2V_1V_2}{V_1+V_2}$

  If it covers the first half time with velocity V_1 and next half time with V_2, then

  $<V>=\frac{V_1+V_2}{2}$

- A particle moved from A to B along a straight line with velocity V_1 and then from B to A with velocity V_2. In this case its average velocity is zero but average speed is $\frac{2V_1V_2}{V_1+V_2}$

- **Acceleration** $\vec{a}=\frac{d\vec{v}}{dt}=\frac{d}{dt}(v_x\hat{i}+v_y\hat{j}+v_z\hat{k})=\frac{dv_x}{dt}\hat{i}+\frac{dv_y}{dt}\hat{j}+\frac{dv_z}{dt}\hat{k}=a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$

- **Average acceleration** = $\frac{\text{Total change in velocity}}{\text{total time taken}}=\vec{a}_{av}=\frac{\Delta\vec{v}}{\Delta t}$

- A particle moves along a straight line with acceleration a_1 for time t_1, a_2 for t_2, then average acceleration = $\frac{a_1t_1+a_2t_2+....+a_nt_n}{t_1+t_2+t_3+....+t_n}$

- If a body starts from rest and moves with an acceleration which increases at a steady rate with time, then $s\propto t^3, V\propto t^2, a\propto t$.

- A particle moving along a straight line with uniform acceleration crosses points A and B with velocities V_1 and V_2. If C is the mid point of AB, it crosses C with a velocity equal to $\sqrt{\frac{V_1^2+V_2^2}{2}}$.

- If a particle starts from rest and moves with uniform acceleration 'a' such that it travels distances S_m and S_n in the m^{th} and n^{th} seconds then $a=\frac{S_m-S_n}{(m-n)}$.

- A particle starts from rest and moves along a straight line with uniform acceleration. If S is the distance travelled by it in n seconds and S_n is the distance travelled in the n^{th} second, then $\frac{S_n}{S}=\frac{(2n-1)}{n^2}$.

- If a body starting from rest moves with acceleration α for certain time and then decceleration at the rate β. Until it stops and 't' is the total time of its motion, maximum velocity of the body $(V) = \dfrac{\alpha\beta t}{(\alpha+\beta)}$

average velocity $=$ V/2

distance travelled by the body $(S) = \dfrac{\alpha\beta t^2}{2(\alpha+\beta)}$

- If a bullet loses $(1/n)^{th}$ of its velocity while passing through a plank, then the minimum number of planks required to stop the bullet is $\left(\dfrac{n^2}{2n-1}\right)$
- Two particles move with constant velocities V_1 and V_2 along X and Y axes. If they are at distances x and y from the origin 'O', they will be closest to each other after time $\left(\dfrac{V_1x+V_2y}{V_1^2+V_2^2}\right)$

4. Problem Solving Strategy :

Motion on a straight Line (one dimensional motion)

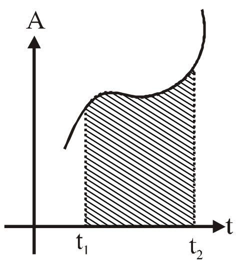
Uniform velocity	Motion with constant acceleration	Motion with variable acceleration
(i) $s = vt$	(i) $s = \left(\frac{u+v}{2}\right)t$	(i) If $a = f(t), a = \frac{dv}{dt}$
(ii) $a = 0$	(ii) $s = ut + \frac{1}{2}at^2$	(ii) If $a = f(s), a = v\frac{dv}{ds}$
	(iii) $v^2 = u^2 + 2as$	(iii) If $a = f(v), a = \frac{dv}{dt}$
	(iv) $v = u + at$	(iv) $v = \frac{ds}{dt}$
	(v) $s_{nth} = u + (2n-1)\frac{a}{2}$	(v) $s = \int vdt$
	(vi) For retardation, 'a' will be negative	(vi) $v = \int adt$

5. Graphical Representation :

- **Area under the curve in 1-D motion**

(i) $a = \frac{dv}{dt} = v\frac{dv}{dx}$

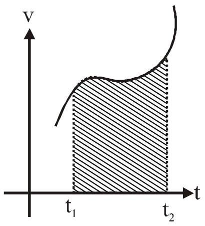
$$\Rightarrow a = \frac{dv}{dt} \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} adt \Rightarrow v_2 - v_1 = \int_{t_1}^{t_2} adt$$



$\Rightarrow$ Change in velocity = Area between acceleration curve and time axis, from t_1 to t_2.

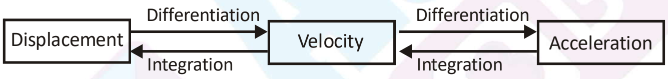
(ii) $v = \dfrac{dx}{dt} \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow x_2 - x_1 = \int_{t_1}^{t_2} v\,dt$

$\Rightarrow$ Change in position = displacement = area between velocity curve and time axis, from t_1 to t_2.

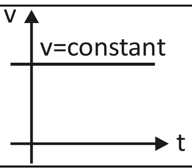
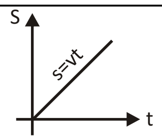
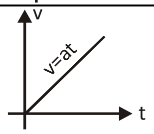
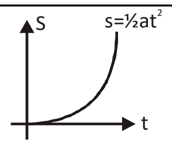
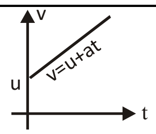
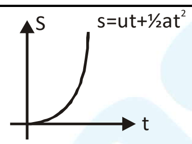
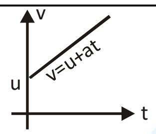
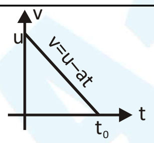
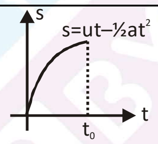
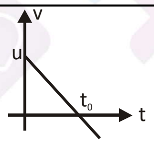
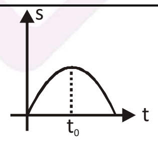


Important point about graphical analysis of motion

- Instantaneous velocity is the slope of position time curve. $\left(v = \dfrac{dx}{dt}\right)$
- Slope of velocity-time curve = instantaneous acceleration $\left(a = \dfrac{dv}{dt}\right)$
- v-t curve area gives displacement. $\left[\Delta x = \int v\,dt\right]$
- a-t curve area gives change in velocity. $\left[\Delta v = \int a\,dt\right]$



6. V-t and S-t graph for different equation of motion

S. No.	Different Cases	v-t Graph	s-t Graph	Important Points
1.	Uniform motion			(i) Slope of v-t graph = v= constant (ii) In s-t graph, s=0 at t=0
2.	Uniformly accelerated motion with u=0 and s=0 at t=0			(i) u=0, i.e. v=0 at t=0 (ii) a or slope of v-t graph is constant (iii) u=0, i.e. slop of s-t graph at t=0, should be zero
3.	Uniformly accelerated motion with u ≠0 but s=0 at t=0			(i) u≠0, i.e., v or slop of v-t graph at t=0 is not zero (ii) s or slope of s-t graph gradually goes on increasing
4.	Uniformly accelerated motion with u ≠0 and s=s_0 at t=0			(i) v=u at t=0 (ii) s=s_0 at t=0
5.	Uniformly retarded motion till velocity becomes zero			(i) slope of s-t graph at t=0 gives u (ii) Slope of s-t graph at t=t_0 becomes zero (iii) In this case u can't be zero
6.	Uniformly retarded then accelerated in opposite direction			(i) At time t=t_0, v=0 or slope of s-t graph is zero (ii) In s-t graph slope or velocity first decreases then increases with opposite sigh.

7. Motion Under Gravity

• If a body is dropped from some height (u=0) -

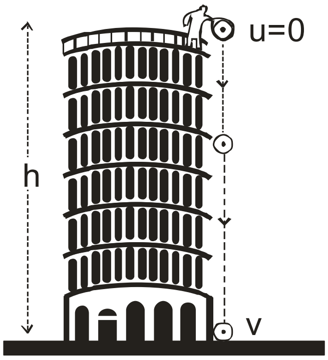
In this case, u=0, a= +g then equations of motion are -

(i) $v = gt$

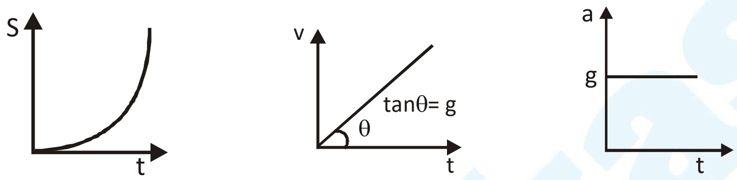
(ii) $h = \frac{1}{2}gt^2$

(iii) $v^2 = 2gh$

(iv) $h_n = \frac{g}{2}(2n-1)$



Graphs-



(v) A body is released from certain height h above the ground. The time after which it reached the ground is $\sqrt{\frac{2h}{g}}$.

Here if g vanishes exactly at the midway during its fall, the time of fall till then will be

$$t_1 = \sqrt{\frac{h}{g}}$$

The further time of fall is $t_2 = \frac{1}{2}\sqrt{\frac{h}{g}}$

Here for time 't_1' it will move with uniform acceleration and for time 't_2' it will move with uniform velocity $\sqrt{gh}$.

Here total time of its fall is $t_1 + t_2 = \frac{3}{2}\sqrt{\frac{h}{g}}$

$$\frac{t_1 + t_2}{t} = \frac{3}{2\sqrt{2}}$$

(vi) When a body is dropped from a certain height above the ground it will reach the ground after time t_1. But after time t_2, the body is stopped and then released. The further time after which it reaches the ground from the instant is t. Then $t^2 = t_1^2 - t_2^2$ or

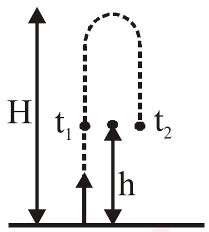
$t = \sqrt{t_1^2 - t_2^2}$

(vii) If the distance travelled by a freely falling body in the first t seconds is equal to the distance travelled inthe last second, the time of its fall is $\left(r^2+1\right)/2$

(viii) A freely falling body acquires a velocity V after falling through a certain distance h. The distance travelled by it in the next second is $V + \frac{g}{2}$ or $\left(\sqrt{2gh} + \frac{g}{2}\right)$.

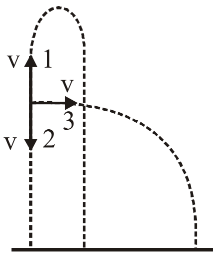
(ix) If air resistance is taken into consideration and two bodies of different masses are dropped simultaneously from the same height, heavier body reaches the ground earlier. If the two bodies have same mass but different sizes, smaller body reaches the ground first.

(x) Gallileo's law of odd numbers : For a freely falling body ratio of successive distance covered in equal time internal 't'



$S_1 : S_2 : S_3 : .... = 1 : 3 : 5 : ... : 2n–1$

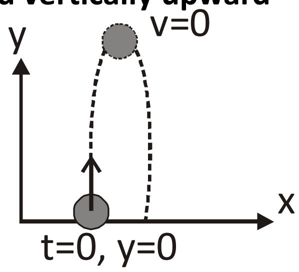
- At any point on its path the body will have same speed for upward journey and downward journey.
- A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ & height from where the particle was throw is $H = \frac{1}{2} g t_1 t_2$



If a body is projected vertically upward

(i) u = gt

(ii) h = ut-gt^2



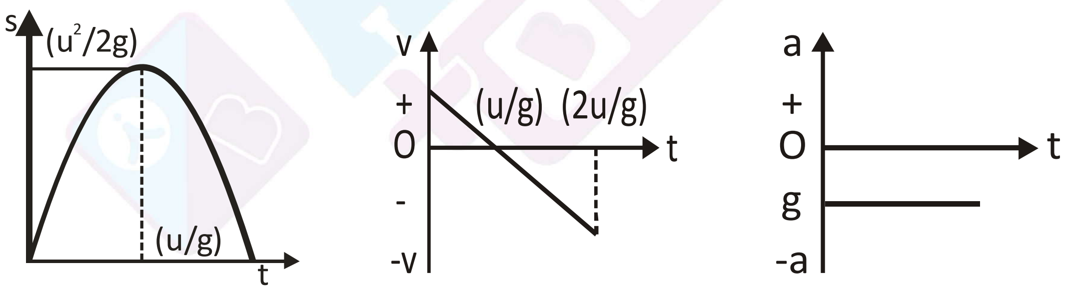
or h_{max} = $ut = -\frac{1}{2}gT^2 \Rightarrow h_{max} = (gT)T - \frac{1}{2}gT^2$

or $\boxed{h_{max} = \frac{1}{2}gT^2}$

(iii) $v^2 = u^2 - 2gh$ or $o = u^2 - 2gh_{max}$

or $\boxed{u^2 = 2gh_{max}}$

Graphs-



Note :-

(i) After attaining maximum height body turns and come back at ground.

(ii) Time taken during up flight and down flight are equal.

(iii) Time for one side T=u/g and

Total time of flight = 2T = 2 u/g

(iv) At each equal height from ground speed of body will be same either going up or coming down.

(v) A body projected vertically up crosses a point P at a height 'h' above the ground at time 't_1' seconds and at time t_2 seconds wile coming down. then total time of its flight $T = t_1+t_2$

Height of P is $h=\frac{1}{2}gt_1t_2$

Maximum height reached above the ground $H=\frac{1}{8}g(t_1+t_2)^2$

Magnitude of velocity wile crossing P is $\frac{g(t_2-t_1)}{2}$

If the body crosses P after t_1 seconds and then reaches the ground after t_2 seconds from that point, the above conditions apply.

(vi) A body is dropped from the top of a tower of height 'h'. Simultaneously another body is projected vertically up with a velocity u from the foot of the tower.

(a) The separation between them after 't' seconds is (h-ut)

(b) The time after which they meet $t=\frac{h}{u}$

(c) The height at which they meet above the ground is $\left(h-\frac{gh^2}{2u^2}\right)$

(d) The time after which their velocities are equal in magnitudes is $t=\frac{u}{2g}$

(e) If their speeds are equal after some time, the ratio of distances travelled by those two will be in the ratio 1:3.

(vii) A body is projected vertically up with velocity u_1 and after 't' seconds body is projected vertically up with a velocity u_2. If $u_2>u_1$, the time after which both the bodies will meet with each other is $\frac{u_2t+\frac{1}{2}gt^2}{(u_2-u_1)+gt}$ for the first body.

In this case if $u_1=u_2=u$ ie the two bodies are projected with same velocity, the time after which they meet is $\left(\frac{u}{g}+\frac{t}{2}\right)$ for the first body and $\left(\frac{u}{g}-\frac{t}{2}\right)$ for the second body.

(viii) A rocket moves up with a resultant acceleration a. If its fuel exhausts completely after time 't' seconds the maximum height reached by the rocked above the ground is

$h=\frac{1}{2}at^2\left(1+\frac{a}{g}\right)$

MOTION IN 2-D

1. Projectile Motion

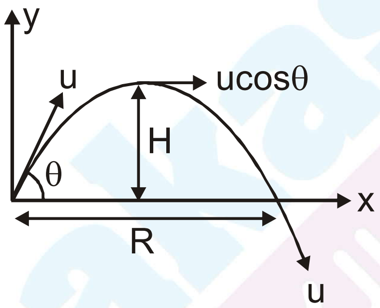
If a constant force (and hence constant acceleration) acts on a particle at an angle $\theta(\neq 0^{\circ} \text{ or } 180^{\circ})$ with the direction of its initial velocity $(\neq \text{zero})$, the path followed by the particle is a parabola and the motion of the particle is called projectile motion. Projectile motion is a two dimensinal motion, i.e., motion of the particle is constrained in a plane.

- **Horizontal Motion**

  $u_x = u\cos\theta$

  $a_x = 0$

  $x = u_x t = (u\cos\theta)t$



- **Vertical Motion**

  $u_y = u\sin\theta$ and $a_y = -g$; $y = u_y t + \frac{1}{2}a_y t^2 = u\sin\theta t - \frac{1}{2}gt^2$

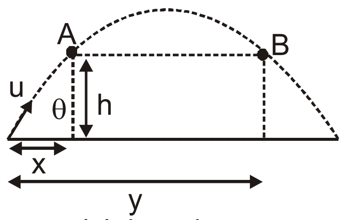
- **Net acceleration** = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

- **At any instant**: $v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

2. Ground to Ground Projectile Motion

2.1 For projectile motion :

A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point so,



(a) $x + y = R$

(b) $t_1 + t_2 = T$

(c) $h = 1/2\ g t_1 t_2$

(d) Average velocity from A to B is $u\cos\theta$

(e) If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2).

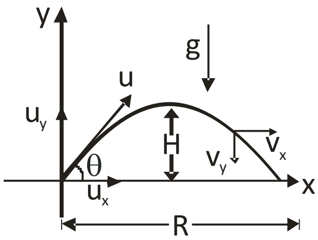
2.2 Velocity of particle at time t :

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ from ground is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

- At highest ponit: $v_y = 0$, $v_x = u\cos\theta$
- Time of flight : $\boxed{T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}}$

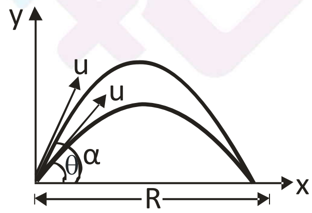


x-axis	y-axis
a_x=o	a_y=g
u_x=ucosθ	u_y=usinθ
x=(ucosθ)t	y=(usinθ)t-$\frac{1}{2}$gt^2
v_x=ucosθ	v_y=usinθ-gt

- Horizontal range : R = (u cos θ).T= $\frac{2(u\cos\theta)(u\sin\theta)}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_xu_y}{g}$

  It is same for θ and (90° – θ) and maximum for θ = 45°
- For objects projected at complementary lauch angle, range will be same

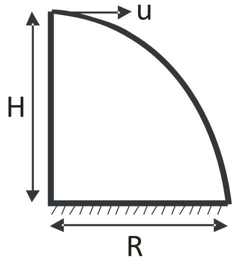
Here, $\alpha + \theta = 90^\circ$



- Maximum height : $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$
- $\frac{H}{R} = \frac{1}{4}\tan\theta$
- Equation of Trajectory : $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

3. Projectile Motion From Some Height

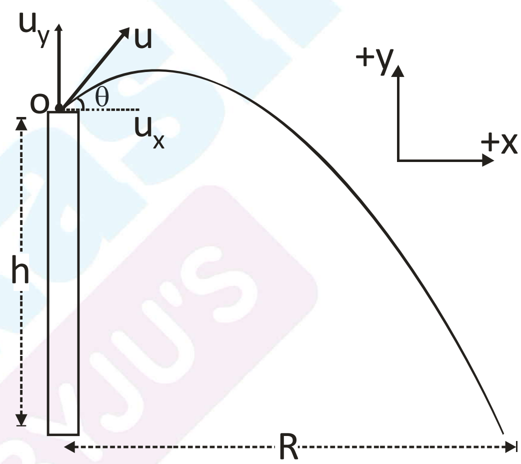
3.1 Projectile thrown horizontally



- Time of flight $T = \sqrt{\frac{2h}{g}}$
- Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$
- Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$

3.2 Projection at an angle θ above horizontal

x-axis	y-axis
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$

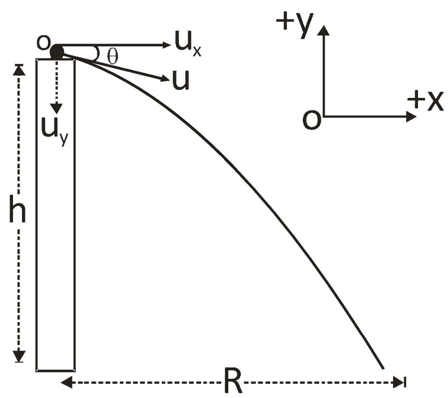


- Time of flight (T) $S_y = u_y t + \frac{1}{2} a_y t^2$

  Putting, $s_y = -h, a_y = -g$ and $t = T$
  we get time of flight T.
- Range, $R = u_x T = (u\cos\theta)T$

3.3 Projection at an angle θ below horizontal

x-axis	y-axis
u_x=ucosθ	u_y=-usinθ
a_x=o	a_y=-g



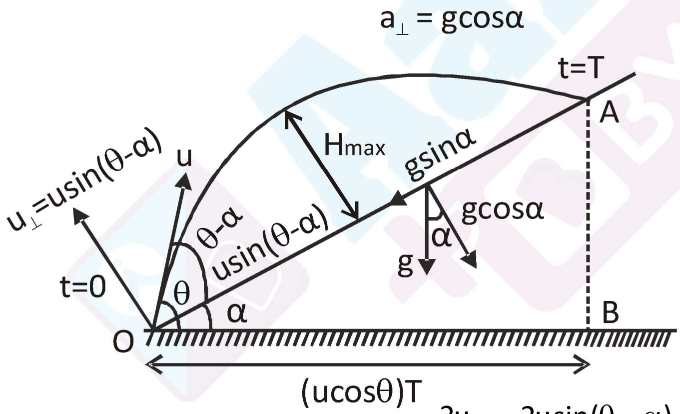
- Time of flight (T) $S_y = u_y t + \frac{1}{2} a_y t^2$

  Putting $s_y = -h, a_y = -g$ and $t = T$, we get time of flight.

- Range, $R = u_x T = (u\cos\theta)T$

4. Projectile Motion on Inclined Plane

4.1 Projectile motion on inclined plane-up motion



- Time of flight $T = \frac{2u_\perp}{g_\perp} = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$

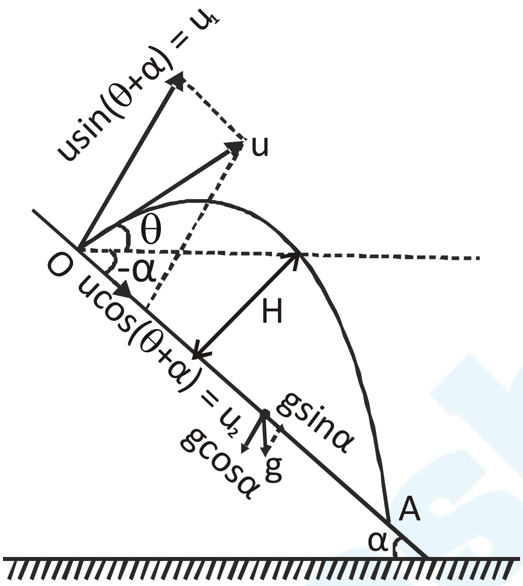
- Maximum height $H_{max} = \frac{u_\perp^2}{2g_\perp} = \frac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha}$

- Range of inclined plane $R = OA = \frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$

- Maximum range $R_{max} = \frac{u^2}{g(1+\sin\alpha)}$ at angle $\theta = \frac{\pi}{4} + \frac{\alpha}{2}$

4.2 Projectile motion of inclined plane (2-D):

Down motion (put $\alpha = -\alpha$ in above)



- Time of flight : $T = 2t_H = \frac{2u_\perp}{a_\perp} = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$
- Maximum height $H = \frac{u_\perp^2}{2a_\perp} = \frac{u^2\sin^2(\theta+\alpha)}{2g\cos\alpha}$
- Range of inclined plane $R = OA = \frac{2u^2\cos\theta\sin(\theta+\alpha)}{g\cos^2\alpha}$
- Maximum range $R_{max} = \frac{u^2}{g(1-\sin\alpha)}$ at angle $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$

5. Comparisonof projection motion

5.1 If a body is projected at angles θ and $(90-\theta)$ with the same initial velocity

Parameter	for θ	for $(90^\circ - \theta)$	Relation
Range	$R_1 = \dfrac{u^2\sin^2\theta}{g}$	$R_2 = \dfrac{u^2\sin 2\theta}{g}$	$R_1 = R_2 = R$
Maximum height	$H_1 = \dfrac{u^2\sin^2\theta}{2g}$	$H_2 = \dfrac{u^2\cos^2\theta}{2g}$	$H_1 + H_2 = \dfrac{u^2}{2g}$ $u = \sqrt{2g(H_1 + H_2}$ $\dfrac{H_1}{H_2} = \tan^2\theta$ $R^2 = 16H_1H_2$
Time of flight	$T_1 = \dfrac{2u\sin\theta}{g}$	$T_2 = \dfrac{2u\cos\theta}{g}$	$\dfrac{T_1}{T_2} = \tan\theta$ $T_1T_2 = \dfrac{2R}{g}$

6. Some important trajectory and theirs results

1. If the equation of trajectory of a projectile is $y = Ax - Bx^2$

   (a) Its angle of projection with horizontal is $\theta = \tan^{-1}(A)$

   (b) Horizontal range $R = (A/B)$

   (c) Maximum height $H = (A^2/4B)$

   (d) Time of flight $T = \left(\sqrt{\dfrac{2}{Bg}}\right)A$

2. The horizontal vertical displacements of a projectile are given by $x = at; y = bt - ct^2$ then

   (a) Velocity of projection is $\sqrt{a^2 + b^2}$

   (b) Angle of projection is $\tan^{-1}(a/b)$

   (c) Acceleration of the projectile = 2c

   (d) Maximum height reached = $b^2/4c$

   (e) Horizontal range = ab/c

3. A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant bullet leaves the gun, the monkey falls under gravity. The bullet hits the monkey (for any muzzle velocity of the bullet).

4. A ship is moving with a constant velocity V_1 in the horizontal direction. If a ball is projected vertically up with a velocity V_2 from its beck, its path is parabola for a person on the ground. If u is resultant velocity, here $u\ \cos\theta = V_1$ and $u\ \sin\theta = V_2$.
   If A is an observer on the ship and B is an observer at rest on the ground.

   for A and B $T = \frac{2V_2}{g}$; $H = \frac{V_2^2}{2g}$

   for A, horizontal range = 0; for B, horizontal range = $\frac{2V_1V_2}{g}$
5. If a body is projected horizontally from certain height above the ground and another is released simultaneously from the same height, both reach the ground after the same time but with different speeds.
6. A body at certain height exploded into two fragments of unequal masses and they move in opposite direction with velocities u_1 and u_2.
   The time after which their displacement vectors (ie then position vecotrs with respect to initial position)are perpendicular to each other is $\frac{2\sqrt{u_1u_2}}{g}$

   The time after which their velocity vectors are perpendicular to each other $\sqrt{\frac{u_1u_2}{g}}$
7. An object is projected vertically up from the top of a tower which reaches the ground after t_2 sec. then

   height of that tower = $\frac{1}{2}gt_1t_2$

   velocity of projction = $\frac{g(t_1 - t_2)}{2}$

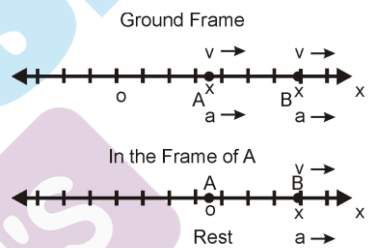
   If that object is projected horizontally with the same velocity or if it is released from the same point, it reaches the ground after $\sqrt{t_1t_2}$ sec.

RELATIVE MOTION

1. Relative Motion :

- Motion is a combined property of the object under study as well as the observer.
- Motion is always defined with respect to an **observer or reference frame**.
- It is always relative, there is no such thing as **absolute motion or absolute rest**.
- All parameters like position, velocity and acceleration are **“frame dependent”**.

2. Relative Motion in 1D :



Relative position : $\vec{x}_{BA} = \vec{x}_B - \vec{x}_A$

Differentiating both sides, we get

$$\frac{d}{dt}\vec{x}_{BA} = \frac{d}{dt}\vec{x}_B - \frac{d}{dt}\vec{x}_A$$

Relative velocity : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Differentiating again on both sides, we get

$$\frac{d}{dt}\vec{v}_{BA} = \frac{d}{dt}\vec{v}_B - \frac{d}{dt}\vec{v}_A$$

Relative acceleration : $\vec{a}_{-BA} = \vec{a}_B - \vec{a}_A$

3. Relative Acceleration :

It is the rate at which relative velocity is changing.

$$a_{AB} = \frac{dv_{AB}}{dt} = \frac{dv_A}{dt} - \frac{dv_B}{dt} = a_A - a_B$$

Equations of motion when relative acceleration is constant.

$$v_{rel} = u_{rel} + a_{rel}t$$

$$s_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$$

$$v^2{}_{rel} = u^2{}_{rel} + 2a_{rel}s_{rel}$$

4. Relative Motion in 2D :

Relative position : $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$

Differentiating both sides, we get

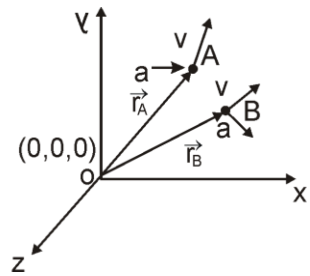
$$\frac{d}{dt}\vec{r}_{AB} = \frac{d}{dt}\vec{r}_A - \frac{d}{dt}\vec{r}_B$$

Relative velocity : $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Differentiating again on both sides, we get

$$\frac{d}{dt}\vec{v}_{AB} = \frac{d}{dt}\vec{v}_A - \frac{d}{dt}\vec{v}_B$$

Relative acceleration : $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$



5. Relative Motion in Lift :

Projectile motion in a lift moving with acceleration a upwards

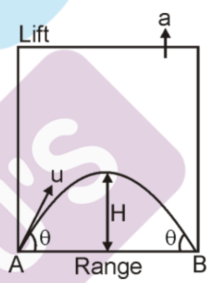
(1) In the reference frame of lift, acceleration of a freely falling object is g+a

(2) Velocity at maximum height = $u\cos\theta$

(3) $T = \dfrac{2u\sin\theta}{g+a}$

(4) Maximum height (H) = $\dfrac{u^2\sin^2\theta}{2(g+a)}$

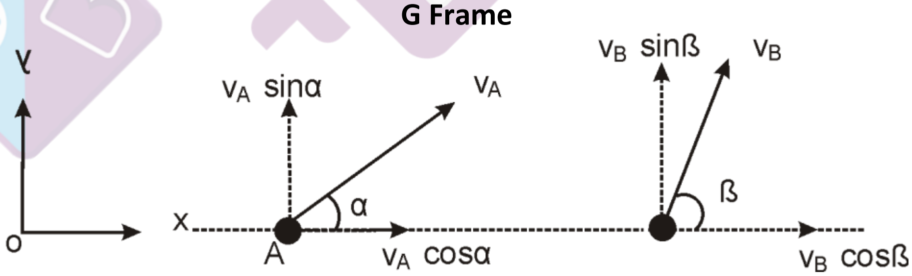
(5) Range = $\dfrac{u^2\sin 2\theta}{g+a}$



6. Velocity of Approach or Separation :

- It is the component of relative velocity of one particle w.r.t. to another, along the line joining them.
- If the separation is decreasing, it is called "velocity of approach" and if separation is increasing, then it is called "velocity of separation".

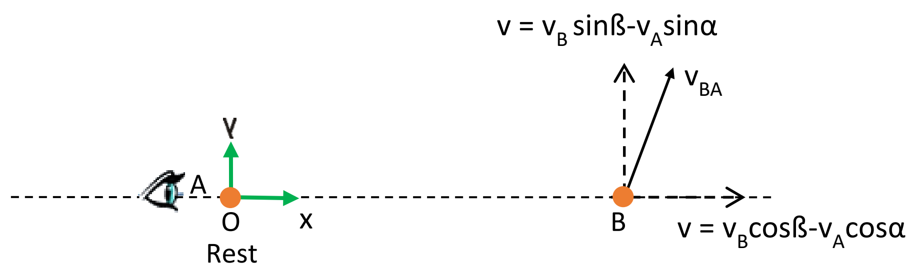
G Frame



$$\overrightarrow{v_{BA}} = \overrightarrow{v_B} - \overrightarrow{v_A}$$

$$\overrightarrow{v_{BA}} = (v_B\cos\beta\,\hat{i} + v_B\sin\beta\,\hat{j}) - (v_A\cos\alpha\,\hat{i} + v_A\sin\alpha\,\hat{j})$$

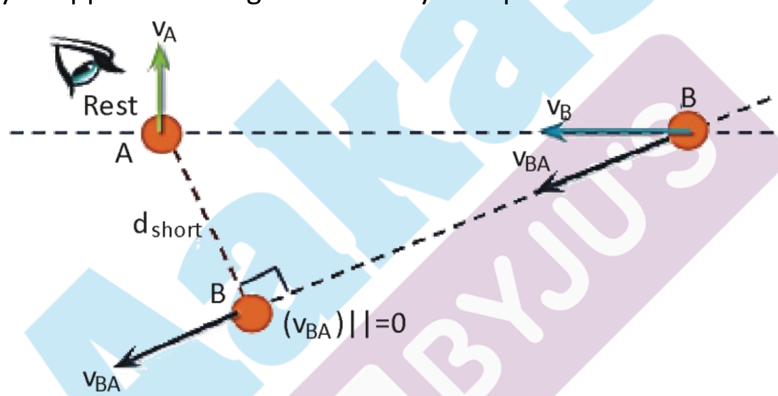
A Frame



v represents velocity of approach or separation

7. Minimum Distance :

- If the separation between two particles decreases and then increases, there will be a minimum distance at the transition.
- Velocity of approach changes to velocity of separation.



S_{AB} is minimum when $\frac{ds_{BA}}{dt} = 0$

8. Relative Motion in River Flow :

If a man can swim relative to water with velocity $\vec{v}_{mr}$ and water is flowing relative to ground with velocity $\vec{v}_R$, velocity of man relative to ground $\vec{v}_m$ will be given by :

$$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$$

or $$\vec{v}_m = \vec{v}_{mR} + \vec{v}_R$$

If $\vec{v}_R = 0$, then $\vec{v}_m = \vec{v}_{mR}$

in words, velocity of man in still water = velocity of man w.r.t. river

9. River Problem in One Dimension :

- Velocity of river is u & velocity of man in still water is v.

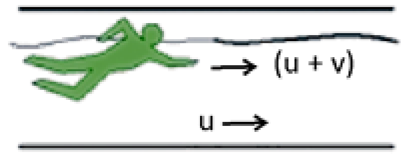
Case-1

Man swimming downstream (along the direction of river flow)

In this case velocity of river $v_R = +u$

velocity of man w.r.t. river $v_{mR} = +v$

now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = u + v$



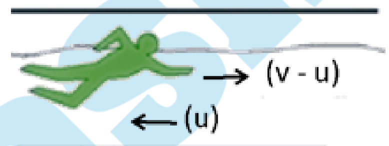
Case-2

Man swimming upstream (opposite to the direction of river flow)

In this case velocity of river $\vec{v}_R = -u$

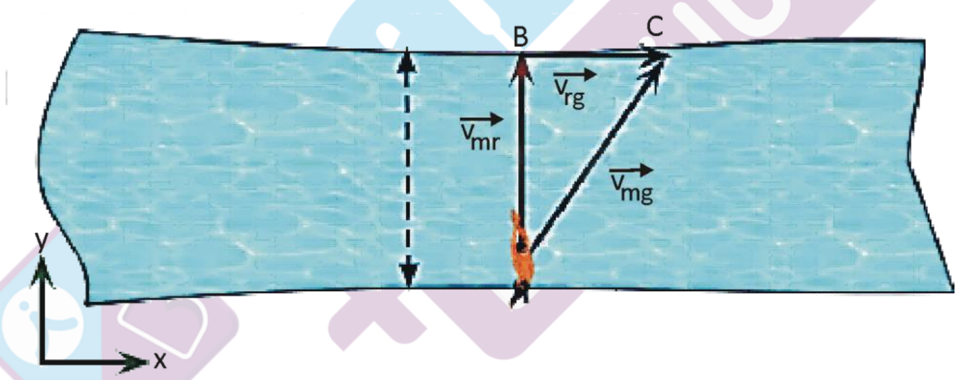
velocity of man w.r.t. river $\vec{v}_{mR} = +v$

now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = (v - u)$



10. Crossing of River (River problem in two dimension :)

Let the speed of the river be u, and speed of the man in still water is v.



$\vec{v}_{rg}$ = velocity of the river w.r.t. ground

$\vec{v}_{mr}$ = velocity of man relative to river

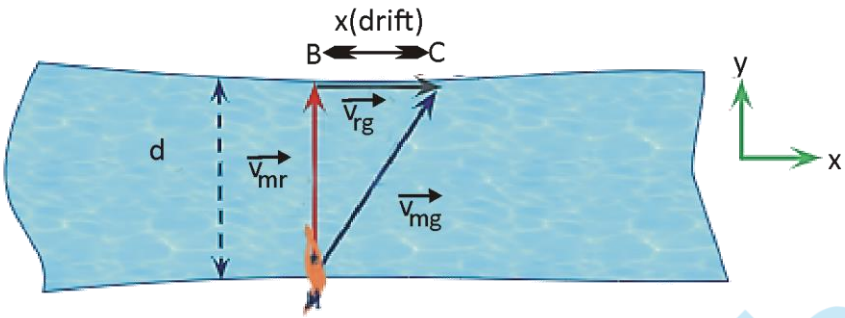
= velocity of man w.r.t. river

= velocity of man in still water

$\vec{v}_{mg}$ = velocity of the man w.r.t. ground

$$\boxed{\overrightarrow{v_{mg}} = \overrightarrow{v_{mr}} + \overrightarrow{v_{rg}}}$$

- **Case-I :** When the man swims perpendicular to the river flow:



- The velocity of man w.r.t. ground, $(\overrightarrow{v_{mg}})$

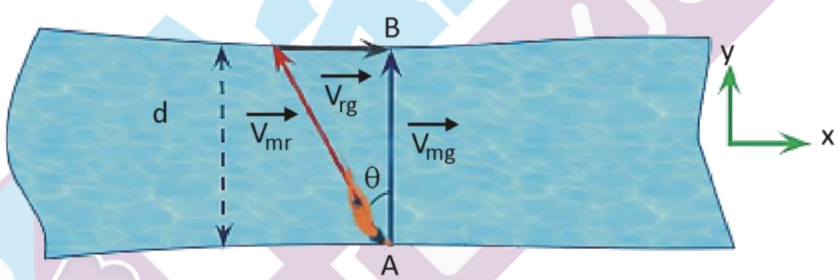
  $\overrightarrow{v_{mg}} = \overrightarrow{v_{mr}} + \overrightarrow{v_{rg}} = v\hat{j} + u\hat{i}$
- The time required to cross the river,

  $t = \dfrac{d}{v_y} = \dfrac{d}{v}$

  The drift of swimmer,

  $x = v_x \times t = u \times \dfrac{d}{v}$

- **Case-II :** When the man crosses river perpendicularly. $(v_{rg} < v_{mr})$



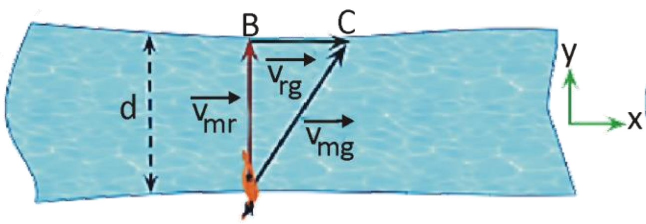
The velocity of man w.r.t ground, $(\overrightarrow{v_{mg}})$

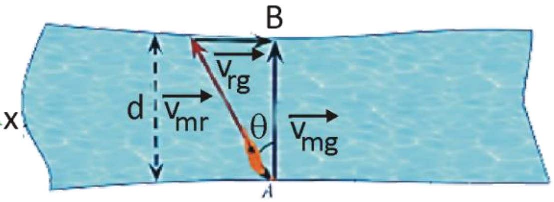
$\overrightarrow{v_{mg}} = \overrightarrow{v_{mr}} + \overrightarrow{v_{rg}} = -v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j} + u\,\hat{i} = (u - v\sin g\theta)\hat{i} + v\cos\theta\,\hat{j}$

The time required to cross the river

$t = \dfrac{d}{v_y} = \dfrac{d}{v\cos\theta}$

- Angle for zero drift $\theta = \sin^{-1}\left(\dfrac{u}{v}\right)$





Case I : Represents the case of minimum time

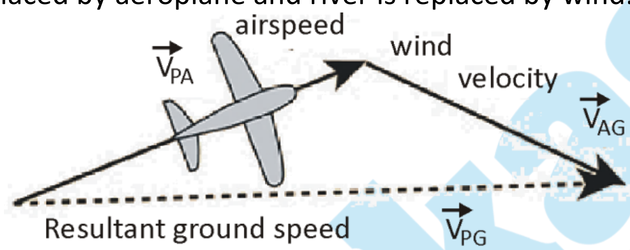
$$t_{min} = \frac{d}{v}$$

Case II : Represents the case of shortest path or zero drift.

$$\theta = \sin^{-1}\left(\frac{u}{v}\right)$$

11. Wind-Aeroplane problems :

This is very similar to boat river flow problems. The only difference is that boat is replaced by aeroplane and river is replaced by wind.



So, $V_{PG} = V_{PA} + V_{AG}$

Here V_{PA} =Aeroplane speed with respect to air.

V_{AG} =Wind velocity with respect to ground.

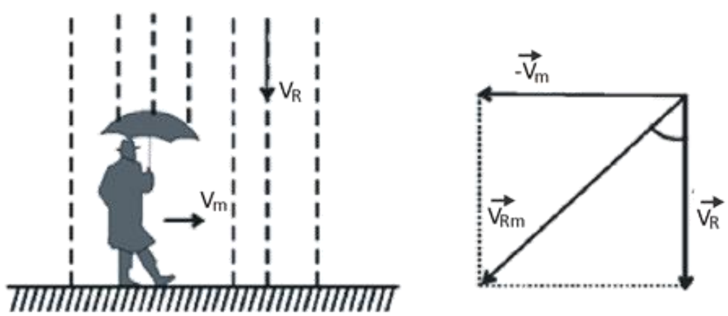
V_{PG} = Aeroplane speed with respect to ground.

12. Rain Problem :

If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :

$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$ or $v_{Rm} = \sqrt{v_R^2 + v_m^2}$

and direction = $\tan^{-1}\frac{v_m}{v_R}$ with the vertical as shown in figure.



LAWS OF MOTION

1. Force :

A push or pull that one object exerts on another.

1.1 Forces in nature

There are four fundamental forces in nature :

1. Gravitational force 2. Electromagnetic force
3. Strong nuclear force 4. Weak force

Electromagnetic force

- Electrical force
  - Coulomb force
  - Electrical force in electrical field
- Magnetic force
- Force of tension
- Spring force
- Normal reaction
- Force of friction
- Viscous force
- Buoyancy force

1.2 Types of forces on macroscopic objects :

(a) Field Forces or Range Forces:

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies.

(ii) Electrostatic force between two charges.

(b) Contact Forces:

Contact forces exist only as long as the objects are touching each other.

Ex. (i) Normal force. (ii) Frictional force

(c) Attachment to Another Body:

Tension (T) in a string and spring force (F = kx) comes in this group.

2. Newton's first law of motion (Galileo's law of Inertia) :

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Inertia : Inertia is the property of the body due to which body oppose the change in its state. Inertia of a body is measured by mass of the body.

$$\boxed{\text{inertia} \propto \text{mass}}$$

3. Momentum :

It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$

SI Unit : kg m s^{-1} **Dimensions** : $[M\, L\, T^{-1}]$

4. Newton's second law :

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} \text{ (Linear momentum } \vec{p} = m\vec{v}\text{)}$$

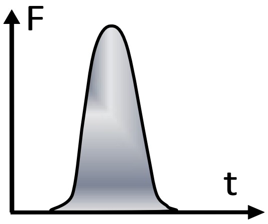
*For constant mass system $\vec{F} = m\vec{a}$

5. Impulse :

1. When a large force acts on a body for a short interval of time, the change in momentum of the body is called impulse.
2. Impulse of a force is the product of force and time interval during which it acts.
3. Impulse $\vec{I} = \vec{F}t$
4. If force $\vec{F}_1$ acts on a body for time t_1, $\vec{F}_2$ for t_2, ...then total change in momentum is equal to $\vec{F}_1 t_1 + \vec{F}_2 t_2 + \vec{F}_3 t_3 + ...$
5. Force $\vec{F}_1$ acts on a body at rest for time t_1 and then force $\vec{F}_2$ brings the body to rest in time t_2. Then $\vec{F}_1 t_1 = \vec{F}_2 t_2$
6. Impulse, change in momentum and force will be along the direction.

5.1 Force time graph and impulse :

Impulse is the area under the curve of the force versus time graph.



impulse = area under curve

Impluse = project of force with time.

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$

If constant force acts for an interval Δt then Impulse = $\vec{F}\Delta t$

5.2 Impulse - Momentum theorem

Impulse of a force is equal to the change of momentum

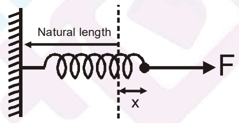
$$\boxed{\vec{F}\,\Delta t = \Delta\vec{p}}$$

6. Newton's third law of motion :

Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude and in opposite direction.

7. Spring Force (According to Hooke's law) :

In equilibrium F = kx (k is spring constant)

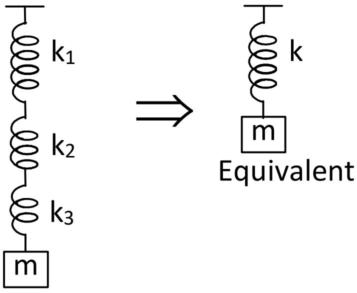


Note : Spring force is non impulsive in nature.

7.1 Combination of springs

(i) Spring in series :

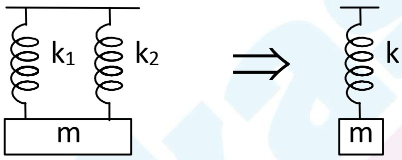
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$



where k is equivalent spring constant.

In general, $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}+...$

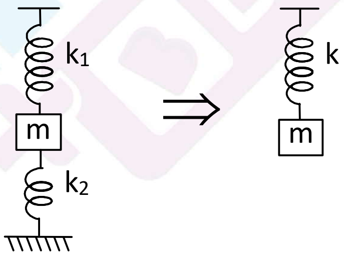
(ii) Spring in parallel :



$k=k_1+k_2$

In general, $k=k_1+k_2+k_3+...$

(iii)



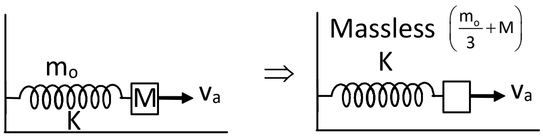
$k=k_1+k_2$

(iv) If spring of spring constant k and length l is cut into two pieces of length l_1 and l_2, then

$$k \propto \frac{1}{l} \; ; \quad k_1 \propto \frac{1}{l_1} \; ; \quad k_2 \propto \frac{1}{l_2}$$

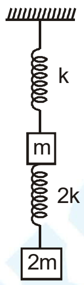
$$\boxed{kl=\text{constant}}$$

(v) If spring is massive then the effective mass of spring is $\frac{m_0}{3}$, where m_0 is mass of spring.



Massive Spring

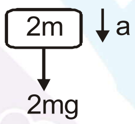
Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.



Sol.: Initial stretches $x_{upper} = 3mg/k$ and $x_{lower} = mg/k$

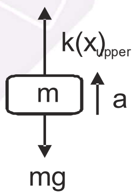
On cutting the lower spring, by virture of non-impulsive nature of spring the stretch in upper spring remains same immediately after cutting the spring, Thus,

Lower Block :



$2mg = 2ma \Rightarrow a = g$

Upper Block :



$k\left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2g$

8. Motion of bodies in contact :

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of Contact. These two bodies will move with same acceleration a.

(i) When the force F acts on the body with mass m_1 as shown in fig. (1)

$F = (m_1 + m_2)a.$

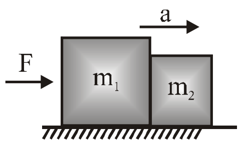


Fig.(1). When the force F acts on mass m_1

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1 : $(F - f_1) = m_1 a$
For body m_2 : $f_1 = m_2 a$

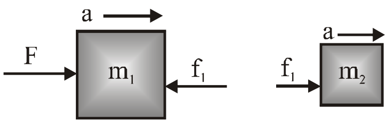


Fig.-1(a) : F.B.D. representation of action and reaction forces

For body $m_2 : f_1 = m_2 a \Rightarrow$ action of m_1 on $m_2 : f_1 = \dfrac{m_2 F}{m_1 + m_2}$

9. Dependent Motion of Connected Bodies :

Method I: Method of constraint equations

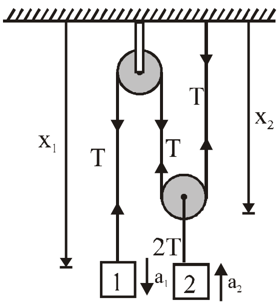
$$\sum x_i = \text{constant} \Rightarrow \sum \dot{x}_i = 0 \Rightarrow \sum \ddot{x}_i = 0$$

*For n moving bodies we have $x_1, x_2, \ldots . x_n$
*No. of constraint equations = no. of strings

Method II: Method of virtual work

The sum of scalar products of forces applied by connecting links of constant length and displacement of corresponding contact points equal to zero.

$$\boxed{\sum \vec{F}_i \cdot \delta \vec{r}_i = 0 \Rightarrow \sum \vec{F}_i \cdot \vec{v}_i = 0 \Rightarrow \sum \vec{F}_i \cdot \vec{a}_i = 0}$$



Here $2a_2 = a_1$

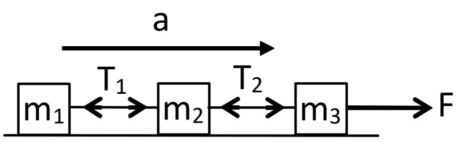
9.1 Connected bodies

1. Masses m_1, m_2, m_3 are connected by light string and are pulled as shown. Then

   For m_1 : $T_1 = m_1 a$

   For m_2 : $T_2 - T_1 = m_2 a$

   For m_3 : $F - T_2 = m_3 a$



   a) acceleration of the system $a = \frac{F}{(m_1 + m_2 + m_3)}$

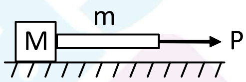
   b) $T_2 = m_1 a$

   c) $T_2 = (m_1 + m_2)a$

   (T_1 and T_2 are tensions in the strings)

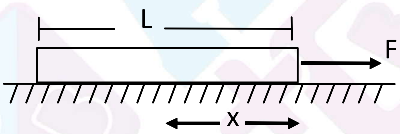
2. A block of mass M is pulled by a rope of mass m by a force P on a smooth horizontal plane.

   Acceleration of the block $a = \frac{P}{(M+m)}$



   Force exerted by the rope on the block is $\frac{MP}{(M+m)}$

3. A rope of length L (uniform rope) is pulled by a constant force F. The tension in the rope at a distance x from the end where the force applied is $F\left(1 - \frac{x}{L}\right)$.

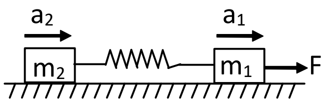


4. If two bodies are connected by a spring and the system is pulled as shown, then accelerations of both bodies are different.

   for m_1, $F - f = m_1 a_1$

   for m_2, $f = m_2 a_2$

   (f is spring force)



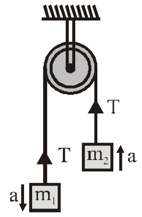
9.2 Pulley system

A single fixed pulley changes the direction of force only and in general assumed to be mass-less and frictionless.

(a) Let $m_1 > m_2$

now for mass m_1, $m_1 g - T = m_1 a$

for mass m_2, $T - m_2 g = m_2 a$



$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g \quad \& \quad T = \frac{2m_1 m_2}{(m_1 + m_2)} g$$

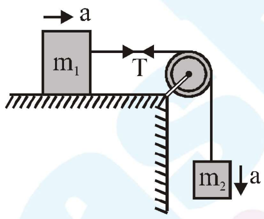
$$\text{Acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

Reaction at the suspension of pulley $R = 2T = \dfrac{4m_1 m_2 g}{(m_1 + m_2)}$

(b) For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2 g - T = m_2 a$



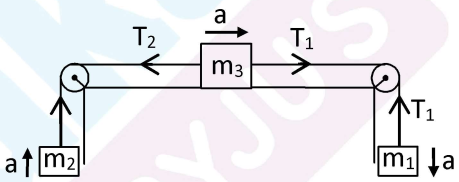
Acceleration $a = \dfrac{m_2 g}{(m_1 + m_2)}$

and $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$

(c) For m_1 : $m_1 g - T_1 = m_1 a$

For m_2 : $T_2 - m_2 g = m_2 a$

For m_3 : $T_1 - T_2 = m_3 a$



$\Rightarrow$ Acceleration of the system is

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + m_3}$$

9.3 Monkey climbing a rope

Case-I : When monkey moves up with constant speed :

In this case, $T = mg$

whee m = mass of the monkey,

T = tension in the rope.

Case-II : When monkey is accelerated upwards

$T - mg = ma$

Case-III : When monkey accelerated downwards,

$mg - T = ma$

10. Frame of Reference :

Any co-ordinate system relative to which the motion, position of a body can be described is called a frame of reference.

- **Inertial frames of reference :** A reference frame which is either at rest or in uniform motion along the straight line. A non-accelerating frame of reference is called an inertial frame of reference.

(i) All the fundamental laws of physics have been formulated in respect of inertial frame of reference.

(ii) In inertial frame, acceleration of a body is caused by real forces.

(iii) We may consider the earth as inertial frame if we neglect the small acceleratin due to rotation of the earth around itself and around the sun.

(iv) In inertial frame events are described by real forces only :

- **Non-inertial frame of reference :** An accelerating frame of reference is called a non-inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force

(i) All the accelerated and rotating frames are non-inertial frames of reference.

(ii) Even if no external force is acting on a particle, in the accelerated frame, it will appear that a force is acting on it which is known as pseudo force or fictitious force.

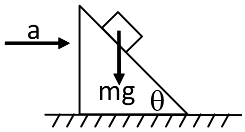
(iii) In non inertial frame events are described by both real and pseudo forces.

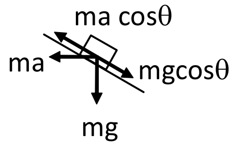
11. Pseudo force:

The force on a body due to acceleration of non-inertial frame is called ficti tious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$ · where $\vec{a}_0$ is acceleration of non-iner tial frame with respect to an inertial frame and m is mass of the particle or body, The direction of pseudo force must be opposite to the direction of acceleration of the non inertial frame.

When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a noninertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

(i) An object kept on a smooth inclined plane can be kept stationary to the incline by giving a horizontal acceleration of g tan θ as shown.





Here ma cosθ = mg sinθ

a = g tanθ

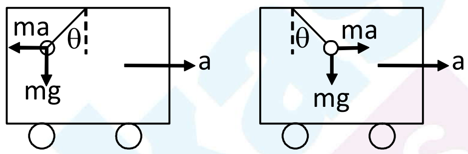
(Force ma on the object is inertial or pseudo force)

Here F=(M+m)a=(M+m)gtanq (M

(ii) A pendulum is suspended from the roof a moving car.

a) If the car is moving with uniform velocity, the position of the bob doe not change.

b) If the car moves with accleration or retardation, the bob moves in opposite direc tion to that of acceleration.

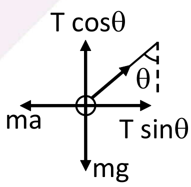


The forces acting on the bob are, weight mg, tension in the string T and pseudo force ma

For the equilibrium of bob.

T sinθ = ma and T cosθ=mg

$\Rightarrow$ tanθ ==a/g or a=g tanθ and T=m $\sqrt{a^2+g^2}$



(iii) In the above case if the car is moving along a horizontal circular track, pseudo force is mrω^2

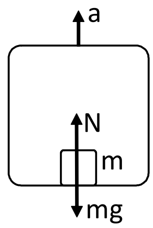
Then $\tan\theta = \frac{ma}{mg} = \frac{mr\omega^2}{mg}$ or rω^2=g tanθ

Here mrω^2 centrifugal force.

12. Man in a Lift :

(a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated' motion hence no pseudo force experienced by observer inside the lift. So apparent weight W'=Mg=Actual weight.

(b) If the lift is accelerated upward with constant acceleration a. Then forces acting on the man w.r.t. observed inside the lift are

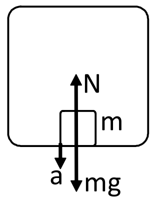


(i) Weight W=Mg downward

(ii) Fictitious force F_0=Ma downward.

So apparent weight $W'=W+F_0=Mg+Ma=M(g+a)$

(c) If the lift is accelerated downward with acceleration a < g.



Then w. r.t. observer inside the lift fictitious force F_0 =Ma acts upward while weight of man W =Mg always acts downward.

So apparent weight $W' = W - F_0 = Mg - Ma = M(g-a)$

Special Case:

If a = g then W' =0 (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration a > g . Then as in Case (c). Apparent weight W' =M(g-a) is negative, i.e., the man will be accelerated up ward and will stay at the ceiling of the lift.

13. Variable Mass System :

1. For any variable mass system force is rate of change of momentum $\Rightarrow F = \left(\frac{dm}{dt}\right)v$

2. If the fuel burns in a rocket at the rate of $\left(\frac{dm}{dt}\right)$ and hot gases eject with a speed V relative to rocket, thrust on the rocket is $F = \left(\frac{dm}{dt}\right)V$

   here $\frac{dm}{dt}$ is mass rat of fuel burnt.

If the rocket moves up with an acceleration 'a', then $\left(\frac{dm}{dt}\right)V = M(g+a)$ here M is mass of the rocket.

If a = 0, here $\left(\frac{dm}{dt}\right)V = Mg$.

3. Gravel is dropped on a conveyor belt at the rate of $\frac{dm}{dt}$. Extra force required to keep the belt moving with velocity V is $\left(\frac{dm}{dt}\right)V$.
4. A jet of water from a tube of area of cross section A strikes a wall normally with a velocity V and then flows down along the wall.

   The force exerted on the wall is $F = AV^2\rho$

   If water bounces back with velocity u, then $F = A\rho V(v+u)$

   If water bounce back with same velocity V, then $F = 2A\rho V^2$

   If water strick at an angle θ to the surface of wall and reflects, then $F=2A\rho V^2\sin\theta$

KEY POINTS

- Aeroplanes always fly at low altitudes because according to Newton's III law of motion as aeroplane displaces air & at low altitude density of air is high.
- Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.
- Pulling a lawn roller is easier than pushing it because pushing increases the apparent weight and hence friction.