Topic : Unit and Dimension

1. If e is the electric charge of an electron, c is the speed of light in free space, and h is planck's constant, ϵ_0 is permittivity of free space. The dimension of $\frac{e^2}{4\pi\epsilon_0 hc}$ is

   A. $[LC^{-1}]$

   B. $[M^0L^0T^0]$

   C. $[MLT^0]$

   D. $[MLT^{-1}]$

Given,
$e =$ Electric charge of an electron $= [TI]$
$c =$ Speed of Light $= [M^0L^1T^{-1}]$
$h =$ Planck's constant $= [ML^2T^{-1}]$
$\epsilon_0 = [M^{-1}L^{-3}T^4I^2]$
Therefore dimension of $\frac{e^2}{4\pi\epsilon hc}$ is $[M^0L^0T^0]$

2. The work done by a gas molecule in an isolated system is given by,

$$W = \alpha\beta^2 e^{-x^2/\alpha kT}$$

Where x is the displacement, k is the Boltzmann constant, T is the temperature, α and β are constants, then the dimension of β will be :

A. $[M^0LT^0]$

B. $[M^2LT^2]$

C. $[MLT^{-2}]$

D. $[ML^2T^{-2}]$

Given:
Work done, $W = \alpha\beta^2 e^{-x^2/\alpha kT}$

Where,
$x =$ Displacement
$k =$ Boltzmann constant
$T =$ Temperature
$\alpha, \beta =$ Constants

We know that,
$-\dfrac{x^2}{\alpha kT} =$ dimensionless

$$\Rightarrow \left[\frac{x^2}{\alpha kT}\right] = [M^0L^0T^0]$$

$[kT] = [PV] = [ML^2T^{-2}]$
$[x^2] = [L^2]$

$\therefore [\alpha] = [M^{-1}T^2]$

Also,
$[W] = [\alpha\beta^2]$
$\Rightarrow [ML^2T^{-2}] = [M^{-1}T^2][\beta^2]$
$\Rightarrow [\beta^2] = [M^2L^2T^{-4}]$
$\Rightarrow [\beta] = [MLT^{-2}]$

3. Match List - I with List - II.

List - I	List - II
$a.\ h$ (Planck's Constant)	$i.\ [MLT^{-1}]$
$b.\ E$ (Kinetic Energy)	$ii.\ [ML^2T^{-1}]$
$c.\ V$ (Electric Potential)	$iii.\ [ML^2T^{-2}]$
$d.\ P$ (Linear Momentum)	$iv.\ [ML^2I^{-1}T^{-3}]$

Choose the correct answer from the options given below.

A. $(a) \rightarrow (ii), (b) \rightarrow (iii), (c) \rightarrow (iv), (d) \rightarrow (i)$

B. $(a) \rightarrow (i), (b) \rightarrow (ii), (c) \rightarrow (iv), (d) \rightarrow (iii)$

C. $(a) \rightarrow (iii), (b) \rightarrow (ii), (c) \rightarrow (iv), (d) \rightarrow (i)$

D. $(a) \rightarrow (iii), (b) \rightarrow (iv), (c) \rightarrow (ii), (d) \rightarrow (i)$

(i). Planck Constant, $h = \frac{E}{f}$

$\Rightarrow [h] = \left[\frac{E}{f}\right] = [ML^2T^{-1}]$

(ii). Kinetic Energy, $E = \frac{1}{2}mv^2$

$\Rightarrow [E] = \left[\frac{1}{2}mv^2\right] = [ML^2T^{-2}]$

(iii). Electric Potential, $V = \frac{W}{q} = \frac{W}{It}$

$\Rightarrow [V] = \left[\frac{W}{It}\right] = [ML^2I^{-1}T^{-3}]$

(iv). Linear Momentum, $P = mv$

$\Rightarrow [P] = [mv] = [MLT^{-1}]$

4. The pitch of the screw gauge is $1 \mathrm{~mm}$ and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72^{nd} division on circular scale coincides with the reference line. The radius of the wire is :

A. $1.64 \mathrm{~mm}$

B. $1.80 \mathrm{~mm}$

C. $0.82 \mathrm{~mm}$

D. $0.90 \mathrm{~mm}$

Least count, $\mathrm{LC}=\dfrac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

$\Rightarrow \mathrm{LC}=\dfrac{1}{100}=0.01 \mathrm{~mm}$

Also, positive zero error $e=+8 \times \mathrm{LC}=+8 \times 0.01=+0.08 \mathrm{~mm}$

Now,
True reading
$=$ Measured reading $-e$
$=1+72 \times 0.01-0.08=1.64 \mathrm{~mm}$

Hence, radius $=\dfrac{1.64}{2}=0.82 \mathrm{~mm}$

5. In a typical combustion engine, the work done by a gas molecule is given by,

$$W = \alpha^2 \beta e^{-\beta x^2/kT}$$

Where x is the displacement, k is the Boltzmann constant, T is the temperature, α and β are constants, then the dimension of α will be :

A. $[M^0 L T^0]$

B. $[M^2 L T^0]$

C. $[M L T^{-2}]$

D. $[M L^2 T^{-2}]$

Given:
Work done, $W = \alpha^2 \beta e^{-\beta x^2/kT}$

Where,
$x =$ Displacement
$k =$ Boltzmann constant
$T =$ Temperature
$\alpha, \beta =$ Constants

We know that,
$-\dfrac{\beta x^2}{kT} =$ dimensionless

$$\Rightarrow \left[\frac{\beta x^2}{kT}\right] = [M^0 L^0 T^0]$$

$[kT] = [PV] = [M L^2 T^{-2}]$
$[x^2] = [L^2]$

$\therefore [\beta] = [M^1 T^{-2}]$

Also,
$[W] = [\alpha^2 \beta]$
$\Rightarrow [M L^2 T^{-2}] = [M^1 T^{-2}][\alpha^2]$
$\Rightarrow [\alpha^2] = [L^2]$
$\Rightarrow [\alpha] = [M^0 L T^0]$

6. If C and V represent capacitance and voltage respectively then what is the dimension of λ where $\lambda = \frac{C}{V}$?

A. $[M^{-2}L^{-4}I^{3}T^{7}]$

B. $[M^{-2}L^{-3}I^{2}T^{6}]$

C. $[M^{-1}L^{-3}I^{-2}T^{-7}]$

D. $[M^{-3}L^{-4}I^{3}T^{7}]$

We know that,

$$V = \frac{w}{q} \text{ and } C = \frac{q}{V} = \frac{q^2}{w}$$

$$\text{So, } \lambda = \frac{C}{V} = \frac{\frac{q^2}{w}}{\frac{w}{q}}$$

$$\Rightarrow \lambda = \frac{q^3}{w^2}$$

$$\Rightarrow \lambda = \frac{I^3t^3}{w^2} \quad [\because I = q/t]$$

$$\Rightarrow [\lambda] = \left[\frac{I^3t^3}{w^2}\right]$$

$$\Rightarrow [\lambda] = \left[\frac{I^3T^3}{(ML^2T^{-2})^2}\right]$$

$$\Rightarrow [\lambda] = [M^{-2}L^{-4}I^{3}T^{7}]$$

7. In order to determine the Young`s Modulus of a wire of radius $0.2\ \mathrm{cm}$ (measured using a scale of least count $= 0.001\ \mathrm{cm}$) and length $1\ \mathrm{m}$ (measured using a scale of least count $= 1\ \mathrm{mm}$), a weight of mass $1\ \mathrm{kg}$ (measured using a scale of least count $= 1\ \mathrm{g}$) was hanged to get the elongation of $0.5\ \mathrm{cm}$ (measured using a scale of least count $0.001\ \mathrm{cm}$). What will be the fractional error in the value of Young`s Modulus determined by this experiment.

A. 9%

B. 1.4% ✔

C. 0.9%

D. 0.14%

In this case, Young's modulus will be given as:

$$\gamma = \frac{mgL}{A.l}$$

$$\frac{\Delta\gamma}{\gamma} = \left(\frac{\Delta m}{m}\right) + \left(\frac{\Delta g}{g}\right) + \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta l}{l}\right) + \left(\frac{\Delta L}{L}\right)$$

$$\frac{\Delta\gamma}{\gamma} = \left(\frac{1\ \mathrm{g}}{1\ \mathrm{kg}}\right) + 0 + 2\left(\frac{\Delta r}{r}\right) + \left(\frac{\Delta l}{l}\right) + \left(\frac{\Delta L}{L}\right)$$

$$= \left(\frac{1\ \mathrm{g}}{1\ \mathrm{kg}}\right) + 2\left(\frac{0.001\ \mathrm{cm}}{0.2\ \mathrm{cm}}\right) + \left(\frac{0.001\ \mathrm{cm}}{0.5\ \mathrm{cm}}\right) + \left(\frac{0.001\ \mathrm{m}}{1\ \mathrm{m}}\right)$$

$$= \left(\frac{1}{1000}\right) + 2\left(\frac{1 \times 10}{2 \times 10^3}\right) + \left(\frac{1}{5} \times \frac{10}{10^3}\right) + \left(\frac{1}{10^3}\right)$$

$$= \frac{1}{1000} + \frac{1}{100} + \frac{2}{10^3} + \frac{1}{10^3}$$

$$= \frac{1 + 10 + 2 + 1}{1000} = \frac{14}{1000}$$

$$= 1.4\%$$

8. The vernier scale used for measurement has a positive zero error of $0.2\ \mathrm{mm}$. If while taking a measurement it was noted that 0 on the vernier scale lies between $8.5\ \mathrm{cm}$ and $8.6\ \mathrm{cm}$, vernier coincidence is 6, then the correct value of measurement is ________cm.

(Least count $=\ 0.01\ \mathrm{cm}$)

A. $8.36\ \mathrm{cm}$

B. $8.56\ \mathrm{cm}$

C. $8.58\ \mathrm{cm}$

D. $8.54\ \mathrm{cm}$

Correct Reading
$= (\mathrm{MSR} + \mathrm{VSR} \times \mathrm{LC}) - \text{Zero Error}$

$= (8.5 + 6 \times 0.01) - 0.2 \times 10^{-1}$

$= 8.54\ \mathrm{cm}$

9. In the experiment of Ohm's law, a potential difference of $5.0 \mathrm{~V}$ is applied across the end of a conductor of length $10.0 \mathrm{~cm}$ and diameter of $5.00 \mathrm{~mm}$. The measured current in the conductor is $2.00 \mathrm{~A}$. The maximum permissible percentage error in the resistivity of the conductor is:

A. 7.5

B. 3.9

C. 8.4

D. 3.0

We know that $V = IR$ and $R = \rho \frac{l}{A}$

So, $V = I\left(\frac{\rho l}{A}\right)$

$\Rightarrow \rho = \frac{VA}{Il}$

Also, $A = \frac{\pi d^2}{4}$

So $\Rightarrow \rho = \frac{V\pi d^2}{4Il}$

Now, $\frac{\Delta\rho}{\rho} = \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$

Since $V = 5.0 \mathrm{~V}$ There is one zero after decimal, so $\Delta V = 0.1 \mathrm{~V}$

$$\frac{\Delta\rho}{\rho} = \frac{0.1}{5} + \frac{0.01}{2} + \frac{0.1}{10} + 2 \times \frac{0.01}{5}$$

$$\frac{\Delta\rho}{\rho} = 0.02 + 0.005 + 0.01 + 0.004$$

$$\frac{\Delta\rho}{\rho} = 0.039$$

$$\frac{\Delta\rho}{\rho} \times 100 = 0.039 \times 100 = 3.9\%$$

10. The time period of a simple pendulum is given by $T=2\pi\sqrt{\frac{l}{g}}$. The measured value of the length of pendulum is $10\ \mathrm{cm}$ known to $1\ \mathrm{mm}$ accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of $1\ \mathrm{s}$ resolution. The percentage accuarcy in the determination of g using this pendulum is x. The value of x to the nearest integer is.

A. 5%

B. 4%

C. 3%

D. 2%

$T=2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow g=4\pi^2\frac{l}{T^2}$

Now, $\frac{\Delta g}{g}=\frac{\Delta l}{l}+\frac{2\Delta T}{T}$

$\Rightarrow \frac{\Delta g}{g}=\frac{1\times 10^{-3}}{10\times 10^{-2}}+\frac{2\times 1}{100}$

$\Rightarrow \frac{\Delta g}{g}=0.01+0.02=0.03$

$\Rightarrow \frac{\Delta g}{g}\times 100=0.03\times 100=3\%$

11. The radius of a sphere is measured to be $(7.50 \pm 0.85)\ \mathrm{cm}$. Suppose the percentage error in its volume is $x\%$. The value of x, to the nearest integer, is ______.

Accepted Answers

34 34.0 34.00

Solution:

We know that, the volume of the sphere,

$V = \frac{4}{3}\pi r^3$

$\Rightarrow \ln V = 3 \ln \frac{4}{3}\pi r$

$\Rightarrow \frac{\Delta V}{V} = 3 \times \frac{\Delta r}{r}$

$\Rightarrow \frac{\Delta V}{V} \times 100 = 3 \times \frac{\Delta r}{r} \times 100$

$= 3 \times \left(\frac{0.85}{7.50}\right) \times 100 = 34\%$

12. Match List-I with List-II

List-I	List-II
(a) Capacitance, C	(i) $M^1L^1T^{-3}A^{-1}$
(b) Permitivity of free space, ϵ_0	(ii) $M^{-1}L^{-3}T^4A^2$
(c) permeabilityof free space, μ_0	(iii) $M^{-1}L^{-2}T^4A^2$
(d) electric field, E	(iv) $M^1L^1T^{-2}A^{-2}$

Choose the correct answer from the options given below.

- **A.** $(a) \rightarrow (iii), (b) \rightarrow (ii), (c) \rightarrow (iv), (d) \rightarrow (i)$
- **B.** $(a) \rightarrow (iii), (b) \rightarrow (iv), (c) \rightarrow (ii), (d) \rightarrow (i)$
- **C.** $(a) \rightarrow (iv), (b) \rightarrow (ii), (c) \rightarrow (iii), (d) \rightarrow (i)$
- **D.** $(a) \rightarrow (iv), (b) \rightarrow (iii), (c) \rightarrow (ii), (d) \rightarrow (i)$

We know that, $Q = CV$

$$[C] = \left[\frac{Q}{V}\right] = \frac{[A \times T]}{[ML^2T^{-3}A^{-1}]}$$

$= [M^{-1}L^{-2}T^4A^2]$

Similarly,

$$[E] = \left[\frac{F}{q}\right] = \frac{[MLT^{-2}]}{[AT]}$$

$= [MLT^{-3}A^{-1}]$

From the Coloumb's law, we have

$$F = \frac{q_1q_2}{4\pi\epsilon_0 r^2}$$

$$[\epsilon_0] = \frac{[q_1q_2]}{[Fr^2]} = \frac{[AT]^2}{[MLT^{-2}][L^2]}$$

$= [M^{-1}L^{-3}T^4A^2]$

We know that, speed of light $c = \dfrac{1}{\sqrt{\mu_0\epsilon_0}}$

$$\mu_0 = \frac{1}{\epsilon_0 c^2}$$

$$[\mu_0] = \frac{1}{[M^{-1}L^{-1}T^4A^2]\,[LT^{-1}]^2}$$

$= \left[M^1L^1T^{-2}A^{-2}\right]$

Hence, option (a) is corrcet.

13.

A physical quantity $'y'$ is represented by the formula $y = m^2 r^{-4} g^x l^{-\frac{3}{2}}$. If the percentage errors found in y, m, r, l and g are $18, 1, 0.5, 4$ and p respectively, then find the value of x and p

A. 5 and ± 2

B. 4 and ± 3

C. $\frac{16}{3}$ and $\pm \frac{3}{2}$

D. 8 and ± 2

Given, $y = m^2 r^{-4} g^x l^{-\frac{3}{2}}$

As we know that, relative errors can be written as

$$\frac{\Delta y}{y} = \frac{2 \triangle m}{m} + \frac{4 \triangle r}{r} + \frac{x \triangle g}{g} + \frac{3 \triangle l}{2 \quad l}$$

On multiplying each term by 100, each term will represent percentage error.

So, putting the given values in the above equation we get,

$$18 = 2(1) + 4(0.5) + xp + \frac{3}{2}(4)$$

$\Rightarrow \ 18 = 10 + xp$

$\Rightarrow \ 8 = xp$

By trial and error, only option (c) is valid for this scenario. Thus,

$$x = \frac{16}{3}, p = \pm \frac{3}{2}$$

14. In a screw gauge, the fifth division of the circular scale coincides with the reference line, when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation, the reading on the main scale is 5 mm and the 20^{th} division of the circular scale coincides with the reference line. Calculate the true reading.

A. 5.20 mm

B. 5.25 mm

C. 5.15 mm

D. 5.00 mm

The distance covered by the main scale in one complete rotation is equal to pitch.
$\therefore \quad \text{pitch} = 0.5 \text{ mm}$

The least count of the screw gauge is,

$$LC = \frac{\text{pitch of the screw}}{\text{total no. of div. on circular scale}}$$

$$\therefore LC = \frac{0.5}{50} = 0.01 \text{ mm}$$

$$\text{Zero error} = (CSR \times LC) = 5 \times 0.01 = 0.05 \text{ mm}$$

Now, the fifth division of the circular scale coincides with the reference line, when the ratchet is closed.
$\Rightarrow$ The zero error should be subtracted from the reading obtained, to get the true reading.

$$\therefore \text{True reading} = MSR + (CSR \times LC) - \text{zero error}$$

$$= 5 + (20 \times 0.01) - 0.05 = 5.15 \text{ mm}$$

Hence, option (C) is correct.

15. Assertion A : If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R :

$$\text{Least Count} = \frac{\text{Pitch}}{\text{Total divisions on circular scale}}$$

In the light of the above statements, choose the most appropriate answer from the options given below :

- **A.** A is not correct but R is correct.
- **B.** Both A and R are correct and R is the correct explanation of A.
- **C.** A is correct but R is not correct.
- **D.** Both A and R are correct and R is NOT the correct explanation of A.

For the screw gauge, the least count (LC) is given by the following formula,

$$\text{Least Count} = \frac{\text{Pitch}}{\text{total division on circular scale}}$$

In 5 revolution, distance travel, 5 mm

So, 1 revolution, it will travel 1 mm.

Therefore, least count of screw gauge will be,

$$\text{LC} = \frac{1}{50} = 0.02 \text{ mm} = 0.002 \text{ cm}$$

It means the mentioned reason is correct, but the assertion is incorrect for the same.

Therefore option (A) is correct.

16. The entropy of any system is given by, $S = \alpha^2 \beta \ln\left[\frac{\mu k R}{J \beta^2} + 3\right]$ where α and β are the constants. μ, J, k and R are the number of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively. Choose the incorrect option.

Take $S = \frac{dQ}{T}$

A. α and J have the same dimension.

B. S, β, k and μR have the same dimension.

C. S and α have different dimensions.

D. α and k have the same dimension. ✔

$$S=\alpha^2\beta\ln\left[\frac{\mu kR}{J\beta^2}+3\right]$$

$$\therefore\left[\frac{\mu kR}{J\beta^2}\right]=\text{Dimensionless}\quad\ldots(1)$$

$$\alpha^2\beta=S=\frac{dQ}{T}\Rightarrow[S]=[\alpha^2\beta]=[\mathrm{JK}^{-1}]$$

$$\Rightarrow[\alpha^2\beta]=[\mathrm{JK}^{-1}]\quad\ldots(2)$$

Further,

$PV=\mu RT\Rightarrow$ the unit of $[\mu RT]$ is Joules.

$$\Rightarrow[R]=[\mathrm{JK}^{-1}\mathrm{mol}^{-1}]$$

And, $[\mu R]=[\mathrm{JK}^{-1}]$

Also,

$$k=\frac{R}{N_A}$$

$$\Rightarrow[k]=[\mathrm{JK}^{-1}]$$

From (1), as J is dimensionless,

$$\Rightarrow[\beta]=[\mathrm{JK}^{-1}]\quad\ldots(3)$$

From (2) and (3),

$$[\alpha]=[\text{Dimensionless}]$$

Hence, option (D) is the correct answer.

17. If time (t), velocity (v), and angular momentum (l) are taken as the fundamental units. Then the dimension of mass (m) in terms of $t,\ v$ and l is :

A. $[t^{-1}v^{1}l^{-2}]$

B. $[t^{1}v^{2}l^{-1}]$

C. $[t^{-2}v^{-1}l^{1}]$

D. $[t^{-1}v^{-2}l^{1}]$

Given:
$t,\ v$ and l are the fundamental units.

$m\ \propto\ t^{a}v^{b}l^{c}$

Using dimensions

$[M^{1}L^{0}T^{0}]\ =\ [T]^{a}[LT^{-1}]^{b}[ML^{2}T^{-1}]^{c}$

$M^{1}L^{0}T^{0} = M^{c}\,L^{b+2c}\,T^{a-b-c}$

After comparing powers, we get

$c = 1,\ b = -2,\ a = -1$

$\therefore m\ = [\,t^{-1}v^{-2}l^{1}]$

Hence, option (D) is correct.

18. The force is given in terms of time t and displacement x by the equation $F = A\cos Bx + C\sin Dt$. The dimensional formula of $\frac{AD}{B}$ is:

A. $[M^0LT^{-1}]$

B. $[ML^2T^{-3}]$

C. $[M^1L^1T^{-2}]$

D. $[M^2L^2T^{-3}]$

The argument of the trigonometric functions should be dimensionless.

$\therefore$ The dimensions of

$[B] = [L^{-1}]$

$[D] = [T^{-1}]$

As the trigonometric functions are dimensionless,

A and C should have the dimension of force.

$\therefore$ The dimension of

$[A] = [MLT^{-2}]$

$$\Rightarrow \left[\frac{AD}{B}\right] = \frac{[MLT^{-2}][T^{-1}]}{[L^{-1}]}$$

$$\Rightarrow \left[\frac{AD}{B}\right] = [ML^2T^{-3}]$$

Hence, option (B) is correct.

19. If E, M, l and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = E\,l^2 M^{-5} G^{-2}$ are

A. $[M^1 L^1 T^{-2}]$

B. $[M^{-1} L^{-1} T^2]$

C. $[M^0 L^1 T^0]$

D. $[M^0 L^0 T^0]$

From the question, following are the dimensions,

Energy, $[E] = [ML^2T^{-2}]$,
Mass $[m] = [M]$,
Angular momentum $[l] = [ML^2T^{-1}]$,
Gravitational constant $[G] = [M^{-1}L^3T^{-2}]$

So, for $P = El^2M^{-5}G^{-2}$

We have,

$$[P] = [ML^2T^{-2}] \times [ML^2T^{-1}]^2 \times [M]^{-5} \times [M^{-1}L^3T^{-2}]^{-2}$$

Combining like terms,

$$[P] = [M^{1+2-5+2} L^{2+4-6} T^{-2-2+4}]$$

$$[P] = [M^0 L^0 T^0]$$

It shows that, P is a dimensionless quantity.

20. Match List - I with List - II :

	List - I		List - II
(a)	Magnetic Induction	(i)	$ML^2T^{-2}A^{-1}$
(b)	Magnetic Flux	(ii)	$M^0L^{-1}A$
(c)	Magnetic Permeability	(iii)	$MT^{-2}A^{-1}$
(d)	Magnetization	(iv)	$MLT^{-2}A^{-2}$

Choose the most appropriate answer from the options given below:

A. $(a)-(ii),(b)-(iv),(c)-(i),(d)-(iii)$

B. $(a)-(ii),(b)-(i),(c)-(iv),(d)-(iii)$

C. $(a)-(iii),(b)-(i),(c)-(iv),(d)-(ii)$

D. $(a)-(iii),(b)-(ii),(c)-(iv),(d)-(i)$

Dimensions of the quantities given in list- I are :
Magnetic induction : $MT^{-2}A^{-1}$
Magnetic flux : $ML^2T^{-2}A^{-1}$
Magnetic permeability : $MLT^{-2}A^{-2}$
Magnetization : $M^0L^{-1}A$

Hence, the correct match is :
$(a)-(iii),(b)-(i),(c)-(iv),(d)-(ii)$

21. Which of the following is not a dimensionless quantity?

A. Quality factor

B. Power factor

C. Relative magnetic permeability (μ_r)

D. Permeability of free space (μ_0)

Permeability of free space is not a dimensionless quantity.

22. If E and H represents the intensity of electric field and magnetizing field respectively, then the unit of E/H will be:

A. mho

B. ohm

C. joule

D. newton

Unit of E is V/m and unit of H is A/m.
Hence, the unit of E/H is :

$$\frac{V/m}{A/m} = \frac{V}{A} = \Omega = \text{ohm}$$

23. Match List - I with List - II.

	List _ I		List - II
(a)	R_H(Rydberg constant)	(i)	$kgm^{-1}s^{-1}$
(b)	h(Planck's constant)	(ii)	kgm^2s^{-1}
(c)	μ_B(Magnetic field energy density)	(iii)	m^{-1}
(d)	η(coefficient of viscocity)	(iv)	$kgm^{-1}s^{-2}$

Choose the most appropriate answer from the options given below :

A. $(a)-(ii),(b)-(iii),(c)-(iv),(d)-(i)$

B. $(a)-(iii),(b)-(ii),(c)-(i),(d)-(iv)$

C. $(a)-(iv),(b)-(ii),(c)-(i),(D)-(iii)$

D. $(a)-(iii),(b)-(ii),(c)-(iv),(d)-(i)$

The units of given quantities are :
R_H(Rydberg constant) : m^{-1}
h(Planck's constant) : kgm^2s^{-1}
μ_B(Magnetic field energy density): $kgm^{-1}s^{-2}$
η(coefficient of viscocity) : $kgm^{-1}s^{-1}$
Hence, the correct answer is :
$(a)-(iii),(b)-(ii),(c)-(iv),(d)-(i)$

24. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density :

A. $[FL^{-3}T^{3}]$

B. $[FL^{-5}T^{2}]$

C. $[FL^{-4}T^{2}]$

D. $[FL^{-3}T^{2}]$

Let the dimensions of the density in new unit is :

$[\text{Density}] = [F^{a}L^{b}T^{c}]$

$[ML^{-3}] = [M^{a}L^{a}T^{-2a}L^{b}T^{c}]$

$[ML^{-3}] = [M^{a}L^{a+b}T^{c-2a}]$

From the above equation, we can say that :

$a = 1$

$a + b = -3 \Rightarrow b = -4$

$c - 2a = 0 \Rightarrow c = 2$

Therefore, the new dimensions of density is :

$[FL^{-4}T^{2}]$

25. If velocity $[\mathrm{V}]$ time $[\mathrm{T}]$ and force $[\mathrm{F}]$ are chosen as the base quantities, the dimensions of the mass will be

A. $\left[\mathrm{F\ T^{-1}\ V^{-1}}\right]$

B. $[\mathrm{F\ V\ T^{-1}}]$

C. $[\mathrm{F\ T^{2}\ V}]$

D. $\left[\mathrm{F\ T\ V^{-1}}\right]$

According to Newton's second law of motion, force is given by,
$F = ma$
where m denotes mass and a denotes acceleration

Applying dimensional analysis
$[\mathrm{F}] = [\mathrm{m}][\mathrm{a}] \ldots (i)$

We know that acceleration is defined as rate of change of velocity
i.e $a = \dfrac{\text{velocity}}{\text{time}} = \dfrac{V}{T}$

Dimensional formula for acceleration is $[\mathrm{a}] = \dfrac{[\mathrm{V}]}{[\mathrm{T}]} \ldots (ii)$

By substituting (ii) in (i)

$$[\mathrm{F}] = [\mathrm{m}]\frac{[\mathrm{V}]}{[\mathrm{T}]}$$

$$[\mathrm{m}] = [\mathrm{F}]\frac{[\mathrm{T}]}{[\mathrm{V}]}$$

$$[\mathrm{m}] = [\mathrm{F\ T\ V^{-1}}]$$

Topic : Motion in 1-D

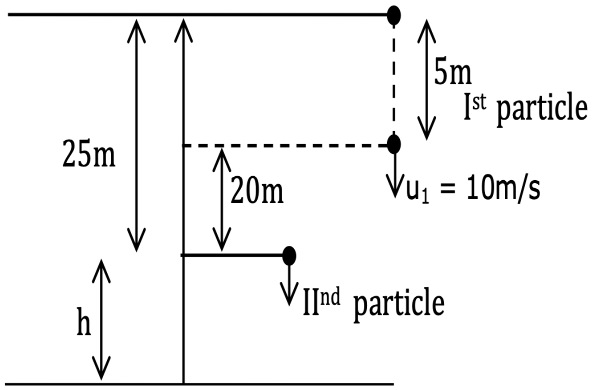
1. A particle is dropped from the top of a building . When it crosses a point $5 \mathrm{~m}$ below the top , another particles starts to fall from a point $25 \mathrm{~m}$ below the top , both particles reach the bottom of the building simultaneously. The height of the building is :
$\left[g=10 \mathrm{~m} / \mathrm{s}^{2}\right]$

A. $45 \mathrm{~m}$

B. $35 \mathrm{~m}$

C. $25 \mathrm{~m}$

D. $50 \mathrm{~m}$



Let the speed of the particle 1 be u_1 , 5 m below the top of the building .
Using $v^2 - u^2 = 2\,a\,s$

$v^2 = (\,2\,)\,(\,10\,)\,(\,5\,) \qquad [\,u = 0\,]$

$v = 10 \text{ m/s}$

For particle 1 using second kinematic equation we have :

$20 + h = 10t + g\,t^2 \ldots\ldots\ldots\ldots(1)$

For particle 2 , using second kinematic equation we have :

$h = g\,t^2 \ldots\ldots\ldots\ldots\ldots(2)$

Using equations (1) and (2) we have

$20 + gt^2 = 10t + gt^2$

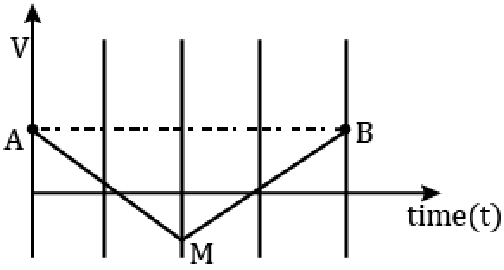
$t = \;2 \text{ sec}$

Using this value in equation (1) we have
$h = 20 \text{ m}$

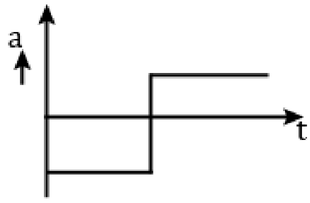
The height of the building is,
$25 + 20 = 45 \text{ m}$

Hence, (A) is the correct answer.

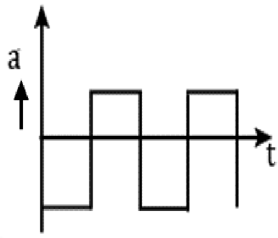
2. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?



✔ **A.**

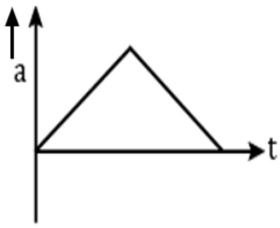


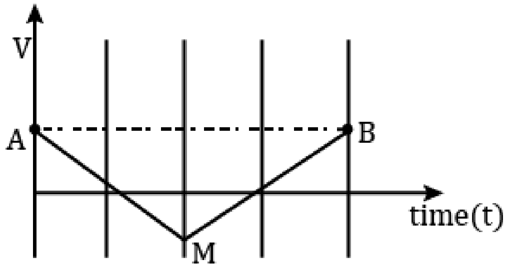
✖ **B.**



✖ **C.**

✖ **D.**





We know that the slope of $v-t$ curve gives acceleration.

For AM, slope is constant and negative. Therefore, acceleration is constant and negative.

Similarly, for MB, slope is constant and positive. Therefore, acceleration is constant and positive.

This situation is best represented by the curve in option (A).

3. An engine of a train, moving with uniform acceleration, passes the signal post with velocity u and the last compartment passes the signal post with velocity v. The velocity with which the middle point of the train passes the signal post is :

A. $\sqrt{\dfrac{v^2-u^2}{2}}$

B. $\sqrt{\dfrac{v-u}{2}}$

C. $\sqrt{\dfrac{v^2+u^2}{2}}$

D. $\sqrt{\dfrac{v+u}{2}}$

Suppose the length of the train is l.

So, $v^2-u^2=2al$

$\Rightarrow l=\dfrac{v^2-u^2}{2a} \quad \ldots.(1)$

Also, let the speed of the middle point of the train is v_m.

Then, $v_m^2-u^2=2a\times\dfrac{l}{2}=al$

$\Rightarrow v_m^2-u^2=a\times\dfrac{v^2-u^2}{2a}$
[From (1)]

$\Rightarrow v_m=\sqrt{\dfrac{v^2+u^2}{2}}$

Hence, (C) is the correct answer.

4. A scooter accelerates from rest for time t_1 at a constant rate α_1 and then retards at constant rate α_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be -

A. $\frac{\alpha_1+\alpha_2}{\alpha_2}$

B. $\frac{\alpha_2}{\alpha_1}$

C. $\frac{\alpha_1+\alpha_2}{\alpha_1}$

D. $\frac{\alpha_1}{\alpha_2}$

During acceleration,

$v=0+\alpha_1 t_1=\alpha_1 t_1$

During retardation,

$0=v-\alpha_2 t_2$

$\Rightarrow v=\alpha_2 t_2$

$\Rightarrow \alpha_1 t_1=\alpha_2 t_2$

$\Rightarrow \frac{t_1}{t_2}=\frac{\alpha_2}{\alpha_1}$

Hence, (B) is the correct answer.

5. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time $(t = 1\ \mathrm{s})$ is:
(g and F are constants)

A. $v_0 + \frac{g}{2} + F$

B. $v_0 + 2g + 3F$

C. $v_o + g + F$

D. $v_0 + \frac{g}{2} + \frac{F}{3}$

Given, $v = v_0 + gt + Ft^2$

We know that, $v = \frac{dx}{dt}$

$$\Rightarrow \frac{dx}{dt} = v_0 + gt + Ft^2$$

$$\Rightarrow \int_{x=0}^{x} dx = \int_{t=0}^{t=1} (v_0 + gt + Ft^2)dt$$

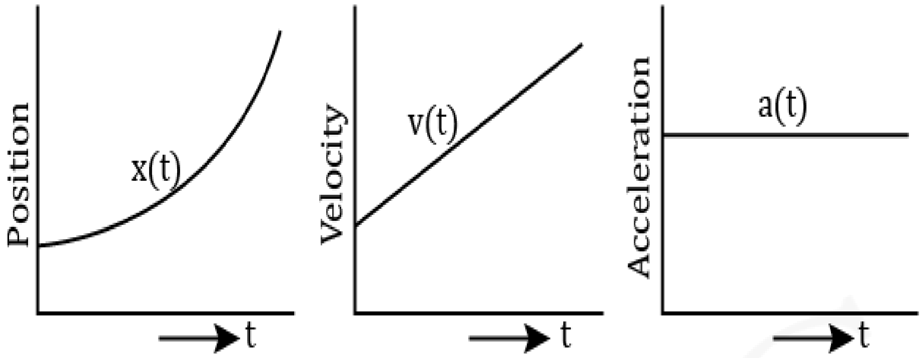
$$\Rightarrow x = \left[v_o t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_{t=0}^{t=1}$$

$$\Rightarrow x = v_o + \frac{g}{2} + \frac{F}{3}$$

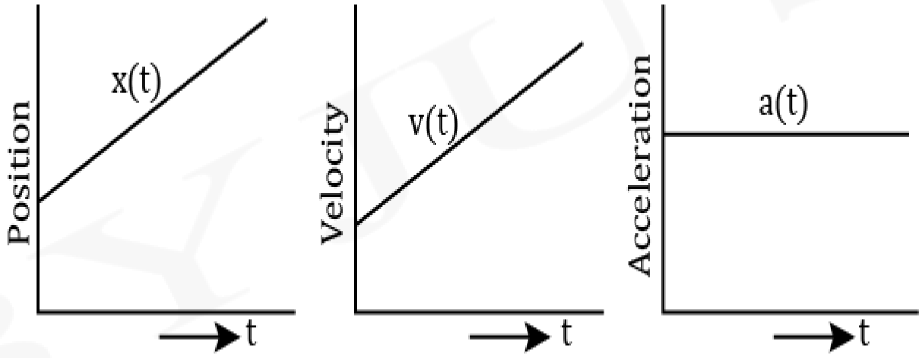
Hence, (D) is the correct answer.

6. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by:

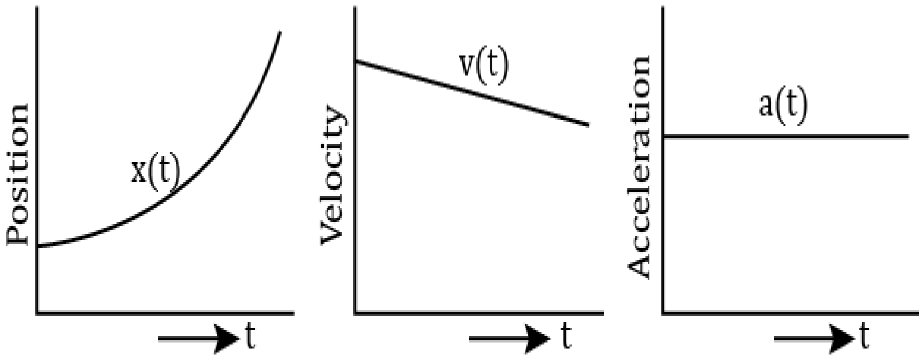
A.



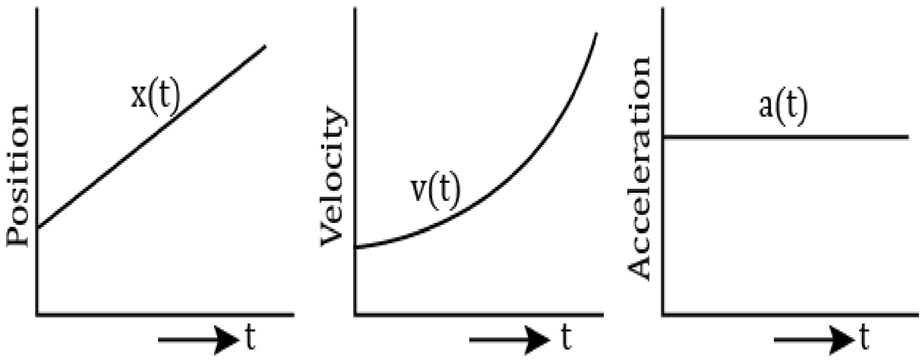
B.



C.



D.



In all the options, acceleration is constant and positive.

Acceleration $a = \frac{dv}{dt}$ is the slope of the $v - t$ graph.

So, $v - t$ graph should be linear and it should be increasing (to have positive slope).

Velocity $v = \frac{dx}{dt}$ is slope of the $x - t$ graph. So $x - t$ graph should be quadratic.

$x \propto t^2$ (parabolic graph)

Hence, (A) is the correct answer.

7. Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4^{th} second after its fall, to the next droplet, is $34.3\ \mathrm{m}$. At what rate the droplets are coming from the tap ? (Take $g = 9.8\ \mathrm{m/s^2}$)

- ✗ **A.** $3\ \mathrm{drops/s}$
- ✗ **B.** $2\ \mathrm{drops/s}$
- ✓ **C.** $1\ \mathrm{drop/s}$
- ✗ **D.** $\frac{1}{7}\mathrm{drops/s}$

The distance travelled by a freely falling drop is,

$$h = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2 \qquad (\because u = 0)$$

In $4\ \mathrm{sec}$, 1^{st} drop will travel,

$$h_1 = \frac{1}{2} \times (9.8) \times (4)^2 = 78.4\ \mathrm{m}$$

$\therefore\ 2^{\mathrm{nd}}$ drop would have travel,

$$h_2 = 78.4 - 34.3 = 44.1\ \mathrm{m}$$

So, time taken by 2^{nd} drop is,

$$h_2 = \frac{1}{2}(9.8)t^2 = 44.1$$

$\therefore t = 3\ \mathrm{sec}$

It means each drop have time gap of $1\ \mathrm{sec}$.

So, drops are falling at a rate of $1\ \mathrm{drop/s}$

Hence, (C) is the correct answer.

8. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is :
(Where v stands for velocity)

A. $2mv^3$

B. $2mnv^3$

C. $2nv^3$

D. $2n^2v^3$

Given that,

$t = mx^2 + nx$

Differentiating w.r.t. x, we get

$$\frac{dt}{dx} = \frac{1}{v} = 2mx + n$$

$$v = \frac{1}{2mx + n} \quad \ldots\ldots.(1)$$

Again differentiating w.r.t. t, we get

$$a = \frac{dv}{dt} = -\frac{2m}{(2mx+n)^2}\left(\frac{dx}{dt}\right)$$

Substituting (1) in above equation,

$a = -(2m)v^3$

So, the retardation will be,

$-a = 2mv^3$

Hence, option (A) is correct.

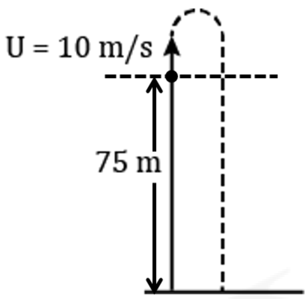
9. A balloon was moving upwards with a uniform velocity of $10 \mathrm{~ms}^{-1}$. An object of finite mass is dropped from the balloon when it was at a height of $75 \mathrm{~m}$ from the ground level. The height of the balloon from the ground when the object strikes the ground will be around :
$(\text{Take } g=10 \mathrm{~ms}^{-2})$

A. $300 \mathrm{~m}$

B. $200 \mathrm{~m}$

C. $125 \mathrm{~m}$ ✔

D. $250 \mathrm{~m}$



$u=10 \mathrm{~m/s}$; $a=-g=-10 \mathrm{~m/s}^2$; $s=-75 \mathrm{~m}$

Object is projected as shown, so as per equation of motion under gravity,

Time taken by the object to reach the ground is given by,

$$s=ut+\frac{1}{2}at^2$$

$$-75=+10t+\frac{1}{2}(-10)t^2$$

Since, t can't be negative

$\therefore t=5 \mathrm{~s}$

As the balloon is undergoing uniform motion,

Height of balloon from ground when the object reaches the ground is

$H=75+ut$

$\Rightarrow H=75+(10\times 5)=125 \mathrm{~m}$

Hence, option (C) is correct.

10. The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between $1\ \mathrm{s}$ and $2\ \mathrm{s}$ is

A. $3\alpha + 7\beta$

B. $\frac{3}{2}\alpha + \frac{7}{3}\beta$

C. $\frac{\alpha}{2} + \frac{\beta}{3}$

D. $\frac{3}{2}\alpha + \frac{7}{2}\beta$

Given that,

$$v = \alpha t + \beta t^2$$

$$\Rightarrow \frac{ds}{dt} = \alpha t + \beta t^2$$

Integrating both side within suitable limit

$$\int_{S_1}^{S_2} ds = \int_1^2 (\alpha t + \beta t^2) dt$$

$$S_2 - S_1 = \left[\frac{\alpha t^2}{2} + \frac{\beta t^3}{3}\right]_1^2$$

As the velocity is always positive, the particle does not change the direction.

So, distance = displacement.

$$\therefore \text{Distance},\ D = \left[\frac{\alpha[4-1]}{2} + \frac{\beta[8-1]}{3}\right]$$

$$D = \frac{3\alpha}{2} + \frac{7\beta}{3}$$

Hence, option (B) is correct.

11. A ball is thrown up with a certain velocity so that it reaches a height $'h'$. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

A. $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

B. $\frac{1}{3}$

C. $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

D. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Using the conservation of energy theorem,

Projected speed, $u = \sqrt{2gh}$

Now, when ball is moving upward

$$S = \frac{h}{3} : \ a = -g$$

Using, $S = ut + \frac{1}{2}at^2$

$$\frac{h}{3} = \sqrt{2gh}\,t + \frac{1}{2}(-g)t^2$$

$$t^2\left(\frac{g}{2}\right) - \sqrt{2gh}\,t + \frac{h}{3} = 0$$

From quadratic equation,

$$t_1, t_2 = \frac{\sqrt{2gh} \pm \sqrt{2gh - \frac{4g}{2}\frac{h}{3}}}{g}$$

So, the ratio of two different times of the ball reaching $h/3$ is

$$\frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}} = \frac{\sqrt{6} - \sqrt{4}}{\sqrt{6} + \sqrt{4}}$$

$$\therefore \frac{t_1}{t_2} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

Hence, the correct option is (C).

12. Water drops are falling from a nozzle of a shower onto the floor, from a height of $9.8\ \mathrm{m}$. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of the second drop from the floor when the first drop strikes the floor.

A. $2.45\ \mathrm{m}$

B. $7.35\ \mathrm{m}$

C. $2.94\ \mathrm{m}$

D. $4.18\ \mathrm{m}$

Time taken by the first drop to reach the ground is :

$$t=\sqrt{\frac{2h}{g}}=\sqrt{2}\ \mathrm{s}$$

When the first drop reaches the ground, the third drop begins to fall.
Therefore, the time interval between the drops is :

$$\Delta t=\frac{t}{2}=\frac{1}{\sqrt{2}}$$

Therefore, the distance covered by the second drop is :

$$s=\frac{1}{2}g\Delta t^2$$

$$=\frac{1}{2}\times 9.8\times\frac{1}{2}=2.45\ \mathrm{m}$$

Therefore, the height of the second drop is :

$$h=H-s=9.8-2.45=7.35\ \mathrm{m}$$

Hence, (B) is the correct answer.

13. A helicopter is rising up from ground with an acceleration of $g\ \mathrm{m/s^2}$, starting from rest after raising a height h it attains a velocity of $v\ \mathrm{m/s}$. At this instant, a particle is now released from the helicopter. Take $t = 0$ at releasing time, calculate the time t when particle reaches to the ground.

A. $\sqrt{\frac{2h}{g}}$

B. $2\sqrt{\frac{2h}{g}}$

C. $\left(1+\sqrt{2}\right)\sqrt{\frac{2h}{g}}$

D. $4\sqrt{\frac{2h}{g}}$

For upward motion of helicopter:

From third equation of motion,

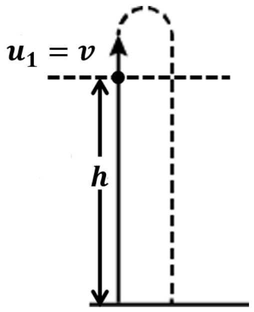
$v^2 = u^2 + 2as$

Here, $u = 0$; $v = v$; $a = g$; $s = h$

$v^2 = 0 + 2as$

$v = \sqrt{2gh}$

Now, for the particle, motion will be as shown in figure, and it starts moving under gravity.



So, from second equation of motion,

$s_1 = u_1 t + \frac{1}{2} a_1 t^2$

Here, $u_1 = v = \sqrt{2gh}$; $s_1 = -h$; $a_1 = -g$

$\Rightarrow -h = \sqrt{2gh}t - \frac{1}{2}gt^2$

$\frac{1}{2}gt^2 - \sqrt{2gh}t - h = 0$

So, value of t will be,

$$t = \frac{\sqrt{2gh} \pm \sqrt{2gh - \left(4 \times \frac{g}{2} \times (-h)\right)}}{2 \times \frac{g}{2}}$$

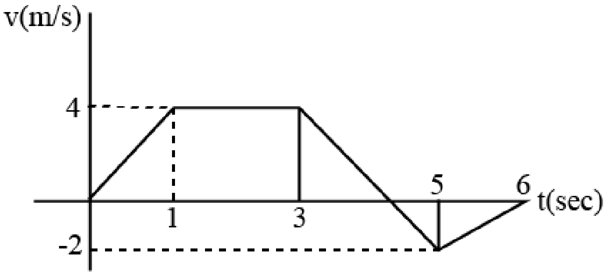
$t = \sqrt{\frac{2h}{g}}(1 \pm \sqrt{2})$

Since, t can't be negative,

$\therefore t = \sqrt{\frac{2h}{g}}(1 + \sqrt{2})$

Hence, option (C) is correct.

14. Velocity-time graph of the particle is shown in figure. Find the displacement of the particle.

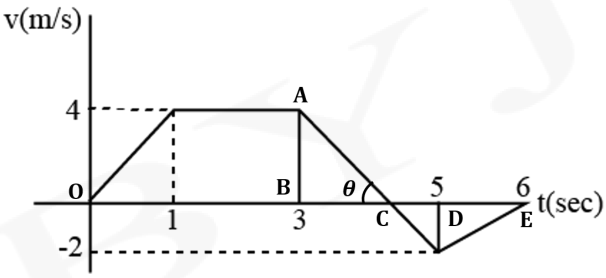


A. 7 m

B. 11 m

C. 5 m

D. 15 m



From diagram,

$$\tan\theta = \frac{4}{BC} = \frac{2}{CD}$$

$$\frac{CD}{BC} = \frac{1}{2} \Rightarrow \frac{CD}{BC} + 1 = \frac{1}{2} + 1$$

$$\therefore BC = \frac{2}{3}BD = \frac{2}{3} \times 2 = \frac{4}{3}$$

As we know that net area of $v - t$ graph gives displacement.

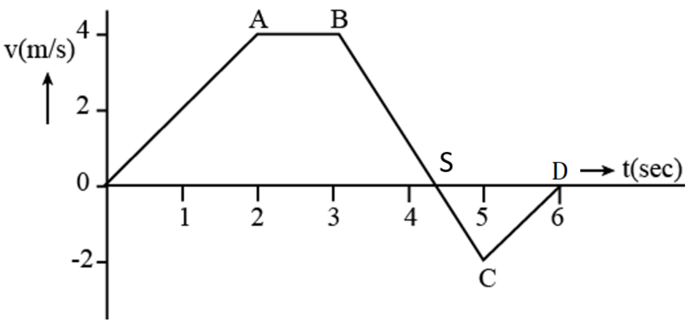
$\therefore s =$ area of graph

$$s = \left[\frac{1}{2} \times 4\left(2 + \frac{13}{3}\right)\right] - \left[\frac{1}{2} \times 2 \times \frac{5}{3}\right]$$

$$\therefore s = \frac{38}{3} - \frac{5}{3} = 11 \text{ m}$$

Hence, option (B) is correct.

15. The velocity v and time t graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 sec. The total distance covered by the body in 6 s is



A. $\frac{37}{3}$ m

B. 11 m

C. $\frac{49}{4}$ m

D. 12 m

As we know that total distance covered by the body is given by,

$s = \int |V| dt =$ sum of magnitude of area under $v - t$ graph

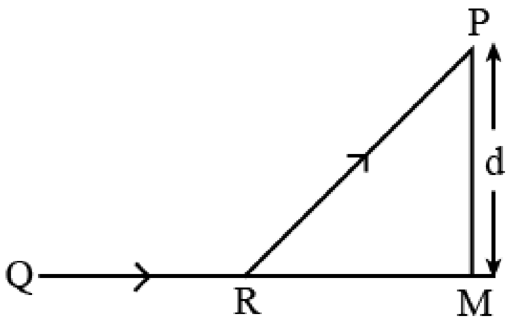
$s = |\text{area of OABSO}| + |\text{area of SCDS}$

$$s = \frac{1}{2}(4)(4.333 + 1) + \frac{1}{2}(6 - 4.333)(2)$$

$$\therefore s = 12.333 = \frac{37}{3} \text{m}$$

Hence, option (A) is correct.

16. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum ?



A. $\dfrac{d}{\sqrt{3}}$

B. $\dfrac{d}{2}$

C. $\dfrac{d}{\sqrt{2}}$

D. d

Let the car take the turn of the highway at a distance $'x'$ from the point M.

So, $\mathrm{RM} = x$

Given the speed of the car at Q is v, so the time taken by the car to cover the distance $QR\ (= QM - x)$ on the highway is,

$$t_1 = \frac{QM - x}{v} \quad ....(1)$$

Time taken to travel the distance $'RP'$ in the field is,

$$t_2 = \frac{\sqrt{d^2 + x^2}}{v/2} \quad ....(2)$$

Total time elapsed to move the car from Q to P is,

$$t = t_1 + t_2 = \frac{QM - x}{v} + \frac{\sqrt{d^2 + x^2}}{v/2}$$

For t to be minimum, $\frac{dt}{dx} = 0$

$$\frac{1}{v}\left[-1 + \frac{2 \times (2x)}{2\sqrt{d^2 + x^2}}\right] = 0$$

$$\Rightarrow \frac{1}{2} = \frac{x}{\sqrt{d^2 + x^2}}$$

$$\Rightarrow d^2 + x^2 = 4x^2$$

$$\therefore x = \frac{d}{\sqrt{3}}$$

Hence, option (A) is correct.

17. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is

- **A.** $v_0 + g/2 + f$
- **B.** $v_0 + 2g + 3f$
- **C.** $v_0 + g/2 + f/3$
- **D.** $v_0 + g/2 + 2f$

We know that, $v = \dfrac{dx}{dt}$

$\Rightarrow dx = vdt$

Integrating, $\displaystyle\int_0^x dx = \int_0^t vdt$

$$\Rightarrow x = \int_0^t (v_0 + gt + ft^2)dt = \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$$

$$\Rightarrow x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$$

At $t = 1$,

$$x = v_0 + \frac{g}{2} + \frac{f}{3}$$

Hence, (C) is the correct answer.

18. In a car race on straight road, car A takes a time t less than car B at the finishing point and passes the finishing point with a speed $'v'$ more than that car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then v is equal to

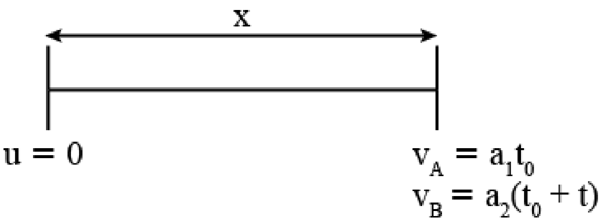
A. $\frac{a_1 + a_2}{2}$

B. $\sqrt{2a_1 a_2} t$

C. $\frac{2a_1 a_2}{a_1 + a_2} t$

D. $\sqrt{a_1 a_2} t$ ✓

Let the total time taken by car A is t_0.



From question,

$$v_A - v_B = v = a_1 t_0 - (t + t_0)a_2$$

$$\Rightarrow v = (a_1 - a_2)t_0 - a_2 t \quad \ldots.(i)$$

And,

$$x_B = x_A$$

$$\Rightarrow \frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$$

$$\sqrt{a_1}t_0 = \sqrt{a_2}(t_0 + t)$$

$$(\sqrt{a_1} - \sqrt{a_2})t_0 = \sqrt{a_2}t$$

$$t_0 = \frac{\sqrt{a_2}t}{(\sqrt{a_1} - \sqrt{a_2})} \quad \ldots.(ii)$$

Putting t_0 in equation (i),

$$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}} - a_2 t$$

$$= (\sqrt{a_1} + \sqrt{a_2})\sqrt{a_2}t - a_2 t$$

$$\Rightarrow v = \sqrt{a_1 a_2}t$$

Hence, (D) is the correct answer.

19. A particle moves from the point $(2.0\hat{i} + 4.0\hat{j})$ m, at $t = 0$, with an initial velocity $(5.0\hat{i} + 4.0\hat{j})\ \mathrm{ms^{-1}}$. It is acted upon by a constant acceleration $(4.0\hat{i} + 4.0\hat{j})\ \mathrm{ms^{-2}}$. What is the distance of the particle from the origin at time 2 s?

A. $20\sqrt{2}$ m

B. $10\sqrt{2}$ m

C. 5 m

D. 15 m

Given,

Initial position vector, $\vec{r}_i = (2.0\hat{i} + 4.0\hat{j})$ m

Initial velocity, $\vec{u} = (5.0\hat{i} + 4.0\hat{j})\ \mathrm{ms^{-1}}$

Acceleration, $\vec{a} = (4.0\hat{i} + 4.0\hat{j})\ \mathrm{ms^{-2}}$

Applying the equation of motion,

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

Where, $\vec{S} = \vec{r_f} - \vec{r_i}$

Substituting the values,

$$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})2^2$$

$$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$$

$$\Rightarrow \vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$$

$$\Rightarrow \vec{r}_f - (2\hat{i} + 4\hat{j}) = 18\hat{i} + 16\hat{j}$$

$$\Rightarrow \vec{r}_f = 20\hat{i} + 20\hat{j}$$

$$\therefore |\vec{r}_f| = 20\sqrt{2}\ \mathrm{m}$$

Hence, (A) is the correct answer.

20. A particle is moving with speed $v = b\sqrt{x}$ along positive $x-$axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$)

A. $\frac{b^2\tau}{4}$

B. $\frac{b^2\tau}{2}$

C. $b^2\tau$

D. $\frac{b^2\tau}{\sqrt{2}}$

Given,

$$v = b\sqrt{x}$$

$$\Rightarrow a = \frac{dv}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt}$$

$$\Rightarrow a = \frac{bv}{2\sqrt{x}} = \frac{b \times b\sqrt{x}}{2\sqrt{x}} \quad \left[\because \frac{dx}{dt} = v\right]$$

$$\Rightarrow a = \frac{b^2}{2}$$

As, $a = \frac{dv}{dt}$

$$\Rightarrow \frac{dv}{dt} = \frac{b^2}{2}$$

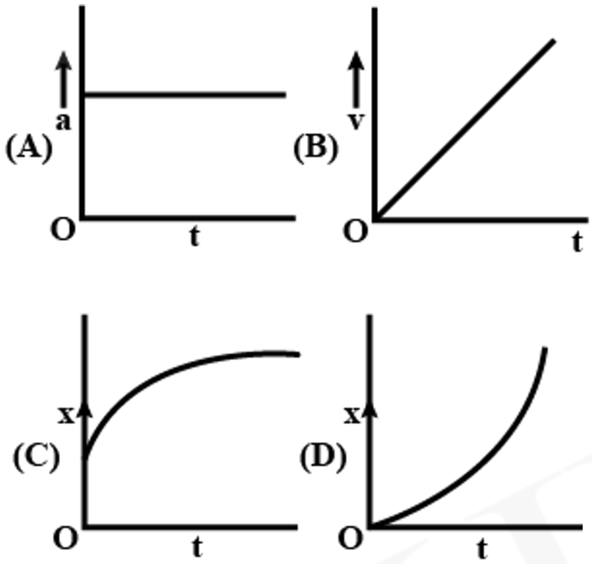
Rearranging and integrating the above equation,

$$\int_0^v dv = \int_0^\tau \frac{b^2}{2}dt$$

$$\therefore v = \frac{b^2}{2}\tau$$

Hence, (B) is the correct answer.

21. A particle starts from origin O from rest and moves with a uniform acceleration along the positive $x-$axis. Identify all figures that correctly represent the motion quantitatively.
($a =$ acceleration, $v =$ velocity, $x =$ displacement, $t =$ time)



A. $(\mathrm{A}),(\mathrm{B}),(\mathrm{C})$

B. (A)

C. $(\mathrm{A}),(\mathrm{B}),(\mathrm{D})$

D. $(\mathrm{B}),(\mathrm{C})$

Given,

initial velocity, $u = 0$ and

acceleration, $a = \text{constant}$

So, graph (A) is correct.

At time t, using equations of motion

$v = 0 + at$

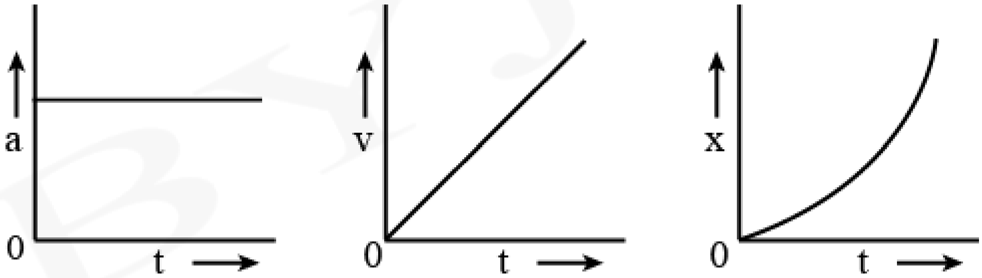
$\Rightarrow v = at \quad \ldots.(1)$

Also,

$x = 0(t) + \frac{1}{2}at^2$

$\Rightarrow x = \frac{1}{2}at^2 \quad \ldots.(2)$

Using equations (1) and (2) we get the graphs (A), (B) and (D) .



Hence, (C) is the correct answer.

22. The position of a particle as a function of time, t is given by $x(t) = at + bt^2 - ct^3$, where, $a,\ b$ and c are constants. When the particle attains zero acceleration, then its velocity will be:

A. $a + \frac{b^2}{4c}$

B. $a + \frac{b^2}{c}$

C. $a + \frac{b^2}{2c}$

D. $a + \frac{b^2}{3c}$

Given,

$$x = at + bt^2 - ct^3$$

$$\Rightarrow v = \frac{dx}{dt} = a + 2bt - 3ct^2$$

$$\Rightarrow a = \frac{dv}{dt} = 2b - 6ct$$

When the particle attains zero acceleration,

$$2b - 6ct = 0 \Rightarrow t = \frac{b}{3c}$$

$$\therefore v_{\left(t=\frac{b}{3c}\right)} = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = a + \frac{b^2}{3c}$$

Hence, (D) is the correct answer.

23. The position vector of a particle changes with time according to the relation $\overrightarrow{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$?

A. 40

B. 100

C. 50

D. 25

The position vector of particle is given as,

$$\overrightarrow{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$$

Velocity of particle is,

$$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt} = \frac{d}{dt}[15t^2\hat{i} + (4 - 20t^2)\hat{j}]$$

$$= 30t\hat{i} - 40t\hat{j}$$

$$\text{Acceleration, } \overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = 30\hat{i} - 40\hat{j}$$

$$\therefore a_{t=1} = \sqrt{30^2 + (-40)^2} = 50$$

Hence, (C) is the correct answer.

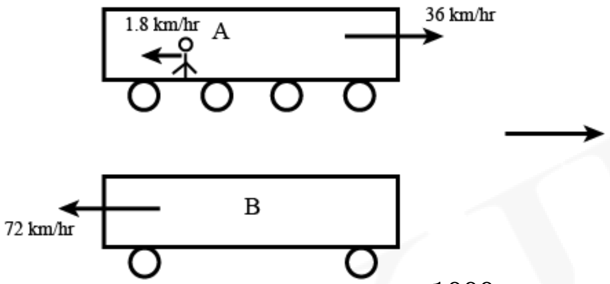
24. Two trains A and B moving with speed of 36 km/hr and 72 km/hr respectively in opposite direction. A man moving in train A with speed of 1.8 km/hr opposite to the direction of train A. Find velocity of man as seen from train B (in m/s).

A. 32 m/s

B. 29.5 m/s

C. 32.5 m/s

D. 28 m/s



Here, $V_A = 36 \text{ km/hr} = 36 \times \frac{1000}{3600} \text{m/s} = 10 \text{ m/s}$

$V_B = -72 \text{ km/hr} = -72 \times \frac{1000}{3600} \text{m/s} = -20 \text{ m/s}$

$V_{\text{Man, A}} = -1.8 \text{ km/hr} = -0.5 \text{ m/s}$

Velocity of man with respect to train B,

$V_{\text{man, B}} = V_{\text{man, A}} + V_{\text{A B}}$

$= V_{\text{man, A}} + (V_A - V_B)$

$=$ -0.5 $+$ 10 - (-20)

$=$ -0.5 $+$ 30$=$ 29.5 m/s

Hence, (B) is the correct answer.

Topic : Motion in 2-D

1. The trajectory of a projectile in a vertical plane is, $y = \alpha x - \beta x^2$, where α, β are constants, x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection and the maximum height attained are respectively given by :

A. $\tan^{-1}\alpha,\ \dfrac{\alpha^2}{4\beta}$ ✔

B. $\tan^{-1}\beta,\ \dfrac{\alpha^2}{2\beta}$ ✗

C. $\tan^{-1}\alpha,\ \dfrac{\alpha^2}{2\beta}$ ✗

D. $\tan^{-1}\beta,\ \dfrac{\alpha^2}{4\beta}$ ✗

Given:
$y = \alpha x - \beta x^2$

Also, $y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}x^2$

On comparing, we get,

$\tan\theta = \alpha \quad \dots(1),$ and

$$\frac{g}{2u^2\cos^2\theta} = \beta \Rightarrow \frac{u^2}{g} = \frac{1}{2\beta\cos^2\theta} \quad \dots(2)$$

So, angle of projection, $\theta = \tan^{-1}\alpha$
[From (1)]

Now, maximum height attained,
$$H_{\max} = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow H_{\max} = \frac{\sin^2\theta}{4\beta\cos^2\theta}$$
[From (2)]

$$\Rightarrow H_{\max} = \frac{\tan^2\theta}{4\beta} = \frac{\alpha^2}{4\beta}$$
[From (1)]

2. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ?

A. $R^2 = 4\ h_1 h_2$

B. $R^2 = 16\ h_1 h_2$

C. $R^2 = 2\ h_1 h_2$

D. $R^2 = h_1 h_2$

For the same range, the angle of projection will be θ and $90^\circ - \theta$. So

$h_1 = \dfrac{u^2 \sin^2\theta}{2g}$ and

$$h_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$$

$$\text{Also, R} = \frac{u^2 \sin 2\theta}{g}$$

So, $h_1 h_2 = \dfrac{u^2 \sin^2\theta}{2g} \times \dfrac{u^2 \cos^2\theta}{2g}$

$$h_1 h_2 = \frac{u^4}{16g^2}(2\sin\theta\cos\theta)^2 = \frac{1}{16}\left(\frac{u^2 \sin 2\theta}{g}\right)^2$$

$$h_1 h_2 = \frac{R^2}{16}$$

$$\therefore R^2 = 16 h_1 h_2$$

Hence, option (B) is correct.

3. The co-ordinates of a moving particle at any time $'t'$ are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time $'t'$ is given by

A. $3t\sqrt{\alpha^2 + \beta^2}$

B. $3t^2\sqrt{\alpha^2 + \beta^2}$

C. $t^2\sqrt{\alpha^2 + \beta^2}$

D. $\sqrt{\alpha^2 + \beta^2}$

Coordinate of moving particle at time t are $x = \alpha t^3$ and $y = \beta t^3$.

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$\Rightarrow v = 3t^2\sqrt{\alpha^2 + \beta^2}$$

Hence, (B) will be the correct option.

4. The position of a projectile launched from the origin at $t=0$ is given by $\overrightarrow{r}=(40\hat{i}+50\hat{j})$ m at $t=2$ s. If the projectile was launched at an angle θ from the horizontal, then θ is
(Take $g=10\ \mathrm{ms}^{-2}$)

A. $\tan^{-1}\frac{2}{3}$

B. $\tan^{-1}\frac{3}{2}$

C. $\tan^{-1}\frac{7}{4}$

D. $\tan^{-1}\frac{4}{5}$

As we know that horizontal component of projected velocity remains constant.

So, from question,
Horizontal velocity, $u_x=\frac{40-0}{2-0}=20\ \mathrm{m/s}$

And, initial vertical velocity (u_y),

$$s_y=u_yt+\frac{1}{2}at^2$$

$$50=u_y(2)-\frac{1}{2}(10)(2)^2$$

$$\Rightarrow u_y=\frac{70}{2}=35\ \mathrm{m}/s$$

$$\therefore \tan\theta=\frac{u_y}{u_x}=\frac{35}{20}=\frac{7}{4}$$

$$\Rightarrow \text{Angle, }\ \theta=\tan^{-1}\frac{7}{4}$$

Hence, option (B) is correct.

5. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ \mathrm{m/s^2}$, the equation of its trajectory is :

A. $y = x - 5x^2$

B. $4y = 2x - 5x^2$

C. $4y = x - 5x^2$

D. $y = 2x - 5x^2$

From equation, $\vec{v} = \hat{i} + 2\hat{j}$

$u_x = 1\ \mathrm{m/s}\ ;\ \ u_y = 2\ \mathrm{m/s}$

Now apply equation of motion along x & y axis,

$$x = u_x t = 1 \times t = t \quad (1)$$

And,

$$y = 2t - \frac{1}{2}(10)t^2 \quad (2)$$

From eqs. (1) & (2),

$y = 2x - 5x^2$

Therefore, option (D) is correct.

6. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40 \text{ m}$. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity $v = 20 \text{ m/s}$ on a horizontal surface ?
(Take $g = 10 \text{ m/s}^2$)

A. 60°

B. 30°

C. 75°

D. 45°

Given:
$R_0 = 40 \text{ m}\ ;\ \ v = 20 \text{ m/s}$

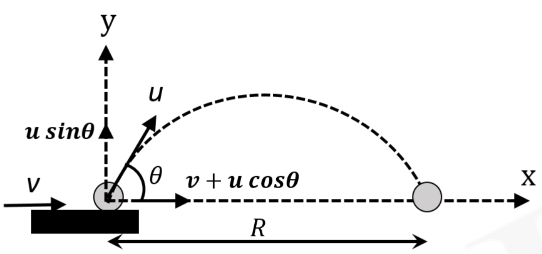
Let u be the projected speed of the bullet.

As we know that,

$$R_{max} = R_0 = \frac{u^2}{g}$$

$$\Rightarrow 40 = \frac{u^2}{10}$$

$$\therefore u = 20 \text{ m/s}$$



Let θ be the inclination of the piston for maximum range when car is moving.

$$\therefore R = \frac{2u_x u_y}{g} = \frac{2(v + u\cos\theta)u\sin\theta}{g}$$

For maximum range, $\dfrac{dR_{max}}{d\theta} = 0$

$$\frac{2u}{g}[(-u\sin\theta)\sin\theta + (v + u\cos\theta)\cos\theta] = 0$$

$$-u\sin^2\theta + v\cos\theta + u\cos^2\theta = 0$$

$$-(1 - \cos^2\theta) + \cos\theta + \cos^2\theta = 0 \qquad (\because u = v = 10 \text{ m/s})$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$\cos\theta = \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)} = -1, \frac{1}{2}$$

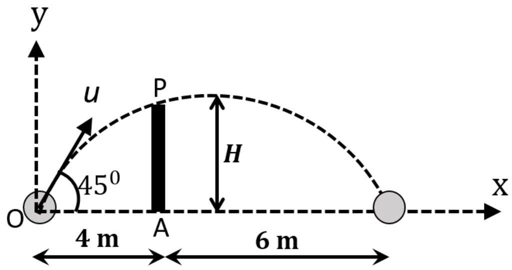
For acute angle,

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

Hence, option (A) is correct.

7. A ball projected from the ground at an angle of 45°just clears a wall in front. If the point of projection is $4\ \mathrm{m}$ from the foot of the wall and the ball strikes the ground at a distance of $6\ \mathrm{m}$ on the other side of the wall, the height of the wall is :

- ✗ **A.** $4.4\ \mathrm{m}$
- ✓ **B.** $2.4\ \mathrm{m}$
- ✗ **C.** $3.6\ \mathrm{m}$
- ✗ **D.** $1.6\ \mathrm{m}$



Here,
$\text{Range} = 4\text{ m} + 6\text{ m} = 10\text{ m}$

As the ball is projected at an angle 45° to the horizontal,

Therefore, $\text{Range} = 4H$

$$10 = 4H \Rightarrow H = \frac{10}{4} = 2.5\text{ m}$$

Maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g}$

$$\therefore u^2 = \frac{H \times 2g}{\sin^2\theta} = \frac{2.5 \times 2 \times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$$

$$u = \sqrt{100} = 10\text{ ms}^{-1}$$

From trajectory formula,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

So, the height of the wall PA is,

$$h = (OA)\ \tan\theta - \frac{g(OA)^2}{2u^2\cos^2\theta}$$

$$h = (4 \times 1) - \frac{1}{2} \times \frac{10 \times 4^2}{10^2 \times (1/\sqrt{2})^2}$$

$$\therefore h = 2.4\text{ m}$$

Hence, option (B) is correct.

8. A boy can throw a stone up to a maximum height of $10\ \mathrm{m}$. The maximum horizontal distance that the boy can throw the same stone up to will be

A. $20\sqrt{2}\ \mathrm{m}$

B. $10\ \mathrm{m}$

C. $10\sqrt{2}\ \mathrm{m}$

D. $20\ \mathrm{m}$

Let u be the projected velocity of stone. So,

$$R = \frac{u^2 \sin 2\theta}{g} \;\&\; H = \frac{u^2 \sin^2 \theta}{2g}$$

H_{max} will be at $\theta = 90^\circ$,

$$H_{max} = \frac{u^2}{2g}$$

$$\Rightarrow u^2 = 20g \qquad (\because H_{max} = 10\ \mathrm{m})$$

Now,

$$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R_{max} = \frac{u^2}{g}$$

$$R_{max} = \frac{20g}{g} = 20\ \mathrm{metre}$$

Therefore, option (D) will be the correct answer.

9. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be times of flight in two cases, then the product of the two times of flight is directly proportional to

A. R

B. $\frac{1}{R}$

C. $\frac{1}{R^2}$

D. R^2

We know that projectile has the same range for two complementary angles θ and $90^\circ - \theta$.

So the times of flight for these motion will be,

$$T_1 = \frac{2u \sin\theta}{g} \; \& \; T_2 = \frac{2u \cos\theta}{g}$$

Then,

$$T_1 T_2 = \frac{4u^2 \sin\theta \cos\theta}{g} = 2R$$

$$\left(\because R = \frac{u^2 \sin 2\theta}{g}\right)$$

Thus, it is proportional to R (Range).

Hence, option (A) is correct.

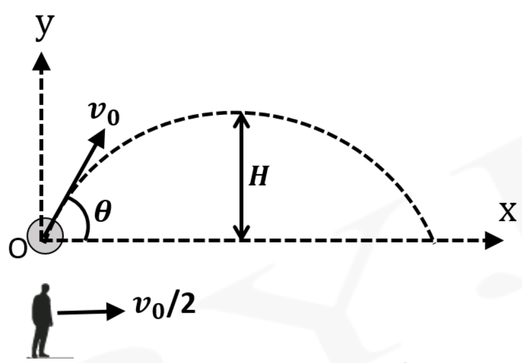
10. A ball is thrown from a point with a speed v_0 at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ ?

A. No

B. yes, 30°

C. yes, 60°

D. yes, 45°



Yes, man will catch the ball, if the horizontal component of velocity of ball becomes equal to the speed of man.

$$\frac{v_0}{2} = v_0 \cos\theta$$

$$\therefore \theta = 60^\circ$$

Hence, option (C) is correct.

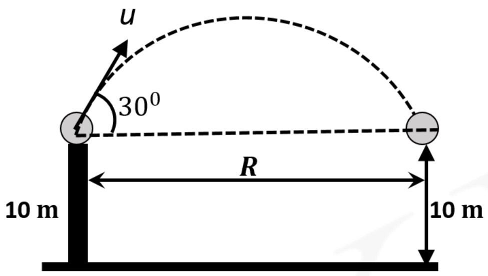
11. A boy playing on the roof of a $10 \mathrm{~m}$ high building throws a ball with a speed of $10 \mathrm{~m/s}$ at the angle of 30° with the horizontal. How far from the throwing point will be the ball be at the height of $10 \mathrm{~m}$ from the ground ?
$(\text{Take, } g = 10 \mathrm{~m/s^2})$

A. $5.20 \mathrm{~m}$

B. $4.33 \mathrm{~m}$

C. $2.60 \mathrm{~m}$

D. $8.66 \mathrm{~m}$



Horizontal range is required

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 30^{\circ})}{10}$$

$$\therefore R = 5\sqrt{3} = 8.66 \mathrm{~m}$$

Hence, option (D) is correct.

12. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion ? $(\text{Take } g = 10 \text{ ms}^{-2})$

A. $h_{max} = 10 \text{ m}, \ T = 2.5 \text{ s}$

B. $h_{max} = 15.625 \text{ m}, \ T = 1.77 \text{ s}$

C. $h_{max} = 15.625 \text{ m}, \ T = 3.54 \text{ s}$

D. $h_{max} = 3.54 \text{ m}, \ T = 0.125 \text{ s}$

Given :
$u = 25 \text{ m/s}$
$\theta = 45^\circ$

The maximum height is given by :

$$h_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$h_{max} = \frac{25^2 \sin^2 45^\circ}{2 \times 10} = 15.625 \text{ m}$$

The time taken to reach the maximum height is :

$$T = \frac{u \sin \theta}{g} = \frac{25 \sin 45^\circ}{10} = 1.77 \text{ s}$$

Hence, (B) is the correct answer.

13. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}\ \mathrm{m/s}$ and moves in the $x - y$ plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})\ \mathrm{m/s^2}$. The $x-$coordinate of the particle at the instant when its $y-$coordinate is $32\ \mathrm{m}$ is D meters. The value of D is:

A. 32

B. 50

C. 60

D. 40

Given, $\vec{u} = (3\hat{i} + 0\hat{j})\ \mathrm{m/s}$ and

$\vec{a} = (6\hat{i} + 4\hat{j})\ \mathrm{m/s^2}$

Using, $s = ut + \frac{1}{2}at^2$, for $y - \text{axis}$

$y = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow 32 = 0 \times t + \frac{1}{2}(4)t^2$

$\Rightarrow \frac{1}{2} \times 4 \times t^2 = 32$

$\Rightarrow t = 4\ \mathrm{s}$

For x-axis,

$x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 60\ \mathrm{m}$

$\therefore D = 60$

Hence, (C) is the correct answer.

14. A particle of mass m is projected with a speed of u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

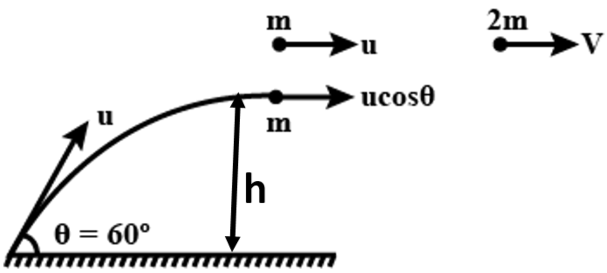
A. $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$

B. $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

C. $\frac{5}{8}\frac{u^2}{g}$

D. $2\sqrt{2}\frac{u^2}{g}$

Image for the given problem,



Using the principle of conservation of linear momentum for horizontal motion, we have

$p_i = p_f$

$mu + mu\cos 60^\circ = 2mv$

$\therefore v = \dfrac{3u}{4}$

For vertical motion,

$h = 0 + \dfrac{1}{2}gT^2 \Rightarrow T = \sqrt{\dfrac{2h}{g}}$

Using maximum height formula,

$$h = \frac{u^2\sin^2 60^0}{2g} = \frac{u^2\left(\frac{\sqrt{3}}{2}\right)^2}{2g} = \frac{3u^2}{8g}$$

Let R is the horizontal distance travelled by the body.

$R = vT + \dfrac{1}{2}(0)(T)^2$ (For horizontal motion, a=0)

$= vT = \dfrac{3u}{4} \times \sqrt{\dfrac{2h}{g}}$

$$\Rightarrow R = \frac{3u}{4} \times \sqrt{\frac{2 \times \frac{3u^2}{8g}}{g}} = \frac{3\sqrt{3}u^2}{8g}$$

Hence, (A) is the correct answer.

15. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}\ \mathrm{ms}^{-1}$, the particle moves in the $x - y$ plane with a constant acceleration of $(10\hat{i} + 4\hat{j})\ \mathrm{m/s}^2$. At time t, its coordinates are $(20\ \mathrm{m},\ y_0\ \mathrm{m})$. The values of t and y_0 are, respectively:

A. $2\ \mathrm{s}$ and $18\ \mathrm{m}$

B. $4\ \mathrm{s}$ and $52\ \mathrm{m}$

C. $2\ \mathrm{s}$ and $24\ \mathrm{m}$

D. $5\ \mathrm{s}$ and $25\ \mathrm{m}$

Given: $\vec{u} = 5\hat{j}\ \mathrm{m/s}$
Acceleration, $\vec{a} = 10\hat{i} + 4\hat{j}$ and final coordinates $(20,\ y_0)$ in time t.

Using equation of motion along $\mathrm{x\text{-}axis}$,

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$[\because u_x = 0]$$

$$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow t = 2\ \mathrm{s}$$

Using equation of motion along $\mathrm{y\text{-}axis}$,

$$y = u_y \times t + \frac{1}{2}a_y t^2$$

$$y = y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18\ \mathrm{m}$$

Hence, (A) is the correct answer.

16. Two guns A and B can fire bullets at speeds $1 \mathrm{~km/s}$ and $2 \mathrm{~km/s}$ respectively, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is:

A. $1:16$

B. $1:2$

C. $1:4$

D. $1:8$

As we know,

Range, $R_{max} = \dfrac{u^2}{g}$

and, area $A = \pi R^2$

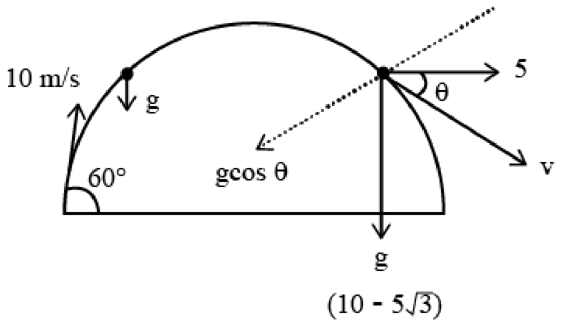
$\therefore A \propto R^2 \text{ or } A \propto u^4$

$\therefore \dfrac{A_1}{A_2} = \dfrac{u_1^4}{u_2^4} = \left[\dfrac{1}{2}\right]^4 = \dfrac{1}{16}$

Hence, (A) is the correct answer.

17. A body is projected at $t=0$ with a velocity $10\ \mathrm{ms}^{-1}$ at an angle 60° with the horizontal. The radius of curvature of its trajectory at $t=1$ is R. Neglecting air resisteance and taking acceleration due to gravity $g=10\ \mathrm{m/s}^2$, the value of R is:

- **A.** $10.3\ \mathrm{m}$
- **B.** $2.8\ \mathrm{m}$
- **C.** $2.5\ \mathrm{m}$
- **D.** $5.1\ \mathrm{m}$



Horizontal component of velocity

$v_x = 10 \cos 60^\circ = 5 \text{ m/s}$

vertical component of velocity,

$v_y = 10 \cos 30^\circ = 5\sqrt{3} \text{ m/s}$

After $t = 1 \text{ sec}$

Horizontal component of velocity $v_{x1} = 5 \text{ m/s}$

Vertical component of velocity,

$v_{1y} = |(5\sqrt{3} - 10)| \text{ m/s} = 10 - 5\sqrt{3}$

Centripetal acceleration, $a_n = \dfrac{v^2}{R}$

$$\Rightarrow R = \frac{v_x^2 + v_y^2}{a_n} = \frac{25 + 100 + 75 - 100\sqrt{3}}{10 \cos \theta} \qquad \ldots(i)$$

So from the given diagram,

$$\tan \theta = \frac{10 - 5\sqrt{3}}{5} = 2 - \sqrt{3}$$

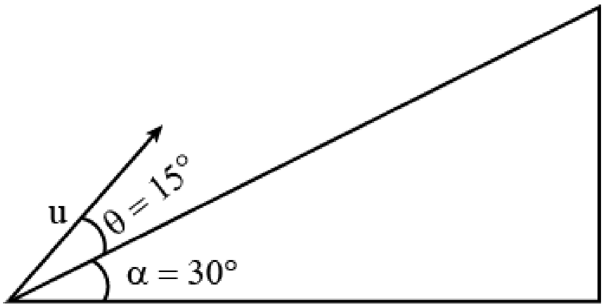
So, $\cos \theta = \dfrac{1}{4(2 - \sqrt{3})}$

Putting the value of $\cos \theta$ in eq. (i),

$$\therefore R = \frac{100(2 - \sqrt{3})}{10 \times \dfrac{1}{4(2 - \sqrt{3})}} = 2.8 \text{ m}$$

Hence, (B) is the correct answer.

18. A plane is inclined at an angle $\alpha = 30^\circ$ with respect to the horizontal. A particle is projected with a speed $u = 2\ \mathrm{m/s}$, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane, is close to:
(Take $g = 10\ \mathrm{m/s^2}$)



A. $20\ \mathrm{cm}$

B. $180\ \mathrm{cm}$

C. $26\ \mathrm{cm}$

D. $14\ \mathrm{cm}$

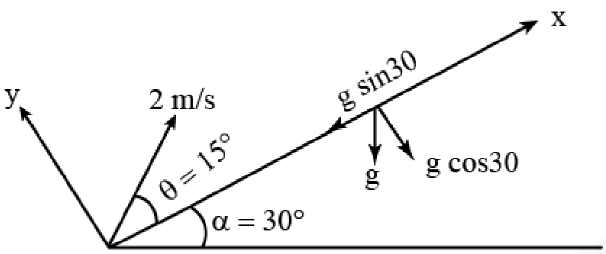
On an inclined plane, time of flight (T) is given by,

$$T=\frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T=\frac{(2)(2\sin 15^\circ)}{g\cos 30^\circ}=\frac{4\sin 15^\circ}{10\cos 30^\circ}$$

$$\text{Distance, S}=(2\cos 15^\circ)T-\frac{1}{2}g\sin 30^\circ(T)^2$$



$$=(2\cos 15^\circ)\frac{4\quad\sin 15^\circ}{10\;10\cos 30^\circ}-\left(\frac{1}{2}\times 10\sin 30^\circ\right)\frac{16\sin^2 15^\circ}{100\cos^2 30^\circ}$$

$$\left[\because \sin 15^0=\frac{\sqrt{3}-1}{2\sqrt{2}}\right]$$

$$=\frac{16\sqrt{3}-16}{60}\simeq 0.1952\text{ m}\simeq 20\text{ cm}$$

Hence, (A) is the correct answer.

19. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is:

A. $\frac{R}{4g}$

B. $\frac{R}{g}$

C. $\frac{R}{2g}$

D. $\frac{2R}{g}$

R will be same for θ and $90^\circ - \theta$.

Time of flights,

$t_1 = \frac{2u\sin\theta}{g}$ and

$$t_2 = \frac{2u\sin(90^\circ)}{g} = \frac{2u\cos\theta}{g}$$

$$\text{Now, } t_1 t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right)$$

$$= \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g} \qquad \left[\because R = \frac{u^2\sin 2\theta}{g}\right]$$

Hence, (D) is the correct answer.

20. The trajectory of the projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it was launched at an angle θ_0 with speed v_0 then $(g = 10\ \mathrm{m/s^2})$

A. $\theta_0 = \sin^{-1}\dfrac{1}{\sqrt{5}}$ and $v_0 = \dfrac{3}{5}\mathrm{ms}^{-1}$

B. $\theta_0 = \cos^{-1}\dfrac{2}{\sqrt{5}}$ and $v_0 = \dfrac{3}{5}\mathrm{ms}^{-1}$

C. $\theta_0 = \cos^{-1}\dfrac{1}{\sqrt{5}}$ and $v_0 = \dfrac{5}{3}\mathrm{ms}^{-1}$

D. $\theta_0 = \sin^{-1}\dfrac{2}{\sqrt{5}}$ and $v_0 = \dfrac{3}{5}\mathrm{ms}^{-1}$

Given, $y = 2x - 9x^2$

On comparing with,

$$y = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$$

We have,

$$\tan\theta = 2 \text{ or } \cos\theta = \frac{1}{\sqrt{5}} \qquad \left[\because \cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}\right]$$

$$\text{And, } \frac{g}{2v_0^2\cos^2\theta} = 9$$

$$\Rightarrow \frac{10}{2v_0^2(1/\sqrt{5})^2} = 9$$

$$\therefore v_0 = \frac{5}{3}\mathrm{m/s}$$

Hence, (C) is the correct answer.

Topic : Relative Motion

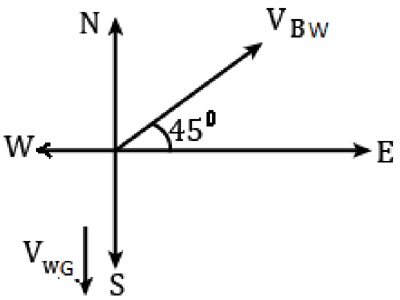
1. A butterfly is flying with a velocity $4\sqrt{2}\ \mathrm{m/s}$ in North-East direction. Wind is slowly blowing at $1\ \mathrm{m/s}$ from North to South. The resultant displacement of the butterfly in 3 seconds is :

A. $3\ \mathrm{m}$

B. $20\ \mathrm{m}$

C. $12\sqrt{2}\ \mathrm{m}$

D. $15\ \mathrm{m}$



In the figure,

$\vec{V}_{BW}$ = Velocity of butterfly in the frame of wind

$\vec{V}_{WG}$ = Velocity of wind in the frame of ground

We know that,

$$\vec{V}_{BW} = \vec{V}_{BG} - \vec{V}_{WG}$$

$$\Rightarrow \vec{V}_{BG} = \vec{V}_{BW} + \vec{V}_{WG}$$

$$\therefore V_{BG} = \sqrt{V_{BW}^2 + V_{WG}^2 + 2V_{BW}V_{WG}\cos\theta}$$

$$\Rightarrow V_{BG} = \sqrt{(4\sqrt{2})^2 + 1^2 + 2(4\sqrt{2})(1)\cos(90^\circ + 45^\circ)}$$

$$\Rightarrow V_{BG} = 5\ \mathrm{m/s}$$

So, displacement in 3 seconds,

$$d = V_{BG}t = 5 \times 3 = 15\ \mathrm{m}$$

Hence, option (D) is the correct answer.

2. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator, then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be:

A. $\dfrac{t_1 t_2}{t_2 - t_1}$

B. $\dfrac{t_1 + t_2}{2}$

C. $\dfrac{t_1 t_2}{t_2 + t_1}$

D. $t_2 - t_1$

Let L be the length of the escalator.

When only boy is walking,

$$v_b = \frac{L}{t_1}$$

And when only escalator is moving,

$$v_{esc} = \frac{L}{t_2}$$

Now let t be the time taken by boy to walk up on the moving escalator.

$$\therefore t = \frac{L}{v_b + v_{esc}} = \frac{L}{(L/t_1) + (L/t_{esc})}$$

$$\Rightarrow t = \frac{t_1 t_2}{t_1 + t_2}$$

Hence, option (C) is correct.

3. Ship A is sailing towards north-east with velocity vector $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in,

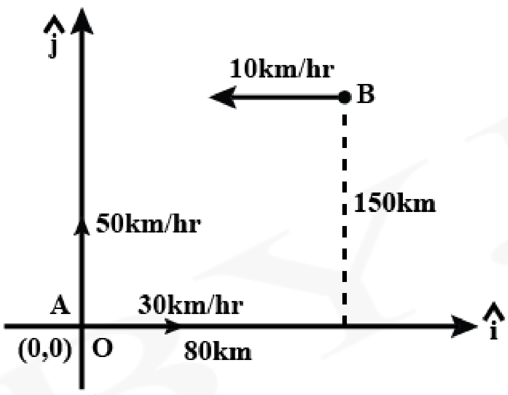
A. 2.2 hrs

B. 4.2 hrs

C. 2.6 hrs

D. 3.2 hrs

Given data is represented in the figure,



From figure,

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = (30\hat{i} + 50\hat{j}) - (-10\hat{i})$

$\Rightarrow \vec{V}_{AB} = 40\hat{i} + 50\hat{j}$ and

$\vec{r}_{AB} = -80\hat{i} - 150\hat{j}$

Now, A will be at minimum distance from B in time,

$$t = \frac{\vec{r}_{AB}.\vec{V}_{AB}}{|\vec{V}_{AB}|^2}$$

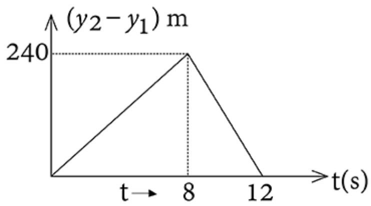
$$\Rightarrow t = -\frac{(-80\hat{i} - 150\hat{j}).(40\hat{i} + 50\hat{j})}{(\sqrt{40^2 + 50^2})^2}$$

$$\Rightarrow t = -\frac{-3200 - 7500}{4100} = 2.6 \text{ hr}$$

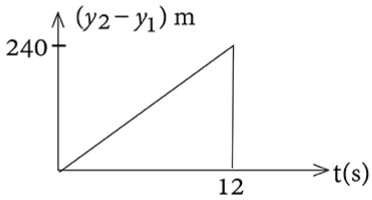
Hence, (C) is the correct answer.

4. Two stones are thrown up simultaneously from the edge of a cliff $240\ \mathrm{m}$ high with initial speed $10\ \mathrm{m/s}$ and $40\ \mathrm{m/s}$ respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ \mathrm{m/s^2}$)
(The figures are schematic and not drawn to scale.)

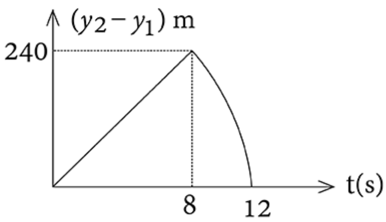
A.



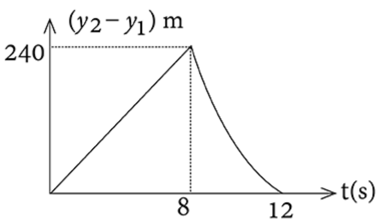
B.



C. ✔



D.



Using equation of motion for the first stone,

$$-240 = 10t - \frac{1}{2}gt^2$$

$$\Rightarrow 5t^2 - 10t - 240$$

$$\Rightarrow t^2 - 2t - 48 = 0$$

$$\Rightarrow t = 8, -6$$

$\therefore$ The 1st stone will reach the ground in 8 secs.

Upto 8 secs, the relative velocity between the particles is 30 m/sec and the relative acceleration is zero.

For the 2nd stone,

$$-240 = 40t - 5t^2$$

$$\Rightarrow t^2 - 8t - 48 = 0$$

$$t = 12 \text{ secs}$$

The second stone will strike the ground in 12 secs.

Now, relative position

$$\Delta y = y_1 - y_2 = 40t - \frac{1}{2}gt^2 - 10t + \frac{1}{2}gt^2 = 30t$$

$$\Delta y = 30t$$

After 8 second stone 1 reaches ground
$y_1 = -240 \text{ m}$

$$\therefore \Delta y = y_2 - y_1$$

$$= 40t - \tfrac{1}{2}gt^2 + 240$$

$\therefore$ It will be a parabolic curve after 8 secs.

So, the correct option is (C).

5. When a car is at rest, its driver sees raindrops falling on it vertically. When driving the car with speed v, he sees that raindrops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1+\beta)v$, this angle changes to 45°. The value of β is close to :

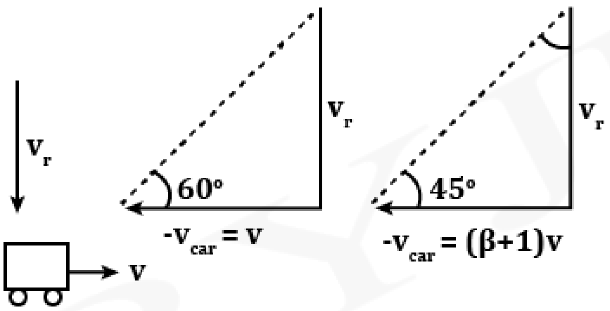
A. 0.50

B. 0.41

C. 0.37

D. 0.73

The given situation is shown in the diagram. Here v_r be the velocity of rain drop.



When car is moving with speed v,

$$\tan 60^\circ = \frac{v_r}{v} \quad \ldots(i)$$

When car is moving with speed $(1+\beta)v$,

$$\tan 45^\circ = \frac{v_r}{(\beta+1)v} \quad \ldots(ii)$$

Dividing (i) by (ii) we get,

$$\sqrt{3} = (\beta+1)$$

$$\Rightarrow \beta = \sqrt{3} - 1 = 0.732.$$

Hence, (D) is the correct answer.

6. A passenger train of length $60 \mathrm{~m}$ travels at a speed of $80 \mathrm{~km} / \mathrm{hr}$. Another freight train of length $120 \mathrm{~m}$ travels at a speed of $30 \mathrm{~km} / \mathrm{h}$. The ratio of times taken by the passenger train to completely cross the freight train when :
(i)they are moving in the same direction, and
(ii) in the opposite directions is:

A. $\frac{11}{5}$

B. $\frac{5}{2}$

C. $\frac{3}{2}$

D. $\frac{25}{11}$

Given:
$l_p=60 \mathrm{~m}=0.06 \mathrm{~km}$; $V_p=80 \mathrm{~km} / \mathrm{h}$
$l_f=120 \mathrm{~m}=0.12 \mathrm{~km}$; $V_f=30 \mathrm{~km} / \mathrm{h}$

(i) When they are moving in the same direction, the time taken to completely cross the freight train by the passenger train be

$$t_1=\frac{d_{total}}{V_{relative}}$$

$$=\frac{0.06+0.12}{80-30}=\frac{0.18}{50} \quad \ldots .(1)$$

(ii) When they are moving in the opposite direction, the time taken to completely cross the freight train by the passenger train be

$$t_2=\frac{d_{total}}{V'_{relative}}$$

$$=\frac{0.18}{80+30}=\frac{0.18}{110} \quad \ldots .(2)$$

Using equations (1) and (2),

$$\frac{t_1}{t_2}=\frac{110}{50}=\frac{11}{5}$$

Hence, (A) is the correct answer.

7. The stream of a river is following with a speed of $2\ \mathrm{km/h}$. A swimmer can swim at a speed of $4\ \mathrm{km/h}$. What should be the direction of swimmer with respect to the flow of the river to cross the river straight?

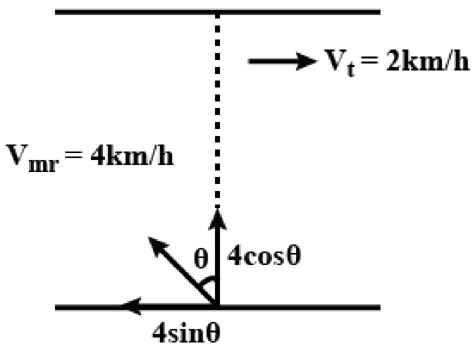
A. 90°

B. 150°

C. 120°

D. 60°

The given situation is shown in the figure below



To cross the river straight,

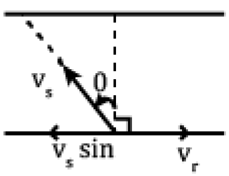
$V_{mr} \sin\theta = V_r$

$\therefore \sin\theta = \dfrac{V_r}{V_{mr}} = \dfrac{2}{4} = \dfrac{1}{2}$

$\therefore \theta = 30^\circ$

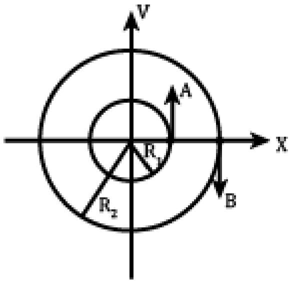
Direction of swimmer with respect to flow,

$= 90^\circ + \theta = 90^\circ + 30^\circ = 120^\circ$



Hence, (C) is the correct answer.

8. Two particles A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t=0$, their positions and direction of motion are shown in the figure :



The relative velocity $\overrightarrow{v_A}-\overrightarrow{v_B}$ at $t=\frac{\pi}{2\omega}$ is given by :

A. $\omega(R_1+R_2)\hat{i}$

B. $-\omega(R_1+R_2)\hat{i}$

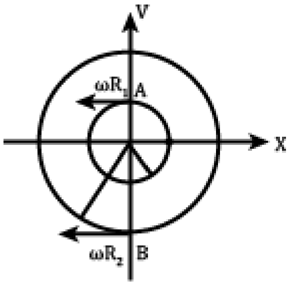
C. $\omega(R_2-R_1)\hat{i}$

D. $\omega(R_1-R_2)\hat{i}$

Using, $\theta=\omega t$ we get,

$$\theta=\omega\frac{\pi}{2\omega}=\frac{\pi}{2}$$

So, both have completed quarter circle



Relative velocity,

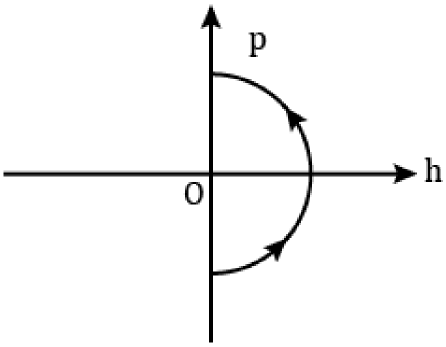
$$\overrightarrow{v}_A-\overrightarrow{v}_B=\omega R_1(-\hat{i})-\omega R_2(-i)$$

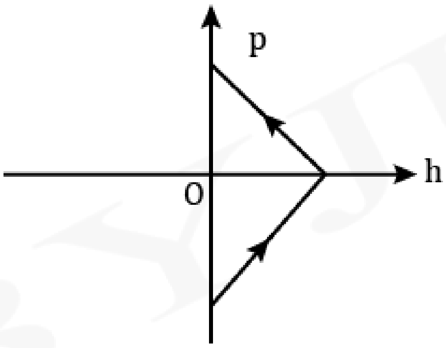
$$\therefore \overrightarrow{v}_A-\overrightarrow{v}_B=\omega(R_2-R_1)i$$

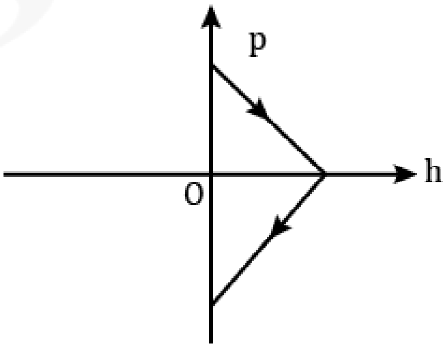
Hence, (C) is the correct answer.

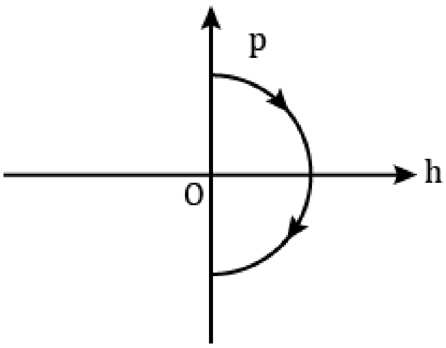
Topic : Newton law of Motion

1. A ball is thrown vertically up (taken as $+z$ - axis) from the ground. The correct momentum-height $(p$ - $h)$ diagram is: (Given $p = mv$)

A.


B.


C.


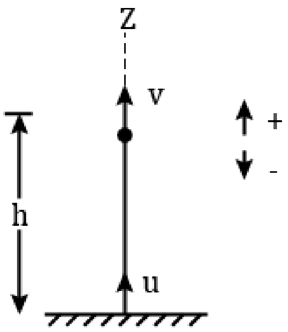
D.


Using the $v^2 = u^2 - 2gh$, we get

$v = \sqrt{u^2 - 2gh}$

Now we know that, momentum $p = mv$

$\therefore p = m\sqrt{u^2 - 2gh}$



Therefore, the graph between p and h cannot be a straight line.

(b) and (c) are not possible.

During upward journey as h increases, p decreases and in downward journey as h decreases p increases.

Therefore (D) is the correct option.

2. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively

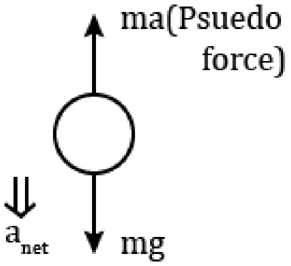
A. $g,\ g$

B. $g-a,\ g-a$

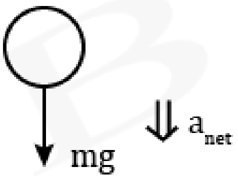
C. $g-a,\ g$

D. $a,\ g$

Man in lift is in non-inertial frame. By applying psuedoforce on the ball,we get net acceleration $= g-a$ w.r.t. lift,



As earth is assumed to be inertial frame, no pseudo force acts. So FBD is



Hence $a_{\text{net}} = g$

3. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the magnitude of acceleration of the particle is

A. $\frac{F_1}{m}$

B. $\frac{F_2 F_3}{m F_1}$

C. $\frac{(F_2 - F_3)}{m}$

D. $\frac{F_2}{m}$

Given,

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0 = F_{\text{net}}$$

Now if F_1 is removed, let acceleration be $'a'$.

Then, $\vec{F_2} + \vec{F_3} = m\vec{a}$

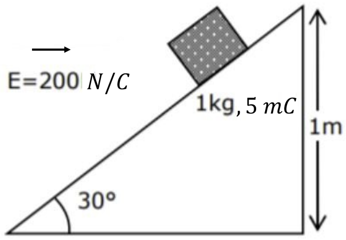
$$\Rightarrow -\vec{F_1} = m\vec{a}$$

$$\Rightarrow \vec{a} = -\frac{\vec{F_1}}{m}$$

$$\therefore \left|\vec{a}\right| = \left|\frac{\vec{F_1}}{m}\right|$$

Hence, (A) is the correct answer.

4. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\ \mathrm{N/C}$ as shown in the figure. A body of mass $1\ \mathrm{kg}$ and charge $5\ \mathrm{mC}$ is allowed to slide down from rest from a height of $1\ \mathrm{m}$. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.



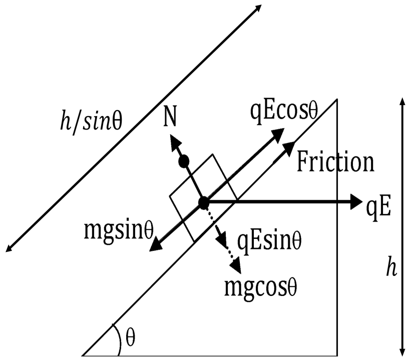
$$\left[g = 9.8\ \mathrm{m/s^2};\ \sin 30^\circ = \frac{1}{2};\ \cos 30^\circ = \frac{\sqrt{3}}{2}\right]$$

A. $2.3\ \mathrm{s}$

B. $0.46\ \mathrm{s}$

C. $1.3\ \mathrm{s}$ ✔

D. $0.92\ \mathrm{s}$



Net force along the inclined plane.

$F = mg\sin\theta - qE\cos\theta - f$

$\Rightarrow F = mg\sin\theta - qE\cos\theta - \mu N$

$\Rightarrow F = mg\sin\theta - qE\cos\theta - \mu(qE\sin\theta + mg\cos\theta)$

$\Rightarrow F = 1 \times 9.8 \times \frac{1}{2} - 5 \times 10^{-3} \times 200 \times \frac{\sqrt{3}}{2} - 0.2\left(5 \times 10^{-3} \times 200 \times \frac{1}{2} + 1 \times 9.8 \times \frac{\sqrt{3}}{2}\right)$

$\Rightarrow F \approx 2.24\ \mathrm{N}$

So, $a \approx F/m \approx 2.24\ \mathrm{m/s^2}$

Further,

$\frac{h}{\sin\theta} = 0 + \frac{1}{2}at^2$

$\Rightarrow \frac{1}{1/2} = \frac{1}{2} \times 2.24 \times t^2$

$\Rightarrow t \approx 1.3\ \mathrm{s}$

5. A particle of mass M orginally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation $F = F_0\left[1-\left(\frac{t-T}{T}\right)^2\right]$ where F_0 and T are constants. The force acts only for the time interval $2T$. The velocity v of the particle after time $2T$ is :

A. $\frac{2F_0T}{M}$

B. $\frac{F_0T}{2M}$

C. $\frac{4F_0T}{3M}$ ✔

D. $\frac{F_0T}{3M}$

From the given problem we can infer that
at $t=0, u=0$

also, $F = F_0\left[1-\left(\frac{t-T}{T}\right)^2\right]$

or, $F = F_0 - \frac{F_0(t-T)^2}{T^2}$

Therefore, we have

acceleration, $a = \frac{F_0}{M} - \frac{F_0(t-T)^2}{MT^2} = \frac{dv}{dt}$

or, $\int_0^v dv = \int_{t=0}^{2T}\left(\frac{F_0}{M} - \frac{F_0(t-T)^2}{MT^2}\right)dt$

$\Rightarrow\ v = \left[\frac{F_0}{M}t\right]_0^{2T} - \frac{F_0}{MT^2}\left[\frac{t^3}{3} - t^2T + T^2t\right]_0^{2T}$

$\Rightarrow\ v = \frac{4F_0T}{3M}$

The particle will have constant velocity, as force will not act after $t = 2T$, which will be equal to v.
Hence, option (c) is correct.

6. A particle is projected with velocity v_o along $x-$axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin, $i.e.\ ma = -\alpha x^2$. The distance at which the particle stops :

A. $\left(\frac{2mv_o^2}{3\alpha}\right)^{\frac{1}{3}}$

B. $\left(\frac{3mv_o^2}{2\alpha}\right)^{\frac{1}{2}}$

C. $\left(\frac{3mv_o^2}{2\alpha}\right)^{\frac{1}{3}}$

D. $\left(\frac{2mv_o^2}{3\alpha}\right)^{\frac{1}{2}}$

For damping force, acceleration of the particle is,

$$a = \frac{-\alpha x^2}{m}$$

Here, acceleration of the particle is varying with distance.
So that,

$$a = v\frac{dv}{dx} = \frac{-\alpha x^2}{m}$$

$$\Rightarrow\ vdv = \frac{-\alpha x^2}{m}dx$$

$$\Rightarrow\ \int_{v_i}^{v_f} vdv = \frac{-\alpha}{m}\int_{x_i}^{x_f} x^2 dx$$

$$\Rightarrow\ \left[\frac{v_f^2 - v_i^2}{2}\right] = \frac{-\alpha}{m}\left[\frac{x_f^3 - x_i^3}{3}\right]$$

Here,
$v_f = 0, \quad v_i = v_0$
$x_f = x, \quad x_i = 0$

$$\Rightarrow\ \left[\frac{-v_o^2}{2}\right] = \frac{-\alpha}{m}\left[\frac{x^3}{3}\right]$$

$$\Rightarrow\ \left[\frac{v_o^2}{2}\right] = \frac{\alpha}{m}\left[\frac{x^3}{3}\right]$$

$$\Rightarrow\ \left[\frac{3mv_o^2}{2\alpha}\right] = x^3$$

$$\Rightarrow\ x = \left(\frac{3mv_o^2}{2\alpha}\right)^{\frac{1}{3}}$$

Hence, (C) is the correct answer.

7. Two forces P and Q, of magnitude $2F$ and $3F$ respectively are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then the angle θ is

A. 120°

B. 60°

C. 90°

D. 30°

Using Parallelogram law,

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$
$$4F^2 + 9F^2 + 12F^2\cos\theta = R^2$$
$$13F^2 + 12F^2\cos\theta = R^2$$

When force Q is doubled,
$$4F^2 + 36F^2 + 24F^2\cos\theta = 4R^2$$
$$4F^2 + 36F^2 + 24F^2\cos\theta = 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$$
$$\therefore \cos\theta = -\frac{12F^2}{24F^2} = -\frac{1}{2}$$
$$\Rightarrow \theta = 120^\circ$$

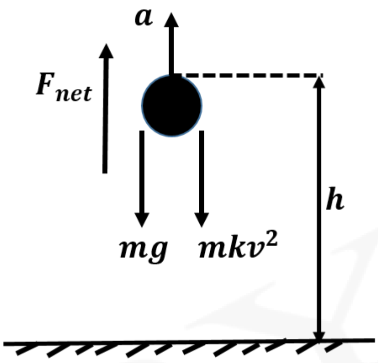
8. A small ball, of mass m, is thrown upward with a velocity u, from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is :

A. $\frac{1}{2k}\tan^{-1}\left(\frac{ku^2}{g}\right)$

B. $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$

C. $\frac{1}{k}\tan^{-1}\frac{ku^2}{g}$

D. $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$



From free body diagram, the net force acting on the ball,

$\Rightarrow F_{net} = -mkv^2 - mg$

So, acceleration

$$\Rightarrow a = \frac{F_{net}}{m} = -[kv^2 + g]$$

$$\Rightarrow v.\frac{dv}{dh} = -[kv^2 + g]\left(\because a = v\frac{dv}{dh}\right)$$

Integrating both sides,

$$\Rightarrow \int_u^0 \frac{v.dv}{kv^2+g} = \int_0^h dh$$

$$\Rightarrow \frac{1}{2k}\ln\left[kv^2+g\right]_u^0 = h$$

$$\Rightarrow \frac{1}{2k}\ln\left(\frac{ku^2+g}{g}\right) = h.$$

Hence, (D) is the correct answer.

9. A mass of $10\ \mathrm{kg}$ is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is $(g = 10\ \mathrm{ms}^{-2})$

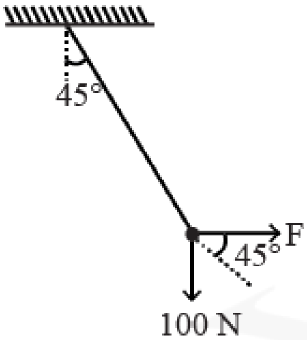
A. $200\ \mathrm{N}$

B. $140\ \mathrm{N}$

C. $70\ \mathrm{N}$

D. $100\ \mathrm{N}$ ✔

Freebody diagram of the problem is draw as



At equilibrium,

$$\tan 45^\circ = \frac{mg}{F} = \frac{100}{F}$$

$$\therefore F = 100\ \mathrm{N}$$

Hence, (D) is the correct answer.

10. A particle of mass m is moving in a straight line. Starting at time $t=0$, a force $F=kt$ acts in the same direction on the moving particle during time interval T so that its momentum changes from p to $3p$. Here k is a constant. The value of T is :

A. $2\sqrt{\frac{k}{p}}$

B. $2\sqrt{\frac{p}{k}}$

C. $\sqrt{\frac{2k}{p}}$

D. $\sqrt{\frac{2p}{k}}$

From Newton's second law

$$\frac{dp}{dt}=F=kt$$

Integrating both sides we get,

$$\int_p^{3p} dp=\int_0^T kt\,dt$$

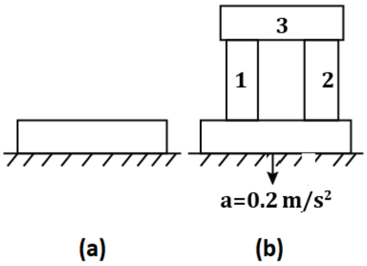
$$\Rightarrow [p]_p^{3p}=k\left[\frac{t^2}{2}\right]_0^T$$

$$\Rightarrow 2p=\frac{kT^2}{2}$$

$$\Rightarrow T=2\sqrt{\frac{p}{k}}$$

Hence, (B) is the correct answer.

11. A steel block of mass $10 \mathrm{~kg}$ rests on a horizontal floor as shown in the figure (a). When three iron cylinders are placed on it as shown in the figure (b), the block and cylinders go down with an acceleration $0.2 \mathrm{~m} / \mathrm{s}^2$. The normal reaction R exerted by the floor, if masses of the iron cylinders are $20 \mathrm{~kg}$ each, is -

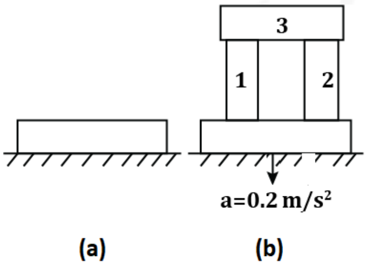


A. $716 \mathrm{~N}$

B. $686 \mathrm{~N}$

C. $714 \mathrm{~N}$

D. $684 \mathrm{~N}$



Assuming all four blocks are as a system.

Writing the force equation on the system in vertical direction,

$M g-R=M a$

$\Rightarrow(10+(3 \times 20)) 10-R=(10+(3 \times 20)) \times 0.2$

$\Rightarrow R=70(10-0.2)=686 \mathrm{~N}$

Hence, option (B) is the correct answer.

12. A force $\vec{F} = (40\hat{i} + 10\hat{j})\ \mathrm{N}$ acts on a body of mass $5\ \mathrm{kg}$. If the body starts from rest, its position vector $\vec{r}$ at time $t = 10\ \mathrm{s}$, will be :

A. $(100\hat{i} + 400\hat{j})\ \mathrm{m}$

B. $(100\hat{i} + 100\hat{j})\ \mathrm{m}$

C. $(400\hat{i} + 100\hat{j})\ \mathrm{m}$

D. $(400\hat{i} + 400\hat{j})\ \mathrm{m}$

Given that,

$\vec{F} = (40\hat{i} + 10\hat{j})\ \mathrm{N}$
$m = 5\ \mathrm{kg}$

As we know that,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{\vec{F}}{m} = (8\hat{i} + 2\hat{j})\ \mathrm{ms^{-2}}$$

Doing double integrating both side w.r.t. t from $t = 0$ to $t = t$ and we get

$$\Rightarrow \frac{d\vec{r}}{dt} = \vec{v} = (8t\hat{i} + 2t\hat{j})\ \mathrm{ms^{-1}}$$

$$\Rightarrow \vec{r} = (8\hat{i} + 2\hat{j})\frac{t^2}{2}\mathrm{m}$$

$\therefore$ At $t = 10\ \mathrm{s}$

$$\vec{r} = (8\hat{i} + 2\hat{j})50\ \mathrm{m}$$

$$\Rightarrow \vec{r} = (400\hat{i} + 100\hat{j})\ \mathrm{m}$$

Hence, option (C) is correct.

13. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2}. the gases come out at a relative speed of 500 ms^{-1} with respect to the rocket : [use $g = 10 \text{ ms}^{-2}$]

A. 60 kg s^{-1}

B. $6.0 \times 10^2 \text{ kg s}^{-1}$

C. 500 kg s^{-1}

D. 10 kg s^{-1}

From the question it is clear that we have to calculate the rate of change of mass with respect to time. The rocket thrust is equal to the burnt mass times relative velocity of the rocket.

So, $F_{\text{thurst}} = \frac{dm}{dt} v_{\text{rel}}$

The net force acting on the rocket is given by,

$F_{\text{thurst}} - mg = ma$

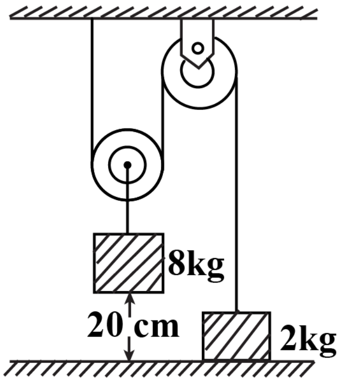
i.e. $\frac{dm}{dt} v_{\text{rel}} - mg = ma$

$\frac{dm}{dt} \times 500 - 1000 \times 10 = 1000 \times 20$

Therefore, $\frac{dm}{dt} = 60 \text{ kg s}^{-1}$

Hence, option (A) is correct.

14. The boxes of masses $2\ \mathrm{kg}$ and $8\ \mathrm{kg}$ are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass $8\ \mathrm{kg}$ to strike the ground starting from rest. $(\text{Use } g = 10\ \mathrm{m/s^2})$:



A. $0.2\ \mathrm{s}$

B. $0.34\ \mathrm{s}$

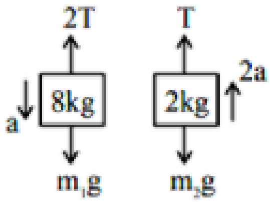
C. $0.25\ \mathrm{s}$

D. $0.4\ \mathrm{s}$

Let tension in the string attached to 2 kg mass be T,
Tension in the string attached to 8 kg mass is $2T$

Using constraing equations,
If acceleration of 8 kg mass is a, then acceleration of 2 kg mass is $2a$

Free body diagram of masses can be drawn as,



From Newtons second law,

$8g - 2T = 8a \ldots (i)$

$T - 2g = 4a \ldots (ii)$

on substitution, we get,

$8g - 2(2g + 4a) = 8a$

$4g = 16a$

$a = 2.5 \text{ ms}^{-2}$

Now, from $S = ut + \frac{1}{2}at^2$

$0.2 = \frac{1}{2}(2.5)t^2$

$\frac{4}{25} = t^2$

$t = \frac{2}{5} = 0.4 \text{ s}$

Hence, option (D) is correct.

15. Statement I:

If three forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are represented by three sides of a triangle and $\overrightarrow{F_1}+\overrightarrow{F_2}=-\overrightarrow{F_3}$, then these forces are concurrent forces and satisfy the condition for equilibrim.

Statement II :

A triangle made up of three forces $\overrightarrow{F_1}, \overrightarrow{F_2}$ and $\overrightarrow{F_3}$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from options given below.

A. Both Statement I and Statement II are true.

B. Both Statement I and Statement II are false

C. Statement I is true but Statement II is false.

D. Statement I is false but Statement II is true

Let three forces $\overrightarrow{F_1}, \overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are represented by three sides of a triangle

Statement I

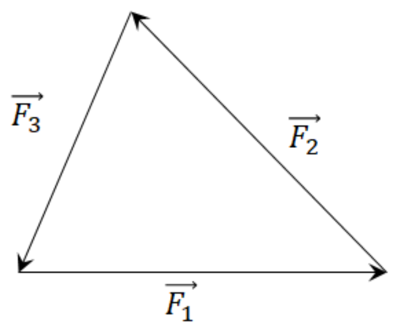
Given that $\overrightarrow{F_1}+\overrightarrow{F_2}=-\overrightarrow{F_3}$

$\overrightarrow{F_1}+\overrightarrow{F_2}+\overrightarrow{F_3}=0$

ie Net force is zero, hence these concurrent forces, satisfy condition of equilibrium.

Statement II

Forces are taken in same order as shown below.



Using triangle law of vectors we get

$\overrightarrow{F_1}+\overrightarrow{F_2}+\overrightarrow{F_3}=0$

Hence these forces satisfy condition of translatory equilibrium.

Hence, both Statement I and Statement II are true.

Therefore, option (A) is correct.

16. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time is taken by the ball to rise to its zenith is:

A. $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

B. $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

C. $\frac{1}{\sqrt{\gamma g}}\ln^{-1}\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

D. $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

Net upward force acting on the ball,

$$F_{net} = -mg - m\gamma v^2$$

So, the net acceleration

$$\frac{dv}{dt} = a = \frac{F_{net}}{m} = -(g+\gamma v^2)$$

Let time t required to rise to its zenith, where $v = 0$, so,

$$\int_{V_0}^{0}\frac{-dv}{g+\gamma v^2} = \int_0^t dt$$

Using the integration formula we get,

$$-\left[\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\frac{\sqrt{\gamma}v}{\sqrt{g}}\right)\right]_{V_0}^{0} = t$$

$$\Rightarrow t = \frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\frac{\sqrt{\gamma}V_0}{\sqrt{g}}\right)$$

Hence, (A) is the correct solution

17. A body of mass $5 \mathrm{~kg}$ under the action of constant force $\vec{F}=F_x \hat{i}+F_y \hat{j}$ has velocity at $t=0 \sec$ as $\vec{v}=(6 \hat{i}-2 \hat{j}) \mathrm{m/s}$ and at $t=10 \sec$ as $\vec{v}=+6 \hat{j} \mathrm{~m/s}$. The force $\vec{F}$ is

A. $(-3 \hat{j}+4 \hat{j}) \mathrm{N}$

B. $\left(-\frac{3}{5} \hat{i}+\frac{4}{5} \hat{j}\right) \mathrm{N}$

C. $(3 \hat{j}-4 \hat{j}) \mathrm{N}$

D. $\left(-\frac{3}{4} \hat{i}-\frac{4}{5} \hat{j}\right) \mathrm{N}$

From question,
Mass of body, $m=5 \mathrm{~kg}$
Velocity at $t=0$ is $u=6 \hat{i}-2 \hat{j} \mathrm{~m/s}$
Velocity at $t=10 \mathrm{~s}$ is $v=+6 \hat{j} \mathrm{~m/s}$

Acceleration, $a=\frac{v-u}{t}$

$$=\frac{6 \hat{j}-(6 \hat{i}-2 \hat{j})}{10}=\frac{-3 \hat{i}+4 \hat{j}}{5} \mathrm{m/s}^2$$

Now, using, Newton's second law,

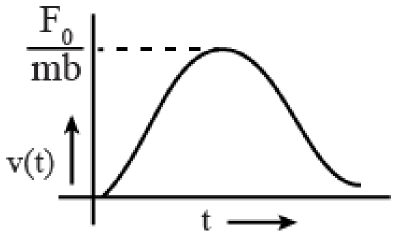
$F=m a$

$$=5 \times \frac{-3 \hat{i}+4 \hat{j}}{5}=(-3 \hat{i}+4 \hat{j}) \mathrm{N}$$

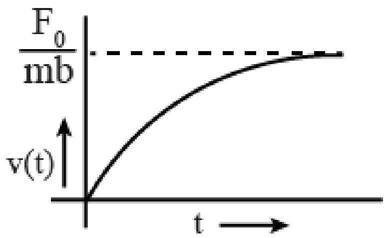
Hence, (A) is the correct answer.

18. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves?

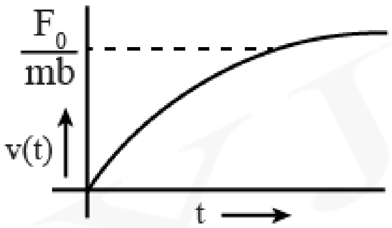
A.



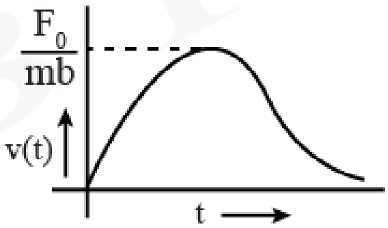
B.



C.



D.



Given that,

$$F(t) = F_0 e^{-bt}$$

Using newton's second law of motion,

$$F(t) = m\frac{dv}{dt}$$

$$\Rightarrow m\frac{dv}{dt} = F_0 e^{-bt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{F_0}{m}e^{-bt}$$

Rearranging and integrating the above equation,

$$\int_0^v dv = \frac{F_0}{m}\int_0^t e^{-bt}$$

$$\Rightarrow v = \frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t = \frac{F_0}{mb}\left[-\left(e^{-bt} - e^{-0}\right)\right]$$

$$\Rightarrow v = \frac{F_0}{mb}[1 - e^{-bt}]$$

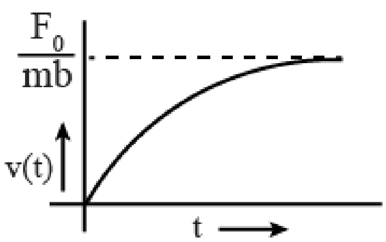
At $t = 0$,

$$v = \frac{F_0}{mb}[1 - e^{-b\times 0}] = \frac{F_0}{mb}[1 - 1] = 0$$

At, $t = \infty$

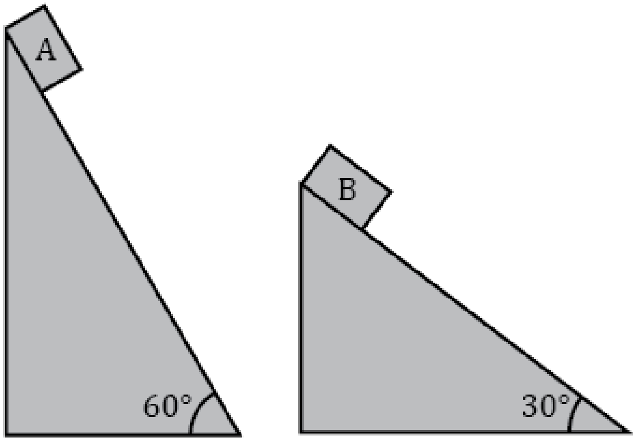
$$v = \frac{F_0}{mb}[1 - e^{-b\times\infty}] = \frac{F_0}{mb}[1 - 0] = \frac{F_0}{mb}$$

On plotting the given data we get,



Hence, (B) is the correct answer.

19. Two fixed frictionless inclined plane making an angle 30° and 60° with the vertical as shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?

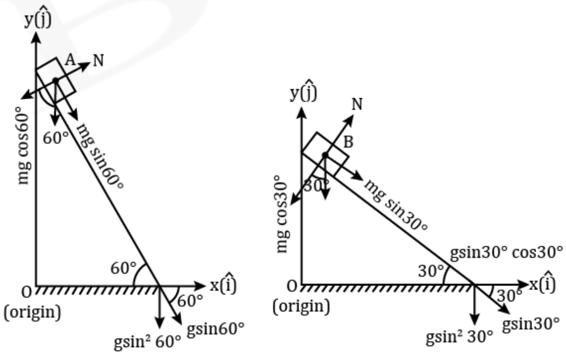


A. $4.9\ \mathrm{ms^{-2}}$ in horizontal direction

B. $9.8\ \mathrm{ms^{-2}}$ in vertical direction

C. Zero

D. $4.9\ \mathrm{ms^{-2}}$ in vertical direction



From FBD of mass A and B, we get

$\vec{a}_A = g\sin 60^\circ \cos 60^\circ \hat{i} - g\sin^2 60^\circ \hat{j}$

$\vec{a}_A = \frac{\sqrt{3}}{4} g\hat{i} - \frac{3}{4} g\hat{j}$

$\vec{a}_B = g\sin 30^\circ \cos 30^\circ \hat{i} - g\sin^2 30^\circ \hat{j}$

$\vec{a}_B = \frac{\sqrt{3}}{4} g\hat{i} - \frac{1}{4} g\hat{j}$

The relative acceleration of block A with respect to B.

$\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B = \frac{-g}{2}\hat{j}$

$\Rightarrow \vec{a}_{A/B} = 4.9\ \mathrm{ms^{-2}}$ in vertical direction

20. A ball of mass $2.0 \mathrm{~kg}$ is thrown vertically upwards by applying a force by hand. If the hand moves $0.2 \mathrm{~m}$ while applying the force and the ball goes up to $2 \mathrm{~m}$ height further, find the magnitude of the force. (Consider $g=10 \mathrm{~m/s}^2$)

A. $4 \mathrm{~N}$

B. $16 \mathrm{~N}$

C. $20 \mathrm{~N}$

D. $22 \mathrm{~N}$

For the motion of ball, just after the throwing
$v=0, s=2 \mathrm{~m}, a=-g=-10 \mathrm{~m/s}^{-2}$

Using equation of motion for upward journey,

$v^2-u^2=2as$

$\Rightarrow -u^2=2(-10)\times 2$

$\Rightarrow u^2=40$

When the ball is in the hands of the thrower

$u=0, v=\sqrt{40} \mathrm{~ms}^{-1},\ s=0.2 \mathrm{~m}$

$\Rightarrow 40-0=2(a)0.2$

$\Rightarrow a=100 \mathrm{~m/s}^2$

From Newton's second law of motion,

$F=ma=0.2\times 100=20 \mathrm{~N}$

$\Rightarrow N-mg=20$

$\Rightarrow N=20+2=22 \mathrm{~N}$

Alternate Solution:

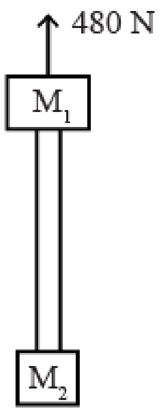
Using work-energy theorem,

$W_{hand}-W_{gravity}=\Delta K$

$\Rightarrow F(0.2)+(0.2)(10)(2.2)=0 \Rightarrow F=22\ N$

Hence, (D) is the correct answer.

21. Two blocks of mass $M_1 = 20 \text{ kg}$ and $M_2 = 12 \text{ kg}$ are connected by a metal rod of mass 8 kg. The system is pulled vertically up by applying a force of 480 N as shown. The tension at the mid-point of the rod is:



A. 144 N

B. 96 N

C. 240 N

D. 192 N

Acceleration produced in upward direction,

$$a = \frac{F}{M_1 + M_2 + \text{Mass of metal rod}}$$

$$= \frac{480}{20 + 12 + 8} = 12 \text{ ms}^{-2}$$

Tension at the midpoint,

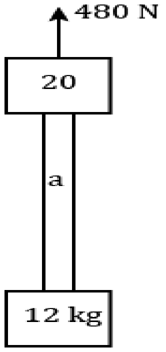
$$T = \left(M_2 + \frac{\text{Mass of rod}}{2}\right) a$$

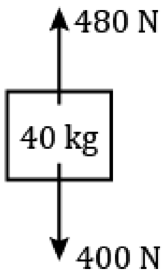
$$= (12 + 4) \times 12 = 192 \text{ N}$$

Hence, (D) is the correct answer.

Alternate solution:

FBD of system:



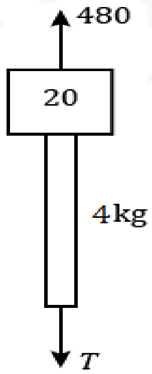


Using equilibrium condition,

$\Rightarrow 480-400=40\text{ a}$

$\Rightarrow \text{a}=2\text{ m/s}^2$

Now at midpoint,



$F_{\text{net}}=480-24\text{ g}-T=24\text{ a}$

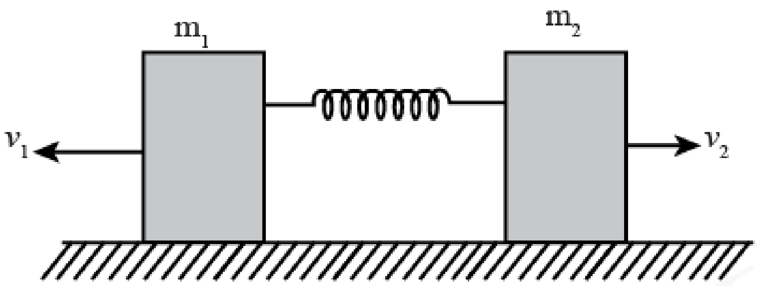
$\Rightarrow 480-240-T=48$

$\therefore T=240-48=192\text{ N}$

Hence, (D) is the correct answer.

22. A spring is compressed between two blocks of masses m_1 and m_2 placed on a horizontal frictionless surface as shown in the figure. When the blocks are released, they have initial velocity of v_1 and v_2 as shown. The blocks travel distances x_1 and x_2 respectively before coming to rest.

The ratio $\left(\frac{x_1}{x_2}\right)$ is



A. $\frac{m_2}{m_1}$

B. $\frac{m_1}{m_2}$

C. $\sqrt{\frac{m_2}{m_1}}$

D. $\sqrt{\frac{m_1}{m_2}}$

Initial momentum of the system is zero, i.e.

$P_i = 0$

Given, the velocity acquire by masses m_1 and m_2 just after they are released be v_1 and v_2.

Final momentum of the system,

$P_f = m_2 v_2 - m_1 v_1$

Using conservation of momentum,

$P_i = P_f$

$\therefore 0 = m_2 v_2 - m_1 v_1$

$\Rightarrow m_2 x_2 - m_1 x_1 = 0 \qquad \left[\because v = \frac{dx}{dt}\right]$

$\Rightarrow \frac{x_1}{x_2} = \frac{m_2}{m_1}$

Hence, (A) is the correct answer.

23. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $\frac{g}{8}$, then the ratio of the masses is

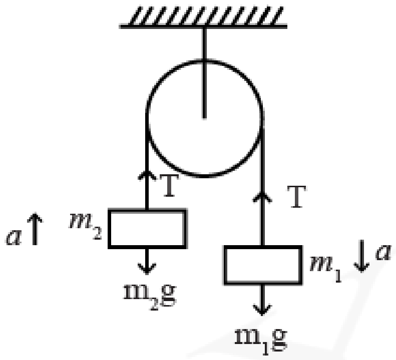
A. $8:1$

B. $9:7$

C. $4:3$

D. $5:3$

Considering $m_1 > m_2$, drawing the FBD of the situation,



For mass m_1 :

$m_1 g - T = m_1 a \quad \ldots.(i)$

For mass m_2 :

$T - m_2 g = m_2 a \quad \ldots.(ii)$

Adding the equations we get

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

Here, $a = \frac{g}{8}$

$$\therefore \frac{1}{8} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}$$

$$\Rightarrow \frac{m_1}{m_2} + 1 = 8\frac{m_1}{m_2} - 8$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{9}{7}$$

Hence, (B) is the correct answer.