Date: 31/03/2022

Subject: Mathematics   Class: Standard XII

1. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is:

A. $\frac{3}{4} + \cos 20^\circ$

B. $\frac{3}{4}$

C. $\frac{3}{2}(1 + \cos 20^\circ)$

D. $\frac{3}{2}$

2. The value of $\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8}$ is :

A. $\frac{1}{4}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

3. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for $k = 1, 2, 3, \ldots$. Then for all $x \in \mathbb{R}$, the value of $f_4(x) - f_6(x)$ is equal to:

A. $\frac{1}{12}$

B. $\frac{-1}{12}$

C. $\frac{1}{4}$

D. $\frac{5}{12}$

4. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is :

A. $15(3-\sqrt{3})$

B. $15(3+\sqrt{3})$

C. $15(1+\sqrt{3})$

D. $15(5-\sqrt{3})$

5. Let $P=\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q=\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then :

A. $P \subset Q$ and $Q - P \neq \phi$

B. $P = Q$

C. $Q \not\subset P$

D. $P \not\subset Q$

6. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is:

A. $\frac{\sqrt{79}}{2}$

B. $\sqrt{19}$

C. $\sqrt{31}$

D. $\sqrt{34}$

7. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:

A. $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

B. $13 - 4\cos^2\theta + 6\cos^4\theta$

C. $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

D. $13 - 4\cos^6\theta$

8. If $\cos(\alpha+\beta) = \dfrac{3}{5}, \sin(\alpha-\beta) = \dfrac{5}{13}$ and $0 < \alpha, \beta < \dfrac{\pi}{4}$, then $\tan(2\alpha)$ is equal to:

A. $\dfrac{21}{16}$

B. $\dfrac{63}{16}$

C. $\dfrac{63}{52}$

D. $\dfrac{33}{52}$

9. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is

A. 108

B. $54\sqrt{3}$

C. $36\sqrt{3}$

D. 54

10. A bird is sitting on the top of a vertical pole $20\ m$ high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is

A. $40(\sqrt{2}-1)$

B. $40(\sqrt{3}-\sqrt{2})$

C. $20\sqrt{2}$

D. $20(\sqrt{3}-1)$

11. The expression $\dfrac{\tan A}{1-\cot A}+\dfrac{\cot A}{1-\tan A}$ can be written as :

A. $\sin A\cdot\cos A+1$

B. $\sec A\cdot \operatorname{cosec} A+1$

C. $\tan A+\cot A$

D. $\sec A+\operatorname{cosec} A$

12. The value of $\cos\dfrac{\pi}{2^2}\cdot\cos\dfrac{\pi}{2^3}\cdot\ \ldots\ \cdot\cos\dfrac{\pi}{2^{10}}\cdot\sin\dfrac{\pi}{2^{10}}$ is :

A. $\dfrac{1}{1024}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{512}$

D. $\dfrac{1}{256}$

13. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is:

A. $\frac{-3}{5}$

B. $\frac{1}{3}$

C. $\frac{2}{9}$

D. $\frac{-7}{9}$

14. Consider a triangular plot ABC with sides $AB = 7m, BC = 5m$ and $CA = 6m$. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is :

A. $\frac{3}{2}\sqrt{21}$

B. $7\sqrt{3}$

C. $2\sqrt{21}$

D. $\frac{2}{3}\sqrt{21}$

15. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6\alpha + 8\,\mathrm{cosec}^6\alpha$ is equal to

A. 250

B. 500

C. 400

D. 350

16. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval:

A. $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

B. $\left[-1, -\frac{1}{2}\right]$

C. $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

D. $\left(-\frac{5}{4}, -1\right)$

17. The minimum value of $2^{\sin x} + 2^{\cos x}$ is

A. $2^{1-\sqrt{2}}$

B. $2^{1-\frac{1}{\sqrt{2}}}$

C. $2^{-1+\sqrt{2}}$

D. $2^{-1+\frac{1}{\sqrt{2}}}$

18. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then:

A. $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

B. $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

C. $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

D. $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

19. The value of $\cot \frac{\pi}{24}$ is

A. $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

B. $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$

C. $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$

D. $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

20. If $\sin\theta+\cos\theta=\frac{1}{2}$, then $16(\sin(2\theta)+\cos(4\theta)+\sin(6\theta))$ is equal to

A. 23

B. -23

C. 27

D. -27

21. Two poles, AB of length a meters and CD of length $a+b$ $(b\neq a)$ meters are erected at the same horizontal level with bases at B and D. If $BD=x$ and $\tan\angle ACB=\frac{1}{2}$, then

A. $x^2-2ax+a(a+b)=0$

B. $x^2+2(a+2b)x-b(a+b)=0$

C. $x^2+2(a+2b)x+a(a+b)=0$

D. $x^2-2ax+b(a+b)=0$

22. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is

A. $8(2+2\sqrt{3}+\sqrt{2})$

B. $8(\sqrt{6}-\sqrt{2}+2)$

C. $8(\sqrt{2}+2+\sqrt{3})$

D. $8(\sqrt{6}+\sqrt{2}+2)$

23. The range of the function
$$f(x)=\log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$$
is

A. $[0,2]$

B. $[-2,2]$

C. $(0,\sqrt{5})$

D. $\left[\frac{1}{\sqrt{5}},\sqrt{5}\right]$

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1. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}, \alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to

2. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is

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1. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$ also satisfy the equation :

   A. $\sin x = 2\sin y$

   B. $2\sin x = \sin y$

   C. $\sin x = |\sin y|$

   D. $2|\sin x| = 3\sin y$

2. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x, \quad x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

   A. 7

   B. 3

   C. 4

   D. 5

3. The triangle of maximum area that can be inscribed in a given circle of radius $'r'$ is:

   A. A right-angle triangle having two of its sides of length $2r$ and r

   B. An equilateral triangle of height $\frac{2r}{3}$

   C. Isosceles triangle with base equal to $2r$

   D. An equilateral triangle having each of its side of length $\sqrt{3}r$

4. If in a triangle ABC, $AB = 5$ units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of $\triangle ABC$ is 5 units, then the area (in sq. units) of $\triangle ABC$ is

A. $10 + 6\sqrt{2}$

B. $6 + 8\sqrt{3}$

C. $8 + 2\sqrt{2}$

D. $4 + 2\sqrt{3}$

5. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

A. $\frac{1+\sqrt{3}}{2}$

B. $\frac{1-\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

6. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is :

A. 60

B. 50

C. 45

D. 42

7. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :

A. $5:6:7$

B. $5:9:13$

C. $4:5:6$

D. $3:4:5$

8. $ABCD$ is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta, BC = p$ and $CD = q$, then AB is equal to :

A. $\dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

B. $\dfrac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

C. $\dfrac{p^2+q^2}{p\cos\theta + q\sin\theta}$

D. $\dfrac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

9. The angles A, B and C of a triangle ABC are in $A.P.$ and $a:b = 1:\sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is:

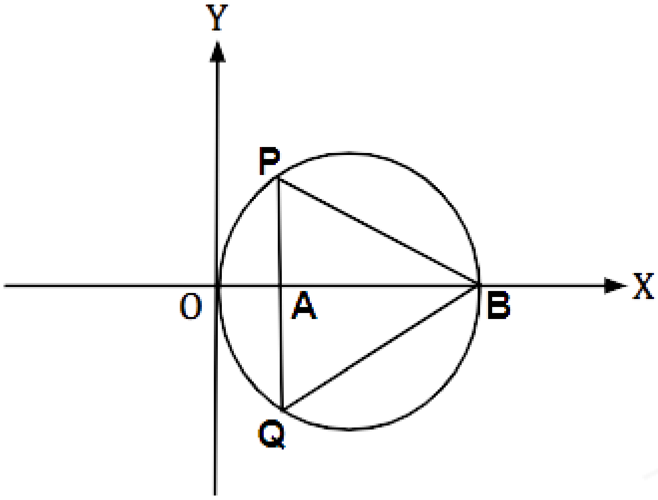
A. $4\sqrt{3}$

B. $2\sqrt{3}$

C. $\dfrac{4}{\sqrt{3}}$

D. $\dfrac{2}{\sqrt{3}}$

10. In the circle given below, let $OA = 1$ unit, $OB = 13$ unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is :



A. $26\sqrt{3}$

B. $24\sqrt{2}$

C. $24\sqrt{3}$

D. $26\sqrt{2}$

11. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to:

A. 3

B. 2

C. 4

D. 8

12. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

A. $10(\sqrt{3}-1)$

B. $10\sqrt{3}$

C. 10

D. $10(\sqrt{3}+1)$

13. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to:

A. $\frac{1}{\sqrt{3}}$

B. $\sqrt{3}$

C. $2\sqrt{3}$

D. $\frac{2\sqrt{3}}{3}$

14. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to

A. $200\sqrt{3}$

B. $400\sqrt{3}$

C. 400

D. 100

15. A vertical pole fixed to the horizontal ground is divided in the ratio $3:7$ by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is:

A. $8\sqrt{10}$

B. $12\sqrt{10}$

C. $12\sqrt{15}$

D. $6\sqrt{10}$

16. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

A. $\dfrac{\sqrt{5}+1}{4}$

B. $\dfrac{\sqrt{2}-1}{2}$

C. $\dfrac{\sqrt{5}-1}{2}$

D. $\dfrac{\sqrt{5}-1}{4}$

17. If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$$

and S is the sum of all these solutions, then the ordered pair (n, S) is :

A. $\left(3, \dfrac{5\pi}{3}\right)$

B. $\left(3, \dfrac{13\pi}{9}\right)$

C. $\left(2, \dfrac{2\pi}{3}\right)$

D. $\left(2, \dfrac{8\pi}{9}\right)$

18. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0,\ 2\pi]$ is :

A. 5

B. 2

C. 4

D. 3

19. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is

A. $\frac{\pi}{10}$

B. $-\frac{7\pi}{30}$

C. $-\frac{11\pi}{30}$

D. $-\frac{\pi}{15}$

20. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

A. $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

B. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

C. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

D. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

21. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

A. π

B. $\frac{\pi}{2}$

C. $\frac{5\pi}{4}$

D. $\frac{3\pi}{8}$

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1. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of $2R + r$ (in cm) is equal to

2. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is

3. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If $AD = 8m, BC = 11m$ and $AB = 10m$; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is

4. Let S be the sum of all solutions (in radians) of the equation $\sin^4 \theta + \cos^4 \theta - \sin\theta \cos\theta = 0$ in $[0, 4\pi]$. Then $\dfrac{8S}{\pi}$ is equal to