

Date: 31/03/2022

Subject: Mathematics

Class: Standard XII

1. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is:

A. $\frac{3}{4} + \cos 20^\circ$

B. $\frac{3}{4}$

C. $\frac{3}{2}(1 + \cos 20^\circ)$

D. $\frac{3}{2}$

$$\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$$

$$= \frac{1}{2}[2 \cos^2 10^\circ - 2 \cos 10^\circ \cos 50^\circ + 2 \cos^2 50^\circ]$$

$$= \frac{1}{2}[1 + \cos 20^\circ + 1 + \cos 100^\circ - (\cos 60^\circ + \cos 40^\circ)]$$

$$= \frac{1}{2}[2 - \cos 60^\circ + \cos 20^\circ + \cos 100^\circ - \cos 40^\circ]$$

$$= \frac{1}{2}\left[\frac{3}{2} + \cos 20^\circ + 2 \sin 70^\circ \sin(-30^\circ)\right]$$

$$= \frac{1}{2}\left[\frac{3}{2} + \cos 20^\circ - \sin 70^\circ\right]$$

$$= \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

2. The value of $\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8}$ is :

- A. $\frac{1}{4}$
- B. $\frac{1}{2\sqrt{2}}$
- C. $\frac{1}{2}$
- D. $\frac{1}{\sqrt{2}}$

$$\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8}$$

$$= \cos^3 \frac{\pi}{8} \left[4 \cos^3 \frac{\pi}{8} - 3 \cos \frac{\pi}{8} \right] + \sin^3 \frac{\pi}{8} \left[3 \sin \frac{\pi}{8} - 4 \sin^3 \frac{\pi}{8} \right]$$

$$= 4 \left[\cos^6 \frac{\pi}{8} - \sin^6 \frac{\pi}{8} \right] + 3 \left[\sin^4 \frac{\pi}{8} - \cos^4 \frac{\pi}{8} \right]$$

$$= 4 \left[\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right] \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} \sin^2 \frac{\pi}{8} \right] - 3 \left[\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right]$$

$$= \left[\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right] \left[4 \left(1 - \cos^2 \frac{\pi}{8} \sin^2 \frac{\pi}{8} \right) - 3 \right]$$

$$= \cos \frac{\pi}{4} \left[1 - \sin^2 \frac{\pi}{4} \right]$$

$$= \frac{1}{2\sqrt{2}}$$

3. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \dots$. Then for all $x \in \mathbb{R}$, the value of $f_4(x) - f_6(x)$ is equal to:

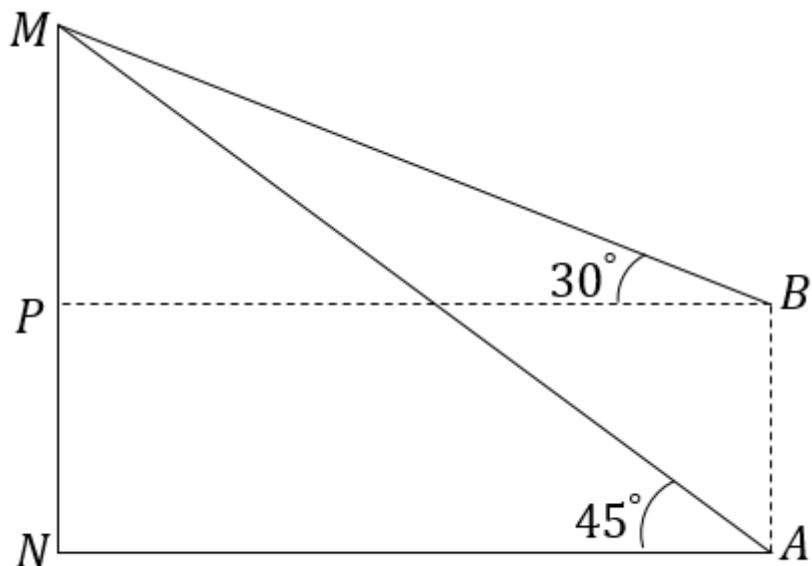
- A. $\frac{1}{12}$
- B. $\frac{-1}{12}$
- C. $\frac{1}{4}$
- D. $\frac{5}{12}$

$$f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$$

$$\begin{aligned} f_4(x) - f_6(x) &= \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x) \\ &= \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}\sin^2 2x\right) \\ &= \frac{1}{4} - \frac{1}{6} - \frac{1}{8}\sin^2 2x + \frac{1}{8}\sin^2 2x \\ &= \frac{1}{12} \end{aligned}$$

4. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A . If the angle of elevation of the top of the tower from B be 30° , then the distance (in m) of the foot of the tower from the point A is :

- A. $15(3 - \sqrt{3})$
- B. $15(3 + \sqrt{3})$
- C. $15(1 + \sqrt{3})$
- D. $15(5 - \sqrt{3})$



$$AB = 30 = NP$$

In $\triangle ANM$

$$\tan 45^\circ = \frac{MN}{AN} = 1 \Rightarrow MN = AN$$

$$PM = MN - 30 = AN - 30$$

In $\triangle BPM$

$$\tan 30^\circ = \frac{PM}{PB} = \frac{AN - 30}{AN}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AN - 30}{AN}$$

$$\Rightarrow AN = \sqrt{3}AN - 30\sqrt{3}$$

$$\Rightarrow AN = \frac{30\sqrt{3}}{\sqrt{3} - 1} = 15(3 + \sqrt{3})$$

5. Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and $Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then :

- A. $P \subset Q$ and $Q - P \neq \emptyset$
- B. $P = Q$
- C. $Q \not\subset P$
- D. $P \not\subset Q$

$$P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$$

$$\Rightarrow \sin \theta = (1 + \sqrt{2}) \cos \theta$$

$$\Rightarrow \tan \theta = (1 + \sqrt{2})$$

$$Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$$

$$\Rightarrow \cos \theta = (\sqrt{2} - 1) \sin \theta$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{2}-1} = (1 + \sqrt{2})$$

$$\therefore P = Q$$

6. The maximum value of $3 \cos \theta + 5 \sin \left(\theta - \frac{\pi}{6} \right)$ for any real value of θ is:

- A. $\frac{\sqrt{79}}{2}$
- B. $\sqrt{19}$
- C. $\sqrt{31}$
- D. $\sqrt{34}$

$$\begin{aligned}
\text{Let } \mu &= 3 \cos \theta + 5 \sin \left(\theta - \frac{\pi}{6} \right) \\
&= 3 \cos \theta + 5 \sin \theta \cdot \cos \frac{\pi}{6} - 5 \cos \theta \cdot \sin \frac{\pi}{6} \\
&= \frac{1}{2} \cos \theta + \frac{5\sqrt{3}}{2} \sin \theta \\
\therefore \max \mu &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2} \\
&= \sqrt{19}
\end{aligned}$$

7. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4 \sin^6 \theta$ equals:

- A. $13 - 4 \cos^2 \theta + 6 \sin^2 \theta \cos^2 \theta$
- B. $13 - 4 \cos^2 \theta + 6 \cos^4 \theta$
- C. $13 - 4 \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta$
- D. $13 - 4 \cos^6 \theta$

$$\begin{aligned}
& 3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4 \sin^6 \theta \\
&= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4 \sin^6 \theta \\
&= 3(1 + \sin^2 2\theta - 2 \sin 2\theta) + 6(1 + \sin 2\theta) + 4 \sin^6 \theta \\
&= 3 + 3 \sin^2 2\theta - 6 \sin 2\theta + 6 + 6 \sin 2\theta + 4 \sin^6 \theta \\
&= 9 + 4 \sin^6 \theta + 3 \sin^2 2\theta \\
&= 9 + 4 - 4 + 4 \sin^6 \theta + 3 \times (2 \sin \theta \cos \theta)^2 \\
&= 13 + 4(\sin^6 \theta - 1) + 12 \sin^2 \theta \cos^2 \theta \\
&= 13 + 4(\sin^2 \theta - 1)(\sin^4 \theta + \sin^2 \theta + 1) + 12 \sin^2 \theta \cos^2 \theta \\
&= 13 - 4 \cos^2 \theta (\sin^4 \theta + \sin^2 \theta + 1) + 12 \sin^2 \theta \cos^2 \theta \\
&= 13 - 4 \cos^2 \theta (\sin^4 \theta + \sin^2 \theta + 1 - 3 \sin^2 \theta) \\
&= 13 - 4 \cos^2 \theta (\sin^4 \theta - 2 \sin^2 \theta + 1) \\
&= 13 - 4 \cos^2 \theta (1 - \sin^2 \theta)^2 \\
&= 13 - 4 \cos^2 \theta \cos^4 \theta \\
&= 13 - 4 \cos^6 \theta
\end{aligned}$$

8. If $\cos(\alpha + \beta) = \frac{3}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to:

- A. $\frac{21}{16}$
- B. $\frac{63}{16}$
- C. $\frac{63}{52}$
- D. $\frac{33}{52}$

$$\cos(\alpha + \beta) = \frac{3}{5} \Rightarrow \tan(\alpha + \beta) = \frac{4}{3}$$

$$\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

Now,

$$\Rightarrow \tan(2\alpha) = \tan((\alpha + \beta) + (\alpha - \beta))$$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)}$$

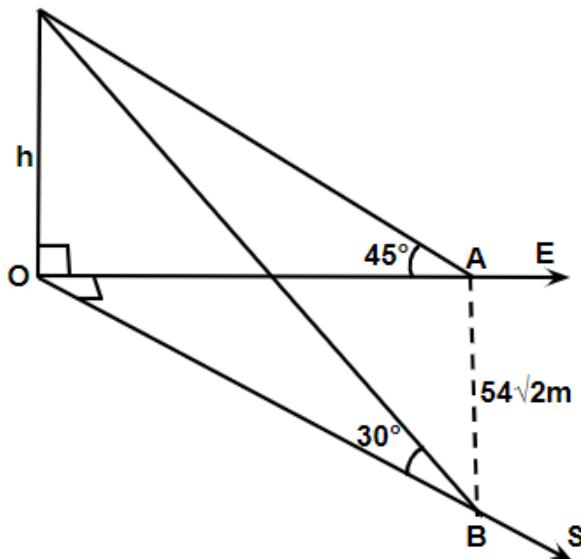
$$= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}}$$

$$= \frac{63}{16}$$

9. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45° . The angle of elevation of the top of the same tower from a point B, due south of A is 30° . If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is

- A. 108
- B. $54\sqrt{3}$
- C. $36\sqrt{3}$
- D. 54

According to the question the figure is



Now from the figure

$$OA = h$$

$$OB = \sqrt{3}h$$

$$OB^2 = OA^2 + AB^2$$

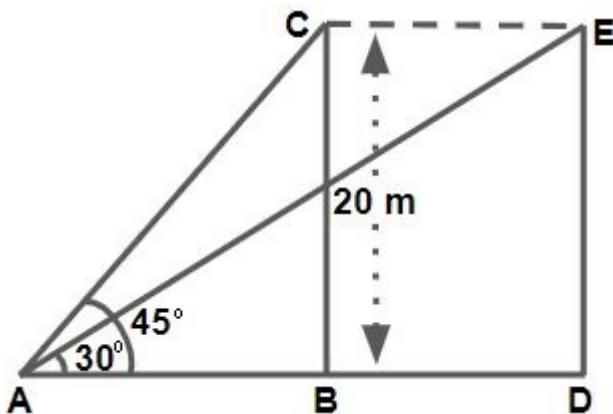
$$3h^2 = h^2 + (54\sqrt{2})^2$$

$$\Rightarrow h = 54m$$

10. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45° . It flies off horizontally straight away from the point O . After one second, the elevation of the bird from O is reduced to 30° . Then the speed (in m/s) of the bird is

- A. $40(\sqrt{2} - 1)$
- B. $40(\sqrt{3} - \sqrt{2})$
- C. $20\sqrt{2}$
- D. $20(\sqrt{3} - 1)$

$\angle BAC = 45^\circ$, $\angle DAE = 30^\circ$ and $BC = DE = 20$ m



$$AB = 20 \tan 45^\circ = 20 \text{ and } AD = 20 \cot 30^\circ = 20\sqrt{3} \text{ m}$$

$$BD = AD - AB = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$$

$$\therefore \text{speed of bird} = 20(\sqrt{3} - 1) \text{ m/s}$$

11. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as :

- A. $\sin A \cdot \cos A + 1$
- B. $\sec A \cdot \operatorname{cosec} A + 1$
- C. $\tan A + \cot A$
- D. $\sec A + \operatorname{cosec} A$

$$\begin{aligned}
& \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} \\
&= \frac{\frac{\sin A}{\cos A}}{\frac{\cos A(\sin A - \cos A)}{\sin^2 A}} + \frac{\frac{\cos A}{\sin A}}{\frac{\sin A(\cos A - \sin A)}{\cos^2 A}} \\
&= \frac{1}{\sin A - \cos A} \left[\frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A} \right] \\
&= \frac{1}{\sin A - \cos A} \cdot \frac{\sin^3 A - \cos^3 A}{\sin A \cos A} \\
&= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{(\sin A - \cos A) \sin A \cos A} \\
&= \frac{1 + \sin A \cos A}{\sin A \cos A} \\
&= \sec A \cdot \operatorname{cosec} A + 1
\end{aligned}$$

12. The value of $\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}}$ is :

- A. $\frac{1}{1024}$
- B. $\frac{1}{2}$
- C. $\frac{1}{512}$
- D. $\frac{1}{256}$

$$A = \cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}}$$

$$\text{Put } \frac{\pi}{2^{10}} = x \Rightarrow \pi = 2^{10}x$$

$$\Rightarrow A = (\cos x \cdot \cos 2x \cdot \dots \cdot \cos 2^8x) \sin x$$

As we know

$$\frac{\sin 2^n \theta}{2^n \sin \theta} = \cos \theta \cdot \cos 2\theta \cdot \dots \cdot \cos 2^{n-1}\theta.$$

$$\text{Here, } n - 1 = 8 \Rightarrow n = 9$$

$$\Rightarrow A = \frac{\sin 2^9 x}{2^9 \sin x} \cdot \sin x$$

$$\Rightarrow A = \frac{\sin 2^9 \frac{\pi}{2^{10}}}{2^9}$$

$$\Rightarrow A = \frac{1}{512}$$

13. If $5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$, then the value of $\cos 4x$ is:

- A. $\frac{-3}{5}$
- B. $\frac{1}{3}$
- C. $\frac{2}{9}$
- D. $\frac{-7}{9}$

$$5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$$

$$5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$$

Let $\cos^2 x = t$

$$5\left(\frac{1}{t} - 1 - t\right) = 2(2t - 1) + 9$$

$$5(1 - t - t^2) = 4t^2 + 7t$$

$$\Rightarrow 9t^2 + 12t - 5 = 0$$

$$t = \frac{1}{3}, \frac{-5}{3}$$

$$\Rightarrow \cos^2 x = \frac{1}{3}$$

$$\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$$

$$\Rightarrow \cos 4x = -\frac{7}{9}$$

14. Consider a triangular plot ABC with sides $AB = 7m$, $BC = 5m$ and $CA = 6m$. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B . The height (in m) of the lamp-post is :

- A. $\frac{3}{2}\sqrt{21}$
- B. $7\sqrt{3}$
- C. $2\sqrt{21}$
- D. $\frac{2}{3}\sqrt{21}$

$$\text{Length of median } BD = \frac{1}{2}\sqrt{2((BC)^2 + (AB)^2) - (AC)^2}$$

$$= \frac{1}{2}\sqrt{2(25 + 49) - 36}$$

$$= \sqrt{28}$$

$$= 2\sqrt{7}$$

Let h be the height of the tower

So, from ΔTBD

$$\because \angle TBD = 30^\circ$$

$$\therefore \tan 30^\circ = \frac{h}{2\sqrt{7}}$$

$$h = 2\sqrt{\frac{7}{3}} = \frac{2}{3}\sqrt{21}m$$

15. If $15 \sin^4 \alpha + 10 \cos^4 \alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27 \sec^6 \alpha + 8 \operatorname{cosec}^6 \alpha$ is equal to

- A. 250
- B. 500
- C. 400
- D. 350

$$\begin{aligned}
 & 15 \sin^4 \alpha + 10 \cos^4 \alpha = 6 \\
 & \Rightarrow 15 \sin^4 \alpha + 10(1 - \sin^2 \alpha)^2 = 6 \\
 & \Rightarrow 15 \sin^4 \alpha + 10(\sin^4 \alpha - 2 \sin^2 \alpha + 1) = 6 \\
 & \Rightarrow 25 \sin^4 \alpha - 20 \sin^2 \alpha + 4 = 0 \\
 & \Rightarrow (5 \sin^2 \alpha - 2)^2 = 0 \\
 & \therefore \sin^2 \alpha = \frac{2}{5} \Rightarrow \cos^2 \alpha = \frac{3}{5} \\
 & \text{Now, } \sec^6 \alpha = \frac{1}{\cos^6 \alpha} = \frac{125}{27} \\
 & \text{and } \operatorname{cosec}^6 \alpha = \frac{125}{8} \\
 & \therefore 27 \sec^6 \alpha + 8 \operatorname{cosec}^6 \alpha = 250
 \end{aligned}$$

16. If the equation $\cos^4 \theta + \sin^4 \theta + \lambda = 0$ has real solutions for θ , then λ lies in the interval:

- A. $\left(-\frac{1}{2}, -\frac{1}{4}\right]$
- B. $\left[-1, -\frac{1}{2}\right]$
- C. $\left[-\frac{3}{2}, -\frac{5}{4}\right]$
- D. $\left(-\frac{5}{4}, -1\right)$

$$\begin{aligned}
 & \cos^4 \theta + \sin^4 \theta + \lambda = 0 \\
 & \lambda = -\left\{1 - \frac{1}{2} \sin^2 2\theta\right\}
 \end{aligned}$$

$$2(\lambda + 1) = \sin^2 2\theta$$

$$0 \leq 2(\lambda + 1) \leq 1$$

$$0 \leq \lambda + 1 \leq \frac{1}{2}$$

$$\Rightarrow -1 \leq \lambda \leq -\frac{1}{2}$$

17. The minimum value of $2^{\sin x} + 2^{\cos x}$ is

- A. $2^{1-\sqrt{2}}$
- B. $2^{1-\frac{1}{\sqrt{2}}}$
- C. $2^{-1+\sqrt{2}}$
- D. $2^{-1+\frac{1}{\sqrt{2}}}$

Let $2^{\sin x}, 2^{\cos x}$ be the two positive numbers.
Then, using A.M. \geq G.M., we have

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} \cdot 2^{\cos x}} \\ 2^{\sin x} + 2^{\cos x} \geq 2 \cdot \sqrt{2^{\sin x + \cos x}} \quad \dots (1)$$

Now, $2^{\sin x + \cos x} = 2^{\sqrt{2} \cos\left(\frac{\pi}{4}-x\right)}$

we know that $\cos x \in [-1, 1]$

$$\Rightarrow 2^{\sqrt{2} \cos\left(\frac{\pi}{4}-x\right)} \geq 2^{-\sqrt{2}}$$

From (1), we have

$$2^{\sin x} + 2^{\cos x} \geq 2 \cdot 2^{\frac{-\sqrt{2}}{2}} \\ \therefore 2^{\sin x} + 2^{\cos x} \geq 2^{1-\frac{1}{\sqrt{2}}}$$

Hence, the minimum value of $2^{\sin x} + 2^{\cos x}$ is $2^{1-\frac{1}{\sqrt{2}}}$.

18. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then:

- A. $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$
- B. $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$
- C. $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$
- D. $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

$$L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$$

$$\left(\because \sin^2 \theta = \frac{1 - \cos 2\theta}{2}\right)$$

$$L = \left(\frac{1 - \cos \frac{\pi}{8}}{2}\right) - \left(\frac{1 - \cos \frac{\pi}{4}}{2}\right)$$

$$L = \frac{-1}{2} \left[\cos \frac{\pi}{8} - \cos \frac{\pi}{4} \right]$$

$$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$$

$$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$$

$$M = \left(\frac{1 + \cos \frac{\pi}{8}}{2}\right) - \left(\frac{1 - \cos \frac{\pi}{4}}{2}\right)$$

$$M = \frac{1}{2} \left[\cos \frac{\pi}{4} + \cos \frac{\pi}{8} \right]$$

$$M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$$

19. The value of $\cot \frac{\pi}{24}$ is

- A. $3\sqrt{2} - \sqrt{3} - \sqrt{6}$
- B. $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$
- C. $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$
- D. $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

$$\begin{aligned}
\cot \frac{\pi}{24} &= \frac{2 \cos \frac{\pi}{24} \cdot \sin \frac{\pi}{24}}{2 \sin \frac{\pi}{24} \cdot \sin \frac{\pi}{24}} \\
&= \frac{\sin \frac{\pi}{12}}{1 - \cos \frac{\pi}{12}} = \frac{\frac{\sqrt{3} - 1}{2\sqrt{2}}}{1 - \frac{\sqrt{3} + 1}{2\sqrt{2}}} \\
&= \frac{\sqrt{3} - 1}{2\sqrt{2} - (\sqrt{3} + 1)} \times \frac{2\sqrt{2} + (\sqrt{3} + 1)}{2\sqrt{2} + (\sqrt{3} + 1)} \\
&= \frac{2\sqrt{6} + 3 + \sqrt{3} - 2\sqrt{2} - \sqrt{3} - 1}{8 - (3 + 1 + 2\sqrt{3})} \\
&= \frac{\sqrt{6} - \sqrt{2} + 1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \\
&= 2\sqrt{6} + 3\sqrt{2} - 2\sqrt{2} - \sqrt{6} + 2 + \sqrt{3} \\
&= \sqrt{6} + \sqrt{2} + 2 + \sqrt{3} \\
&= \sqrt{2} + \sqrt{3} + 2 + \sqrt{6}
\end{aligned}$$

20. If $\sin \theta + \cos \theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to

- A. 23
- B. -23
- C. 27
- D. -27

Given: $\sin \theta + \cos \theta = \frac{1}{2}$

squaring on both sides, we get

$$\sin 2\theta = \frac{1}{4} - 1$$

$$\therefore \sin 2\theta = -\frac{3}{4}$$

$$\text{and } \cos 4\theta = 1 - 2(\sin 2\theta)^2$$

$$\Rightarrow \cos 4\theta = 1 - \frac{9}{8}$$

$$\therefore \cos 4\theta = -\frac{1}{8}$$

$$\text{and } \sin 6\theta = 3 \sin 2\theta - 4 \sin^3 2\theta$$

$$\Rightarrow \sin 6\theta = -\frac{9}{4} + \frac{27}{16}$$

$$\therefore \sin 6\theta = -\frac{9}{16}$$

Now,

$$16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$$

$$= 16 \left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16} \right)$$

$$= -23$$

21. Two poles, AB of length a meters and CD of length $a + b$ ($b \neq a$) meters are erected at the same horizontal level with bases at B and D . If $BD = x$ and $\tan \angle ACB = \frac{1}{2}$, then

- A. $x^2 - 2ax + a(a + b) = 0$
- B. $x^2 + 2(a + 2b)x - b(a + b) = 0$
- C. $x^2 + 2(a + 2b)x + a(a + b) = 0$
- D. $x^2 - 2ax + b(a + b) = 0$

$$\therefore \tan \theta = \frac{1}{2}$$

$$\tan \phi = \frac{b}{x}$$

$$\text{and } \tan(\theta + \phi) = \frac{a+b}{x}$$

$$\begin{aligned} & \Rightarrow \frac{\frac{1}{2} + \frac{b}{x}}{1 - \frac{b}{2x}} = \frac{a+b}{x} \\ & \Rightarrow 2bx + x^2 = (a+b)2x - b(a+b) \\ & \Rightarrow x^2 - 2ax + b(a+b) = 0 \end{aligned}$$

22. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75° . Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is
- A. $8(2 + 2\sqrt{3} + \sqrt{2})$
 - B. $8(\sqrt{6} - \sqrt{2} + 2)$
 - C. $8(\sqrt{2} + 2 + \sqrt{3})$
 - D. $8(\sqrt{6} + \sqrt{2} + 2)$

Let O be the

centre of circle.

P, Q points of contact tangents from A

T be the topmost point on ellipse and R be the foot of perpendicular.

In ΔOAP

$$OA = 16 \operatorname{cosec} 30^\circ = 32$$

In ΔABO

$$OA = OA \sin 75^\circ = 32 \times \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$OA = 16 \times \frac{\sqrt{3} + 1}{2} \times \sqrt{2} = 8\sqrt{2}(\sqrt{3} + 1) = 8(\sqrt{6} + \sqrt{2})$$

So, topmost point is $= OR + OT$
 $= 8(\sqrt{6} + \sqrt{2} + 2)m$

23. The range of the function

$$f(x) = \log_{\sqrt{5}} \left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right) \right)$$

is

- A. $[0, 2]$
- B. $[-2, 2]$
- C. $(0, \sqrt{5})$
- D. $\left[\frac{1}{\sqrt{5}}, \sqrt{5} \right]$

$$\begin{aligned} f(x) &= \log_{\sqrt{5}} \left(3 + 2 \sin\left(\frac{3\pi}{4}\right) \sin(-x) + 2 \cos\left(\frac{\pi}{4}\right) \cdot \cos(x) \right) \\ &= \log_{\sqrt{5}}(3 + \sqrt{2}(\cos x - \sin x)) \end{aligned}$$

\therefore Range of $\cos x - \sin x$ is $[-\sqrt{2}, \sqrt{2}]$

Then range of $f(x)$ is $[0, 2]$

Date: 31/03/2022

Subject: Mathematics

Class: Standard XII

1. If $\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to

Accepted Answers

1 1.0 1.00

Solution:

$$\begin{aligned} \frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} &= \frac{1}{7} \\ \Rightarrow \frac{\sqrt{2} \sin \alpha}{\sqrt{2} \cos \alpha} &= \frac{1}{7} \\ \Rightarrow \tan \alpha &= \frac{1}{7} \end{aligned}$$

$$\begin{aligned} \sqrt{\frac{1 - \cos 2\beta}{2}} &= \frac{1}{\sqrt{10}} \\ \Rightarrow \frac{\sqrt{2} \sin \beta}{\sqrt{2}} &= \frac{1}{\sqrt{10}} \\ \Rightarrow \sin \beta &= \frac{1}{\sqrt{10}} \Rightarrow \tan \beta = \frac{1}{3} \end{aligned}$$

$$\tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta}$$

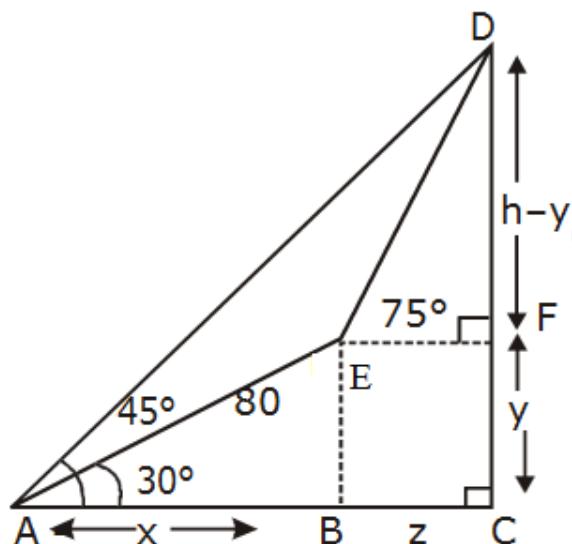
$$\tan(\alpha + 2\beta) = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} = 1$$

2. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45° . After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75° . Then the height of the hill (in meters) is

Accepted Answers

80 80.0 80.00

Solution:



$$x = 80 \cos 30^\circ = 40\sqrt{3}$$

$$y = 80 \sin 30^\circ = 40$$

In $\triangle ADC$

$$\tan 45^\circ = \frac{h}{x+z} \Rightarrow h = x + z$$

$$\Rightarrow h = 40\sqrt{3} + z \dots (1)$$

In $\triangle EDF$

$$\tan 75^\circ = \frac{h-y}{z}$$

$$2 + \sqrt{3} = \frac{h - 40}{z} \Rightarrow z = \frac{h - 40}{2 + \sqrt{3}} \dots (2)$$

Put the value of z from (1)

$$h - 40\sqrt{3} = \frac{h - 40}{2 + \sqrt{3}}$$

$$h(1 + \sqrt{3}) = 40(2\sqrt{3} + 3 - 1)$$

$$h(1 + \sqrt{3}) = 80(1 + \sqrt{3})$$

$$h = 80$$

Date: 31/03/2022

Subject: Mathematics

Class: Standard XII

1. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2 \sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$ also satisfy the equation :

- A. $\sin x = 2 \sin y$
- B. $2 \sin x = \sin y$
- C. $\sin x = |\sin y|$
- D. $2|\sin x| = 3 \sin y$

$$2^{\sqrt{\sin^2 x - 2 \sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$$

$$\Rightarrow 2^{\sqrt{\sin^2 x - 2 \sin x + 5}} \leq 2^{2 \sin^2 y}$$

$$\Rightarrow \sqrt{\sin^2 x - 2 \sin x + 5} \leq 2 \sin^2 y \quad \dots (1)$$

$$\sqrt{\sin^2 x - 2 \sin x + 5} = \sqrt{(\sin x - 1)^2 + 4}$$

$$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \in [2, 2\sqrt{2}]$$

But

$$2 \sin^2 y \in [0, 2]$$

Equation (1) holds true only when

$$\sqrt{\sin^2 x - 2 \sin x + 5} = 2 \sin^2 y = 2$$

$$\Rightarrow (\sin x - 1)^2 + 4 = 4 \Rightarrow \sin x = 1$$

and $2 \sin^2 y = 2 \Rightarrow |\sin y| = 1$

$$\Rightarrow \sin x = |\sin y|$$

2. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

- A. 7
- B. 3
- C. 4
- D. 5

$$1 + \sin^4 x = \cos^2 3x, \quad x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$$

As we know, range of $\cos^2 3x$: $[0, 1]$

whereas $1 + \sin^4 x \geq 1$

Therefore only possibilities will be

$$1 + \sin^4 x = 1 = \cos^2 3x$$

No of solution of $\sin^4 x = 0$; $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is $(-2\pi, -\pi, 0, \pi, 2\pi)$... (1)

and solution of $\cos^2 3x = 1$; $3x \in \left[-\frac{15\pi}{2}, \frac{15\pi}{2}\right]$ is

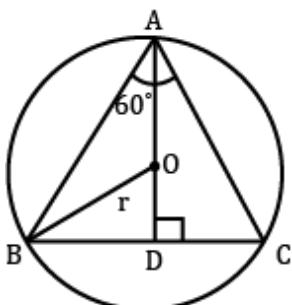
$$3x = (-7\pi, \dots, -\pi, 0, \pi, \dots, 7\pi) \quad \dots (2)$$

Hence, according to (1) and (2) there are 5 number of solutions.

3. The triangle of maximum area that can be inscribed in a given circle of radius ' r ' is:

- A. A right-angle triangle having two of its sides of length $2r$ and r
- B. An equilateral triangle of height $\frac{2r}{3}$
- C. Isosceles triangle with base equal to $2r$
- D. An equilateral triangle having each of its side of length $\sqrt{3}r$

Triangle of maximum area that can be inscribed in a circle is an equilateral triangle. Let $\triangle ABC$ be inscribed in circle,



Now, in $\triangle OBD$

$$OD = r \cos 60^\circ = \frac{r}{2}$$

$$\text{Height} = AD = \frac{3r}{2}$$

Again in $\triangle ABD$

$$\text{Now } \sin 60^\circ = \frac{3r/2}{AB}$$

$$\Rightarrow AB = \sqrt{3}r$$

4. If in a triangle ABC , $AB = 5$ units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of $\triangle ABC$ is 5 units, then the area (in sq. units) of $\triangle ABC$ is

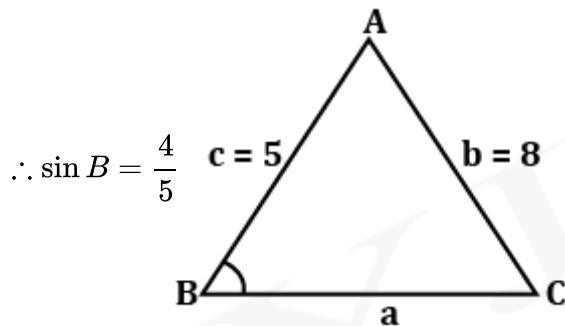
A. $10 + 6\sqrt{2}$

B. $6 + 8\sqrt{3}$

C. $8 + 2\sqrt{2}$

D. $4 + 2\sqrt{3}$

$$\text{In } \triangle ABC \cos B = \frac{3}{5}, R = 5$$



$$\therefore \sin B = \frac{4}{5}$$

$$\Rightarrow C = 30^\circ \text{ and } b = 8$$

$$\therefore \sin A = \sin(B + C)$$

$$\Rightarrow \sin A = \sin B \cos C + \cos B \sin C$$

$$\Rightarrow \sin A = \frac{4}{5} \cdot \frac{\sqrt{3}}{2} + \frac{3}{5} \cdot \frac{1}{2} = \frac{3 + 4\sqrt{3}}{10}$$

$$\therefore \triangle = \frac{1}{2}bc \sin A$$

$$= \frac{1}{2} \cdot 8 \cdot 5 \cdot \left(\frac{3 + 4\sqrt{3}}{10} \right) = 6 + 8\sqrt{3}$$

5. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

A. $\frac{1+\sqrt{3}}{2}$

B. $\frac{1-\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

$$\cos x + \cos y - \cos(x+y) = \frac{3}{2}$$

$$\Rightarrow 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) - \left[2 \cos^2\left(\frac{x+y}{2}\right) - 1\right] = \frac{3}{2}$$

$$\Rightarrow 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) - 2 \cos^2\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

$$\Rightarrow 4 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) - 4 \cos^2\left(\frac{x+y}{2}\right) = 1 = \cos^2\left(\frac{x-y}{2}\right) + \sin^2\left(\frac{x-y}{2}\right)$$

$$\Rightarrow 4 \cos^2\left(\frac{x+y}{2}\right) \cos^2\left(\frac{x-y}{2}\right) - 4 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) + \sin^2\left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \left(\cos\left(\frac{x-y}{2}\right) - 2 \cos\left(\frac{x+y}{2}\right) \right)^2 + \sin^2\left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \sin \frac{x-y}{2} = 0 \Rightarrow x = y$$

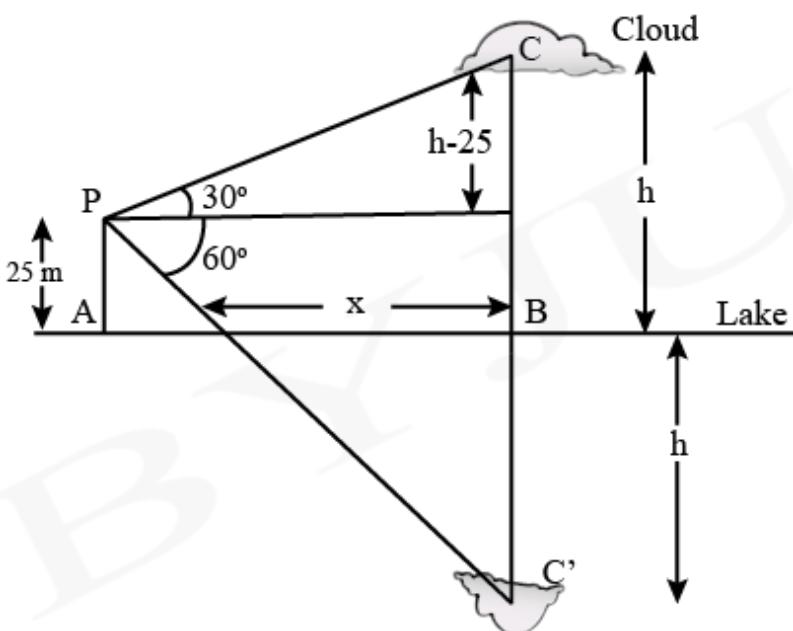
$$\text{and } \cos \frac{x-y}{2} = 2 \cos \frac{x+y}{2}$$

$$\Rightarrow \cos x = \frac{1}{2} = \cos y$$

$$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3} + 1}{2}$$

6. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60° , then the height of the cloud (in meters) from the surface of the lake is :

- A. 60
- B. 50
- C. 45
- D. 42



Let the height of cloud above the lake is h metres.

$$\tan 30^\circ = \frac{h - 25}{x} \quad \dots (1)$$

$$\tan 60^\circ = \frac{h + 25}{x} \quad \dots (2)$$

From (1) and (2),

$$\sqrt{3} = \frac{h + 25}{(h - 25) \times \sqrt{3}}$$

$$\Rightarrow 3h - 75 = h + 25 \\ \Rightarrow h = 50 \text{ m}$$

7. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :

A. 5 : 6 : 7

B. 5 : 9 : 13

C. 4 : 5 : 6

D. 3 : 4 : 5

Let the smallest angle be A

and let the sides in ascending order be a, b, c

Then the corresponding angles are $A, 180^\circ - 3A, 2A$

a, b, c are in A.P.

$$\therefore 2b = a + c$$

$$\Rightarrow 2 \times 2R \sin(180^\circ - 3A) = 2R \sin A + 2R \sin 2A$$

$$\Rightarrow 2 \sin 3A = \sin A + \sin 2A \quad \dots (1)$$

$$\Rightarrow 6 \sin A - 8 \sin^3 A = \sin A + 2 \sin A \cos A$$

$$\Rightarrow 5 \sin A - 2 \sin A \cos A - 8 \sin^3 A = 0$$

$$\Rightarrow \sin A(-8 \sin^2 A - 2 \cos A + 5) = 0$$

$$\Rightarrow 8 \cos^2 A - 2 \cos A - 3 = 0$$

$$\Rightarrow \cos A = \frac{3}{4}$$

$$\text{or } \cos A = \frac{-1}{2} \text{ (not possible)}$$

$$\therefore \sin A = \frac{\sqrt{7}}{4}$$

$$\sin 2A = 2 \sin A \cos A = \frac{3\sqrt{7}}{8}$$

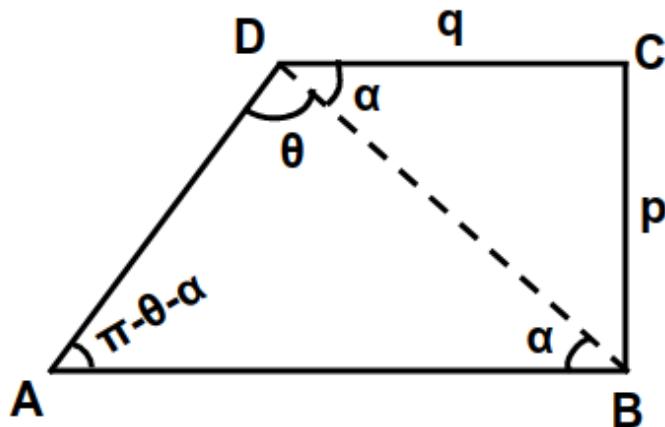
$$\sin 3A = \frac{\sin A + \sin 2A}{2} = \frac{5\sqrt{7}}{16}$$

$$\begin{aligned}\therefore a : b : c &= \sin A : \sin 3A : \sin 2A \\ &= \frac{\sqrt{7}}{4} : \frac{5\sqrt{7}}{16} : \frac{3\sqrt{7}}{8} \\ &= 4 : 5 : 6\end{aligned}$$

8. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to :

- A. $\frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$
- B. $\frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$
- C. $\frac{p^2 + q^2}{p \cos \theta + q \sin \theta}$
- D. $\frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$

Consider ABCD is a trapezium
 $\angle ADB = \theta$, $BC = p$, $CD = q$



Let $\angle BDC = \alpha$ then $\angle ABD = \angle BDC = \alpha$
 $\angle DAB = \pi - \theta - \alpha$

Applying sine rule on $\triangle ABD$

$$\frac{AB}{\sin \theta} = \frac{BD}{\sin(\pi - \theta - \alpha)}$$

$$AB = \frac{BD \sin \theta}{\sin \theta \cdot \cos \alpha + \cos \theta \cdot \sin \alpha} \dots (1)$$

Now, from $\triangle DCB$

$$\sin \alpha = \frac{p}{BD}, \cos \alpha = \frac{q}{BD}$$

Put the value of $\sin \alpha$ and $\cos \alpha$ in equation (1)

$$\begin{aligned} AB &= \frac{BD \sin \theta}{\sin \theta \cdot \frac{q}{BD} + \cos \theta \cdot \frac{p}{BD}} \\ &= \frac{BD^2 \sin \theta}{q \sin \theta + p \sin \theta} \\ &= \frac{(p^2 + q^2) \sin \theta}{p \sin \theta + q \cos \theta} \quad \because BD = \sqrt{p^2 + q^2} \end{aligned}$$

9. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is:

A. $4\sqrt{3}$

B. $2\sqrt{3}$

C. $\frac{4}{\sqrt{3}}$

D. $\frac{2}{\sqrt{3}}$

Given,

The angles of $\triangle ABC$ are in A.P.

sum of angles of a triangle $= \pi$

$$\therefore A + B + C = \pi \text{ & } 2B = A + C$$

$$\Rightarrow 3B = \pi \Rightarrow B = \frac{\pi}{3}$$

$$\Rightarrow A + C = \frac{2\pi}{3}$$

$$\Rightarrow \frac{a}{b} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{2R \sin A}{2R \sin B} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\sin A}{\sin \frac{\pi}{3}} = \frac{1}{\sqrt{3}}$$

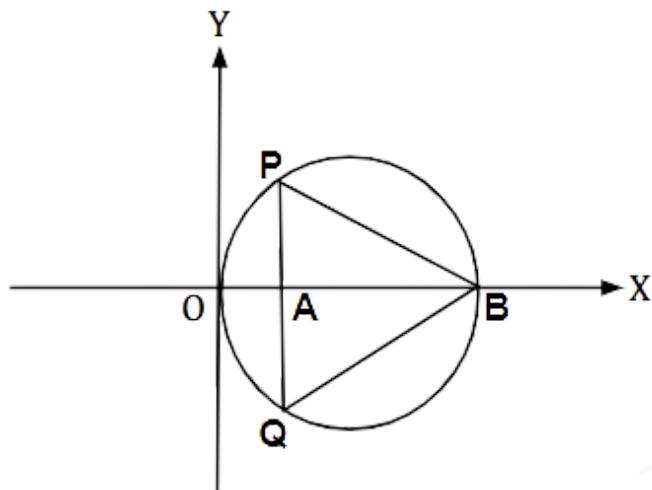
$$\Rightarrow \sin A = \frac{1}{2}$$

$$\therefore A = 30^\circ$$

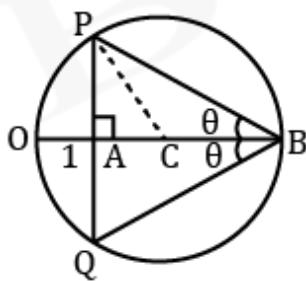
$$\Rightarrow a = 2, b = 2\sqrt{3}, c = 4$$

$$\therefore \text{Area} = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$$

10. In the circle given below, let $OA = 1$ unit, $OB = 13$ unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is :



- A. $26\sqrt{3}$
- B. $24\sqrt{2}$
- C. $24\sqrt{3}$
- D. $26\sqrt{2}$



$$OC = \frac{13}{2} = 6.5$$

$$AC = CO - AO \\ = 6.5 - 1 = 5.5$$

In $\triangle PAC$

$$PA = \sqrt{6.5^2 - 5.5^2}$$

$$\Rightarrow PA = \sqrt{12}$$

$$\Rightarrow PQ = 2PA = 2\sqrt{12}$$

$$\text{Now, area of } \triangle PQB = \frac{1}{2} \times PQ \times AB$$

$$= \frac{1}{2} \times 2\sqrt{12} \times 12$$

$$= 12\sqrt{12}$$

$$= 24\sqrt{3} \text{ sq. units}$$

11. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to:

A. 3

B. 2

C. 4

D. 8

$$(81)^{\sin^2 x} + (81)^{1-\sin^2 x} = 30$$

$$(81)^{\sin^2 x} + \frac{81}{(81)^{\sin^2 x}} = 30$$

$$\text{Let } (81)^{\sin^2 x} = t$$

$$t + \frac{81}{t} = 30 \Rightarrow t^2 + 81 = 30t$$

$$\Rightarrow t^2 - 30t + 81 = 0$$

$$\Rightarrow t^2 - 27t - 3t + 81 = 0$$

$$\Rightarrow (t - 3)(t - 27) = 0$$

$$\Rightarrow t = 3, 27$$

$$\Rightarrow (81)^{\sin^2 x} = 3, 3^3$$

$$\Rightarrow 3^{4\sin^2 x} = 3^1, 3^3$$

$$\Rightarrow 4\sin^2 x = 1, 3$$

$$\Rightarrow \sin^2 x = \frac{1}{4}, \frac{3}{4}$$

$$\text{in } [0, \pi], \sin x \geq 0$$

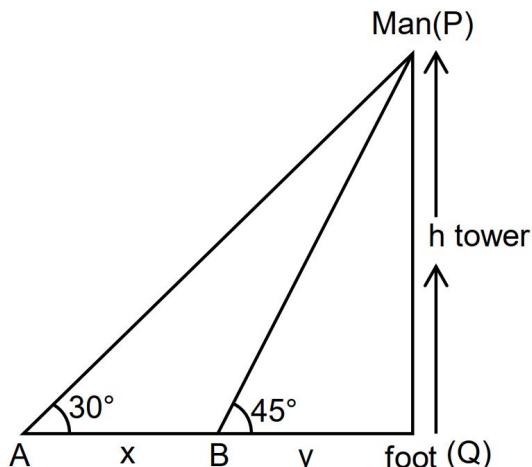
$$\sin x = \frac{1}{2}, \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\text{Number of solutions} = 4$$

12. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A , with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B , where the angle of depression is 45° . Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

- A. $10(\sqrt{3} - 1)$
- B. $10\sqrt{3}$
- C. 10
- D. $10(\sqrt{3} + 1)$



$$\frac{h}{x+y} = \tan 30^\circ$$

$$x+y = \sqrt{3}h \quad \dots (1)$$

also

$$\frac{h}{y} = \tan 45^\circ$$

$$h = y \quad \dots (2)$$

put $h = y$ in equation (1)

$$x+y = \sqrt{3}y$$

$$x = (\sqrt{3}-1)y$$

and

$$\frac{x}{20} = 'v' \quad (v = \text{speed})$$

\therefore time taken to reach foot from B

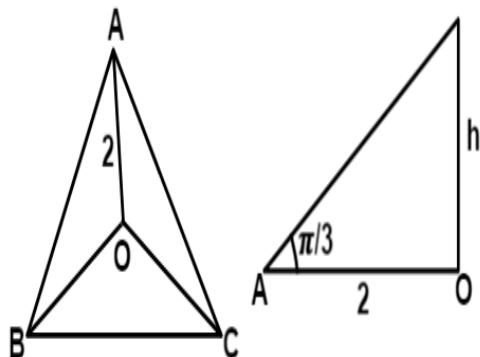
$$= \frac{y}{v}$$

$$= \frac{x}{(\sqrt{3}-1) \times x} \times 20$$

$$= 10(\sqrt{3}+1)$$

13. A pole stands vertically inside a triangular park ABC . Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of $\triangle ABC$ is 2, then the height of the pole is equal to:

- A. $\frac{1}{\sqrt{3}}$
- B. $\sqrt{3}$
- C. $2\sqrt{3}$
- D. $\frac{2\sqrt{3}}{3}$



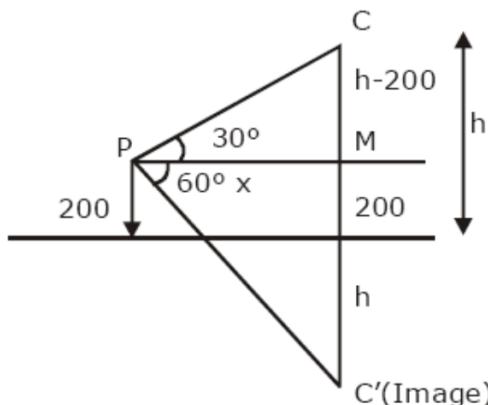
Let the height of the pole be h , then

$$\tan 60^\circ = \frac{h}{2}$$

$$\Rightarrow h = 2\sqrt{3}$$

14. The angle of elevation of a cloud C from a point P , 200 m above a still lake is 30° . If the angle of depression of the image of C in the lake from the point P is 60° , then PC (in m) is equal to

- A. $200\sqrt{3}$
- B. $400\sqrt{3}$
- C. 400
- D. 100



In $\triangle PMC$, we get

$$\tan 30^\circ = \frac{h - 200}{x} \quad \dots (1)$$

In $\triangle PMC'$, we get

$$\tan 60^\circ = \frac{h + 200}{x} \quad \dots (2)$$

From (1) and (2), we get

$$\frac{h + 200}{h - 200} = 3$$

$$\Rightarrow h + 200 = 3h - 600$$

$$\Rightarrow h = 400$$

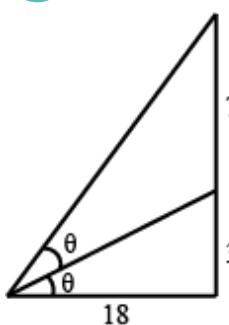
$$\text{Now, } \sin 30^\circ = \frac{h - 200}{PC}$$

$$\Rightarrow PC = 2h - 400$$

$$\therefore PC = 400 \text{ m}$$

15. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is:

- A. $8\sqrt{10}$
- B. $12\sqrt{10}$
- C. $12\sqrt{15}$
- D. $6\sqrt{10}$



$$\begin{aligned}
 \tan \theta &= \frac{3h}{18} = \frac{h}{6} \\
 3h \quad \tan 2\theta &= \frac{10h}{18} = \frac{5h}{9} \\
 \Rightarrow \frac{2 \tan \theta}{1 - \tan^2 \theta} &= \frac{5h}{9} \\
 \Rightarrow \frac{2 \left(\frac{h}{6} \right)}{1 - \frac{h^2}{36}} &= \frac{5h}{9} \\
 \Rightarrow \frac{12}{36 - h^2} &= \frac{5}{9} \\
 \Rightarrow 108 &= 180 - 5h^2 \\
 \Rightarrow h^2 &= \frac{72}{5} \\
 \Rightarrow h &= 6\sqrt{\frac{2}{5}} \quad [:: h > 0] \\
 \therefore 10h &= 60\sqrt{\frac{2}{5}} = 12\sqrt{10}
 \end{aligned}$$

Hence, the height of the pole is $12\sqrt{10}$ meters

16. Let in a right angled triangle, the smallest angle be θ . If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin \theta$ is equal to :

A. $\frac{\sqrt{5} + 1}{4}$

B. $\frac{\sqrt{2} - 1}{2}$

C. $\frac{\sqrt{5} - 1}{2}$

D. $\frac{\sqrt{5} - 1}{4}$

Let a ΔABC having $C = 90^\circ$ and $A = \theta$

$$\frac{\sin \theta}{a} = \frac{\cos \theta}{b} = \frac{1}{c} \dots (i)$$

Also for triangle of reciprocals

$$\cos A' = \frac{\left(\frac{1}{c}\right)^2 + \left(\frac{1}{b}\right)^2 - \left(\frac{1}{a}\right)^2}{2\left(\frac{1}{c}\right)\left(\frac{1}{b}\right)} \text{ and } A' = 90^\circ$$

$$\Rightarrow \frac{1}{c^2} + \frac{1}{(c \cos \theta)^2} = \frac{1}{(c \sin \theta)^2}$$

$$\Rightarrow 1 + \sec^2 \theta = \operatorname{cosec}^2 \theta$$

$$\Rightarrow \frac{1}{4} = \frac{\cos 2\theta}{4 \sin^2 \theta \cos^2 \theta}$$

$$\Rightarrow \frac{1}{4} = \frac{\cos 2\theta}{\sin^2 2\theta}$$

$$\Rightarrow 1 - \cos^2 2\theta = 4 \cos 2\theta$$

$$\cos^2 2\theta + 4 \cos 2\theta - 1 = 0$$

$$\cos 2\theta = \frac{-4 \pm \sqrt{16 + 4}}{2}$$

$$\cos 2\theta = -2 \pm \sqrt{5}$$

$$\cos 2\theta = \sqrt{5} - 2 = 1 - 2 \sin^2 \theta$$

$$\Rightarrow 2 \sin^2 \theta = 3 - \sqrt{5}$$

$$\Rightarrow \sin^2 \theta = \frac{3 - \sqrt{5}}{2} = \frac{6 - 2\sqrt{5}}{4}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{5} - 1}{2}$$

17. If n is the number of solutions of the equation

$$2 \cos x \left(4 \sin \left(\frac{\pi}{4} + x \right) \sin \left(\frac{\pi}{4} - x \right) - 1 \right) = 1, x \in [0, \pi]$$

and S is the sum of all these solutions, then the ordered pair (n, S) is :

- A. $\left(3, \frac{5\pi}{3} \right)$
- B. $\left(3, \frac{13\pi}{9} \right)$
- C. $\left(2, \frac{2\pi}{3} \right)$
- D. $\left(2, \frac{8\pi}{9} \right)$

$$2 \cos x \left(4 \sin \left(\frac{\pi}{4} + x \right) \sin \left(\frac{\pi}{4} - x \right) - 1 \right) = 1$$

$$2 \cos x \left(4 \left(\sin^2 \frac{\pi}{4} - \sin^2 x \right) - 1 \right) = 1$$

$$2 \cos x \left(4 \times \frac{1}{2} - 4 \sin^2 x - 1 \right) = 1$$

$$2 \cos x (1 - 2(1 - \cos 2x)) = 1$$

$$4 \cos x \cos 2x - 2 \cos x = 1$$

$$2[\cos 3x + \cos x] - 2 \cos x = 1$$

$$2 \cos 3x = 1$$

$$\cos 3x = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$3x = 2n\pi \pm \frac{\pi}{3} = (6n \pm 1)\frac{\pi}{3}$$

$$x = (6n \pm 1)\frac{\pi}{9}$$

$$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

$$\text{Sum} = \frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} = \frac{13\pi}{9}$$

18. The number of solutions of the equation $x + 2 \tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is :

A. 5

B. 2

C. 4

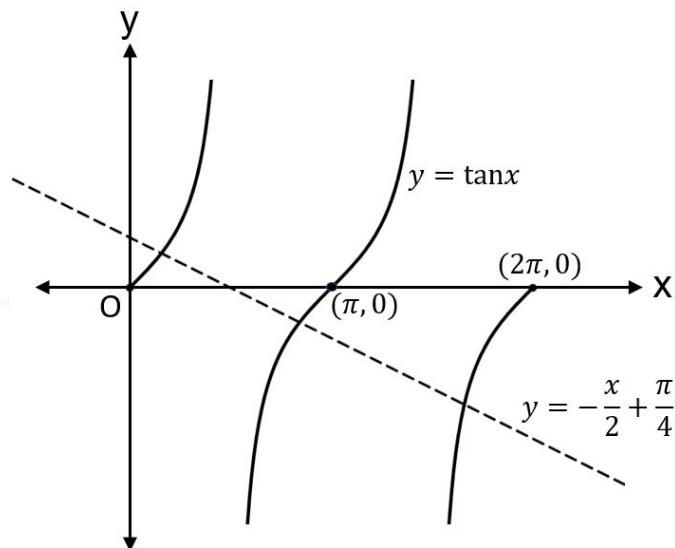
D. 3

$$x + 2 \tan x = \frac{\pi}{2} \text{ in } [0, 2\pi]$$

$$2 \tan x = \frac{\pi}{2} - x$$

$$\tan x = \frac{\pi}{4} - \frac{x}{2}$$

$$y = \tan x \text{ and } y = \frac{-x}{2} + \frac{\pi}{4}$$



From the above graph it can be observed that there are 3 intersection points in $[0, 2\pi]$
 \therefore Number of solutions = 3

19. The sum of solutions of the equation $\frac{\cos x}{1 + \sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is

- A. $\frac{\pi}{10}$
- B. $-\frac{7\pi}{30}$
- C. $-\frac{11\pi}{30}$
- D. $-\frac{\pi}{15}$

$$\frac{\cos x}{1 + \sin x} = |\tan 2x|, x \in \left(\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$$

$$\text{If } x \in \left(0, \frac{\pi}{4}\right), \text{ then } \frac{\cos x}{1 + \sin x} = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\Rightarrow \frac{\cos x}{1 + \sin x} = \frac{2 \sin x \cdot \cos x}{\cos^2 x - \sin^2 x}$$

$$\Rightarrow \cos^2 x - \sin^2 x = 2 \sin x + 2 \sin^2 x \quad (\because \cos x \neq 0)$$

$$\Rightarrow 4 \sin^2 x + 2 \sin x - 1 = 0$$

$$\therefore \sin x = \frac{\sqrt{5} - 1}{4}, \frac{-\sqrt{5} - 1}{4} \text{ (Not applicable)}$$

$$\therefore x = \frac{\pi}{10} \dots (i)$$

If $x \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$, then again

$$\sin x = \frac{\sqrt{5} - 1}{4}, -\left(\frac{\sqrt{5} + 1}{4}\right)$$

$$\therefore x = -\frac{3\pi}{10} \dots (ii)$$

If $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{4}, 0\right)$, then

$$1 - 2 \sin^2 x = -2 \sin x - 2 \sin^2 x$$

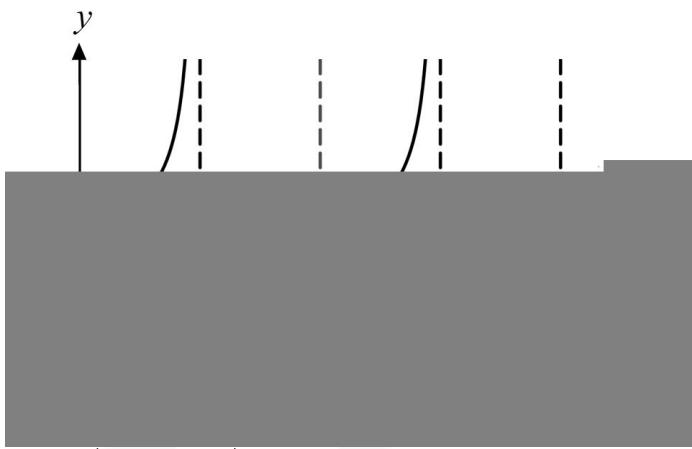
$$\therefore \sin x = -\frac{1}{2}$$

$$\therefore x = -\frac{\pi}{6} \dots (iii)$$

$$\therefore \text{Sum of solution} = \frac{\pi}{10} - \frac{3\pi}{10} - \frac{\pi}{6} = -\frac{11\pi}{30}$$

20. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

- A. $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$
- B. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$
- C. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$
- D. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$



$$\tan 2\theta(1 + \cos 2\theta) > 0$$

$$\tan 2\theta > 0 \quad (\because 1 + \cos 2\theta > 0)$$

$$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\Rightarrow \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

21. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

- A. π
- B. $\frac{\pi}{2}$
- C. $\frac{5\pi}{4}$
- D. $\frac{3\pi}{8}$

$$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$$

$$\Rightarrow \sin^2 2\theta + 1 + \sin^4 2\theta - 2 \sin^2 2\theta = \frac{3}{4}$$

$$\Rightarrow \sin^4 2\theta - \sin^2 2\theta + \frac{1}{4} = 0$$

$$\Rightarrow 4 \sin^4 2\theta - 4 \sin^2 2\theta + 1 = 0$$

$$\Rightarrow (2 \sin^2 2\theta - 1)^2 = 0 \Rightarrow \sin^2 2\theta = \frac{1}{2}$$

$$\Rightarrow \sin 2\theta = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow 2\theta = \frac{\pi}{4}, \frac{3\pi}{4} \quad (\because 0 < 2\theta < \pi)$$

$$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \quad (\because 0 < \theta < \frac{\pi}{2})$$

$$\text{Sum} = \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$$

Date: 31/03/2022

Subject: Mathematics

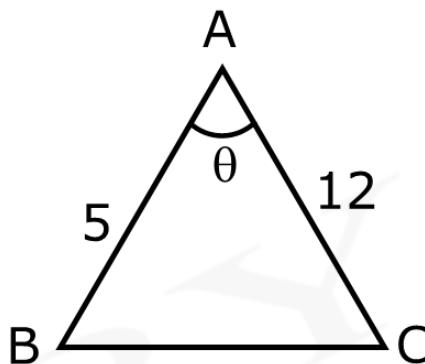
Class: Standard XII

1. In $\triangle ABC$, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of $\triangle ABC$ is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of $\triangle ABC$, then the value of $2R + r$ (in cm) is equal to

Accepted Answers

15 15.0 15.00

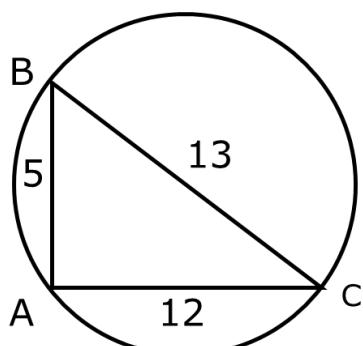
Solution:



$$\text{Area} = \frac{1}{2}(5)(12) \sin \theta = 30$$

$$\Rightarrow \sin \theta = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$\triangle ABC$ is right angled at A.



$$r = (s - a) \tan \frac{A}{2}$$

$$\Rightarrow r = s - a$$

$$\Rightarrow 2R + r = s \quad (\because a = 2R)$$

$$\Rightarrow 2R + r = \frac{30}{2} = 15$$

2. The number of distinct solutions of the equation,
 $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is

Accepted Answers

8 8.0 8.00

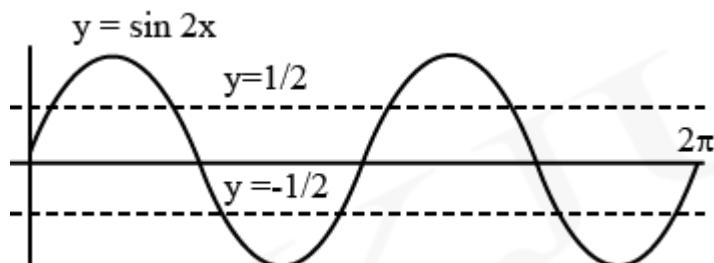
Solution:

$$\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0, 2\pi]$$

$$\Rightarrow \log_{\frac{1}{2}}|\sin x||\cos x| = 2$$

$$\Rightarrow |\sin x \cos x| = \frac{1}{4}$$

$$\therefore \sin 2x = \pm \frac{1}{2}$$



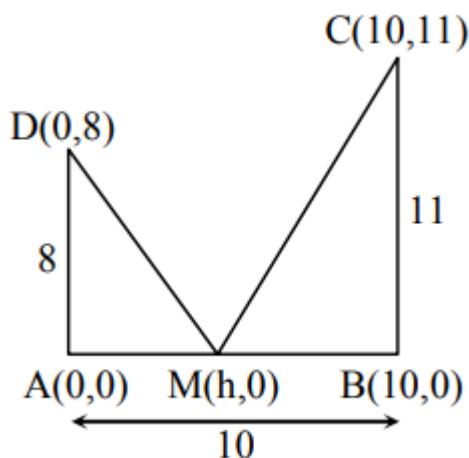
\therefore We have 8 solutions for $x \in [0, 2\pi]$

3. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If $AD = 8m$, $BC = 11m$ and $AB = 10m$; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is

Accepted Answers

5 5.0 5.00

Solution:



$$\begin{aligned}
 (MD)^2 + (MC)^2 &= h^2 + 64 + (h - 10)^2 + 121 \\
 &= 2h^2 - 20h + 64 + 100 + 121 \\
 &= 2(h^2 - 10h) + 285 \\
 &= 2(h - 5)^2 + 235
 \end{aligned}$$

It is minimum if $h = 5$

4. Let S be the sum of all solutions (in radians) of the equation $\sin^4 \theta + \cos^4 \theta - \sin \theta \cos \theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to

Accepted Answers

56 56.0 56.00

Solution:

$$(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta - \sin \theta \cos \theta = 0$$

Let $\sin \theta \cdot \cos \theta = t$,

$$\Rightarrow 1 - 2t^2 - t = 0$$

$$\Rightarrow 2t^2 + t - 1 = 0$$

$$\Rightarrow t = \frac{1}{2}, -1$$

$$\Rightarrow \sin \theta \cos \theta = \frac{1}{2} \text{ or } \sin \theta \cos \theta = -1$$

$\Rightarrow \sin 2\theta = 1$ or $\sin 2\theta = -2$ (Not Possible)

$$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$\therefore S = 7\pi$$

$$\Rightarrow \frac{8S}{\pi} = 56$$