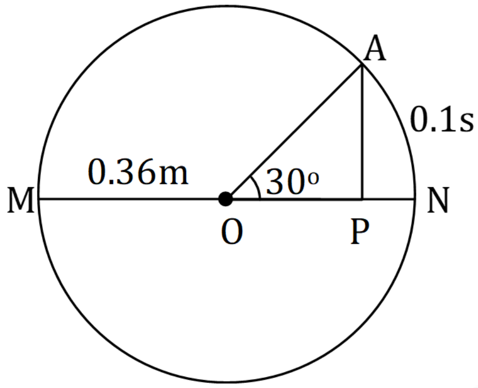
Topic : Circular Motion

1. The point A moves with a uniform speed along the circumference of a circle of radius $0.36\ \mathrm{m}$ and cover 30° in $0.1\ \mathrm{s}$. The perpendicular projection $'P'$ from $'A'$ on the diameter MN represents the simple harmonic motion of $'P'$. The restoring force per unit mass when P touches M will be:

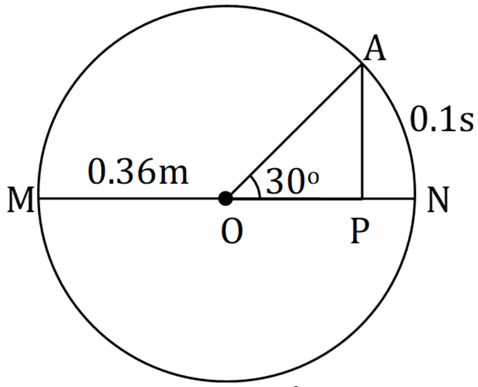


A. $100\ \mathrm{N}$

B. $50\ \mathrm{N}$

C. $9.87\ \mathrm{N}$ ✔

D. $0.49\ \mathrm{N}$



The point A covers 30° in $0.1\ \mathrm{s}$.
From unitary method it means,

$$\frac{\pi}{6} \rightarrow 0.1\ \mathrm{s}$$

$$1 \rightarrow \frac{0.1}{\frac{\pi}{6}}\mathrm{s}$$

$$2\pi \rightarrow \frac{0.1}{\frac{\pi}{6}} \times 2\pi\ \mathrm{s}$$

$$T = 1.2\ \mathrm{s}$$

We know that $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{1.2}$

Restoring force $(F) = m\omega^2 A$

Then, restoring force per unit mass $\left(\dfrac{F}{m}\right) = \omega^2 A = \left(\dfrac{2\pi}{1.2}\right)^2 \times 0.36 \approx 9.87\ \mathrm{N}$

2. Statement I : A cyclist is moving on an unbanked road with a speed of $7\ \mathrm{kmh}^{-1}$ and takes a sharp circular turn along a path of radius of $2\ \mathrm{m}$ without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve.
$(g = 10\ \mathrm{m/s^2})$

Statement II : If the road is banked at an angle of 45°, cyclist can cross the curve of $2\ \mathrm{m}$ radius with the speed of $15\ \mathrm{kmh}^{-1}$ without slipping.

In the light of the above statements, choose the correct answer from the options given below.

- **A.** Both statement I and statement II are false
- **B.** Both statement I and statement II are true
- **C.** Statement I is correct and statement II is incorrect
- **D.** Statement I is incorrect and statement II is correct

Statement 1 :
On a horizontal ground,
$V_{max} = \sqrt{\mu R g} = \sqrt{0.2 \times 2 \times 10}$
$= 2\ \mathrm{m/s} = 2 \times \dfrac{18}{5}$
$= \ 7.2\ \mathrm{km/h}$ > given velocity.
Hence, it can turn safely.

Statement - 2 :
Given, $\theta = 45^\circ \Rightarrow \ \tan\theta = \tan 45^\circ = 1$
Also, $V_{max} = \sqrt{gr\left(\dfrac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$

$$= \left(\sqrt{2 \times 10 \times \frac{1.2}{0.8}}\right) \times \frac{18}{5} = 19.72\ \mathrm{km/h}$$

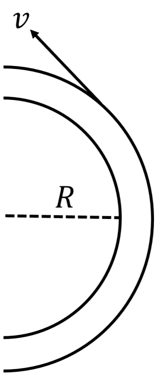
And, $V_{min} = \sqrt{rg\left(\dfrac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$

$$= \left(\sqrt{2 \times 10 \times \frac{0.8}{1.2}}\right) \times \frac{18}{5} = 13.14\ \mathrm{km/h}$$

Since, given velocity is between V_{max} and V_{min}, cyclist can safely turn.

3. A modern Grand Prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_1 acting downwards on the car is -

(Assume forces on the four tyres are identical and $g =$ acceleration due to gravity)



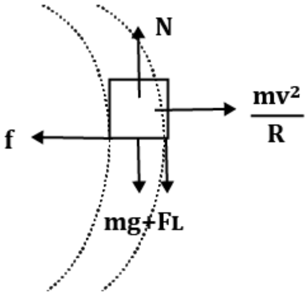
A. $m\left(\frac{v^2}{\mu_s R} - g\right)$

B. $m\left(\frac{v^2}{\mu_s R} + g\right)$

C. $m\left(g - \frac{v^2}{\mu_s R}\right)$

D. $-m\left(g + \frac{v^2}{\mu_s R}\right)$

On drawing the FBD of the car,



Applying the condition of equilibrium along the radial direction,

$$f = \frac{mv^2}{R}$$

$$\Rightarrow \mu_s N = \frac{mv^2}{R}$$

$$\Rightarrow \mu_s (mg + F_L) = \frac{mv^2}{R}$$

$$\Rightarrow F_L = m\left(\frac{v^2}{\mu_s R} - g\right)$$

Hence, option (A) is the correct answer.

4. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

($g = 10\ \mathrm{ms}^{-2}$, radius of earth, $R = 6400 \times 10^3\ \mathrm{m}, \pi = 3.14$)

A. 60 minutes

B. Does not change

C. 84 minutes

D. 1200 minutes

It is given that at the equator, the body starts floating, $i.e.$ condition of weightlessness.

$$\Rightarrow g_e = g - R\omega^2 = 0$$

$$\Rightarrow \omega = \sqrt{\frac{g}{R}}$$

Now, time period,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

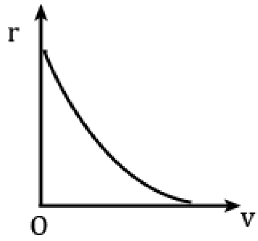
$$\Rightarrow T = 2 \times 3.14 \times \sqrt{\frac{6400 \times 10^3}{10}}$$

$$\Rightarrow T = 5024\ \mathrm{s} \approx 84\ \text{minutes}$$

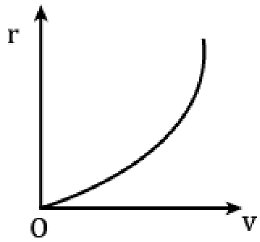
Hence, option (C) is the correct answer.

5. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{-c}{r}$, where c is positive constant. The correct radius(r)-velocity (v) graph of the particle's motion is :

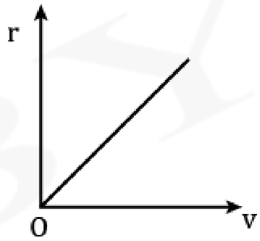
A.



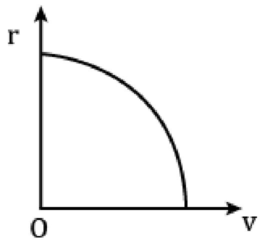
B.



C.



D.



Given, potential field,

$$U(r) = \frac{-c}{r}$$

$$\Rightarrow F = -\frac{dU}{dr} = \frac{c}{r^2}$$

We know that the centripetal force is given by,

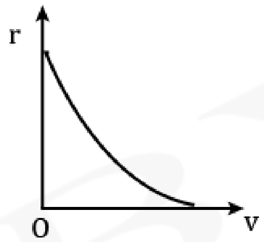
$$F_c = \frac{mv^2}{r}$$

$$\therefore \frac{mv^2}{r} = \frac{c}{r^2}$$

$$\Rightarrow r = \frac{c}{mv^2}$$

$$\Rightarrow r \propto \frac{1}{v^2}$$

This situation is best represented by the graph of option (A).



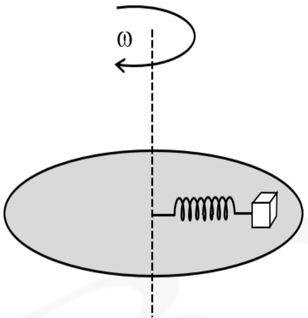
6. A spring mass system (mass m, spring constant k and natural length of spring l) rests in equilibrium, on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc, together with spring mass system, rotates about it's axis with an angular velocity ω, $(k >> m\omega^2)$ the relative change in the length of the spring is best given by the option:

A. $\sqrt{\frac{2}{3}\left(\frac{m\omega}{k}\right)}$

B. $\frac{2m\omega^2}{k}$

C. $\frac{m\omega^2}{k}$

D. $\frac{m\omega^2}{3k}$



The free body diagram in the frame of disc is as shown below.

$\therefore m\omega^2(l+x) = kx$

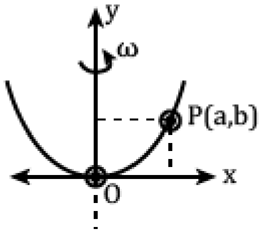
$$\Rightarrow x = \frac{ml\omega^2}{k - m\omega^2}$$

For $k >> m\omega^2$

$$\Rightarrow \frac{x}{l} = \frac{m\omega^2}{k}$$

Hence, (C) is the correct answer.

7. A bead of mass m stays at point $P(a,b)$ on a wire bent in the shape of a parabola $y=4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction)



A. $2\sqrt{2gC}$

B. $2\sqrt{gC}$

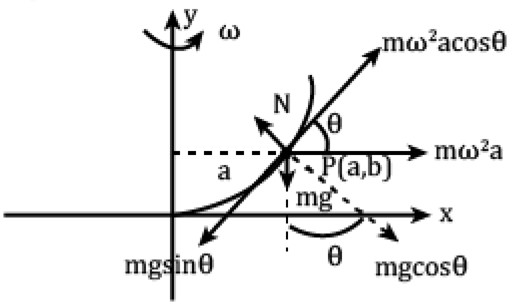
C. $\sqrt{\dfrac{2gC}{ab}}$

D. $\sqrt{\dfrac{2g}{C}}$

Given that, $y=4Cx^2$

$$\Rightarrow \frac{dy}{dx}=\tan\theta=8Cx$$

At $P, \tan\theta=8Ca$



For the bead to stay at point P,

$$m\omega^2 a\cos\theta=mg\sin\theta$$

$$\Rightarrow \omega=\sqrt{\frac{g\tan\theta}{a}}$$

$$\therefore \omega=\sqrt{\frac{g\times 8aC}{a}}=2\sqrt{2gC}.$$

Hence, option (A) is correct.

8. A clock has a continuously moving second's hand of $0.1\ \mathrm{m}$ length. The average accelaration of the tip of the hand (in units of ms^{-2}) is of the order of :

- A. 10^{-3}
- B. 10^{-4}
- C. 10^{-2}
- D. 10^{-1}

Here, $R = 0.1\ \mathrm{m}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105\ \ \mathrm{rad/s}$

Accelaration of the tip of the clock second's hand will be,

$a = \omega^2 R = (0.105)^2 (0.1) = 0.0011 = 1.1 \times 10^{-3}\ \mathrm{m/s}^2$

Hence, average accelaration is of the order of 10^{-3}.

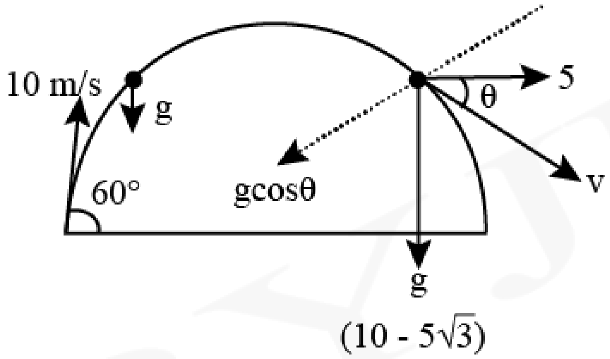
9. A body is projected at $t=0$ with a velocity $10\ \mathrm{ms}^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t=1\ \mathrm{s}$ is R. Neglecting air resistance and taking acceleration due to gravity $g=10\ \mathrm{ms}^{-2}$, the value of R is:
$(\text{Take: } \tan 15^\circ = 0.268; \cos 15^\circ = 0.966)$

A. $10.3\ \mathrm{m}$

B. $2.8\ \mathrm{m}$

C. $2.5\ \mathrm{m}$

D. $5.1\ \mathrm{m}$



At $t=0$,
Horizontal component of velocity
$u_x = 10\cos 60^\circ = 5\ \mathrm{ms}^{-1}$

Vertical component of velocity
$u_y = 10\cos 30^\circ = 5\sqrt{3}\ \mathrm{ms}^{-1}$

At $t=1\ \mathrm{s}$,
Horizontal component of velocity
$v_x = 5\ \mathrm{ms}^{-1}$

Vertical component of velocity
$v_y = u_y - gt = |(5\sqrt{3}-10)|\ \mathrm{ms}^{-1}$

$\Rightarrow v_y = 10 - 5\sqrt{3}\ \mathrm{ms}^{-1}$

$$\text{Centripetal acceleration, } a_c = \frac{v^2}{R}$$

$$\Rightarrow R = \frac{v_x^2 + v_y^2}{a_c}$$

Here, $g\cos\theta$ points towards the centre and acts as the centripetal acceleration,

$$\Rightarrow R = \frac{5^2 + (10-5\sqrt{3})^2}{g\cos\theta}$$

$$\Rightarrow R=\frac{25+(100+75-100\sqrt{3})}{10\cos\theta}$$

$$\Rightarrow R=\frac{100(2-\sqrt{3})}{10\cos\theta}$$

At $t=1\rightarrow\frac{v_y}{v_x}=\tan\theta,$

$$\tan\theta=\frac{10-5\sqrt{3}}{5}=2-\sqrt{3}=0.268$$

$$\Rightarrow\theta=15^\circ$$

$$\Rightarrow R=\frac{100(2-\sqrt{3})}{10\cos 15}=\frac{100\times 0.268}{10\times 0.966}=2.8\text{ m}$$

Hence, option (B) is correct.

10. The normal reaction N for a vehicle of $800\ \mathrm{kg}$ mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _____ $\times 10^3\ \mathrm{kg\text{-}m/s^2}$.

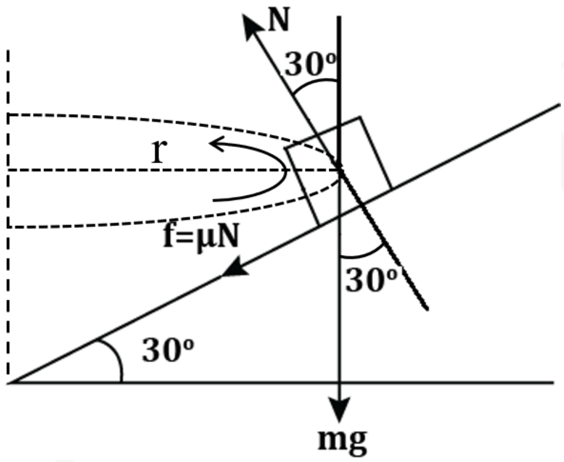
Take $\cos 30^\circ = 0.87$ and $\mu = 0.2$

A. 10.2

B. 7.2

C. 12.4

D. 6.96



At maximum possible speed, friction will be limiting in nature.

$\therefore$ Balancing the forces in vertical direction,

$N\cos 30^\circ - mg - \mu N \cos 60^\circ = 0$

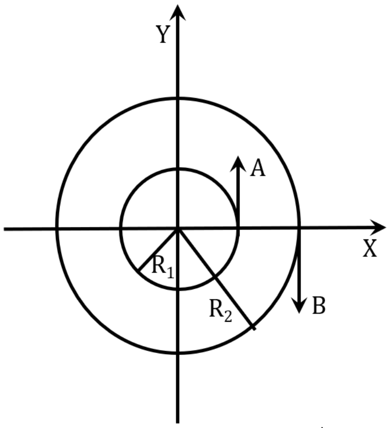
$\Rightarrow N(\cos 30^\circ - \mu \cos 60^\circ) = mg$

$$\Rightarrow N = \frac{mg}{\cos 30^\circ - \mu \cos 60^\circ}$$

$$\Rightarrow N = \frac{800 \times 9.8}{(0.87 - 0.2 \times 0.5)} \approx 10.2 \times 10^3\ \mathrm{kg\text{-}m/s^2}$$

Hence, option (A) is the correct answer.

11. Two particles A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure:



The relative velocity $\overrightarrow{v_A} - \overrightarrow{v_B}$ at $t = \frac{\pi}{2\omega}$ is given by:

- **A.** $\omega(R_1 + R_2)\hat{i}$
- **B.** $-\omega(R_1 + R_2)\hat{i}$
- **C.** $\omega(R_2 - R_1)\hat{i}$ ✔
- **D.** $\omega(R_1 - R_2)\hat{i}$

Angle cover is given by,

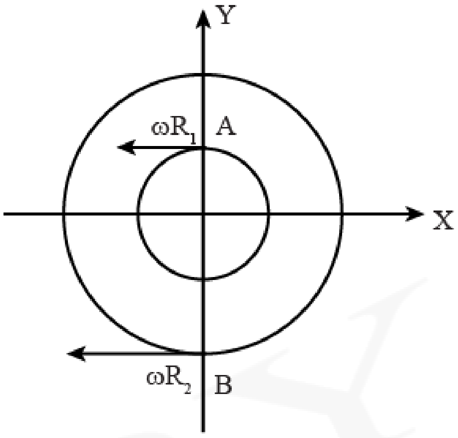
$\theta = \omega t$

At, $t = \dfrac{\pi}{2\omega}$

$\theta = \omega \dfrac{\pi}{2\omega} = \dfrac{\pi}{2}$

So, both particles would have completed a quarter circle.

$\therefore$ the position of the particles at
$t = \dfrac{\pi}{2\omega}$ is shown below,



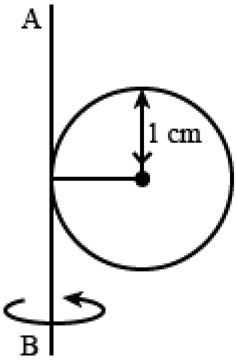
Relative velocity,

$\vec{v}_A - \vec{v}_B = \omega R_1(-\hat{i}) - \omega R_2(-\hat{i})$

$\Rightarrow \vec{v}_A - \vec{v}_B = \omega(R_2 - R_1)\ \hat{i}$

Hence, option (C) is correct.

12. A metal coin of mass $5 \mathrm{~g}$ and radius $1 \mathrm{~cm}$ is fixed to a thin stick AB of negligible mass as shown in the figure The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in $5 \mathrm{~s}$, is close to :



A. $4.0 \times 10^{-6} \mathrm{~Nm}$

B. $1.6 \times 10^{-5} \mathrm{~Nm}$

C. $7.9 \times 10^{-6} \mathrm{~Nm}$

D. $2.0 \times 10^{-5} \mathrm{~Nm}$

Angular accceletration,

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$= \frac{25 \times 2\pi - 0}{5}$$

$$= 10\ \pi \mathrm{~rad/s}^2$$

$$\tau = I\alpha$$

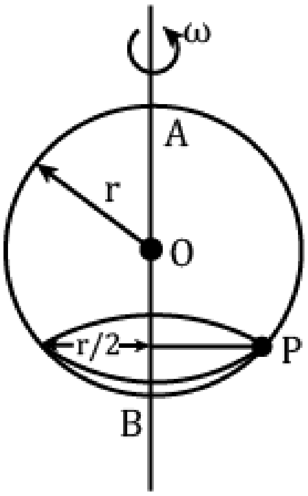
$$\Rightarrow \ \tau = \left(\frac{5}{4} mR^2\right) \alpha$$

$$\approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$$

$$\approx 2.0 \times 10^{-5} \mathrm{~Nm}$$

Hence, option (D) is correct.

13. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring, at position P, as shown. Then the value of ω^2 is equal to :

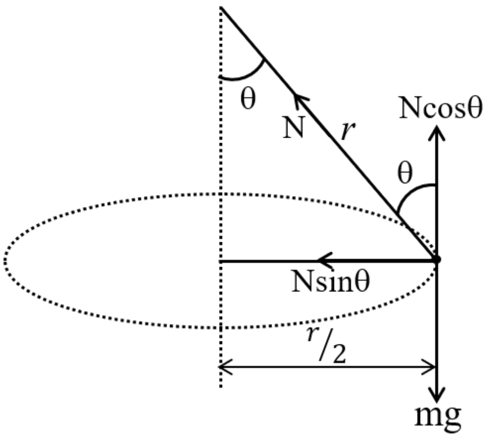


A. $\dfrac{\sqrt{3g}}{2r}$

B. $\dfrac{2g}{(r\sqrt{3})}$ ✔

C. $\dfrac{(g\sqrt{3})}{r}$

D. $\dfrac{2g}{r}$



Now, $\sin\theta = \frac{r/2}{r} = \frac{1}{2} \Rightarrow \theta = 30^\circ$

From the free body diagram, we can write,

$N\sin\theta = m\omega^2\left(\frac{r}{2}\right)$

And, $N\cos\theta = mg$

$\Rightarrow \tan\theta = \frac{\omega^2 r}{2g}$

$\tan 30^\circ = \frac{\omega^2 r}{2g}$

$\frac{1}{\sqrt{3}} = \frac{\omega^2 r}{2g}$

$\therefore \omega^2 = \frac{2g}{r\sqrt{3}}$

Hence, (B) is the correct answer.

14. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then

A. $T \propto R^{(n+1)/2}$

B. $T \propto R^{n/2}$

C. $T \propto R^{3/2}$ for any n

D. $T \propto R^{\frac{n}{2}+1}$

Given that,

$$\mathrm{F} \propto \frac{1}{R^n} \Rightarrow F = \frac{\mathrm{k}}{\mathrm{R^n}}$$

And we know that,

$$\mathrm{F} = \mathrm{m}\omega^2\mathrm{R}$$

$$\therefore \frac{\mathrm{k}}{\mathrm{R^n}} = \mathrm{m}\omega^2\mathrm{R}$$

$$\Rightarrow \omega^2 \propto \frac{1}{\mathrm{R}^{n+1}}$$

So, time period will be,

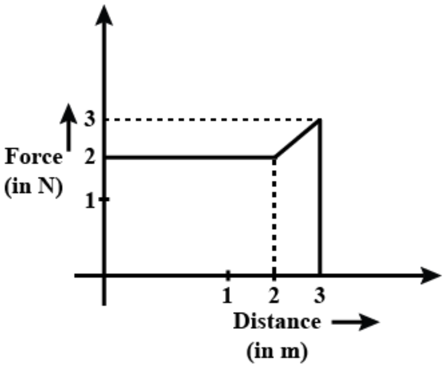
$$\Rightarrow \mathrm{T} = \frac{2\pi}{\omega}$$

So, $T \propto R^{(n+1)/2}$

Hence, option (A) is correct.

Topic : Work Power Energy

1. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled $3\ m$ is



A. $6.5\ J$

B. $2.5\ J$

C. $4\ J$

D. $5\ J$

According to Work Energy Theorem,
Work done by force on the particle $=$ Change in KE
Work done $=$ Area under $F-x$ graph
$=\int F.dx$

$$= 2 \times 2 + \frac{(2+3)\times 1}{2} = 6.5\ J$$

$W = KE_{final} - KE_{initial} = 6.5\ J$
$KE_{initial} = 0$
$\therefore KE_{final} = 6.5\ J$

2. A particle is moving in a circular path of radius a under the action of an attractive potential energy $U=-\frac{k}{2r^2}$. Its total energy is

A. zero

B. $-\frac{3\ \mathrm{k}}{2\ \mathrm{a}^2}$

C. $\frac{k}{4a^2}$

D. $\frac{k}{2a^2}$

$U=-\frac{K}{2r^2}=P.E$

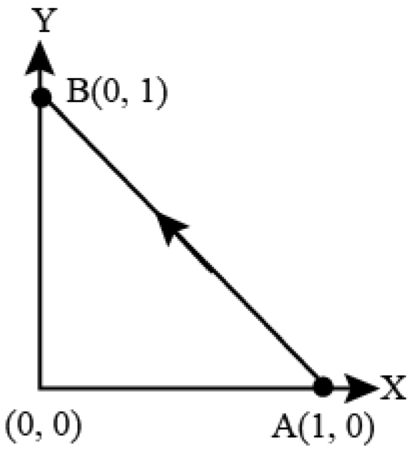
$F=-\frac{du}{dr}=-\left(-\frac{K}{2}\left(-\frac{2}{r^3}\right)\right)=-\frac{K}{r^3}$

$\frac{K}{r^3}=\frac{mv^2}{r} \quad \Rightarrow \quad mv^2=\frac{K}{r^2}$

$K.E.=\frac{1}{2}mv^2=\frac{K}{2r^2}$

$E=P.E.+K.E.=0$

3. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force, in moving a particle, from point $A(1, 0)$ to $B(0, 1)$, along the line segment is: (all quantities are in SI units)



A. 2

B. $\frac{1}{2}$

C. 1

D. $\frac{3}{2}$

Work done, $W = \int \vec{F} \cdot \vec{ds}$

Here, $\vec{F} \cdot \vec{ds} = (-x\hat{i} + y\hat{j}).(dx\hat{i} + dy\hat{j}) = -xdx + ydy$

$$\Rightarrow W = -\int_1^0 xdx + \int_0^1 ydy$$

$$= \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1 \text{ J}$$

Hence, (C) is the correct answer,

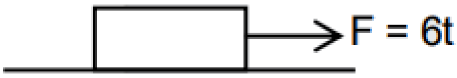
4. A time dependent force $F = 6t$ acts on a particle of mass $1\,kg$. If the particle starts from rest, the work done by the force during the first 1 sec will be

A. $18\,J$

B. $22\,J$

C. $4.5\,J$

D. $9\,J$



$a = \frac{F}{m} = \frac{6t}{1} = 6t$

$\frac{dv}{dt} = 6t$

$\int_0^v dv = 6\int_0^1 tdt$

$\Rightarrow v = 6\left[\frac{t^2}{2}\right]_0^1 = 3$

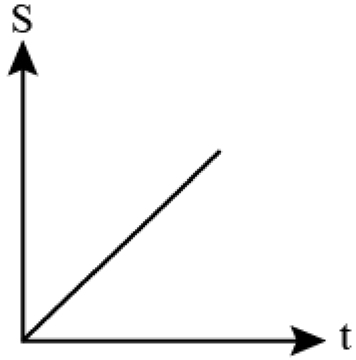
$W = \Delta K.E = K_f - K_i$

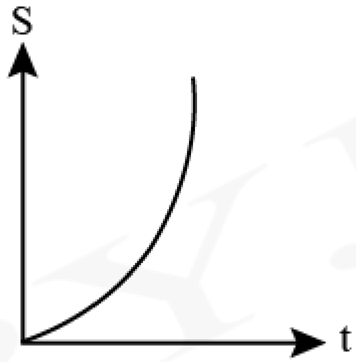
$= \frac{1}{2}mv^2$

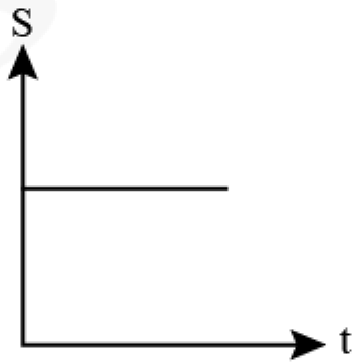
$= \frac{1}{2} \times 1 \times 3^2$

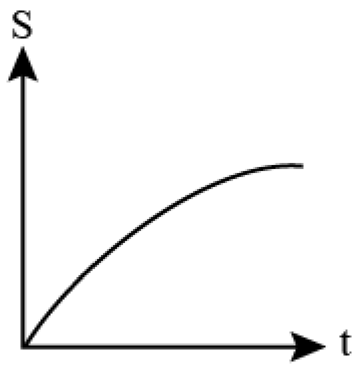
$= 4.5\,J$

5. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement(S) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale):

A. 

B. 

C. 

D. 

We know that
$\text{Power}, P = Fv$

$$\Rightarrow P = mav = m\frac{dv}{dt}v$$

$$\Rightarrow Pdt = mvdv$$

Integrating both sides $\int_0^t P\,dt = m\int_0^v v\,dv$

$$P.t = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

Displacement,
$$S = \int_0^1 v\,dt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$\Rightarrow S = \sqrt{\frac{2P}{m}}.\frac{t^{3/2}}{(3/2)}$$

$$\Rightarrow S = \sqrt{\frac{8P}{9m}}.\ t^{3/2}$$

$$\Rightarrow S \propto t^{3/2}$$

Hence, graph (B) is correct.

6. A body of mass $m = 10^{-2}\ kg$ is moving in a medium and experiences a frictional force $F =- kv^2$. Its initial speed is $v_0 = 10\ ms^{-1}$. If, after $10\ s$, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

A. $10^{-3}\ kg\, m^{-1}$

B. $10^{-3}\ kg\, s^{-1}$

C. $10^{-4}\ kg\, m^{-1}$

D. $10^{-1}\ kg\, m^{-1} s^{-1}$

$m = 10^{-2} kg$

$F = -kv^2$

$v_0 = 10 ms^{-1}$

Also

$u = v_0, v = \dfrac{v_0}{2}$

$a = \dfrac{f}{m} = -\dfrac{kv^2}{m}$

$a = \dfrac{dv}{dt}$

$\dfrac{-kv^2}{m} = \dfrac{dv}{dt}$

$\dfrac{-k}{m}\int_0^{10} dt = \int_{v_0}^{v_0/2} \dfrac{dv}{v^2}$

$\dfrac{-k}{m} \times 10 = -[\dfrac{1}{v}]_{10}^{5}$

$\dfrac{k}{m} \times 10 = \dfrac{1}{5} - \dfrac{1}{10}$

$k \times 10^3 = \dfrac{1}{10}$

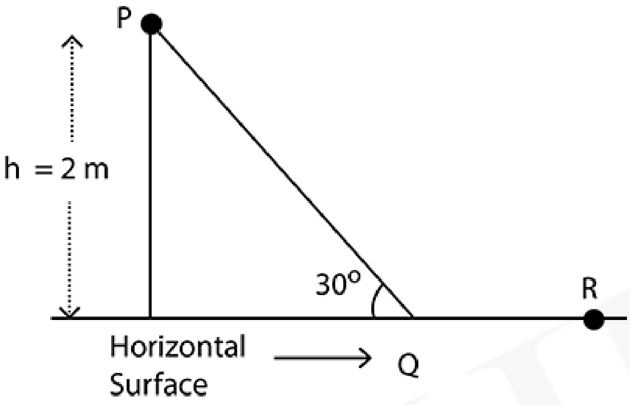
$R = 10^{-4}\ kgm^{-1}$

7. A person trying to lose weight by burning fat lifts a mass of $10\ kg$ upto a height of $1\ m$, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work is done only when the weight is lifted up? Fat supplies $3.8 \times 10^7\ J$ of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$.

- A. $2.45 \times 10^{-3}\ kg$
- B. $6.45 \times 10^{-3}\ kg$
- C. $9.89 \times 10^{-3}\ kg$
- D. $12.89 \times 10^{-3}\ kg$

The net work done by the man will be 1000 times the work done in lifting $10\ kg$ to a height of $1\ m$.
Net work done $= 1000 \times 10 \times 9.8 \times 1\ \ J$
$= 9.8 \times 10^4\ J$
Let's assume $x\ kg$ of fat is burnt in doing this work. Energy balance will give the following equation.
$x \times \dfrac{20}{100} \times 3.8 \times 10^7 =$ Net work done $= 9.8 \times 10^4\ J$
$\Rightarrow x = 12.8947 \times 10^{-3}\ kg$

8. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when the particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x\ (= QR)$, are respectively close to



A. 0.2 and $6.5\ m$

B. 0.2 and $3.5\ m$

C. 0.29 and $3.5\ m$

D. 0.29 and $6.5\ m$

All the potential energy is lost by dissipation due to work done by frictional force.
$P.E\ lost =$ Work done by friction

Work done by friction from P to Q $= \mu mg(\cos\theta)PQ = 2\sqrt{3}\mu mg$
Work done by friction from Q to R $= \mu mg \times QR$
$2mg = 2\sqrt{3}\mu mg + \mu mg \times QR$
$2 = 2\sqrt{3}\mu + \mu QR$ --- (1)
Since equal energies are lost along PQ and QR,
Work done by friction is the same on both path lengths.
$\mu mg\cos\theta \times PQ = \mu mg \times QR$
$PQ\cos\theta = QR$ --- (2)
From (1) and (2) we get,
$\mu = 0.29, QR = 3.5$

9. A $60\ HP$ electric motor lifts an elevator with a maximum total load capacity of $2000\ kg$. If the frictional force on the elevator is $4000\ N$, the speed of the elevator at full load is close to (Given $1\ HP = 746\ W$, $g = 10\ m/s^2$)

A. $1.5\ m/s$

B. $2.0\ m/s$

C. $1.7\ m/s$

D. $1.9\ m/s$ ✔

Friction will oppose the motion

$$\text{Net force} = 2000 \times g + 4000 = 24000\ N$$
$$\text{Power of lift} = 60\ \text{HP}$$
$$\text{Power} = \text{Force} \times \text{Velocity}$$
$$v = \frac{P}{F} = \frac{60 \times 746}{24000}$$
$$v = 1.86 \approx 1.9\ m/s$$

10. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being $68\ \mathrm{kg}$. The mass of the elevator itself is $920\ \mathrm{kg}$ and it moves with a constant speed of $3\ \mathrm{ms}^{-1}$. The frictional force opposing the motion is $6000\ \mathrm{N}$. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator $(g = 10\ \mathrm{ms}^{-2})$ must be at least

A. $56300\ \mathrm{W}$

B. $62360\ \mathrm{W}$

C. $48000\ \mathrm{W}$

D. $66000\ \mathrm{W}$

As the lift is moving with a constant speed,

$$F = (10m + M)g + f$$

where,
$F =$ Force exerted by motor,
$m =$ mass of the person,
$M =$ mass of the elevator,
$f =$ frictional force

$$F = [((10 \times 68) + 920) \times 10] + 6000$$

$$\Rightarrow F = 22000\ \mathrm{N}$$

$$\Rightarrow \text{Power, } P = Fv = 22000 \times 3$$

$$\therefore P = 66000\ \mathrm{W}$$

Hence, option (D) is correct.

11. A person pushes a box on a rough horizontal platform surface. He applies a force of $200 \mathrm{~N}$ over a distance of $15 \mathrm{~m}$. Thereafter, he gets progressively tired, and his applied force reduces linearly with distance to $100 \mathrm{~N}$. The total distance through which the box has been moved, is $30 \mathrm{~m}$. What is the work done by the person, during the total movement of the box ?

A. $3280 \mathrm{~J}$

B. $2780 \mathrm{~J}$

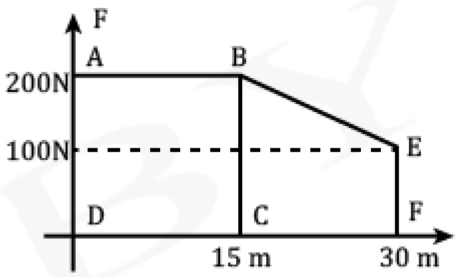
C. $5690 \mathrm{~J}$

D. $5250 \mathrm{~J}$

The given situation can be drawn graphically as shown in figure.

$\text{Work done} = \text{Area under F-x graph}$

$= \text{Area of rectangle ABCD} + \text{Area of trapezium BCFE}$



$$W = (200 \times 15) + \frac{1}{2}(100 + 200) \times 15$$

$$= 3000 + 2250 = 5250 \mathrm{~J}$$

Hence, (D) is the correct answer.

12. If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules and the potential energy respectively are:

A. $\left(\frac{B}{2A}\right)^{\frac{1}{6}}, -\left(\frac{A^2}{2B}\right)$

B. $\left(\frac{B}{A}\right)^{\frac{1}{6}}, 0$

C. $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\left(\frac{A^2}{4B}\right)$

D. $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, \left(\frac{A^2}{2B}\right)$

Given : $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$

For equilibrium,

$F = \frac{dU}{dr} = 0$

$\Rightarrow \; -(A(-6r^{-7})) + B(-12r^{-13}) = 0$

$\Rightarrow 0 = \frac{6A}{r^7} - \frac{12B}{r^{13}} \Rightarrow \frac{6A}{12B} = \frac{1}{r^6}$

$\therefore$ Separation between molecules, $r = \left(\frac{2B}{A}\right)^{1/6}$

Also, Potential energy,

U at $\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$

$= \frac{-A^2}{2B} + \frac{A^2}{4B} = -\left(\frac{A^2}{4B}\right)$

13. A particle moving in the xy plane experiences a velocity dependent force $\vec{F} = k(v_y\hat{i} + v_x\hat{j})$, where v_x and v_y are x and y components of its velocity $\vec{v}$. If $\vec{a}$ is the acceleration of the particle, then which of the following statements is true for the particle?

A. Quantity $\vec{v} \times \vec{a}$ is constant in time

B. $\vec{F}$ arises due to a magnetic field

C. Kinetic energy of particle is constant in time

D. Quantity $\vec{v} . \vec{a}$ is constant in time

Given, $\vec{F} = k(v_y\hat{i} + v_x\hat{j})$

$$\therefore F_x = kv_y\hat{i},\ F_y = kv_x\hat{j}$$

$$\frac{mdv_x}{dt} = kv_y \Rightarrow \frac{dv_x}{dt} = \frac{k}{m}v_y$$

Similarly, $\frac{dv_y}{dt} = \frac{k}{m}v_x$

$$\frac{dv_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dv_y = \int v_x dv_x$$

$$v_y^2 = v_x^2 + C$$

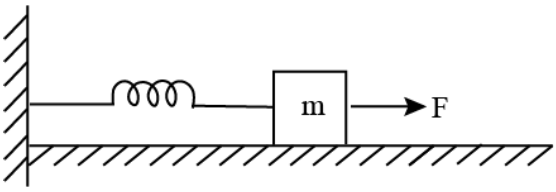
$$v_y^2 - v_x^2 = \text{constant}$$

$$\vec{v} \times \vec{a} = (v_x\hat{i} + v_y\hat{j}) \times \frac{k}{m}(v_y\hat{i} + v_x\hat{j})$$

$$= (v_x^2\hat{k} - v_y^2\hat{k})\frac{k}{m} = (v_x^2 - v_y^2)\frac{k}{m}\hat{k} = \text{ constant}$$

Hence, option (A) is correct.

14. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is:



A. $\frac{2F}{\sqrt{mk}}$

B. $\frac{F}{\pi\sqrt{mk}}$

C. $\frac{\pi F}{\sqrt{mk}}$

D. $\frac{F}{\sqrt{mk}}$

Maximum speed is at mean position or equilibrium .

At extreme position

$F = kx \Rightarrow x = \frac{F}{k}$

From work-energy theorem,

$W_F + W_{sp} = \Delta KE$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{k} = \frac{1}{2}mv^2$$

$$\therefore v_{\max} = \frac{F}{\sqrt{mk}}$$

Hence, option (D) is correct.

15. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?

A. 850 J

B. 950 J

C. 875 J

D. 900 J

Position, $x = 3t^2 + 5$

$$\therefore \text{Velocity}, v = \frac{dx}{dt}$$
$$\Rightarrow v = \frac{d(3t^2+5)}{dt}$$
$$\Rightarrow v = 6t + 0$$

$$\text{At } t = 0 \text{ sec} \quad v_i = 0 \text{ m/s}$$
$$\text{At } t = 5 \text{ sec} \quad v_f = 30 \text{ m/s}$$

According to work-energy theorem, $W = \Delta KE$.

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$W = \frac{1}{2}(2)(30)^2 - 0 = 900 \text{ J}$$

16. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

A. 9 J

B. 12 J

C. 10 J

D. 15 J

Work done by the force is $W = \vec{F}.\vec{d} = (3\vec{i} - 12\vec{j}).(4\vec{i}) = 12$ J

From work energy theorem.

$$W_{net} = \Delta K.E. = K_f - K_i$$
$$\Rightarrow 12 = K_f - 3$$
$$\therefore K_f = 15 \text{ J.}$$

17. A body of mass $1 \mathrm{~kg}$ falls freely from a height of $100 \mathrm{~m}$, on a platform of mass $3 \mathrm{~kg}$ which is mounted on a spring having spring constant $k=1.25 \times 10^{6} \mathrm{~Nm}^{-1}$. The body sticks to the platform, and the spring's maximum compression is found to be x. Given that $g=10 \mathrm{~ms}^{-2}$, the value of x will be close to:

A. $40 \mathrm{~cm}$

B. $4 \mathrm{~cm}$

C. $80 \mathrm{~cm}$

D. $8 \mathrm{~cm}$

By the principle of conservation of energy, the initial gravitational potential energy will be equal to the spring potential energy at maximum compression.

$$\Rightarrow \frac{1}{2} k x^{2}=m g h$$

$$\Rightarrow x^{2}=\frac{2 \times 1 \times 10 \times 100}{1.25 \times 10^{6}}$$

$$\Rightarrow x^{2}=16 \times 10^{-4}$$

$$\Rightarrow x=4 \times 10^{-2} \mathrm{~m}=4 \mathrm{~cm}$$

Hence, option (B) is correct.

18. A uniform cable of mass $'M'$ and length $'L'$ is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be

A. $\frac{MgL}{2n^2}$

B. $\frac{MgL}{n^2}$

C. $\frac{2MgL}{n^2}$

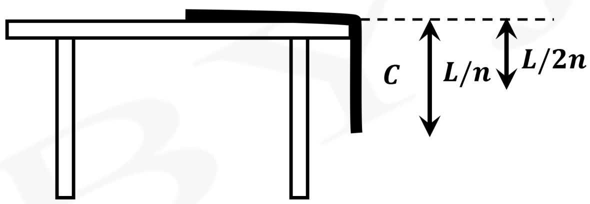
D. $nMgL$

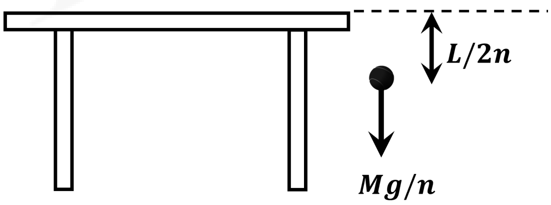
Length of hanging part $= \frac{L}{n}$

Mass of hanging part $= \frac{M}{n}$

Weight of hanging part $= \frac{Mg}{n}$

Let $'C'$ be the centre of mass of the hanging part.





The hanging part can be assued to be a particle of weight $\frac{Mg}{n}$ at a distance $\frac{L}{2n}$ below the table top.

The work done in lifting it to the table top is equal to increase in its potential energy.

$$\therefore\ W = \left(\frac{Mg}{n}\right)\left(\frac{L}{2n}\right)$$

$$\therefore\ W = \frac{MgL}{2n^2}$$

Hence option (A) is correct.

19. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by-

A. $\frac{v^2}{g}$

B. $\frac{2v^2}{7g}$

C. $\frac{2v^2}{5g}$

D. $\frac{v^2}{2g}$

Using conservation of linear momentum,

$$mv = (m+M)v'$$

$$v' = \frac{mv}{m+M} = \frac{mv}{m+4m} = \frac{v}{5}$$

Using conservation of mechanical energy, we have

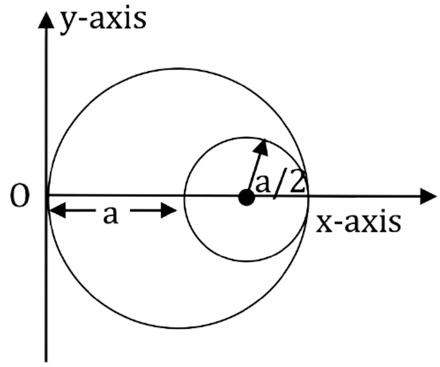
$$\frac{1}{2}mv^2 = \frac{1}{2}(m+4m)\left(\frac{v}{5}\right)^2 + mgh$$

$$\Rightarrow \quad h = \frac{2}{5}\frac{v^2}{g}$$

Hence, (C) is the correct answer.

Topic : Center of mass

1. A circular hole of radius $\frac{a}{2}$ is cut out of a circular disc of radius $'a'$ shown in the figure. The centroid of the remaining circular portion with respect to point $'O'$ will be :

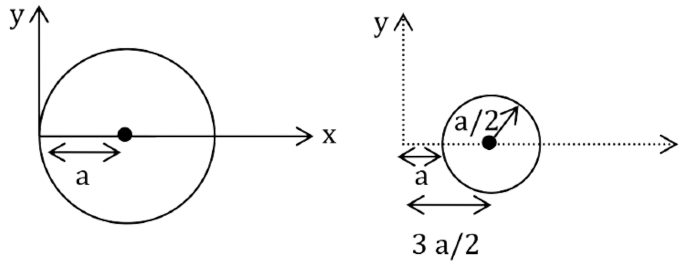


A. $\frac{10}{11}a$

B. $\frac{2}{3}a$

C. $\frac{1}{6}a$

D. $\frac{5}{6}a$



Let σ be the surface mass density of the disc.

$$X_{\text{com}} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Where, $m = \sigma \pi a^2$

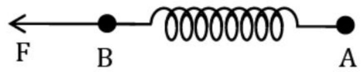
$$X_{\text{com}} = \frac{\sigma \pi a^2 \times a - \sigma \pi \left(\frac{a}{2}\right)^2 \times \frac{3a}{2}}{\sigma \pi a^2 - \sigma \pi \left(\frac{a}{2}\right)^2}$$

$$X_{\text{com}} = \frac{a - \frac{3a}{8}}{1 - \frac{1}{4}}$$

$$X_{\text{com}} = \frac{\frac{5a}{8}}{\frac{3}{4}}$$

$$X_{\text{com}} = \frac{5}{6} a$$

2. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in the figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B will be :

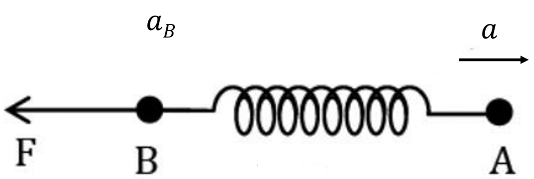


A. $\dfrac{F+Ma}{M}$

B. $\dfrac{F-Ma}{M}$

C. $\dfrac{Ma-F}{M}$

D. $\dfrac{MF}{F+Ma}$



On considering the two mass system, the acceleration of centre of mass,

$$a_{\text{com}} = \frac{F}{2M}$$

Also,

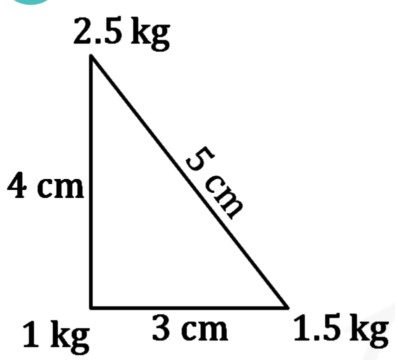
$$a_{\text{com}} = \frac{M_A a_A + M_B a_B}{M_A + M_B}$$

$$\Rightarrow \frac{F}{2M} = \frac{Ma + Ma_B}{M+M}$$

$$\Rightarrow a_B = \frac{F-Ma}{M}$$

3. Three point particles of mass 1 kg, 1.5 kg and 2.5 kg are placed at three corners of a right triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at the point:

- A. 0.9 cm right and 2.0 cm above 1 kg mass
- B. 2.0 cm right and 0.9 cm above 1 kg mass
- C. 1.5 cm right and 1.2 cm above 1 kg mass
- D. 0.6 cm right and 2.0 cm above 1 kg mass

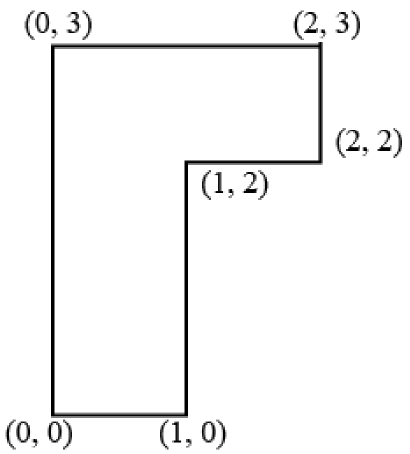


Taking 1 kg as the origin

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$
$$x_{com} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5}$$
$$x_{com} = 0.9$$
$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$
$$y_{com} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5}$$
$$y_{com} = 2$$

Centre of mass is at $(0.9, 2)$

4. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass $4\ \mathrm{kg}$. (The coordinates of the same are shown in figure) are:

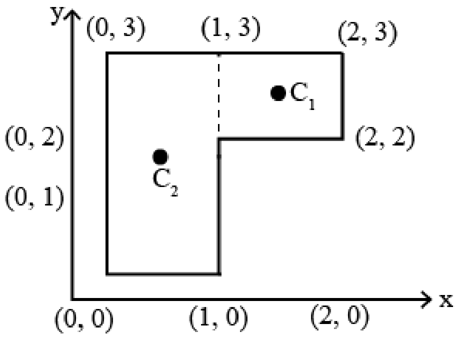


A. $1.25\ \mathrm{m}, 1.50\ \mathrm{m}$

B. $0.75\ \mathrm{m}, 1.75\ \mathrm{m}$

C. $0.75\ \mathrm{m}, 0.75\ \mathrm{m}$

D. $1\ \mathrm{m}, 1.75\ \mathrm{m}$



The given Lamina can be divided into two parts, as shown.

The mass and the position of centre of mass of these parts are given by,

$m_1 = 1, C_1 = (1.5, 2.5)$

$m_2 = 3, C_2 = (0.5, 1.5)$

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\Rightarrow X_{cm} = \frac{1.5 + 1.5}{4} = 0.75$$

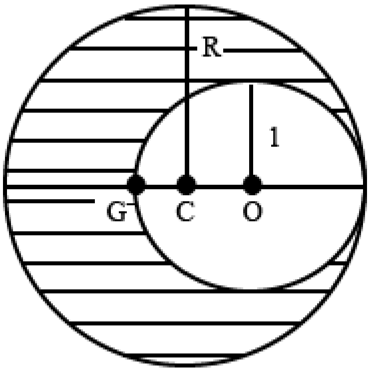
$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$\Rightarrow Y_{cm} = \frac{2.5 + 4.5}{4} = 1.75$$

$\therefore$ Coordinate of the centre of mass of flag shaped lamina $(0.75, 1.75)$

Hence, option (B) is correct.

5. As shown in fig. when a spherical cavity $(\text{centred at } O)$ of radius 1 is cut out of a uniform sphere of radius R $(\text{centred at } C)$, the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation



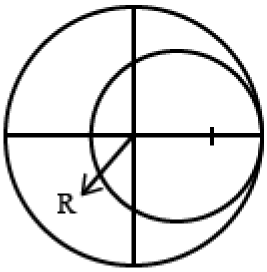
A. $(R^2 + R + 1)\,(2 - R) = 1$

B. $(R^2 - R - 1)\,(2 - R) = 1$

C. $(R^2 - R + 1)\,(2 - R) = 1$

D. $(R^2 + R - 1)\,(2 - R) = 1$

Mass of sphere = volume of sphere x density of sphere $= \frac{4}{3}\pi R^3 \rho$



Mass of cavity

$$M_{cavity} = \frac{4}{3}\pi (1)^3 \rho$$

Mass of remaining

$$M_{(Remaining)} = \frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi (1)^3 \rho$$

Centre of mass of remaining part(considering cavinty as negative amss and applying superposition),

$$X_{COM} = \frac{M_1 r_1 + M_2 r_2}{M_1 + M_2}$$

$$\Rightarrow -(2-R) = \frac{\left[\frac{4}{3}\pi R^3 \rho\right] 0 + \left[\frac{4}{3}\pi (1)^3 (-\rho)\right] [R-1]}{\frac{4}{3}\pi R^3 \rho + \frac{4}{3}\pi (1)^3 (-\rho)}$$

$$\Rightarrow \frac{(R-1)}{(R^3-1)} = 2-R$$

$$\Rightarrow \frac{(R-1)}{(R-1)(R^2+R+1)} = 2-R$$

$$\Rightarrow (R^2+R+1)(2-R) = 1$$

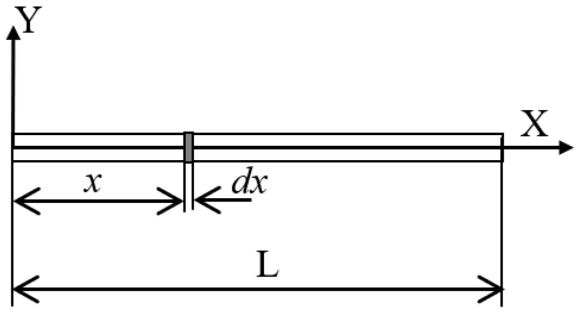
6. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{a}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at:

A. $\frac{3}{2}\left(\frac{a+b}{2a+b}\right) L$

B. $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right) L$

C. $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right) L$

D. $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right) L$



Linear mass density, $\rho(x) = a + b\left(\frac{x}{L}\right)^2$

We know that, $X_{CM} = \frac{\int x dm}{\int dm}$

Now, $\int dm = \int_0^L \rho(x) dx$

$= \int_0^L \left[a + b\left(\frac{x}{L}\right)^2\right] dx = aL + \frac{bL}{3}$

So, $\int_0^L x dm = \int_0^L \left(ax + \frac{bx^3}{L^2}\right) dx$

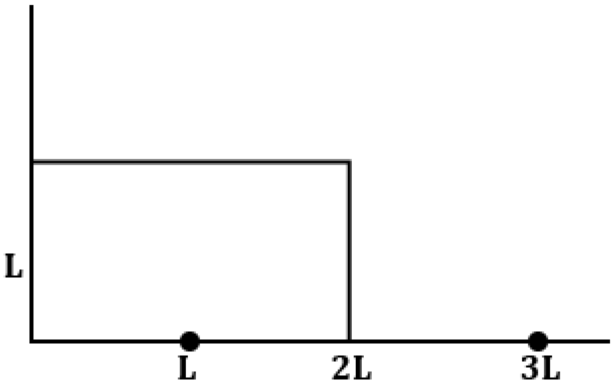
$= \left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)$

$\therefore X_{CM} = \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)}{aL + \frac{bL}{3}}$

$\Rightarrow X_{CM} = \frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$

Hence, (B) is the correct answer.

7. The position vector of the centre of mass r_{cm} of an asymmetric uniform bar of negligible area of cross - section as shown in figure is:

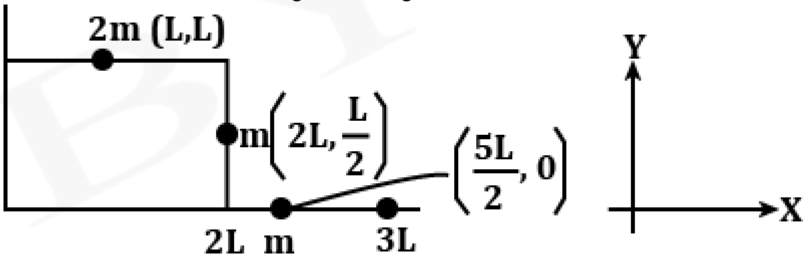


A. $\vec{r}_{\text{cm}} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

B. $\vec{r}_{\text{cm}} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

C. $\vec{r}_{\text{cm}} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

D. $\vec{r}_{\text{cm}} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$



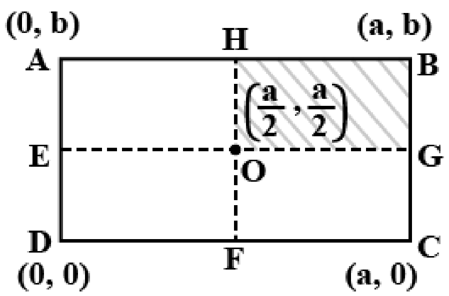
$x-$ coordinate of centre of mass is,

$$X_{\text{cm}} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m} = \frac{13}{8}L$$

$y-$ coordinate of centre of mass is,

$$Y_{\text{cm}} = \frac{2mL + m\left(\frac{L}{2}\right) + m \times 0}{4m} = \frac{5L}{8}$$

8. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be:



A. $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

B. $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

C. $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

D. $\left(\frac{5a}{12}, \frac{5b}{12}\right)$ ✔

With respect to point O, the CM of the cut-off potion will have coordinates, $\left(\frac{a}{4}, \frac{b}{4}\right)$

$\therefore$ With respect to point D, the CM of the cut-off potion will have coordinates, $\left[\left(\frac{a}{2}+\frac{a}{4}\right), \left(\frac{b}{2}+\frac{b}{4}\right)\right]$

So, With respect to point D, the coordinates of CM of the cut-off portion are, $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

Let the mass of the removed portion is m and mass per unit area of the sheet is σ.

$$\Rightarrow \sigma = \frac{M}{ab} = \frac{m}{\frac{a}{2} \times \frac{b}{2}}$$

$$\Rightarrow m = \frac{M}{4}$$

$$\Rightarrow x_{\text{CM}} = \frac{MX - mx}{M - m}$$

$$= \frac{M \times \frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}} = \frac{5a}{12}$$

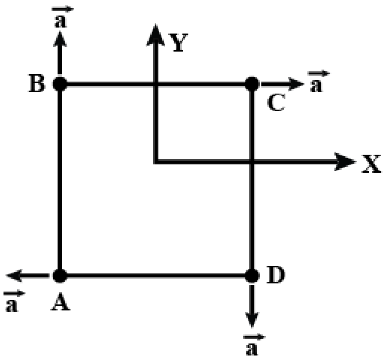
and $y_{\text{CM}} = \frac{MY - my}{M - m}$

$$= \frac{M \times \frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

So, coordinates of CM of the remaining portion are, $\frac{5a}{12}, \frac{5b}{12}$

Hence, (D) is the correct answer.

9. Four particles A, B, C and D with masses $m_A = m, m_B = 2m, m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is

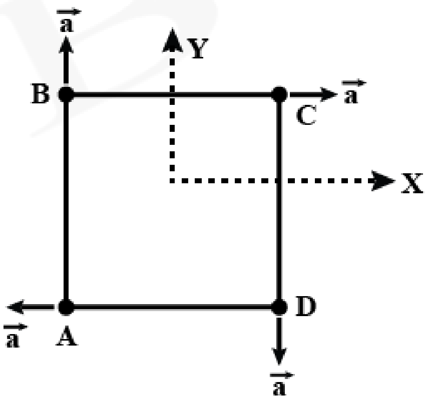


A. $\frac{a}{5}(\hat{i} - \hat{j})$

B. $\frac{a}{5}(\hat{i} + \hat{j})$

C. Zero

D. $a(\hat{i} + \hat{j})$



$\vec{a}_A = -a\hat{i}$
$\vec{a}_B = a\hat{j}$
$\vec{a}_C = a\hat{i}$
$\vec{a}_D = -a\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\vec{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m}$$

$$= \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j}$$

$$= \frac{a}{5}(\hat{i} - \hat{j}).$$

10. When a 5 V potential difference is applied across a wire of length $0.1\ m$, the drift speed of electrons is $2.5 \times 10^{-4}\ ms^{-1}$. If the electron density in the wire is $8 \times 10^{28}\ m^{-3}$, the resistivity of the material is close to:

A. $1.6 \times 10^{-8}\ \Omega m$

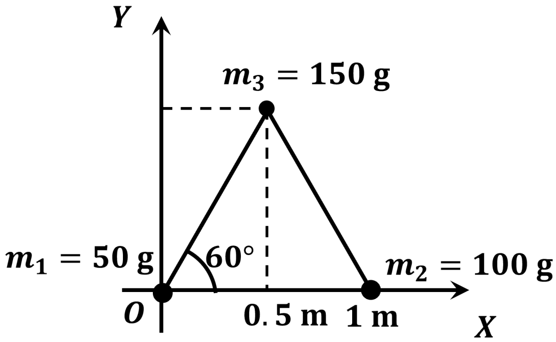
B. $1.6 \times 10^{-7}\ \Omega m$

C. $1.6 \times 10^{-6}\ \Omega m$

D. $1.6 \times 10^{-5}\ \Omega m$

We know,
$i = neAv_d$
where symbol have their usual meaning.
From Ohm's law,
$i = \frac{V}{R} = neAv_d$-------(1)
Where $R = \rho \frac{L}{A}$
Putting value of R in equation (1) we get,
$\frac{V}{\rho \frac{L}{A}} = neAv_d$
$\rho = \frac{V}{nev_d L}$
$\rho = \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$
$\rho = 1.6 \times 10^{-5}\ \Omega m$

11. Three particles of masses $50 \mathrm{~g}, 100 \mathrm{~g}$ and $150 \mathrm{~g}$ are placed at the vertices of an equilateral triangle of side $1 \mathrm{~m}$ (asshown in the figure). The (x, y) coordinates of the centre of mass will be :



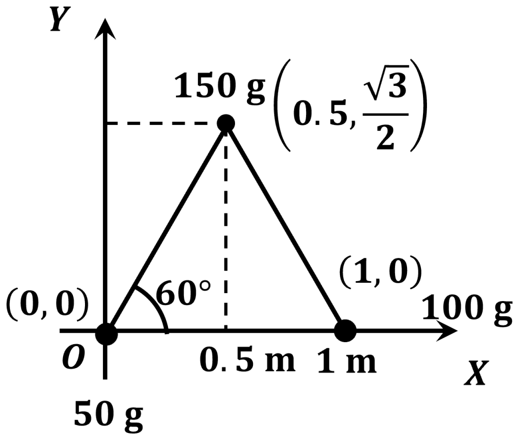
A. $\left(\frac{\sqrt{3}}{4} \mathrm{~m}, \frac{5}{12} \mathrm{~m}\right)$

B. $\left(\frac{7}{12} \mathrm{~m}, \frac{\sqrt{3}}{8} \mathrm{~m}\right)$

C. $\left(\frac{7}{12} \mathrm{~m}, \frac{\sqrt{3}}{4} \mathrm{~m}\right)$

D. $\left(\frac{\sqrt{3}}{8} \mathrm{~m}, \frac{7}{12} \mathrm{~m}\right)$

Given:
masses $m_1 = 50 \text{ g}$,
$m_2 = 100 \text{ g}$
$m_3 = 150 \text{ g}$



For the center of mass of the given system ,
Using formula

$$x_{cm} = \frac{m_1.x_1 + m_2.x_2 + m_3.x_3}{m_1 + m_2 + m_3}$$

$$= \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150}$$

$$= \frac{7}{12} \text{m}$$

Similarly,

$$y_{cm} = \frac{m_1.y_1 + m_2.y_2 + m_3.y_3}{m_1 + m_2 + m_3}$$

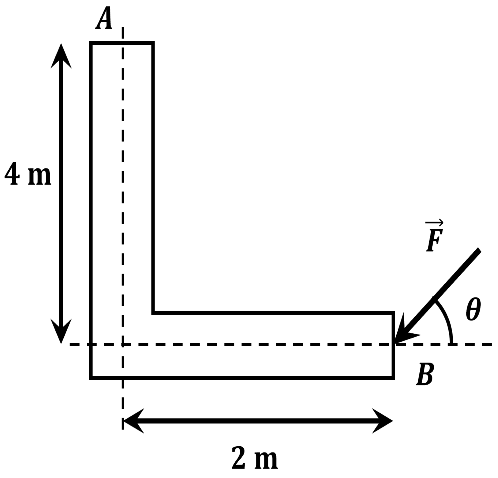
$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150}$$

$$= \frac{\sqrt{3}}{4} \text{m}$$

Hence the coordinates $(x,\ y)$ are $\left(\frac{7}{12}\text{m},\ \frac{\sqrt{3}}{4}\text{m}\right)$

Hence, option (C) is correct.

12. A force of $40\ \mathrm{N}$ acts on a point B at the end of an $L-$shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by:



A. $\tan\theta = \frac{1}{4}$

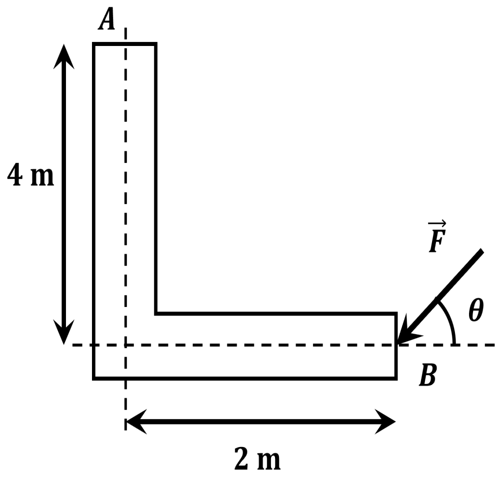
B. $\tan\theta = 2$

C. $\tan\theta = \frac{1}{2}$

D. $\tan\theta = 4$

Given:

Force $= 40\ \mathrm{N}$



To produce maximum moment of force line of action of force must be perpendicular to line AB.

$\therefore \tan\theta = \frac{2}{4}$

$\Rightarrow \tan\theta = \frac{1}{2}$

Hence, option (C) is correct.

13. In a physical balance working on the principle of moments, when $5\ \mathrm{mg}$ weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?

A. Left arm is longer than the right arm

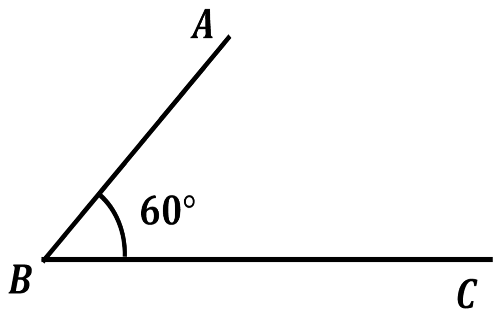
B. Both the arms are of same length

C. Left arm is shorter than the right arm

D. Every object that is weighed using this balance appears lighter than its actual weight.

According to principle of moments when a system is stable or balance, the anti-clockwise moment is equal to
clockwise moment.
i.e., load $\times$ load arm $=$ effort $\times$ effort arm
When $5\ \mathrm{mg}$ weight is placed, load arm shifts to left side,
hence left arm becomes shorter than right arm.
Hence, option (C) is correct.

14. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to :



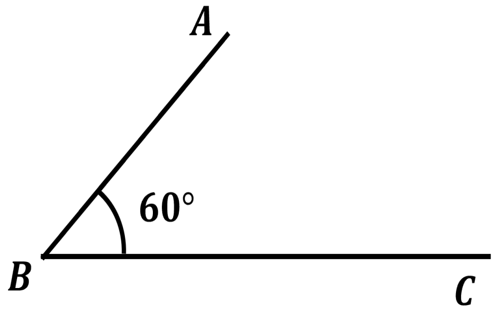
A. 1.85

B. 1.5

C. 1.37

D. 3

Given:
As the center of mass lies below the point A,
So, $X_{cm} = AB\cos 60^\circ = \frac{AB}{2}$
Let $AB = x,\ BC = y$



Using formula of center of mass

$$x_{cm} = \frac{m_1.x_1 + m_2.x_2}{m_1 + m_2}$$

$$\frac{x}{2} = \frac{(\rho x).\left(\frac{x}{2}.\cos 60^\circ\right) + (\rho y).\frac{y}{2}}{\rho(x+y)}$$

$$\frac{x}{2}(x+y) = \frac{x^2}{4} + \frac{y^2}{2}$$

$$\frac{x^2}{4} + \frac{xy}{2} = \frac{y^2}{2}$$

dividing by $\frac{x^2}{2}$ on both sides,

$$\frac{1}{2} + \frac{y}{x} = \frac{y^2}{x^2}$$

Now, calculating the value of $\frac{y}{x}$ using

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A},$$

$$\frac{y}{x} = \frac{1 \pm \sqrt{(-1)^2 - 4\times 1\times(-1/2)}}{2\times 1},$$

$$\frac{y}{x} = \frac{1+\sqrt{3}}{2}$$

$$\frac{y}{x} = 1.37$$

15. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right) L$ from A, then a and b are related as :

A. $a = 2b$

B. $2a = b$

C. $a = b$

D. $3a = 2b$

Centre of mass of the rod is given by:

$$x_{cm} = \frac{\int_0^L \left(ax + \frac{bx^2}{L}\right) dx}{\int_0^L \left(a + \frac{bx}{L}\right) dx}$$

$$= \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{3}\right)}{\left(aL + \frac{bL}{2}\right)} = L.\frac{\left(\frac{a}{2} + \frac{b}{3}\right)}{\left(a + \frac{b}{2}\right)}$$

$$\Rightarrow \frac{7L}{12} = L.\frac{\left(\frac{a}{2} + \frac{b}{3}\right)}{\left(a + \frac{b}{2}\right)}$$

$$7a + \frac{7}{2}b = 6a + 4b \Rightarrow a = 4b - \frac{7}{2}b$$

$$\Rightarrow b = 2a$$

Hence, option (B) is correct.

16. A thin bar of length L has a mass per unit length λ, that increases linearly with distance form one end. If its total mass is M and its mass per unit legth at the lighter end is λ_0, then the distance of the center of mass from the lighter end is:

A. $\frac{L}{2} - \frac{\lambda_0 L^2}{4M}$

B. $\frac{L}{3} + \frac{\lambda_0 L^2}{8M}$

C. $\frac{L}{3} + \frac{\lambda_0 L^2}{4M}$

D. $\frac{2L}{3} - \frac{\lambda_0 L^2}{6M}$

Let the bar be oriented along the $x-$ axis with its lighter end at the origin.
We have the mass per unit length as

$$\lambda(x)=\lambda_0+ax$$

So mass,

$$M=\int_0^L \lambda(x)dx=\int_0^L(\lambda_0+ax)dx$$
$$=\left[\lambda_0 x+\frac{ax^2}{2}\right]_0^L=\lambda_0 L+\frac{aL^2}{2}$$

or $a=\dfrac{2(M-\lambda_0 L)}{L^2}$

Thus we ge the center of mass $\bar{x}$ as

$$\bar{x}=\frac{\int_0^L x\lambda(x)dx}{M}=\frac{\int_0^L x(\lambda_0+ax)dx}{M}$$
$$=\frac{\left[\frac{\lambda_0 x^2}{2}+\frac{ax^3}{3}\right]_0^L}{M}$$
$$=\frac{\left(\frac{\lambda_0 L^2}{2}+\frac{aL^3}{3}\right)}{M}$$

Substitutig for a we get

$$\bar{x}=\frac{\frac{\lambda_0 L^2}{2}+\frac{2(M-\lambda_0 L)}{L^2}\times\frac{L^3}{3}}{M}$$

$$\bar{x}=\frac{2L}{3}-\frac{\lambda_0 L^2}{6M}$$

Hence, option (D) is correct.

17. A boy of mass $20 \mathrm{~kg}$ is standing on a $80 \mathrm{~kg}$ free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing $25 \mathrm{~m}$ from a wall. If he walks $10 \mathrm{~m}$ on the cart towards the wall, then the final distance of the boy from the wall will be

- **A.** $15 \mathrm{~m}$
- **B.** $12.5 \mathrm{~m}$
- **C.** $15.5 \mathrm{~m}$
- **D.** $17 \mathrm{~m}$ ✓

Given:
Mass of boy $m_1 = 20 \mathrm{~kg}$
Mass of cart $m_2 = 80 \mathrm{~kg}$
Initial distance between boy and cart is $25 \mathrm{~m}$
Distance moved by boy towards wall is $10 \mathrm{~m}$

As there is no external force, so displacement of centre of mass of the (cart + boy) system parallel to the surface is zero.

$$\therefore x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0$$

Let when the boy moves $10 \mathrm{~m}$ towards the wall, the cart moves away from the wall a distance x
So, displacement of man w.r.t. ground towards the wall is
$x_1 = 10 - x$
And the displacement of cart w.r.t. ground towards the wall is
$x_2 = -x$

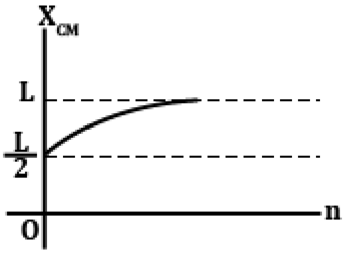
$$20 \times (10 - x) + (80 \times (-x)) = 0$$

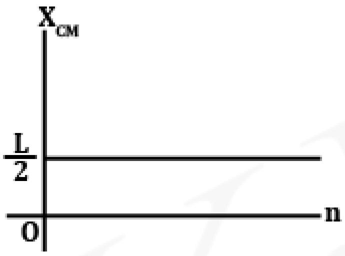
$$\Rightarrow x = 2 \mathrm{~m}$$

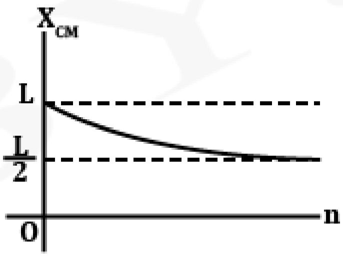
i.e. Final distance between boy and wall,
$= 25 - 10 + 2 = 17 \mathrm{~m}$

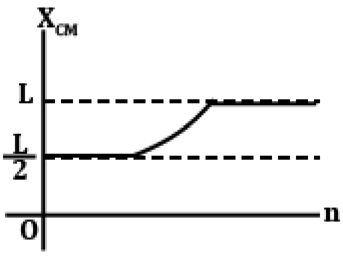
Hence, option (D) is correct.

18. A thin rod of length $'L'$ is lying along the x-axis with its ends at $x=0$ and $x=L$. Its linear density $\left(\frac{\text{mass}}{\text{length}}\right)$ varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{cm} of the center of mass of the rod is plotted against $'n'$ which of the following graphs best approximates the dependence of x_{cm} on n?

A. 

B. 

C. 

D. 

Given:

The linear mass density $\lambda = k\left(\frac{x}{L}\right)^n$

for varying mass,
we can write the equation for the center of mass as,

$$x_{CM} = \frac{\int_0^L xdm}{\int_0^L dm} = \frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda\, dx} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n xdx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{kx^{n+1}}{(n+1)L^n}\right]_0^L} = \frac{L(n+1)}{n+2}$$

For $n = 0,\ \ x_{CM} = \frac{L}{2}$,

$n = 1,\ \ x_{CM} = \frac{2L}{3}$

$n = 2\ \ x_{CM} = \frac{3L}{4}; \dots$

For $n \to \infty,\ \ x_{cm} = L$

So we can observe that the Graph A satisfies the above conditions.

Hence, option (A) is correct.

19. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The center of mass of the new disc is $\frac{\alpha}{R}$ from the center of the bigger disc. The value of α is

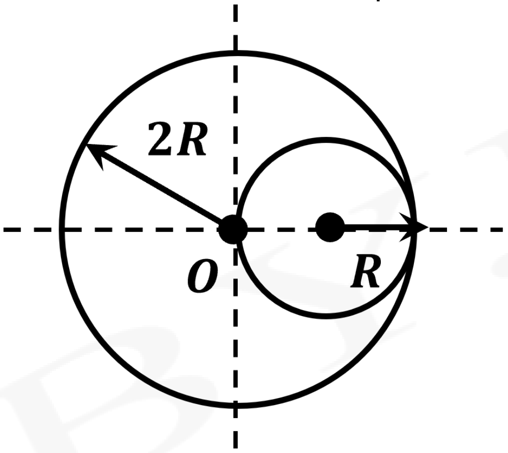
A. $\frac{1}{4}$

B. $\frac{1}{3}$

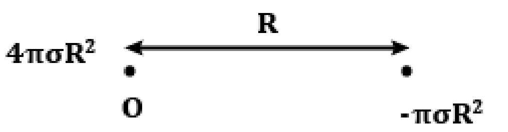
C. $\frac{1}{2}$

D. $\frac{1}{6}$

Let σ be the mass per unit area of the disc.
Then the mass of the complete disc $= \sigma(\pi(2R)^2)$



The mass of the removed disc $= \sigma(\pi R^2) = \pi\sigma R^2$
Let us consider the above situation to be a complete disc of radius $2R$ on which a disc of radius R of negative mass is superimposed.
Let O be the origin.
Then the above figure can be redrawn keeping in mind the concept of centre of mass as :



$$x_{cm} = \frac{(6\pi(2R^2)) \times 0 + (-6(\pi R^2))R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore x_{cm} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore x_{cm} = -\frac{R}{3} = \alpha R \Rightarrow \alpha = \frac{1}{3}$$

Hence, option (B) is correct.

20. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d by what distance should the second particle is moved, so as to keep the centre of mass at the same position?

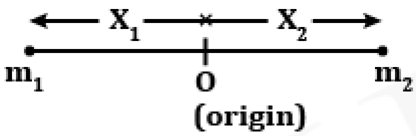
A. $\frac{m_2}{m_1}d$

B. $\frac{m_1}{m_1+m_2}d$

C. $\frac{m_1}{m_2}d$

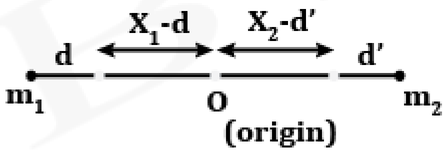
D. d

Given:
Displacement of first particle towards center of mass $= d$

Initially 

$$0=\frac{m_1(-x_1)+m_2x_2}{m_1+m_2}\Rightarrow m_1x_1=m_2x_2....(1)$$

Let second particles is displaced through distance d' towards center of mass



$$\therefore 0=\frac{-m_1(x_1-d)+m_2(x_2-d')}{m_1+m_2}$$

$$\Rightarrow 0=-m_1x_1+m_1d+m_2x_2-m_2d$$

$$\Rightarrow d'=\frac{m_1}{m_2}d$$

Hence, option (C) is correct.

21. A body A of mass M while falling vertically downwards under gravity breaks into two parts; abody B of mass $\frac{1}{3}M$ ad a body C of mass $\frac{2}{3}M$. The center of mass of bodies B and C taken together shifts compared to that of body A towards.

A. does not shift

B. depends on height of breaking

C. body B

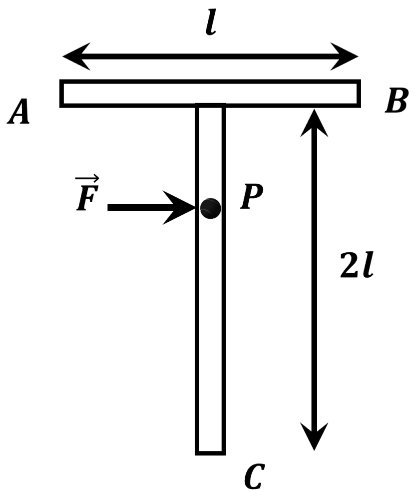
D. body C

Given:
A body A, mass $= M$
body B, mass $= \frac{1}{3}M$
body C, mass $= \frac{2}{3}M$

The centre of mass of bodies B and C taken together does not shift as no external force acts.
The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

Hence, option (A) is correct.

22. A $'T'$ shaped object with dimensions shown in figure, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.

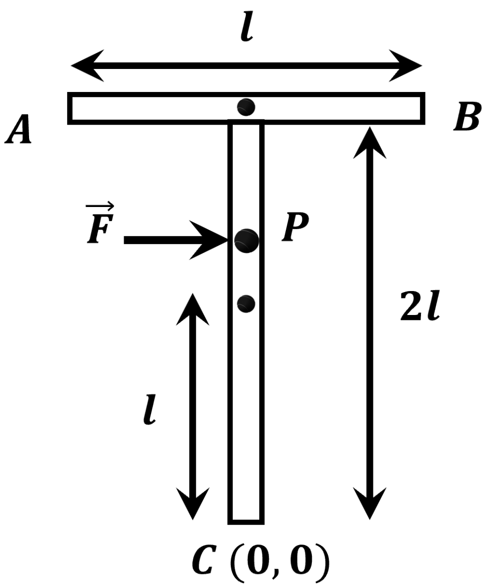


A. $\frac{3}{2}l$

B. $\frac{2}{3}l$

C. l

D. $\frac{4}{3}l$



To have translational motion without rotation, the force $\vec{F}$ has to be applied at COM that lies on $'P'$
Taking point C at the origin

Let $y_1 = 2l, y_2 = l$

and $m_1 = m$ and $m_2 = 2m$

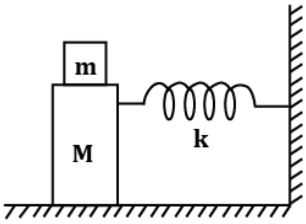
$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$
$$= \frac{m \times 2l + 2m \times l}{3m}$$

$$y_{cm} = \frac{4l}{3}$$

Hence, option (D) is correct.

Topic : Collision

1. In the given figure, a mass M is attached to a horizontal spring, which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it then the new amplitude of oscillation will be :



A. $A\sqrt{\dfrac{M}{M+m}}$

B. $A\sqrt{\dfrac{M}{M-m}}$

C. $A\sqrt{\dfrac{M-m}{M}}$

D. $A\sqrt{\dfrac{M+m}{M}}$

Before placing the mass m, angular frequency,

$$\omega_i = \sqrt{\frac{k}{M}}$$

After placing the mass m, angular frequency,

$$\omega_f = \sqrt{\frac{k}{M+m}}$$

As there is no impulsive force, so, linear momentum will remain conserved.

$$\therefore p_i = p_f$$
$$\Rightarrow M\omega_i A_i = (M+m)\omega_f A_f$$

$$\Rightarrow M \times \sqrt{\frac{k}{M}} \times A = (M+m) \times \sqrt{\frac{k}{M+m}} \times A_f$$

$$\Rightarrow A_f = A\sqrt{\frac{M}{M+m}}$$

2. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Body P having mass M moving with speed u has head-on collision elastically with another body Q having mass m initially at rest. If $m << M$, body Q will have a maximum speed equal to $2u$ after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

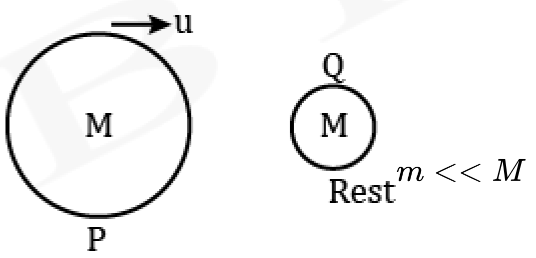
In the light of the above statements, choose the most appropriate answer from the options given below:

A. A is correct but R is not correct.

B. Both A and R are correct but R is NOT the correct explanation of A.

C. A is not correct but R is correct.

D. Both A and R are correct and R is the correct explanation of A.



$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

[Newton's Law of Restitution]

For elastatic collision, $e = 1$

$$1 = \frac{v_2 - u}{u - 0}$$

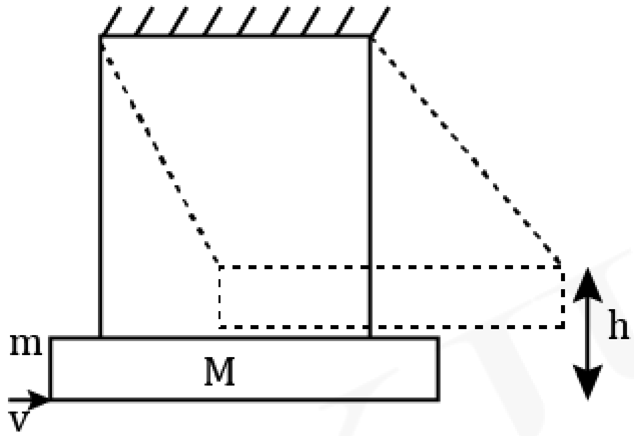
$$u = v_2 - u$$

$$v_2 = 2u$$

In elastic collision kinetic energy & momentum are conserved.

Hence, option (d) is the correct answer.

3. A large block of wood of mass $M = 5.99\ \mathrm{kg}$ is hanging from two long massless cords. A bullet of mass $m = 10\ \mathrm{g}$ is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h = 9.8\ \mathrm{cm}$ before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is: (take $g = 9.8\ \mathrm{ms^{-2}}$)



A. $831.4\ \mathrm{m/s}$

B. $841.4\ \mathrm{m/s}$

C. $811.4\ \mathrm{m/s}$

D. $821.4\ \mathrm{m/s}$

Let the spped of bullet just before collision is u and speed of (block+bullet) just after collision is v.

Using momentum conservation just before and after the collision:
$P_i = 0.01 \times u + 0 = P_f = 6 \times v$
$$v = \frac{0.01u}{6}\mathrm{m/s}$$

Using energy conservation:
$$\frac{1}{2} \times 6 \times v^2 = 6 \times g \times h$$
$$\Rightarrow \frac{1}{2} \times \left(\frac{0.01u}{6}\right)^2 = 9.8 \times 9.8 \times 10^{-2}$$
$$\Rightarrow u = 9.8 \times 6\sqrt{2} \times 10$$
$$\therefore u = 831.4\ \mathrm{m/s}$$

4. A rubber ball is released from a height of $5\ \mathrm{m}$ above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball.
(Take $g = 10\ \mathrm{m\,s^{-2}}$)

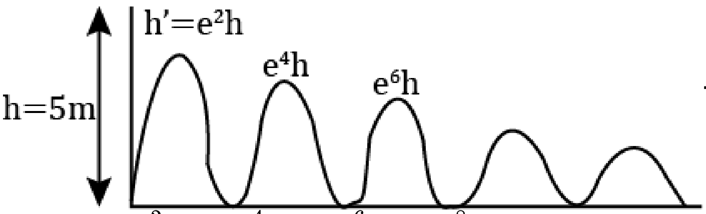
A. $2.50\ \mathrm{m\,s^{-1}}$

B. $3.50\ \mathrm{m\,s^{-1}}$

C. $3.0\ \mathrm{m\,s^{-1}}$

D. $2.0\ \mathrm{m\,s^{-1}}$

Let the situation of the ball be as shown in the below figure.



Total distance:

$$d = h + 2e^2h + 2e^4h + 2e^6h + 2e^8h + \ldots$$

$$d = h + 2e^2h(1 + e^2 + e^4 + e^6 + \ldots)$$

$$d = h + 2e^2h\left(\frac{1}{1-e^2}\right)$$

$$d = \frac{(1-e^2)h + 2e^2h}{1-e^2} = \frac{h(1+e^2)}{1-e^2}$$

Total time: $t = T + 2eT + 2e^2T + 2e^3T + \ldots$ Where $T = \sqrt{\frac{2h}{g}} = 1$

$$t = T + 2eT(1 + e + e^2 + e^3 + \ldots)$$

$$t = T + 2eT\left(\frac{1}{1-e}\right)$$

$$t = \frac{T(1+e)}{1-e}$$

Now, average speed of the ball

$$V_{avg} = \frac{d}{t} = \frac{h\dfrac{(1+e^2)}{(1-e^2)}}{T\left(\dfrac{1+e}{1-e}\right)}$$

$$V_{avg} = \frac{5}{1}\left(\frac{1+e^2}{(1+e)(1-e)}\frac{(1-e)}{(1+e)}\right)$$

$$V_{avg} = \frac{5(1+e^2)}{(1+e)^2}$$

$\because h' = e^2h$

From the question: $\frac{81}{100} = e^2$

$$e = \frac{9}{10} = 0.9$$

$$V_{avg} = \frac{5\left(1 + \frac{81}{100}\right)}{(1+0.9)^2}$$

$$V_{avg} = 2.50 \text{ m/s}$$

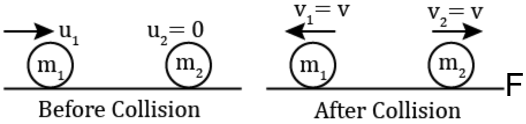
5. An object of mass m_1 collides elastically with another object of mass m_2, which is at rest. After the collision, the objects move with equal speeds in opposite directions. The ratio of the masses, $m_2 : m_1$ is -

A. $2 : 1$

B. $1 : 1$

C. $1 : 2$

D. $3 : 1$ ✓



From conservation of linear momentum;

$$p_i = p_f$$

$$\Rightarrow m_1 u_1 + m_2(0) = m_1(-v) + m_2 v$$

$$\Rightarrow m_1 u_1 = v(m_2 - m_1) \quad .....(i)$$

Also, co-efficient of restitution,

$$e = \frac{v_{\text{separation}}}{v_{\text{approach}}} = \frac{v-(-v)}{u_1} = \frac{2v}{u_1} = 1 \quad [\text{For elastic collision}]$$

$$\Rightarrow u_1 = 2v \quad .....(ii)$$

From (i) and (ii),

$$m_1(2v) = v(m_2 - m_1)$$

$$\Rightarrow 2m_1 = m_2 - m_1$$

$$\Rightarrow 3m_1 = m_2$$

$$\Rightarrow \frac{m_2}{m_1} = \frac{3}{1} = 3 : 1$$

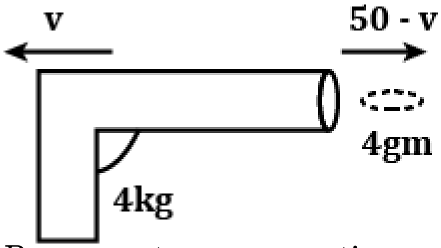
6. A bullet of $4\ \mathrm{g}$ mass is fired from a gun of mass $4\ \mathrm{kg}$. If the bullet moves with the muzzle speed of $50\ \mathrm{ms}^{-1}$, the impulse imparted to the gun and velocity of recoil of gun are :

A. $0.4\ \mathrm{kg\ ms}^{-1}, 0.1\ \mathrm{ms}^{-1}$

B. $0.2\ \mathrm{kg\ ms}^{-1}, 0.05\ \mathrm{ms}^{-1}$

C. $0.2\ \mathrm{kg\ ms}^{-1}, 0.1\ \mathrm{ms}^{-1}$

D. $0.4\ \mathrm{kg\ ms}^{-1}, 0.05\ \mathrm{ms}^{-1}$



By momentum conservation

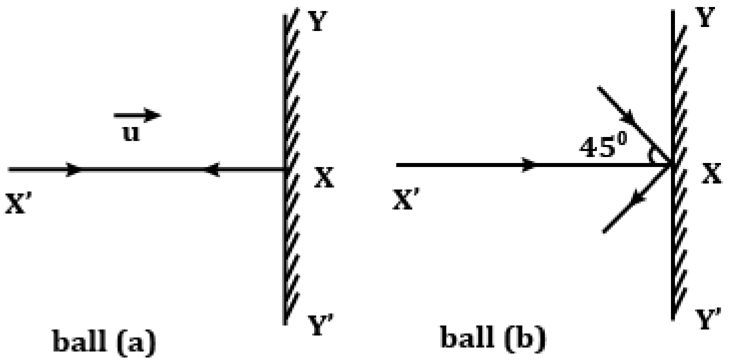
$$4 \times 10^{-3}(50 - v) - 4v = 0$$

$$\Rightarrow v = \frac{4 \times 10^{-3} \times 50}{4 + 4 \times 10^{-3}} \approx 0.05\ \mathrm{ms}^{-1}$$

Impulse $J = mv = 4 \times .05 = 0.2\ \mathrm{kg\ ms}^{-1}$

Hence, (B) is the correct answer.

7. Two billiard balls of equal mass 30 gms strike a rigid wall with the same speed of 108 kmph (as shown) , but at different angles. If the balls get reflected with the same speed, then the ratio of the magnitude of impulses imparted to ball $'a'$ and ball $'b'$ by the wall along $'X'$ direction is :



A. $1 : 1$

B. $\sqrt{2} : 1$ ✔

C. $2 : 1$

D. $1 : \sqrt{2}$

Impulse = change in momentum

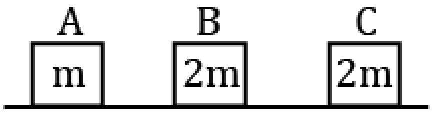
For ball (a) $J_1 = \left|\overrightarrow{\Delta P}\right| = 2mu$

For ball (b) $J_2 = \left|\overrightarrow{\Delta P}\right| = 2mu \ \cos 45^\circ$

$$\therefore \frac{J_1}{J_2} = \frac{1}{\cos 45^\circ} = \sqrt{2}$$

Hence, (B) is the correct answer.

8. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, $2\,m$ and $2\,m$ respectively. A moves towards B with a speed of $9\ \mathrm{m/s}$ and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is :



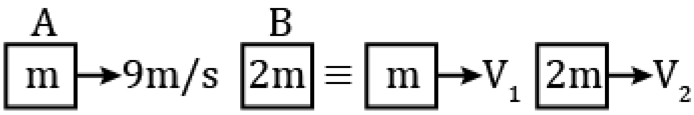
A. $6\ \mathrm{m/s}$

B. $9\ \mathrm{m/s}$

C. $4\ \mathrm{m/s}$

D. $3\ \mathrm{m/s}$

Collision between A and B is elastic, so both momentum and energy will be conserved.



Applying momentum conservation for above collision,

$$m \times 9 = mV_1 + (2m)V_2$$

$$\Rightarrow V_1 + 2V_2 = 9 \quad \ldots\ldots(1)$$

Now, since the collision between m and $2m$ is elastic, so

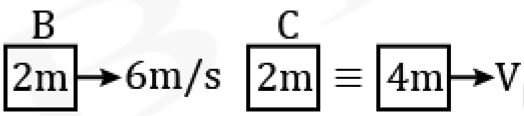
coefficient of restitution, $e = 1 = \dfrac{V_2 - V_1}{9}$

$$\Rightarrow -V_1 + V_2 = 9 \quad \ldots\ldots\ldots(2)$$

From eqs. (1) & (2), we get

$$V_2 = 6\ \mathrm{m/s}, \quad V_1 = -3\ \mathrm{m/s}$$

Collision between B and C is inelastic, so after the collision both the blocks will stick together and move with velocity V.



From momentum conservation,

$$2m \times 6 = 4mV$$

$$\therefore V = 3\ \mathrm{m/s}$$

Therefore, the correct answer is option (D).

9. A block moving horizontally on a smooth surface with a speed of $40 \mathrm{~m/s}$ splits into two parts with masses in the ratio of $1:2$ If the smaller part moves at $60 \mathrm{~m/s}$ in the same direction, then the fractional change in kinetic energy is

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Let initial mass of of block be $3m$ which breaks into two parts of masses $m, 2m$
Given that intial mass of 3m is $4 \mathrm{~m/s}$,mass of m after explosion is $4 \mathrm{~m/s}$
let mass of $2m$ after collision is v
From conservation of momentum
$3m(40) = m(60) + 2m(v)$
$v = 30 \mathrm{~m/s}$
Initial Kinetic energy is $K_i = \frac{1}{2} 3m(40)^2$
Final kinetic energy is $K_f = \frac{1}{2} m(60)^2 + \frac{1}{2} 2m(30)^2$
Fractional change in kinetic energy
$$\frac{K_f - K_i}{K_i} = \frac{60^2 + 2(30)^2 - 3(40)^2}{3(40)^2} = \frac{3600 + 1800 - 4800}{4800} = \frac{600}{4800} = \frac{1}{8}$$

10. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2\,gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is

A. $\sqrt{\frac{1}{2}}$

B. $\sqrt{\frac{3}{4}}$

C. $\frac{1}{2}$

D. $\sqrt{\frac{3}{2}}$

Let S_1 be distance travelled by particle being dropped from height h before collision
and S_2 be distance travelled by vertically projected particle before collision
Here,

$S_1 = \frac{1}{2}gt^2$

$S_2 = ut - \frac{1}{2}gt^2$

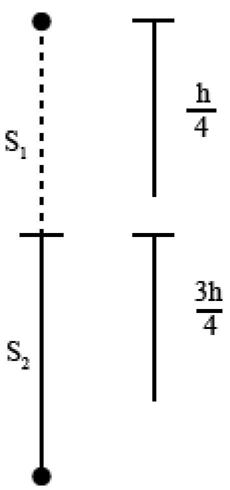
Given that $u = \sqrt{2gh}$

We know that,

$S_1 + S_2 = h$

$\sqrt{2gh}t = h$

$t = \sqrt{\frac{h}{2g}}$



Velocity of dropped particle just before collision is $v_1 = gt = \sqrt{\frac{hg}{2}}$

Velocity of projected particle just before collision is

$$v_2 = u - gt = \sqrt{2gh} - \sqrt{\frac{hg}{2}}$$

For inelastic collission, using principle of conversation of linear momentum
$mv_1 + mv_2 = 2mv_f$

$$\Rightarrow v_f = \frac{m\left(\sqrt{2gh} - \sqrt{\frac{gh}{2}}\right) - m\sqrt{\frac{gh}{2}}}{2m} = 0$$

ie after collision combined mass as zero velocity.
Distance travelled by this combined mass after collision before reaching ground is $S_2 = h - S_1 = h - \frac{h}{4} = \frac{3h}{4}$
After collison, time taken (t_1) for combined mass to reach the ground is

$$\Rightarrow \frac{3h}{4} = \frac{1}{2}gt_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{3h}{2g}}$$

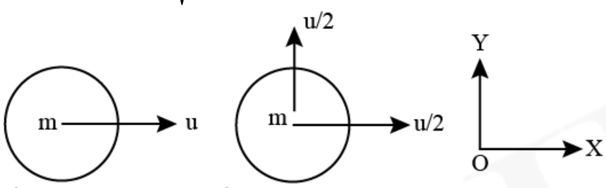
11. Two particles of equal mass m, have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is:

A. $\frac{1}{3}mu^2$

B. $\frac{1}{8}mu^2$

C. $\frac{3}{4}mu^2$

D. $\sqrt{\frac{2}{3}}mu^2$



Conservation of momentum along x-direction:

$$mu + \frac{mu}{2} = 2mv_x \Rightarrow v_x = \frac{3u}{4}$$

Conservation of momentum along y-direction:

$$0 + \frac{mu}{2} = 2mv_y \Rightarrow v_y = \frac{u}{4}$$

$$(K.E.)_i = \frac{1}{2}m\,u^2 + \frac{1}{2}m\left[\left(\frac{u}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right]$$

$$= \frac{1}{2}mu^2 + \frac{mu^2}{4} = \frac{3mu^2}{4}$$

$$(K.E.)_f = \frac{1}{2}(2m)(v_x)^2 + \frac{1}{2}(2m)(v_y)^2$$

$$= \frac{1}{2}2m\left[\left(\frac{3u}{4}\right)^2 + \left(\frac{u}{4}\right)^2\right] = \frac{5}{8}mu^2$$

$$\therefore \text{ Loss of } KE = (KE)_i - (KE)_f$$

$$= mu^2\left(\frac{6}{8} - \frac{5}{8}\right) = \frac{mu^2}{8}$$

Hence, (B) is the correct answer.

12. A particle of mass m is projected with a speed of u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

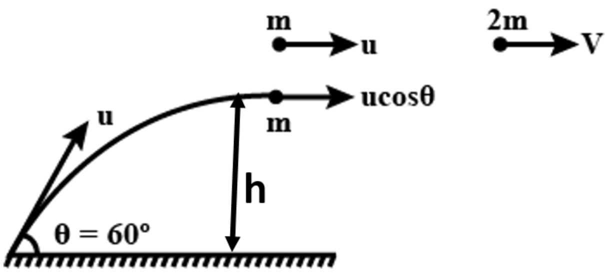
A. $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$

B. $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

C. $\frac{5}{8}\frac{u^2}{g}$

D. $2\sqrt{2}\frac{u^2}{g}$

Image for the given problem,



Using the principle of conservation of linear momentum for horizontal motion, we have

$p_i = p_f$

$mu + mu\cos 60^\circ = 2mv$

$\therefore v = \dfrac{3u}{4}$

For vertical motion,

$h = 0 + \dfrac{1}{2}gT^2 \Rightarrow T = \sqrt{\dfrac{2h}{g}}$

Using maximum height formula,

$$h = \frac{u^2 \sin^2 60^0}{2g} = \frac{u^2\left(\frac{\sqrt{3}}{2}\right)^2}{2g} = \frac{3u^2}{8g}$$

Let R is the horizontal distance travelled by the body.

$R = vT + \frac{1}{2}(0)(T)^2$ (For horizontal motion, a=0)

$= vT = \dfrac{3u}{4} \times \sqrt{\dfrac{2h}{g}}$

$$\Rightarrow R = \frac{3u}{4} \times \sqrt{\frac{2 \times \frac{3u^2}{8g}}{g}} = \frac{3\sqrt{3}u^2}{8g}$$

Hence, (A) is the correct answer.

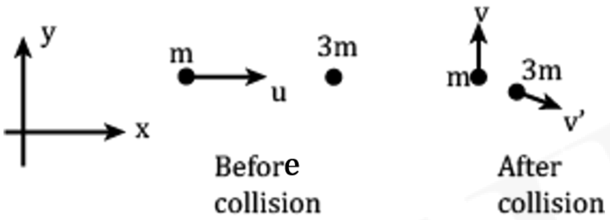
13. A particle of mass m with an initial velocity $u\hat{i}$ collides elastically with a mass $3m$ at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by :

A. $v = \sqrt{\frac{2}{3}}u$

B. $v = \frac{u}{\sqrt{3}}$

C. $v = \frac{u}{\sqrt{2}}$

D. $v = \frac{1}{\sqrt{6}}u$



From the law of conservation of linear momentum,

$$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v'}$$

$$\Rightarrow \vec{v'} = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$$

$$\Rightarrow |v'| = \sqrt{\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2}$$

As the collision is elastic, from the conservation of kinetic energy,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)\left(\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2\right)$$

$$\Rightarrow u^2 = v^2 + \frac{u^2}{3} + \frac{v^2}{3}$$

$$\therefore v = \frac{u}{\sqrt{2}}$$

Hence, option (C) is correct.

14. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 ms^{-1} in the horizontal direction just before the collision then the kinetic energy, just before the combined system strikes the floor, is [Take $g = 10 \text{ ms}^{-2}$]. Assume there is no rotational motion and losses of energy after the collision is negligible.

A. 20 J

B. 21 J

C. 19 J

D. 23 J

Given,
Mass of block, $m_1 = 1.9 \text{ kg}$
Mass of bullet, $m_2 = 0.1 \text{ kg}$
Velocity of bullet, $v_2 = 20 \text{ ms}^{-1}$

It is an inelastic collision.

Let V be the velocity of the combined system.

Using conservation of linear momentum
$m_1 \times 0 + m_2 \times v_2 = (m_1 + m_2)V$

$\Rightarrow 0.1 \times 20 = (0.1 + 1.9) \times V$

$\Rightarrow V = 1 \text{ ms}^{-1}$

Let K be the Kinetic energy of the combined system jus before striking the ground,

Using work energy theorem
$\text{Work done} = \text{Change in Kinetic energy}$

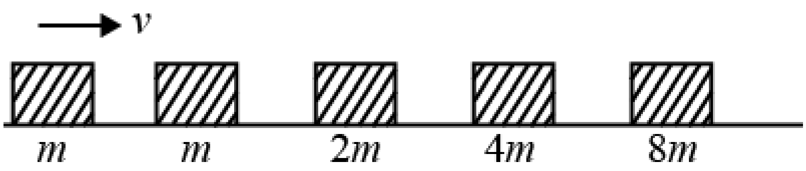
$$(m_1 + m_2)gh = K - \frac{1}{2}(m_1 + m_2)V^2$$

$$\Rightarrow 2 \times 10 \times 1 = K - \left(\frac{1}{2} \times 2 \times 1^2\right)$$

$\Rightarrow K = 21 \text{ J}$

Hence, option (B) is correct.

15. Blocks of masses $m,\ 2m,\ 4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (as shown) collides with mass m in a perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass $8m$ starts moving, the total energy loss is $p\%$ of the original energy. The value of p is close to-



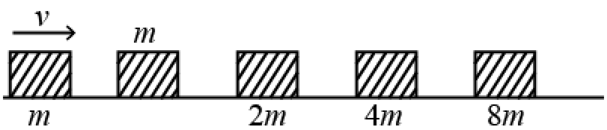
A. 77

B. 94

C. 37

D. 87

According to the question, all collisions are perfectly ineleastic, so after the final collision, all blocks are moving together.



Let the final velocity of the system be v, using momentum conservation,

$$mv = 16mv'$$

$$\Rightarrow v' = \frac{v}{16}$$

Now initial and final energy of the system is,

$$E_i = \frac{1}{2}mv^2$$

$$E_f = \frac{1}{2}m\left(\frac{v}{16}\right)^2 = \frac{1}{2}\frac{mv^2}{16}$$

Energy loss: $E_i - E_f = \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$$\Rightarrow \frac{1}{2}mv^2\left[1 - \frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$$

The total energy loss is $p\%$ of the original energy.

$$\therefore \quad p\% = \frac{\text{Energy loss}}{\text{Original energy}} \times 100$$

$$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2} \times 100 = 93.75\%$$

Hence, value of p is close to 94.

Hence, (B) is the correct answer.

16. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i}+\hat{j})\ \mathrm{ms^{-1}}$ collides with another partice B of mass m_2 which is at rest initially. Let $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\overrightarrow{V_1} = (\hat{i}+\sqrt{3}\hat{j})\mathrm{ms^{-1}}$, the angle between $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ is :

- **A.** 15°
- **B.** 60°
- **C.** -45°
- **D.** 105°

Before collision:
velocity of particle A, $u_1 = (\sqrt{3}\hat{i} + \hat{j})\ \mathrm{m/s}$

Velocity of particle B, $u_2 = 0$

After collision:
Velocity of particle A, $v_1 = (\hat{i} + \sqrt{3}\hat{j})\ \mathrm{m/s}$

Using conservation of linear momentum

$$m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2} = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}$$

$$\Rightarrow 2m_2(\sqrt{3}\hat{i} + \hat{j}) + (m_2 \times 0) = 2m_2(\hat{i} + \sqrt{3}\hat{j}) + (m_2 \times \overrightarrow{v_2})$$

$$\Rightarrow 2\sqrt{3}\hat{i} + 2\hat{j} = 2\hat{i} + 2\sqrt{3}\hat{j} + \overrightarrow{v_2}$$

$$\Rightarrow \overrightarrow{v_2} = 2(\sqrt{3}-1)\hat{i} - 2(\sqrt{3}-1)\hat{j}$$

And, $\overrightarrow{v_1} = \hat{i} + \sqrt{3}\hat{j}$

For angle between $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$,

$$\cos\theta = \frac{\overrightarrow{v_1}.\overrightarrow{v_2}}{|\overrightarrow{v_1}||\overrightarrow{v_2}|} = \frac{2(\sqrt{3}-1) - 2\sqrt{3}(\sqrt{3}-1)}{2 \times 2\sqrt{2}(\sqrt{3}-1)}$$

$$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \theta = 105^\circ$$

So, the angle between $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ is 105°.

Alternate solution:
Here, $\overrightarrow{v_1} = \hat{i} + \sqrt{3}\hat{j}$
$\overrightarrow{v_1}$ will make an agle θ with $x-$ axis.

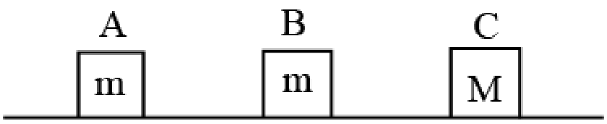
$$\therefore \theta = \tan^{-1}(\sqrt{3}/1) = 60^\circ$$

Similarly, $\overrightarrow{v_2} = 2(\sqrt{3}-1)\hat{i} - 2(\sqrt{3}-1)\hat{j}$ will make angle ϕ with $x-$ axis.

$$\therefore \phi = \tan^{-1}\left(\frac{-2(\sqrt{3}-1)}{2(\sqrt{3}-1)}\right) = -45^\circ$$

So, the angle between $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ will be $60^\circ + 45^\circ = 105^\circ$.

17. Three block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5^{th}}{6}$ of the initial kinetic energy is lost in the whole process. What is the value of $\frac{M}{m}$?



A. 5

B. 2

C. 4

D. 3

Initial Kinetic energy of block A

$$K_1 = \frac{1}{2}mv^2$$

$\therefore$ From principle of linear momentum conservation

$$mv = (2m + M)v_f$$

$$\Rightarrow\ v_f = \frac{mv}{2m + M}$$

According to question, of $\frac{5^{th}}{6}$ the initial kinetic energy is lost in the whole process.

$$\Rightarrow k_f = \frac{1}{6}k_i \Rightarrow \frac{k_i}{k_f} = 6$$

$$\Rightarrow \frac{\frac{1}{2}mv^2}{\frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2} = 6$$

$$\Rightarrow \frac{2m + M}{m} = 6$$

$$\Rightarrow \frac{M}{m} = 4$$

Hence, option (C) is correct.

18. A piece of wood of mass $0.03\ \mathrm{kg}$ is dropped from the top of a $100\ \mathrm{m}$ height building. At the same time, a bullet of mass $0.02\ \mathrm{kg}$ is fired vertically upward, with a velocity $100\ \mathrm{ms}^{-1}$, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is: $(g = 10\ \mathrm{ms}^{-2})$

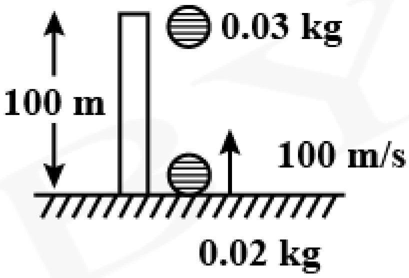
A. $20\ \mathrm{m}$

B. $30\ \mathrm{m}$

C. $40\ \mathrm{m}$

D. $10\ \mathrm{m}$

Given:
Wooden piece of mass $= 0.03\ \mathrm{kg}$
Building height $= 100\ \mathrm{m}$
Bullet mass $= 0.02\ \mathrm{kg}$
Initial velocity of bullet $= 100\ \mathrm{ms}^{-1}$,



Time taken for the particles to collide,

For wooden ball

Let h_1 be the distance which is travelled, then using equation of motion

$$h_1 = 0 + \frac{1}{2}gt^2$$

Similarly, h_2 is the distance covered by the bullet in verticxal direction

$$h_2 = 100.t - \frac{1}{2}gt^2$$

Since, total distance covered by both,

$$h_1 + h_2 = 100\ \mathrm{m}$$

$$100t = 100\ \mathrm{m} \Rightarrow t = 1\ \mathrm{s}$$

Speed of wood just before collision

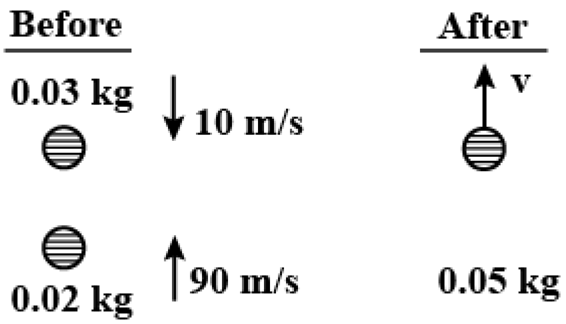
$$u_1 = 0 + g.t = 10\ \mathrm{m/s}$$

and speed of bullet just before collision

$u_2 = v - gt = 100 - 10 \times 1 = 90 \text{ m/s}$

Distance travelled by the bullet (collision point)

$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95 \text{ m}$



Now, using conservation of linear momentum just before and after the collision

$m_1 u_1 + m_2 U_2 = (m_1 + m_2) v$

$-(0.03)(10) + (0.02)(90) = (0.05)v$

$\Rightarrow 150 = 5v$

$\therefore v = 30 \text{ m/s}$

Max. height reached by body,

$v^2 = u^2 - 2gh$

$\Rightarrow h = \frac{v^2}{2g} = \frac{30 \times 30}{2 \times 10} = 45 \text{ m}$

$\therefore$ Height above tower $45 - 5 = 40 \text{ m}$

Hence, option (C) is correct.

19. An alpha-particle of mass m suffers $1-$dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is:

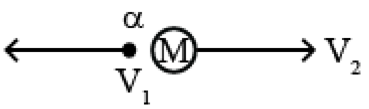
A. $2m$

B. $3.5\ m$

C. $1.5\ m$

D. $4\ m$ ✔

After collision



Using conservation of momentum

$mV_0 = MV_2 - mV_1 \quad ---(1)$

Given that, the mass m loses 64% of its kinetic energy,

$\Rightarrow \frac{1}{2}mV_1^2 = 0.36 \times \frac{1}{2}mV_0^2$

$\Rightarrow V_1 = 0.6V_0$

As the collision is elastic, the 64% kinetic energy losy by m is transferred to M,

$\Rightarrow \frac{1}{2}MV_2^2 = 0.64 \times \frac{1}{2}mV_0^2$

$\Rightarrow V_2 = \sqrt{\frac{m}{M}} \times 0.8V_0$

Substituting the values of V_1 and V_2 in (1)

$\Rightarrow mV_0 = M\sqrt{\frac{m}{M}} \times 0.8V_0 - m \times 0.6V_0$

$\Rightarrow 1.6m = 0.8\sqrt{mM}$

$\Rightarrow 4m^2 = mM$

$\therefore M = 4m$

Hence, option (D) is correct.

20. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.
If $m_2 = 0.5\ \mathrm{m}_1$ and $v_3 = 0.5v_1$ then v_1 is equal to,

A. $v_4 - \frac{v_2}{2}$

B. $v_4 - v_2$

C. $v_4 - \frac{v_2}{4}$

D. $v_4 + v_2$

According to conservation of momentum principle,

$$m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$$

$$\Rightarrow m_1v_1 + (0.5m_1)v_2 = m_1(0.5v_1) + (0.5m_1)v_4$$

$$\Rightarrow v_1 + 0.5v_2 = 0.5v_1 + 0.5v_4$$

$$\therefore v_1 = v_4 - v_2$$

Hence, (B) is the correct answer.

21. A body of mass $2 \mathrm{~kg}$ makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body ?

A. $1 \mathrm{~kg}$

B. $1.5 \mathrm{~kg}$

C. $1.8 \mathrm{~kg}$

D. $1.2 \mathrm{~kg}$

Given:
$m_1 = 2\text{ kg},$
$u_1 = u,$
$u_2 = 0,$
$v_1 = u/4$

By conservation of linear momentum
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$2u + 0 = 2 \times \frac{u}{4} + m_2v_2$
$m_2v_2 = \frac{3u}{2}$

For elastic collision
$v_{\text{seperation}} = v_{\text{approach}}$
$v_2 - \frac{u}{4} = u$
$\Rightarrow v_2 = \frac{5u}{4}$

Now,
$m_2 \times \frac{5u}{4} = \frac{3u}{2}$
$\Rightarrow m_2 = 1.2\text{ kg}$

Alternate Solution:
For head on elastic collision we have
$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$
Here
$m_1 = 2\text{ kg},$
$u_1 = u,$
$u_2 = 0,$
$v_1 = u/4$

$\therefore \frac{u}{4} = \frac{(2 - m_2)u}{2 + m_2}$
$\Rightarrow m_2 = 1.2\text{ kg}$

Hence option (D) is correct.

22. A particle of mass m is moving with speed $2v$ and collides with a mass $2m$ moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be-

A. $\sqrt{2}v$

B. $2\sqrt{2}v$

C. $\frac{v}{2\sqrt{2}}$

D. $\frac{v}{\sqrt{2}}$

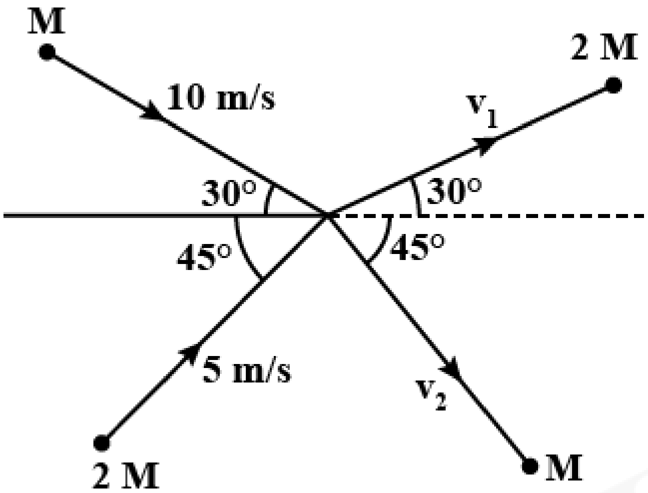
Using, conservation of linear momentum

$$m(2v) + 2mv = 0 + 2mv' \cos 45^\circ$$

$$\Rightarrow \quad v' = 2\sqrt{2}v$$

Hence, (B) is the correct answer.

23. Two particles, of masses M and $2M$, moving, as shown, with speeds of $10\ \mathrm{m/s}$ and $5\ \mathrm{m/s}$, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly



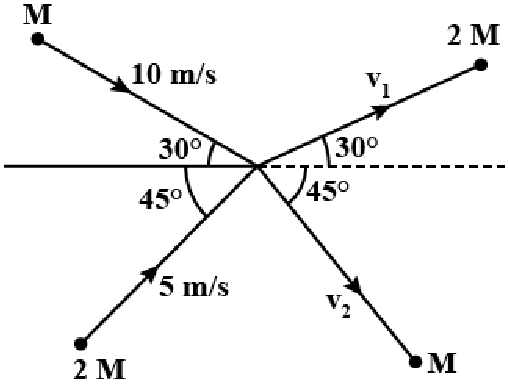
A. $6.5\ \mathrm{m/s}$ and $6.3\ \mathrm{m/s}$

B. $3.2\ \mathrm{m/s}$ and $6.3\ \mathrm{m/s}$

C. $6.5\ \mathrm{m/s}$ and $3.2\ \mathrm{m/s}$

D. $3.2\ \mathrm{m/s}$ and $12.6\ \mathrm{m/s}$

Given:
Speed of mass M particle $= 10 \text{ m/s}$
Speed of mass $2M$ particle $= 5 \text{ m/s}$



Applying conservation of linear momentum in X and Y direction for the system then

$$M(10\cos 30^\circ) + 2M(5\cos 45^\circ) = 2M(v_1\cos 30^\circ) + M(v_2\cos 45^\circ)$$

$$5\sqrt{3} + 5\sqrt{2} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}} \quad \text{... } (1)$$

Also

$$2M(5\sin 45^\circ) - M(10\sin 30^\circ) = 2Mv_1\sin 30^\circ - Mv_2\sin 45^\circ$$

$$5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \quad \text{... } (2)$$

Solving equation (1) and (2)

$$(\sqrt{3}+1)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5$$

$$\Rightarrow v_1 = 6.5 \text{ m/s}$$

$$\Rightarrow v_2 = 6.3 \text{ m/s}$$

Hence option (A) is correct.

24. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

A. 50%

B. 56%

C. 62%

D. 44%

As the collision is inelastic the masses will move together
Assuming the speed of block A and B becomes $v_1\hat{i} + v_2\hat{j}$
Writing momentum equation in x direction

$m(2v) + 2m(0) = 3mv_1 \Rightarrow v_1 = \frac{2v}{3}$

Writing momentum equation in Y direction

$m(0) + 2m(v) = 3mv_2 \Rightarrow v_2 = \frac{2v}{3}$

The velocity of both blocks will be $\frac{2v}{3}\hat{i} + \frac{2v}{3}\hat{j}$

So,

$E_{initial} = \frac{1}{2}m(2v)^2 + \frac{1}{2}2m(v)^2 = 3mv^2$

$E_{final} = \frac{1}{2}3m(\frac{4}{9}v^2 + \frac{4}{9}v^2) = \frac{4}{3}mv^2$

Therefore the fractional loss is

$$\frac{E_{initial} - E_{final}}{E_{initial}} \times 100 = \frac{3 - \frac{4}{3}}{3} \times 100$$

$$= \frac{5}{9} \times 100$$
$$= 56\%$$

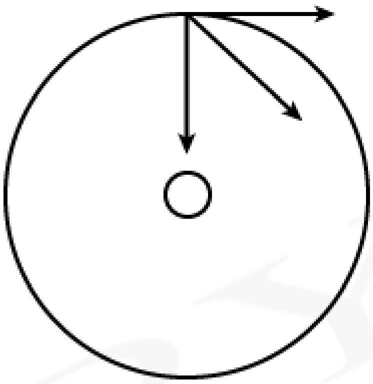
25. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be

A. such that it escape to infinity

B. In an elliptical orbit

C. in the same circular orbit of radius R

D. in a circular orbit of a different radius



$$mv\hat{i} + mv(-\hat{j}) = 2m\vec{v}$$
$$\Rightarrow \vec{v} = \frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$$

$$\Rightarrow \vec{v} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$$

It shows the orbit after the collision is an elliptical.

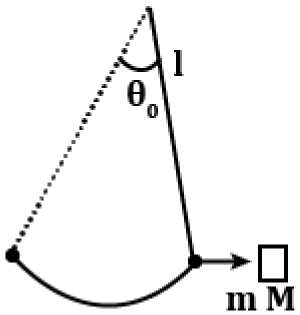
26. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. The M is given by :

A. $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

B. $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

C. $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

D. $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$



Applying conservation of energy theorem,

Velocity before colision,

$$v=\sqrt{2gl(1-cos\theta_0)}$$

And, velocity after colision,

$$v_1=\sqrt{2gl(1-\cos\theta_0)}$$

Using momentum conservation,

$$mv=Mv_m-mv_i$$

$$m\sqrt{2gl(1-\cos\theta_0)}=Mv_m-m\sqrt{2gl(1-\cos\theta)}$$

$$\Rightarrow m\sqrt{2gl}\left\{\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right\}=Mv_m$$

and $e=1=\dfrac{v_m+\sqrt{2gl(1-\cos\theta_1)}}{\sqrt{2gl(1-\cos\theta_0)}}$

$$\sqrt{2gl}\left(\sqrt{1-\cos\theta_0-\sqrt{1-\cos\theta_1}}\right)=v_m...(i)$$

$$m\sqrt{2gl}\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)=Mv_M...(ii)$$

Dividing (ii) by (i) we get

$$\frac{\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right)}=\frac{M}{m}$$

By componendo and dividendo rule,

$$\frac{m-M}{m+M}=\frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}}=\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow\frac{M}{m}=\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\Rightarrow M=m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$$