

Practice Challenge - Objective

Subject: Mathematics

Topic : Arithmetic Progressions

Exam Prep 1

Class: X

1. Which of the following sequences form an AP?

(i) 2, 4, 8, 16.....

(ii) 2, 3, 5, 7, 11.....

(iii) -1, -1.25, -1.5, -1.75.....

(iv) 1, -1, -3, -5.....

☐ A. (i) and (iv)

☐ B. (ii) and (iv)

☒ C. (iii) and (iv)

☐ D. (i),(iii) and (iv)

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Consider each list of numbers:

(i) 2, 4, 8, 16.....

Difference between the first two terms = $4 - 2 = 2$

Difference between the third and second term = $8 - 4 = 4$

Since $a_2 - a_1 \neq a_3 - a_2$, this sequence is not an AP.

(ii) 2, 3, 5, 7, 11.....

Difference between the first two terms = $3 - 2 = 1$

Difference between the third and second term = $5 - 3 = 2$

Since $a_2 - a_1 \neq a_3 - a_2$, this sequence is not an AP.

(iii) -1, -1.25, -1.5, -1.75.....

Difference between the first two terms = $-1.25 - (-1) = -0.25$

Difference between the third and second term = $-1.5 - (-1.25) = -0.25$

Difference between the fourth and third term = $-1.75 - (-1.5) = -0.25$

Since $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$, this sequence is an AP.

(iv) 1, -1, -3, -5.....

Difference between the first two terms = $-1 - 1 = -2$

Difference between the third and second term = $-3 - (-1) = -2$

Difference between the fourth and third term = $-5 - (-3) = -2$

Since $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$, this sequence is an AP

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2. A player who was playing video game was given 20 coins to begin with the game. To go to the next level, he needs to spend 4 coins and if he succeeds the particular level, he earns 6 coins. Find the number of coins he collects after clearing each level (assuming he clears every level).

☐ A. 20, 25, 30, 35, 36.....

☒ B. 20, 22, 24, 26, 28.....

☐ C. 20, 24, 28, 30, 34.....

☐ D. 20, 26, 32, 38, 44.....

Initial coins the player has is 20 coins

$$\begin{aligned}\text{Net no. of coins he gain after clearing a level} &= \text{coins gained} - \text{coins spent} \\ &= 6 - 4 \\ &= 2\end{aligned}$$

$$\therefore \text{Coins he has after clearing first level} = 20 + 2 = 22$$

$$\text{No. of coins after clearing second level} = 22 + 2 = 24$$

$$\text{No. of coins after clearing third level} = 24 + 2 = 26$$

So, the series of coins after clearing each level is 20, 22, 24, 26, 28.....

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3. Find the 20th term of the AP 2, 5, 8, 11, 14 ,...

☐ A. 57

☐ B. 58

☒ C. 59

☐ D. 60

2, 5, 8, 11, 14 ,...

$\therefore n^{\text{th}}$ term of an AP is $a + (n-1)d$

Here, $a = 2$ & $d = 8-5 = 3$

$$\begin{aligned}\therefore 20^{\text{th}} \text{ term} &= 2 + (20-1)3 \\ &= 2 + (19 \times 3) \\ &= 59\end{aligned}$$

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4. An arithmetic sequence has 6^{th} term as 52 and 15^{th} term as 142. Find a & d

☒ A. 2, 10

☐ B. 10, 2

☐ C. 2, 20

☐ D. 20, 2

Given $a_6 = a + (6 - 1)d = 52$

$a + 5d = 52$ -----(i)

and $a_{15} = a + (15 - 1)d = 142$

$a + 14d = 142$ -----(ii)

equation (ii) – equation (i) $\Rightarrow 9d = 90$

$d = 10$

Substituting $d = 10$ in (i)

$a + 50 = 52$

$a = 2$

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5. Does the sequence of odd numbers form an AP?

- ☒ A. Yes, with a common difference of 1.
- ☒ B. No
- ☒ C. Yes, with a common difference of 2.
- ☒ D. Yes, with a common difference of -1.

Odd numbers are 1,3,5,7,...

First term = 1.

Second term = 3

difference = $3 - 1 = 2$

Similarly, $5 - 3 = 7 - 5 = 2$

Observe that the difference between any two consecutive terms is 2 .

Hence it forms an AP.

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6. A twenty storied building was observed from outside, each floor has a kite shaped figure which gets on magnifying as observed from the 1st to 20th floor. The area of first five kites on each floor was calculated and they were found to develop a pattern. These five areas are $10m^2$, $15m^2$, $20m^2$, $25m^2$, $30m^2$ and so on. The area of the kite on the top floor is ____.

- ☐ A. $100m^2$
- ☒ B. $105m^2$
- ☐ C. $110m^2$
- ☐ D. $200m^2$

The sequence of the areas on each floor forms an Arithmetic Progression, with first term as 10 and common difference as 5.

$$\begin{aligned} \text{General term } T_n &= a + (n - 1)d \\ &= 10 + (n - 1)5 \end{aligned}$$

$$\text{We require, } T_{20} = 10 + (20 - 1)5 = 10 + 19 \times 5 = 105$$

7. If the first term of an AP is -8 and the common difference is 4, then the sum of the first ten terms is

- ☐ A. 92
- ☐ B. 96
- ☒ C. 100
- ☐ D. 104

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}(2(-8) + 9(4))$$

$$S_{10} = 100$$

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8. The sides of a right triangle are in an AP. The area and the perimeter of the triangle are numerically equal. Find its perimeter.

- ☒ A. 24 units
- ☐ B. 34 units
- ☐ C. 44 units
- ☐ D. 66 units

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Let us take the sides of the triangle to be $(a - d)$, a and $(a + d)$, with a and d both positive. We assume this so that we have positive values for the sides.

$$\text{Perimeter } P = (a - d) + a + (a + d) = 3a$$

Since the triangle is right angled, $(a + d)$ is the hypotenuse as it is the largest side.

So, by Pythagoras theorem

$$(a + d)^2 = a^2 + (a - d)^2$$

Simplifying the equation, we get $2ad = a^2 - 2ad$

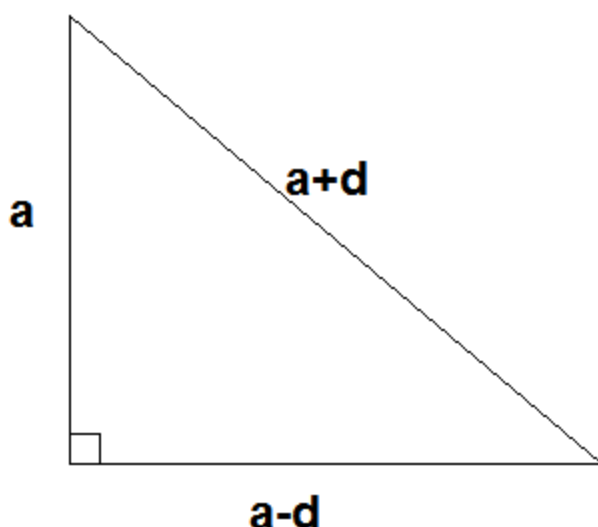
$$\Rightarrow a^2 - 4ad = 0$$

$$\Rightarrow a(a - 4d) = 0$$

$$\Rightarrow a = 0 \text{ or } a = 4d$$

we cannot take $a = 0$ as side of a triangle cannot be zero.

Since $(a + d)$ is the hypotenuse, the other sides a and $(a - d)$ form the base and altitude of the right angled triangle, as shown below:



So, area

$$= \frac{1}{2} \times a \times (a - d)$$

$$= \frac{1}{2} \times 4d \times (4d - d)$$

$$= 6d^2$$

Given that area and perimeter are numerically equal,

$$\Rightarrow 3a = 6d^2$$

We have $a = 4d$ and hence

$$12d = 6d^2$$

$$\Rightarrow 6d^2 - 12d = 0$$

$$\Rightarrow d(6d - 12) = 0$$

$$\Rightarrow d = 0 \text{ or } d = 2$$

(we cannot take $d = 0$, because it will make all sides equal)

Hence, $d = 2$ and $a = 4d = 8$

Therefore, the sides are 6, 8 and 10

and perimeter = 24 units

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9. The sum of 'n' terms of a finite AP is $\frac{13}{2}$ times the sum of its first and last terms. Which term would be the middle term in this AP?

- ☐ A. 3rd term
- ☐ B. 5th term
- ☒ C. 7th term
- ☐ D. 9th term

For a finite AP having first term as a and last term as l , the formula for the sum of n terms would become $\frac{n}{2}(a + l) = \frac{n}{2}(\text{first term} + \text{last term})$.

Now, in the question it is given that the sum of n terms is $\frac{13}{2}$ times the sum of its first and last terms, which means,
 $n = 13$

Since, the AP has 13 terms.

So, the 7th term is the middle term.

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10. If 8 times the 8th term of an AP is equal to 15 times the 15th term of the AP, then the 23rd term of the AP is ____.

☐ A. 144

☐ B. 1

☒ C. 0

☐ D. 8

Given,

$$8a_8 = 15a_{15}.$$

Since the n^{th} term of an AP of first term a and common difference d is given by

$$a_n = a + (n - 1)d, \text{ we have}$$

$$8[a + (8 - 1)d] = 15[a + (15 - 1)d].$$

$$\Rightarrow 8(a + 7d) = 15(a + 14d)$$

$$\Rightarrow 8a + 56d = 15a + 210d$$

$$\Rightarrow 7a + 154d = 0$$

$$\Rightarrow a + 22d = 0$$

$$\Rightarrow a + (23 - 1)d = 0$$

Hence, the 23rd term of the AP is zero.