1. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first \( n \) terms.
It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, \( S_n = \frac{n}{2}(2a + (n-1)d) \) to find sum of n terms of AP, we get

\[
49 = \frac{7}{2}(2a + (7-1)d)
\]

\[
\Rightarrow 98 = 7(2a + 6d)
\]

\[
\Rightarrow 98 = 14a + 42d
\]

\[
\Rightarrow 7 = a + 3d \tag{1}
\]

And, \( 289 = \frac{17}{2}(2a + (17-1)d) \)

\[
\Rightarrow 578 = 17(2a + 16d)
\]

\[
\Rightarrow 34 = 2a + 16d
\]

\[
\Rightarrow 17 = a + 8d
\]

Putting equation (1) in the above equation, we get

\[
17 = 7 - 3d + 8d
\]

\[
\Rightarrow 10 = 5d
\]

\[
\Rightarrow d = \frac{10}{5} = 2
\]

Putting value of \( d \) in equation (1), we get

\[
a = 7 - 3d = 7 - 3 (2) = 7 - 6 = 1
\]

Again applying formula, \( S_n = \frac{n}{2}(2a + (n-1)d) \) to find sum of n terms of AP, we get

\[
S_n = \frac{n}{2}[2(1) + (n-1)(2)]
\]

\[
\Rightarrow S_n = \frac{n}{2}[2 + 2n - 2]
\]

\[
\Rightarrow S_n = \frac{n}{2}[2n]
\]

\[
\Rightarrow S_n = n^2
\]

Therefore, sum of n terms of AP is equal to \( n^2 \)
2. Check whether -150 is a term of the AP: 11, 8, 5, 2...

Let -150 is the \( n \)th of AP 11, 8, 5, 2... Which means that \( a_n = -150 \)

Here, First term = \( a = 11 \)

Common difference = \( d = 8 - 11 = -3 \)

Using formula \( a_n = a + (n-1)d \), to find nth term of arithmetic progression, we get

\[-150 = 11 + (n-1)(-3)\]

\[\Rightarrow -150 = 11 - 3n + 3\]

\[\Rightarrow 3n = 164\]

\[\Rightarrow n = \frac{164}{3}\]

But, \( n \) cannot be in fraction. Therefore, our supposition is wrong. -150 cannot be term in AP.
3. Write first four terms of the AP, when the first term $a$ and common difference $d$ are given as follows:

(i) $a = -2$, $d = 0$

Here, difference between any two consecutive terms which is also called common difference is equal to 0.

First term = $a = -2$, $d = 0$

Second term = $a + d = -2 + 0 = -2$

Third term = second term + $d = -2 + 0 = -2$

Fourth term = third term + $d = -2 + 0 = -2$

Therefore, first four terms are: -2, -2, -2, -2

(ii) $a = 4$, $d = -3$

First term = $a = 4$, $d = -3$

Second term = $a + d = 4 -3 = 1$

Third term = second term + $d = 1 -3 = -2$

Fourth term = third term + $d = -2 -3 = -5$

Therefore, first four terms are: 4, 1, -2, -5
(iii)

First term = \(a = -1.25\), \(d = -0.25\)

Second term = \(a + d = -1.25 - 0.25 = -1.50\)

Third term = second term + \(d = -1.50 - 0.25 = -1.75\)

Fourth term = third term + \(d = -1.75 - 0.25 = -2\)

Therefore, first four terms are: -1.25, -1.50, -1.75, -2
4. Determine the AP whose third term is 16 and the $7^{th}$ term exceeds the $5^{th}$ term by 12.

Let first term of AP = $a$

Let common difference of AP = $d$

It is given that its term is equal to 16. It means $a_3=16$, where $a_3$ is the $3^{rd}$ term of AP.

Using formula $a_n=a+(n-1)d$, to find $n^{th}$ term of arithmetic progression, we can say that

$16 = a + (3 - 1)(d)$

$\Rightarrow 16 = a + 2d$  

It is also given that $7^{th}$ term exceeds $5^{th}$ term by 12. Again using formula $a_n=a+(n-1)d$, which is used to find $n^{th}$ term of arithmetic progression, we can say that

$a_7 = a + (7 - 1)d = a + 6d$ and, $a_5 = a + (5 - 1)d = a + 4d$    \hspace{1cm} (1)

According to the given condition, we can say that

$a_7 = a_5 + 12$

Putting (1) in the above equation, we get

$\Rightarrow a + 6d = a + 4d + 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = \frac{12}{2} = 6$

Putting value of $d$ in equation $16=a+2d$, we get

$16 = a + 2(6)$

$\Rightarrow a = 4$

Therefore first term = $a = 4$

And, common difference = $d = 6$

Therefore, AP is 4, 10, 16, 22....
5. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the $m^{th}$ and $n^{th}$ term is $(2m-1) : (2n-1)$.

Let $a$ be the first term and $d$ the common difference of the given A.P. Then, the sums of m and n terms are given by

$$S_m = \left(\frac{m}{2}\right) [2a + (m - 1) d], \text{ and}$$

$$S_n = \left(\frac{n}{2}\right) [2a + (n - 1) d]$$

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

$$\frac{\frac{m}{2}[2a + (m - 1) d]}{\frac{n}{2}[2a + (n - 1) d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a + (m - 1) d}{2a + (n - 1) d} = \frac{m}{n}$$

$$\Rightarrow [2a + (m - 1) d]n = [2a + (n - 1) d]m$$

$$\Rightarrow 2a(n - m) = d((n - 1)m - (m - 1)n)$$

$$\Rightarrow 2a(n - m) = d(n - m)$$

$$\Rightarrow d = 2a$$

$$\frac{T_m}{T_n} = \frac{a + (m - 1) \times 2a}{a + (n - 1) \times 2a} = \frac{a + (m - 1) \times 2a}{a + (n - 1) \times 2a} = \frac{2m - 1}{2n - 1}$$

($\therefore d = 2a$)
6. In the following APs, find the missing term in the boxes.

(i) 2, □, 26

For this A.P.,
\[ a = 2 \]
\[ a_3 = 26 \]

We know that, \[ a_n = a + (n - 1)d \]
\[ a_3 = 2 + (3 - 1)d \]
\[ 26 = 2 + 2d \]
\[ 24 = 2d \]
\[ d = 12 \]
\[ a_2 = 2 + (2 - 1)12 \]
\[ = 14 \]
Therefore, 14 is the missing term.

7. If the sum of the first 2n terms of the AP 2, 5, 8, ... is equal to the sum of the first n terms of A.P. 57, 59, 61, ... then what is the value of n?

Given: \[ 2 + 5 + 8 + ... \text{ 2n terms} = 57 + 59 + 61 + ... \text{ n terms} \]

\[ S_n = \frac{n}{2}2a + (n - 1)d \]
\[ \Rightarrow \frac{2n}{2}[2(2) + (2n - 1)3] = \frac{n}{2}[2(57) + (n - 1)2] \]
\[ \Rightarrow n[4 + 6n - 3] = n[57 + n - 1] \]
\[ \Rightarrow 6n + 1 = n + 56 \]
\[ \Rightarrow 5n = 55 \]
\[ \Rightarrow n = 11 \]
8. The sum of n terms of an A.P is written as \( S_n = pn + qn^2 \). What is the common difference \( d \) of the A.P.?

Let \( a \) be the first term of the AP. We have

\[
S_n = \frac{n}{2}(2a + (n - 1)d)
\]

\[
S_n = an + \frac{n(n - 1)d}{2}
\]

\[
S_n = \left(a - \frac{d}{2}\right)n + \frac{d}{2}n^2
\]

\[
S_n = pn + qn^2
\]

\[
 pn + qn^2 = \left(a - \frac{d}{2}\right)n + \frac{d}{2}n^2
\]

On comparing the coefficients of \( n \) and \( n^2 \), we get

\[
\frac{d}{2} = q
\]

\[
\Rightarrow d = 2q
\]