Practice Challenge - Subjective

Subject: Mathematics

Topic : Arithmetic Progressions
Exam Prep 1 Class: X

1. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

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It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, $S_n=\frac{n}{2}$(2a+ (n−1) d) to find sum of n terms of AP , we get

49=$\frac{7}{2}$(2a + (7−1) d)

⇒98=7(2a+6d)

⇒98=14a+42d

⇒7=a+3d

⇒a=7−3d (1)

And, 289=$\frac{17}{2}$(2a+ (17−1) d)

⇒578=17(2a+16d)

⇒34=2a+16d

⇒17=a+8d

Putting equation (1) in the above equation, we get

17=7−3d+8d

⇒10=5d

⇒d=$\frac{10}{5}$=2

Putting value of d in equation (1), we get

a=7−3d=7−3 (2) =7−6=1

Again applying formula, $S_n=\frac{n}{2}$(2a+ (n−1) d) to find sum of n terms of AP , we get

$S_n=\frac{n}{2}$[2(1) + (n−1) (2)]

⇒$S_n=\frac{n}{2}$[2+2n−2]

⇒$S_n=\frac{n}{2}$[2n]

⇒$S_n= n^2$

Therefore, sum of n terms of AP is equal to n^2

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2. Check whether -150 is a term of the AP: 11,8,5,2…

   Let −150 is the n^{th} of AP 11,8,5,2... Which means that a_n =−150

   Here, First term = a = 11

   Common difference = d = 8 - 11 = -3

   Using formula a_n=a+ (n−1) d, to find nth term of arithmetic progression, we get

   −150=11+ (n−1) (−3)

   ⇒ −150=11−3n+3

   ⇒ 3n=164

   ⇒ n= $\frac{164}{3}$

   But, n cannot be in fraction. Therefore, our supposition is wrong. −150 cannot be term in AP.

3. Write first four terms of the AP, when the first term a and common difference d are given as follows:
(i) a= -2, d=0
(ii) a=4, d=-3
(iii) a= -1.25, d= -0.25

(i)

Here, difference between any two consecutive terms which is also called common difference is equal to 0.

First term = a = -2, d=0

Second term = a+d = -2 + 0 = -2

Third term = second term + d = -2 + 0 = -2

Fourth term = third term + d = -2 + 0 = -2

Therefore, first four terms are: -2, -2, -2, -2

(ii)

First term = a = 4, d=-3

Second term = a+ d = 4 -3 = 1

Third term = second term + d = 1-3 = -2

Fourth term = third term + d = -2 -3 = -5

Therefore, first four terms are: 4, 1, -2, -5

(iii)

First term = a =-1.25, d = -0.25

Second term = a+ d = -1.25 - 0.25 = -1.50

Third term = second term + d = -1.50 -0.25 = -1.75

Fourth term = third term + d = -1.75 -0.25 = -2

Therefore, first four terms are: -1.25, -1.50, -1.75,-2

4. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Let first term of AP = a

Let common difference of AP = d

It is given that its term is equal to 16. It means a_3=16, where a_3 is the 3^{rd} term of AP.

Using formula a_n=a+ (n−1) d, to find n^{th} term of arithmetic progression, we can say that

$16 = a + (3-1)(d)$

$\Rightarrow 16 = a + 2d$

It is also given that 7^{th} term exceeds 5^{th} term by12. Again using formula a_n = a + (n−1) d, which is used to find n^{th} term of arithmetic progression, we can say that

$a_7 = a + (7-1)d = a + 6d$ and, $a_5 = a + (5-1)d = a + 4d$ (1)

According to the given condition, we can say that

a_7 = a_5+12

Putting (1) in the above equation, we get

$\Rightarrow a + 6d = a + 4d + 12$
$\Rightarrow 2d = 12$
$\Rightarrow d = \frac{12}{2} = 6$

Putting value of d in equation 16=a+2d, we get

$16 = a + 2(6)$

$\Rightarrow a = 4$

Therefore first term = a = 4

And, common difference = d = 6

Therefore, AP is 4, 10, 16, 22....

5. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m^{th} and n^{th} term is $(2m-1) : (2n-1)$.

   Let a be the first term and d the common difference of the given A.P. Then, the sums of m and n terms are given by

   S_m = ($\frac{m}{2}$) [2a + (m – 1) d], and

   S_n = ($\frac{n}{2}$) [2a + (n – 1) d]

   $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

   $\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

   $\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

   $\Rightarrow [2a+(m-1)d]n = [2a+(n-1)d]m$

   $\Rightarrow 2a(n-m) = d((n-1)m-(m-1)n)$

   $\Rightarrow 2a(n-m) = d(n-m)$

   $\Rightarrow d = 2a$

   $\frac{T_m}{T_n} = \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a} = \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a} = \frac{2m-1}{2n-1}$
   $(\because d = 2a)$

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6. In the following APs, find the missing term in the boxes.

(i) 2, $\boxed{}$, 26

For this A.P.,
$a = 2$
$a_3 = 26$
$We\ know\ that,\ a_n = a + (n-1)d$
$a_3 = 2 + (3-1)d$
$26 = 2 + 2d$
$24 = 2d$
$d = 12$
$a_2 = 2 + (2-1)12$
$= 14$
Therefore, 14 is the missing term.

7. If the sum of the first 2n terms of the AP 2, 5, 8, ... is equal to the sum of the first n terms of A.P. 57, 59, 61, ... then what is the value of n?

Given: 2 + 5 + 8 + ... 2n terms = 57 + 59 + 61 + ... n terms

$S_n = \frac{n}{2}[2a + (n-1)d$
$\Rightarrow \frac{2n}{2}[2(2) + (2n-1)3] = \frac{n}{2}[2(57) + (n-1)2]$
$\Rightarrow n[4 + 6n - 3] = n[57 + n - 1]$
$\Rightarrow 6n + 1 = n + 56$
$\Rightarrow 5n = 55$
$\Rightarrow n = 11$

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8. The sum of n terms of an A.P is written as $S_n = pn + qn^2$. What is the common difference d of the A.P.?

   Let a be the first term of the AP. We have

   $S_n = \frac{n}{2}(2a + (n-1)d)$

   $S_n = an + \frac{n(n-1)d}{2}$

   $S_n = \left(a - \frac{d}{2}\right) n + \frac{d}{2} n^2$

   $S_n = pn + qn^2$

   $pn + qn^2 = \left(a - \frac{d}{2}\right) n + \frac{d}{2} n^2$

   On comparing the coefficients of n and n^2, we get

   $\frac{d}{2} = q$

   $\Rightarrow d = 2q$