Practice Challenge - Subjective

Subject: Mathematics

Topic : Circles Exam Prep 1 Class: X

1. In Fig. 8.64, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at M. Prove that PL+ML=PN+MN.

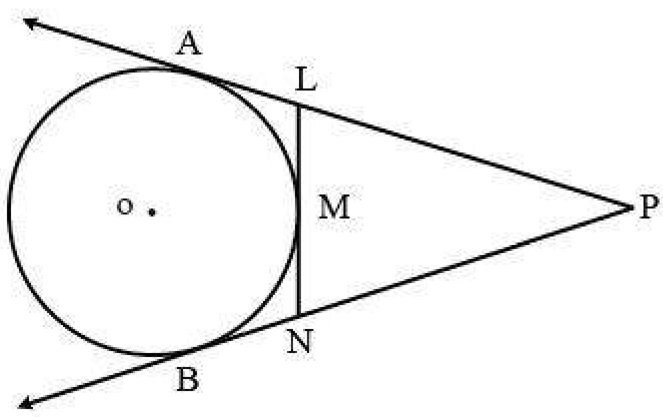

Fig.8.64

1. PA = PB [TANGENTS DRAWN FROM AN EXTERNAL POINT TO A CIRCLE ARE EQUAL IN LENGTH]
2. PL + AL = PN + BN [P-L-A, AND P-N-B]
3. AL = ML & BN = MN [SAME REASON AS 1]
4. Thus, PL + ML = PN + MN [FROM 2 AND 3]

Hence, proved.

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2. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that ΔAPB is equilateral.

∠OAP = 90° (PA and PB are the tangents to the circle.)

In ΔOPA,

sin ∠OPA = $\frac{OA}{OP} = \frac{r}{2r}$ [OP is the diameter = 2*radius]

sin ∠OPA = $\frac{1}{2}$ = sin 30 °

∠OPA = 30°

Similarly, ∠OPB = 30°.

∠APB = ∠OPA + ∠OPB = 30° + 30° = 60°

In ΔPAB,

PA = PB (tangents from an external point to the circle)

⇒ ∠PAB = ∠PBA ....(1) (angles opp.to equal sides are equal)

⇒ ∠PAB + ∠PBA + ∠APB = 180° [Angle sum property]

⇒ ∠PAB + ∠PAB = 180° – 60° = 120° [Using (1)]

⇒ 2∠PAB = 120°

⇒ ∠PAB = 60° ....(2)

From (1) and (2)

∠PAB = ∠PBA = ∠APB = 60°

All angles are equal in an equilateral triangle. (60°)

ΔPAB is an equilateral triangle

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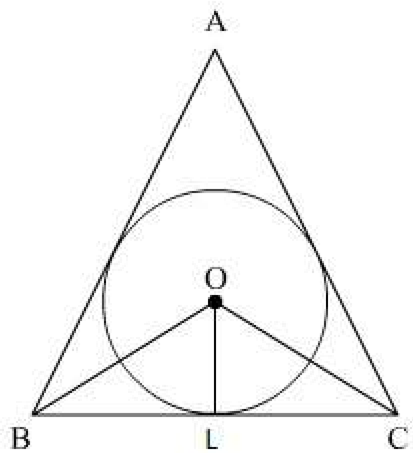
3. If ΔABC is isosceles with AB = AC and C(O,r) is the incircle of the ΔABC touching BC at L, prove that L bisects BC.



Given: ABC is an isosceles triangle.

C(O,r) is the incircle of ΔABC.

$\therefore$ O is the point of intersection of angle bisector.

(i,e.,) OB bisects B and OC bisects C

In triangle ABC,

$AB = BC$ (GIven)

$\Rightarrow \angle C = \angle B$ (Since two sides are equal angle between them also equal)

$\Rightarrow \Delta OCL = \Delta OBL$ (OB bisects triangle(B) and OC bisects triangle(C))

In ΔOCL and ΔOBL,

ΔOLB = ΔOLC

ΔOBL = ΔOCL

$BL = LC$

Thus, L bisects the side BC

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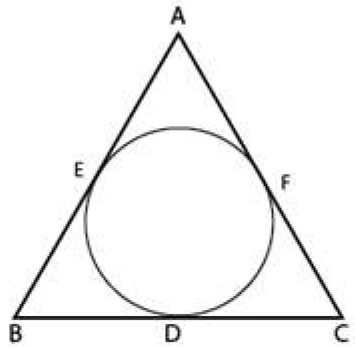
4. Let s denotes the semi – perimeter of a Δ ABC, in which BC=a, CA=b and AB=c , if a circle touches the sides BC, CA , AB at D,E,F respectively prove that BD = s - b.

A circle is inscribed in the Δ ABC, which touches the BC, CA and AB.



Given, BC = a. CA= b and AB =c.

By using the property , tangents are drawn from an external point to the circles are equal in length.

$\therefore$ BD = BE = x

DC = CF = y

And AF = AE = z

Now, BC + CA + AB= a + b+ c

$\Rightarrow$ (BD + DC) + (CF + FA) + (AE + EB) = a + b + c

$\Rightarrow$ (x+y) + (y+z) + (z+x) = a+b+c

$\Rightarrow$ 2 (x + y + z) = 2s

[$\because$ 2s = a + b + c= perimeter of Δ ABC]

$\Rightarrow$ s= x + y + z

$\Rightarrow$ x= s - (y + z) [$\because$ b = AE + EC = z + y]

$\Rightarrow$ BD = s - b

Hence proved

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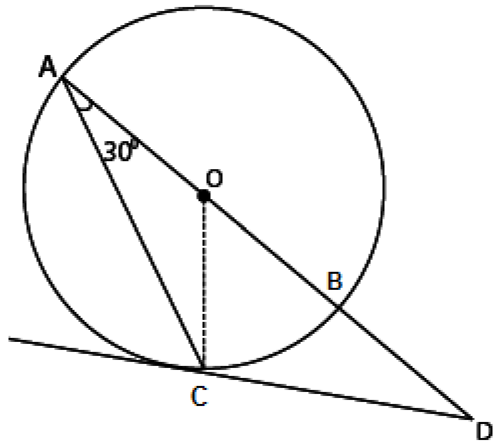
5. AB is a diameter of a circle and AC is the chord such that $\angle$ BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD.

True
To prove, BC = BD



Join BC and OC

$Given \angle BAC = 30^\circ$
$\angle BCD = 30^\circ$
$\Rightarrow$ [angle between tangent and chord is equal to angle made by chord in the alternate segment]
$\therefore \angle ACD = \angle ACO + \angle OCD = 30^\circ + 90^\circ = 120^\circ$
$[OC \perp CD \, and \, OA = OC = radius \Rightarrow \angle OAC = \angle OCA = 30^\circ$
$In \Delta ACD, \angle CAD + \angle ACD + \angle ADC = 180^\circ$
[Since sum of all interior angles of a triangle is 180°]
$\Rightarrow 30^\circ + 120^\circ + \angle ADC = 180^\circ$
$\Rightarrow \angle ADC = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$
$Now, \; in \; \Delta BCD \; \angle BCD = \angle BDC = 30^\circ$
$\Rightarrow BC = BD$
[Since , sides opposite to equal angles are equal]

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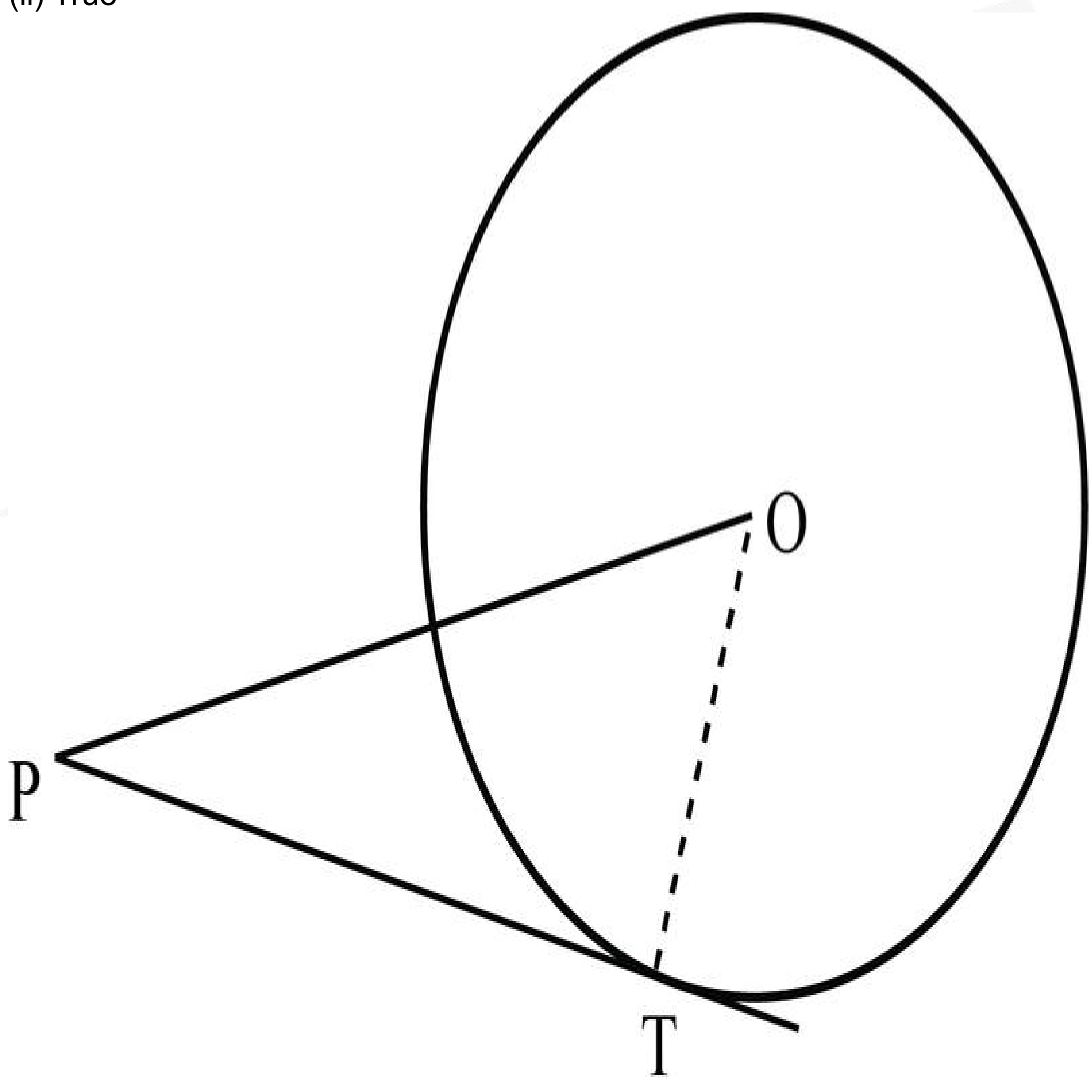
6. Write 'True' or 'False' and justify your answer in each of the following :
   (i) The length of tangents from an external point P on a circle is always greater than the radius of the circle.
   (ii) The length of tangents from an external point P on a circle with centre O is always less than OP.

(i) False
Because the length of tangents from an external point P on a circle may or may not be greater than the radius of the circle.
(ii) True



PT is a tangents drawn from external point P . Join OT
$\therefore OT \perp PT$
So, OPT is a right angled triangle formed
In right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.
$\therefore$ OP > PT
Or PT < OP

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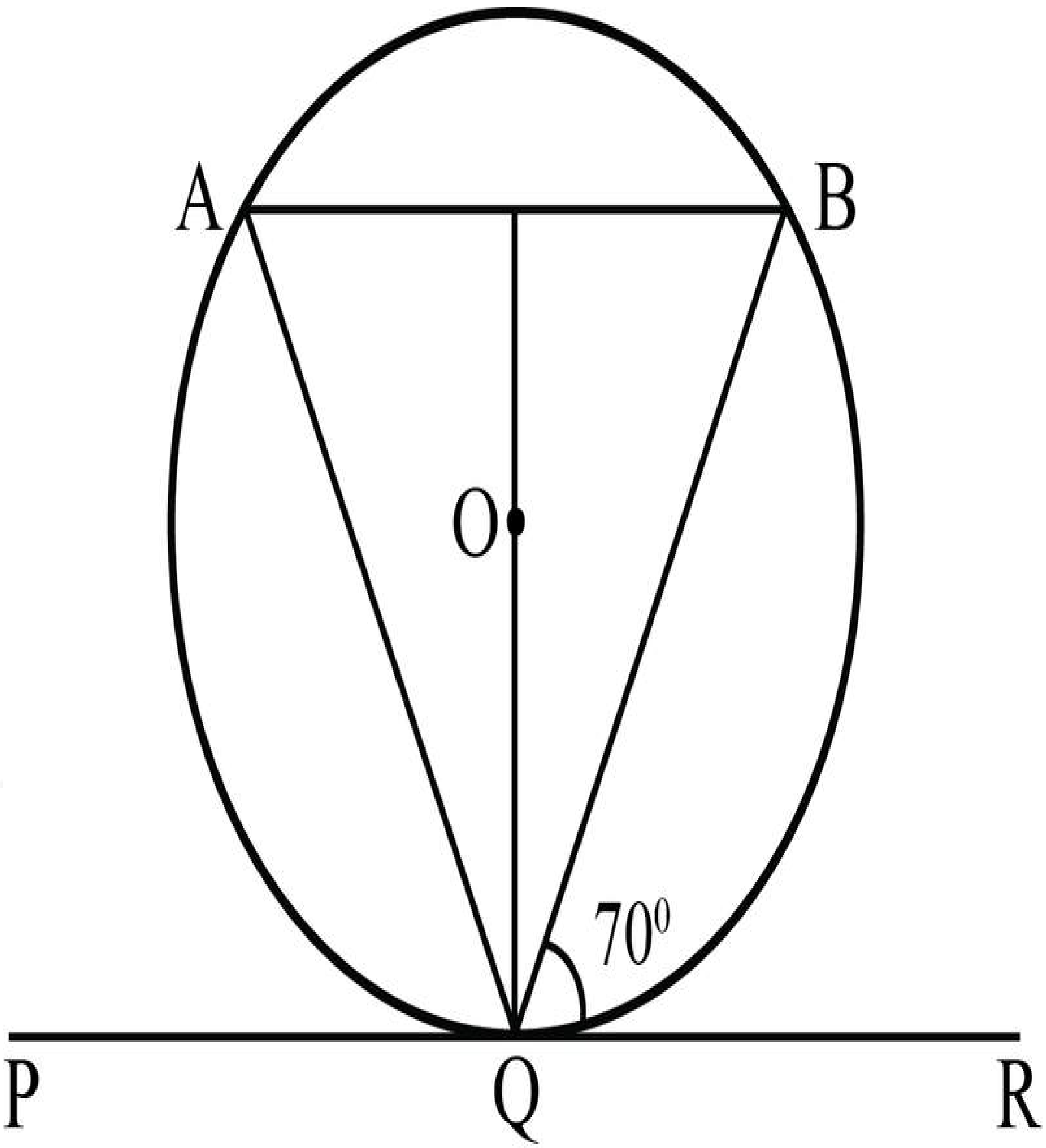
7. In figure. If PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle$AQB is equal to



Given , AB $||$ PR

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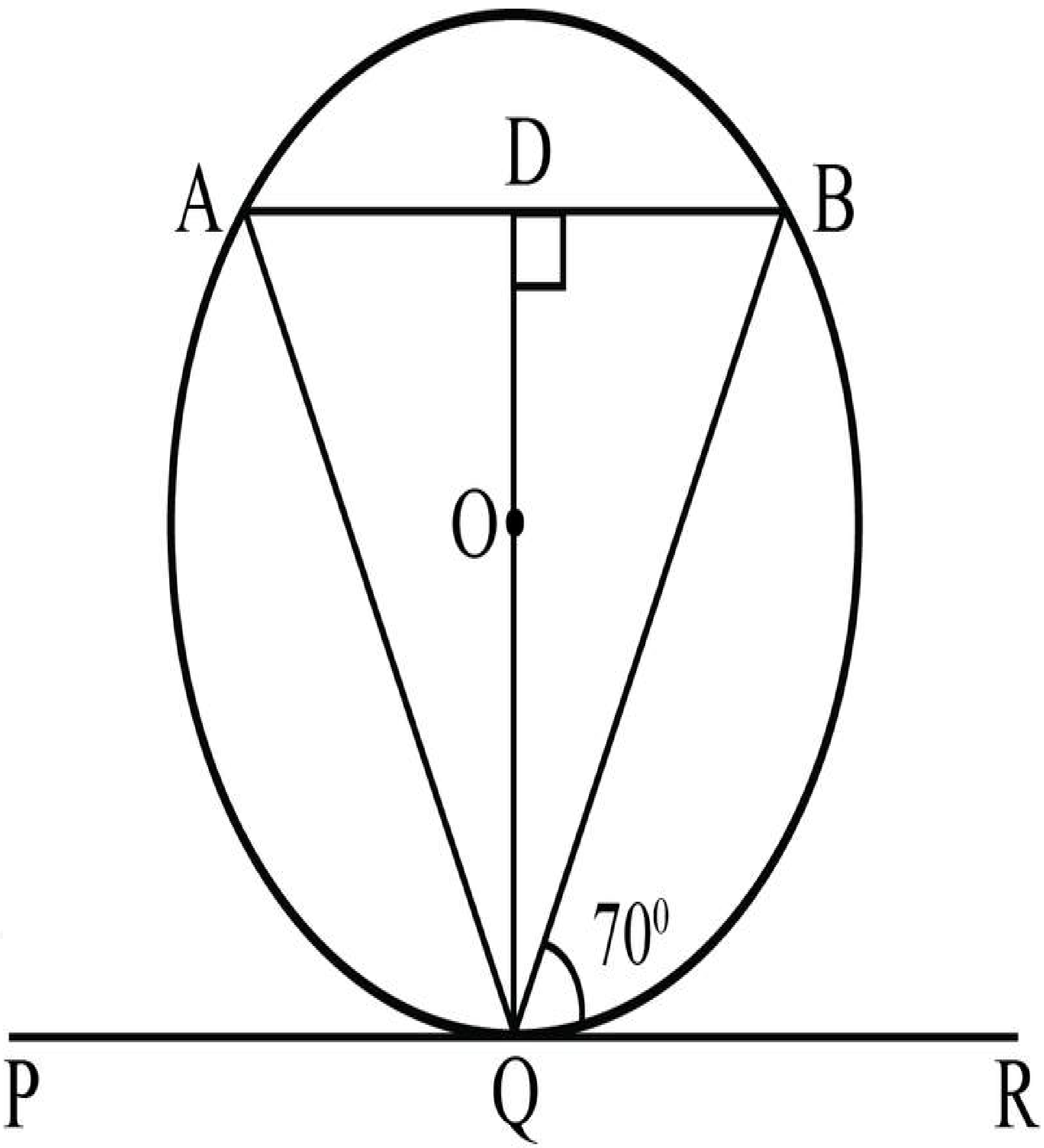

$\therefore \angle ABQ = \angle BQR = 70^\circ$ [*Alternate angles*]

Also, QD is perpendicular to AB and QD bisects AB.

In ΔQDA *and* $\Delta QDB, \angle QDA = \angle QDB$ [*Each* 90°]

$AD = BD$

$QD = QD$ [*Common sides*]

$\therefore \Delta ADQ \cong \Delta BDQ$ [*By SAS similarly criterion*]

Then $\angle QAD = \angle QBD$ [*CPTC*](i)

Also $\angle ABQ = \angle BQR$ [*Alternate interior angle*]

$\therefore \angle ABQ = 70^\circ$ [$\angle BQR = 70^\circ$]

Hence $\angle QAB = 70^\circ$

Now in $\Delta ABQ, \angle A + \angle B + \angle Q = 180^\circ$

$\Rightarrow \angle Q = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$

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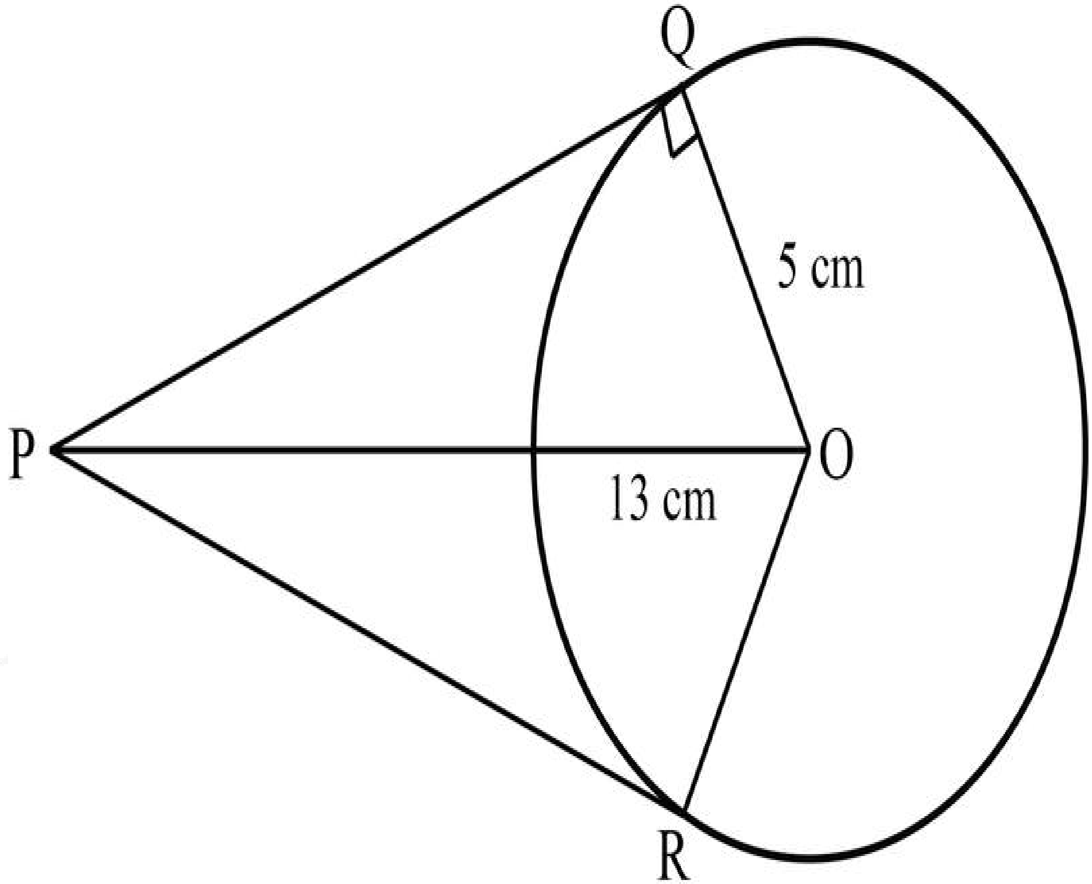
8. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle is drawn. Then, the area of the quadrilateral PQOR is

Firstly, draw a circle of radius 5 cm having centre O . P is a point at a distance of 13 cm from O. A pair of tangents PQ and PR are drawn.



Thus, Quadrilateral PQOR is formed.
[Since , QP is a tangent line]
$\therefore OQ \perp QP$
In right angled ΔPQO,
$OP^2 = OQ^2 + QP^2$
$\Rightarrow 13^2 = 5^2 + QP^2$
$\Rightarrow QP^2 = 169 - 25 = 144$
$QP = 12cm$
$Now,\ area\ of\ \Delta OQP = \frac{1}{2} \times QP \times QO$
$= \frac{1}{2} \times 12 \times 5 = 30cm^2$
$\therefore Area\ of\ quadrilateral\ QORP = 2 \times\ Area\ of\ \Delta OQP$
$= 2 \times 30 = 60cm^2$