1. In Fig. 8.64, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at M. Prove that PL+ML=PN+MN.

![Diagram of a circle with tangents PA, PB, and secant LN]

**Fig.8.64**

1. PA = PB \[\text{TANGENTS DRAWN FROM AN EXTERNAL POINT TO A CIRCLE ARE EQUAL IN LENGTH}\]
2. PL + AL = PN + BN \[P-L-A, \text{ AND } P-N-B]\]
3. AL = ML & BN = MN \[\text{SAME REASON AS 1}\]
4. Thus, PL + ML = PN + MN \[\text{FROM 2 AND 3}\]

Hence, proved.
2. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that \( \triangle APB \) is equilateral.

\[ \angle OAP = 90^\circ \quad \text{(PA and PB are the tangents to the circle.)} \]

In \( \triangle OPA \),

\[ \sin \angle OPA = \frac{OA}{OP} = \frac{r}{2r} \quad \text{[OP is the diameter = 2*radius]} \]

\[ \sin \angle OPA = \frac{1}{2} = \sin 30^\circ \]

\[ \angle OPA = 30^\circ \]

Similarly, \( \angle OPB = 30^\circ \).

\[ \angle APB = \angle OPA + \angle OPB = 30^\circ + 30^\circ = 60^\circ \]

In \( \triangle PAB \),

\( PA = PB \) (tangents from an external point to the circle)

\[ \Rightarrow \angle PAB = \angle PBA \ldots \ldots \ldots (1) \quad \text{(angles opp.to equal sides are equal)} \]

\[ \Rightarrow \angle PAB + \angle PBA + \angle APB = 180^\circ \quad \text{[Angle sum property]} \]

\[ \Rightarrow \angle PAB + \angle PAB = 180^\circ - 60^\circ = 120^\circ \quad \text{[Using (1)]} \]

\[ \Rightarrow 2\angle PAB = 120^\circ \]

\[ \Rightarrow \angle PAB = 60^\circ \ldots \ldots \ldots \ldots (2) \]

From (1) and (2)

\[ \angle PAB = \angle PBA = \angle APB = 60^\circ \]

All angles are equal in an equilateral triangle. \( (60^\circ) \)

\( \triangle PAB \) is an equilateral triangle
3. If \( \triangle ABC \) is isosceles with \( AB = AC \) and \( C(O,r) \) is the incircle of the \( \triangle ABC \) touching \( BC \) at \( L \), prove that \( L \) bisects \( BC \).

Given: \( ABC \) is an isosceles triangle.
\( C(O,r) \) is the incircle of \( \triangle ABC \).
\[ \therefore \] \( O \) is the point of intersection of angle bisector.
(i,e.,) \( OB \) bisects \( B \) and \( OC \) bisects \( C \)

In triangle \( ABC \),
\[
AB = BC \text{ (Given)}
\]
\[ \Rightarrow \angle C = \angle B \text{ (Since two sides are equal angle between them also equal)} \]
\[ \Rightarrow \triangle OCL = \triangle OBL \text{ (OB bisects triangle(B) and OC bisects triangle(C))} \]

In \( \triangle OCL \) and \( \triangle OBL \),
\[
\triangle OLB = \triangle OLC
\]
\[
\angle OBL = \angle OCL
\]
\[
BL = LC
\]
Thus, \( L \) bisects the side \( BC \)
4. Let \( s \) denotes the semi – perimeter of \( \triangle ABC \), in which \( BC=a \), \( CA=b \) and \( AB=c \), if a circle touches the sides \( BC \), \( CA \), \( AB \) at \( D \), \( E \), \( F \) respectively prove that \( BD = s - b \).

A circle is inscribed in the \( \triangle ABC \), which touches the \( BC \), \( CA \) and \( AB \).

Given, \( BC = a \), \( CA = b \) and \( AB = c \).

By using the property, tangents are drawn from an external point to the circles are equal in length.

\[
\therefore \ BD = BE = x \\
DC = CF = y \\
And \ AF = AE = z \\
\]

Now, \( BC + CA + AB = a + b + c \)

\[ \Rightarrow (BD + DC) + (CF + FA) + (AE + EB) = a + b + c \]

\[ \Rightarrow (x+y) + (y+z) + (z+x) = a+b+c \]

\[ \Rightarrow 2(x+y+z) = 2s \]

\[ \therefore 2s = a + b + c\text{=} \text{ perimeter of } \triangle ABC \]

\[ \Rightarrow s = x + y + z \]

\[ \Rightarrow x = s - (y + z) \text{ [ } \therefore b = AE + EC = z + y \text{]} \]

\[ \Rightarrow BD = s - b \]

Hence proved
5. AB is a diameter of a circle and AC is the chord such that $\angle BAC = 30^\circ$. If the tangent at C intersects AB extended at D, then BC = BD.

True

To prove, BC = BD

Given $\angle BAC = 30^\circ$

$\angle BCD = 30^\circ$

$\Rightarrow [\text{angle between tangent and chord is equal to angle made by chord in the alternate segment}]$

$\therefore \angle ACD = \angle ACO + \angle OCD = 30^\circ + 90^\circ = 120^\circ$

$[OC \perp CD \text{ and } OA = OC = \text{radius} \Rightarrow \angle OAC = \angle OCA = 30^\circ]$

In $\triangle ACD$, $\angle CAD + \angle ACD + \angle ADC = 180^\circ$

$[\text{Since sum of all interior angles of a triangle is } 180^\circ]$

$\Rightarrow 30^\circ + 120^\circ + \angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$

Now, in $\triangle BCD$, $\angle BCD = \angle BDC = 30^\circ$

$\Rightarrow BC = BD$

[Since, sides opposite to equal angles are equal]
Practice Challenge - Subjective

6. Write ‘True’ or ‘False’ and justify your answer in each of the following:

(i) The length of tangents from an external point P on a circle is always greater than the radius of the circle.

(ii) The length of tangents from an external point P on a circle with centre O is always less than OP.

(i) False
   Because the length of tangents from an external point P on a circle may or may not be greater than the radius of the circle.

(ii) True

\[ \text{PT is a tangents drawn from external point P. Join OT} \]
\[ \therefore OT \perp PT \]
So, OPT is a right angled triangle formed
In right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.
\[ \therefore OP > PT \]
Or PT < OP
7. In figure. If PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and \( \angle BQR = 70^\circ \), then \( \angle AQB \) is equal to

Given, AB \parallel PR
\[ \therefore \angle ABQ = \angle BQR = 70^\circ \ [\text{Alternate angles}] \]

Also, \( QD \) is perpendicular to \( AB \) and \( QD \) bisects \( AB \).

In \( \triangle QDA \) and \( \triangle QDB \), \( \angle QDA = \angle QDB \ [\text{Each } 90^\circ] \)

\[ AD = BD \]

\[ QD = QD \ [\text{Common sides}] \]

\[ \therefore \triangle ADQ \cong \triangle BDQ \ [\text{By SAS similarly criterion}] \]

Then \( \angle QAD = \angle QBD \ [\text{CPTC}](i) \)

Also \( \angle ABQ = \angle BQR \ [\text{Alternate interior angle}] \)

\[ \therefore \angle ABQ = 70^\circ \ [\angle BQR = 70^\circ] \]

Hence \( \angle QAB = 70^\circ \)

Now in \( \triangle ABQ \), \( \angle A + \angle B + \angle Q = 180^\circ \)

\[ \Rightarrow \angle Q = 180^\circ - (70^\circ + 70^\circ) = 40^\circ \]
8. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle is drawn. Then, the area of the quadrilateral PQOR is

Firstly, draw a circle of radius 5 cm having centre O. P is a point at a distance of 13 cm from O. A pair of tangents PQ and PR are drawn.

Thus, Quadrilateral PQOR is formed.

[Since , QP is a tangent line]
\[ \therefore OQ \perp QP \]

In right angled \( \triangle PQO \),
\[ OP^2 = OQ^2 + QP^2 \]
\[ \Rightarrow 13^2 = 5^2 + QP^2 \]
\[ \Rightarrow QP^2 = 169 - 25 = 144 \]
\[ QP = 12\text{ cm} \]

Now, \[ \text{area of } \triangle OQP = \frac{1}{2} \times QP \times QO \]
\[ = \frac{1}{2} \times 12 \times 5 = 30\text{ cm}^2 \]
\[ \therefore \text{Area of quadrilateral } QORP = 2 \times \text{Area of } \triangle OQP \]
\[ = 2 \times 30 = 60\text{ cm}^2 \]