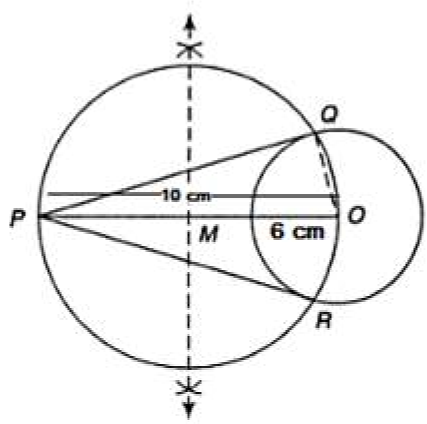
Practice Challenge - Subjective

Subject: Mathematics

Topic : Constructions Theory
Session 1 Class: X

1. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct a pair of tangents to the circle and measure their lengths.

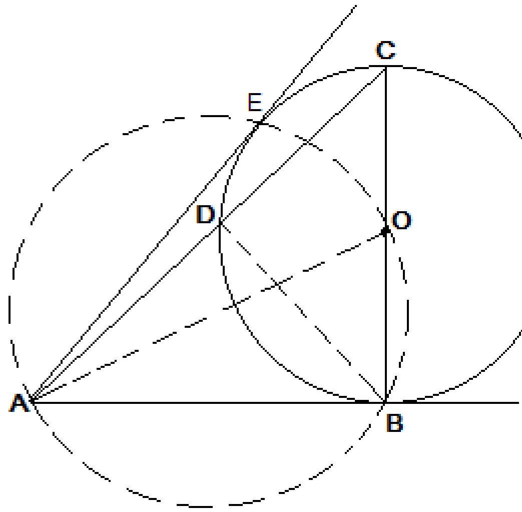
Steps of Construction:
Step I: With O as a centre and radius equal to 6 cm, a circle is drawn.
Step II: A point P at a distance of 10 cm from the centre O is taken. OP is joined.



Step III: Perpendicular bisector of OP is drawn and it intersects OP at M.
Step IV: With M as a centre and OM as a radius, a circle is drawn intersecting the previous circle at Q and R.
Step V: PQ and PR are joined.
Thus, PQ and PR are the tangents to the circle.
On measuring the length, tangents are equal to 8 cm.
PQ = PR = 8cm.
Justification:
OQ is joined.
$\angle PQO = 90^\circ$ (Angle in the semi-circle)
$\therefore OQ \perp PQ$
Therefore, OQ is the radius of the circle then PQ has to be a tangent of the circle.
Similarly, PR is a tangent of the circle.

2. Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. A circle through B, C, D is drawn. Construct the tangents from A to this circle.

Steps of Construction:
Step I: ΔABC is drawn.
Step II: Perpendicular to AC is drawn to point B which intersected it at D.



Step III: With O as a center and OC as a radius, a circle is drawn. The circle through B, C, D is drawn.
Step IV: OA is joined and a circle is drawn with diameter OA which intersected the previous circle at B and E.
Step V: AE is joined.
Thus, AB and AE are the required tangents to the circle from A.
Justification:
$\angle OEA = 90^\circ$ (Angle in the semi-circle)
$\therefore OE \perp AE$
Therefore, OE is the radius of the circle then AE has to be a tangent of the circle.
Similarly, AB is another tangent to the circle.

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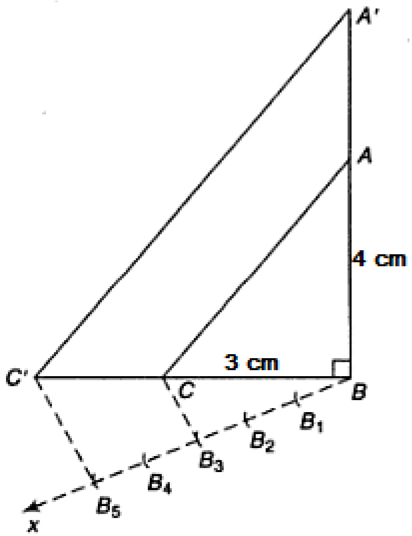
3. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

Steps of Construction:
Step I: BC = 3 cm is drawn.
Step II: At B, A ray making an angle of 90° with BC is drawn.
Step III: With B as centre and radius equal to 4 cm, an arc is made on the previous ray intersecting it at point A.



Step IV: AC is joined to form ΔABC

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Step V: A ray BX is drawn making an acute angle with BC opposite to vertex A.
Step VI: 5 points $B_1, B_2, B_3, B_4 and B_5$ are marked at equal distances on BX.
Step VII: B_3C is joined B_5C' is made parallel to B_3C.
Step VIII: A'C' is drawn parallel to AC.
Thus, $\Delta A'BC'$ is the required triangle.

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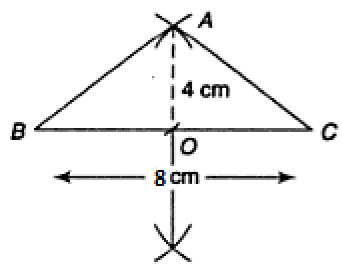
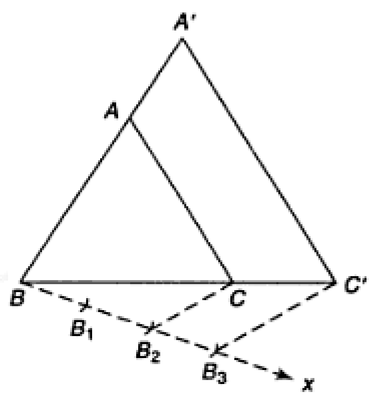
4. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are 1.5 times the corresponding sides of the isosceles triangle.

Steps of Construction:
Step I: BC = 8 cm is drawn.
Step II: Perpendicular bisector of BC is drawn and it intersects BC at O.



Step III: At a distance of 4 cm, a point A is marked on the perpendicular bisector of BC.

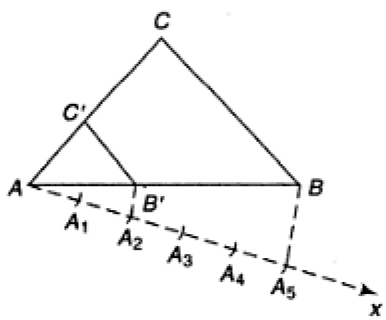


Step IV: AB and AC are joined to form ΔABC.

Step V: A ray BX is drawn making an acute angle with BC opposite to vertex A.
Step VI: 3 points $B_1, B_2 and B_3$ are marked on BX.
Step VII: B_2 is joined with C to form B_2C.
Step VIII: B_3C' is drawn parallel to B_2C and C'A' is drawn parallel to CA.
Thus, A'BC' is the required triangle formed.
Justification:
$\Delta AB'C' \sim \Delta ABC$ by AA similarity condition.
$\therefore \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$
Also,
$\frac{AB}{AB'} = \frac{AA_2}{AA_3} = \frac{2}{3}$
$\Rightarrow AB' = \frac{3}{2}AB, B'C' = \frac{3}{2}$
$BC \ and \ AC' = \frac{3}{2}AC$

5. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Steps of Construction:
Step I: AB = 6 cm is drawn.
Step II: With A as a centre and radius equal to 4 cm, an arc is drawn.
Step III: Again, with B as a centre and radius equal to 5 cm an arc is drawn on the same side of AB intersecting the previous arc at C.
Step IV: AC and BC are joined to form ΔABC.
Step V: A ray AX is drawn making an acute angle with AB below it.
Step VI: 5 equal points (sum of the ratio = 2 + 3 =5) is marked on AX as $A_1, A_2 \ldots A_5$
Step VII: A_5B is joined. A_2B' is drawn parallel to A_5B and B'C' is drawn parallel to BC.
$\Delta AB'C'$ is the required triangle



Justification:

$\angle A (Common)$
$\angle C = \angle$ C' and $\angle B = \angle$ B' (corresponding angles)
Thus ΔAB'C' ~ ΔABC by AAA similarity condition
From the figure,
$\frac{AB'}{AB} = \frac{AA2}{AA5} = \frac{2}{3}$
$AB' = \frac{2}{3}AB$
$AC' = \frac{2}{3}AC$

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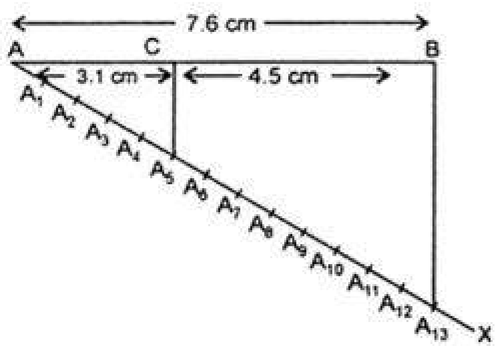
6. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8 Measure the two parts.

1) Draw a line segment AB = 7.6 cm.

2) Draw a ray AX making an acute angle $\angle$ BAX along with AB.



3) On AX make 5 + 8 i.e. 13 equal parts and mark them as $A_1, A_2, A_3, A_4, \ldots A_{13}$

4) Join B to A_{13}. From A_5 draw $A_5C \parallel A_{13}B$.

C is the required point of division and AC: CB = 5 : 8.

On measuring, we get

AC = 3.1 cm,

CB = 4.5 cm

Justification

$A_5C \parallel A_{13}B$

$\frac{A_5C}{A_5A_{13}} = \frac{AC}{CB}$

[Using basic proportionality theorem]

But $\frac{A_5C}{A_5A_{13}} = \frac{5}{8}$

This shows that C divides AB in the ratio 5 : 8.

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7. Draw an isosceles triangle ABC in which AB=AC = 6 cm and BC=5 cm Construct a triangle PQR similar to ΔABC in which PQ = 8 cm, Also justify the construction.

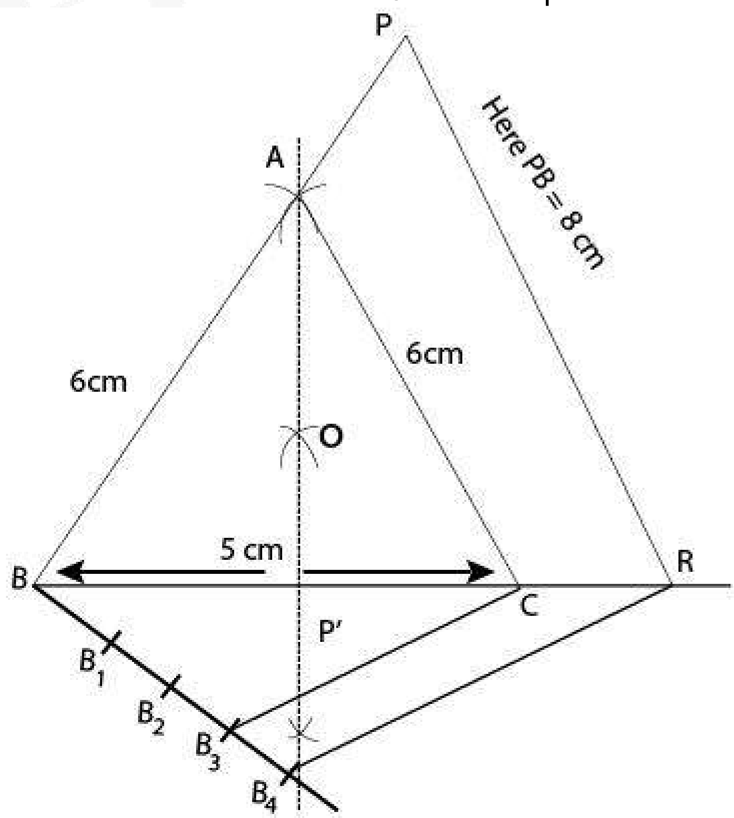
Thinking process
(i)Here, for making two similar triangles with one vertex is base We assume that In ΔABC and ΔPQR, vertex B = vertex Q.

(ii) In ΔABC and ΔPQR, vertex B = vertex Q.
So, we get the required scale factor.

Now, construct a ΔABC and then a ΔPBR, similar to ΔABC whose sides are $\frac{PQ}{AB}$ of the corresponding sides of the ΔABC.

Let ΔPQR and ΔABC are similar triangle, then its scale factor between the corresponding sides is $\frac{PQ}{AB} = \frac{8}{6} = \frac{4}{3}$

Steps of construction
1. Draw a line segment BC = 5 cm
2. Construct OQ the perpendicular bisector of line segment BC meeting BC at P'
3. Taking B and C as centers draw two arcs of equal radius 6cm intersecting each other at A
4. Join BA and CA. So. ΔABC is the required isosceles triangle.



5. From B, draw a ray BX making an acute $\angle CBX$
6. Locate four points B_1, B_2, B_3, B_4 on BX Such that $BB_1 = B_1B_2 = B_3B_4$

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7. Join B_3C and from B_4 draw a line $B_4R \parallel B_3C$ intersecting the extended line segment BC at R.
8. From point R draw $RP \parallel CA$ meeting BA produced at p

Then, ΔPBR is the required triangle.

Justification

$\because B_4R \parallel B_3C$

$\therefore \frac{BC}{CR} = \frac{3}{1}$

Now $\frac{BR}{BC} = \frac{BC+CR}{BC}$

$= 1 + \frac{CR}{BC} = 1 + \frac{1}{3} = \frac{4}{3}$

Also $RP \parallel CA$

$\therefore \Delta ABC \cong \Delta PBR$

And $\frac{PB}{AB} = \frac{RP}{CA} = \frac{BR}{BC} = \frac{4}{3}$

Hence the new triangle is similar to the given triangle whose sides are $\frac{4}{3}$ times of the corresponding sides of the isosceles ΔAC.

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8. Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is 60°. Also justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents.

In order to draw the pair of tangents, we follow the following steps

Steps of construction
1. Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm
2. Produce OA to B such that OA = AB = 4 cm
3.Taking A as the center draw a circle of radius OA = AB = 4 cm
Suppose it cuts the circle drawn in step 1 at P and Q
4. Join BP and BQ to get desired tangents.
Justification in ΔOAP, we have

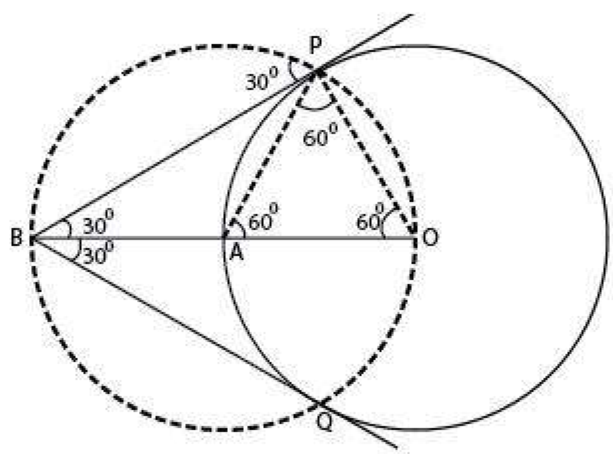
OA = OP = 4 cm (∵ Radius)

Also, AP = 4 cm (∵ Radius of circle with centre)

ΔOAP is equilateral

$\Rightarrow \angle PAO = 60^\circ$

$\Rightarrow \angle BAP = 120^\circ$



In ΔBAP we have

BA = AP and $\angle BAP = 120^\circ$

$\therefore \angle ABP = \angle APB = 30^\circ$

$\Rightarrow \angle PBQ = 60^\circ$

Alternate method

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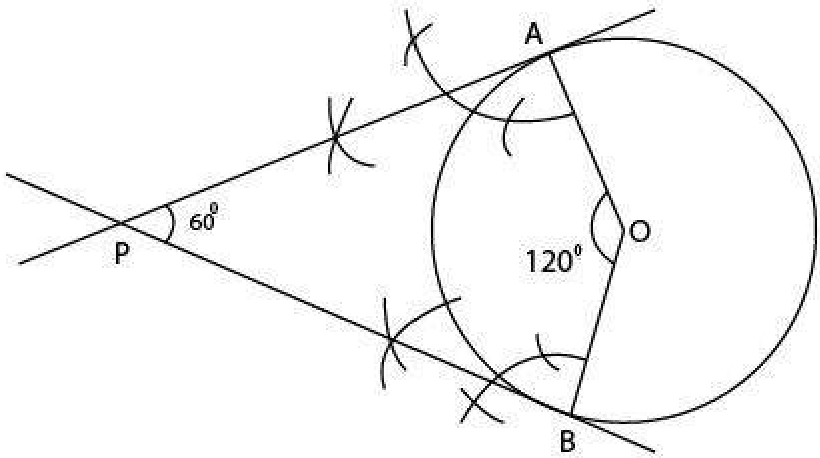
Steps of construction

1. Take a point O on the plane of the paper and draw a circle with centre O and radius OA = 4cm
2. AT O construct radii OA and OB such that to $\angle AOB$ equal 120° i.e supplement of the angle between the tangents.
3. Draw perpendicular to OA and OB at A and B respectively Suppose these Perpendicular intersects at P. Then PA and PB are required tangents.



Justification

In quadrilateral OAPB we have

$\angle OAP = \angle OBP = 90^{\circ}$

And $\angle ABO = 120^{\circ}$

$\therefore \angle OAP + \angle OBP + \angle AOB + \angle APB = 360^{\circ}$

$\angle APB = 360^{\circ} - (90^{\circ} + 90^{\circ} + 120^{\circ})$

$= 360^{\circ} - 300^{\circ} = 60^{\circ}$