Practice Challenge - Subjective

Subject: Mathematics

Topic : Quadratic Equations Exam Prep 1

Class: X

1. Find the roots of the equation $5x^2 - 6x - 2 = 0$ by the method of completing the square.
[3 MARKS]

Concept : 1 Mark
Application : 2 Marks

We have, $5x^2 - 6x - 2 = 0$

$\Rightarrow x^2 - \frac{6}{5}x - \frac{2}{5} = 0$

$\Rightarrow x^2 - \frac{6}{5}x = \frac{2}{5}$ [Dividing throughout by 5]

$\Rightarrow x^2 - 2\left(\frac{3}{5}\right)x + \left(\frac{3}{5}\right)^2 = \frac{2}{5} + \left(\frac{3}{5}\right)^2$

[Adding $\left(\frac{b}{2a}\right)^2$ on both sides]

$\Rightarrow \left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$

$\Rightarrow x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$

$\Rightarrow x = \frac{3}{5} \pm \frac{\sqrt{19}}{5} = \frac{3 \pm \sqrt{19}}{5}$

$\Rightarrow x = \frac{3+\sqrt{19}}{5} or,\ x = \frac{3-\sqrt{19}}{5}$

Hence, the roots of the given equation are $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$

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2. Using quadratic formula solve the following quadratic equation: [3 MARKS]

$p^2x^2 + (p^2 - q^2)x - q^2 = 0, p \neq 0$

Concept: 1 Mark
Application: 2 Marks

We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0, p \neq 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2, b = p^2 - q^2 \; and \; c = -q^2$

$\therefore D = b^2 - 4ac = (p^2 - q^2)^2 - 4 \times p^2 \times -q^2$

$\Rightarrow D = (p^2 - q^2)^2 + 4p^2q^2$

$\Rightarrow D = (p^2 + q^2)^2$

$\Rightarrow D > 0$

So, the given equation has real roots given by

$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{q^2}{p^2}$

and, $\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = -1$

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3. Find the values of k for which the given equation has real and equal roots: [3 MARKS]

$x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$

Formula: 1 Mark
Steps: 1 Mark
Answer: 1 Mark

The given equation is $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$

Here, $a = 1, b = -2(1 + 3k)\ and\ c = 7(3 + 2k)$

$\therefore D = b^2 - 4ac$

$\Rightarrow D = 4(3k + 1)^2 - 4 \times 1 \times 7(3 + 2k)$

$\Rightarrow D = 4(9k^2 + 6k + 1 - 21 - 14k)$

$\Rightarrow D = 4(9k^2 - 8k - 20)$

The given equation will have equal roots, if

$D = 0$

$\Rightarrow 4(9k^2 - 8k - 20) = 0$

$\Rightarrow 9k^2 - 8k - 20 = 0$

$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow (k - 2)(9k + 10) = 0$

$\Rightarrow k - 2 = 0\ or,\ 9k + 10 = 0 \Rightarrow k = 2\ or,\ k = -\frac{10}{9}$

4. Solve the following quadratic equations by factorization method: [4 MARKS]

$4x^2 - 4ax + (a^2 - b^2) = 0$

Concept : 1 Mark
Application : 1 Mark
Calculation : 2 Marks

We have,

$4x^2 - 4ax + (a^2 - b^2) = 0$

Here, Constant term $= (a^2 - b^2) = (a - b)(a + b)$

and, coefficient of middle term $= -4a$

Also, Coefficient of the middle term $-4a = -\{2(a + b) + 2(a - b)\}$

$\therefore 4x^2 - 4ax + (a^2 - b^2) = 0$

$\Rightarrow 4x^2 - \{2(a + b) + 2(a - b)\}x + (a + b)(a - b) = 0$

$\Rightarrow 4x^2 - 2(a + b)x - 2(a - b)x + (a + b)(a - b) = 0$

$\Rightarrow \{4x^2 - 2(a + b)x\} - \{2(a - b)x - (a + b)(a - b)\} = 0$

$\Rightarrow 2x\{2x - (a + b)\} - (a - b)\{2x - (a + b)\} = 0$

$\Rightarrow \{2x - (a + b)\}\{2x - (a - b)\} = 0$

$\Rightarrow \{2x - (a + b)\} = 0 \; or, \; \{2x - (a - b)\} = 0$

$\Rightarrow 2x = a + b \; or, \; 2x = a - b \Rightarrow x = \frac{a+b}{2} or, \; x = \frac{a-b}{2}$

5. Find the values of k for which the following equation has equal roots: [4 MARKS]

$(k-12)x^2+2(k-12)x+2=0$

Concept: 1 Mark
Steps: 2 Marks
Answer: 1 Mark

We have,

$(k-12)x^2+2(k-12)x+2=0$

Here, $a=k-12, b=2(k-12)\ and\ c=2$

$\therefore D=b^2-4ac=4(k-12)^2-4(k-12)\times 2$

$\Rightarrow D=4(k-12)\{(k-12)-2\}$

$\Rightarrow D=4(k-12)(k-14)$

The given equation will have equal roots, if

$D=0\Rightarrow 4(k-12)(k-14)=0$

$\Rightarrow k-12=0\ or,\ k-14=0$

$\Rightarrow k=12\ or,\ k=14$

But, if $k=12$, then the given equation will not be quadratic.
Hence, $k=14$ is the only solution.

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6. A plane left 30 minutes later than the schedule time and in order to reach its destination 1500 km away in time it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed. [4 MARKS]

Framing the equation: 1 Mark
Steps: 2 Marks
Answer: 1 Mark

Let the usual speed of the plane be x km/hr. Then,

Time taken to cover 1500 km with the usual speed $= \frac{1500}{x} hrs$

Time taken to cover 1500 m with the speed of $(x + 250) km/hr = \frac{1500}{x+250}$

$\therefore \frac{1500}{x} = \frac{1500}{x+250} + \frac{1}{2}$

$\Rightarrow \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

$\Rightarrow \frac{1500x+1500\times250-1500x}{x(x+250)} = \frac{1}{2}$

$\Rightarrow \frac{1500\times250}{x^2+250x} = \frac{1}{2}$

$\Rightarrow 750000 = x^2 + 250x$

$\Rightarrow x^2 + 250x - 750000 = 0$

$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow x(x + 1000)(x - 750) = 0$

$\Rightarrow x = -1000 \; or, \; x = 750 \Rightarrow x = 750$ [$\because$ speed cannot be negative]

Hence, the usual speed of the plane is 750 km/hr.

7. Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.

Let x and (16 - x) are two parts of 16 where (16 - x) is longer and x is smaller.
According to the question,

2 $\times$ square of longer = square of smaller + 164

$\Rightarrow 2 \times (16 - x)^2 = x^2 + 164$

$\Rightarrow \ 2 \times (256 + x^2 - 32x) = x^2 + 164$

$\Rightarrow \ 512 + 2x^2 - 64x = x^2 + 164$

$\Rightarrow \ x^2 - 64x + 512 - 164 = 0$

$\Rightarrow \ x^2 - 64x + 348 = 0$

$\Rightarrow x^2 - 58x - 6x + 348 = 0$

$\Rightarrow \ x(x - 58) - 6(x - 58) = 0$

$\Rightarrow (x - 6)(x - 58) = 0$

$\Rightarrow \ x = 6 \ and \ 58$

But $x \neq 58$ because $x < 16$

so, $x = 6$ and $16 - x = 10$

Hence, the answer is 6 and 10

8. If one root of the quadratic equation $2x^2 + kx - 6 = 0$ is 2, find the value of k. Also, find the other root.
[2 MARKS]

Concept : 1 Mark
Application :1 Mark

Since $x = 2$ is a root of the equation

$2x^2 + kx - 6 = 0$

$\therefore 2 \times 2^2 + 2k - 6 = 0$

$\Rightarrow 8 + 2k - 6 = 0 \Rightarrow 2k + 2 = 0 \Rightarrow k = -1$

Substituting $k = -1$ in the equation $2x^2 + kx - 6 = 0$, we get

$2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow (x - 2)(2x + 3) = 0$

$\Rightarrow x - 2 = 0, 2x + 3 = 0 \Rightarrow x = 2, x = -\frac{3}{2}$

Hence, the other root is $-\frac{3}{2}$