

M A T H E M A T I C S

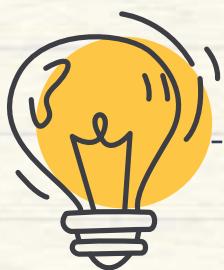


Quadratic Equations



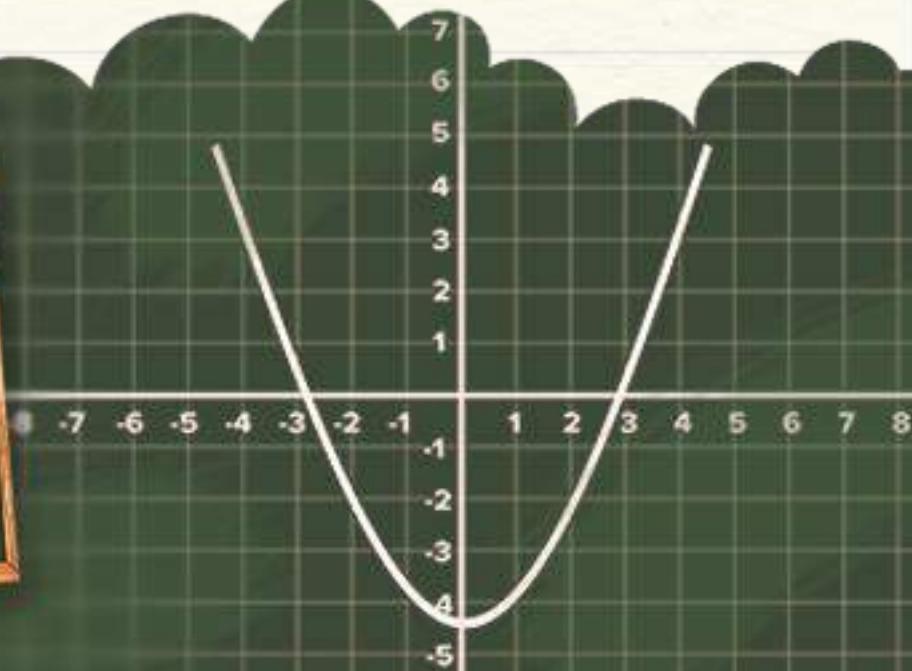


Topics



- 1. Standard Form of Quadratic Equations
- 2. Methods to Solve
- 3. Zeroes, Roots and Solutions
- 4. Nature of Roots

$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$





Standard Form

$$ax^2 + bx + c = 0$$

Degree

 Real numbers and $a \neq 0$

Important Terms

Zeroes

Zeroes are for quadratic polynomial $P(x)$

$$P(x) = (x - 2)(x - 2)$$

Zeroes, $x = 2 \& 2$

Roots

Roots are for quadratic equation

$$(x - 2)(x - 2) = 0$$

Roots, $x = 2 \& 2$

Solutions

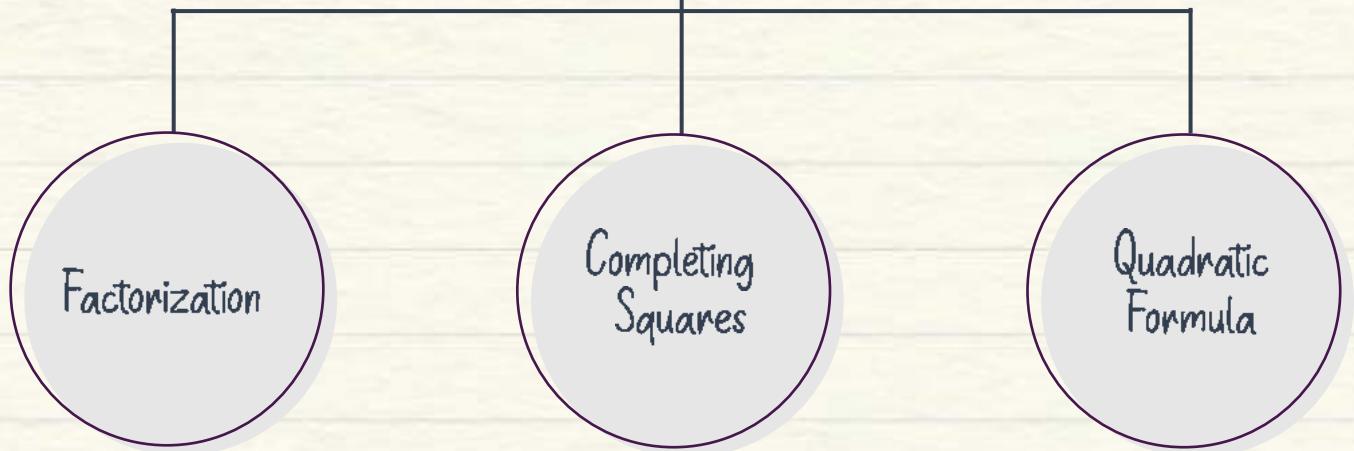
Quadratic equation having equal and identical roots will have a unique solution.

$$(x - 2)(x - 2) = 0$$

$x = 2$ is the solution of the given equation



Methods to Solve Quadratic Equations



Factorization

General form:
 $ax^2 + bx + c = 0.$

1. Split the middle term.

Product of split terms = $(a \times c)$



$$9x^2 - 3x - 2 = 0.$$

$$9x^2 - 6x + 3x - 2 = 0.$$



2. Factorize the equation



$$3x(3x - 2) + 1(3x - 2) = 0.$$

$$(3x - 2)(3x + 1) = 0.$$



3. Equate each factor to 0



$$(3x - 2)(3x + 1) = 0$$

$$x = \frac{2}{3} \text{ or } x = -\frac{2}{3}$$



Completing Squares

1. Divide both sides by
the coefficient of x^2



$$ax^2 + bx + c = 0$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$



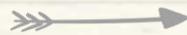
2. Divide and multiply the
coefficient of x by 2



$$x^2 + \frac{2bx}{2a} + \frac{c}{a} = 0$$



3. Add the square of
half of the coefficient
of x on both sides.



$$x^2 + \frac{2bx}{2a} + \frac{c}{a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2$$



4. Take square root on
both sides



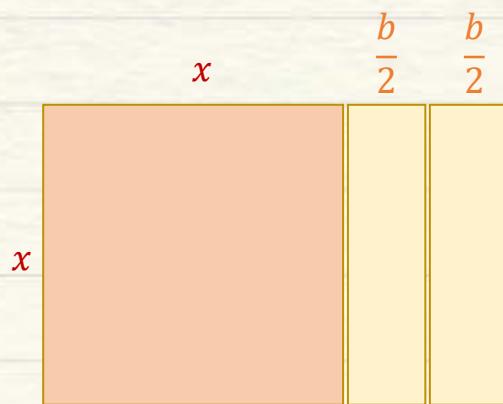
$$\sqrt{(2ax + b)^2} = \sqrt{b^2 - 4ac}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

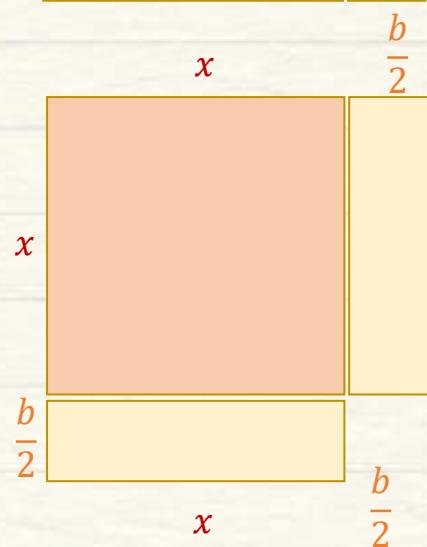


Visualizing Completing the Squares

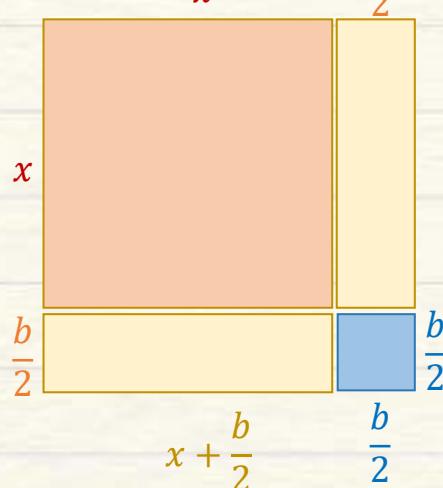
$$x^2 + bx = 0$$



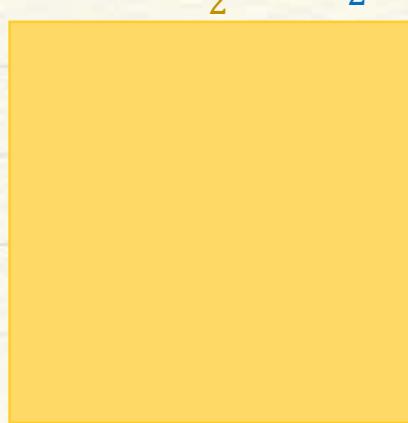
→ $x^2 + bx$



→ $x^2 + bx$



→ $x^2 + bx + \frac{b^2}{4} - \frac{b^2}{4} = 0$



→ $\left(x + \frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2$



Quadratic Formula

$$ax^2 + bx + c = 0$$

$$\text{Roots } (x) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i.e.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

where, $b^2 - 4ac \geq 0$

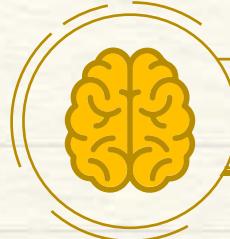


Quadratic formula is used where factorization method is difficult to apply.

Nature of Roots

$$\text{Discriminant } (D) = "b^2 - 4ac".$$

Nature of Roots		
Type of Roots	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$
Value of Roots	$\frac{-b - \sqrt{D}}{2a}, \frac{-b + \sqrt{D}}{2a}$	$\frac{-b}{2a}, \frac{-b}{2a}$
		No Real Roots Not Valid



Mind Map

