Practice Challenge - Objective

Subject: Mathematics

Topic : Some Applications of Trignometry Exam Prep 1

Class: X

1. The value of $\cos 30^\circ$ is

A. $\frac{1}{2}$

B. $\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{\sqrt{2}}$

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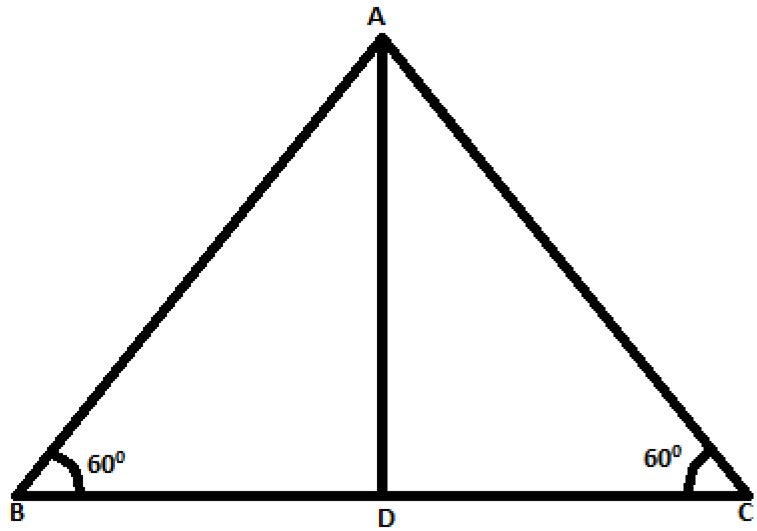


Consider an equilateral ΔABC with sides of 2 units as shown in fig. Let there be a perpendicular dropped from A to side BC cuttng BC at D.

Now, in $\Delta ABD\ and \Delta ACD$:

AD is common

AB = AC (sides of an equilateral ΔABC)

and $\angle ADC = \angle ADB = 90^\circ$

$\therefore \Delta ABD \cong \Delta ACD \quad (RHS\ congruency)$

$\Rightarrow \angle BAD = \angle CAD \quad (corresponding\ angles\ of\ congruent\ \Delta s)$

$\Rightarrow \angle BAD + \angle CAD = \angle BAC = 60^\circ$

$\Rightarrow \angle BAD = \angle CAD = 30^\circ$

$and\ BD = DC = \frac{1}{2}BC = 1$

$In\ \Delta ABD,$

$AB^2 = AD^2 + BD^2$

$\Rightarrow 2^2 = 1^2 + AD^2$

$\Rightarrow AD = \sqrt{3}$

$cos(\angle BAD) = cos30^\circ = \frac{AD}{BA} = \frac{\sqrt{3}}{2}$

$Also,\ cos(\angle ABD) = cos60^\circ = \frac{BD}{AB} = \frac{1}{2}$

2. The height of a tree is $10\sqrt{3}$ m, if a boy looks at the top of the tree with an angle of elevation of 30°, find the distance between the boy and the tree.

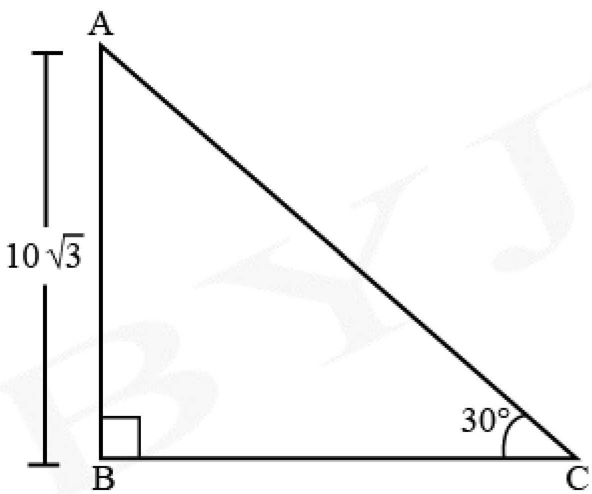
A. 10 m

B. $10\sqrt{3}$ m

C. 30 m

D. 20 m

The given figure illustrates the scenario mentioned in the question :



Given, the tree is having a height of $10\sqrt{3}\, m$.

So, AB = $10\sqrt{3}\, m$.

In Triangle ABC,

$\tan 30^\circ = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$

$\Rightarrow BC = 10\sqrt{3} \times \sqrt{3}$

$\Rightarrow BC = 30\, m.$

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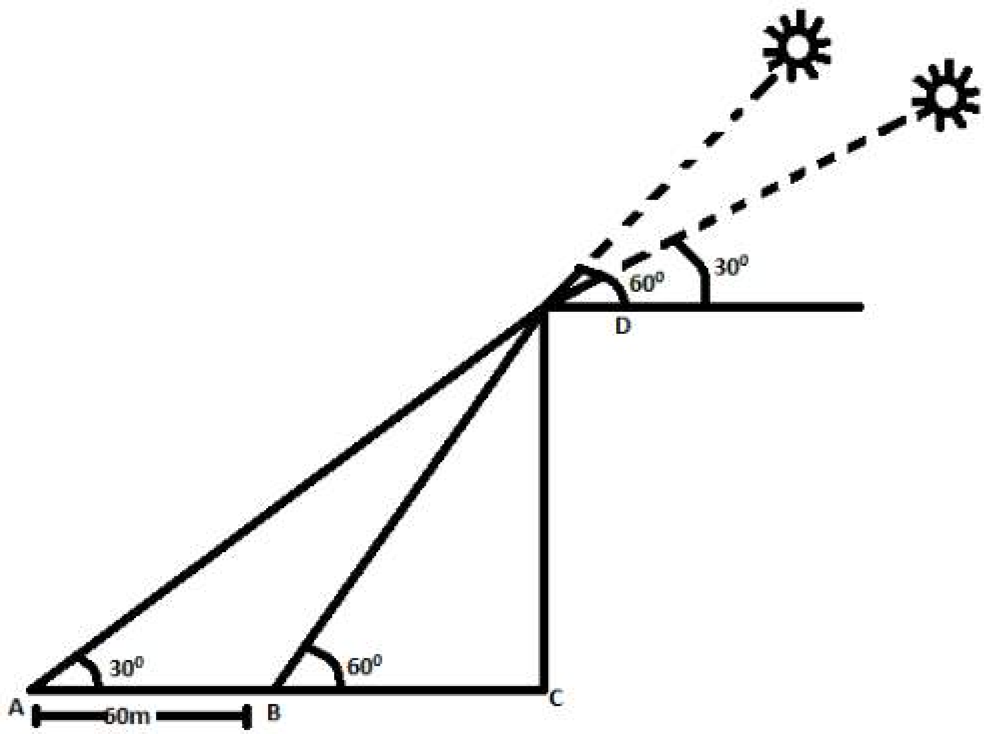
3. The shadow of a tower standing on a level ground is found to be 60 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

- **A.** 30 m
- **B.** 20 m
- **C.** $30\sqrt{3}$ m ✓
- **D.** $20\sqrt{3}$ m

The given situation is represented aptly by the figure below



$tan30^\circ = \frac{DC}{AC} = \frac{DC}{AB+BC}$

$\Rightarrow (AB+BC) = \frac{DC}{tan30^\circ} \ldots\ldots (1)$

$Also,\ tan60^\circ = \frac{DC}{BC}$

$\Rightarrow BC = \frac{DC}{tan60^\circ} \ldots\ldots (2)$

Subtracting eq(2) from (1)

$AB = \frac{DC}{tan30^\circ} - \frac{DC}{tan60^\circ}$

$\Rightarrow 60 = DC\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) \Rightarrow DC = 30\sqrt{3}m$

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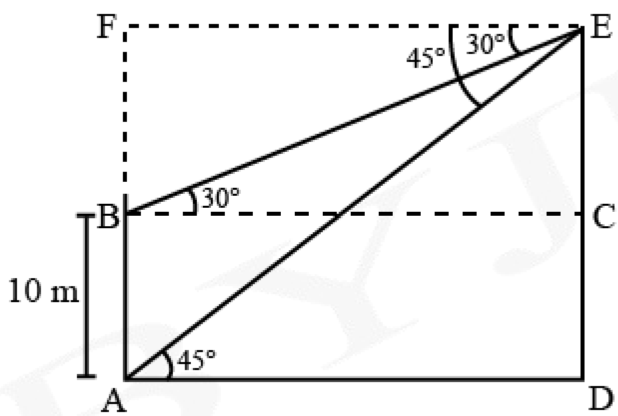
4. The angles of depression of the top and the bottom of a 10 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building.

A. 10 m

B. 15 m

C. $5(\sqrt{3} + 3)$ m

D. 5 m



The above figure represents the situation aptly

$\angle CBE = \angle BEF \; and \; \angle DAE = \angle AEF \quad$ (alternate angles)

$tan(\angle EAD) = tan45^\circ = \frac{ED}{AD} = \frac{EC+CD}{AD}$

$\Rightarrow AD \times tan45^\circ = EC + CD \ldots\ldots(1)$

$and \; tan(\angle EBC) = tan30^\circ = \frac{EC}{CB}$

$\Rightarrow CB \times tan30^\circ = EC \ldots\ldots(2)$

Subtracting eq(2) from (1)

$\Rightarrow AD \times tan45^\circ - CB \times tan30^\circ = CD$

$\Rightarrow AD(tan45^\circ - tan30^\circ) = CD \qquad (\because AD = CB)$

$\Rightarrow AD\left(1 - \frac{1}{\sqrt{3}}\right) = 10$

$\Rightarrow AD = 5(3 + \sqrt{3})m$

$\Rightarrow ED = AD(tan45^\circ = 1)$

ED is the height of the building.

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5. The value of $tan 58^\circ \ tan 32^\circ \ tan 57^\circ \ tan 33^\circ$ is

- **A.** 0
- **B.** 1
- **C.** 2
- **D.** 3

$$tan 32^\circ = tan(90^\circ - 58^\circ)$$
$$= cot 58^\circ$$

$$tan 33^\circ = tan(90^\circ - 57^\circ)$$
$$= cot 57^\circ$$

Therefore,

$tan 58^\circ \ tan 32^\circ \ tan 57^\circ \ tan 33^\circ$

= $tan 58^\circ \ cot 58^\circ \ tan 57^\circ \ cot 57^\circ$

= $tan 58^\circ \ \frac{1}{tan 58^\circ} tan 57^\circ \frac{1}{tan 57^\circ}$

[since cot A=$\frac{1}{tan A}$]

=1

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6. From the top of a lighthouse H m tall, a person observes the angle of depression of a boat to be 60°. Another person who is $\frac{H}{3}$ m from the top of a lighthouse observes the angle of depression of another boat directly behind the first boat to be 45°. Find the distance between the two boats.

(Take $\sqrt{3}$ = 1.7)

- ✗ **A.** $\frac{H}{10}$
- ✓ **B.** $\frac{H}{3}(2-\sqrt{3})$
- ✗ **C.** 0.6 H
- ✗ **D.** 1.7H

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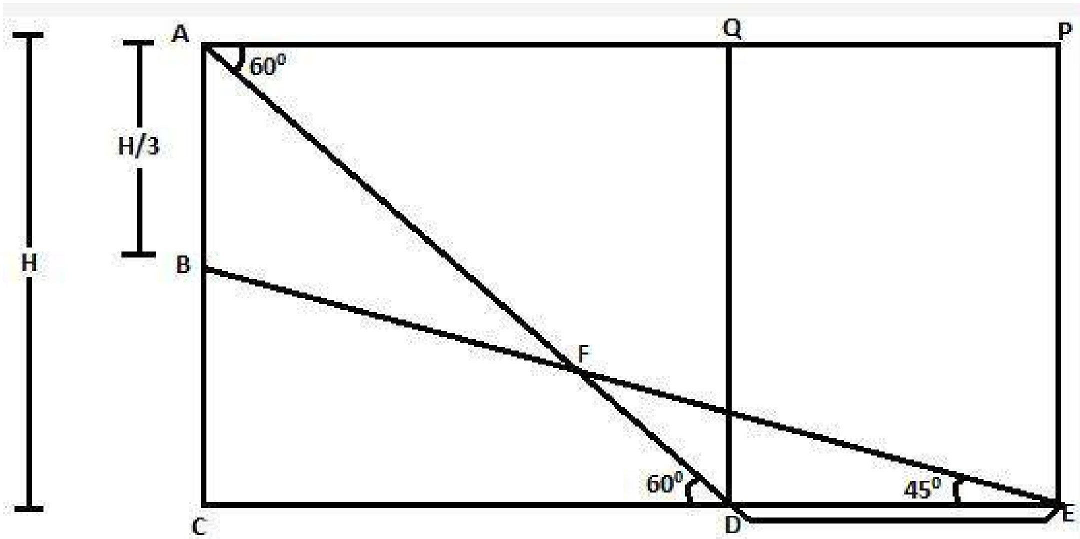

$\angle QAD = \angle ADC$ (alternate angles)

$\therefore tan60^\circ = \frac{AC}{CD} = \frac{H}{CD}$

$\Rightarrow CD = \frac{H}{tan60^\circ} = \frac{H}{\sqrt{3}} \ldots\ldots(1)$

$also\ tan45^\circ = \frac{BC}{CE} = \frac{AC-AB}{CD+DE} = \frac{H-\frac{H}{3}}{CD+DE} = \frac{\frac{2H}{3}}{CD+DE}$

$\Rightarrow CD + DE = \frac{2H}{3} \ldots\ldots(2)$

$Subtracting\ (1)\ from\ (2)$

$DE = \frac{2H}{3} - \frac{H}{\sqrt{3}}$

$= \frac{2H - H\sqrt{3}}{3}$

$= \frac{H}{3}(2 - \sqrt{3})$

$\therefore distance\ between\ boats\ is\ \frac{H}{3}(2 - \sqrt{3})$

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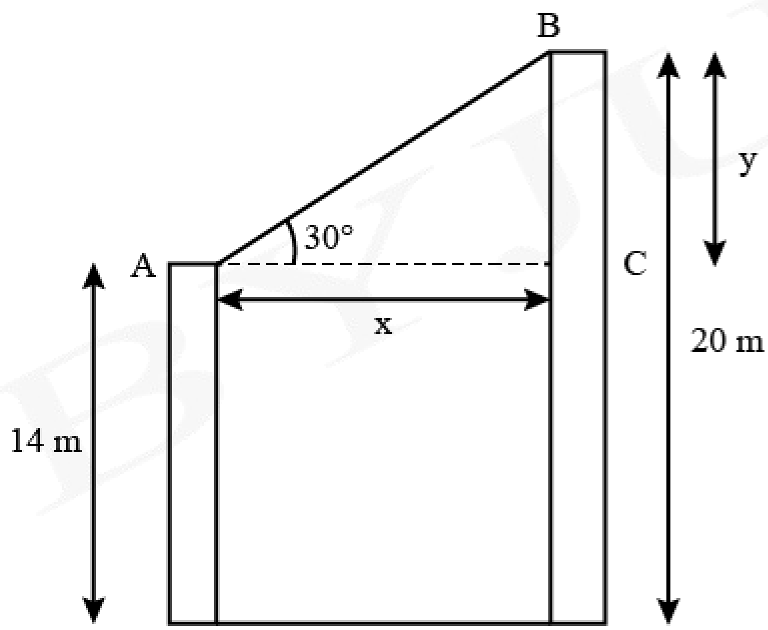
7. The tops of two poles of height 14 m and 20 m are connected by a wire which makes an angle of 30° with the horizontal. What is the length of the wire?

- **A.** 6 m
- **B.** 10 m
- **C.** 8 m
- **D.** 12 m



In triangle ABC,
BC = y = 20 - 14 = 6m

Let AB = x

$\Rightarrow \sin 30° = \frac{BC}{AB}$

$\Rightarrow \frac{1}{2} = \frac{6}{x}$

$\Rightarrow x = 12$

Thus, the length of the string is 12 m.

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8. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The angle of elevation of the sun is _________ .

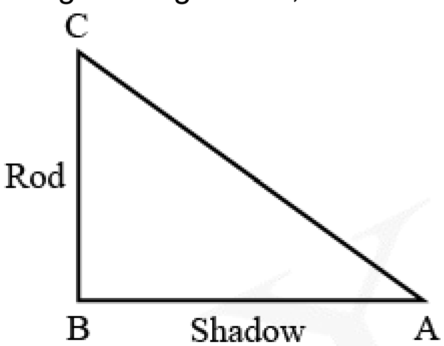
A. 30°

B. 45°

C. 60°

D. 90°

In right triangle ABC,



$tanA = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

$\Rightarrow A = 30^\circ$

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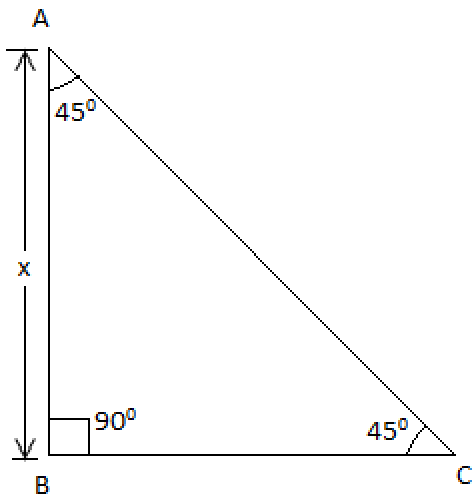
9. In the given figure, ABC is an isosceles right angle triangle, right angled at B. The ratio of the sides AB: BC : AC is _________.

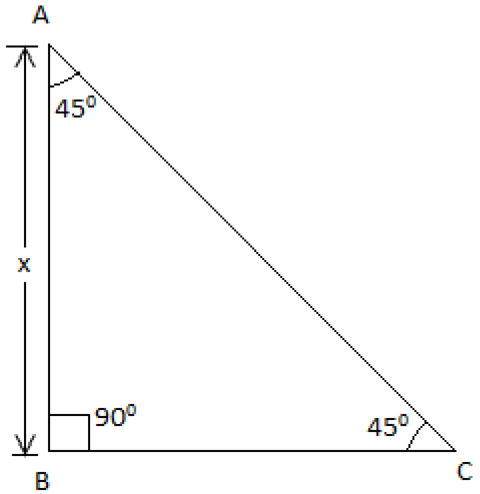


A. $1 : 1 : \sqrt{2}$

B. $\sqrt{2} : 1 : 1$

C. $\sqrt{3} : 2 : 1$

D. $1 : 2 : \sqrt{3}$



Let us assume the length of side AB of the triangle to be x units.
Applying trignometric ratios to the sides, we get :
$sin\ 45^\circ = (\frac{x}{AC})$
$\Rightarrow \frac{1}{\sqrt{2}} = (\frac{x}{AC})$
$\Rightarrow AC = x\sqrt{2} \quad \ldots(i)$
$Similarly,$
$tan\ 45^\circ = (\frac{x}{BC})$
$\Rightarrow 1 = (\frac{x}{BC})$
$\Rightarrow BC = x \quad \ldots(ii)$
So, the ratios of the sides of the triangle with angles
$45^\circ, 45^\circ \& \ 90^\circ = x : BC : AC$
$= x : x : x\sqrt{2}(from\ (i) \& (ii))$
$= 1 : 1 : \sqrt{2}$

An alternate and shortcut method of solving this question is:

For the given triangle, as two angles are equal; the two sides opposite to these angles will also be equal.
And as the third angle is 90°, the triangle is right - angled triangle.
Let us assume the length of the equal sides is equal to x units.
So, length of the hypotenuse = $\sqrt{(x^2 + x^2)}$ = $x\sqrt{2}$
So, Ratio of the sides of the triangle = x : x : $x\sqrt{2}$
$\Rightarrow$ Ratio of the sides of the triangle = 1 : 1 : $\sqrt{2}$

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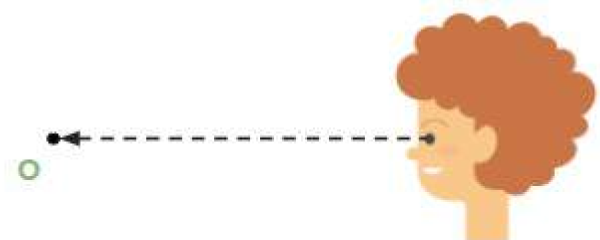
10. What is the line drawn from the eye of the observer to the the object viewed by the observer?

- **A.** Line of sight ✔
- **B.** Vertical line ✗
- **C.** Transversal line ✗
- **D.** Horizontal line ✗

The line drawn from the eye of an observer to a point on the object viewed is called as line of sight.



The dashed line shown is the the line of sight.