Practice Challenge - Subjective

Subject: Mathematics

Topic : Some Applications of Trignometry Exam Prep 1

Class: X

1. Two poles are erected from the ground. The length of the longer pole is 10m. If the distance between the tips of the poles is 6m, and the angle of elevation of the tip of the longer pole from that of the shorter pole is 30°, find the length of the second pole.

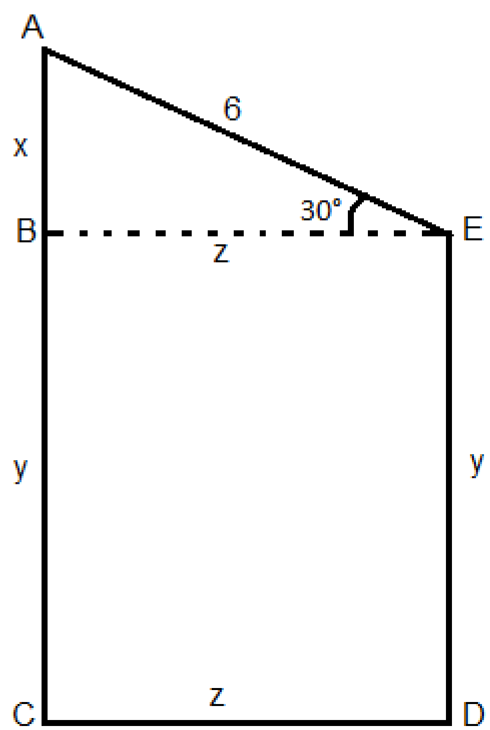
   Refer to the following figure:

   AB is the longer pole of length 10m. ED is the shorter pole. Let the lengths be broken down as shown.

   In ΔABE, sin30° $\Rightarrow$ x=3

   Since x + y = 10 and x = 3, y = 7m.

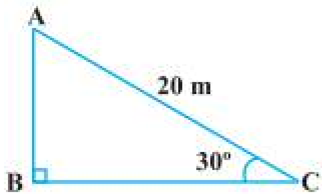
   Observe that BC = y = ED = length of the shorter pole = 7m.



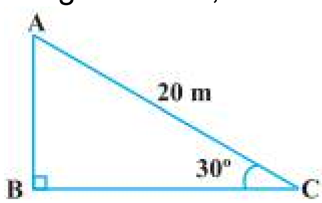
Copyright © Think and Learn Pvt. Ltd.

Practice Challenge - Subjective

2. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.



Let AB be the vertical pole and AC be 20 m long rope tied to point C.
In right ΔABC,

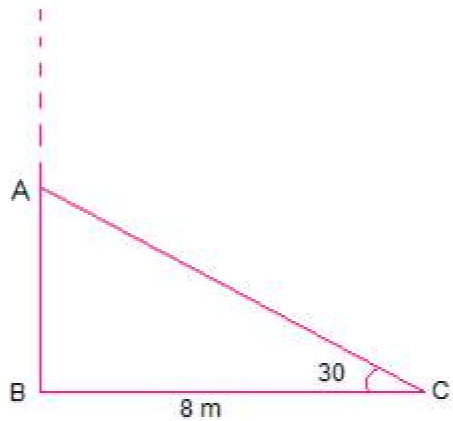


$sin30^{\circ} = \frac{AB}{AC}$
$\Rightarrow \frac{1}{2} = \frac{AB}{20}$
$\Rightarrow AB = \frac{20}{2}$
$\Rightarrow AB = 10$
Thus, the height of the pole is 10 m.

Practice Challenge - Subjective

3. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Let AC be the broken part of the tree.
$\therefore$Total height of the tree $= AB + AC$
In right ΔABC,
cos $30^{\circ} = \frac{BC}{AC}$

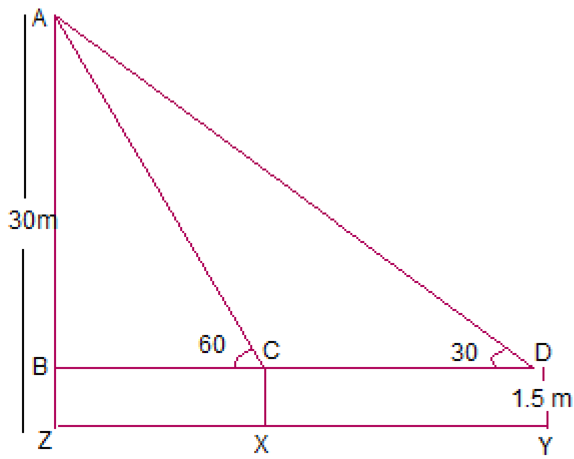


$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC}$

$\Rightarrow AC = \frac{16}{\sqrt{3}}$
Also, $tan 30^{\circ} = \frac{AB}{BC}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$
$\Rightarrow AB = \frac{8}{\sqrt{3}}$
Therefore, the total height of the tree $= AB + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$ = $8\sqrt{3}$ m.

Practice Challenge - Subjective

4. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from $30^\circ\ to\ 60^\circ$ as he walks towards the building. Find the distance he walked towards the building.



Let the boy initially stands at point Y with inclination 30° and then he approaches the building to the point X with inclination 60°.
$\therefore$ XY is the distance he walked towards the building.
Also, XY = CD
Height of the building = AZ = 30 m
$AB = AZ - BZ = (30 - 1.5) = 28.5m$
As per the question,
In right ΔABD,
tan $30^\circ = \frac{AB}{BD}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{BD}$
$\Rightarrow BD = 28.5\sqrt{3}m$
Also,
In right ΔABC,
tan $60^\circ = \frac{AB}{BC}$
$\Rightarrow \sqrt{3} = \frac{28.5}{BC}$
$\Rightarrow BC = \frac{28.5}{\sqrt{3}} = \frac{28.5\sqrt{3}}{3}$ m
$\therefore XY = CD = BD - BC$

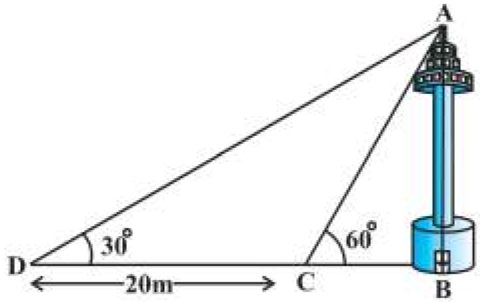
$$= (28.5\sqrt{3} - \frac{28.5\sqrt{3}}{3})$$
$$= 28.5\sqrt{3}(1 - \frac{1}{3})$$
$$= 28.5\sqrt{3} \times \frac{2}{3}$$
$$= \frac{57}{\sqrt{3}}$$
$$= 19\sqrt{3}\ m$$

Thus, the distance boy walked towards the building is $\frac{19}{\sqrt{3}}$ m.

Copyright © Think and Learn Pvt. Ltd.

Practice Challenge - Subjective

5. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

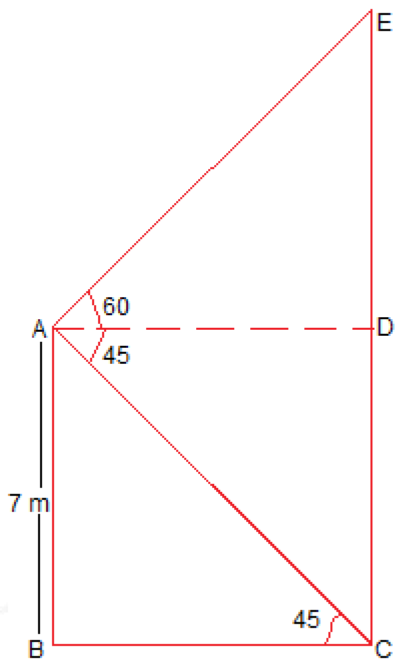


Here, AB is the height of the tower and BC is the width of canal.
CD = 20 m
As per question,
In right ΔABD,
tan $30^\circ = \frac{AB}{BD}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{(20+BC)}$
$\Rightarrow AB = \frac{(20+BC)}{\sqrt{3}} \ldots (i)$
Also,
In right ΔABC,
tan $60^\circ = \frac{AB}{BC}$
$\Rightarrow \sqrt{3} = \frac{AB}{BC}$
$\Rightarrow AB = \sqrt{3}BC \ldots (ii)$
From equation (i) and (ii)
$AB = \sqrt{3}BC = \frac{(20+BC)}{\sqrt{3}}$
$\Rightarrow 3BC = 20 + BC$
$\Rightarrow 2BC = 20 \Rightarrow BC = 10m$
Putting the value of BC in equation (ii)
$AB = 10\sqrt{3}m$
Thus, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

Copyright © Think and Learn Pvt. Ltd.

6. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Let AB be the building of height 7 m and EC be the height of tower.
A is the point from where elevation of tower is 60° and the angle of depression of its foot is 45°.



$EC = DE + CD$

Also, $CD = AB = 7m$ and $BC = AD$
According to question,
In right ΔABC,
tan $45^\circ = \frac{AB}{BC}$
$\Rightarrow 1 = \frac{7}{BC}$
$\Rightarrow BC = 7m = AD$
Also,
In right ΔADE,
tan $60^\circ = \frac{DE}{AD}$
$\Rightarrow \sqrt{3} = \frac{DE}{7}$
$\Rightarrow DE = 7\sqrt{3}m$
Height of the tower $= EC = DE + CD = (7\sqrt{3} + 7)m = 7(\sqrt{3} + 1)m$

Copyright © Think and Learn Pvt. Ltd.

Practice Challenge - Subjective

7. A window of a house is h m above the ground. Form the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be α and β, respectively. Prove that the height of the other house is $h(1 + tan\ \alpha\ cot\ \beta)m$.

   Let the height of the other house = OQ = H

   And, OB = MW = x m

   Given that, height of the first house = WB = h = MO

   And $\angle QWM = \alpha,\ \angle OWM = \beta = \angle WOB$

   [alternate angle]

   Now, in $\Delta WOB, tan\ \beta = \frac{WB}{OB} = \frac{h}{x}$ (window)

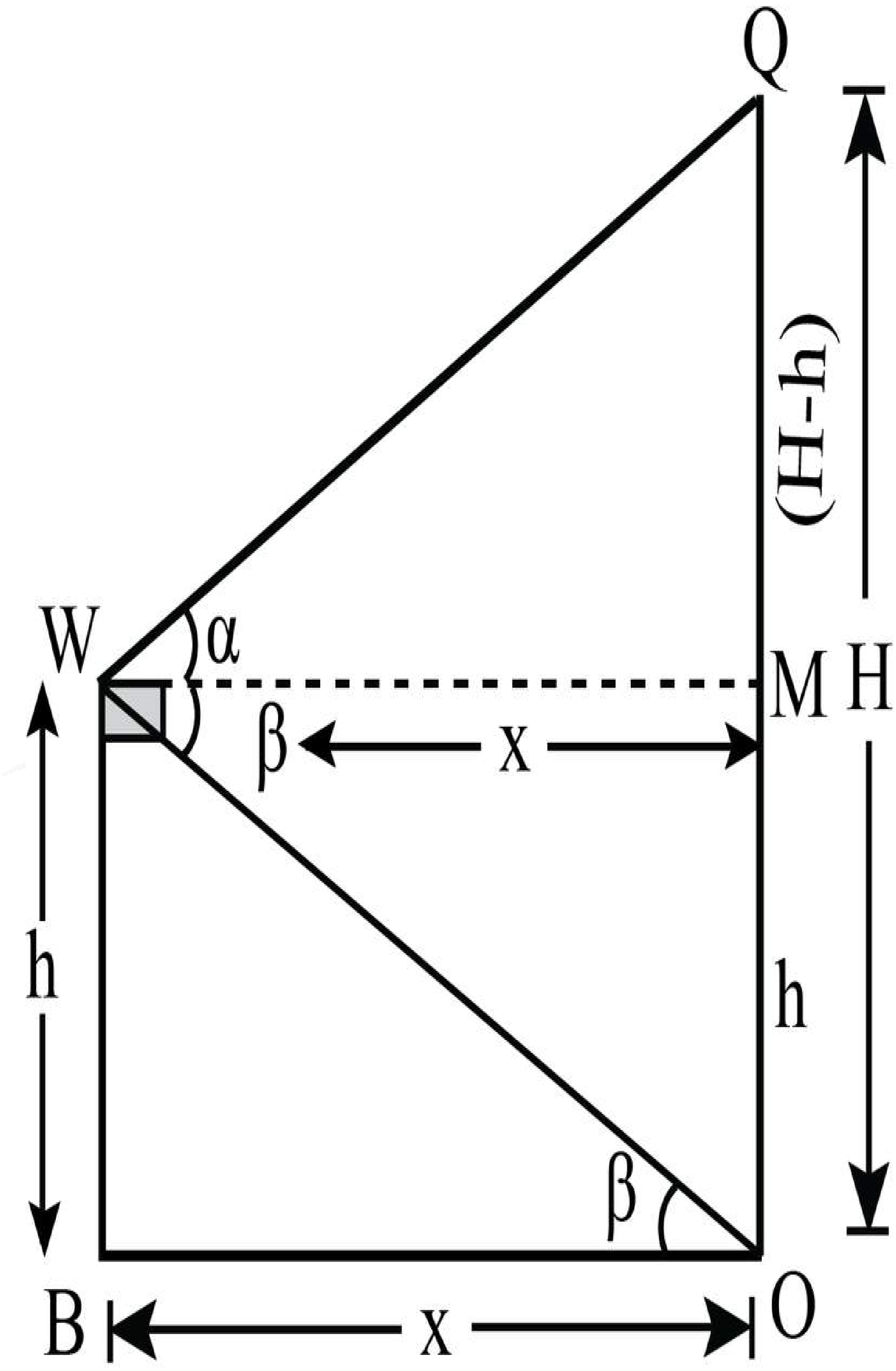
   $\Rightarrow x = \frac{h}{tan\ \beta} \ldots\ldots (i)$

   And in $\Delta QWM,\ tan\ \alpha = \frac{QM}{WM} = \frac{OQ - MO}{WM}$

   $\Rightarrow tan\ \alpha = \frac{H-h}{x}$

   $\Rightarrow x = \frac{H-h}{tan\ \alpha}$ ...(ii)

Copyright © Think and Learn Pvt. Ltd.



From Eq. (i) and (ii),

$$\frac{h}{\tan\beta} = \frac{H-h}{\tan\alpha}$$

$\Rightarrow h\ tan\ \alpha = (H - h) tan\ \beta$

$\Rightarrow h\ tan\ \alpha = H\ tan\ \beta - h\ tan\ \beta$

$\Rightarrow H\ tan\ \beta = h(tan\ \alpha + tan\ \beta)$

$\therefore H = h\left(\frac{tan\ \alpha + tan\ \beta}{tan\ \beta}\right)$

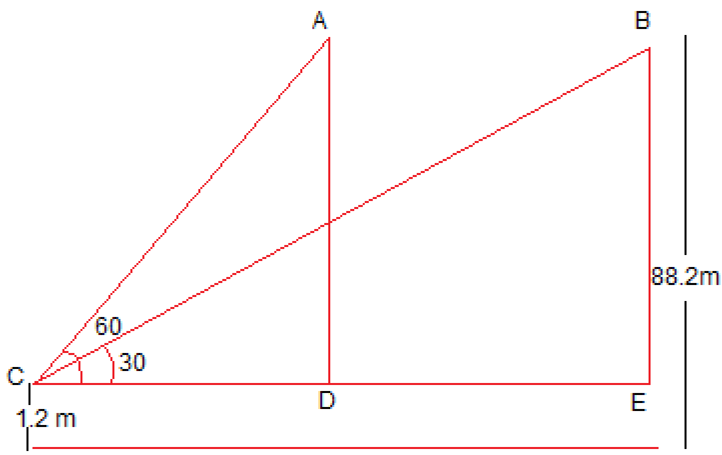
$= h\left(1 + tan\ \alpha.\frac{1}{tan\ \beta}\right) = h(1 + tan\ \alpha. cot\ \beta) \qquad \left[\because cot\ \theta = \frac{1}{tan\ \theta}\right]$

Hence, the required height of the other house is $h(1 + tan\ \alpha. cot\ \beta)$

BYJU'S

8. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

Let the initial position of the balloon be A and final position be B.
Height of balloon above the girl height $= 88.2m - 1.2m = 87m$
Distance travelled by the balloon = $DE = CE - CD$
As per question,



In right ΔBEC,

$\tan 30^\circ = \frac{BE}{CE}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{CE}$
$\Rightarrow CE = 87\sqrt{3}m$
Also,
In right ΔADC,
$\tan 60^\circ = \frac{AD}{CD}$
$\Rightarrow \sqrt{3} = \frac{87}{CD}$
$\Rightarrow CD = \frac{87}{\sqrt{3}}m = 29\sqrt{3}m$
Distance travelled by the balloon
$= DE = CE - CD = (87\sqrt{3} - 29\sqrt{3})m = 29\sqrt{3}(3-1)m$ =100.45 m

Copyright © Think and Learn Pvt. Ltd.