Practice Challenge - Subjective

Subject: Mathematics

Topic : Statistics Exam Prep 1

Class: X

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	$0-2$	$2-4$	$4-6$	$6-8$	$8-10$	$10-12$	$12-14$
Number of Houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Let us find class marks (x_i) for each interval by using the relation:

$$\text{Class Mark (x\_i)} = \frac{\text{Upper class limit } + \text{lower class limit}}{2}$$

Now we may compute $x_i \; and \; f_i x_i$ as following

Number of plants	Number of Houses (f_i)	x_i	$f_i x_i$
$0-2$	1	1	$1 \times 1 = 1$
$2-4$	2	3	$2 \times 3 = 6$
$4-6$	1	5	$1 \times 5 = 5$
$6-8$	5	7	$5 \times 7 = 35$
$8-10$	6	9	$6 \times 9 = 54$
$10-12$	2	11	$2 \times 11 = 22$
$12-14$	3	13	$3 \times 13 = 39$
$Total$	20		162

From the table we may observe that:

$$Mean\ (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

So, mean number of plants per house is 8.1.

We have used the direct method as values of class marks $(x_i) \; and \; f_i$ are small.

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2. If the mean of the following frequency distibution is 24, find the value of p.

Class	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$
Frequency	3	4	p	3	2

The given data is shown as follows:

Class	Frequency (f_i)	Class mark (x_i)	(f_ix_i)
0 - 10	3	5	15
10 - 20	4	15	60
20 - 30	p	25	25p
30 - 40	3	35	105
40 - 50	2	45	90
Total	$\sum f_i$=12+p		$\sum f_ix_i$=270+25p

The mean of given data is given by

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 24 = \frac{270+25p}{12+p}$

$\Rightarrow 24(12 + p) = 270 + 25p$

$\Rightarrow 288 + 24p = 270 + 25p$

$\Rightarrow 25p - 24p = 288 - 270$

$\Rightarrow p = 18$

Hence, the value of p is 18.

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3. An incomplete distribution is given as follows:

Variable: 0 -10 10 - 20 20 - 30 30 - 40 40 - 50 50 - 60 60 - 70

Frequency: 10 20 ? 40 ? 25 15

You are given that the median value is 35 and the sum of all the frequencies is 170. Using the median formula, fill up the missing frequencies.

Class interval	Frequency	Cumulative frequency
0 – 10	10	10
10 – 20	20	30
20 – 30	f_1	30 + f_1 (F)
30 – 40	40 (F)	70 + f_1
40 – 50	f_2	70 + f_1 + f_2
50 – 60	25	95 + f_1 + f_2
60 – 70	15	110 + f_1 + f_2
	N = 170	

Given

Median = 35

Then median class = 30 – 40

L = 30, h = 40 – 30 = 10, f = 40, F = 30 + f_1

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$35 = 30 + \frac{\frac{170}{2} - (30 + f_1)}{40} \times 10$$

$$35 - 30 = \frac{85 - 30 - f_1}{40} \times 10$$

$$5 = \frac{55 - f}{4}$$

$$20 = 55 - f_1$$

$f_1 = 55 – 20 = 35$

Given

Sum of frequencies = 170

10 + 20 + f_i + 40 + f_2 + 25 + 15 = 170

10 + 20 + 35 + 40 + f_2 + 25 + 15 = 170

$f_2 = 25$

$f_1 = 35$ and $f_2 = 25$

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4. A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained:

Height in cm	Number of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Find the median height.

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To calculate the median height, we need to find the class intervals and their corresponding frequencies.

The given distribution being of the less than type, 140, 145, 150, 155, 160, 165 give the upper limits of the corresponding class intervals. So, the classes should be below 140, 140-145, 145-150, 150-155, 155-160, 160-165. Observe that from the given distribution, we find that there are 4 girls with height less than 140, i.e. the frequency of class interval below 140 is 4. Now, there are 11 girls with heights less than 145 and 4 girls with height less than 140. Therefore, the number of girls with height in the interval 140 – 145 is 11 – 4 = 7. Similarly, the frequency of 145 – 150 is 29 – 11 = 18, for 150 – 155, it is 40 – 29 = 11, and so on. So, our frequency distribution table with given cumulative frequencies becomes:

Class interval	Frequency	Cumulative frequency
Below 140	4	4
140 – 145	7	11
145 – 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51

Now n = 51. an odd number that is n/ 2 = 51/ 2 =25.5 this observation lies in the class 145 – 150

Then,

L (the lower limit) = 145

cf (the cumulative frequency of the class preceding 145 – 150) = 11

f (the frequency of the median class 145 – 150) = 18

h (the class size) = 5

$$\text{Using the formula, Median } = l+\left(\frac{\frac{n}{2}-cf}{f}\right)\times h$$

$$\text{Median } = 145+\left(\frac{25.5-11}{18}\right)\times 5$$

$$= 145 + \frac{72.5}{18}$$

$$= 149.03$$

So, the median height of the girls is 149.03 cm

This means that the height of about 50% of the girls in less than this height, and 50% are taller than this height.

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5. A survey was conducted on 20 families in a locality by a group of students .What will be the mode of the data?

Age of family member	$0-20$	$20-40$	$40-60$	$60-80$	$80-100$
Number of students	7	8	2	2	1

Using, Mode = $l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$

where,

Maximum class frequency, $f_1=8$

The class corresponding to the frequency $=20-40$

Lower limit of the modal class, $l=20$

Frequency of class preceding the modal class, $f_0=7$

Frequency of class succeeding the modal class, $f_2=2$

$Now, Mode=20+\frac{8-7}{16-7-2}\times 20$

$Mode=20+\frac{1}{7}\times 20$

$\therefore Mode=22.86$

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6. The marks obtained by the sudents of a class, in an exam which was out of 50 marks,is given below.

Marks	Number of students
5	1
7	2
11	2
16	3
21	5
24	6
28	3
32	5
34	8
39	6
43	4
45	2
49	3

(i)Find mode for the ungrouped data.

(ii) Represent the same as grouped data, with class intervals of width = 10 and find the mode for the grouped data.

The mode for the ungrouped data is the most frequent observation, which is 34.

Grouped data:

Class Intervals	Frequency
$0-10$	3
$10-20$	5
$20-30$	14
$30-40$	19
$40-50$	9

Modal Class is 30-40.

Mode$= I+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

$=30+\frac{19-14}{38-14-9}\times 10=33.33$

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7. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

$Monthly\ consumption\ (in\ units)$	$Number\ of\ consumers$
$65-85$	4
$85-105$	5
$105-125$	13
$125-145$	20
$145-165$	14
$165-185$	8
$185-205$	4

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We may find the class marks by using the relation:

$Class\ mark = \frac{upper\ class\ limit + lower\ class\ limit}{2}$

Taking 135 as assumed mean a, we may find $d_i, u_i, f_i u_i$, according to step deviation method as following:

$Monthly\ consumption\ (in\ units)$	$Number\ of\ consumers\ (f_i)$	$x_i\ class\ mark$	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
$65 - 85$	4	75	-60	-3	-12
$85 - 105$	5	95	-40	-2	-10
$105 - 125$	13	115	-20	-1	-13
$125 - 145$	20	135	0	0	0
$145 - 165$	14	155	20	1	14
$165 - 185$	8	175	40	2	16
$185 - 205$	4	195	60	3	12
$Total$	68				7

From the table, we may observe that:

$\sum f_i u_i = 7$, $\sum f_i = 68$, class size h = 20

$Mean\ \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$

$= 135 + \frac{7}{68} \times 20$

$= 135 + \frac{140}{68}$

$= 137.06$

Now, from table, it is clear that maximum class frequency is 20 belonging to class interval 125 - 145.

Modal class = 125 - 145

Lower limit l of modal class = 125

Class size h = 20

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 13

Frequency (f_2) of class succeeding the modal class = 14

$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 125 + \left[\frac{20-13}{2(20)-13-14}\right] \times 20$

$= 125 + \frac{7}{13} \times 20$

$= 125 + \frac{140}{13} = 135.77$

We know that:

3 median = mode + 2 mean

= 135.77 + 2 (137.06)

= 135.77 + 274.12 = 409.89

Median = 409.9 / 3 = 136.63

So, median, mode, mean of given data is 136.63, 135.77, 137.06 respectively.

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8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	$0-6$	$6-10$	$10-14$	$14-20$	$20-28$	$28-38$	$38-40$
Number of students	11	10	7	4	4	3	1

We can find the class mark of each interval by using the relation:

$x_i = \frac{\text{Upper class limit } + \text{lower class limit}}{2}$

Now, taking 17 as assumed mean a, we can calculate d_i and $f_i d_i$ as following:

Number of days	Number of students f_i	x_i	$d_i = x_i - 16$	$f_i d_i$
$0-6$	11	3	-14	-154
$6-10$	10	8	-9	-90
$10-14$	7	12	-5	-35
$14-20$	4	17	0	0
$20-28$	4	24	7	28
$28-38$	3	33	16	48
$38-40$	1	39	22	22
Total	40			-181

Now, we observe that:

$\sum f_i = 40$

$\sum f_i d_i = -181$

$\text{mean } \bar{x} = a + \left(\frac{\sum f_i d_i}{\sum f_i}\right)$

$= 17 + \left(\frac{-181}{40}\right)$

=17-4.525

=12.475

=12.48

So, mean number of days for which a student was absent is 12.48 days.