Practice Challenge - Objective

Subject: Mathematics

Topic : Surface areas and Volumes
Exam Prep 1 Class: X

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1. A cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid(in cm^2).

   A. 332.5 cm^2

   B. 346.8 cm^2

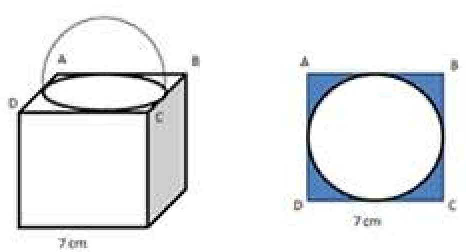
   C. 312.5 cm^2

   D. 320 cm^2

The hemisphere surmounts the cube, the maximum diameter the hemisphere can have = side of cube as shown in the figure below. The hemisphere will just touch the sides of top face of the cube.

Therefore, maximum diameter Hemisphere can have = 7 cm

$\Rightarrow$ Radius of Hemisphere $=$ r $= \frac{7}{2} = 3.5\ cm$



Surface area of solid = Surface area of 5 Faces of cube + Surface area of Hemisphere + Surface area left uncovered on the top face of the cube (shown in blue)

$= 5l^2 + 2.\pi.r^2 +$ Area of Square ABCD - Area of inscribed circle in square ABCD

$= (5 \times 7 \times 7) + (2 \times \frac{22}{7} \times 3.5 \times 3.5) + (49 - (\frac{22}{7} \times 3.5 \times 3.5))$

$= 245 + 77 + (49 - 38.5) = 322 + 10.5 = 332.5\ cm^2$

$\therefore$ The surface area of combined solid = $332.5\ cm^2$

Practice Challenge - Objective

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made . (Assume the outer and inner dimensions of the model to be nearly the same.)

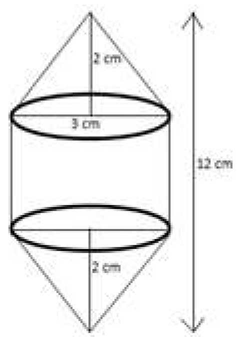
- A. 50 cm^3
- B. 66 cm^3
- C. 62 cm^3
- D. 75 cm^3

Practice Challenge - Objective

Diameter of Cylinder = Diameter of Cones = 3 cm

$\Rightarrow$ Radius of Cylinder = Radius of Cones $= r = \frac{3}{2} = 1.5\ cm$

Height of each cone $= h_1 = 2\ cm$



Length of model = 12 cm

Height of Cylinder, h
= Total Height of Model - height of 1st Cone - height of 2nd Cone
$= 12 - 2 - 2$
$= 8\ cm$

Volume of air in the model
= Volume of Solid
= Volume of Cylinder + Volume of 1st Cone + Volume of 2nd Cone

We know that, volume of a cylinder of radius r and height h is $\pi r^2 h$ and volume of a cone of radius R and height H is given by $\frac{1}{3}\pi R^2 H$.

Therefore, volume of air in the model
$= \pi r^2 h + \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_1$

$= \pi r^2 (h + \frac{h_1}{3} + \frac{h_1}{3})$

$= \frac{22}{7} \times 1.5 \times 1.5 (8 + \frac{4}{3})$

$= \frac{22}{7} \times 1.5 \times 1.5 (\frac{28}{3})$

$= 66\ cm^3$

Practice Challenge - Objective

3. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere(in cm).

- A. 12
- B. 21
- C. 18
- D. 15

Radius of first sphere $= r_1 = 6\ cm$
Radius of second sphere $= r_2 = 8\ cm$
Radius of third sphere $= r_3 = 10\ cm$
Let radius of resulting sphere be $r\ cm$.
According to given condition, we have
Volume of first sphere + Volume of second sphere + Volume of third sphere = Volume of resulting sphere

$\Rightarrow \frac{4}{3}\pi(r_1)^3 + \frac{4}{3}\pi(r_2)^3 + \frac{4}{3}\pi(r_3)^3 = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{4}{3}[(r_1)^3 + (r_2)^3 + (r_3)^3] = \frac{4}{3}r^3$

$\Rightarrow (6^3 + 8^3 + (10)^3) = r^3$

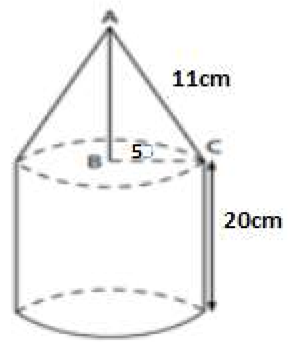
$\Rightarrow (216 + 512 + 1000) = r^3$

$\Rightarrow r^3 = 1728$

$\Rightarrow r = 12\ cm$

Practice Challenge - Objective

4. Find the surface area of the given below figure having dimension in cm as shown.



A. $900\ cm^2$

B. $880\ cm^2$

C. $650\ cm^2$

D. $400\ cm^2$

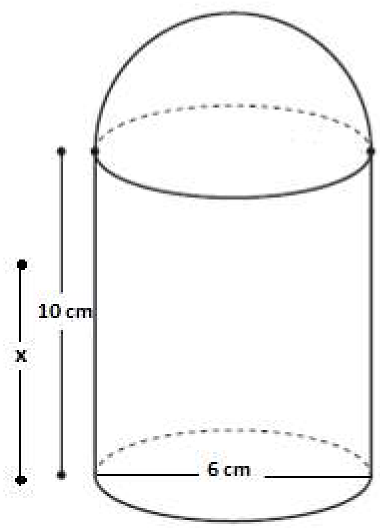
Surface area = CSA of cylinder + CSA of cone + Base area

$$= 2\pi rh + \pi rl + \pi r^2$$
$$= 2\pi(5)(20) + \pi(5)(11) + \pi(5)^2$$
$$= \pi \times 5(40 + 11 + 5)$$
$$= \frac{22}{7} \times 5 \times 56 = 880\ \text{cm}^2$$

$\therefore$ Surface area of the figure is $880\ \text{cm}^2$.

Practice Challenge - Objective

5. Anita buys a new salt cellar in the shape of a cylinder topped by a hemisphere as shown below. The cylinder has a diameter of 6 cm and a height of 10 cm. She pours the salt into the salt cellar, so that it takes up half the total volume of the cellar. Find the depth of the salt, marked with x in the diagram



A. 3 cm

B. 9 cm

C. 6 cm

D. 12 cm

Let the depth of the salt in the cellar be $'x'$

Volume of salt cellar = volume of cylinder + volume of hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \pi \times 3^2 \times 10 + \frac{2}{3}\pi \times 3^3$$
$$= \pi[90 + 18] = 108\pi \text{ cm}^3$$

So height x will come on the cylinder.

Half of total volume $= 54\pi \text{ cm}^3$

$$\pi r^2 x = 54\pi$$
$$\pi \times 3^2 \times x = 54\pi$$
$$9x = 54$$
$$\Rightarrow x = 6$$

$\therefore$ The salt will be 6 cm deep.

6. What is the diameter (in cm) of a sphere whose surface area is 616 cm^2?

- **A.** 28
- **B.** 21
- **C.** 7
- **D.** 14 ✔

Surface area of sphere =

$4\pi r^2 = 616$

$\Rightarrow r^2 = 616 \times \frac{7}{22} \times \frac{1}{4}$

$\Rightarrow r^2 = 49$

$\Rightarrow r = \sqrt{49}$

$\Rightarrow r = 7 \text{ cm}$

$\therefore$ The diameter, $D = 2r = 2 \times 7 = 14 \text{ cm}$

Practice Challenge - Objective

7. There is hollow cube. Its external edge is 5 cm long and its internal edge is 3 cm long. What is its surface area in cm^2 ?

- ✔ **A.** 150
- ✖ **B.** 36
- ✖ **C.** 186
- ✖ **D.** 114

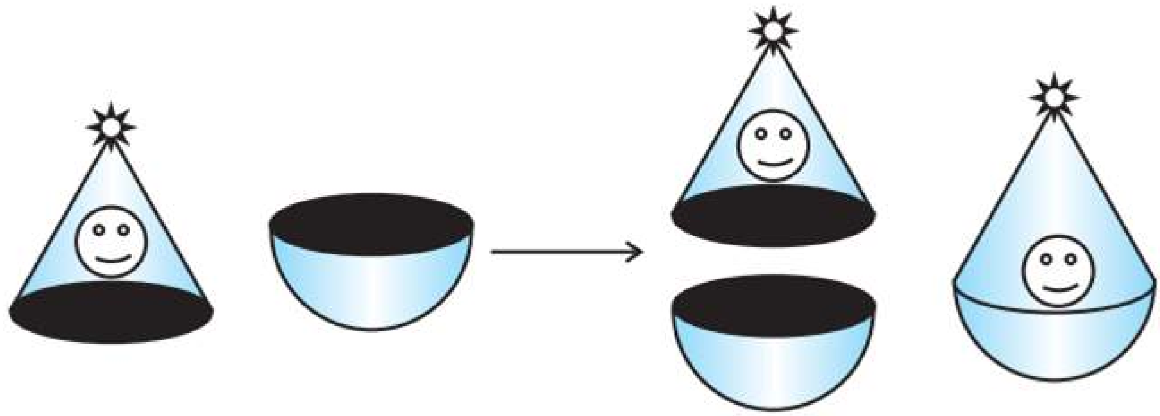
Here, it is necessary to realise that the total surface area of an object is the total area of surface of an object that is visible to human eye. In case of a hollow cube, the internal surface is hidden and is completely invisible to us.
This means that for hollow cubes, surface area is just simply the external surface area. This is nothing but the sum of the areas of each face of cube (which is nothing but the total surface area of the cube).
We know that if 'a' is the edge of cube, area of one face is a^2.
Then the sum of areas of 6 faces will be $6a^2$.
Therefore, surface area of cube
$= 6 \times 5^2$
$= 150 \text{ cm}^2$

8. A toy has a hemispherical base with a conical top attached to it. Radius of the hemisphere = 6 cm. Height of the cone = 8 cm. What is the surface area of the toy?

- **A.** $132\pi\ cm^2$
- **B.** $60\pi\ cm^2$
- **C.** $82\pi\ cm^2$
- **D.** $100\pi\ cm^2$

Practice Challenge - Objective

The diagram given below can represents the toy.



We can see from the diagram that the bottom circular surface of the cone and the top circular surface of hemisphere come together. These 2 surfaces have been darkened. Once they come together, they become invisible to the eye as observable in the final diagram. In other words these 2 surfaces will not be a part of the surface of the combined body which is the toy.

Hence the surface area of the toy = C.S.A of cone + C.S.A of hemisphere

Cone Radius r = 6 cm, Height h = 8 cm

Slant height l = $\sqrt{r^2 + h^2}$ = $\sqrt{6^2 + 8^2}$ = 10 cm

C.S.A of cone = πrl = π × 6 × 10 = 60π cm^2

C.S.A of hemisphere = 2πr^2 = 2$\pi(6)^2$ = 72π cm^2

Surface area of the toy = C.S.A of cone + C.S.A of hemisphere

= 72π + 60π

= 132π cm^2

Practice Challenge - Objective

9. A factory is designed as shown in the figure. There are 150 people working in the factory and each occupy a space of 0.5 m^3. Find the maximum volume available to place machinery in the factory. (Take $\pi = \frac{22}{7}$)

A. $5880m^3$

B. $8960m^3$

C. $8885m^3$

D. $5805m^3$

The factory is a combination of a cuboid of dimensions 40m x 14m x 5m and half a cylinder of radius 7m and height 40m.

Volume of the factory $= lbh + \frac{1}{2}\pi r^2 h$

$$= 40 \times 14 \times 5 + \frac{1}{2}\pi(7)^2 \times 40.$$

= $5880m^3$

Volume occupied by people = $150 \times 0.5 = 75\ m^3$

Maximum volume permissible for machinery = $5805m^3$.

Practice Challenge - Objective

10. A milk carrying container has the shape of a cylinder mounted on a frustum. The radius of the cylinder is 14 cm and height is 20 cm. The other diameter of the frustum is 7 cm and its height is 5 cm. What is the curved surface area of the container?

A. $1880\ cm^2 (approx)$

B. $201.12\ cm^2 (approx)$

C. $1060.5\ cm^2 (approx)$

D. $2399.65\ cm^2 (approx)$

Curved surface area of the required container = Surface area of cylinder + Surface area of frustum.

Radius of the cylinder, R = 14 cm
Height of the cylinder, h = 20 cm
Curved surface area of cylinder = $2\pi Rh$
= $2 \times \frac{22}{7} \times 14 \times 20$
= $1760\ cm^2$

Radii of the furstum are R and r, where R is 14 cm and r is 3,5 cm.
Slant height of frustum, l = $\sqrt{(14 - 3.5)^2 + 5^2} = 11.63$

Curved surface area of the frustum = $\pi l(R + r)$
= $\frac{22}{7} \times 17.5 \times 11.63 = 639.65\ cm^2$

$\therefore$ Total curved surface area of the container
= $1760\ cm^2 + 639.65\ cm^2 = 2399.65\ cm^2$