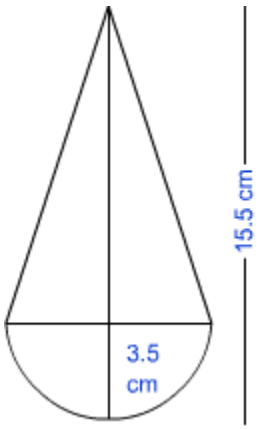
Practice Challenge - Subjective

Subject: Mathematics

Topic : Surface areas and Volumes
Exam Prep 1 Class: X

1. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.



Radius of cone $= 3.5\ cm$,
Total height of the toy = 15.5cm , height of cone $= 15.5 - 3.5 = 12$ cm
Slant height of cone can be calculated as follows:
$l = \sqrt{h^2 + r^2}$
$= \sqrt{12^2 + 3.5^2}$
$= \sqrt{144 + 12.25}$
$= \sqrt{156.25} = 12.5\ cm$
Curved surface area of cone can be calculated as follows:
$= \pi r l$
$= \frac{22}{7} \times 3.5 \times 12.5$
$= 137.5\ cm^2$
Curved surface area of hemispherical portion can be calculated as follows:
$= 2\pi r^2$
$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$
$= 77\ cm^2$
Hence, total surface area $= 137.5 + 77 = 214.5\ cm^2$

2. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

Height of cylinder $= 14 - 5 = 9$ mm,
Radius, r $= 2.5$ mm
Curved surface area of cylinder
$= 2\pi rh$
$= 2\pi \times 2.5 \times 9$
$= 45\pi\ mm^2$
Curved surface area of two hemispheres
$= 2 \times 2\pi r^2$
$= 4\pi \times 2.5^2$
$= 25\pi\ mm^2$
Total surface area:
=Curved surface area of cylinder + Curved surface area of two hemispheres
$=45\pi + 25\pi$
$= 70\pi = 220\ mm^2$

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3. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Radius of cylinder = 6 cm, height of cylinder = 15 cm
Radius of cone = 3 cm, height of cone = 12 cm
Radius of hemispherical top on ice cream = 3 cm

Volume of cylinder
$= \pi r^2 h$
$= \pi \times 6 \times 6 \times 15$
$= 540\pi \ cm^3$

Volume of cone
$= \frac{1}{3} \times \pi \times 3^2 \times 12$
$= 36\pi \ cm^3$

Volume of hemisphere
$= \frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \pi \times 3^3$
$= 18\pi \ cm^3$
Volume of ice cream
$= (36 + 18)\pi$
$= 54\pi \ cm^3$

$\Rightarrow \text{Number of ice creams}$
$= \frac{\text{Volume of cylinder}}{\text{Volume of ice cream}}$
$= \frac{540\pi}{54\pi}$
$= 10$

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4. Two cubes each of volume 125 cm^3 are joined end to end to form a solid. Find the surface area of the resulting cuboid.

Volume of each cube = 125 cm^3

So, edge of each cube = 5 cm

For new cuboid formed, l = 5+5 = 10 cm

b = 5 cm

h = 5 cm

Therefore, surface area of the resulting cuboid = 2 (lb + bh + lh)

= 2 (10 × 5 + 5 × 5 + 10 × 5) cm^2

= 250 cm^2

5. 150 spherical marbles, each of diameter 14 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

Sol:
Diameter of the spherical marble = 1.4 cm
Radius of the marble = 0.7 cm
$Volume of each marble = 43 \times \pi \times r^3 = 43 \times \frac{22}{7} \times (0.7)^3 = 1.44$
Volume of 150 marbles = 1.44 × 150 = 216 cm^3
Let the rise in level of water in the cylindrical vessel = 'h' cm
Diameter of the vessel = 7 cm
Radius of the vessel = 3.5 cm
Volume of the increased level of the water = $\pi \times r^2 \times h = \frac{22}{7} \times (3.5)^2 \times h$
Volume of the increased level of the water = Volume of 150 marbles
$\frac{22}{7} \times (3.5)^2 \times h = 216$
⇒ h = 5.6 cm
Therefore, rise in level of water in the vessel = 5.6 cm.

6. A farmer connects a pipe of internal diameter 25 cm from a canal into a cylindrical tank in his field, which is 12 m in diameter and 2.5 m deep. If water flows through the pipe at the rate of 3.6 km/hr, in how much time will the tank be filled? Also, find the cost of water if the canal department charges at the rate of Rs 0.07 per m^3.

R = radius of cylinder = 6m

H = height of cylinder = 2.5m

r = radius of pipe = $\frac{25}{2}cm = \frac{1}{8}$m

Rate of flow of water = $\frac{3.6km}{hr}$

In 1 hr water upto a length of 3.6km = 3600m will come out of pipe.

let the tank be filled in 'x' hrs.

volume of water coming out of pipe in x hrs = volume of cylindrical tank.

$\frac{(22}{7} \times \frac{1}{8} \times \frac{1}{8} \times 3600 \times (x) = \frac{22}{7} \times 6 \times 6 \times 2.5$

$\frac{1}{8}x\frac{1}{8} \times 3600 \times x$ = 6 x 6 x 2.5

$$3600 \times (x) = 36x2.5x8x8$$

$$100\times = 8x8x2.5$$

$$1000 \times (x) = 8x8x25$$

$$x = \frac{16}{10} = 1.6hrs$$

$$\rightarrow x = 1hr.36min.$$

Now,

$$costofwater = volumeofcylindricaltank \times 0.07$$

$$= 22/7x6x6x2.5x0.07$$

= Rs.19.80.

The tank will be filled in 1hr.36 min and the cost of water will be Rs.19.80

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7. If the perimeter of each face of a cube is 32 cm, find its lateral surface area.

$\text{Perimeter of each face of a cube } = 32\ cm$

$\therefore \text{ Length of edge } = \frac{32}{4} = 8\ cm$

$\text{and lateral surface area of the cube}$

$= 4 \times (side)^2 = 4 \times 8 \times 8 = 256\ cm^2$

8. A hemispherical bowl is made of steel, 0.25 cm thick.The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

$\left[Assume\ \pi = \frac{22}{7}\right]$

Given:

Inner radius of the hemispherical bowl = 5 cm

Thickness of the bowl = 0.25 cm

$\therefore$ Outer radius (r) of the hemispherical bowl = inner radius+thickness = (5 + 0.25) cm

= 5.25 cm

Outer CSA of hemispherical bowl $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times (5.25cm)^2 = 173.25\ cm^2$