Mock Board Exam
Mathematics
Solutions

SECTION - A

1. $\int \cos^4 x\, dx = \int (\cos^2 x)^2\, dx$

$$= \int \left\{\frac{1+\cos 2x}{2}\right\}^2 dx$$ [1]

$$= \frac{1}{4}\int \{1 + 2\cos 2x + \cos^2 2x\}\, dx$$

$$= \frac{1}{4}\int \left\{1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right\} dx$$

$$= \frac{1}{4}\int \left\{\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right\} dx$$

$$= \frac{1}{4}\left\{\frac{3}{2}x + 2\frac{\sin 2x}{2} + \frac{1}{2}\frac{\sin 4x}{4}\right\} + c$$ [1]

OR

$$I = \int \frac{dx}{\sqrt{2 - 4x + x^2}}$$

$$= \int \frac{dx}{\sqrt{(x-2)^2 - (\sqrt{2})^2}} = \int \frac{dy}{\sqrt{y^2 - (\sqrt{2})^2}}$$; [Put $x – 2 = y$] [1]

$$= \log|\, y + \sqrt{y^2 - 2})\,| + c \text{ (using } \int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c)$$

$$= \log|\,(x-2) + \sqrt{x^2 - 4x + 2})\,| + c$$ [1]

2. $\frac{dy}{dx} + \frac{1}{x}\,.\, y = 1$

Above equation is first order linear differential equation.

Integrating factor (I.F.) = $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$ [1]

Multiply by x on both side of DE

$$x\frac{dy}{dx} + y = x$$

$\frac{d}{dx}(xy) = x \Rightarrow \int d(xy) = \int x dx$

$xy = \frac{x^2}{2} + c$

[1]

3. $|\vec{a} \times \vec{b}| = 35$

$\Rightarrow |\vec{a}||\vec{b}| \sin\theta = 35$

$\Rightarrow \sin\theta = \frac{35}{\sqrt{26} \times 7}$

$\Rightarrow \sin\theta = \frac{5}{\sqrt{26}}$

[1]

$\therefore \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{25}{26}} = \frac{1}{\sqrt{26}}$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$= \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$

[1]

Therefore $\vec{a} \cdot \vec{b} = 7$

4. Clearly d.r's are <4, 1, 8> [1]

$\therefore$ d.r's $= \pm\frac{4}{\sqrt{81}}, \pm\frac{1}{\sqrt{81}}, \pm\frac{8}{\sqrt{81}}$

i.e. $< \frac{4}{9}, \frac{1}{9}, \frac{8}{9} >$ or $< -\frac{4}{9}, \frac{-1}{9}, \frac{-8}{9} >$ [1]

5. There are 4 aces in the pack of 52 cards. [1]

So, the required probability $\frac{{}^{4}C_2}{{}^{52}C_2} = \frac{1}{221}$ [1]

6. Number of ways of selecting 3 white balls out of 8 white balls = ${}^{8}C_3$ [1]

So, Required probability $= \frac{{}^{8}C_3}{{}^{14}C_3} = \frac{8 \times 7 \times 6}{14 \times 13 \times 12} = \frac{2}{13}$ [1]

SECTION - B

7. Let $I = \int x\sqrt{x^4 - 1}\,dx$

Put $x^2 = t$

$\Rightarrow$ $2x\,dx = dt$

$\therefore$ $I = \frac{1}{2}\int \sqrt{t^2 - 1}\,dt$

[1]

$$= \frac{1}{2}\left[\frac{t\sqrt{t^2-1}}{2} - \frac{1}{2}\log(t + \sqrt{t^2-1})\right] + c$$

$$= \frac{1}{4}\left[x^2\sqrt{x^4-1} - \log(x^2 + \sqrt{x^4-1})\right] + c$$

[2]

(Using $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2-a^2})$)

8. Clearly the differential equation is variable separable [1]

$\Rightarrow$ $y\,dy = (1 + x^2)dx$

$$\Rightarrow \int y\,dy = \int (1 + x^2)dx$$

$$\Rightarrow \frac{y^2}{2} = x + \frac{x^3}{3} + c$$

[2]

OR

$$\frac{dy}{dx} + \frac{x}{1-x^2}y = \frac{ax}{1-x^2}$$

The above equation is first order linear differential equation, whose integrating factor is given by

$$\text{I.F.} = e^{\int \frac{x}{1-x^2}dx} = \frac{1}{\sqrt{1-x^2}}$$

[1]

Multiply by I.F.

$$\frac{d}{dx}\left(y.\frac{1}{\sqrt{1-x^2}}\right) = \frac{ax}{(1-x^2)^{3/2}}$$

$$\frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{(1-x^2)^{3/2}}dx$$

[1]

To evaluate the integral substitute

$1 - x^2 = t$

$-2xdx = dt$

$$\frac{y}{\sqrt{1-x^2}} = -\frac{a}{2}\int \frac{dt}{t^{3/2}} = -\frac{a}{2} \times \frac{1}{\frac{-1}{2}\sqrt{t}} + c$$

$$y\frac{1}{\sqrt{1-x^2}} = -\frac{a}{2}\times\frac{1}{-\frac{1}{2}\sqrt{1-x^2}}+c$$

$$y = a + c\sqrt{1-x^2}$$

[1]

9. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$ [1]

$$\Rightarrow \vec{a}\times\vec{b} = 0\hat{i} - 0\hat{j} + (21+20)\hat{k}$$

$$\Rightarrow \vec{a}\times\vec{b} = 41\hat{k}$$

[1]

We know that,

the area of a triangle $= \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{41}{2}$sq.unit. [1]

10. $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (3x^3 - x\cos x + \sec^2 x)dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (3x^3 - x\cos x)dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx$ [1]

As we know that if $f(x)$ is an odd function

then $\int_{-a}^{a} f(x)dx = 0$

[1]

$$I = 0 + [\tan x]_{-\pi/4}^{\pi/4} = 2$$

[1]

SECTION - C

11. Let $y = e^x \Rightarrow e^x dx = dy$ [1]

putting values,

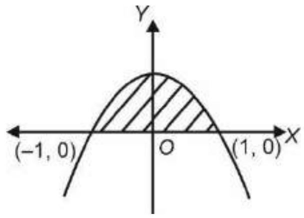
$$\int_1^e \frac{e^x dx}{1+e^{2x}} = \int_1^e \frac{dt}{1+t^2}$$

[1]

$$= [\tan^{-1} t]_1^e = \tan^{-1} e - \frac{\pi}{4}.$$

[2]

(As we know $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$)

12. 

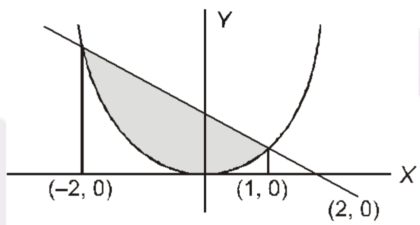
[1]

Required area = $2\int_0^1 (1-x^2)\,dx$ [1]

$= 2\left(x - \frac{x^3}{3}\right)_0^1$ [1]

$= 2\left(1 - \frac{1}{3}\right) = \frac{4}{3} =$ sq. units [1]

OR



[1]

⇒ $x^2 = 2 - x$

⇒ $x^2 + x - 2 = 0$

⇒ $(x + 2)(x - 1) = 0$

⇒ $x = -2, 1$ [1]

Required area $= \int_{-2}^{1} \{(2-x) - x^2\}\,dx$ [1]

$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1}$

$= 2(3) - \frac{1}{2}(1-4) - \frac{1}{3}(1+8)$

$= 6 + \frac{3}{2} - 3$

$= \frac{9}{2}$ sq. units

[1]

13. $x = y = z$ … (i)

$2x + y + z = 1$ … (ii)

$3x + y + 2z = 2$ … (iii)

Normal vector to plane (ii) is $\vec{n_2} = 2\hat{i} + \hat{j} + \hat{k}$

Similarly normal vector to the plane (iii) is $\vec{n_3} = 3\hat{i} + \hat{j} + 2\hat{k}$

Vector parallel to the line of intersection is $\vec{n_2} \times \vec{n_3}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= \hat{i}$ (2 - 1) - $\hat{j}$ (4 - 3) + $\hat{k}$ (2 - 3)

$= (\hat{i} - \hat{j} - \hat{k})$, hence DR's of the line are (1, -1, -1)

[1]

To find point on the line of intersection, we put $z = 0$ in (ii) and (iii)

$\Rightarrow$ $2x + y = 1$ …(iv)

$3x + y = 2$ …(v)

Solving (iv) and (v)

$x = 1, y = -1, z = 0$

Equation of line is

$$\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-0}{-1}$$

…(vi) [1]

Let one point on the line (i) is $\vec{a_1} = (0\hat{i} + 0\hat{j} + 0\hat{k})$

Similarly the point on the line is $\vec{a_2} = (\hat{i} - \hat{j} + 0\hat{k})$

Vector parallel to line (i) is $\vec{b_1} = \hat{i} + \hat{j} + \hat{k}$

Similarly the vector parallel to the line (vi) is $\vec{b_2}$ = (1, -1, -1)

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 1 & -1 & -1 \end{vmatrix}$$

$\Rightarrow$ $\hat{i}(0) - \hat{j}(-2) + \hat{k}(-2) = 2(j - k)$

[1]

Shortest distance $= \left|\dfrac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right|$

Shortest distance $= \left|\dfrac{(\hat{i} - \hat{j}) \cdot (2\hat{j} - 2\hat{k})}{2\sqrt{2}}\right| = \left|\dfrac{2}{2\sqrt{2}}\right| = \dfrac{1}{\sqrt{2}}$ [1]

OR

Here $\vec{a_1} = -3\hat{i} + 6\hat{j},\ \vec{a_2} = -2\hat{i} + 7\hat{k},\ \vec{b_1} = -4\hat{i} + 3\hat{j} + 2\hat{k}$ [1]

and $\hat{b}_2 = -4\hat{i} + \hat{j} + \hat{k}$.

$\therefore$ $\vec{a_2} - \vec{a_1} = \hat{i} - 6\hat{j} + 7\hat{k},\ \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix} = i - 4\hat{j} + 8\hat{k}$

[1]

$\therefore$ $\text{S.D.} = \left|\dfrac{(\vec{a_2} - \vec{a_1}) \cdot \vec{b_1} \times \vec{b_2}}{|\vec{b_1} \times \vec{b_2}|}\right|$

[1]

$$= \frac{(1 + 24 + 56)}{\sqrt{1 + 16 + 64}} = \frac{81}{9} = 9 \text{ units}$$

[1]

14. Let E_1 and E_2 be the events of choosing a bicycle from the first and second plants respectively, then

$P(E_1) = \dfrac{60}{100} = \dfrac{3}{5}$ and $P(E_2) = \dfrac{40}{100} = \dfrac{2}{5}$ [1]

Let E be the event of choosing a bicycle of standard quality. Then,

$P(E|E_1)$ = probability of choosing a bicycle of standard quality given that it is produced by the first plant.

$$= \frac{80}{100} = \frac{4}{5}$$

$P(E|E_2)$ = probability of choosing a bicycle of standard quality, given that it is produced by the second plant.

$$= \frac{90}{100} = \frac{9}{10}$$

(i) Using Bayes' theorem, the required probability is

$P(E_1|E)$ = probability of choosing a bicycle from the first plant, given that it is of standard quality.

$$= \frac{P(E_1)P(E \mid E_1)}{P(E_1)P(E \mid E_1) + P(E_2)P(E \mid E_2)}$$

[1]

$$= \frac{\frac{3}{5} \times \frac{4}{5}}{\frac{3}{5} \times \frac{4}{5} + \frac{2}{5} \times \frac{9}{10}} = \frac{4}{7}$$

[1]

(ii) $$P(E_2 \mid E) = \frac{P(E_2),P(E \mid E_2)}{P(E_1)P(E \mid E_1) + P(E_2)P(E \mid E_2)}$$

[1]

$$= \frac{\frac{2}{5} \times \frac{9}{10}}{\frac{3}{5} \times \frac{4}{5} + \frac{2}{5} \times \frac{9}{10}} = \frac{3}{7}$$

[1]

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