BYJU'S EXAM PREP

GATE 2022

Electrical Engineering

Questions with Detailed Solutions

1. As you grow older, an injury to your ________ may take longer to ________.

A. heel/heel B. heal/heel

C. heal/heal D. heel/heal

[MCQ: 1 Mark]

Ans. D

Sol. **Heel:** the back part of a human foot.

Heal: to recover from an injury.

So, the correct sentence will be:

"As you grow older, an injury your heel may take longer to heal".

Hence, option D is correct.

2. In a 500 m race, P and Q have speeds in the ratio of 3 : 4. Q starts the race when P has already covered 140 m. What is the distance between P and Q (in m) when P wins the race?

A. 20 B. 40

C. 60 D. 140

[MCQ: 1 Mark]

Ans. A

Sol. Let distance remaining to be covered by P is $d_1 = 500 - 140 = 360$ m

And the distance remaining to be covered by Q at the same time = d_2

Therefore, the ratio of distances will be same as the ratio of speeds i.e.,

$$\frac{d_1}{d_2} = \frac{s_1}{s_2} = \frac{3}{4}$$

$$\frac{360}{d_2} = \frac{3}{4}$$

So, $d_2 = 480$ m

Therefore, the distance between P and Q will be = (500 – 480) = 20 m behind P.

Hence, option A is correct.

3. Three bells P, Q, and R are rung periodically in a school. P is rung every 20 minutes; Q is rung every 30 minutes and R is rung every 50 minutes. If all the three bells are rung at 12:00 PM, when will the three bells ring together again the next time?

A. 5:00 PM B. 5:30 PM

C. 6:00 PM D. 6:30 PM

[MCQ: 1 Mark]

Ans. A

Sol. It is given that all three bells rang together at 12:00 PM. Now, for the next time when they will ring together will be after 'x' hours.

x = LCM(20, 30, 50) = 300 minutes

Therefore, x = 5 hours

Hence, all three bells will ring together at 5:00 PM.

Hence, option A is correct.

4. Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some bottles are cups.

Statement 2: All cups are knives.

Conclusion I: Some bottles are knives.

Conclusion II: Some knives are cups.

Conclusion III: All cups are bottles.

Conclusion IV: All knives are cups.

Which one of the following options can be logically inferred?

A. Only conclusion I and conclusion II are correct.

B. Only conclusion II and conclusion III are correct.

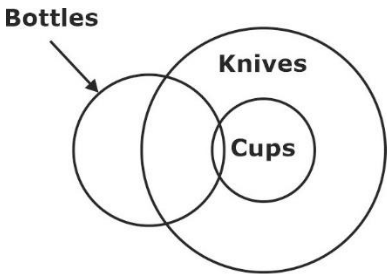
C. Only conclusion II and conclusion IV are correct.

D. Only conclusion III and conclusion IV are correct.

[MCQ: 1 Mark]

Ans. A

Sol. Following Venn diagram is based upon the given statements in the question:



From conclusion 1:

Some bottles are Knives which is true as seen from the diagram.

From conclusion 2:

Some knives are cups as all cups are Knives. Hence, it is also true.

From conclusion 3:

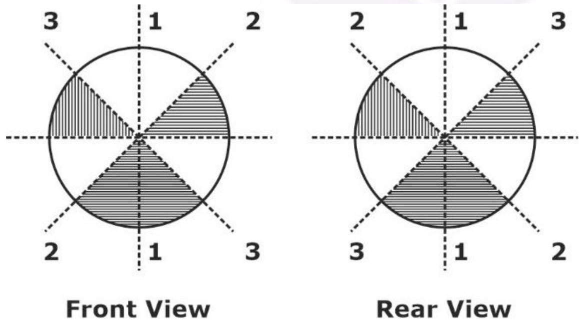
Some bottles are cups so all cups may or may not be the bottles, Thus, it is incorrect.

From conclusion 4:

All knives cannot always be true as seen from the diagram. Thus, it is incorrect.

5. The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes 1-1, 2-2 or 3-3 chosen uniformly at random.

What is the probability that the disc **DOES NOT** retain the same front and rear views after the flipping operation?



A. 0 B. 1/3

C. 2/3 D. 1

[MCQ: 1 Mark]

Ans. C

Sol. By looking at the front view and the rear view, we conclude that the view is symmetrical about 1-1 axis, Thus the disc will retain the same front and rear view after flipping about 1-1 axis out of the three-given axis.

Hence, the probability that the disc does not retain the same front and rear view

= 1 – (probability of retaining same views)

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, option C is correct.

6. Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity, and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

A. Humans engage in altruism due to guilt but not empathy.

B. Humans engage in altruism due to empathy but not guilt.

C. Humans engage in altruism due to group identification but not empathy.

D. Humans engage in altruism due to empathy but not familiarity.

[MCQ: 2 Marks]

Ans. C

Sol. As per the last statement of the passage, "The notion that altruism may be attributed to empathy or guilt has now been rejected", we can conclude that humans does not engage in altruism due to empathy and guilt. Thus, the logical inference based on the passage is, "Humans engage in altruism due to group identification but not empathy" whereas all the other options can be eliminated.

7. There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U & V one in each of the faces. Any of the six outcomes are equally likely. The two dice are thrown once independently at random. What is the probably that outcome on the dice were composed only of any combination of the following possible outcomes Q, U & V ?

A. 1/4 B. 3/4

C. 1/6 D. 5/36

[MCQ: 2 Marks]

Ans. A

Sol. Total number of possible outcome = 36

Favorable outcomes can be composed of any combination of Q, U, & V. i.e., (Q, Q), (U, U), (V, V), (Q, U), (U, Q), (Q, V), (V, Q), (U, V), (V, U).

The favorable outcomes came from 3 outcomes on each dice.

$\therefore$ Probability $= \frac{3}{6} \times \frac{3}{6} = \frac{9}{36} = \frac{1}{4}$

Hence, option A is correct.

8. The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges ₹ 150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved ₹ 100. What is the price of the item at the online store S (in ₹) if there are no other charges than what is described above?

A. 2500 B. 2250

C. 1750 D. 1500

[MCQ: 2 Marks]

Ans. B

Sol. Let the price of an item at store S = S

And the price of an item at store M = M

So, according to the question

S= (1 – 0.1)M = 0.9M

And M – (S + 150) = 100

$\Rightarrow$ M – (0.9M + 150) = 100

0.1M = 250

M = Rs. 2500

Price of item at store S = 0.9M

= 0.9 × 2500 = Rs. 2250

Hence, option B is correct.

9. The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order.

Consider the following statements:

- The line segment joining R and S is longer than the line segment joining P and Q.
- The line segment joining R and S is perpendicular to the line segment joining P and Q.
- The line segment joining R and U is parallel to the line segment joining T and Q.

Based on the above statements, which one of the following options is

CORRECT?

A. The line segment joining R and T is parallel to the line segment joining Q and S.

B. The line segment joining T and Q is parallel to the line joining P and U.

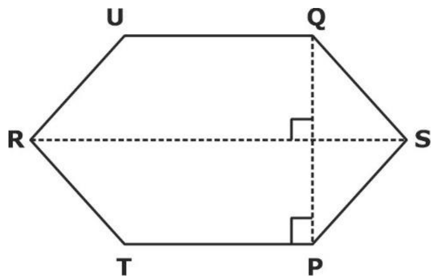
C. The line segment joining R and P is perpendicular to the line segment joining U and Q.

D. The line segment joining Q and S is perpendicular to the line segment joining R and P.

[MCQ: 2 Marks]

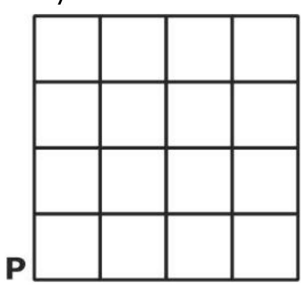
Ans. A

Sol. According to the given statements in the question, the hexagon will look as follows:

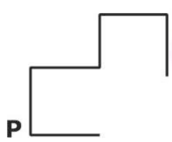


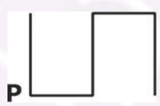
So, from the given statements, options A will be correct.

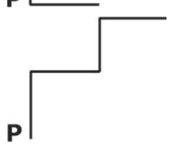
10. An ant is at the bottom-left corner of a grid (point P) as shown above. It aims to move to the top-right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases.

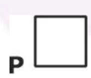


Which one of the following is a part of a possible trajectory of the ant during the movement?

A. 

B. 

C. 

D. 

[MCQ: 2 Marks]

Ans. C

Sol. As per the equation, initial point of the ant is at point P, so option A and B gets eliminated. Also, the ant moves only along those lines of grid such that the distance to the top-right corner strictly decreases which eliminates the options D.

Hence, options C, will be correct.

11. The transfer function of a real system, H(s), is given as:

$$H(s) = \frac{As + B}{s^2 + Cs + D}$$

Where A, B, C and D are positive constants. This system cannot operate as

A. low pass filter. B. high pass filter.

C. band pass filter. D. an integrator.

[MCQ: 1 Mark]

Ans. B

Sol. $H(s) = \frac{As + B}{s^2 + Cs + D}$

Where, A, B, C, D are positive constants,

Put s = 0

$$H(s) = \frac{B}{D}$$

It is a low pas filter.

Put s = ∞

$H(\infty) = 0$

It is not a high pas filter; hence it cannot operate as high pass filter.

Hence, option B is correct.

12. For an ideal MOSFET biased in saturation, the magnitude of the small signal current gain for a common drain amplifier is

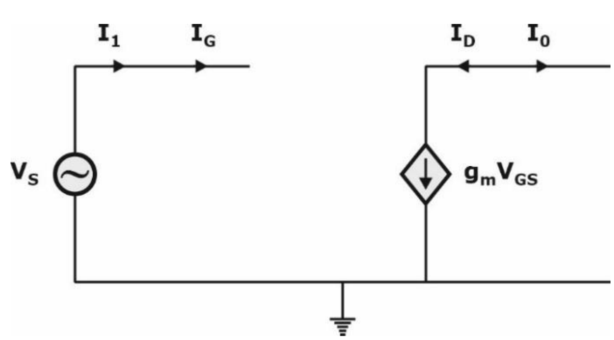
A. 0 B. 1

C. 100 D. infinite

[MCQ: 1 Mark]

Ans. D

Sol.



Current gain $= \frac{I_0}{I_i} = \frac{-I_D}{I_G} = \frac{g_m V_{GS}}{0} = \infty$

Hence, option D is correct.

13. The most commonly used relay, for protection of an alternator against loss of excitation, is

A. offset mho relay B. Over current relay

C. Differential relay D. Buchholz relay

[MCQ: 1 Mark]

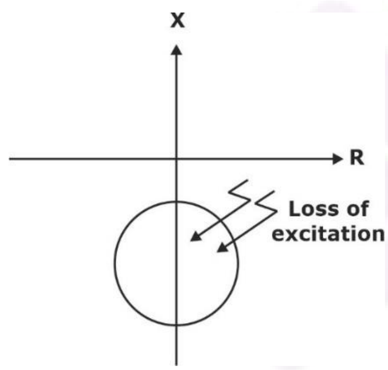
Ans. A

Sol. The offset mho relay is most commonly used relay, for the protection of alternator against loss of excitation.

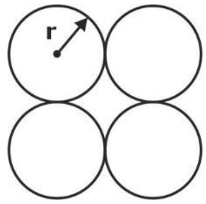
The offset Mho relay is a single phase, single element distance relay which is applied to the generate terminal.

The R-X characteristic is an offset circle which has an angle of maximum torque that falls on the (–X) ordinate.

The relay will operate for any impedance phasor that terminates impedance phasor that terminates inside the circular characteristic.



14. The geometric mean radius of a conductor, having four equal strands with each strand of radius 'r', as shown in the figure below, is



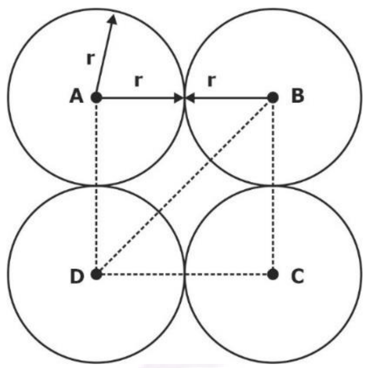
A. 4r B. 1.414r

C. 2r D. 1.723r

[MCQ: 1 Mark]

Ans. D

Sol.



For conductor A:

$GMR_A = (0.7788r \times 2r \times 2r \times 2\sqrt{2}\,r)$

For conductor B:

$GMR_B = (0.7788r \times 2r \times 2r \times 2\sqrt{2}\,r)$

Similarly, for conductor C & D

$GMR_A = GMR_B = GMR_C$

$= GMR_D = (0.7788r \times 2r \times 2r \times 2\sqrt{2}\,r)$

Overall GMR $= (GMR_A \times GMR_B \times GMR_C \times GMR_D)^{1/16} = (GMR_A)^{1/4}$

$= (0.7788r \times 2r \times 2r \times 2\sqrt{2}\,r)^{1/4}$

$= 1.723r$

Hence, option D is correct.

15. The valid positive, negative and zero sequence impedances (in p.u.), respectively, for a 220 kV, fully transposed three-phase transmission line, from the given choices are

A. 1.1, 0.15 and 0.8

B. 0.15, 0.15 and 0.35

C. 0.2, 0.2 and 0.2

D. 0.1, 0.3 and 0.1

[MCQ: 1 Mark]

Ans. B

Sol. For fully transposed transmission line

Positive sequence impedance, $Z_1 = Z_s - Z_m$

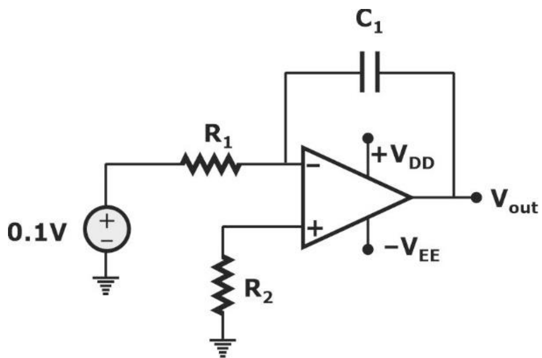
Negative sequence impedance, $Z_2 = Z_s - Z_m$

Zero sequence impedance, $Z_0 = Z_s + 2Z_m$

So, $Z_1 = Z_2 < Z_0$

Hence, option B is correct.

16. The steady state output (V_{out}), of the circuit shown below, will



A. saturate to $+V_{DD}$

B. saturate to $-V_{EE}$

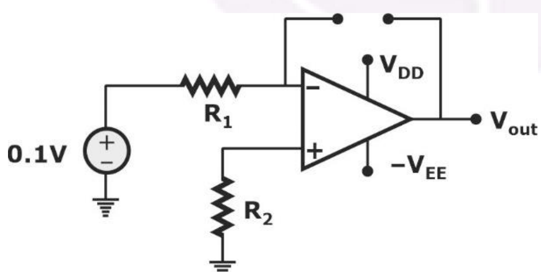
C. become equal to 0.1 V

D. become equal to −0.1 V

[MCQ: 1 Mark]

Ans. B

Sol. Under steady state capacitor acts as open circuit for DC supply.



Op-amp will work as a comparator.

$V^- = 0.1$ V & $V^+ = 0$ V

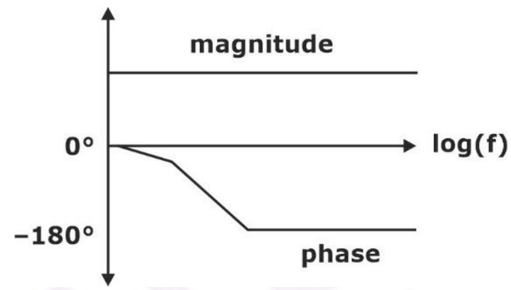
$V_D = V^+ - V^- = 0 - 0.1 = -0.1$ V

Op-amp will be in saturation mode, as inverting terminal is greater than the non-inverting terminal, hence it will saturates at $-V_{EE}$.

Therefore, $V_{out} = -V_{EE}$

Hence, option B is correct.

17. The Bode magnitude plot of a first order stable system is constant with frequency. The asymptotic value of the high frequency phase, for the system, is −180°. This system has



A. one LHP pole and one RHP zero at the same frequency.

B. one LHP pole and one LHP zero at the same frequency.

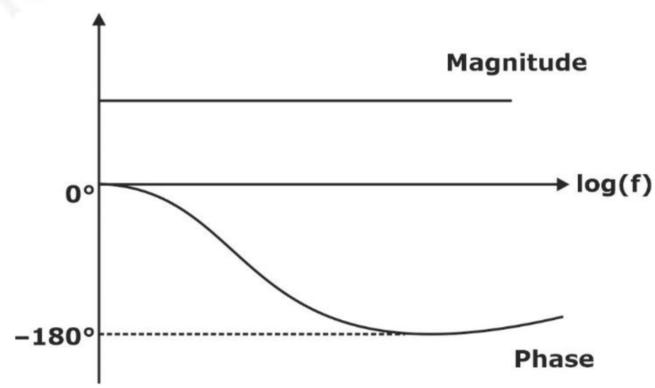
C. two LHP poles and one RHP zero.

D. two RHP poles and one LHP zero.

[MCQ: 1 Mark]

Ans. A

Sol. Bode plot:



The given system is stable.

The transfer function for this system can be assumed like

$$T(s)\Big|_{s=j\omega} = \frac{K(1-s)}{(1+s)} = \frac{k(1-j\omega)}{(1+j\omega)}$$

Magnitude (M) = $|T(j\omega)| = \frac{k\sqrt{1+\omega^2}}{(1+j\omega)}$

$M = |T(j\omega)| = k$

Phase (θ) = $-2\tan^{-1}\omega$

At high frequency, the phase will be –180° only.

So, corner frequencies are:

Pole (P): s = –1 (LHP)

Zero (z): s = +1 (RHP)

So, it has one LHP pole and one RHP zero at same frequency. Hence, option A is correct.

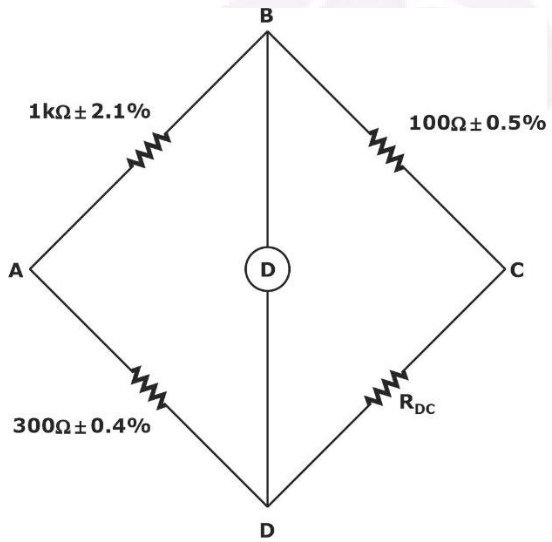
18. A balanced Wheatstone bridge ABCD has the following arm resistances:
R_{AB} = 1 kΩ ± 2.1%; R_{BC} = 100 Ω ± 0.5%
R_{CD} is an unknown resistance
R_{DA} = 300 Ω ± 0.4%. The value of R_{CD} and its accuracy is

A. 30 Ω ± 3 Ω B. 30 Ω ± 0.9 Ω
C. 3000 Ω ± 90 Ω D. 3000 Ω ± 3 Ω

[MCQ: 1 Mark]

Ans. B

Sol. During balancing condition



$$R_{DC} = \frac{R_{AD} \times R_{BC}}{R_{AB}}$$

$$= \frac{(300 \pm 0.4\%)(100 \pm 0.5\%)}{1k\Omega \pm 2.1\%}$$

$$= \frac{(300 \times 100)}{1000} \pm (0.4\% + 0.5\% + 2.1\%)$$

= 30 ± 3%

= 30 ± 0.9 Ω

19. The open loop transfer function of unity gain negative feedback system is given by

$$G(s) = \frac{k}{s^2 + 4s - 5}$$

The range of k for which system is stable, is

A. k > 3 B. k < 3
C. k > 5 D. k < 5

[MCQ: 1 Mark]

Ans. C

Sol. Open loop transfer function:

$$G(s) = \frac{k}{s^2 + 4s - 5}$$

Characteristic equation will be,

1 + G(s)H(s) = 0

$$1 + \frac{k}{s^2 + 4s - 5} = 0$$

$s^2 + 4s + (k - 5) = 0$

It is second order quadratic equation.

So, all coefficients should be positive for the system to be stable.

k – 5 > 0 ⇒ k > 5

Hence, option C is correct.

20. Consider a 3 x 3 matrix A whose (i, j)-th element, $a_{i,j} = (i - j)^3$. Then the matrix A will be

A. symmetric B. skew-symmetric
C. unitary D. null

[MCQ: 1 Mark]

Ans. B

Sol. $A_{3\times3} = a_{ij} = (i - j)^3$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix}$$

Here, $A^T = -A$

$\therefore$ A is skew-symmetric matrix.

Alternate solution:

$A = [a_{ij}]_{3\times3}$, $a_{ij} = (i - j)^3$

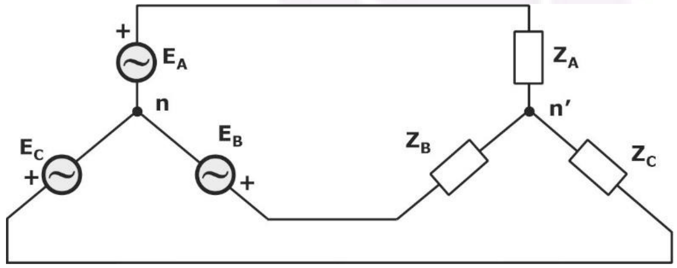
For $i = j \Rightarrow a_{ii} = (i - i)^3 = 0 \ \forall i$

For $i \neq j \Rightarrow a_{ij} = (i - j)^3 = (-(j - i))^3 = (j - i)^3 = -a_{ji}$

$\therefore A_{3\times3}$ is a skew-symmetric matrix.

Hence, option B is correct.

21. In the circuit shown below, a three-phase star-connected unbalanced load is connected to a balanced three-phase supply of $100\sqrt{3}$ V with phase sequence ABC. The star connected load has $Z_A = 10\Omega$ & $Z_B = 20\angle60°\Omega$. The value of Z_C in Ω, for which the voltage difference across the nodes n and n′ is zero, is



A. $20\angle-30°$
B. $20\angle30°$
C. $20\angle-60°$
D. $20\angle60°$

[MCQ: 1 Mark]

Ans. C

Sol. As given voltage difference between n and n′ is zero, then current between n and n′ must be zero.

$\therefore I_A + I_B + I_C = 0$

$$\frac{E_A}{Z_A} + \frac{E_B}{Z_B} + \frac{E_C}{Z_C} = 0$$

Taking per phase voltage,

$$\frac{100\angle0°}{10} + \frac{100\angle-120°}{20\angle60°} + \frac{100\angle-240°}{Z_C} = 0$$

$$\frac{100\angle-240°}{Z_C} = -\left(\frac{100\angle0°}{10} + \frac{100\angle-120°}{20\angle60°}\right)$$

On solving, we get, $Z_C = 20\angle-60°\ \Omega$

Hence, option C is correct.

22. A charger supplies 100 W at 20 V for charging the battery of a laptop. The power devices, used in the converter inside the charger, operate at a switching frequency of 200 kHz. Which power device is best suited for this purpose?

A. IGBT
B. Thyristor
C. MOSFET
D. BJT

[MCQ: 1 Mark]

Ans. C

Sol. **1.** MOSFET is used for high switching frequency & low power applications

2. MOSFET is used for high switching frequency of 100 kHz - 1 MHz. As given frequency is 200 kHz, MOSFET is used.

Extra Information:

Parameter	SCR	Power BJT	Power MOSFET	IGBT
Operating frequency	400-500 Hz	< 20 kHz	< 1 MHz	<50 kHz
On state voltage drop	< 2V	<2V	4-5V	3V

23. A long conducting cylinder having a radius 'b' is placed along the z axis. The current density is $J = J_a r^3 \hat{z}$ for the region r < b where r is the distance in the radial direction. The magnetic field intensity **(H)** for the region inside the conductor (i.e., for r < b) is

A. $\frac{Ja}{4} r^4$

B. $\frac{J_a}{3} r^3$

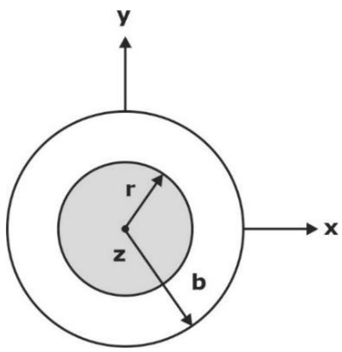
C. $\frac{J_a}{5} r^4$

D. $J_a r^3$

[MCQ: 1 Mark]

Ans. C

Sol. Given that: $\vec{J} = J_a r^3 \hat{z}$



$$I = \int_s \vec{J} \cdot \overrightarrow{ds}; \quad \overrightarrow{ds} = r\,dr\,d\phi\,\hat{z}$$

$$d\vec{s} = r\,dr\,d\phi\,\hat{z}$$

$$\Rightarrow I = \int J_a r^3 \hat{z} \cdot r dr d\phi\ \hat{z}$$

$$= J_a \int_{r=0}^{r} r^4 dr \int_{\phi=0}^{2\pi} d\phi$$

$$= J_a \frac{r^5}{5}\bigg|_0^5 \cdot \phi\big|_0^{2\pi} = \frac{J_a (2\pi) r^5}{5}$$

As $\oint \vec{H} \cdot d\vec{L} = I_{enc} = \int_s \vec{J} \cdot d\vec{s}$

$$\Rightarrow H(2\pi r) = \frac{J_a (2\pi) r^5}{5}$$

$$\Rightarrow H \frac{J_a r^4}{5}$$

Hence, option C is correct.

24. The type of single-phase induction motor, expected to have the maximum power factor during steady state running condition, is

A. split phase (resistance start)

B. shaded pole

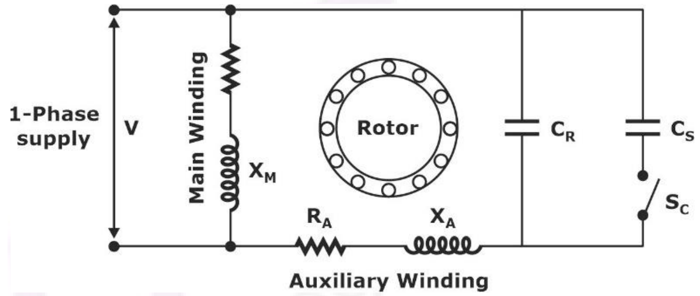
C. capacitor start

D. capacitor start, capacitor run

[MCQ: 1 Mark]

Ans. D

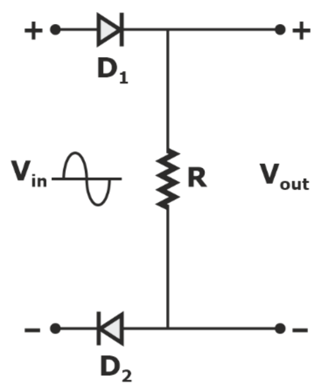
Sol. Capacitor start capacitor run inductor motor have maximum power factor during steady state running condition because at steady state it have capacitor in main winding which improves the power factor.



During steady state condition only main winding taking current.

Hence, option D is correct.

25. For the circuit shown below with ideal diodes, the output will be



A. $V_{out} = V_{in}$ for $V_{in} > 0$

B. $V_{out} = V_{in}$ for $V_{in} < 0$

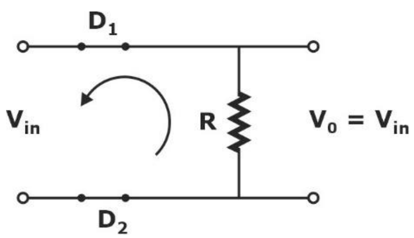
C. $V_{out} = -V_{in}$ for $V_{in} > 0$

D. $V_{out} = -V_{in}$ for $V_{in} < 0$

[MCQ: 1 Mark]

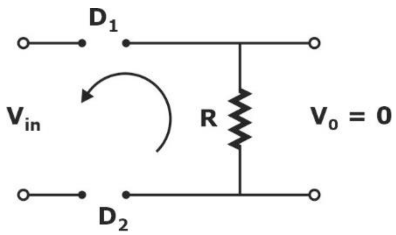
Ans. A

Sol. In positive cycle, diode D_1 and D_2 are ON



Therefore, $V_{out} = V_{in}$

In negative cycle diode D_1 and D_2 are OFF.



Therefore, $V_{out} = 0$ V

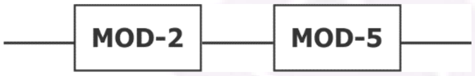
Hence, $V_{out} = V_{in}$, for $V_{in} > 0$ V

26. A MOD-2 and a MOD-5 up-counter when cascaded together results in a MOD ______ counter. (in integer)

[NAT: 1 Mark]

Ans. 10 to 10

Sol.



For overall configuration = Mod (2 × 5) = 10

It will be mod-10 counter.

27. An inductor having a Q-factor of 60 is connected in series with a capacitor having a Q-factor of 240. The overall Q-factor of the circuit is ________. (round off to nearest integer)

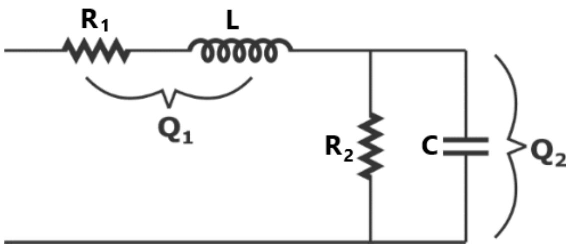
[NAT: 1 Mark]

Ans. 48 to 48

Sol. Quality factor of inductive coil, $Q_1 = 60$

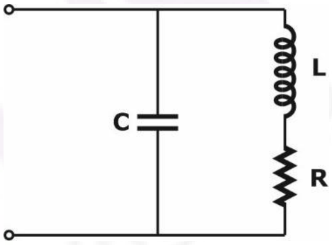
Quality factor of capacitive coil, $Q_2 = 240$

When both are connected in series as shown in figure below,



$$Q = \frac{Q_1 Q_2}{Q_1 + Q_2} = \frac{240 \times 60}{240 + 60} = 48$$

28. The network shown below has a resonant frequency of 150 kHz and bandwidth of 600 Hz. The Q-factor of the network is ______. (round off to nearest integer)



[NAT: 1 Mark]

Ans. 250 to 250

Sol. $\text{Q-factor} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$

$$Q = \frac{150 \times 10^3}{600} = 250$$

29. The maximum clock frequency in MHz of a 4-stage ripple counter, utilizing flip-flop with each flip-flop having propagation delay of 20 ns, is ________. (round off to one decimal place)

[NAT: 1 Mark]

Ans. 12.3 to 12.7

Sol. Given ripple counter 4 flip-flop, t_{pd} = 20 nsec for each flip-flop.

$T = nt_{pd} = 4 \times 20$ nsec = 80 nsec

Clock frequency is calculated as

$$f_{CLK} = \frac{1}{T} = \frac{1}{4 \times 20 \times 10^{-9}} = 12.5 \text{ MHz}$$

30. If only 5% of the supplied power to a cable reaches the output terminal the power loss in the cable, in decibels, is _______. (round off to nearest integer)

[NAT: 1 Mark]

Ans. 13 to 13

Sol. Power loss in the cable = 95% of input power

Let input power = x

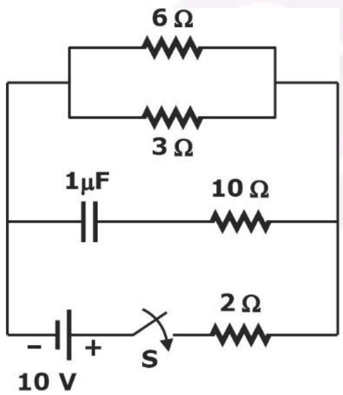
Then output power = 0.05x

$$P_{loss}\big|_{dB} = 10\log_{10} x - 10\log_{10} 0.05x$$

$$= 10\log_{10}\left(\frac{x}{0.05x}\right) = 10\log_{10}\left(\frac{1}{0.05}\right)$$

= 10 $\log_{10}20$ = 10 × 1.30103 dB = 13 dB

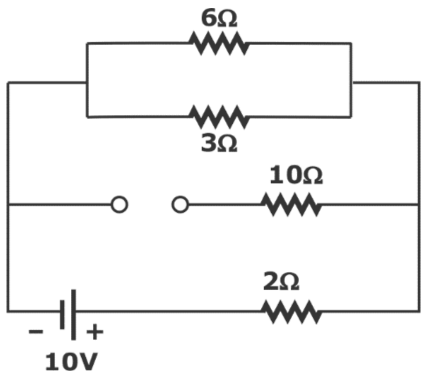
31. In the circuit below, the switch S is closed at t = 0. The magnitude of steady state voltage, in volts, across 6 Ω resistor is ________. (round off to two decimal places)



[NAT: 1 Mark]

Ans. 4.95 to 5.05

Sol. In steady state capacitor acts as open circuit for DC source.



Voltage across 6 Ω is same as voltage across parallel combination of 6 Ω and 3 Ω.

The equivalent resistance across open-terminals replacing independent sources with their internal resistances, i.e., 10 V source will be replaced by short-circuit.

$$R_{eq} = \frac{6\times 3}{6+3} = 2\,\Omega$$

Therefore, the voltage across 6 Ω is

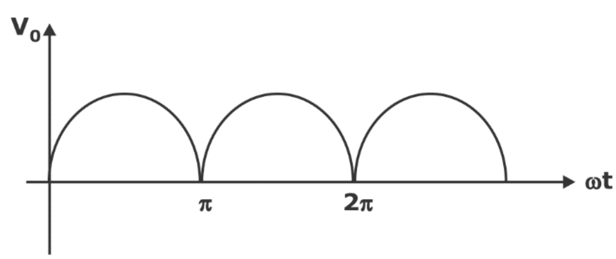
$$V_{6\text{-}\Omega} = \frac{10\times 2}{2+2} = 5\text{ V}$$

32. A single-phase full-bridge diode rectifier feeds a resistive load of 50 Ω from a 200 V, 50 Hz single phase AC supply. If the diodes are ideal, then the active power, in watts, drawn by the load is _________. (round off to nearest integer)

[NAT: 1 Mark]

Ans. 795 to 805

Sol. Output voltage waveform of single-phase full bridge rectifier is shown in figure below:



RMS output voltage $V_0(rms) = \frac{V_m}{\sqrt{2}}$

Power drawn by load $= \frac{V_{or}^2}{R}$

$$= \frac{V_m^2}{2R} = \frac{\left(200\sqrt{2}\right)^2}{2 \times 50} = 800 \text{ W}$$

33. The voltage at the input of AC-DC rectifier is given $v(t) = 230\sqrt{2}\sin\omega t$ where ω = 2π × 50 rad/s. The input current drawn by the rectifier is given by

$$i(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right) + 4\sin\left(3\omega t - \frac{\pi}{6}\right) + 3\sin\left(5\omega t - \frac{\pi}{3}\right)$$

Then find the input power factor, (rounded off to two decimal places) is, _______ lag.

[NAT: 1 Mark]

Ans. 0.43 to 0.47

Sol. Input power factor $= \frac{I_{s1}}{I_s}\cos\phi_1$

$$I_s = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 7.905$$

$$I_{s1} = \frac{10}{\sqrt{2}} = 7.07$$

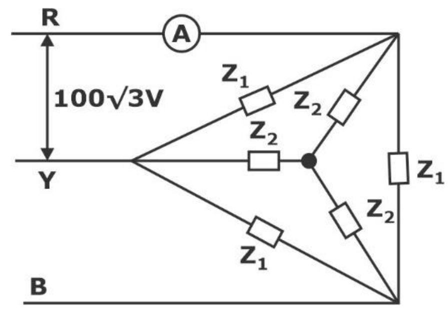
ϕ_1 = angle between V_s and I_{s1} $= \frac{\pi}{3} = 60°$

Input power factor is calculated as

$$= \frac{7.07}{7.9}\cos 60° = 0.447 \text{ lag}$$

34. Two balanced three-phase loads, as shown in the figure, are connected to a 100√3 V, three-phase, 50 Hz main supply.

Given Z_1 = (18 + j24) Ω and Z_2 = (6 + j8) Ω. The ammeter reading, in amperes, is _______. (round off to nearest integer)



[NAT: 1 Mark]

Ans. 20 to 20

Sol. Convert inner star into delta:

Z_2' = (6 + j8) × 3 = (18 + j24) Ω

Now, Z_1 and Z_2' are in parallel.

$Z_{eq} = Z_1 \,||\, Z_2'$

= (18 + j24) || (18 + j24) = 9 + j12

$$I_{Ph\Delta} = \frac{V_{Ph}}{Z_{eq}} = \frac{100\sqrt{3}}{9 + j12}$$

$$|I_{Ph\Delta}| = \frac{20}{\sqrt{3}}$$

Current through Ammeter will be line current

$$(I_L)_\Delta = \sqrt{3}\,|(I_{ph})_\Delta| = \sqrt{3} \times \frac{20}{\sqrt{3}} = 20 \text{ A}$$

35. The frequencies of the stator and rotor currents flowing in a three-phase 8-pole induction motor are 40 Hz and 1 Hz, respectively. The motor speed, in rpm, is _________. (round off to nearest integer)

[NAT: 1 Mark]

Ans. 580 to 590

Sol. Given: 8-pole Induction motor

Stator frequency, f_s = 40 Hz

Rotor frequency, f_r = 1 Hz

We know that $f_r = sf_s$

$$s = \frac{1}{40} = 0.025$$

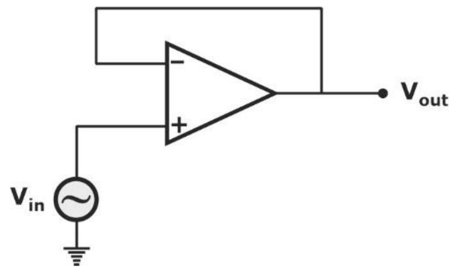
Synchronous speed is given as

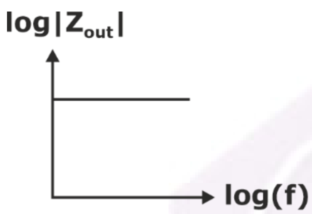
$$N_S = \frac{120f}{P} = \frac{120 \times 40}{8} = 600 \text{ rpm}$$

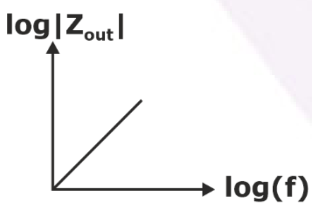
Motor speed, $N_r = (1 - s)N_s$

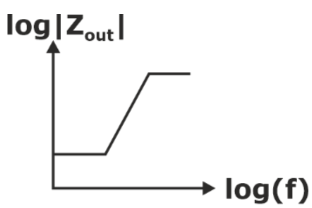
$N_r = (1 - 0.025) \times 600 = 585$ rpm

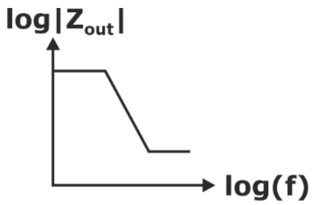
36. The output impedance of a non-ideal operational amplifier is denoted by Z_{out}. The variation in the magnitude of Z_{out} with increasing frequency, f, in the circuit shown below, is best represented by



A.


B.


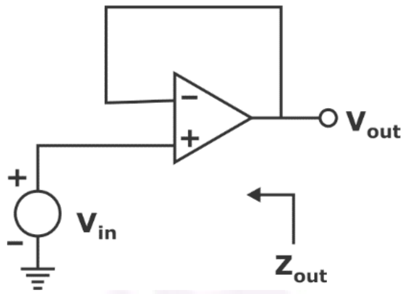
C.


D.


[MCQ: 2 Marks]

Ans. C

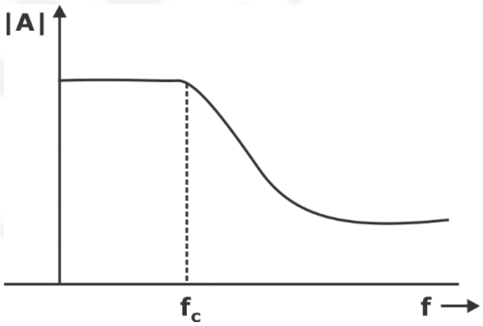
Sol.



It has voltage series feedback.
Output Impedance is given by

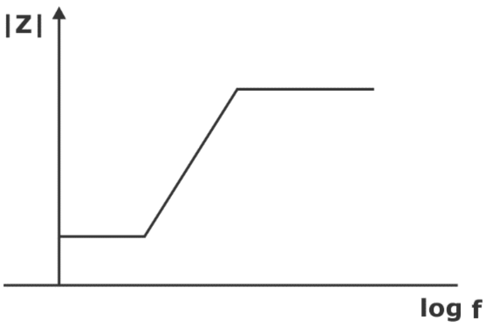
$$Z_{out} = \frac{Z_0}{1 + \beta A_V}$$

Op-amp behaves as law pass filter.



$$A_v(f) = \frac{AV_0}{1 + j\frac{f}{f_c}}$$

Therefore, if frequency (f) increases, the magnitude $|A_v(f)|$ decreases.

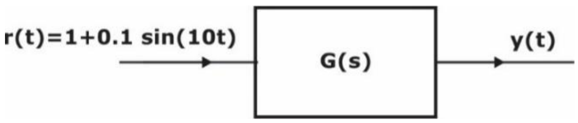


37. An LTI system is shown in the figure where $G(s)=\dfrac{100}{s^2+0.1s+10}$.

The steady state output of the system, to the input r(t) is given as

y(t) = a + b sin(10t + θ)

The values of 'a' and 'b' will be



A. a = 1, b = 10 B. a = 10, b = 1

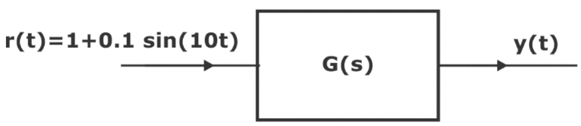
C. a = 1, b = 100 D. a = 100, b = 1

[MCQ: 2 Marks]

Ans. A

Sol. Given that: $G(s)=\dfrac{100}{s^2+0.1s+100}$

r(t) = 1 + 0.1sin(10t)



Output y(t) = a + b sin(10t + θ)

To calculate, a and b = ?

Steady state output, $y(t) = G(s)|_{s=j\omega}\, x(t)$

Case 1: When ω = 10 rad/sec

$$y_1(t)=\left.\frac{100}{s^2+0.1s+100}\right|_{s=j10} x(t)$$

$$y_1(t)=\left.\frac{100}{-100+j1+100}\right|_{s=j10} x(t)$$

$y_1(t) = 100\angle-90°\ x(t)$

$y_1(t) = 100\angle-90°\ [0.1\sin(10t)]$

$y_1(t) = 10\sin(10t - 90°)$

Case 2: When ω = 0, (DC value)

$G(s)|_{\omega=0} = 1$

$y_2(t) = 1 \times 1 = 1$

Output $y(t) = y_1(t) + y_2(t)$

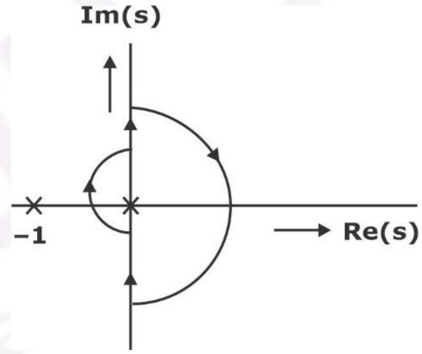
$y(t) = 1 + 10\sin(10t - 90°)$

On comparion, we get, a = 1 and b = 10

Hence, option A is correct.

38. The open loop transfer function of a unity gain negative feedback system is given as $G(s)=\dfrac{1}{s(s+1)}$.

The Nyquist contour in the s-plane encloses the entire right half plane and a small neighborhood around the origin in the left half plane, as shown in the figure below. The number of encirclements of the point (−1 + j0) by the Nyquist plot of G(s), corresponding to the Nyquist contour, is denoted as N. Then N equals to



A. 0 B. 1

C. 2 D. 3

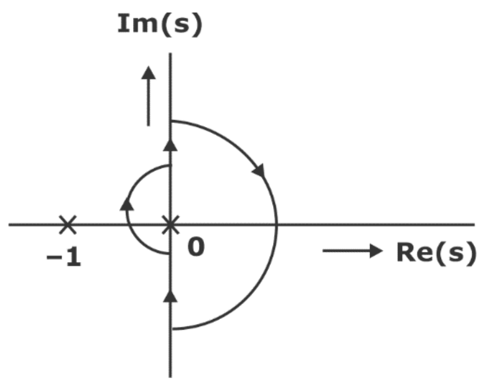
[MCQ: 2 Marks]

Ans. B

Sol. Open loop transfer function is given as

$G(s)=\dfrac{1}{s(s+1)}$

Nyquist contour,



$$N = (P - Z) \qquad ...(1)$$

P = No. of poles of open loop lying in RHS in s-plane

Z = No. of closed loop poles lying in RHS in s-plane

N = No. of encirclements origin is lying inside the contour as shown in the given diagram,

So, $P = 1$

Characteristic equation,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{1}{s(s+1)} = 0$$

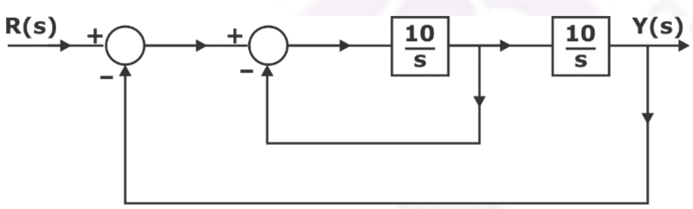
$$s^2 + s + 1 = 0$$

All the coefficients are positive, so Z = 0

From equation (1), no. of encirclements (N) $= P - Z = 1 - 0 = 1$

Hence, option B is correct.

39. The damping ratio and undamped natural frequency of a closed loop system as shown in the figure, are denoted as ξ and ω_n, respectively. The values of ξ and ω_n are



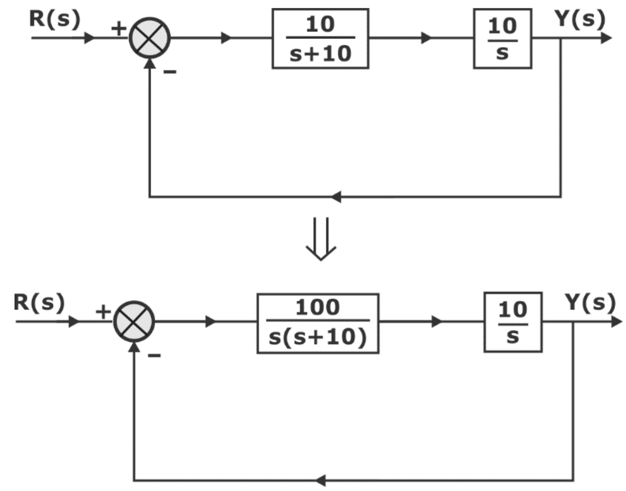
A. $\xi = 0.5$, $\omega_n = 10$ rad/s
B. $\xi = 0.1$, $\omega_n = 10$ rad/s
C. $\xi = 0.707$, $\omega_n = 10$ rad/s
D. $\xi = 0.707$, $\omega_n = 100$ rad/s

[MCQ: 2 Marks]

Ans. A

Sol. To find damping ratio and undamped natural frequency.

By block diagram reduction,



$$\frac{Y(s)}{R(s)} = \frac{100}{s^2 + 10s + 100}$$

On comparison with 2nd order standard transfer function.

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

$$\omega_n^2 = 100 \Rightarrow \omega_n = 10 \text{ rad/sec}$$

Also, $2\xi\omega_n = 10$

$$\xi = \frac{10}{2 \times 10} = 0.5$$

So, $\xi = 0.5$ and $\omega_n = 10$ rad/sec

Hence, option A is correct.

40. e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and v is the corresponding eigen-vector of matrix A.
Consider the following two statements:
Statement 1: e^λ is an eigenvalue of e^A.
Statement 2: v is an eigen-vector of e^A.
Which one of the following options is correct?

A. Statement 1 is true and statement 2 is false.

B. Statement 1 is false and statement 2 is true.

C. Both the statements are correct.

D. Both the statements are false.

[MCQ: 2 Marks]

Ans. C

Sol. Given λ is an eigen value of A.

Then eigen value of

$$\left(e^A = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots\right) \text{is}$$

$$\left(1 + \lambda + \frac{\lambda^2}{2!} + \ldots = e^{\lambda}\right)$$

Therefore, statement (1) is true.

We know that eigen vector of A and polynomial matrix in A is same.

Eigne vector of A and e^4 is same.

Therefore, statement (2) is true.

Hence, option C is correct.

41. Let $f(x) = \int_0^x e^t(t-1)(t-2)\,dt$.

Then f(x) decreases in the interval

A. $x \in (1, 2)$ B. $x \in (2, 3)$

C. $x \in (0, 1)$ D. $x \in (0.5, 1)$

[MCQ: 2 Marks]

Ans. A

Sol.

$$f(x) = \int_0^x e^t (t-1)(t-2)\,dt$$

$$f(x) = \int_0^x \underset{\text{II}}{e^t} \underset{\text{I}}{(t^2 - 2t + 2)}\,dt$$

$$f(x) = [(t^2 - 3t + 2)e^t - (2t - 3)\,e^t + 2e^t]_0^x$$

$$= \left\{e^t \left[t^2 - 3t + 2 - 2t + 3 + 2\right]\right\}_{t=0}^{t=x}$$

$$= \left[e^t (t^2 - 5t + 7)\right]_0^x$$

$$f(x) = e^x (x^2 - 5x + 7) - 7$$

Now differentiating w.r.t 'x', we get,

$$f'(x) = e^x (2x - 5) + (x^2 - 5x + 7)\,e^x$$

$$f'(x) = e^x[x^2 - 3x + 2]$$

For decreasing function, $f'(x) < 0$

$$e^x (x^2 - 3x + 2) < 0$$

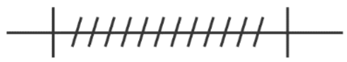
$$e^x (x - 1)(x - 2) < 0$$

$$(x - 1)(x - 2) < 0$$

$$x - 1 > 0 \;\&\; x - 2 < 0$$

$$x > 1 \text{ and } x < 2$$

Using wavy curve method,



$$x - 1 < 0 \text{ and } x - 2 > 0$$

$$x < 1 \;\&\; x > 2$$

$$x \in (1, 2)$$

Hence, option A is correct.

42. Consider a matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars, and I is the identity matrix. Then (c + d) is equal to

A. 5 B. 17

C. −6 D. 11

[MCQ: 2 Marks]

Ans. A

Sol. Given that: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -z \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$

$(1 - \lambda)[(\lambda - 4)(1 - \lambda) + 2] = 0$

$(1 - \lambda)(\lambda^2 - 5\lambda + 6) = 0$

$\lambda^2 - 5\lambda + 6 - \lambda^3 + 5\lambda^2 - 6\lambda = 0$

$-\lambda^3 - 6\lambda^2 - 11\lambda + 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

Applying Cayley Hamilton theorem, i.e., λ = A in the above equation, we get,

$A^3 - 6A^2 + 11A - 6I = 0$

Left multiply by A^{-1}

$A^2 - 6A + 11I - 6A^{-1} = 0$

$6A^{-1} = A^2 - 6A + 11I = A^2 + cA + dI$

On comparing, we get, $c = -6$, $d = 11$

$(c + d) = 11 + (-6) = 5$

Hence, option A is correct.

43. The fuel cost functions in rupees/hour for two 600 MW thermal power plants are given by Plant 1: $C_1 = 350 + 6P_1 + 0.004P_1^2$

Plant 2: $C_2 = 450 + aP_2 + 0.003P_2^2$

where P_1 and P_2 are power generated by plant 1 and plant 2, respectively, in MW and a is constant. The incremental cost of power (λ) is 8 rupees per MWh. The two thermal power plants together meet a total power demand of 550 MW. The optimal generation of plant 1 and plant 2 in MW, respectively, are

A. 200, 350

B. 250, 300

C. 325, 225

D. 350, 200

[MCQ: 2 Marks]

Ans. B

Sol. Given that:

Fuel cost of 2 generating units are:

$C_1 = 350 + 6P_1 + 0.004P_1^2$

$C_2 = 450 + aP_1 + 0.003P_2^2$

Incremental cost, λ = 8 ₹ MWh

Total demand, $P_1 + P_2 = 550$ MW

Incremental cost of plant 1 is

$$\frac{\partial C_1}{\partial P_1} = 6 + 0.008P_1$$

For economic load dispatch,

$$\frac{\partial C_1}{\partial P_1} = \frac{\partial C_2}{\partial P_2} = \lambda$$

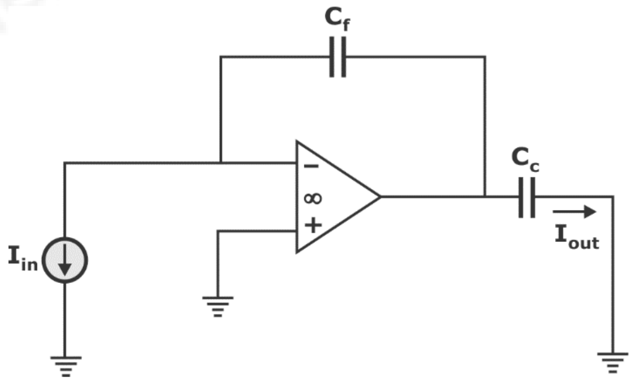
$6 + 0.008P_1 = 8$

$0.008P_1 = 2$

$P_1 = 250$ W & $P_2 = 550 - 250 = 300$ W

Hence, option B is correct.

44. The current gain (I_{out}/I_{in}) in the circuit with an ideal current amplifier given below is



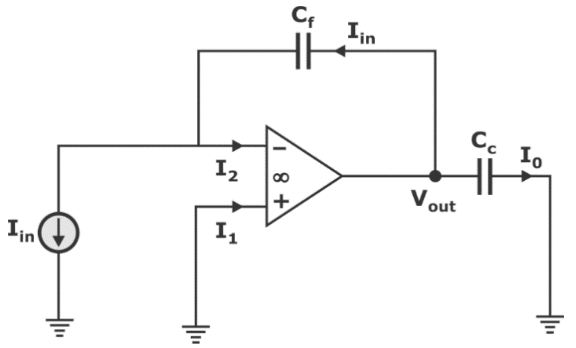
A. $\frac{C_f}{C_c}$ B. $\frac{-C_f}{C_c}$

C. $\frac{C_c}{C_f}$ D. $\frac{-C_c}{C_f}$

[MCQ: 2 Marks]

Ans. C

Sol. From the figure shown below,



According to virtual ground, $V_P = V_n = 0$ V

$I_1 = I_2 = 0 \because R_i = \infty$

$V_{out} = I_{in} \times Z_{cf}$

$$I_{out} = \frac{V_{out}}{Z_{cc}} = \frac{I_{in} Z_{cf}}{Z_{cc}}$$

$$\frac{I_{out}}{I_{in}} = \frac{\frac{1}{sC_f}}{\frac{1}{sC_c}} = \frac{C_c}{C_f}$$

45. If the magnetic field intensity (**H**) in a conducting region is given by the expression, $H = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k}$ A/m. The magnitude of the current density, in A/m^2, at x = 1 m, y = 2 m, and z = 1 m, is

A. 8 B. 12

C. 16 D. 20

[MCQ: 2 Marks]

Ans. B

Sol. Given that:

$$\vec{H} = (x^2)\hat{i} + (x^2y^2)\hat{j} + (x^2y^2z^2)\hat{k} \quad A/m$$

To evaluate current density (J),

We know that $\nabla \times \vec{H} = \vec{J}$

$$\therefore \nabla \times \vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & x^2y^2 & x^2y^2z^2 \end{vmatrix}$$

$$= \hat{i}\left[2yx^2z^2 - 0\right] - \hat{j}\left[2xy^2z^2 - 0\right] + \hat{k}\left[2xy^2 - 0\right]$$

$$\nabla \times \vec{H} = 2yx^2z^2\hat{i} - 2xy^2z^2\hat{j} + 2xy^2\hat{k}$$

At x = 1 m, y = 2 m, and z = 1 m

$$\vec{J} = 4\hat{i} - 8\hat{j} + 8\hat{k}$$

$$\left|\vec{J}\right| = \sqrt{16+64+64} = \sqrt{144}$$

$$\left|\vec{J}\right| = 12 \text{ A/m}^2$$

Hence, option B is correct.

46. Let a causal LTI system be governed by following differential equation

$y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$, where x(t) and y(t) are input & output respectively. Its impulse response is

A. $2e^{-\frac{1}{4}t}u(t)$ B. $2e^{-4t}u(t)$

C. $8e^{-\frac{1}{4}t}u(t)$ D. $8e^{-4t}u(t)$

[MCQ: 2 Marks]

Ans. D

Sol. Given differential equation as

$$y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$$

Taking Laplace transform on both sides,

$$Y(s) + \frac{1}{4}sY(s) = 2X(s)$$

$$Y(s)\left[1 + \frac{s}{4}\right] = 2X(s)$$

So, impulse response is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{8}{s+4}$$

Taking inverse Laplace transform of the impulse response, we get,

$h(t) = 8e^{-4t}\,u(t)$

Hence, option D is correct.

47. Let an input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$ is passed through an LTI system having an impulse response,

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t)$$

The output of the system is

A. $2\sin(10\pi t) + 5\cos(15\pi t)$

B. $5\cos(15\pi t) + 7\sin(42\pi t)$

C. $7\sin(42\pi t) + 4\cos(45\pi t)$

D. $2\sin(10\pi t) + 4\cos(45\pi t)$

[MCQ: 2 Marks]

Ans. C

Sol. Given, impulse response is

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t)$$

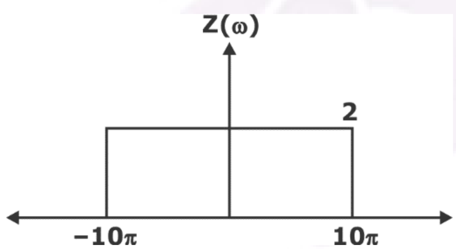
Consider $z(t) = 20\,\mathrm{Sa}(10\pi t)$

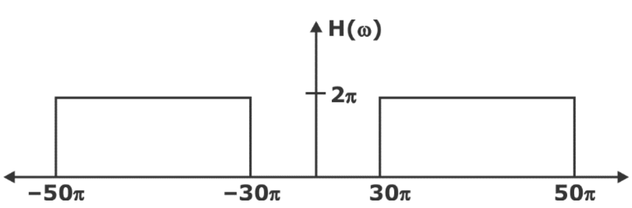
$$\text{So, } Z(\omega) = \frac{20\pi}{10\pi}\mathrm{rect}\left(\frac{\omega}{20\pi}\right) = 2\,\mathrm{rect}\left(\frac{\omega}{20\pi}\right)$$

Also, $h(t) = 20\,\mathrm{Sa}(10\pi t)\cos(40\pi t)$

$$h(t) = z(t)\cos(40\pi t)$$

$$H(\omega) = \pi\left[Z(\omega + 40\pi) + Z(\omega - 40\pi)\right]$$





When input x(t) passes through the system h(t), the frequencies of 42π and 45π will pass through the band.

So, output = $7\sin(42\pi t) + 4\cos(45\pi t)$

Hence, option C is correct.

48. Consider the system as shown below

where $y(t) = x(e^t)$. The system is

A. linear & causal

B. linear & non-causal

C. non-linear & causal

D. non-linear & non-causal

[MCQ: 2 Marks]

Ans. B

Sol. Given that: $y(t) = x(e^t)$

Checking for causality:

At $t = 0$, $y(0) = x(e^0) = x(1)$

So, the output value depends upon future values, hence it is not causal i.e., non-causal.

Now, checking for linearity, we can check for homogeneity and superposition principle,

i.e., $ay(t) = ax(e^t)$

∴ It follows homogeneity.

and $y_3(t) = [y_1(t) = x_1(e^t)] + [y_2(t) = x_2(e^t)]$

$\Rightarrow y_3(t) = x_1(e^t) + x_2(e_t)$

∴ It follows superposition.

Hence, it is linear.

So, the system will be linear and non-causal.

49. The discrete time Fourier series representation of a signal x[n] with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j\left(\frac{2kn\pi}{N}\right)}$. A discrete time periodic signal with period N = 3, has the non-zero Fourier series coefficients: $a_{-3} = 2$ and $a_4 = 1$. The signal is

A. $2+2e^{-\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

B. $1+2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

C. $1+2e^{\left(j\frac{2\pi}{3}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

D. $2+2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

[MCQ: 2 Marks]

Ans. B

Sol. Given that: a_{-3} = 2, a_4 = 1

We know the periodicity principle i.e.,

$a_k = a_{k\pm N}$

So, $a_0 = a_{-3} = a_{-3} = 2$ and,

$a_4 = a_{4\pm3} = a_1 = a_7 = 1$

a_2 = ?

$\because$ It is told that x(n) has non-zero Fourier series coefficient a_{-3} = 2 and a_4 = 1

$\Rightarrow a_0 = 2,\ a_1 = 1$

$\Rightarrow a_2$ is a zero coefficient i.e., $a_2 = 0$

Now, the signal, $x[n]=\sum_{k=0}^{2}a_k e^{\frac{j2\pi kn}{N}}$

$$=a_0 e^0 + a_1 e^{\frac{j2\pi 1}{3}n} + a_z e^{\frac{j2\pi 2}{3}n}$$

$$=2+e^{\frac{j2\pi}{3}n}+0=2+e^{\frac{j2\pi}{3}n}$$

This is the correct answer of the signal x(n), but as it is not present in the options, we need to rearrange it to reach the given options.

$$=1+1+e^{\frac{j2\pi}{3}n}=1+e^{\frac{j2\pi}{6}n}\left[e^{-\frac{j2\pi}{6}n}+e^{\frac{j2\pi}{6}n}\right]$$

Hence, $x(n)=1+2e^{\left(\frac{j2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

50. Let $f(x,y,z)=4x^2+7xy+3xz^2$. The direction in which the function f(x, y, z) increases most rapidly at point P = (1, 0, 2) is

A. $20\hat{i}+7\hat{j}$

B. $20\hat{i}+7\hat{j}+12\hat{k}$

C. $20\hat{i}+12\hat{k}$

D. $20\hat{i}$

[MCQ: 2 Marks]

Ans. B

Sol. Given that: $f(x,y,z)=4x^2+7xy+3xz^2$

$$\vec{\nabla}\cdot f=\hat{i}\frac{\partial f}{\partial x}+\hat{j}\frac{\partial f}{\partial y}+\hat{k}\frac{\partial f}{\partial z}$$

$$\vec{\nabla}\cdot f=\hat{i}\left(8x+7y+3z^2\right)+\hat{j}(7x)+\hat{k}(6xz)$$

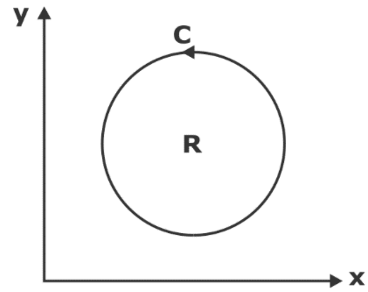
At point P(1, 0, 2), we get,

$$\vec{\nabla}\cdot f=\hat{i}(8+0+12)+\hat{j}(7)+\hat{k}(12)$$

$$\vec{\nabla}\cdot f=20\hat{i}+7\hat{j}+12\hat{k}$$

Hence, option B is correct.

51. Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?



A. $\iint_R dx\,dy$

B. $\oint_C x\,dy$

C. $\oint_C y\,dx$

D. $\frac{1}{2}\oint_C (x\,dy - y\,dx)$

[MCQ: 2 Marks]

Ans. C

Sol. **For Option A:**

By Green's theorem,

$$\iint_R dx\,dy = \text{Area of region R bounded by C}$$

For Option B:

By Green's theorem,

$$\oint_C xdy = \iint_R (1-0)\,dx\,dy = \text{Area of R bounded by C}$$

For Option C:

By Green's theorem,

$$\oint_C (Mdx + Ndy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \iint_R dx\,dy$$

$M = y$, $N = 0$

$$\frac{\partial M}{\partial y} = 1, \frac{\partial N}{\partial x} = 0$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0 - 1 = -1$$

$$\oint_C ydx = \iint_R (1-0)\,dx\,dy = -[\text{Area of R}]$$

Hence, option C is correct.

For Option D:

By Green's theorem,

$$\frac{1}{2}\oint_C xdy = \frac{1}{2}\iint_R [1-(-1)]\,dxdy = \text{Area of R}$$

52. Let $\vec{E}(x,y,z) = 2x^2 i + 5y\hat{j} + 3z\hat{k}$. The value of $\iiint_V (\vec{\nabla}.\vec{E})dV$, where V is the volume enclosed by the unit cube defined by $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$, is

A. 3 B. 8

C. 10 D. 5

[MCQ: 2 Marks]

Ans. C

Sol. Given that: $\vec{E} = 2x^2\hat{i} + 5y\hat{j} + 3z\hat{k}$

Now, to calculate $\int_V (\bar{\nabla}\cdot\vec{E})dv$

Given V is the volume of cube for $0 \le x, y, z \le 1$

Now,

$$\bar{\nabla}\cdot\vec{E} = \frac{\partial}{\partial x}2x^2 + \frac{\partial}{x\partial}5y + \frac{\partial}{\partial z}3z = 4x + 5 + 3$$

$$\bar{\nabla}\cdot\vec{E} = 4x + 8 = 4(x+2)$$

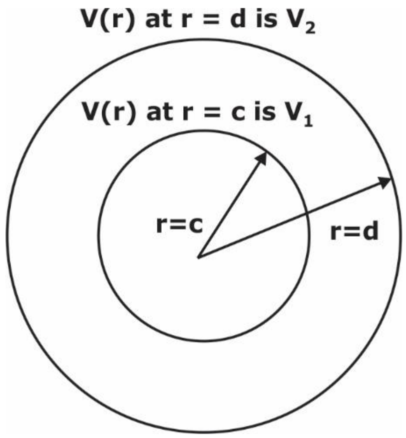
$$\int_V (\vec{\nabla}\cdot\vec{E})dv = 4\int_{x=0}^{x=1}(x+2)\int_{y=0}^{y=1}\int_{z=0}^{z=1} 1\cdot dz\,dy\,dx$$

$$= 4\int_0^1 (x+2)\,dx$$

$$= 4\left[\frac{x^2}{2} + 2x\right]_0^1 = 10$$

Hence, option C is correct.

53. As shown in the figure below, two concentric conducting spherical shells, centered at r = 0 and having radii r = c and r = d are maintained at potentials such that the potential V(r) at r = c is V_1 and V(r) at r = d is V_2. Assume that V(r) depends only on r, where r is the radial distance. The expression for V(r) in the region between r = c and r = d is



A. $V(r)=\frac{cd(V_2-V_1)}{(d-c)r}-\frac{V_1c+V_2d-2V_1d}{d-c}$

B. $V(r)=\frac{cd(V_1-V_2)}{(d-c)r}+\frac{V_2d-V_1c}{d-c}$

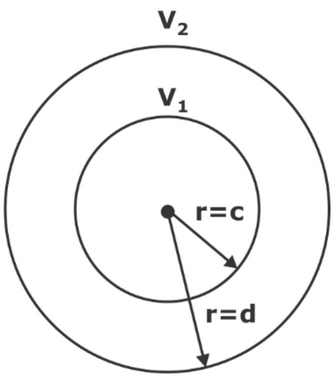
C. $V(r)=\frac{cd(V_1-V_2)}{(d-c)r}-\frac{V_1c-V_2c}{d-c}$

D. $V(r)=\frac{cd(V_2-V_1)}{(d-c)r}-\frac{V_2c-V_1c}{d-c}$

[MCQ: 2 Marks]

Ans. B

Sol. From Laplace's equation, $\nabla^2V=0$



For spherical co-ordinate system,

$$\nabla^2V=\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(\frac{r^2\sin\theta}{1}\cdot\frac{\partial V}{\partial r}\right)\right]=0$$

$$\Rightarrow\quad r^2\frac{dV}{dr}=A;\ A=\text{constant}$$

$$\Rightarrow\quad \frac{dV}{dr}=\frac{A}{r^2}$$

$$\Rightarrow V(r)=\frac{-A}{r}+B;\quad B=\text{constant}$$

At r = c; $V=V_1$

$$\Rightarrow V_1=\frac{-A}{c}+B$$

At r = d; $V=V_2$

$$\Rightarrow V_2=\frac{-A}{d}+B$$

Subtracting equation (1) – (2), we get,

$$V_1-V_2=\frac{-A}{c}+\frac{A}{d}$$

$$\Rightarrow\quad V_1-V_2=A\left[\frac{c-d}{cd}\right]$$

$$\Rightarrow\quad A=\left(\frac{V_1-V_2}{c-d}\right)cd$$

From (1), we get, $B=V_1+\frac{A}{c}$

$$\Rightarrow B=V_1+\left(\frac{V_1-V_2}{c-d}\right)d$$

$$=\frac{V_1(c-d)+(V_1-V_2)d}{c-d}$$

$$=\frac{V_1c-V_1d+V_1d-V_2d}{c-d}$$

$$\therefore\ B=\frac{V_1c-V_2d}{c-d}$$

$$V(r)=\frac{-\left(\frac{V_1-V_2}{c-d}\right)cd}{r}+\frac{V_1c-V_2d}{c-d}$$

$$V(r)=\frac{(V_1-V_2)cd}{r(d-c)}+\frac{V_2d-V_1c}{d-c}$$

Hence, option B is correct.

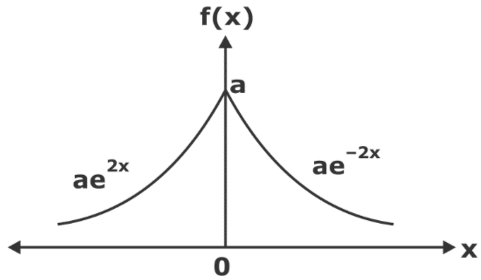
54. Let the probability density function of a random variable x be given as $f(x)=ae^{-2|x|}$ The value of 'a' is ________.

[NAT: 2 Marks]

Ans. 0.99 to 1.01

Sol. Given that: $f(x)=ae^{-2|x|}$

$$\because\int_{-\infty}^{\infty}f(x).dx=1$$



And f(x) is even function.

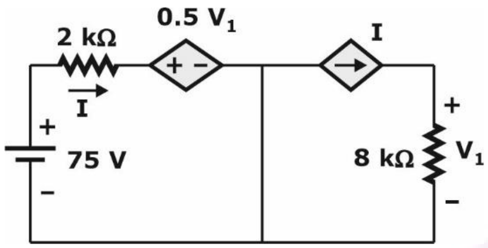
$$\Rightarrow 2\int_0^{\infty} f(x)\cdot dx = 1$$

$$\Rightarrow 2\int_0^{\infty} a e^{-zx}\cdot dx = 1$$

$$\Rightarrow 2a\left[\frac{e^{-2x}}{-2}\right]_0^{\infty} = 1$$

$$\Rightarrow a\left[\frac{0-1}{1}\right] = 1 \Rightarrow a = 1$$

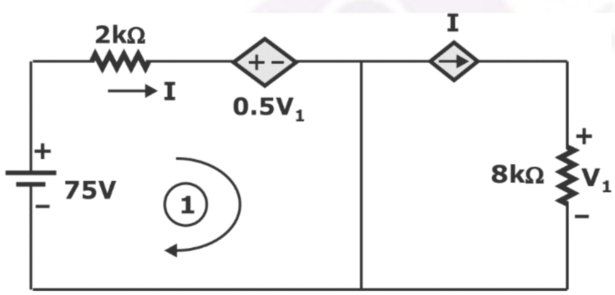
55. In the circuit shown below, the magnitude of the voltage V_1 in volts, across the 8kΩ resistor is ________. (round off to nearest integer)



[NAT: 2 Marks]

Ans. 98 to 102

Sol.



$V_1 = 8000I$ ......(i)

Apply KVL in loop (1)

$-75 + 2000I + 0.5V_1 = 0$

$-75 + 2000I + 0.5(8000I) = 0$

$6000I = 75$

$$I = \frac{75}{6000}$$

Put the value of I in equation (i), we get,

$$V_1 = 8000I = 8000 \times \frac{75}{6000} = 100\ V$$

56. Two generating units rated for 250 MW and 400 MW have governor speed regulations of 6% and 6.4%, respectively, from no load to full load. Both the generating units are operating in parallel to share a load of 500 MW. Assuming free governor action, the load shared in MW, by the 250 MW generating unit is __________. (round off to nearest integer)

[NAT: 2 Marks]

Ans. 198 to 202

Sol. Given: Two generators rated at 250 MW and 400 MW

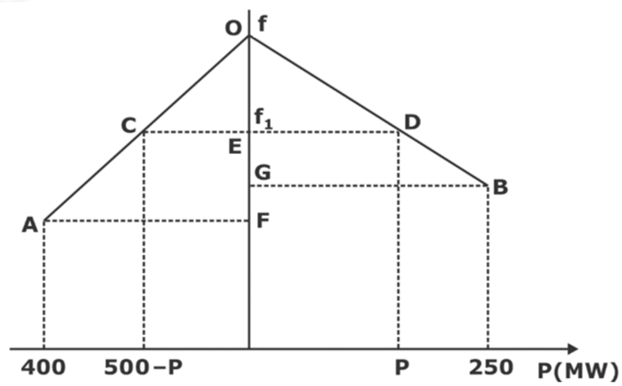
Speed regulation are 6% and 6.4%

Total load = 500 MW

From triangle, ΔOCE and ΔOAF

$$\frac{f-f_1}{500-P} = \frac{OF}{400} = \frac{0.064f}{400}$$

$$\frac{f-f_1}{f} = \frac{(500-P)0.064}{400} \qquad \ldots (1)$$



From triangle ΔODE and ΔOBG

$$\frac{f-f_1}{P} = \frac{(0.06)f}{250}$$

$$\frac{f-f_1}{f} = \frac{0.06P}{250} \qquad \ldots (2)$$

From equation (1) and (2), we get,

$$\frac{0.06\,P}{250} = \frac{(500-P)}{400} \times 0.064$$

$1.5 = 500 - P$

$$P = \frac{500}{2.5} = 200\ \text{W}$$

57. A 20 MVA, 11.2 kV, 4-pole, 50 Hz alternator has an inertia constant of 15 MJ/MVA. If the input and output powers of the alternator are 15 MW and 10 MW, respectively, the angular acceleration in mechanical degree/s^2 is __________. (round off to nearest integer)

[NAT: 2 Marks]

Ans. 74 to 76

Sol. Given that: Machine rating, G = 20 MVA

Pole, P = 4

Frequency, f = 50 Hz

Inertia constant, H = 15 MJ/MVA

Input Power, P_{in} = 15 MW

Output Power, P_{out} = 10 MW

Angular acceleration $= \frac{\Delta P}{M}$

$$\alpha = \frac{P_{in} - P_{out}}{\frac{GH}{180f}} \text{ electrical deg/sec}^2$$

$$\alpha = \frac{(15-10)\times 180 \times 50}{15 \times 20} = 150 \text{ electrical}$$
deg/sec^2

From the relation, $\theta_e = \frac{2}{P}\theta_m$

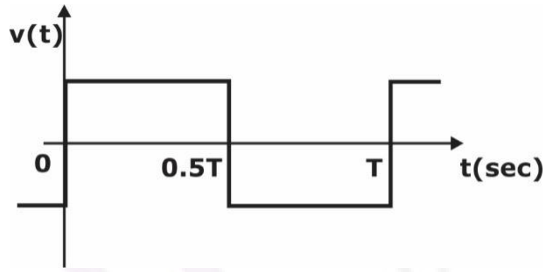
Acceleration in mechanical degrees is

$$\alpha = \frac{2}{4} \times 150 = 75 \text{ mech-deg/sec}^2$$

58. Consider an ideal full-bridge single-phase DC-AC inverter with a DC bus voltage magnitude of 1000 V. The inverter output voltage v(t) shown below, is obtained when diagonal switches of the inverter are switched with 50% duty cycle. The inverter feeds a load with a sinusoidal current given by

$i(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right)$ A, where $\omega = \frac{2\pi}{T}$.

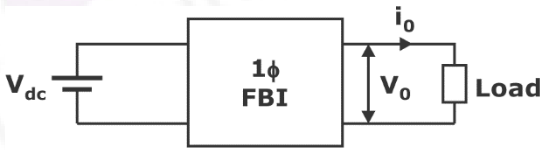
The active power, in watts, delivered to the load is ______. (round off to nearest integer)

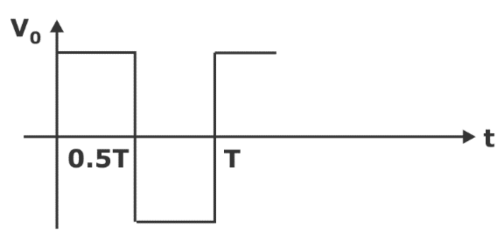


[NAT: 2 Marks]

Ans. 3170 to 3190

Sol.





Fourier series of output voltage is

$$V_o(t) = \sum_{n=1,3,5....} \frac{4V_{dc}}{n\pi} \sin n\omega t$$

As i(t) contains only fundamental, we take V_{01} in power calculation.

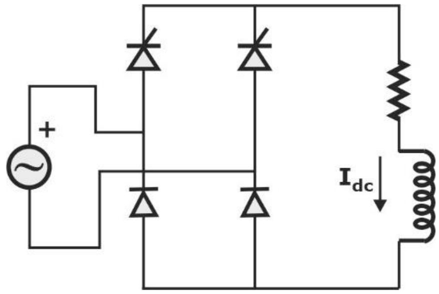
$$V_{01}(t) = \frac{4\,V_{dc}}{\pi}\sin\omega t$$

$$i_{01}(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right)$$

$$P = \frac{1}{2}\left[\frac{4V_{dc}}{\pi}\cdot 10\cos\frac{\pi}{3}\right]$$

$$= \frac{1}{2}\left[\frac{4000}{\pi}\cdot 10\cos\frac{\pi}{3}\right] = 3183.098 \text{ Watts}$$

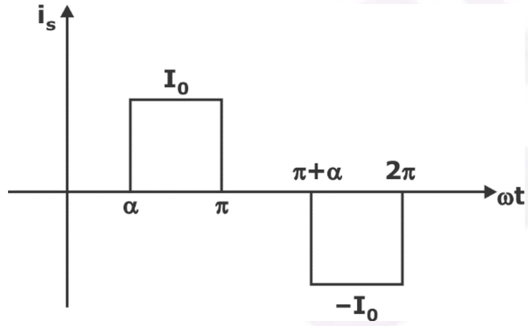
59. For the ideal AC-DC rectifier circuit shown in the figure below, the load current magnitude is I_{dc} = 15 A and is ripple free. The thyristors are fired with a delay angle of 45°. The amplitude of the fundamental component of the source current, in amperes, is ________. (round off to two decimal places)



[NAT: 2 Marks]

Ans. 17.3 to 18.0

Sol. Single Phase Semi converter:



Fourier series of above waveform

$$i_s = \sum_{n=1,3,5}\frac{4I_0}{n\pi}\cos\frac{n\alpha}{2}\sin\left(n\omega t - \frac{n\alpha}{2}\right)$$

Fundamental source RMS $I_{S1} = \frac{2\sqrt{2}}{\pi} I_0 \cos\frac{\alpha}{2}$

$I_{S1}(\text{rms}) = 0.9 \times 15\cos\frac{45^\circ}{2} = 12.472$ A

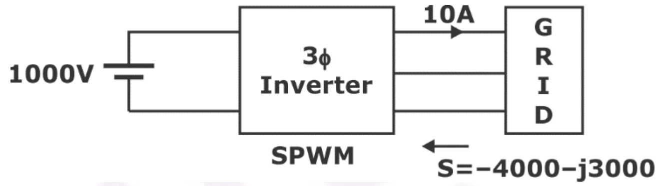
$I_{S1}(\text{max}) = 12.472\sqrt{2} = 17.638$ A

60. A 3-phase grid-connected voltage source converter with DC link voltage of 1000 V is switched using sinusoidal Pulse Width Modulation (PWM) technique. If the grid phase current is 10 A and the 3-phase complex power supplied by the converter is given by (−4000−j3000) VA, then the modulation index used in sinusoidal PWM is ________. (round off to two decimal places)

[NAT: 2 Marks]

Ans. 0.46 to 0.48

Sol.



In, SPWM

Phase voltage output (fundamental) = $\frac{mV_s}{2\sqrt{2}}$

m = modulation index

We know, $S = 3V_{ph}I_{ph}$

$$S = \sqrt{4000^2 + 3000^2} = 5000$$

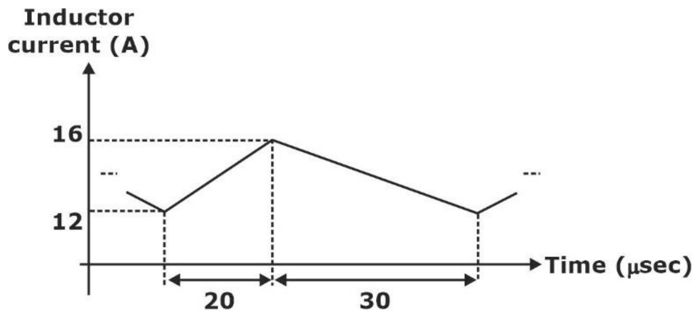
So, 5000 = 3 × V_{ph} × 10

$$V_{Ph} = \frac{5000}{30} = 166.67V$$

$$\text{So}, 166.67 = \frac{mV_s}{2\sqrt{2}}$$

$$m = \frac{166.6 \times 2\sqrt{2}}{1000} = 0.4712$$

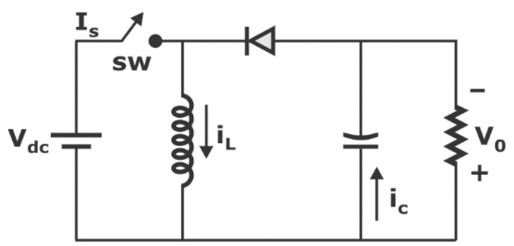
61. The steady state current flowing through the inductor of a DC-DC buck boost converter is given in the figure below. If the peak-to-peak ripple in the output voltage of the converter is 1 V, then the value of the output capacitor, in μF, is ________. (round off to nearest integer)

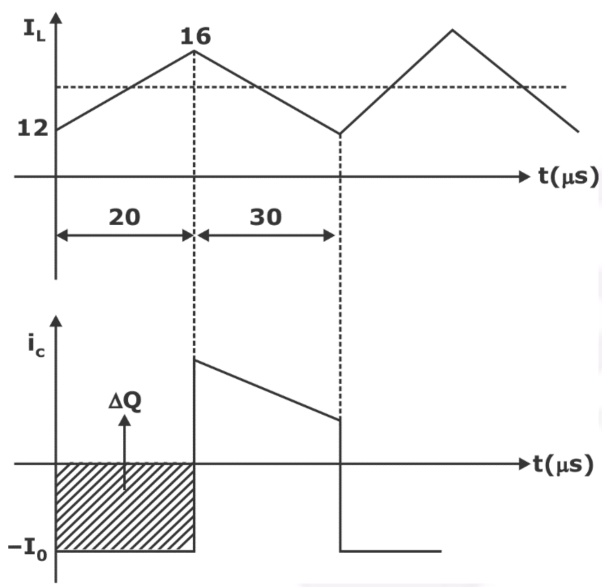


[NAT: 2 Marks]

Ans. 165 to 171

Sol. Buck boost converter is





$$\Delta V_0 = \frac{\Delta Q}{C}$$

$$\Delta V_0 = \frac{1}{C} I_0 DT = 1V$$

$I_{L,(min)} = 12$ A & $I_{L,(max)} = 16$ A

$$I_{L(avg)} \frac{I_{L(min)} + I_{L(max)}}{2} = \frac{12+16}{2} = 14 \text{ A}$$

$T_{ON} = 20$ μsec, $T_{OFF} = 30$ μsec

$T = T_{ON} + T_{OFF} = 50$ μsec

$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}} = \frac{2}{5}$$

$$I_{L(avg)} = \frac{I_0}{1-D}$$

$$I_0 = 14\left(1 - \frac{2}{5}\right) = 8.4 \text{ A}$$

We know that,

$$\Delta V_0 = \frac{1}{C} I_0 DT$$

$$C = 8.4 \times \frac{2}{5} \times 50\,\mu\text{sec} = 168\,\mu\text{F}$$

62. A 280 V, separately excited DC motor with armature resistance of 1 Ω and constant field excitation drives a load. The load torque is proportional to the speed. The motor draws a current of 30 A when running at a speed of 1000 rpm. Neglect frictional losses in the motor. The speed, in rpm, at which the motor will run, if an additional resistance of value 10 Ω is connected in series with the armature, is ___________. (round off to nearest integer)

[NAT: 2 Marks]

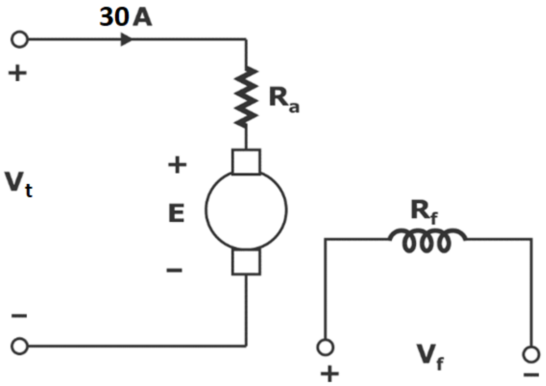
Ans. 480 to 485

Sol. Given that: In separately excited motor,

Terminal voltage V_t = 280 V, armature resistance R_a = 1 Ω

Armature current I_a = 30 A at speed N = 1000 rpm

If R = 10 Ω inserted in series with armature, then speed = ?



$E = V_t - I_a R_a$

$E = 280 - 30 \times 1 = 250$ V

Now, 10 Ω is inserted.

$E = 280 - I(1 + 10) = 280 - 11I$ ... (1)

We know that Torque $T \propto \phi I$

$\because \phi$ is constant in separately-excited motor.

So, $T \propto I$ & $T \propto N$ (given)

So, $N \propto I$

$$\frac{N_1}{N_2} = \frac{I_1}{I_2}$$

$$\frac{1000}{N_2} = \frac{30}{I_2} \Rightarrow I_2 = 0.03N_2$$

Put in equation (1),

$E = 280 - 11 \times 0.03N_2 = 280 - 0.33N_2$

Also, we know that

$E \propto N$ (for separately-excited motor)

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

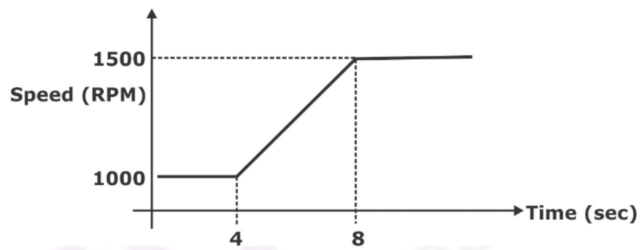
$$\frac{250}{280 - 0.33N_2} = \frac{1000}{N_2}$$

$N_2 = 1120 - 1.32N_2$

$2.32N_2 = 1120$

$N_2 = 482.75$ rpm

63. A 4-pole induction motor with inertia of 0.1 kg-m^2 drives a constant load torque of 2 Nm. The speed of the motor is increased linearly from 1000 rpm to 1500 rpm in 4 seconds as shown in the figure below. Neglect losses in the motor. The energy, in joules, consumed by the motor during the speed change is ________. (round off to nearest integer)

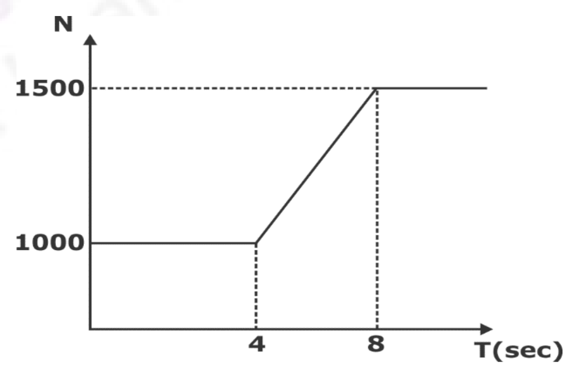


[NAT: 2 Marks]

Ans. 1725 to 1740

Sol. Given that: Moment of inertia J = 0.1 kg/m^2

Torque $T_L = 2$ N-m



$$\omega_1 = 1000 \times \frac{2\pi}{60} = 104.7 \text{ rad/sec}$$

$$\omega_2 = 1500 \times \frac{2\pi}{60} = 157.08 \text{ rad/sec}$$

From equation of line

$$N = \left(\frac{1500 - 1000}{8 - 4}\right)t + c = 125\,t + c$$

At t = 4, N = 1000

So, c = 500

$\therefore$ N = 125t + 500

We know that

$$J\frac{d\omega}{dt} = T_e - T_L$$

$$T_e = T_L + J\frac{d\omega}{dt}$$

$$P_e = T_e\omega = T_L\omega + J\omega\frac{d\omega}{dt}$$

Change in energy, $dE = T_L\,\omega dt + J\omega d\omega$

So, Total change energy is

$$= \int_4^8 T_L\omega dt + \int_{\omega_1}^{\omega_2} J\omega\, d\omega$$

$$\Delta E = 2\int_4^8 \frac{2\pi N}{60} dt + 0.1\int_{104.7}^{157.08} \omega\, d\omega$$

$$\Delta E = \frac{4\pi}{60}\int_4^8 (125t + 500)\,dt + 0.1 \times \frac{1}{2}\left[\omega^2\right]_{104.7}^{157.08}$$

$$\Delta E = \frac{\pi}{15} \times \left[\frac{125}{2}\left[t^2\right]_4^8 + 500\left[t\right]_4^8\right]$$

$$+\frac{0.1}{2}\left[157.08^2 - 104.7\right]$$

$$\Delta E = \frac{\pi}{15} \times \left[\frac{125}{2} \times 48 + 500 \times 4\right] + 685.4$$

$\Delta E =$ 1732.5 J

64. A star-connected 3-phase, 400 V, 50 kVA, 50 Hz synchronous motor has a synchronous reactance of 1 ohm per phase with negligible armature resistance. The shaft load on the motor is 10 kW while the power factor is 0.8 leading. The loss in the motor is 2 kW. The magnitude of the per phase excitation emf of the motor, in volts, is ________. (round off to nearest integer)

[NAT: 2 Marks]

Ans. 240 to 248

Sol. Given star-connected 3-phase synchronous motor

Line voltage = 400 V

Synchronous reactance X = 1 Ω/phase

Shaft load = 10 kW; Losses 2 kW

Power factor = 0.8 leading

$\cos\phi = 0.8 \Rightarrow \phi = 36.87°$

Input power = Shaft load + losses

P_{in} = 10 + 2 = 12 kW

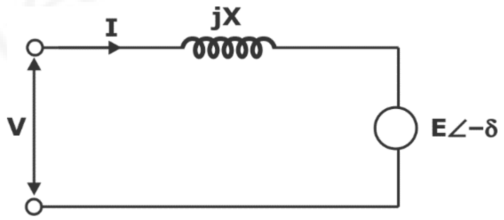
$$\text{Current, } I = \frac{P_{in}}{\sqrt{3} \times V_L \times \cos\phi}$$

$$= \frac{12 \times 10^3}{\sqrt{3} \times 100 \times 0.8}$$

$= 21.65$ A

$$\text{Terminal voltage} = \frac{400}{\sqrt{3}}$$

= 230.9 V/phase



$E\angle{-\delta} = V - jX(I\angle\phi)$

$E\angle{-\delta} = 230.9 - j \times 21.65\angle 36.87°$

$E\angle{-\delta} = 230.9 - 21.65\angle 126.87°$

$E\angle{-\delta} = 230.9 - 21.65(\cos 126.87° + j\sin 126.87°)$

$E\angle{-\delta} = 230.9 - (-13.02 + j\,17.32)$

$E\angle{-\delta} = 243.92 - j17.32$

$E = \sqrt{(243.93)^2 + (17.32)^2} = 244.5 \approx 245$ V

Alternate Solution:

$$E_{Ph} = \sqrt{(V\cos\phi - I_aR_a)^2 + (V\sin\phi + I_aX_S)^2}$$

$$V_{Ph} = \frac{400}{\sqrt{3}}$$

$X_S = 1\ \Omega$

$R_a = 0$

$\cos\varphi = 0.8$

$\Rightarrow \sin\varphi = 0.6$

$\therefore P_{in} = P_{sh} + P_{loss} = 10 + 2 = 12$ kW

$$= \sqrt{3}V_LI_L\cos\phi$$

$$I_L = \frac{12000}{\sqrt{3}\times 400\times 0.8} = 21.6A = I_a$$

$$E_{Ph} = \sqrt{\left(\frac{400}{\sqrt{3}}\times 0.8 - 21.6\times 0\right)^2 + \left(\frac{400}{\sqrt{3}}\times 0.6 + 21.6\times 1\right)^2}$$

$E_{Ph} = 244.52$ V

65. A 3-phase, 415 V, 4-pole, 50 Hz induction motor draws 5 times the rated current at rated voltage at starting. It is required to bring down the starting current from the supply to 2 times of the rated current using a 3-phase autotransformer. If the magnetizing impedance of the induction motor and no-load current of the autotransformer is neglected, then the transformation ratio of the autotransformer is given by ________. (round off to two decimal places)

[NAT: 2 Marks]

Ans. 0.61 to 0.65

Sol. Given that: Starting current $I_{SC} = 5I_R$

It is required to bring down starting current to $2I_R$ by using autotransformer.

We know that in autotransformer starting method.

$I_S = x^2\ I_{SC}$

$2I_R = x^2 \times 5I_R$

$$x^2 = \frac{2}{5} = 0.4$$

$x = 0.632$

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