BYJU'S EXAM PREP

GATE 2022

Mechanical Engineering

Afternoon Shift

Questions with Detailed Solution

1. Writing too many things on the ________ while teaching could make the students get ________.

[MCQ: 1 Mark]

A. bored / board

B. board / bored

C. board / board

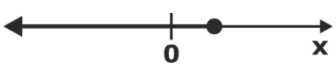
D. bored / bored

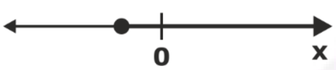
Ans. B

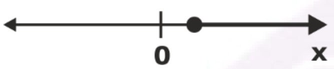
Sol. Writing too many things on the board while teaching could make the students get bored.

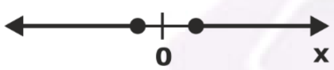
2. Which one of the following is a representation (not to scale and in bold) of all values of satisfying the inequality $2 - 5x \le -\frac{6x-5}{3}$ on the real number line?

[MCQ: 1 Mark]

A. 

B. 

C. 

D. 

Ans. C

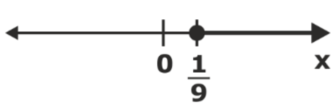
Sol. Given,

$$2 - 5x \le -\left(\frac{6x-5}{3}\right)$$

$$(2-5x)3 \le -6x + 5$$

$$9x \ge 1$$

$$x \ge \frac{1}{9}$$



3. If $f(x) = 2\ln(\sqrt{e^x})$, what is the area bounded by f(x) for the interval [0, 2] on the x axis?

[MCQ: 1 Mark]

A. 1/2 B. 1

C. 2 D. 4

Ans. C

Sol. $f(x) = 2\ln(\sqrt{e^x}) = 2\ \ln\ e^{x/2} = \frac{x}{2} \times 2$

$$f(x) = x$$

$$\text{Area} = \int_0^2 f(x)dx = \int_0^2 xdx = \frac{x^2}{2} = \frac{4}{2} = 2$$

4. A person was born on the fifth Monday of February in a particular year. Which one of the following statements is correct based on the above information?

[MCQ: 1 Mark]

A. The 2nd February of that year is a Tuesday

B. There will be five Sundays in the month of February in that year

C. The 1st February of that year is a Sunday

D. All Mondays of February in that year have even dates

Ans. A

Sol. Given,

February has 5 Mondays.

Possibility of number of days in February month are 28 or 29.

To have 5th Monday February should have 5 weeks, 4 complete weeks and minimum one Monday.

So,

29 = 4 × 7 + 1

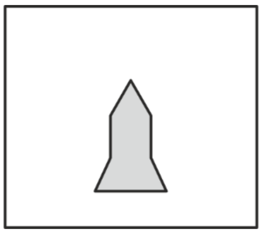
So,

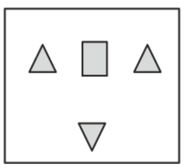
First day will be Monday and second day will be Tuesday.

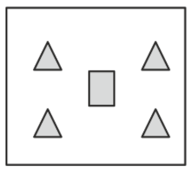
5. Which one of the groups given below can be assembled to get the shape that is shown above using each piece only once without overlapping with each other?

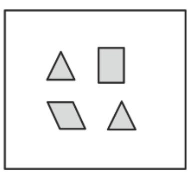
(Rotation and translation operations may be used).

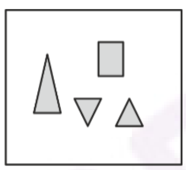
[MCQ: 1 Mark]



A. 

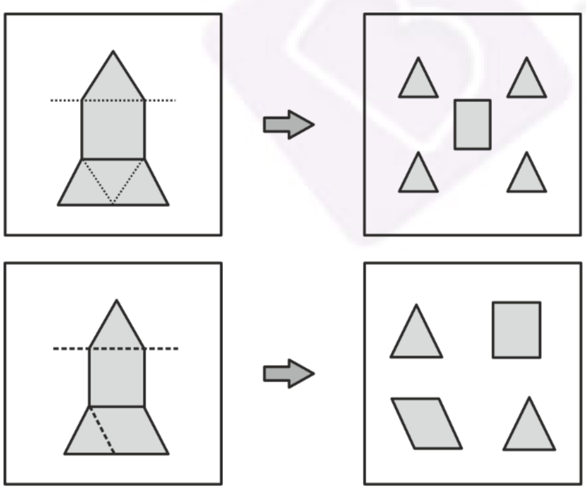
B. 

C. 

D. 

Ans. B, C

Sol.



Both options B and C are correct but only option B is given in the GATE 2022 official answer key

6. Fish belonging to species S in the deep sea have skins that are extremely black (ultra-black skin). This helps them not only to avoid predators but also sneakily attack their prey. However, having this extra layer of black pigment results in lower collagen on their skin, making their skin more fragile.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

[MCQ: 2 Marks]

A. Having ultra-black skin is only advantageous to species S

B. Species S with lower collagen in their skin are at an advantage because it helps them avoid predators

C. Having ultra-black skin has both advantages and disadvantages to species S

D. Having ultra-black skin is only disadvantageous to species S but advantageous only to their predators

Ans. C

Sol. Fragile means easily broken or damaged which is a disadvantage.

So, ultra-black skin has both advantage and disadvantage.

7. For the past days, the average daily production at a company was 100 units per day.

If today's production of 180 units changes the average to 110 units per day, what is the value of m?

[MCQ: 2 Marks]

A. 18 B. 10

C. 7 D. 5

Ans. C

Sol. Given,

For m days average is 100

For (m + 1) days average is 110 with $(m + 1)^{th}$ term 180.

So,

$$\frac{m \times 100 + 180}{m+1} = 110$$

$$100m + 180 = 110m + 110$$

$$70 = 10m$$

$$m = 7$$

8. Consider the following functions for non-zero positive integers, p and q

$$f(p,q) = \underbrace{p \times p \times p \times \ldots \ldots \ldots \times p}_{q\ \text{terms}} = p^q; \quad f(p, 1) = p$$

$$g(p,q) = p; \; g(p, 1) = p$$

Which one of the following options is correct based on the above?

[MCQ: 2 Marks]

A. f(2, 2) = g(2, 2)

B. f(g(2, 2), 2) < f(2, g(2, 2))

C. g(2, 1) $\neq$ f(2, 1)

D. f(3, 2) > g(3, 2)

Ans. A

Sol. $f(2, 2) = 2^2 = 4$

$$g(2, 2) = 2^2 = 4$$

$$f(2, 1) = 2^1 = 2$$

$$g(2, 1) = 2$$

$$f\left(g(2, 2), 2\right) = f(4, 2) = 4^2 = 16$$

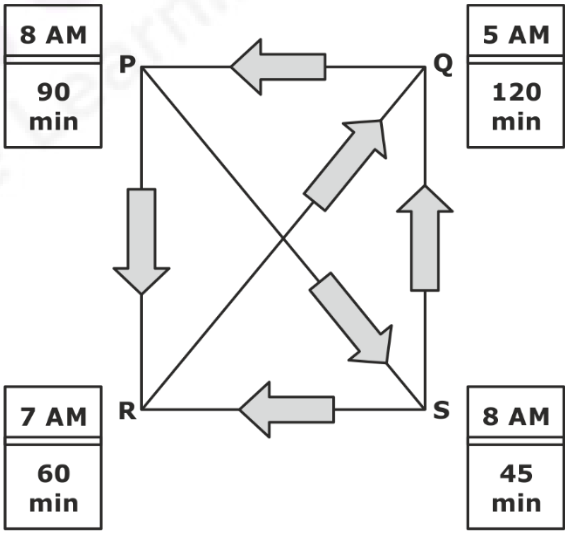
$$f\left(2, g(2, 2)\right) = f(2, 4) = 2^4 = 16$$

So, answer A will be correct.

9. Four cities P, Q, R and S are connected through one-way routes as shown in the figure. The travel time between any two connected cities is one hour. The boxes beside each city name describe the starting time of first train of the day and their frequency of operation. For example, from city P, the first trains of the day start at 8 AM with a frequency of 90 minutes to each of R and S. A person does not spend additional time at any city other than the waiting time for the next connecting train.

If the person starts from R at 7 AM and is required to visit S and return to R, what is the minimum time required?

[MCQ: 2 Marks]



A. 6 hours 30 minutes

B. 3 hours 45 minutes

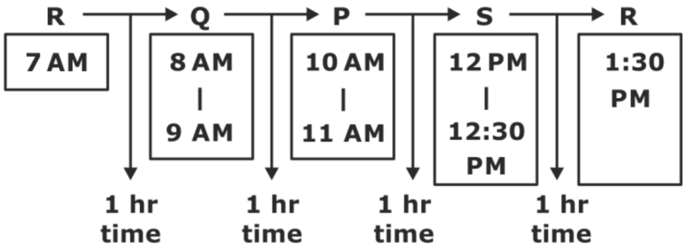
C. 4 hours 30 minutes

D. 5 hours 15 minutes

Ans. A

Sol. Required answer is

$R \to S \to R$



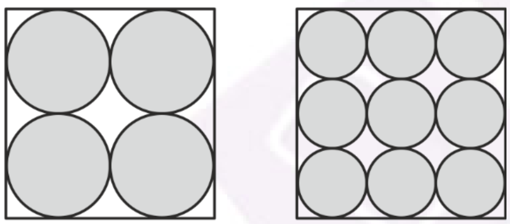
1 + 1 + 1 + 1 + 1 + (30 min) + 1

= 6 hr. 30 min.

10. Equal sized circular regions are shaded in a square sheet of paper of 1 cm side length. Two cases, case M and case N, are considered as shown in the figures below. In the case M, four circles are shaded in the square sheet and in the case N, nine circles are shaded in the square sheet as shown.

What is the ratio of the areas of unshaded regions of case M to that of case N?

[MCQ: 2 Marks]



Case M **Case N**

A. 2 : 3

B. 1 : 1

C. 3 : 2

D. 2 : 1

Ans. B

Sol. Case 1

$6r = 1$

$r = \frac{1}{6}$

Case 2

$4R = 1$

$R = \frac{1}{4}$

Area of square $= 1^2 = 1$

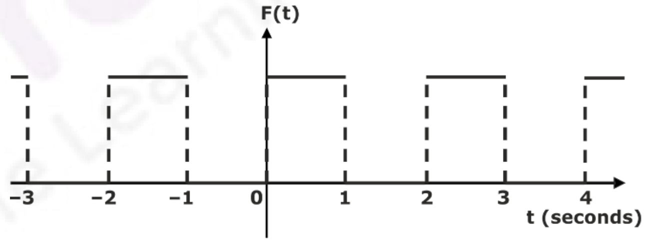
Area of small circle $= 9 \times \pi \times \left(\frac{1}{6}\right)^2 = \frac{\pi}{4}$

Area of large circle $= 4 \times \pi \times \left(\frac{1}{4}\right)^2 = \frac{\pi}{4}$

Shaded area is same, so ratio of unshaded part will also be same.

11. F(t) is a periodic square wave function as shown. It takes only two values 4 and 0, and stays at each of these values for 1 second before changing. What is the constant term in the Fourier series expansion of F(t)?

[MCQ: 1 Mark]



A. 1 B. 2

C. 3 D. 4

Ans. B

Sol. Given, f(t)

Constant term from Fourier series

$$\frac{a_0}{2} = \frac{1}{2}\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\,dx$$

$$= \frac{1}{2}\frac{1}{\ell}\int_{\alpha}^{\alpha+2\ell} f(x)\,dx$$

$$\alpha = 0,\ \alpha + 2\ell = 2$$

$$\ell = 1$$

$$\frac{a_0}{2} = \frac{1}{2} \times \frac{1}{1} \times \int_0^2 f(x)\,dx$$

$$= \frac{1}{2} \times \left[\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx \right]$$

$$= \frac{1}{2} \left[\int_0^1 4\,dx + \int_1^2 0\,dx \right]$$

$$= \frac{1}{2} \times 4 = 2$$

12. Consider a cube of unit edge length and sides parallel to co-ordinate axes, with its centroid at the point (1, 2, 3). The surface integral $\int_A \vec{F} \cdot d\vec{A}$ of a vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$ over the entire surface A of the cube is ______.

[MCQ: 1 Mark]

A. 14
B. 27
C. 28
D. 31

Ans. A

Sol. Given,

Edge length = 1 m

Centroid, (1, 2, 3)

$$\int_A \vec{f} \cdot d\vec{A}$$

Only Gauss divergence theorem

$$\int_A \vec{f} \cdot dA = \iiint_V \nabla \cdot \vec{f}\, dV$$

$$= \iiint_V \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) (3x\hat{i} + 5y\hat{j} + 6z\hat{k})dV$$

$$= \iiint_V (3 + 5 + 6)\,dV$$

$$= 14 \times \iiint_V dV$$

$$= 14 \times 1^3 = 14$$

13. Consider the definite integral

$$\int_1^2 (4x^2 + 2x + 6)\,dx$$

Let I_e be the exact value of the integral. If the same integral is estimated using Simpson's rule with 10 equal subintervals, the value is I_S. The percentage error is defined as $e = 100 \times (I_e - I_S)/I_e$. The value of e is

[MCQ: 1 Mark]

A. 2.5 B. 3.5
C. 1.2 D. 0

Ans. D

Sol. Given,

$$\int_1^2 (4x^2 + 2x + 6)\,dx$$

$$f(x) = 4x^2 + 2x + 6$$

Since f(x) is quadratic polynomial and we know that Simpson rule give exact value for quadratic polynomial so error will be zero.

14. Given $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$. If a and b are positive integers, the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$ is ______.

[MCQ: 1 Mark]

A. $\sqrt{\pi a}$ B. $\sqrt{\pi / a}$
C. $b\sqrt{\pi a}$ D. $b\sqrt{\pi / a}$

Ans. B

Sol. Given,

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$$

Let, $a(x+b)^2 = t^2$

$$a \times 2(x+6)dx = 2t\,dt$$

$$\frac{a \times 2(x+b)}{2t} dx = dt$$

$$\frac{a \times (x+b)}{\sqrt{a}(x+b)} dx = dt$$

$$\sqrt{a}\,dx = dt$$

$$dx = \frac{dt}{\sqrt{a}}$$

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$= \sqrt{\frac{\pi}{a}}$$

15. A polynomial $\phi(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots. + a_1 s + a_0$ of degree $n > 3$ with constant real coefficients $a_n, a_{n-1}, \ldots a_0$ has triple roots at $s = -\sigma$. Which one of the following conditions must be satisfied?

[MCQ: 1 Mark]

A. $\phi(s) = 0$ at all the three values of s satisfying $s^3 + \sigma^3 = 0$

B. $\phi(s) = 0,\ \frac{d\phi(s)}{ds} = 0,$ and $\frac{d^2\phi(s)}{ds^2} = 0$ at $s = -\sigma$

C. $\phi(s) = 0,\ \frac{d^2\phi(s)}{ds^2} = 0,$ and $\frac{d^4\phi(s)}{ds^4} = 0$ at $s = -\sigma$

D. $\phi(s) = 0,$ and $\frac{d^3\phi(s)}{ds^3} = 0$ at $s = -\sigma$

Ans. B

Sol. Given,

$$\phi(s) = a_n s^n + a_{n-1} s^{n+} + \ldots\ldots + a_1 s + a_0$$

has three triple roots at s = −σ

So,

$$\phi(s) = (s+\sigma)^3(\phi(s))\ (n > 3)$$

$$\phi(s) = 0 \text{ at } s = -\sigma$$

$$\phi'(s) = 3(s+\sigma)^2(\phi(s)) + \phi'(s)(s+\sigma)^3$$

So,

$$\phi'(s) = 0 \text{ at } s = -\sigma$$

$$\phi''(s) = 3 \times 2(s+\sigma)(\phi(s)) + 3(s+\sigma)^2(\phi'(s))$$

$$+ \phi''(s)(s+\sigma)^3 + 3(s+\sigma)^2\,\phi'(s)$$

So,

$$\phi''(s) = 0 \text{ at } s = -\sigma$$

16. Which one of the following is the definition of ultimate tensile strength (UTS) obtained from a stress-strain test on a metal specimen?

[MCQ: 1 Mark]

A. Stress value where the stress-strain curve transitions from elastic to plastic behavior

B. The maximum load attained divided by the original cross-sectional area

C. The maximum load attained divided by the corresponding instantaneous cross-sectional area

D. Stress where the specimen fractures

Ans. B

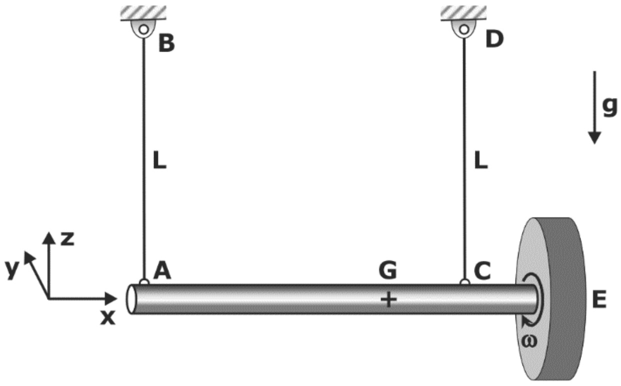
Sol. From stress strain test on a metal specimen, we obtain UTS which is engineering stress.

So, ultimate tensile strength will be obtained using

$$\text{UTS} = \frac{\text{Maximum tensile load}}{\text{original cross - sectional area}}$$

17. A massive uniform rigid circular disc is mounted on a frictionless bearing at the end E of a massive uniform rigid shaft AE which is suspended horizontally in a uniform gravitational field by two identical light inextensible strings AB and CD as shown, where G is the center of mass of the shaft-disc assembly and is the acceleration due to gravity. The disc is then given a rapid spin ω about its axis in the positive x-axis direction as shown, while the shaft remains at rest. The direction of rotation is defined by using the right-hand thumb rule. If the string AB is suddenly cut, assuming negligible energy dissipation, the shaft AE will

[MCQ: 1 Mark]



A. rotate slowly (compared to ω about the negative z-axis direction

B. rotate slowly (compared to ω about the positive z-axis direction

C. rotate slowly (compared to ω about the positive y-axis direction

D. rotate slowly (compared to ω about the negative y-axis direction

Ans. A

Sol. When the wire AB, is out as the centre of gravity is on opposite side of rotor and between them the string is attached.

So, point A will try to go down and this will only happen if precision axis is negative z-axis.

18. A structural member under loading has a uniform state of plane stress which in usual notations is given by $\sigma_x = 3P$, $\sigma_y = -2P$ and $\tau_{xy} = \sqrt{2}\,P$ where P > 0. The yield strength of the material is 350 MPa. If the member is designed using the maximum distortion energy theory, then the value of at which yielding starts (according to the maximum distortion energy theory) is

[MCQ: 1 Mark]

A. 70 MPa B. 90 MPa

C. 120 MPa D. 75 MPa

Ans. A

Sol. Given

$$\sigma_x = 3P$$

$$\sigma_4 = -2P$$

$$\tau_{xy} = \sqrt{2}\,P$$

$$P > 0$$

Strength = 350 MPa

Using maximum distortion energy theory

$$\sqrt{\sigma_1^2 + \sigma_1^2 - \sigma_1\,\sigma_2} = \frac{S_{yt}}{N}$$

$$\sigma_1, \sigma_2 = \frac{1}{2}\left[(3P - 2P) \pm \sqrt{(3P + 2P)^2 + 4(\sqrt{2}P)^2}\right]$$

$$\sigma_{1,2} = 3.375\text{ P}, \; -2.375P$$

$$\sqrt{(3.375P)^2 + (-2.375P)^2 + 2.375 \times 3.375P^2}$$

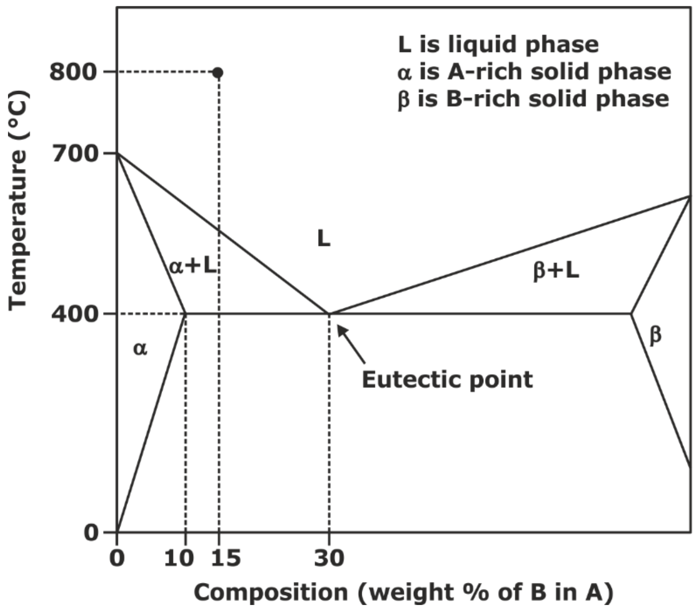
$$= \frac{S_{yt}}{N}$$

$$N = 1, \; S_{yt} = 350\,\text{MPa}$$

$$P = 70.06\,\text{N}$$

19. Fluidity of a molten alloy during sand casting depends on its solidification range. The phase diagram of a hypothetical binary alloy of components A and B is shown in the figure with its eutectic composition and temperature. All the lines in this phase diagram, including the solidus and liquidus lines, are straight lines. If this binary alloy with 15 weight % of B is poured into a mould at a pouring temperature of 800°C, then the solidification range is

[MCQ: 1 Mark]

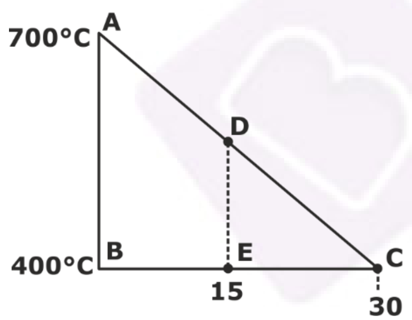


A. 400°C B. 250°C

C. 800°C D. 150°C

Ans. D

Sol. Using triangle



From similar triangles

$$\frac{AB}{BC} = \frac{DE}{EC}$$

$$\frac{300}{30} = \frac{DE}{15}$$

$$150 = DE$$

Solidification range is 150°C.

20. A shaft of diameter $25^{-0.04}_{-0.07}$ mm is assembled in a hole of diameter $25^{+0.02}_{-0.00}$ mm.

Match the allowance and limit parameter in Column I with its corresponding quantitative value in Column II for this shaft-hole assembly.

Allowance and limit parameter (Column I)		**Quantitative value (Column II)**	
P	Allowance	1	0.09 mm
Q	Maximum clearance	2	24.96 mm
R	Maximum material limit for hole	3	0.04 mm
		4	25.0 mm

[MCQ: 1 Mark]

A. P-3, Q-1, R-4

B. P-1, Q-3, R-2

C. P-1, Q-3, R-4

D. P-3, Q-1, R-2

Ans. A

Sol. Given,

Shaft $25^{-0.04}_{-0.07}$

Hole $25^{+0.02}_{-0.00}$

Allowance = Minimum clearance

= 25 – 0.00 – (25 – 0.04)

= 0.04 mm

Maximum clearance = 25.02 – 25 + 0.07

= 0.09 mm

Maximum material limit for hole = 25.0 mm

21. Match the additive manufacturing technique in Column I with its corresponding input material in Column II.

Additive manufacturing technique (Column I)		Input material (Column II)	
P	Fused deposition modelling		Photo sensitive liquid resin
Q	Laminated object manufacturing	2	Heat fusible powder
R	Selective laser sintering	3	Filament of polymer
		4	Sheet of thermoplastic or green compacted metal sheet

[MCQ: 1 Mark]

A. P-3, Q-4, R-2 B. P-1, Q-2, R-4

C. P-2, Q-3, R-1 D. P-4, Q-1, R-4

Ans. A

Sol. Fixed deposition modelling – Filament of polymer

Laminated object manufacturing – Sheet of thermos plastic green compacted metal sheet

Selective laser sintering – Heat fusible powder

22. Which one of the following CANNOT impart linear motion in a CNC machine?

[MCQ: 1 Mark]

A. Linear motor B. Ball screw

C. Lead screw D. Chain and sprocket

Ans. D

Sol. Only chain and sprocket cannot import linear motion in a CNC machine. It is used for power transmission.

23. Which one of the following is an intensive property of a thermodynamic system?

[MCQ: 1 Mark]

A. Mass B. Density

C. Energy D. Volume

Ans. B

Sol.

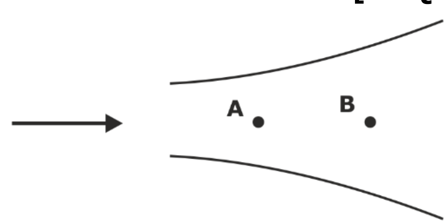
Intensive property is independent of mass of system

Density is independent of mass.

24. Consider a steady flow through a horizontal divergent channel, as shown in the figure, with supersonic flow at the inlet. The direction of flow is from left to right.

Pressure at location B is observed to be higher than that at an upstream location A. Which among the following options can be the reason?

[MCQ: 1 Mark]



A. Since volume flow rate is constant, velocity at B is lower than velocity at A

B. Normal shock

C. Viscous effect

D. Boundary layer separation

Ans. B

Sol. Due to normal shock pressure at location.

B is observed to be higher.

25. Which of the following non-dimensional terms is an estimate of Nusselt number?

[MCQ: 1 Mark]

A. Ratio of internal thermal resistance of a solid to the boundary layer thermal resistance

B. Ratio of the rate at which internal energy is advected to the rate of conduction heat transfer

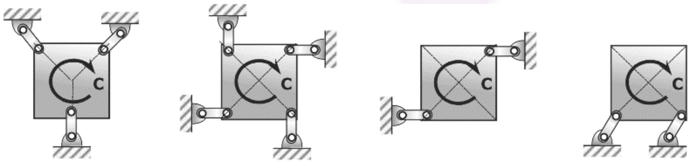
C. Non-dimensional temperature gradient

D. Non-dimensional velocity gradient multiplied by Prandtl number

Ans. C

Sol. The Nusselt number describes the ratio of convective heat transfer compared to pure heat conduction. The Nusselt number can therefore be interpreted as a measure of a dimensionless temperature gradient on the wall.

26. A square plate is supported in four different ways (configurations (P) to (S) as shown in the figure). A couple moment is applied on the plate. Assume all the members to be rigid and mass-less, and all joints to be frictionless. All support links of the plate are identical.



The square plate can remain in equilibrium in its initial state for which one or more of the following support configurations?

[MSQ: 1 Mark]

A. Configuration (P) B. Configuration (Q)

C. Configuration (R) D. Configuration (S)

Ans. B, C, D

Sol. The moment in Q, R and S due to the reaction forces will act in opposite direction to the applied moment. But in P as line of action of all the reaction forces passes through the centre of the plate. So, no reactive moment acts due to these forces, thus Plate P will not be in equilibrium.

27. Consider sand casting of a cube of edge length a. A cylindrical riser is placed at the top of the casting. Assume solidification time, $t_s \propto V/A$, where V is the volume and A is the total surface area dissipating heat. If the top of the riser is insulated, which of the following radius/radii of riser is/are acceptable?

[MSQ: 1 Mark]

A. a/3 B. a/2

C. a/4 D. a/6

Ans. A, B

Sol. Given,

$$ts \propto \left(\frac{V}{A}\right)$$

$$(t_s)_{riser} \geq (t_s)_{casting}$$

$$\left(\frac{V}{A}\right)_{riser} \geq \left(\frac{V}{A}\right)_{casting}$$

$$\frac{\pi r^2 \times h}{2\pi r \times h} \geq \frac{a^3}{6a^2}$$

$$\frac{r}{2} \geq \frac{a}{6}$$

$$\boxed{r \geq \frac{a}{3}}$$

Only true options in range are $\frac{a}{3}, \frac{a}{2}$.

28. Which of these processes involve(s) melting in metallic workpieces?

[MSQ: 1 Mark]

A. Electrochemical machining

B. Electric discharge machining

C. Laser beam machining

D. Electron beam machining

Ans. B, C, D

Sol. Processes that involve melting in metallic work pieces are

- EDM
- LBM
- EBM

29. The velocity field in a fluid is given to be $\vec{V} = (4xy)\hat{i} + 2(x^2 - y^2)\hat{j}$.

Which of the following statement(s) is/are correct?

[MSQ: 1 Mark]

A. The velocity field is one-dimensional.

B. The flow is incompressible.

C. The flow is irrotational.

D. The acceleration experienced by a fluid particle is zero at (x = 0, y = 0).

Ans. B, C, D

Sol. $V = 4xy\hat{i} + 2(x^2 - y^2)\hat{j}$

$$u = 4xy$$

$$v = 2(x^2 - y^2)$$

$$\nabla \cdot V = \left\{\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right\}[4xy\hat{i} + 2(x^2 - y^2)\hat{j}]$$

$$\nabla \cdot V = 4y - 2 \times 2y = 0$$

So, the flow is incompressible.

$$\nabla \cdot V = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 4x - 4x = 0$$

So, the flow is irrotational.

$$\vec{a}\,(0,0) = \vec{a}_x\hat{i} + \vec{a}_y\hat{j}$$

$$= \left[\frac{u\,\partial u}{\partial x} + \frac{v\,\partial u}{\partial y}\right]\hat{i} + \left[\frac{u\,\partial v}{\partial x} + \frac{v\,\partial v}{xy}\right]\hat{j}$$

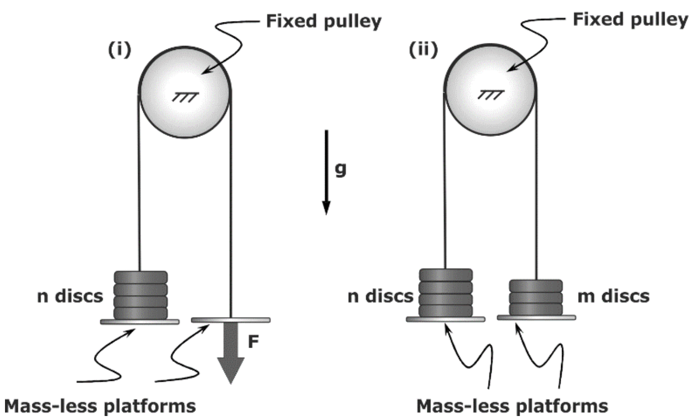
$$= (4xy \times 4y + 2(x^2 - y^2)\,4x)\hat{i}$$

$$+ (4xy \times 2 \times 2x + 2(x^2 - y^2) \times [-2y \times 2]\hat{j}$$

$$\boxed{\vec{a}_{(0,0)} = 0}$$

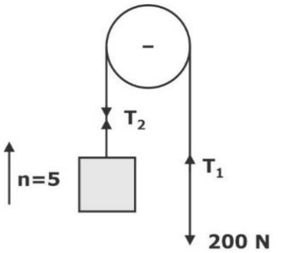
30. A rope with two mass-less platforms at its two ends passes over a fixed pulley as shown in the figure. Discs with narrow slots and having equal weight of 20 N each can be placed on the platforms. The number of discs placed on the left side platform is and that on the right side platform is m It is found that for n = 5 and m = 0, a force F = 200 N (refer to part (i) of the figure) is just sufficient to initiate upward motion of the left side platform. If the force is removed, then the minimum value of (refer to part (ii) of the figure) required to prevent downward motion of the left side platform is______ (in integer).

[NAT: 1 Mark]



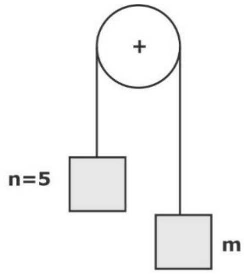
Ans. 3

Sol. Range (3 to 3)



(1)

Just about to go up



(2)

Keeping n to go down

Case 1:

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

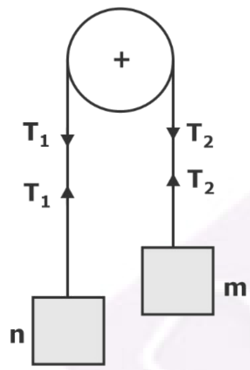
$$\frac{200}{20 \times 5} = e^{\mu\theta}$$

$$\boxed{e^{\mu\theta} = 2}$$

Case 2:

As 'n' mass is just about to go down

[or restrictly to go down]



$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{20 \times 5}{T_2} = e^{\mu\theta}$$

$$\frac{20 \times 5}{T_2} = 2$$

$$\boxed{T_2 = 50N}$$

$$m \times 20 = 50$$

$$m = 2.5 \text{ disc}$$

$\Rightarrow$ if m = 2 disc then n side disc will go down.

Hence, we will place 3 discs so that the left side platform does not go down.

31. For a dynamical system governed by the equation,

$$\ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t) = 0,$$

the damping ratio ξ is equal to $\frac{1}{2\pi}\log_e 2$.

The displacement x of this system is measured during a hammer test. A displacement peak in the positive displacement direction is measured to be 4 mm. Neglecting higher powers (>1) of the damping ratio, the displacement at the next peak in the positive direction will be ____________ mm (in integer).

[NAT: 1 Mark]

Ans. 2

Range (2 to 2)

Sol.

$$\ddot{x}(t) + 2\,\zeta\, w_n\ \dot{x}(t) + \omega_n^2\ x(t) = 0$$

Given,

$$\boxed{\xi = \frac{1}{2\,\pi}\ln 2} << 1$$

$$x_1 = 4 \text{ mm}$$

Logarithmic decrement factor, δ

$$= \left(\frac{2\,\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \frac{\left(2\pi \times \frac{1}{2\,\pi}\ln 2\right)}{\sqrt{1-\left(\frac{1}{2\,\pi}\ln 2\right)^2}} = 0.6974$$

$$\text{Decrement ratio} = \frac{x_0}{x_1} = e^{\delta} = e^{0.6974} = 2$$

The displacement at the next peak in the positive direction, $x_1 = x_0/2 = 4/2 = 2$ mm

32. An electric car manufacturer underestimated the January sales of car by 20 units, while the actual sales was 120 units. If the manufacturer uses exponential smoothing method with a smoothing constant of $\alpha = 0.2$, then the sales forecast for the month of February of the same year is ___________units (in integer).

[NAT: 1 Mark]

Ans. 104

Sol. Range (104 to 104)

Given,

$$D_t - F_t = 20 \text{ units}$$

$$D_t = 120 \text{ units}$$

$$F_t = 100 \text{ units}$$

$$\alpha = 0.2$$

$$F_{t+1} = F_t + \alpha(D_t - F_t)$$

$$F_{t+1} = F_t + \alpha(D_t - F_t)$$

$$F_{t+1} = 100 + 0.2(20)$$

$$F_{t+1} = 104 \text{ units}$$

33. The demand of a certain part is 1000 parts/year and its cost is ₹1000/part. The orders are placed based on the economic order quantity (EOQ). The cost of ordering is ₹100/order and the lead time for receiving the orders is 5 days. If the holding cost is ₹20/part/year, the inventory level for placing the orders is __________ parts (round off to the nearest integer).

[NAT: 1 Mark]

Ans. 14

Sol. Range (13 to 15)

Given

$D = 1000$ parts / year

$d = \frac{1000}{365}$ parts / day

Load time = 5 days

Reorder level = LT × $d = \frac{5 \times 1000}{365}$

= 13.69 = 14 units

34. Consider 1 kg of an ideal gas at 1 bar and 300 K contained in a rigid and perfectly insulated container. The specific heat of the gas at constant volume C_v is equal to 750 $J \cdot kg^{-1} K^{-1}$. A stirrer performs 225 kJ of work on the gas. Assume that the container does not participate in the thermodynamic interaction. The final pressure of the gas will be ___________ bar (in integer).

[NAT: 1 Mark]

Ans. 2

Range (2 to 2)

Sol. Given,

$m = 1$ kg

$p_1 = 1$ bar

$T_1 = 300$ K

$C_V = 750$ J/K

Work done = −225 kJ

$$\Delta Q = \Delta U + W$$

$$\Delta U = -W$$

$$\Delta U = 225$$

$$mC_v \Delta T = 225$$

$$0.75 \times \Delta T = 225$$

$$\Delta T = \frac{225}{0.75} = 225 \times \frac{4}{3} = 75 \times 4$$

$\Delta T = 300\ K$

$T_2 = 600\ K$

Now,

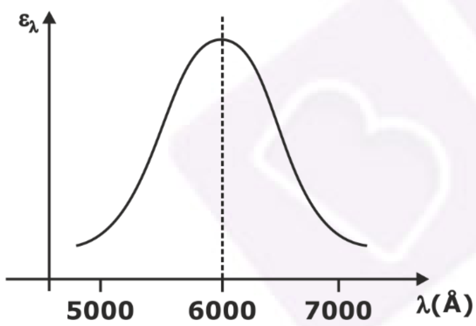
$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$

$$V_2 = V_1$$

$$P_2 = \frac{T_2}{T_1} \times P_1 = 2 P_1$$

$$P_2 = 2\ bar$$

35. Wien's law is stated as follows: $\lambda_m T = C$, where C is 2898 μm·K and λ_m is the wavelength at which the emissive power of a black body is maximum for a given temperature T. The spectral hemispherical emissivity (ε_λ) of a surface is shown in the figure below (1Å = 10^{-10}m). The temperature at which the total hemispherical emissivity will be highest is ___________ K (round off to the nearest integer)



[NAT: 1 Mark]

Ans. 4830

Range (4825 to 4835)

Sol.

Given,

$$\lambda_m T = 2898\ \mu m\text{-}K$$

$$\lambda_m = 6000\ A^\circ = 6000 \times 10^{-10} m$$

$$\lambda_m = 6000 \times 10^{-4} \mu m$$

$$\lambda_m \times T = 2898$$

$$6000 \times 10^{-4} \times T = 2898$$

$$T = \frac{2898}{0.6}$$

$$T = 4830\ K$$

36. For the exact differential equation,

$$\frac{du}{dx} = \frac{-xu^2}{2 + x^2 u}$$

which one of the following is the solution?

[MCQ: 2 Mark]

A. $u^2 + 2x^2$ = constant

B. $xu^2 + u$ = constant

C. $\frac{1}{2}x^2u^2 + 2u =$ constant

D. $\frac{1}{2}us^2 + 2x =$ constant

Ans. C

Sol. Option A:

$$u^2 + 2x^2 = C$$

Differentiate w.r.t. x.

$$2u.\frac{du}{dx} + 4x = 0$$

$$\frac{du}{dx} = -\frac{2x}{u}$$

Option B:

$$xu^2 + u = const$$

Differentiate w.r.t. x

$$u^2 + 2xu.\frac{du}{dx} + \frac{du}{dx} = 0$$

$$\frac{du}{dx}(1 + 2xu) = -u^2$$

$$\frac{du}{dx} = -\frac{u^2}{1 + 2xu}$$

Option C:

$$\frac{1}{2}x^2u^2 + 2u = c$$

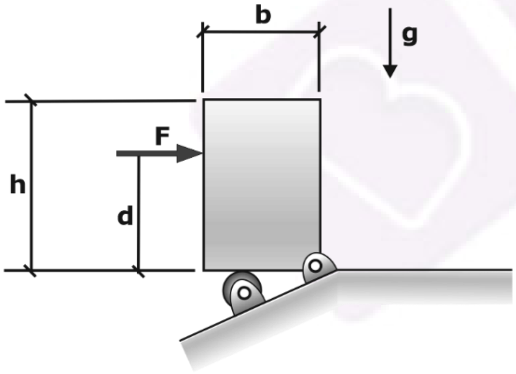
$$\frac{1}{2}\left[2x.u^2+2x^2u\frac{du}{dx}\right]+2\frac{du}{dx}=0$$

$$\frac{du}{dx}\left[x^2u+2\right]=-xu^2$$

$$\frac{du}{dx}=\frac{-xu^2}{x^2u+2}$$

Hence 'C' option is correct

37. A rigid homogeneous uniform block of mass 1 kg, height h = 0.4 m and width b = 0.3 m are pinned at one corner and placed upright in a uniform gravitational field (g = 9.81 m/s^2), supported by a roller in the configuration shown in the figure. A short duration (impulsive) force F, producing an impulse I_F, is applied at a height of d = 0.3 m from the bottom as shown. Assume all joints to be frictionless. The minimum value of I_F required to topple the block is



[MCQ: 2 Mark]

A. 0.953 Ns

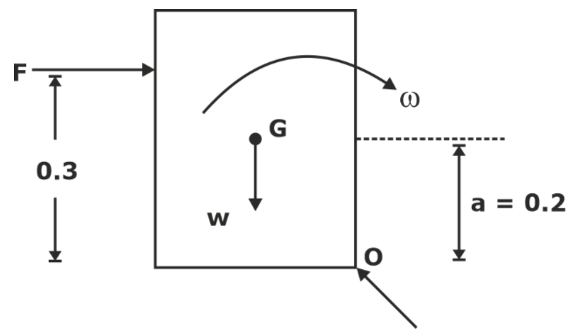
B. 1.403 Ns

C. 0.814 Ns

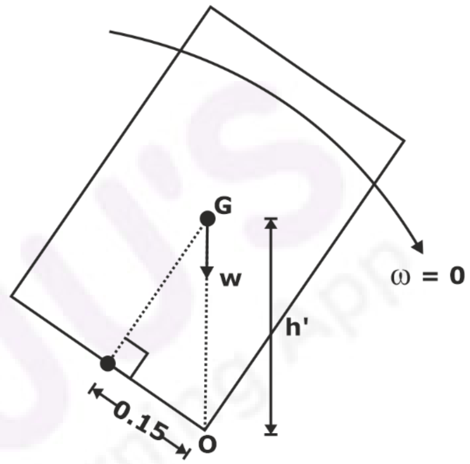
D. 1.172 Ns

Ans. A

Sol. I_F applied at d = 0.3 m



(i)



(ii)

For minimum value of F, the line of action of c.g should pass through the point-O (pin joint) as if it you beyond that the block will topple so, taking limiting case.

$$h'=OG'=\sqrt{0.2^2+0.15^2}\quad=0.25\text{ m}$$

$$I_a=\frac{1}{12}m\left(b^2+a^2\right)$$

$$I_0=\frac{1}{3}m\left(b^2+a^2\right)$$

So,

Decrease in KE = Increase in PE

$$\frac{1}{2}I_o\left(\omega^2-0\right)=mg\left(h'-h/2\right)$$

$$\frac{1}{2}\times\frac{1}{3}\times1\times\left(0.4^2+0.3^2\right)\omega=mg\left(0.25-0.2\right)$$

$$\omega=3.431\text{ rad / s}$$

Angular impulse = Change in angular moments

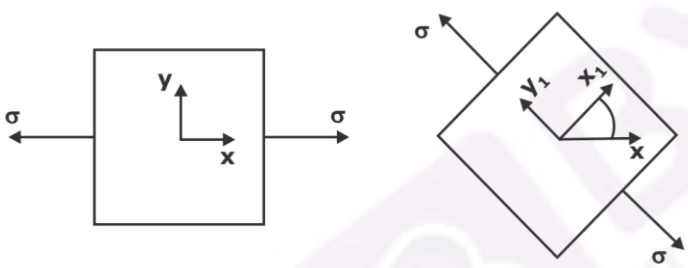
Impulse × d $= I_o(\omega - 0)$

Impulse × 0.3 $= \frac{1}{3} \times 1\left(0.4^2 + 0.3^2\right) \times 3.43$

Impulse = 0.9528 Ns

= 0.953 Ns

38. A linear elastic structure under plane stress condition is subjected to two sets of loading, I and II. The resulting states of stress at a point corresponding to these two loadings are as shown in the figure below. If these two sets of loading are applied simultaneously, then the net normal component of stress σ_{xx} is ________.



Loading - I   Loading - II

[MCQ: 2 Mark]

A. 3σ/2

B. $\sigma(1 + 1/\sqrt{2})$

C. σ/2

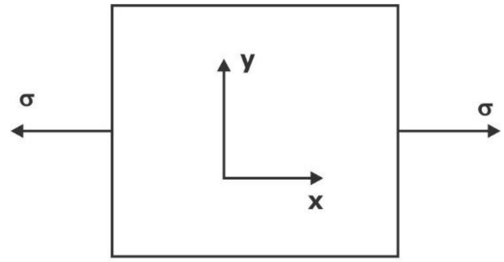
D. $\sigma(1 - 1/\sqrt{2})$

Ans. A

Sol. Given,

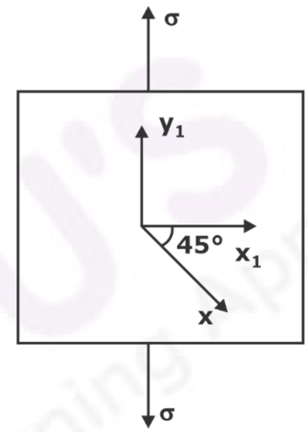
Due to loading – I normal stress

In x direction is $\sigma_{x1} = \sigma$



Loading - I

Due to loading – II normal stress in x direction



Loading - II

$$\sigma_{x2} = \frac{\sigma_x' + \sigma_y'}{2} + \frac{\sigma_x' - \sigma_y'}{2}\cos 90^\circ + \tau_{xy}\sin 90^\circ$$

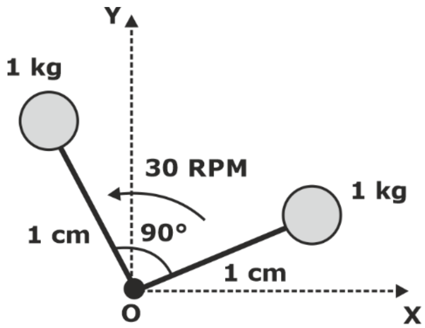
$$\sigma_{x2} = \frac{u + \sigma}{2} + \frac{O - \sigma}{2} \times O + O$$

$$\sigma_{x2} = \frac{\sigma}{2}$$

Normal stress $\sigma_{xx} = \sigma_{x1} + \sigma_{x2}$

$$= \sigma + \frac{\sigma}{2} = \frac{3\sigma}{2}$$

39. A rigid body in the X-Y plane consists of two point masses (1 kg each) attached to the ends of two massless rods, each of 1 cm length, as shown in the figure. It rotates at 30 RPM counterclockwise about the Z-axis passing through point O. A point mass of $\sqrt{2}$ kg, attached to one end of a third massless rod, is used for balancing the body by attaching the free end of the rod to point O. The length of the third rod is _________ cm.



[MCQ: 2 Mark]

A. 1

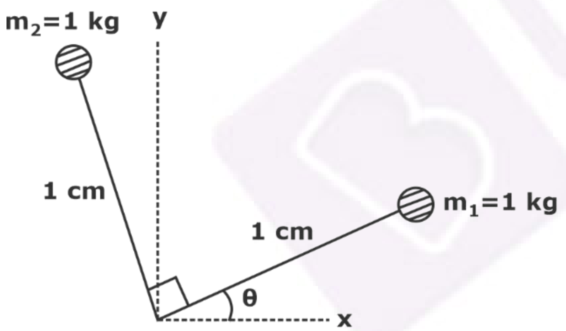
B. $\sqrt{2}$

C. $1 / \sqrt{2}$

D. $1 / 2\sqrt{2}$

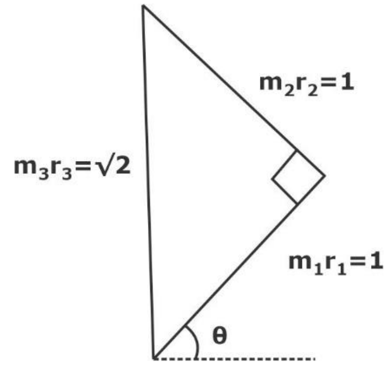
Ans. A

Sol.



A point mass $m_3 = \sqrt{2}$ kg is used for balancing the body, so we draw force polygon for balance system

Mass	r	m.r
$m_1 = 1$kg	1	1
$m_2 = 1$kg	1	1
$m_3 = \sqrt{2}$ kg	r	$\sqrt{2}$ r

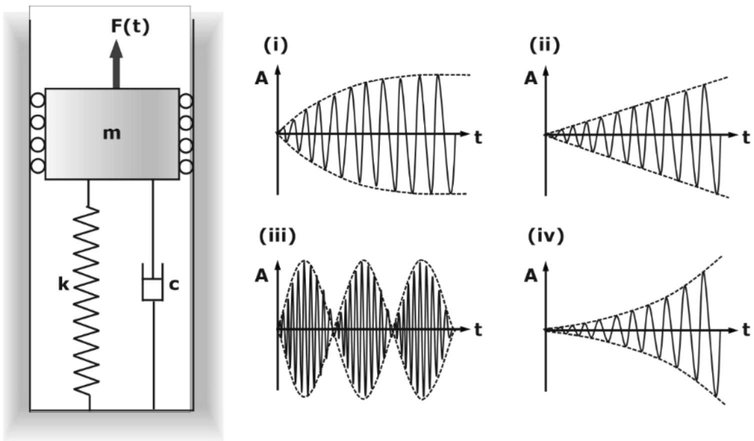


From force polygon, $m_3r_3 = \sqrt{2}$

$$\sqrt{2} \times r_3 = \sqrt{2}$$

$$r_3 = 1 \text{ cm}$$

40. A spring mass damper system (mass m, stiffness k, and damping coefficient c) excited by a force F(t) = B sin ωt, where B, ω and t are the amplitude, frequency and time, respectively, is shown in the figure. Four different responses of the system (marked as (i) to (iv)) are shown just to the right of the system figure. In the figures of the responses, A is the amplitude of response shown in red color and the dashed lines indicate its envelope. The responses represent only the qualitative trend and those are not drawn to any specific scale.



Four different parameter and forcing conditions are mentioned below.

(P) $c > 0$ and $\omega = \sqrt{k/m}$

(Q) $c < 0$ and $\omega \neq 0$

(R) $c = 0$ and $\omega = \sqrt{k/m}$

(S) $c = 0$ and $\omega \cong \sqrt{k/m}$

Which one of the following options gives correct match (indicated by arrow $\rightarrow$) of the parameter and forcing conditions to the responses?

[MCQ: 2 Mark]

A. (P) $\rightarrow$ (i), (Q) $\rightarrow$ (iii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (ii)

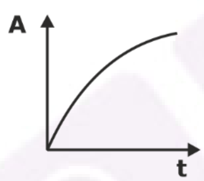
B. (P) $\rightarrow$ (ii), (Q) $\rightarrow$ (iii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (i)

C. (P) $\rightarrow$ (i), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (ii), (S) $\rightarrow$ (iii)

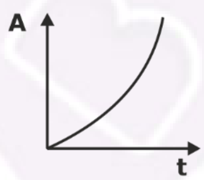
D. (P) $\rightarrow$ (iii), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (ii), (S) $\rightarrow$ (i)

Ans. C

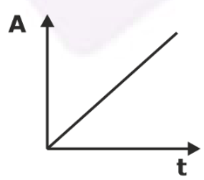
Sol. If C > 0, Amplitude



If C < 0,



If C = 0



P(i), Q(iv), R(ii), S(iii)

41. Parts P1-P7 are machined first on a milling machine and then polished at a separate machine. Using the information in the following table, the minimum total completion time required for carrying out both the operations for all 7 parts is __________ hours.

Part	Milling (hours)	Polishing (hours)
P_1	8	6
P_2	3	2
P_3	3	4
P_4	4	6
P_5	5	7
P_6	6	4
P_7	2	1

[MCQ: 2 Mark]

A. 31B. 33

C. 30D. 32

Ans. B

Sol.

Part	Milling (hours)	Polishing (hours)
P_1	8	6
P_2	3	2
P_3	3	4
P_4	4	6
P_5	5	7
P_6	6	4
P_7	2	1

Optimal sequence P_3 P_4 P_5 P_1 P_6 P_2 P_7

	Milling		Polishing	
	In	Out	In	Out
P_3	0	3	3	7
P_4	3	7	7	13
P_5	7	12	13	20
P_1	12	20	20	26
P_6	20	26	26	30
P_2	26	29	30	32
P_7	29	31	32	33

Hence minimum total completion time required = 33 hours

42. A manufacturing unit produces two products P1 and P2. For each piece of P1 and P2, the table below provides quantities of materials M1, M2, and M3 required, and also the profit earned. The maximum quantity available per day for M1, M2 and M3 is also provided. The maximum possible profit per day is ₹__________.

	M1	**M2**	**M3**	**Profit per piece (₹)**
P1	2	2	0	150
P2	3	1	2	100
Maximum quantity available per day	70	50	40	

[MCQ: 2 Mark]

A. 5000
B. 4000
C. 3000
D. 6000

Ans. B

Sol. Maximum, z = 150 x + 100 y

let $p_1 = x$

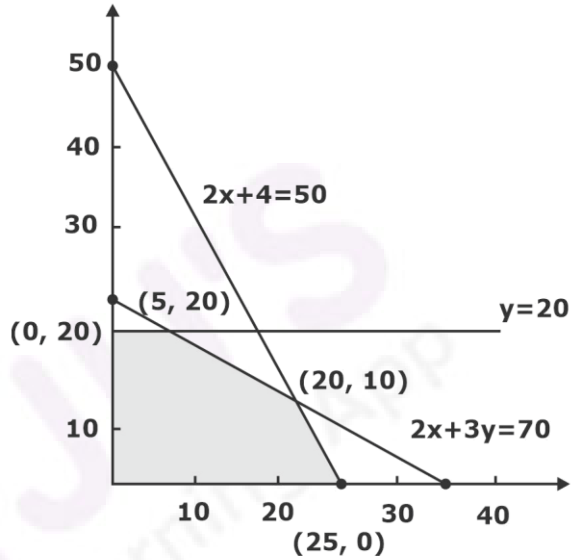
$p_2 = y$

subjected to

$2x + 3y \le 70$

$2x + y \le 50$

$2y \le 40$



Corner points are (σ, 20), (5, 20), (20, 10), (25, 0) and (0, 0)

at (0, 20) z = 150 × 0 + 100 × 20 = 2000

at (5, 20) z = 150 × 5 + 100 × 20 = 2750

at (20, 10) z = 150 × 20 + 100 × 10

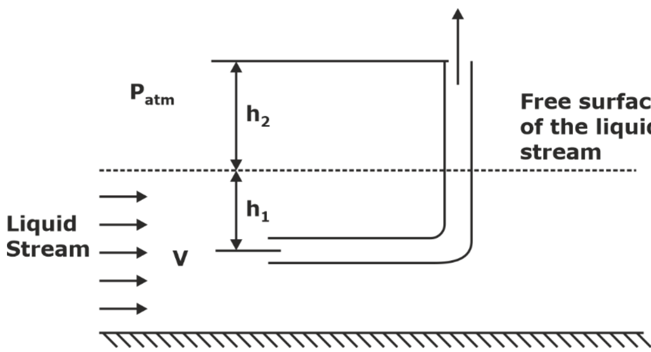
= 4000

at (25, 0) z = 150 × 25 + 100 × 0 = 3750

at (0, 0) z = 0

hence maximum possible profit per day is 4000 ₹

43. A tube of uniform diameter D is immersed in a steady flowing inviscid liquid stream of velocity V, as shown in the figure. Gravitational acceleration is represented by g. The volume flow rate through the tube is _______.



[MCQ: 2 Mark]

A. $\frac{\pi}{4}D^2V$

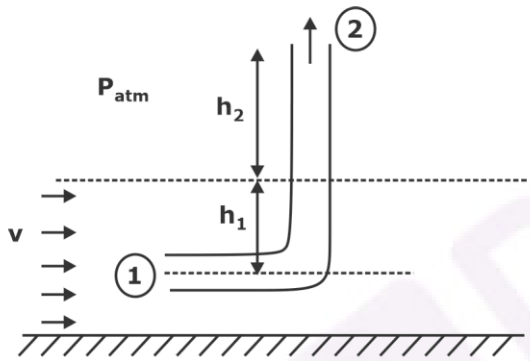
B. $\frac{\pi}{4}D^2\sqrt{2gh_2}$

C. $\frac{\pi}{4}D^2\sqrt{2g(h_1+h_2)}$

D. $\frac{\pi}{4}D^2\sqrt{V^2-2gh_2}$

Ans. D

Sol.



Applying Bernoulli between (1) & (2)

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_1^2}{2g}+z_2$$

$$\left(\frac{p_{atm}+\rho gh_1}{\rho g}\right)+\frac{V^2}{2g}+0=\frac{p_{atm}}{\rho g}+\frac{V_2^2}{2g}+(h_1+h_2)$$

$$\frac{V_2^2}{2g}=\frac{V^2}{2g}-h_2$$

$$V_2=\sqrt{V^2-2gh_2}$$

The volume flow rate through the tube is

$$Q=A_2V_2$$

$$=\frac{\pi}{4}\times D^2\times\sqrt{V^2-2gh_2}$$

$$Q=\frac{\pi}{4}D^2\sqrt{V^2-2gh_2}$$

44. The steady velocity field in an inviscid fluid of density 1.5 is given to be $\vec{V}=(y^2-x^2)\hat{i}+(2xy)\hat{j}$. Neglecting body forces, the pressure gradient at $(x=1, y=1)$ is ______ .

A. $10\hat{j}$

B. $20\hat{i}$

C. $-6\hat{i}-6\hat{j}$

D. $-4\hat{i}-4\hat{j}$

Ans. C

Sol. Given,

Velocity field, $\vec{V}=(y^2-x^2)\hat{i}+2xy\hat{j}$

Density of fluid, $\rho=1.5$

Momentum equation:

$$\rho\frac{D\vec{v}}{Dt}=-\nabla P+\rho\vec{g}+\mu\nabla^2\vec{v}$$

Given that body force neglected so $\rho\vec{g}=0$

Inviscid fluid, so $\mu\nabla^2\vec{v}=0$

Now momentum equation reduces to

$$\rho\left(\frac{d\vec{v}}{dt}+(v.\nabla)v\right)=-\nabla P$$

Steady flow is given, so

$$\frac{d\vec{v}}{dt}=0$$

Now momentum equation in required to-

$\nabla P=-\rho(\vec{v}.\nabla)\vec{v}$

x – momentum equation

$$\frac{\partial P}{\partial x}=-\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial P}{\partial x}=-\rho\left[(y^2-x^2)(-2x)+2xy(2y)\right]$$

$$\left.\frac{\partial P}{\partial x}\right|_{(1,1)}=-1.5[0+4]$$

$$\left.\frac{\partial p}{\partial x}\right|_{(1,1)} = -6 = -6\hat{i}$$

y-momentum equation

$$\frac{\partial p}{\partial x} = -\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)$$

$$= -\rho\left[(y^2 - x^2)(2y) + 2xy(2x)\right]$$

$$\left.\frac{\partial p}{\partial y}\right|_{(1,1)} = -1.5[0 + 4] = -6 = -6\hat{j}$$

Pressure gradient $\nabla\vec{P} = -6\hat{i} - 6\hat{j}$

45. In a vapour compression refrigeration cycle, the refrigerant enters the compressor in saturated vapour state at evaporator pressure, with specific enthalpy equal to 250 kJ/kg. The exit of the compressor is superheated at condenser pressure with specific enthalpy equal to 300 kJ/kg. At the condenser exit, the refrigerant is throttled to the evaporator pressure. The coefficient of performance (COP) of the cycle is 3. If the specific enthalpy of the saturated liquid at evaporator pressure is 50 kJ/kg, then the dryness fraction of the refrigerant at entry to evaporator is __________.

[MCQ: 2 Mark]

A. 0.2

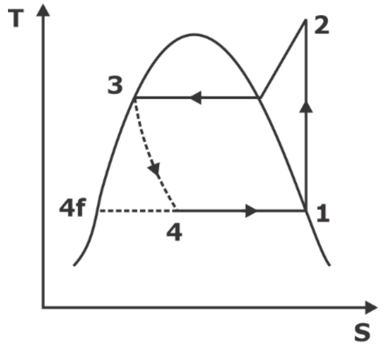
B. 0.25

C. 0.3

D. 0.35

Ans. B

Sol.



Given,

$h_1 = 250$ kJ/kg

$h_2 = 300$ kJ/kg

$h_{4f} = 50$ kJ/kg

Cop = 3

$$\text{Cop} = \frac{\text{desire output}}{\text{work input}} = \frac{h_1 - h_4}{h_2 - h_1}$$

$$3 = \frac{250 - h_4}{300 - 250}$$

$h_4 = 100$ kJ/kg

$h_4 = h_f + x\,(h_{fg})$

$100 = 50 + x\,(250 - 50)$

$$x = \frac{50}{200} = 0.25$$

46. A is a 3 × 5 real matrix of rank 2. For the set of homogeneous equations Ax = 0, where 0 is a zero vector and x is a vector of unknown variables, which of the following is/are true?

[MSQ: 2 Mark]

A. The given set of equations will have a unique solution.

B. The given set of equations will be satisfied by a zero vector of appropriate size.

C. The given set of equations will have infinitely many solutions.

D. The given set of equations will have many but a finite number of solutions.

Ans. B & C

Sol. Given

Ax = 0 (homogeneous equation)

A is 3 × 5 real matrix, so rank of A can be

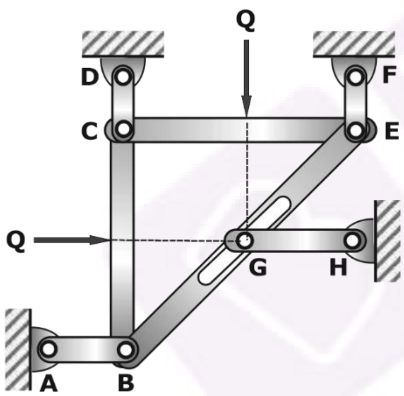
$r(A) \le \min\{3, 5\}$

$r(A) \le 3$

given that the rank of A is 2.

Hence r(A) < n (number of variable) then given set of equation will have infinity many solution and this set of equation will also satisfied by a zero vector.

47. The lengths of members BC and CE in the frame shown in the figure are equal. All the members are rigid and lightweight, and the friction at the joints is negligible. Two forces of magnitude $Q > 0$ are applied as shown, each at the mid-length of the respective member on which it acts.



Which one or more of the following members do not carry any load (force)?

[MSQ: 2 Mark]

A. AB

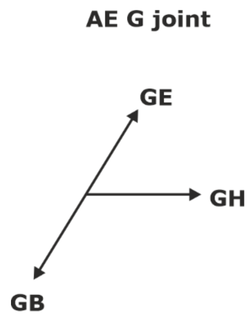
B. CD

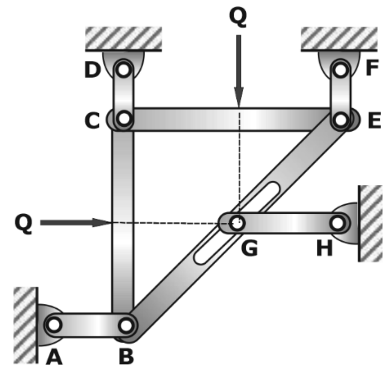
C. EF

D. GH

Ans. B, D

Sol.





The component of GH perpendicular to BE link does not carry any load so GH link will not carry any resistive force and CD also do not carry any resistive load.

48. If the sum and product of eigenvalues of a 2 × 2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and −1 respectively, then |p| is ______ (in integer).

[NAT: 2 Mark]

Ans. 2

Sol. Range (2 to 2)

Given,

$$2 \times 2 \text{ matrix } A = \begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$$

Sum of eigen values = 4

Product of eigen values = − 1

Sum of eigen values = trace of matrix

$4 = 3 + q$

$q = 1$

product of eigen values = determinant of matrix

$-1 = 3q - p^2$

$-1 = 3 \times 1 - p^2$

$p^2 = 4$

$p = \pm 2$

Hence $|p| = 2$

49. Given $z = x + iy$, $i = \sqrt{-1}$. C is a circle of radius 2 with the centre at the origin.

If the contour C is traversed anticlockwise, then the value of the integral $\frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz$ is ________ (round off to one decimal place).

[NAT: 2 Mark]

Ans. 0.2

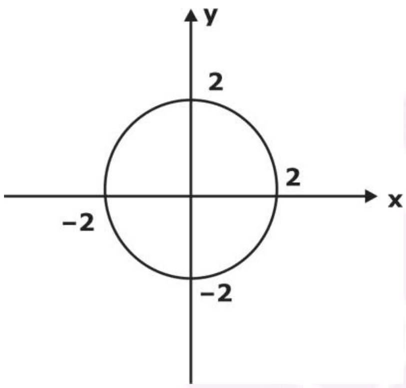
Sol. Range (0.2 to 0.2)

Given,

$z = x + iy$

c is a circle of radius 2 with center at the origin

$$\frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}$$



Only z = i singular point lies inside the circle

$$\frac{1}{2\pi}\int_C \frac{\left(\frac{1}{z+4i}\right)}{(z-i)}$$

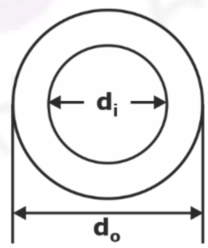
Hence, $f(z) = \frac{1}{2\pi}\left(\frac{1}{z+4i}\right)$

$= 2\pi i \quad f(i)$

$= 2\pi i \left[\frac{1}{2\pi} \times \frac{1}{5i}\right]$

$= 0.2$

50. A shaft of length L is made of two materials, one in the inner core and the other in the outer rim, and the two are perfectly joined together (no slip at the interface) along the entire length of the shaft. The diameter of the inner core is d_i and the external diameter of the rim is d_o, as shown in the figure. The modulus of rigidity of the core and rim materials are G_i and G_o, respectively. It is given that $d_o = 2d_i$ and $G_i = 3G_o$. When the shaft is twisted by application of a torque along the shaft axis, the maximum shear stress developed in the outer rim and the inner core turn out to be τ_o and τ_i, respectively. All the deformations are in the elastic range and stress strain relations are linear. Then the ratio τ_i / τ_o is ______ (round off to 2 decimal places).

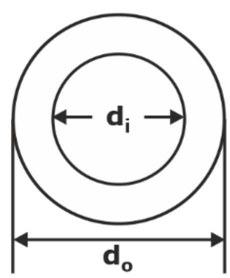


Shaft cross-section

[NAT: 2 Mark]

Ans. 1.50

Sol. Range (1.48 to 1.52)



Given,

$$\frac{d_o}{d_i} = 2$$

$$\frac{G_i}{G_o} = 3$$

Since, the shaft is same with different material so, angle of twist is also same

$\theta_i = \theta_o$

Torsion formula

$$\frac{\tau}{y} = \frac{G\theta}{L}$$

$\tau \propto Gy$

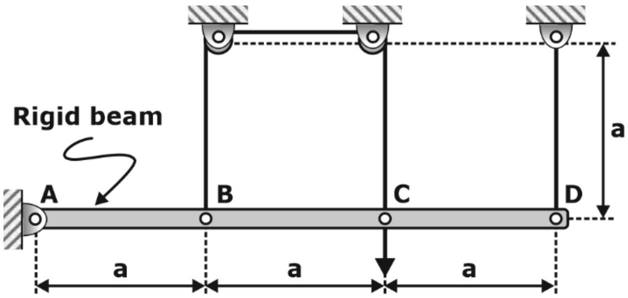
$$\frac{\tau_o}{\tau_i} = \frac{G_o \;\; d_o / 2}{G_i \;\; d_i / 2}$$

$$\frac{\tau_o}{\tau_i} = \frac{1}{3} \times 2 = \frac{2}{3}$$

$$\frac{\tau_i}{\tau_0} = \frac{3}{2} = 1.5$$

51. A rigid beam AD of length $3a$ = 6 m is hinged at frictionless pin joint A and supported by two strings as shown in the figure. String BC passes over two small frictionless pulleys of negligible radius. All the strings are made of the same material and have equal cross-sectional area. A force F = 9 kN is applied at C and the resulting stresses in the strings are within linear elastic limit. The self-weight of the beam is negligible with respect to the applied load. Assuming small deflections, the tension developed in the string at C is __________ kN (round off to 2 decimal places).

[NAT: 2 Mark]



Ans. 1.50

Sol. Range (1.48 to 1.52)

Given,

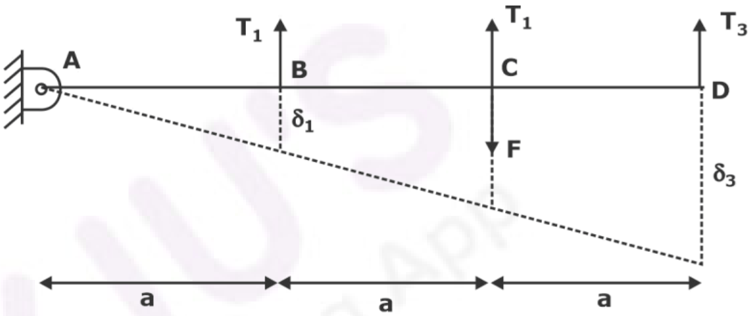
3a = 6m

a = 2m

F = 9 kN

strings have same material some cross section and same length.

$$\frac{\delta_1}{\delta_3} = \frac{a}{3a}$$



$$\delta_3 = 3\delta_1$$

$$\frac{T_3 L_3}{A_3 E_3} = \frac{3T_1 L_1}{A_1 E_1} \qquad \left(\begin{aligned} \because \; L_3 &= L_1 \\ A_3 &= A_1 \\ E_3 &= E_1 \end{aligned}\right)$$

$T_3 = 3T_1$

Taking moment about A (hinge point)

$T_1 \times a + (T_1 - F)2a + T_3 \times 3a = 0$

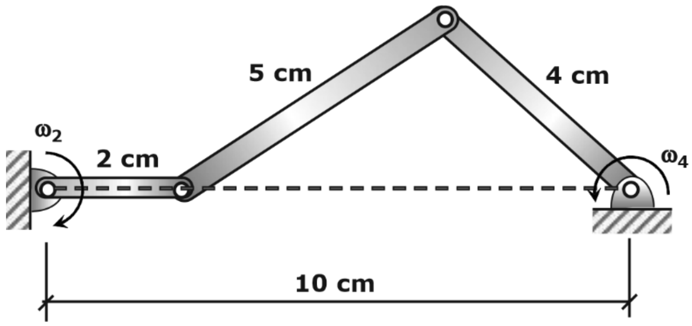
$T_1 + (T_1 - F)2 + 3T_1 \times 3 = 0$

$2F = 12\ T_1$

$T_1 = F/6$

$$T_1 = \frac{9}{6} = 1.5\,\text{kN}$$

52. In the configuration of the planar four-bar mechanism at a certain instant as shown in the figure, the angular velocity of the 2 cm long link is ω_2 = 5 rad/s. Given the dimensions as shown, the magnitude of the angular velocity ω_4 of the 4 cm long link is given by ______ rad/s (round off to 2 decimal places).



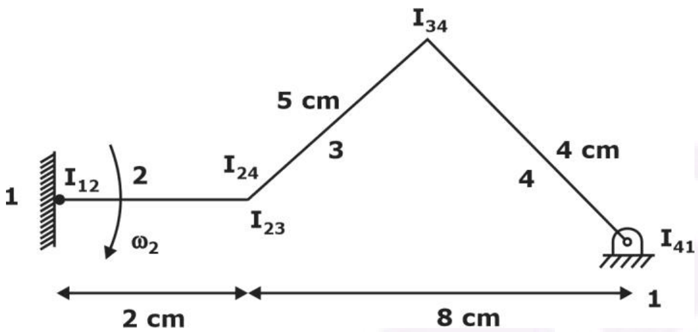
[NAT: 2 Mark]

Ans. 1.25

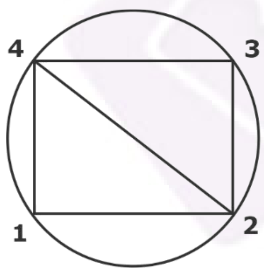
Sol. Range (1.24 to 1.26)

Given,

$\omega_2 = 5$ rad/s



Applying Kennedy theorem

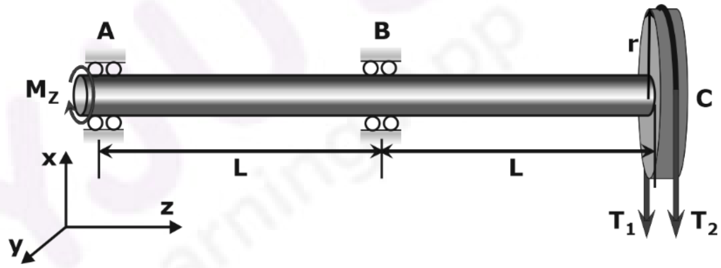


$\omega_2(I_{12}\ I_{24}) = \omega_4\ (I_{14}\ I_{24})$

$5\ (2) = \omega_4(8)$

$\omega_4 = 1.25$ rad/s

53. A shaft AC rotating at a constant speed carries a thin pulley of radius $r = 0.4$ m at the end C which drives a belt. A motor is coupled at the end A of the shaft such that it applies a torque Mz about the shaft axis without causing any bending moment. The shaft is mounted on narrow frictionless bearings at A and B where AB = BC = L = 0.5 m. The taut and slack side tensions of the belt are T_1 = 300 N and T_2 = 100 N, respectively. The allowable shear stress for the shaft material is 80 MPa. The self-weights of the pulley and the shaft are negligible. Use the value of π available in the on-screen virtual calculator. Neglecting shock and fatigue loading and assuming maximum shear stress theory, the minimum required shaft diameter is _______ mm (round off to 2 decimal places).



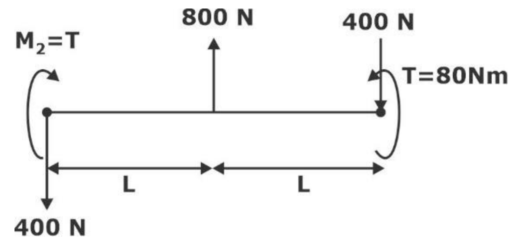
[NAT: 2 Mark]

Ans. 23.94

Sol. Range (23.60 to 24.20)

Support will not experience any torsional reaction moment

Given that motor does not Cause any bending so M = O



So,

Torsional moment,

$T = (300 - 100) \times 0.4 = 80$ N.m.

Maximum bending moment,

$$M = 400 \times 0.5 = 200 \text{ Nm}$$

Maximum combined shear stress

$$= \tau_{max} = \frac{16}{\pi d^3}\left(\sqrt{M^2 + T^2}\right)$$

$$\tau_{max} = \frac{16 \times \left(\sqrt{200^2 + 80^2}\right) \times 10^3}{\pi \times d^3} = \frac{1097 \times 10^3}{d^3}$$

(Where d = diameter of shaft)

According to maximum shear stress theory

$$\tau_{max} \leq \tau_{per}$$

$$\frac{1097 \times 10^3}{d^3} \leq 80$$

$$d \geq 23.94$$

Minimum required diameter is 23.94 mm

54. A straight-teeth horizontal slab milling cutter is shown in the figure. It has 4 teeth and diameter (D) of 200 mm. The rotational speed of the cutter is 100 rpm and the linear feed given to the workpiece is 1000 mm/minute. The width of the workpiece (w) is 100 mm, and the entire width is milled in a single pass of the cutter. The cutting force/tooth is given by $F = Ktcw$, where specific cutting force K = 10 N/mm^2, w is the width of cut, and tc is the uncut chip thickness.

The depth of cut (d) is $D/2$, and hence the assumption of d/D ≪ 1 is invalid. The maximum cutting force required is ________ kN (round off to one decimal place).

[NAT: 2 Mark]

Ans. 2.5

Sol. Range (2.4 to 2.6)

Number of teeth, Z = 4

Diameter of cutter, D = 200 mm

RPM of cutter, N = 100

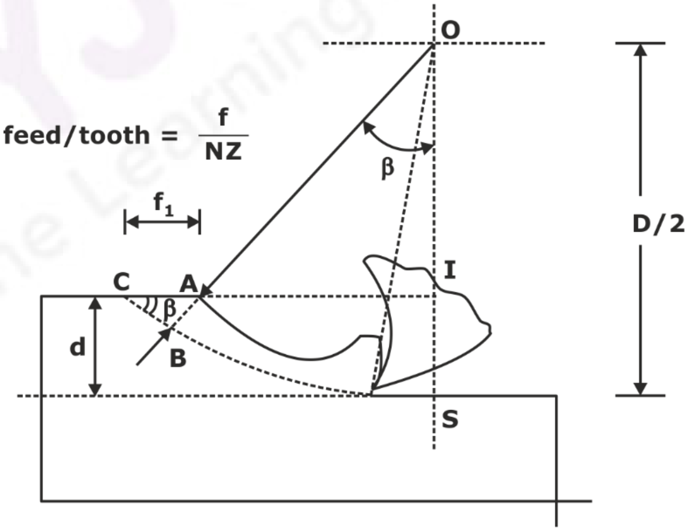
Linear feed, f_3 = 1000 mm/min

Width, w = 100 mm

and Force, $F = kt_cw$

d = depth of cut $= \frac{D}{2}$

and $\frac{d}{D} << 1$ is not valid.



$$\therefore \text{ In } \Delta OIA, \cos\beta = \frac{OI}{OA} = \frac{\left(\frac{D}{2} - d\right)}{\frac{D}{2}}$$

$$\cos\beta = \left(1 - \frac{2d}{D}\right)$$

$$\therefore \sin^2\beta + \cos^2\beta = 1$$

$$\therefore \ \sin^2\beta = 1-\cos^2\beta = 1-\left(1-\frac{2d}{D}\right)^2$$

$$\therefore \ \sin\beta = \sqrt{1-\left(1-\frac{2d}{D}\right)^2} \qquad ...(1)$$

Here $\frac{d}{D} <<< 1$ is invalid and $d = \frac{D}{2}$, put in equation (1)

$$\therefore \ \sin\beta = \sqrt{1-\left(1-\frac{2\times D}{2\times D}\right)^2} = \sqrt{1-0} = 0$$

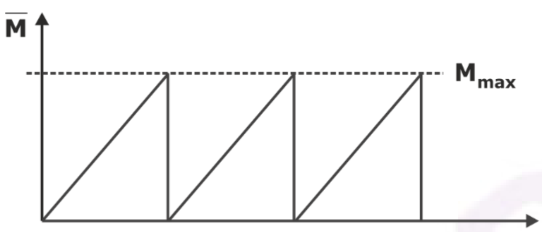
$$\therefore \ \sin\beta = 1$$

$$\therefore \ \boxed{\beta=90°}$$

and angle between two consecutive teeth is

$$\frac{2\pi}{4} = \frac{360}{4} = 90°$$

$\therefore$ Torque during milling is



$$\beta = \frac{2\pi}{4} = 90°$$

In right angle ΔABC

AB = t_{1max} = maximum uncut chip thickness

$$\therefore \ \sin\beta = \frac{AB}{AC}$$

$$\therefore \ t_{1max} = AC\sin\beta = f_1\times\sin\beta$$

Here, $f_3 = f_1\times Z\times N$

$$\therefore \ f_1 = \frac{f_3}{NZ} = \frac{1000}{1000\times 4} = 2.5 \text{ mm/tooth}$$

$$\therefore \ t_{1max} = 2.5\times 1 = 2.5\text{mm}$$

(as sin β = 1)

Now, from given equation

$$F = kt_cw = 10\times 2.5\times 100 = 2500\text{N}$$

$$\boxed{F = 2.5 \text{ kN}}$$

55. In an orthogonal machining operation, the cutting and thrust forces are equal in magnitude. The uncut chip thickness is 0.5 mm and the shear angle is 15°. The orthogonal rake angle of the tool is 0° and the width of cut is 2 mm. The workpiece material is perfectly plastic and its yield shear strength is 500 MPa. The cutting force is __________ N (round off to the nearest integer).

[NAT: 2 Mark]

Ans. 2732

Sol. Range (2700 to 2750)

Given

Uncut chip thickness, t = 0.5 mm

Shear angle, ϕ = 15°

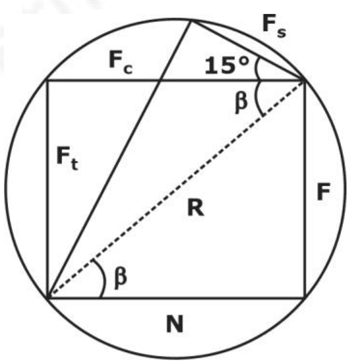
Rates angle, α = 0°

Width of cut, b = 2 mm

Yield shear strength, τ_{yt} = 500 MPa

Cutting force = thrust force

$F_C = F_t$



From merchant circle

$F_c = N$

$F_t = F$

$\because F_c = F_t$

$\therefore F = N$

Hence β = 45°

$$\text{Shear force } F_s = \tau_{yt}\frac{bt}{\sin\phi}$$

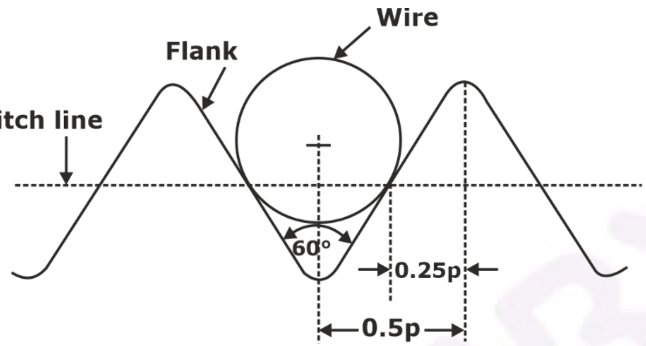
$$F_s = 500 \times \frac{2\times 0.5}{\sin 15} = 1931.85 \text{ N}$$

$$\frac{F_s}{\cos(15+\beta)} = \frac{F_C}{\cos\beta}$$

$$\frac{1931.85}{\cos(15+45)} = \frac{F_c}{\cos 45^\circ}$$

$$F_c = 2732 \text{ N}$$

56. The best size wire is fitted in a groove of a metric screw such that the wire touches the flanks of the thread on the pitch line as shown in the figure. The pitch (p) and included angle of the thread are 4 mm and 60º, respectively. The diameter of the best size wire is ___________ mm (round off to 2 decimal places).



[NAT: 2 Mark]

Ans. 2.31

Sol. Range (2.29 to 2.33)

Given,

Pitch, p = 4 mm

Included angle, α= 60°

The diameter of the best wire size

$$d = \frac{p}{2}\sec\frac{\alpha}{2}$$

$$= \frac{4}{2}\sec 30°$$

= 2.309 mm

57. In a direct current arc welding process, the power source has an open circuit voltage of 100 V and short circuit current of 1000 A. Assume a linear relationship between voltage and current. The arc voltage (V) varies with the arc length (l) as V = 10 + 5l, where V is in volts and l is in mm. The maximum available arc power during the process is _________ kVA (in integer).

[NAT: 2 Mark]

Ans. 25

Sol. Range (24 to 26)

Given,

Open circuit voltage, V_o = 100 V

Short circuit current, I_o = 1000 V

For the maximum available arc power

Optimum voltage, $V_t = \frac{V_o}{2} = 50$ V

Optimum current, $I_t = \frac{I_o}{2} = 500$ A

Maximum power, $p_{max} = I\,V$

= 50 × 500 = 25000 W

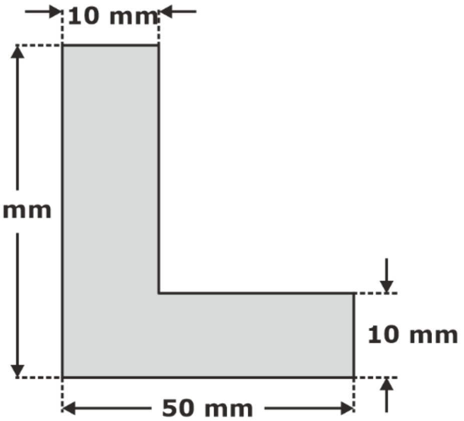
= 25 kW = 25 kVA

58. A cylindrical billet of 100 mm diameter and 100 mm length is extruded by a direct extrusion process to produce a bar of L-section. The cross sectional dimensions of this L-section bar are shown in the figure. The total extrusion pressure (p) in MPa for the above process is related to extrusion ratio (r) as

$$p = K_s\sigma_m\left[0.8 + 1.5\ln(r) + \frac{2l}{d_0}\right]$$

where σ_m is the mean flow strength of the billet material in MPa, l is the portion of the billet length remaining to be extruded in mm, d_0 is the initial diameter of the billet in mm, and K_s is the die shape factor.

If the mean flow strength of the billet material is 50 MPa and the die shape factor is 1.05, then the maximum force required at the start of extrusion is _________ kN (round off to one decimal place).



[NAT: 2 Mark]

Ans. 2429.2

Sol. Range (2426.0 to 2432.0)

Given,

Initial cylinder billet diameter, d_t = 100 mm

Length, t = 100 mm

Final cross-section after extrusion is L section

Area of L-section

$A_f = 60 \times 100 + 40 \times 10$

1000 mm^2

$$\text{Extrusion ratio, } r = \frac{A_i}{A_f} = \frac{\frac{\pi}{4} \times 100^2}{1000}$$

$$= 7.853$$

$$\text{Pressure (P)} = K_s\, \sigma_m \left(0.8 + 1.5 \ln r + \frac{2l}{d_o}\right)$$

$$= -1.05 \times 50 \left[0.8 + 1.5 \ln 7.853 + \frac{2 \times 100}{100}\right]$$

P = 309.3 MPa

Maximum force required at the start of extrusion

$$F_{max} = p \times A_i$$

$$= 309.3 \times \frac{\pi}{4} \times 100^2$$

$$= 2429.2 \text{ kN}$$

59. A project consists of five activities (A, B, C, D and E). The duration of each activity follows beta distribution. The three-time estimates (in weeks) of each activity and immediate predecessor(s) are listed in the table. The expected time of the project completion is ___________ weeks (in integer).

	Time estimate (in weeks)			
Activity	**Optimistic time**	**Most likely time**	**Pessimistic time**	**Immediate predecessor (s)**
A	4	5	6	None
B	1	3	5	A
C	1	2	3	A
D	2	4	6	C
E	3	4	5	B,D

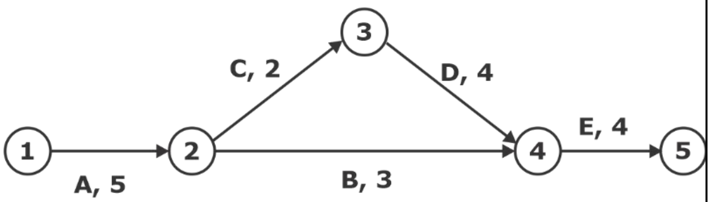
[NAT: 2 Mark]

Ans. 15

Sol. Range (15 to 15)

Activity	**Time (weeks)**			$t_E = \frac{t_o + 4t_m + t_p}{6}$
	t_o	t_m	t_p	
A	4	5	6	5
B	1	3	5	3
C	1	2	3	2
D	2	4	6	4
E	3	4	5	4

Network diagram



Critical path is A → C → D → E and path completion time is 15 weeks

The expected time of the project completion is 15 weeks

60. A rigid tank of volume of 8 m^3 is being filled up with air from a pipeline connected through a valve. Initially the valve is closed and the tank is assumed to be completely evacuated. The air pressure and temperature inside the pipeline are maintained at 600 kPa and 306 K, respectively. The filling of the tank begins by opening the valve and the process ends when the tank pressure is equal to the pipeline pressure. During the filling process, heat loss to the surrounding is 1000 kJ. The specific heats of air at constant pressure and at constant volume are 1.005 kJ/kg.K and 0.718 kJ/kg.K, respectively. Neglect changes in kinetic energy and potential energy.

The final temperature of the tank after the completion of the filling process is ________ K (round off to the nearest integer).

[NAT: 2 Mark]

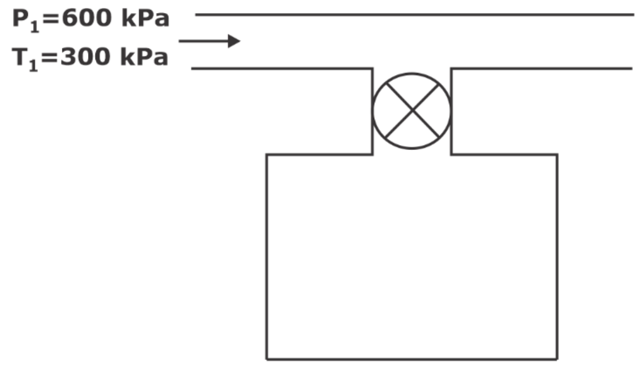
Ans. 395

Range [385 to 405]

Sol. Given,

Volume of tank = $8m^3$

Heat loss to surrounding = 1000 kJ



$$\frac{dm_{CV}}{dt} = \dot{m}_i - \dot{m}_e$$

$$\because \dot{m}_e = 0 \qquad \begin{pmatrix} i = \text{inlet} \\ e = \text{exit} \end{pmatrix}$$

$$\frac{dm_{CV}}{dt} = \dot{m}_i \qquad \text{.....(i)}$$

$$\frac{dE}{dt} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{w}$$

$$\because \Delta kE = 0, \quad \Delta PE = 0 \text{ and } \dot{w} = 0$$

$$\frac{dU}{dt} = \dot{m}_i h_i + \dot{Q} \qquad \text{.....(ii)}$$

From equation (i) and (ii)

$$\frac{dU}{dt} = \frac{dm_{CV}}{dt} h_i + \dot{Q}$$

$$dU = dm_{CV} h_i + \dot{Q}dt$$

$$m_2u_2 - m_1u_1 = (m_2 - m_1)h_i + Q \begin{bmatrix} 1 \to \text{initial} \\ 2 \to \text{final} \end{bmatrix}$$

$\because$ initially tank is evacuated, so $m_1 = 0$

$m_2u_2 = m_2h_i + Q$

ideal gas equation,

$$m_2 = \frac{p_2v_2}{RT_2} = \frac{600 \times 8}{0.257\ T_2} = \frac{16724.73}{T_2}$$

$$\frac{16724.73}{T_2} \times C_v T_2 = \frac{16724.73}{T_2} \times C_p T_i + (-1000)$$

$$16724.73 \times 0.718 =$$

$$\frac{16724.73}{T_2} \times 1.005 \times 30^6 - 1000$$

$$T_2 = 395.38\,K$$

61. At steady state, 500 kg/s of steam enters a turbine with specific enthalpy equal to 3500 kJ/kg and specific entropy equal to 6.5 kJ·kg^{-1}·K^{-1}. It expands reversibly in the turbine to the condenser pressure. Heat loss occurs reversibly in the turbine at a temperature of 500 K. If the exit specific enthalpy and specific entropy are 2500 kJ/kg and 6.3 kJ·kg^{-1}·K^{-1}, respectively, the work output from the turbine is ________ MW (in integer).

[NAT: 2 Mark]

Ans. 450

Sol. Range (450 to 450)

Given,

m_1 = 500 kg/s

h_1 = 3500 kJ/kg

s_1 = 6.5 kJ/kgK

T_o = 500 K

h_2 = 2500 kJ/kg

s_1 = 63 kJ/kgK

T_o = 500 K

h_2 = 2500 kJ/kg

s_1 = 63 kJ/kgK

using SFEE equation

$$m\left(h_1 + \frac{V_1^2}{2} + gz_1 + \dot{q}\right) = m\left(h_2 + \frac{V_2^2}{2} + g\dot{z}_2\right) + \dot{w}$$

$\because \Delta KE = 0$ $\qquad \because \Delta PE = 0$

$$m(h_1 + T_o \Delta S) = mh_2 + \dot{w}$$

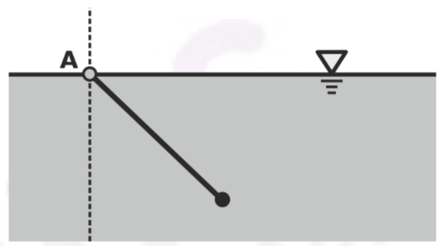
500(3500 + 500 (63 – 65))

= 500 × 2500 + w

$\dot{w}$ = 450 MW

62. A uniform wooden rod (specific gravity = 0.6, diameter = 4 cm and length = 8 m) is immersed in the water and is hinged without friction at point A on the waterline as shown in the figure. A solid spherical ball made of lead (specific gravity = 11.4) is attached to the free end of the rod to keep the assembly in static equilibrium inside the water. For simplicity, assume that the radius of the ball is much smaller than the length of the rod. Assume density of water = 10^3 kg/m^3 and π = 3.14.

Radius of the ball is _______ cm (round off to 2 decimal places).



[NAT: 2 Mark]

Ans. 3.58

Sol. Range (3.48 to 3.7)

Given,

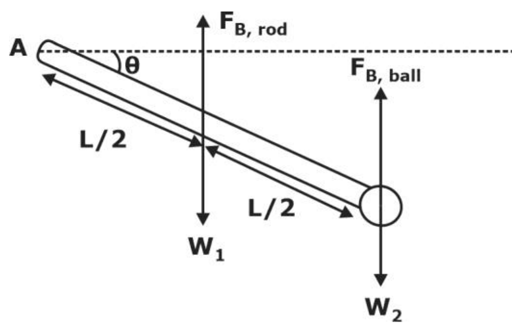
Specific gravity of wooden rod, S_{rod} = 0.6

Diameter = 4 cm

Length = 8 m

Specific gravity of ball, S_b = 11.4

Taking moment about A



$$\left(W_1 - F_{B,rod}\right)\frac{L}{2}\cos\theta + \left(W_2 - F_{B,Ball}\right)\times L\cos\theta = 0$$

$$\left(\rho_{rod} gV - \rho_w gV\right)\frac{L}{2} + \left(\rho_b gV_b - \rho_w gV_b\right)L = 0$$

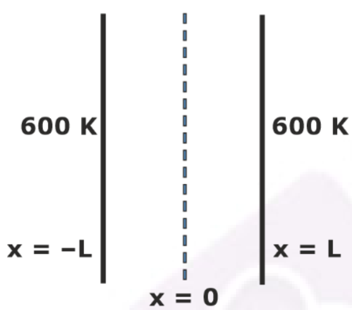
$[0.6 \times 1000 \times (\pi/4 \times 0.04^2 \times 8) - 10^3 \times \pi/4 \times 0.04^2 \times 8]1/2 + (11.4 \times 10^3 \times 4/3 \times \pi r^3 - 10^3 \times 4/3 \times \pi r^3) = 0$

$-2.01 + 43563.42\ r^3 = 0$

$r^3 = 4.61 \times 10^{-5}$

$r = 0.0358\ m = 3.58\ cm$

63. Consider steady state, one-dimensional heat conduction in an infinite slab of thickness 2L (L = 1 m) as shown in the figure. The conductivity (k) of the material varies with temperature as k = CT, where T is the temperature in K, and C is a constant equal to 2 $W \cdot m^{-1} \cdot K^{-2}$. There is a uniform heat generation of 1280 kW/m^3 in the slab. If both faces of the slab are maintained at 600 K, then the temperature at x = 0 is _________ K (in integer).



[NAT: 2 Mark]

Ans. 1000

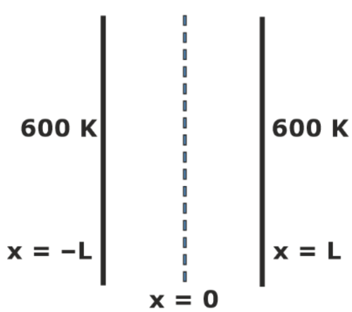
Sol. Range (1000 to 1000)

Given,

Thickness of slab, 2L = 2m

Thermal conductivity, k = CT

k = 2T



q_g, heat generation = 1250 kW/m^3

steady state one dimensional heat conduction equation

$$\frac{\partial}{\partial x}\left(2T\frac{\partial T}{\partial x}\right) = -q_g$$

After integrating

$$2T\frac{\partial T}{\partial x} = -q_g x + c_1$$

Again, integrating both side

$$T^2 = -\frac{q_g x^2}{2} + c_1 x + c_2$$

At x = L,

T = 600 K

$$600^2 = -q_g\frac{xL^2}{2} + c_1 L + c_2 \qquad .....(i)$$

At x = – L, T – 600 K

$$600^2 = \frac{q_g L^2}{2} - c_1 L + c_2 \qquad ....(ii)$$

From equation (i) and (ii)

$$c_2 = 600^2 + \frac{q_g L^2}{2}$$

$$c_2 = 600^2 + \frac{1280 \times 10^3 \times 1^2}{2}$$

$$c_2 = 10^6$$

Now at x = 0,

$$T^2 = \frac{q_g(0)}{2} + c_1 \times 0 + c_2$$

$T^2 = C_2$

$T^2 = 10^6$

T = 1000 K

64. Saturated vapor at 200 °C condenses to saturated liquid at the rate of 150 kg/s on the shell side of a heat exchanger (enthalpy of condensation h_{fg} = 2400 kJ/kg). A fluid with c_p = 4 kJ·kg^{-1}·K^{-1} enters at 100 °C on the tube side. If the effectiveness of the heat exchanger is 0.9, then the mass flow rate of the fluid in the tube side is _________ kg/s (in integer).

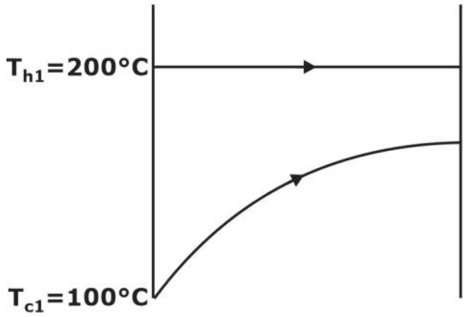
[NAT: 2 Mark]

Ans. 1000

Sol. Range (1000 to 1000)

Given,

Effectiveness of heat exchanger = 0.9



$$\varepsilon = \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_{pc}(T_{h1} - T_{c1})}$$

$$0.9 = \frac{150 \times 2400}{\dot{m}_c \times 4 \times (200 - 100)}$$

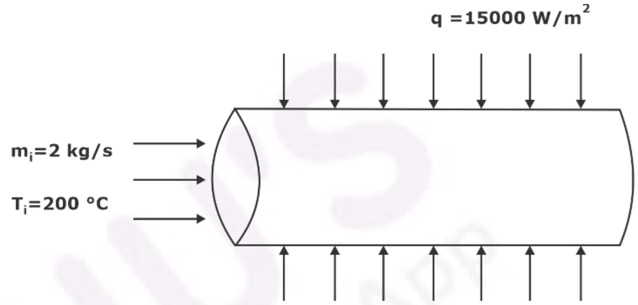
$$\dot{m}_c = 1000 \text{ kg / s}$$

65. Consider a hydrodynamically and thermally fully developed, steady fluid flow of 1 kg/s in a uniformly heated pipe with diameter of 0.1 m and length of 40 m. A constant heat flux of magnitude 15000 W/m^2 is imposed on the outer surface of the pipe. The bulk-mean temperature of the fluid at the entrance to the pipe is 200 °C. The Reynolds number (Re) of the flow is 85000, and the Prandtl number (Pr) of the fluid is 5. The thermal conductivity and the specific heat of the fluid are 0.08 W·m^{-1}·K^{-1} and 2600 J·kg^{-1}·K^{-1}, respectively. The correlation Nu = 0.023 Re$^{0.8}$.Pr$^{0.4}$ is applicable, where the Nusselt Number (Nu) is defined on the basis of the pipe diameter. The pipe surface temperature at the exit is _________ °C (round off to the nearest integer).

[NAT: 2 Mark]

Ans. 321

Sol. Range (317 to 324)



Given,

Reynold number of flow= Re = 85000

Constant heat flux, q = 15000 w/m^2

Prandtl number, Pr = 5

Nu = 0023 Re$^{0.3}$ Pr$^{0.4}$

$$\frac{h \times 0.1}{0.08} = 0.023\,(85000)^{0.8}\,(5)^{0.4}$$

h = 307.56 W/m^{2}K

Applying energy hance in pipe

$m_i c T_1 + qA = m_e c T_e$

1 × 2600 × 200 + 15000 × (π × 0.1 × 40)

= 1 × 2600 × T_e

T_e = 272.5 °C

Bulk mean temperature of fluid at the exit of pipe is T_e = 272.5 °C

At exit of pipe

q = h (T_s – T_e)

15000 = 307.56 × (T_s – 272.5)

T_s = 321.27 °C