

EXERCISE 3.1

Choose the correct answer from the given four options:

1. Graphically, the pair of equations

$$6x - 3y + 10 = 0$$

$$2x - y + 9 = 0$$

represents two lines which are

- (A) Intersecting at exactly one point.
(C) Coincident

- (B) Intersecting at exactly two points.
(D) parallel.

Solution:

(D) Parallel

Explanation:

The given equations are,

$$6x - 3y + 10 = 0$$

dividing by 3

$$\Rightarrow 2x - y + 10/3 = 0 \dots (i)$$

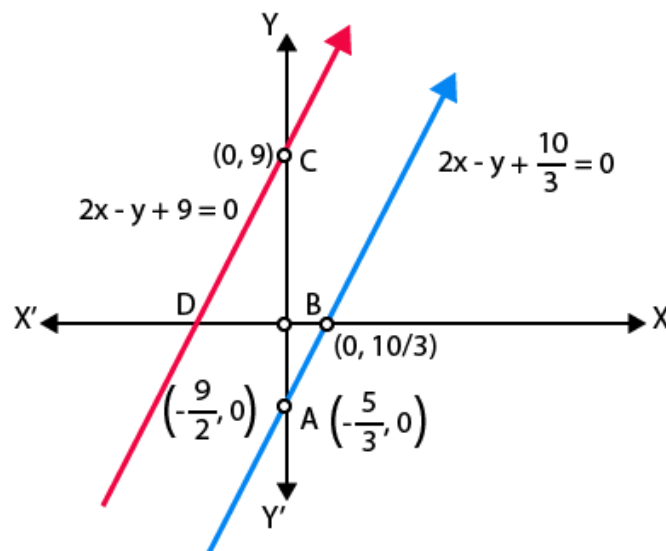
$$\text{And } 2x - y + 9 = 0 \dots (ii)$$

Table for $2x - y + 10/3 = 0$,

x	0	-5/3
y	10/3	0

Table for $2x - y + 9 = 0$

x	0	-9/2
y	9	0



Hence, the pair of equations represents two parallel lines.

2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have

- (A) a unique solution
(C) infinitely many solutions

- (B) exactly two solutions
(D) no solution

Solution:

(D) No solution

Explanation:

The equations are:

$$x + 2y + 5 = 0$$

$$-3x - 6y + 1 = 0$$

$$a_1 = 1; b_1 = 2; c_1 = 5$$

$$a_2 = -3; b_2 = -6; c_2 = 1$$

$$a_1/a_2 = -1/3$$

$$b_1/b_2 = -2/6 = -1/3$$

$$c_1/c_2 = 5/1 = 5$$

Here,

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

Therefore, the pair of equation has no solution.

3. If a pair of linear equations is consistent, then the lines will be

- (A) parallel
(C) intersecting or coincident
- (B) always coincident
(D) always intersecting

Solution:

(C) intersecting or coincident

Explanation:

Condition for a pair of linear equations to be consistent are:

Intersecting lines having unique solution,

$$a_1/a_2 \neq b_1/b_2$$

Coincident or dependent

$$a_1/a_2 = b_1/b_2 = c_1/c_2$$

4. The pair of equations $y = 0$ and $y = -7$ has

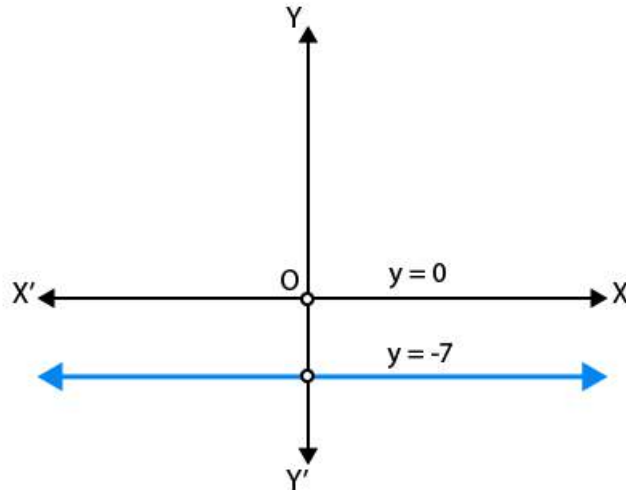
- (A) one solution
(C) infinitely many solutions
- (B) two solutions
(D) no solution

Solution:

(D) no solution

Explanation:

The given pair of equations are $y = 0$ and $y = -7$.



Graphically, both lines are parallel and have no solution

5. The pair of equations $x = a$ and $y = b$ graphically represents lines which are
- (A) parallel
 - (B) intersecting at (b, a)
 - (C) coincident
 - (D) intersecting at (a, b)

Solution:

(D) intersecting at (a, b)

Explanation:

Graphically in every condition,

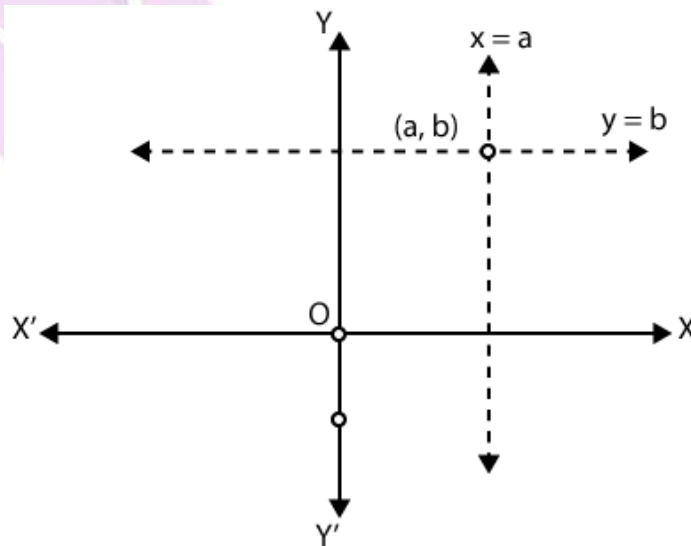
$a, b > 0$

$a, b < 0$

$a > 0, b < 0$

$a < 0, b > 0$ but $a = b \neq 0$.

The pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b) .



Hence, the cases two lines intersect at (a, b) .

EXERCISE 3.2

PAGE NO: 21

1. Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$

$12y + 6x = 6$

(ii) $x = 2y$

$y = 2x$

(iii) $3x + y - 3 = 0$

$2x + 2/3y = 2$

Solution:

The Condition for no solution = $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (parallel lines)

(i) Yes.

Given pair of equations are,

$2x + 4y - 3 = 0$ and $6x + 12y - 6 = 0$

Comparing the equations with $ax + by + c = 0$;

We get,

$a_1 = 2, b_1 = 4, c_1 = -3$;

$a_2 = 6, b_2 = 12, c_2 = -6$;

$a_1/a_2 = 2/6 = 1/3$

$b_1/b_2 = 4/12 = 1/3$

$c_1/c_2 = -3/-6 = 1/2$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e parallel lines

Hence, the given pair of linear equations has no solution.

(ii) No.

Given pair of equations,

$x = 2y$ or $x - 2y = 0$

$y = 2x$ or $2x - y = 0$;

Comparing the equations with $ax + by + c = 0$;

We get,

$a_1 = 1, b_1 = -2, c_1 = 0$;

$a_2 = 2, b_2 = -1, c_2 = 0$;

$a_1/a_2 = 1/2$

$b_1/b_2 = -2/-1 = 2$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has unique solution.

(iii) No.

Given pair of equations,

$3x + y - 3 = 0$

$2x + 2/3 y = 2$

Comparing the equations with $ax + by + c = 0$;

We get,

$a_1 = 3, b_1 = 1, c_1 = -3$;

$a_2 = 2, b_2 = 2/3, c_2 = -2$;

$a_1/a_2 = 3/2 = 3/2$

$$b_1 / b_2 = 4/12 = 3/2$$

$$c_1 / c_2 = -3/-2 = 3/2$$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e coincident lines

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + 1/7y = 3$

$$7x + 3y = 7$$

(ii) $-2x - 3y = 1$

$$6y + 4x = -2$$

(iii) $x/2 + y + 2/5 = 0$

$$4x + 8y + 5/16 = 0$$

Solution:

Condition for coincident lines,

$$a_1/a_2 = b_1/b_2 = c_1/c_2;$$

(i) No.

Given pair of linear equations are:

$$3x + 1/7y = 3$$

$$7x + 3y = 7$$

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = 3$, $b_1 = 1/7$, $c_1 = -3$;

And $a_2 = 7$, $b_2 = 3$, $c_2 = -7$;

$$a_1 / a_2 = 3/7$$

$$b_1 / b_2 = 1/21$$

$$c_1 / c_2 = -3/-7 = 3/7$$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has unique solution.

(ii) Yes,

Given pair of linear equations.

$$-2x - 3y - 1 = 0 \text{ and } 4x + 6y + 2 = 0;$$

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = -2$, $b_1 = -3$, $c_1 = -1$;

And $a_2 = 4$, $b_2 = 6$, $c_2 = 2$;

$$a_1 / a_2 = -2/4 = -1/2$$

$$b_1 / b_2 = -3/6 = -1/2$$

$$c_1 / c_2 = -1/2$$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e. coincident lines

Hence, the given pair of linear equations is coincident.

(iii) No,

Given pair of linear equations are

$$x/2 + y + 2/5 = 0$$

$$4x + 8y + 5/16 = 0$$

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = 1/2$, $b_1 = 1$, $c_1 = 2/5$;

And $a_2 = 4$, $b_2 = 8$, $c_2 = 5/16$;

$$a_1/a_2 = 1/8$$

$$b_1/b_2 = 1/8$$

$$c_1/c_2 = 32/25$$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e. parallel lines

Hence, the given pair of linear equations has no solution.

3. Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$

$$4y + 3x = 12$$

(ii) $(3/5)x - y = 1/2$

$$(1/5)x - 3y = 1/6$$

(iii) $2ax + by = a$

$$4ax + 2by - 2a = 0; a, b \neq 0$$

(iv) $x + 3y = 11$

$$2(2x + 6y) = 22$$

Solution:

Conditions for pair of linear equations to be consistent are:

$$a_1/a_2 \neq b_1/b_2. \text{ [unique solution]}$$

$$a_1/a_2 = b_1/b_2 = c_1/c_2 \text{ [coincident or infinitely many solutions]}$$

(i) No.

The given pair of linear equations

$$-3x - 4y - 12 = 0 \text{ and } 4y + 3x - 12 = 0$$

Comparing the above equations with $ax + by + c = 0$;

We get,

$$a_1 = -3, b_1 = -4, c_1 = -12;$$

$$a_2 = 3, b_2 = 4, c_2 = -12;$$

$$a_1/a_2 = -3/3 = -1$$

$$b_1/b_2 = -4/4 = -1$$

$$c_1/c_2 = -12/-12 = 1$$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Hence, the pair of linear equations has no solution, i.e., inconsistent.

(ii) Yes.

The given pair of linear equations

$$(3/5)x - y = 1/2$$

$$(1/5)x - 3y = 1/6$$

Comparing the above equations with $ax + by + c = 0$;

We get,

$$a_1 = 3/5, b_1 = -1, c_1 = -1/2;$$

$$a_2 = 1/5, b_2 = 3, c_2 = -1/6;$$

$$a_1/a_2 = 3$$

$$b_1/b_2 = -1/3 = 1/3$$

$$c_1/c_2 = 3$$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has unique solution, i.e., consistent.

(iii) Yes.

The given pair of linear equations –

$$2ax + by - a = 0 \text{ and } 4ax + 2by - 2a = 0$$

Comparing the above equations with $ax + by + c = 0$;

We get,

$$a_1 = 2a, b_1 = b, c_1 = -a;$$

$$a_2 = 4a, b_2 = 2b, c_2 = -2a;$$

$$a_1/a_2 = 1/2$$

$$b_1/b_2 = 1/2$$

$$c_1/c_2 = 1/2$$

$$\text{Here, } a_1/a_2 = b_1/b_2 = c_1/c_2$$

Hence, the given pair of linear equations has infinitely many solutions, i.e., consistent

(iv) No.

The given pair of linear equations

$$x + 3y = 11 \text{ and } 2x + 6y = 11$$

Comparing the above equations with $ax + by + c = 0$;

We get,

$$a_1 = 1, b_1 = 3, c_1 = 11$$

$$a_2 = 2, b_2 = 6, c_2 = 11$$

$$a_1/a_2 = 1/2$$

$$b_1/b_2 = 1/2$$

$$c_1/c_2 = 1$$

$$\text{Here, } a_1/a_2 = b_1/b_2 \neq c_1/c_2.$$

Hence, the given pair of linear equations has no solution.

EXERCISE 3.3

PAGE NO: 25

1. For which value(s) of λ , do the pair of linear equations

$$\lambda x + y = \lambda^2 \text{ and } x + \lambda y = 1 \text{ have}$$

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

Solution:

The given pair of linear equations is

$$\lambda x + y = \lambda^2 \text{ and } x + \lambda y = 1$$

$$a_1 = \lambda, b_1 = 1, c_1 = -\lambda^2$$

$$a_2 = 1, b_2 = \lambda, c_2 = -1$$

The given equations are;

$$\lambda x + y - \lambda^2 = 0$$

$$x + \lambda y - 1 = 0$$

Comparing the above equations with $ax + by + c = 0$;

We get,

$$a_1 = \lambda, b_1 = 1, c_1 = -\lambda^2;$$

$$a_2 = 1, b_2 = \lambda, c_2 = -1;$$

$$a_1/a_2 = \lambda/1$$

$$b_1/b_2 = 1/\lambda$$

$$c_1/c_2 = \lambda^2$$

(i) For no solution,

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

$$\text{i.e. } \lambda = 1/\lambda \neq \lambda^2$$

$$\text{so, } \lambda^2 = 1;$$

$$\text{and } \lambda^2 \neq \lambda$$

Here, we take only $\lambda = -1$,

Since the system of linear equations has no solution.

(ii) For infinitely many solutions,

$$a_1/a_2 = b_1/b_2 = c_1/c_2$$

$$\text{i.e. } \lambda = 1/\lambda = \lambda^2$$

$$\text{so } \lambda = 1/\lambda \text{ gives } \lambda = \pm 1;$$

$$\lambda = \lambda^2 \text{ gives } \lambda = 1, 0;$$

Hence satisfying both the equations

$\lambda = 1$ is the answer.

(iii) For a unique solution,

$$a_1/a_2 \neq b_1/b_2$$

$$\text{so } \lambda \neq 1/\lambda$$

$$\text{hence, } \lambda^2 \neq 1;$$

$$\lambda \neq \pm 1;$$

So, all real values of λ except ± 1

2. For which value(s) of k will the pair of equations

$$kx + 3y = k - 3$$

$$12x + ky = k$$

have no solution?

Solution:

The given pair of linear equations is

$$kx + 3y = k - 3 \dots(i)$$

$$12x + ky = k \dots(ii)$$

On comparing the equations (i) and (ii) with $ax + by = c = 0$,

We get,

$$a_1 = k, b_1 = 3, c_1 = -(k - 3)$$

$$a_2 = 12, b_2 = k, c_2 = -k$$

Then,

$$a_1 / a_2 = k/12$$

$$b_1 / b_2 = 3/k$$

$$c_1 / c_2 = (k-3)/k$$

For no solution of the pair of linear equations,

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

$$k/12 = 3/k \neq (k-3)/k$$

Taking first two parts, we get

$$k/12 = 3/k$$

$$k^2 = 36$$

$$k = \pm 6$$

Taking last two parts, we get

$$3/k \neq (k-3)/k$$

$$3k \neq k(k - 3)$$

$$k^2 - 6k \neq 0$$

$$\text{so, } k \neq 0, 6$$

Therefore, value of k for which the given pair of linear equations has no solution is $k = -6$.

3. For which values of a and b , will the following pair of linear equations have infinitely many solutions?

$$x + 2y = 1$$

$$(a - b)x + (a + b)y = a + b - 2$$

Solution:

The given pair of linear equations are:

$$x + 2y = 1 \dots(i)$$

$$(a-b)x + (a + b)y = a + b - 2 \dots(ii)$$

On comparing with $ax + by = c = 0$ we get

$$a_1 = 1, b_1 = 2, c_1 = -1$$

$$a_2 = (a - b), b_2 = (a + b), c_2 = -(a + b - 2)$$

$$a_1 / a_2 = 1/(a-b)$$

$$b_1 / b_2 = 2/(a+b)$$

$$c_1 / c_2 = 1/(a+b-2)$$

For infinitely many solutions of the, pair of linear equations,

$$a_1/a_2 = b_1/b_2 = c_1/c_2 (\text{coincident lines})$$

so, $1/(a-b) = 2/(a+b) = 1/(a+b-2)$

Taking first two parts,

$$1/(a-b) = 2/(a+b)$$

$$a + b = 2(a - b)$$

$$a = 3b \dots(\text{iii})$$

Taking last two parts,

$$2/(a+b) = 1/(a+b-2)$$

$$2(a + b - 2) = (a + b)$$

$$a + b = 4 \dots(\text{iv})$$

Now, put the value of a from Eq. (iii) in Eq. (iv), we get

$$3b + b = 4$$

$$4b = 4$$

$$b = 1$$

Put the value of b in Eq. (iii), we get

$$a = 3$$

So, the values $(a,b) = (3,1)$ satisfies all the parts. Hence, required values of a and b are 3 and 1 respectively for which the given pair of linear equations has infinitely many solutions.

4. Find the value(s) of p in (i) to (iv) and p and q in (v) for the following pair of equations:

(i) $3x - y - 5 = 0$ and $6x - 2y - p = 0$, if the lines represented by these equations are parallel.

Solution:

Given pair of linear equations is

$$3x - y - 5 = 0 \dots(\text{i})$$

$$6x - 2y - p = 0 \dots(\text{ii})$$

On comparing with $ax + by + c = 0$ we get

We get,

$$a_1 = 3, b_1 = -1, c_1 = -5;$$

$$a_2 = 6, b_2 = -2, c_2 = -p;$$

$$a_1/a_2 = 3/6 = 1/2$$

$$b_1/b_2 = 1/2$$

$$c_1/c_2 = 5/p$$

Since, the lines represented by these equations are parallel, then

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

Taking last two parts, we get $1/2 \neq 5/p$

$$\text{So, } p \neq 10$$

Hence, the given pair of linear equations are parallel for all real values of p except 10.

(ii) $-x + py = 1$ and $px - y = 1$, if the pair of equations has no solution.

Solution:

Given pair of linear equations is

$$-x + py = 1 \dots(\text{i})$$

$$px - y - 1 = 0 \dots(\text{ii})$$

On comparing with $ax + by + c = 0$,

We get,

$$a_1 = -1, b_1 = p, c_1 = -1;$$

$$a_2 = p, b_2 = -1, c_2 = -1;$$

$$a_1/a_2 = -1/p$$

$$b_1/b_2 = -p$$

$$c_1/c_2 = 1$$

Since, the lines equations has no solution i.e., both lines are parallel to each other.

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

$$-1/p = -p \neq 1$$

Taking last two parts, we get

$$p \neq -1$$

Taking first two parts, we get

$$p^2 = 1$$

$$p = \pm 1$$

Hence, the given pair of linear equations has no solution for $p = 1$.

(iii) $-3x + 5y = 7$ and $2px - 3y = 1$, if the lines represented by these equations are intersecting at a unique point.

Solution:

Given, pair of linear equations is

$$-3x + 5y = 7$$

$$2px - 3y = 1$$

On comparing with $ax + by + c = 0$, we get

$$\text{Here, } a_1 = -3, b_1 = 5, c_1 = -7;$$

$$\text{And } a_2 = 2p, b_2 = -3, c_2 = -1;$$

$$a_1/a_2 = -3/2p$$

$$b_1/b_2 = -5/3$$

$$c_1/c_2 = 7$$

Since, the lines are intersecting at a unique point i.e., it has a unique solution

$$a_1/a_2 \neq b_1/b_2$$

$$-3/2p \neq -5/3$$

$$p \neq 9/10$$

Hence, the lines represented by these equations are intersecting at a unique point for all real values of p except $9/10$

(iv) $2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$, if the pair of equations has a unique solution.

Solution:

Given, pair of linear equations is

$$2x + 3y - 5 = 0$$

$$px - 6y - 8 = 0$$

On comparing with $ax + by + c = 0$ we get

$$\text{Here, } a_1 = 2, b_1 = 3, c_1 = -5;$$

$$\text{And } a_2 = p, b_2 = -6, c_2 = -8;$$

$$a_1/a_2 = 2/p$$

$$b_1/b_2 = -3/6 = -1/2$$

$$c_1/c_2 = 5/8$$

Since, the pair of linear equations has a unique solution.

$$a_1/a_2 \neq b_1/b_2$$

$$\text{so } 2/p \neq -1/2$$

$$p \neq -4$$

Hence, the pair of linear equations has a unique solution for all values of p except -4 .

(v) $2x + 3y = 7$ and $2px + py = 28 - qy$, if the pair of equations have infinitely many solutions.

Solution:

Given pair of linear equations is

$$2x + 3y = 7$$

$$2px + py = 28 - qy$$

$$\text{or } 2px + (p + q)y - 28 = 0$$

On comparing with $ax + by + c = 0$,

We get,

$$\text{Here, } a_1 = 2, b_1 = 3, c_1 = -7;$$

$$\text{And } a_2 = 2p, b_2 = (p + q), c_2 = -28;$$

$$a_1/a_2 = 2/2p$$

$$b_1/b_2 = 3/(p+q)$$

$$c_1/c_2 = 1/4$$

Since, the pair of equations has infinitely many solutions i.e., both lines are coincident.

$$a_1/a_2 = b_1/b_2 = c_1/c_2$$

$$1/p = 3/(p+q) = 1/4$$

Taking first and third parts, we get

$$p = 4$$

Again, taking last two parts, we get

$$3/(p+q) = 1/4$$

$$p + q = 12$$

$$\text{Since } p = 4$$

$$\text{So, } q = 8$$

Here, we see that the values of $p = 4$ and $q = 8$ satisfies all three parts.

Hence, the pair of equations has infinitely many solutions for all values of $p = 4$ and $q = 8$.

5. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.

Solution:

Given linear equations are

$$x - 3y - 2 = 0 \dots(i)$$

$$-2x + 6y - 5 = 0 \dots(ii)$$

On comparing with $ax + by + c = 0$,

We get

$$a_1 = 1, b_1 = -3, c_1 = -2;$$

$$a_2 = -2, b_2 = 6, c_2 = -5;$$

$$a_1/a_2 = -1/2$$

$$b_1/b_2 = -3/6 = -1/2$$

$$c_1/c_2 = 2/5$$

i.e., $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ [parallel lines]

Hence, two straight paths represented by the given equations never cross each other, because they are parallel to each other.

6. Write a pair of linear equations which has the unique solution $x = -1, y = 3$. How many such pairs can you write?

Solution:

Condition for the pair of system to have unique solution

$$a_1/a_2 \neq b_1/b_2$$

Let the equations be,

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Since, $x = -1$ and $y = 3$ is the unique solution of these two equations, then

It must satisfy the equations –

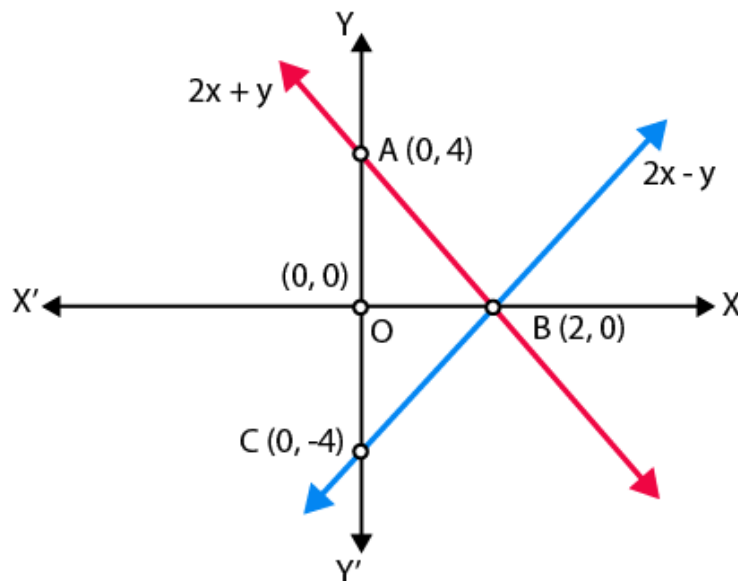
$$a_1(-1) + b_1(3) + c_1 = 0$$

$$-a_1 + 3b_1 + c_1 = 0 \dots(i)$$

$$\text{and } a_2(-1) + b_2(3) + c_2 = 0$$

$$-a_2 + 3b_2 + c_2 = 0 \dots(ii)$$

Since for the different values of a_1, b_1, c_1 and a_2, b_2, c_2 satisfy the Eqs. (i) and (ii).



Hence, infinitely many pairs of linear equations are possible.

7. If $2x + y = 23$ and $4x - y = 19$, find the values of $5y - 2x$ and $y/x - 2$.

Solution:

Given equations are

$$2x + y = 23 \dots(i)$$

$$4x - y = 19 \dots(ii)$$

On adding both equations, we get

$$6x = 42$$

$$\text{So, } x = 7$$

Put the value of x in Eq. (i), we get

$$2(7) + y = 23$$

$$y = 23 - 14$$

so, $y = 9$

Hence $5y - 2x = 5(9) - 2(7) = 45 - 14 = 31$

$y/x - 2 = 9/7 - 2 = -5/7$

Hence, the values of $(5y - 2x)$ and $y/x - 2$ are 31 and $-5/7$ respectively.

8. Find the values of x and y in the following rectangle [see Fig. 3.2].

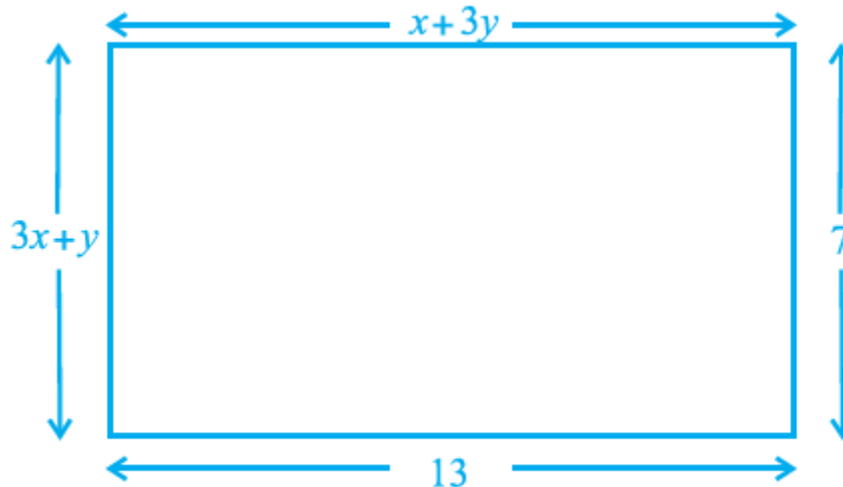


Fig. 3.2

Solution:

Using property of rectangle,

We know that,

Lengths are equal,

i.e., $CD = AB$

Hence, $x + 3y = 13 \dots(i)$

Breadth are equal,

i.e., $AD = BC$

Hence, $3x + y = 7 \dots(ii)$

On multiplying Eq. (ii) by 3 and then subtracting Eq. (i),

We get,

$$8x = 8$$

So, $x = 1$

On substituting $x = 1$ in Eq. (i),

We get,

$$y = 4$$

Therefore, the required values of x and y are 1 and 4, respectively.

EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$$2x + y = 6$$

$$2x - y + 2 = 0$$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x -axis and the lines with the y -axis.

Solution:

Given equations are $2x + y = 6$ and $2x - y + 2 = 0$

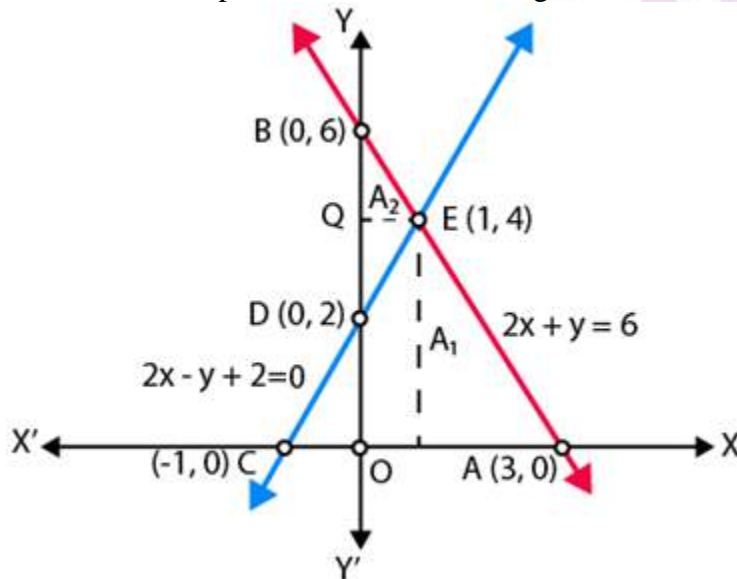
Table for equation $2x + y - 6 = 0$, for $x = 0$, $y = 6$, for $y = 0$, $x = 3$.

x	0	3
y	6	0

Table for equation $2x - y + 2 = 0$, for $x = 0$, $y = 2$, for $y = 0$, $x = -1$

x	0	-1
y	2	0

Let A_1 and A_2 represent the areas of triangles ACE and BDE respectively.



Let, Area of triangle formed with x -axis = T_1

$$T_1 = \text{Area of } \triangle ACE = \frac{1}{2} \times AC \times PE$$

$$T_1 = \frac{1}{2} \times 4 \times 4 = 8$$

And Area of triangle formed with y - axis = T_2

$$T_2 = \text{Area of } \triangle BDE = \frac{1}{2} \times BD \times QE$$

$$T_2 = \frac{1}{2} \times 4 \times 1 = 2$$

$$T_1 : T_2 = 8 : 2 = 4 : 1$$

Hence, the pair of equations intersect graphically at point $E(1, 4)$

i.e., $x = 1$ and $y = 4$.

2. Determine, graphically, the vertices of the triangle formed by the lines

$$y = x, 3y = x, x + y = 8$$

Solution:

Given linear equations are

$$y = x \dots(i)$$

$$3y = x \dots(ii)$$

$$\text{and } x + y = 8 \dots(iii)$$

Table for line $y = x$,

x	0	1	2
y	0	1	2

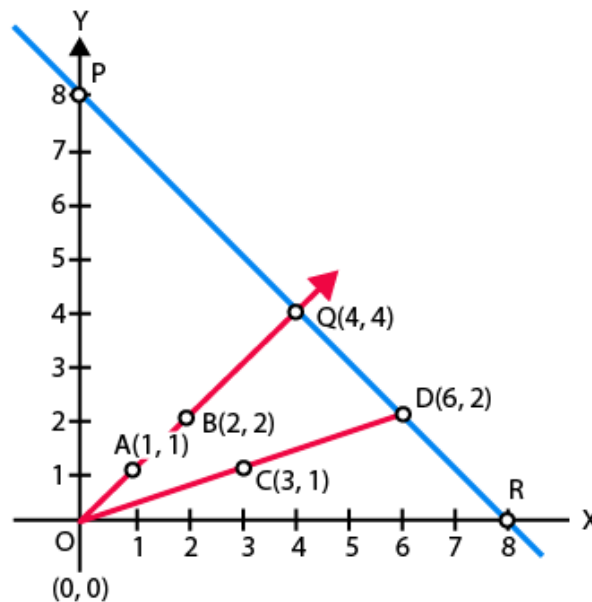
Table for line $x = 3y$,

x	0	3	6
y	0	1	2

Table for line $x + y = 8$

x	0	4	8
y	8	4	0

Plotting the points A (1, 1), B(2,2), C (3, 1), D (6, 2), we get the straight lines AB and CD. Similarly, plotting the point P (0, 8), Q(4, 4) and R(8, 0), we get the straight line PQR. AB and CD intersects the line PR on Q and D, respectively.



So, $\triangle OQD$ is formed by these lines. Hence, the vertices of the $\triangle OQD$ formed by the given lines are O(0, 0), Q(4, 4) and D(6,2).

3. Draw the graphs of the equations $x = 3$, $x = 5$ and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x-axis.

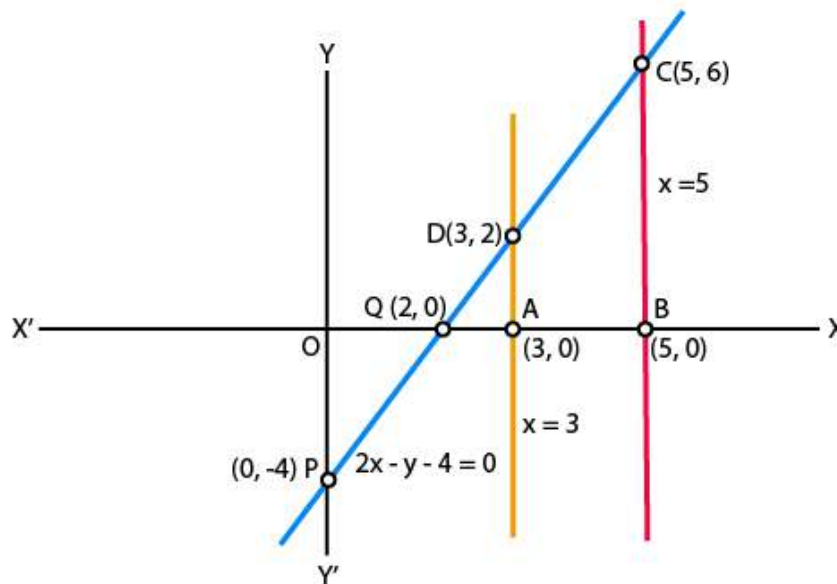
Solution:

Given equation of lines $x = 3$, $x = 5$ and $2x - y - 4 = 0$.

Table for line $2x - y - 4 = 0$

x	0	2
y	-4	0

Plotting the graph, we get,



From the graph, we get,

$$AB = OB - OA = 5 - 3 = 2$$

$$AD = 2$$

$$BC = 6$$

Thus, quadrilateral ABCD is a trapezium, then,

$$\begin{aligned} \text{Area of Quadrilateral ABCD} &= \frac{1}{2} \times (\text{distance between parallel lines}) \times (AD + BC) \\ &= 8 \text{ sq units} \end{aligned}$$

4. The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Solution:

Let the cost of a pen and a pencil box be Rs x and Rs y respectively.

According to the question,

$$4x + 4y = 100$$

Or $x + y = 25 \dots(i)$

$$3x = y + 15$$

Or $3x - y = 15 \dots(ii)$

On adding Equation (i) and (ii), we get,

$$4x = 40$$

So, $x = 10$

Substituting $x = 10$, in Eq. (i) we get

$$y = 25 - 10 = 15$$

Hence, the cost of a pen = Rs. 10
The cost of a pencil box = Rs. 15

5. Determine, algebraically, the vertices of the triangle formed by the lines

$$\begin{aligned}3x - y &= 3 \\2x - 3y &= 2 \\x + 2y &= 8\end{aligned}$$

Solution:

$$\begin{aligned}3x - y &= 2 \dots(i) \\2x - 3y &= 2 \dots(ii) \\x + 2y &= 8 \dots(iii)\end{aligned}$$

Let the equations of the line (i), (ii) and (iii) represent the side of a ΔABC .

On solving (i) and (ii),

We get,

[First, multiply Eq. (i) by 3 in Eq. (i) and then subtract]

$$(9x - 3y) - (2x - 3y) = 9 - 2$$

$$7x = 7$$

$$x = 1$$

Substituting $x=1$ in Eq. (i), we get

$$3 \times 1 - y = 3$$

$$y = 0$$

So, the coordinate of point B is (1, 0)

On solving lines (ii) and (iii),

We get,

[First, multiply Eq. (iii) by 2 and then subtract]

$$(2x + 4y) - (2x - 3y) = 16 - 2$$

$$7y = 14$$

$$y = 2$$

Substituting $y=2$ in Eq. (iii), we get

$$x + 2 \times 2 = 8$$

$$x + 4 = 8$$

$$x = 4$$

Hence, the coordinate of point C is (4, 2).

On solving lines (iii) and (i),

We get,

[First, multiply in Eq. (i) by 2 and then add]

$$(6x - 2y) + (x + 2y) = 6 + 8$$

$$7x = 14$$

$$x = 2$$

Substituting $x=2$ in Eq. (i), we get

$$3 \times 2 - y = 3$$

$$y = 3$$

So, the coordinate of point A is (2, 3).

Hence, the vertices of the ΔABC formed by the given lines are as follows,

A (2, 3), B (1, 0) and C (4, 2).

6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

Solution:

Let the speed of the rickshaw and the bus are x and y km/h, respectively.

Now, she has taken time to travel 2 km by rickshaw, $t_1 = (2/x)$ hr

Speed = distance/ time

she has taken time to travel remaining distance i.e., $(14 - 2) = 12$ km

By bus $t_2 = (12/y)$ hr

By first condition,

$$t_1 + t_2 = \frac{1}{2} = \frac{2}{x} + \frac{12}{y} \dots (i)$$

Now, she has taken time to travel 4 km by rickshaw, $t_3 = (4/x)$ hr

and she has taken time to travel remaining distance i.e., $(14 - 4) = 10$ km, by bus = $t_4 = (10/y)$ hr

By second condition,

$$t_3 + t_4 = \frac{1}{2} + \frac{9}{60} = \frac{1}{2} + \frac{3}{20}$$

$$\frac{4}{x} + \frac{10}{y} = \frac{13}{20} \dots (ii)$$

Let $(1/x) = u$ and $(1/y) = v$

Then Equations. (i) and (ii) becomes

$$2u + 12v = \frac{1}{2} \dots (iii)$$

$$4u + 10v = \frac{13}{20} \dots (iv)$$

[First, multiply Eq. (iii) by 2 and then subtract]

$$(4u + 24v) - (4u + 10v) = 1 - \frac{13}{20}$$

$$14v = \frac{7}{20}$$

$$v = \frac{1}{40}$$

Substituting the value of v in Eq. (iii),

$$2u + 12\left(\frac{1}{40}\right) = \frac{1}{2}$$

$$2u = \frac{2}{10}$$

$$u = \frac{1}{10}$$

$$x = \frac{1}{u} = 10 \text{ km/hr}$$

$$y = \frac{1}{v} = 40 \text{ km/hr}$$

Hence, the speed of rickshaw = 10 km/h

And the speed of bus = 40 km/h.