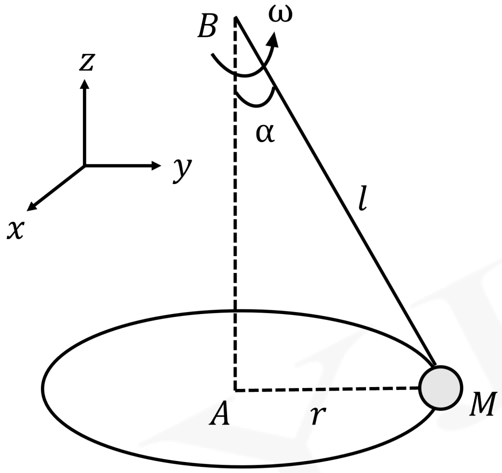
Topic : Rotational Motion

1. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive $z-$direction and the angular momentum of M about point B is L_B. The correct statement for this system is :



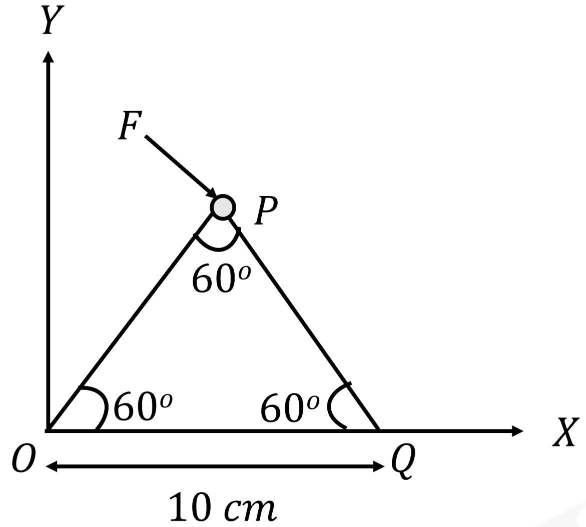
A. L_A and L_B are both constant in magnitude and direction.

B. L_B is constant, both in magnitude and direction.

C. L_A is constant, both in magnitude and direction.

D. L_A is constant in direction with varying magnitude.

2. A triangular plate is shown in the figure. A force $\overrightarrow{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque acting at point P with respect to point O and point Q respectively are :



A. $15 - 20\sqrt{3}\,;\ 15 + 20\sqrt{3}$

B. $15 + 20\sqrt{3}\,;\ 15 - 20\sqrt{3}$

C. $-15 + 20\sqrt{3}\,;\ 15 + 20\sqrt{3}$

D. $-15 - 20\sqrt{3}\,;\ 15 - 20\sqrt{3}$

3. A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become:

A. $\omega \dfrac{M}{M + 2m}$

B. $\omega \dfrac{M}{M + m}$

C. $\omega \dfrac{M + 2m}{M}$

D. $\omega \dfrac{M - 2m}{M + 2m}$

4. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

   A. Solid sphere

   B. Solid cylinder

   C. Hollow cylinder

   D. Ring

5. The moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of the square plate is given by:

   A. $\dfrac{Ml^2}{12}$

   B. $\frac{2}{3}Ml^2$

   C. $\dfrac{Ml^2}{6}$

   D. Ml^2

6. Two discs have moments of inertia $I_1 \; and \; I_2$ about their respective axes perpendicular to the plane and passing through the center. They are rotating with angular speeds, $\omega_1 \; and \; \omega_2$ respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by :

   A. $\dfrac{I_1 I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$

   B. $\dfrac{I_1 I_2}{(I_1+I_2)}(\omega_1-\omega_2)^2$

   C. $\dfrac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$

   D. $\dfrac{(I_1+I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$

7. Mass per unit area of a circular disc of radius $'a'$ depends on the distance r from its centre, as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre, is:

A. $2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5}\right)$

B. $2\pi a^4 \left(\frac{aA}{4} + \frac{B}{5}\right)$

C. $\pi a^4 \left(\frac{A}{4} + \frac{aB}{5}\right)$

D. $2\pi a^4 \left(\frac{A}{4} + \frac{B}{5}\right)$

8. Consider a uniform rod of mass $M = 4m$ and length L pivoted about its centre. A mass m moving with a velocity V making an angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod, and sticks to it. The angular speed of the rod-mass system just after the collision is:

A. $\frac{3}{7\sqrt{2}} \frac{V}{L}$

B. $\frac{3}{7} \frac{V}{L}$

C. $\frac{3\sqrt{2}}{7} \frac{V}{L}$

D. $\frac{4}{7} \frac{V}{L}$

9. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is

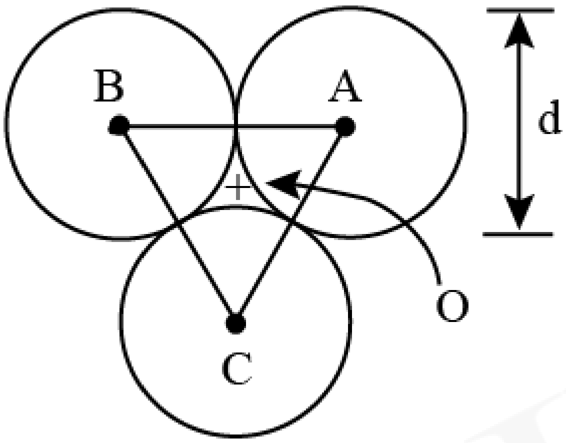
A. $8.75 \times 10^{-4} \text{ J}$

B. $8.75 \times 10^{-3} \text{ J}$

C. $6.25 \times 10^{-4} \text{ J}$

D. $1.13 \times 10^{-3} \text{ J}$

10. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio $\frac{I_0}{I_A}$ of moment of inertia I_0 of the system about an axis passing the centroid and about centre of any of the spheres I_A and perpendicular to the plane of the triangle, is:



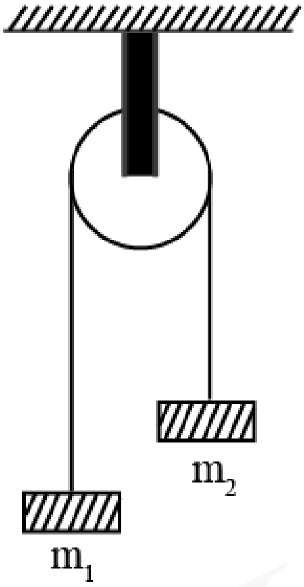
A. $\frac{13}{23}$

B. $\frac{15}{13}$

C. $\frac{23}{13}$

D. $\frac{13}{15}$

11. A uniformly thick wheel, with moment of inertia I and radius R, is free to rotate about its centre of mass (see fig.). A massless string is wrapped over its rim and two blocks of masses m_1 and $m_2 > m_2$ are attached to the ends of the string. The system is released from rest. The angular speed of the wheel, when m_1 descents through a distance h, is

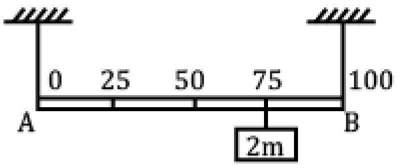


A. $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{1/2}$

B. $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{1/2}$

C. $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{1/2} gh$

D. $\left[\frac{(m_1 + m_2)}{(m_1 + m_2)R^2 + I}\right]^{1/2} gh$

12. 

Shown in the figure is a rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass m and has another weight of mass $2m$ hung at a distance of $75\ \mathrm{cm}$ from A. The tension in the string at A is :

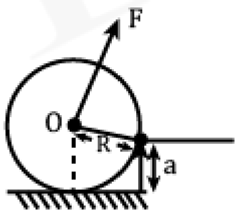
A. $0.5mg$

B. $2mg$

C. $0.75mg$

D. $1mg$

13. A uniform cylinder of mass M and radius R is to be pulled over a step of height a $(a < R)$ by applying a force F at its centre O perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :



A. $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

B. $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

C. $Mg\frac{a}{R}$

D. $Mg\sqrt{1-\frac{a^2}{R^2}}$

14. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder to be made for a given mass of a material, the ratio $\frac{L}{R}$ for it to have minimum possible I is-

A. $\frac{2}{3}$

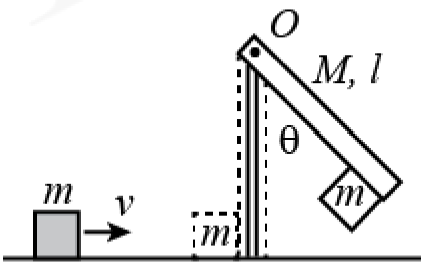
B. $\frac{3}{2}$

C. $\sqrt{\frac{3}{2}}$

D. $\sqrt{\frac{2}{3}}$

15. A block of mass $m = 1\ \mathrm{kg}$ slides with velocity $v = 6\ \mathrm{ms}^{-1}$ on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision, making angle θ before momentarily coming to rest. If the rod has mass $M = 2\ \mathrm{kg}$ and length $l = 1\ \mathrm{m}$, the value of θ is approximately-

[Take $g = 10\ \mathrm{ms}^{-2}$]



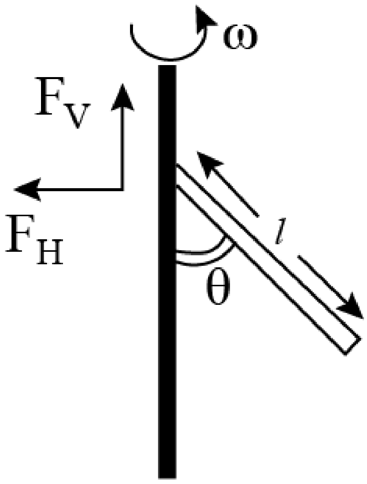
A. 63°

B. 55°

C. 69°

D. 49°

16.



A uniform rod of length l is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ, equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that:

A. $\cos\theta = \frac{2g}{3l\omega^2}$

B. $\cos\theta = \frac{g}{2l\omega^2}$

C. $\cos\theta = \frac{g}{l\omega^2}$

D. $\cos\theta = \frac{3g}{2l\omega^2}$

17. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$, made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$, then the value of α is :

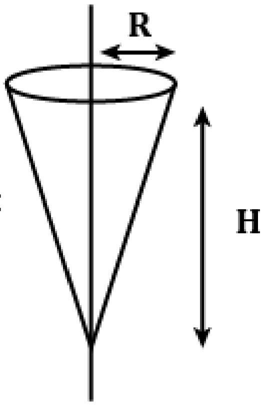
A. $2\sqrt{2}$

B. $\sqrt{2}$

C. 2

D. 4

18. Shown in the figure, is a hollow ice-cream cone (it is open at the top). If its mass is M, radius of its top is R and height H then, its moment of inertia about its axis is:



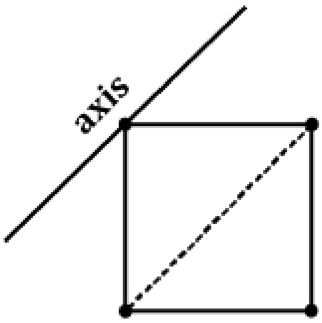
A. $\frac{MR^2}{2}$

B. $\frac{M(R^2+H^2)}{4}$

C. $\frac{MH^2}{3}$

D. $\frac{MR^2}{3}$

19. Four point masses, each of mass m are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is:



A. $ml^2\omega$

B. $4\,ml^2\omega$

C. $3\,ml^2\omega$

D. $2\,ml^2\omega$

20. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is :

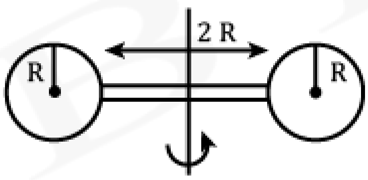
A. $\frac{5}{12}ML^2$

B. $\frac{7}{18}ML^2$

C. $\frac{2}{5}ML^2$

D. $\frac{3}{7}ML^2$

21. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is



A. $\frac{137}{15}MR^2$

B. $\frac{17}{15}MR^2$

C. $\frac{209}{15}MR^2$

D. $\frac{152}{15}MR^2$

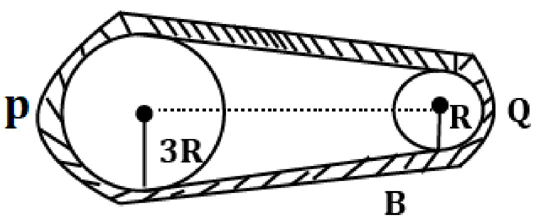
Topic : Rotational Motion

1. A force $\overrightarrow{F} = 4\hat{i} + 3\hat{j} + 4\hat{K}$ is applied on an intersection point of $x = 2$ plane and $x - \text{axis}$. The magnitude of torque of this force about a point $(2, 3, 4)$ is (Round off to the Nearest Integer)

2. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck wheel during this time is ______.

   (Assuming the acceleration to be uniform)

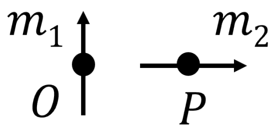
3. In the given figure , two wheels P and q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_P}{I_Q}\right)$ will be $x : 1$. The value of x will be



4. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is ____

5. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc to rest in 10 s is $\pi \times 10^{-1} \text{ Nm}$.

6. Two short magnetic dipoles, m_1 and m_2 each having magnetic moment of $1\ \mathrm{A\text{-}m^2}$ are placed at points O and P respectively. The distance between OP is $1\ \mathrm{m}$. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is ______ $\times 10^{-7}\ \mathrm{N\text{-}m}$.



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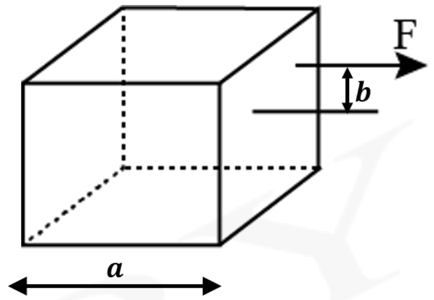
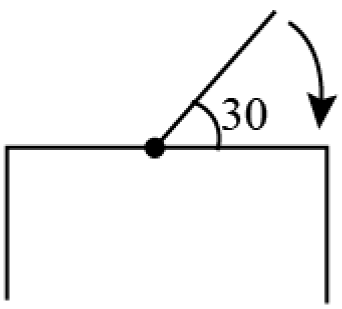


Consider a uniform cubical box of side $'a'$ on a rough floor that is to be moved by applying minimum possible force F at a point $'b'$ above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \dfrac{b}{a}$ for the box not to topple before moving is .

8. One end of a straight, uniform $1\ \mathrm{m}$ long bar is pivoted on a horizontal table. It is released from rest, when it makes an angle 30° from the horizontal (see figure). Its angular speed , when it hits the table, is given as $\sqrt{n}\ \mathrm{s^{-1}}$, where n is an integer. The value of n is .



9. A person of $80\ \mathrm{kg}$ mass is standing on the rim of a circular platform of mass $200\ \mathrm{kg}$ rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _____.