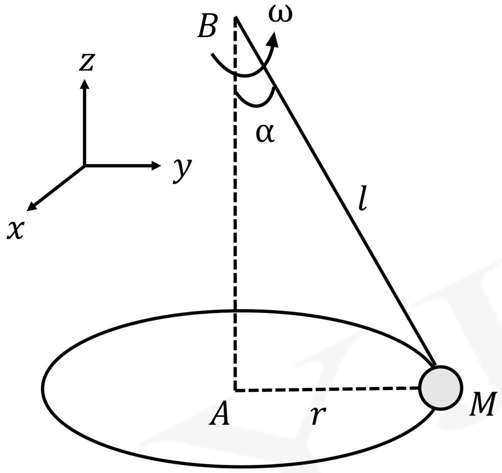
1. A mass $M$ hangs on a massless rod of length $l$ which rotates at a constant angular frequency. The mass $M$ moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity $\omega$. The angular momentum of $M$ about point $\mathrm{A}$ is $L_A$ which lies in the positive $z-$direction and the angular momentum of $M$ about point $\mathrm{B}$ is $L_B$. The correct statement for this system is :



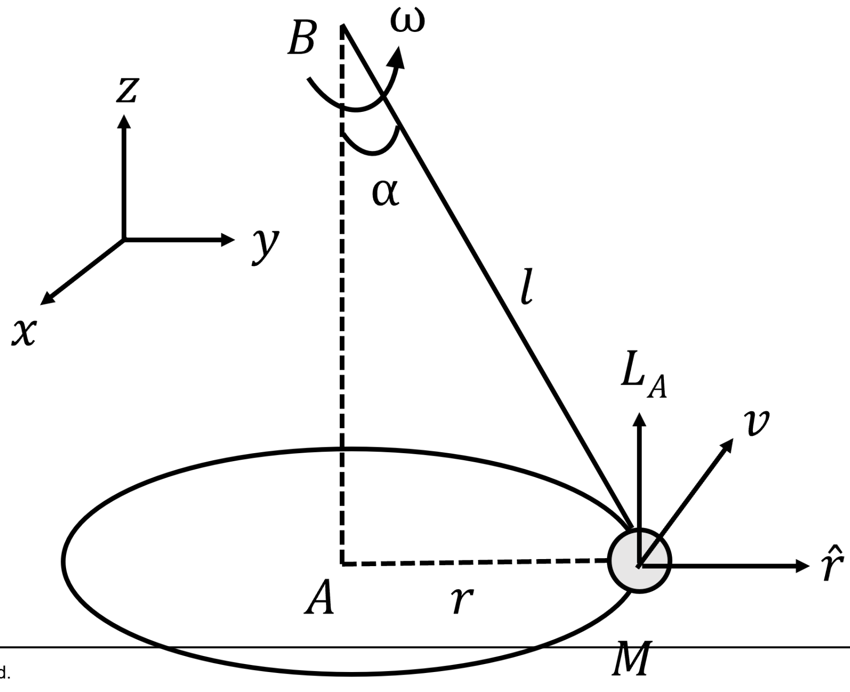

A. $L_A$ and $L_B$ are both constant in magnitude and direction.

B. $L_B$ is constant, both in magnitude and direction.

C. $L_A$ is constant, both in magnitude and direction.

D. $L_A$ is constant in direction with varying magnitude.

About point $\mathrm{A}$ :



Angular momentum is given by,

$\vec{L} = \vec{r} \times \vec{p} = M\left(\vec{r} \times \vec{v}\right)$

So,

$L_A = Mvr\sin\theta = Mvr\sin 90^\circ = Mvr$

Speed $v$ is constant as $\omega$ and $r$ are constant, $\because v = \omega r$.

Also, mass $M$ is constant.

Therefore, $L_A$ is constant in magnitude.

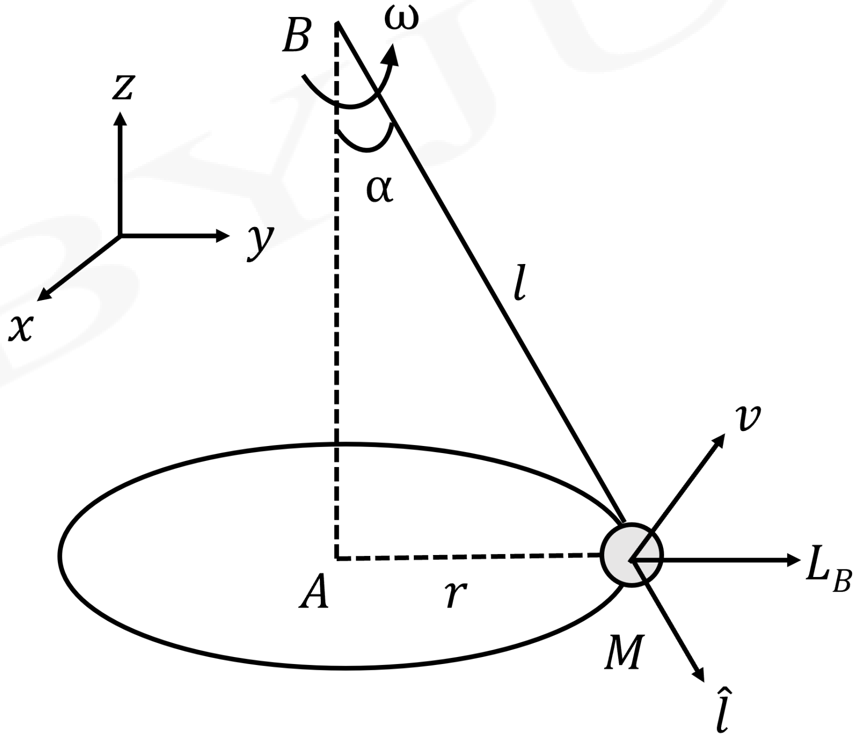
The direction of velocity $\vec{v}$ is always tangential to the path.

$\vec{r} \times \vec{v}$ points in vertically upwards direction.

So, the direction of $\overrightarrow{L_A}$ about $\mathrm{A}$ is also constant.

Hence, $L_A$ is constant, both in magnitude and direction.

About point $\mathrm{B}$ :



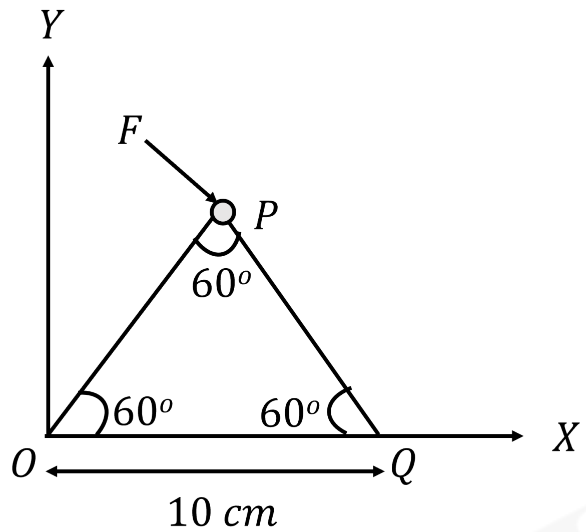

Here, $L_B = Mvl\sin\theta$

The angle between $\vec{l}$ and $\vec{v}$ is always $90^\circ$, so the magnitude is constant.

However, the direction of $\overrightarrow{L_B}$ changes continuously as the mass rotate.

Hence, option $(C)$ is the correct answer.

2. A triangular plate is shown in the figure. A force $\overrightarrow{F} = 4\hat{i} - 3\hat{j}$ is applied at point $P$. The torque acting at point $\mathrm{P}$ with respect to point $\mathrm{O}$ and point $\mathrm{Q}$ respectively are :

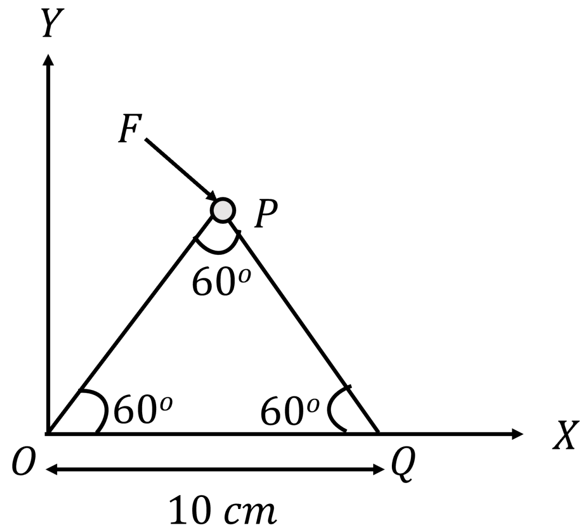



A. $15 - 20\sqrt{3}\ ;\ 15 + 20\sqrt{3}$

B. $15 + 20\sqrt{3}\ ;\ 15 - 20\sqrt{3}$

C. $-15 + 20\sqrt{3}\ ;\ 15 + 20\sqrt{3}$

D. $-15 - 20\sqrt{3}\ ;\ 15 - 20\sqrt{3}$




Given,

$\overrightarrow{F} = 4\hat{i} - 3\hat{j}$

Position vector of $\overrightarrow{F}$ about point O,

$\overrightarrow{r_1} = 10\cos 60^\circ\,\hat{i} + 10\sin 60^\circ\,\hat{j}$

$\Rightarrow \overrightarrow{r_1} = 5\,\hat{i} + 5\sqrt{3}\,\hat{j}$

Now, torque,

$\overrightarrow{\tau_1} = \overrightarrow{r_1} \times \overrightarrow{F}$

$\Rightarrow \overrightarrow{\tau_1} = \left(5\hat{i} + 5\sqrt{3}\hat{j}\right) \times \left(4\hat{i} - 3\hat{j}\right)$

$\Rightarrow \overrightarrow{\tau_1} = (-15 - 20\sqrt{3})\hat{k}$

Similarly, position vector of $\overrightarrow{F}$ about point Q,

$\overrightarrow{r_2} = -10\cos 60^\circ\,\hat{i} + 10\sin 60^\circ\,\hat{j}$

$\Rightarrow \overrightarrow{r_2} = -5\,\hat{i} + 5\sqrt{3}\,\hat{j}$

Now, torque,

$\overrightarrow{\tau_2} = \overrightarrow{r_2} \times \overrightarrow{F}$

$\Rightarrow \overrightarrow{\tau_2} = \left(-5\,\hat{i} + 5\sqrt{3}\,\hat{j}\right) \times \left(4\,\hat{i} - 3\,\hat{j}\right)$

$\Rightarrow \overrightarrow{\tau_2} = \left(15 - 20\sqrt{3}\right)\hat{k}$

3. A thin circular ring of mass $M$ and radius $r$ is rotating about its axis with an angular speed $\omega$. Two particles having mass $m$ each are now attached at diametrically opposite points. The angular speed of the ring will become:

A. $\omega\dfrac{M}{M+2m}$

B. $\omega\dfrac{M}{M+m}$

C. $\omega\dfrac{M+2m}{M}$

D. $\omega\dfrac{M-2m}{M+2m}$

External torque is zero on the system. $\tau_{net}=0$, so angular momentum is conserved.

By angular momentum conservation:
$I_i\omega_i = I_f\omega_f$

$$(MR^2)\omega = (MR^2+2mR^2)\omega_f$$

$$\omega_f = \frac{(MR^2)\omega}{MR^2+2mR^2} = \frac{M\omega}{M+2m}$$

$$\omega_f = \frac{M\omega}{M+2m}$$

4. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is $50\%$ of its translational kinetic energy. The body is :

A. Solid sphere

B. Solid cylinder

C. Hollow cylinder

D. Ring

Give:

$(KE)_R = 50\%$ of $(KE)_T = \frac{1}{2} \times (KE)_T$

$\Rightarrow \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{1}{2} mv^2$

Here, body moves without slipping, $v = R\omega$

$\therefore I = \frac{1}{2} mR^2$

And we know that, $\mathrm{MOI}$ of disc or solid cylinder is $mR^2/2$.

Hence, option $(\mathrm{B})$ is correct.

5. The moment of inertia of a square plate of side $l$ about the axis passing through one of the corner and perpendicular to the plane of the square plate is given by:

A. $\frac{Ml^2}{12}$

B. $\frac{2}{3}Ml^2$

C. $\frac{Ml^2}{6}$

D. $Ml^2$

Moment of inertia of the square plate about an axis passing through its center of mass and perpendicular to its plane is :

$I_{com} = \frac{Ml^2}{6}$

According to the parallel axis theorem, moment of inertia of the plate about the axis passing from the corner and perpendicular to its plane is :

$I = I_{com} + Mx^2$

Here, the distance between this axis from com is equal to half the length of the diagonal of the square.

$i.e. x = \frac{l\sqrt{2}}{2} = \frac{l}{\sqrt{2}}$

$\Rightarrow I = \frac{Ml^2}{6} + \frac{Ml^2}{2}$

Or, $I = \frac{2Ml^2}{3}$

6. Two discs have moments of inertia $I_1 \; and \; I_2$ about their respective axes perpendicular to the plane and passing through the center. They are rotating with angular speeds, $\omega_1 \; and \; \omega_2$ respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by :

A. $\dfrac{I_1 I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$

B. $\dfrac{I_1 I_2}{(I_1+I_2)}(\omega_1-\omega_2)^2$

C. $\dfrac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$

D. $\dfrac{(I_1+I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$

Let their final common angular velocity is $\omega$.
From conservation of angular momentum we have :

$$I_1\omega_1 + I_2\omega_2 = (I_1+I_2)\omega$$

$$\Rightarrow \omega = \frac{(I_1\omega_1 + I_2\omega_2)}{(I_1+I_2)} \ldots (1)$$

Initial kinetic energy of the system :

$$KE_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

Final kinetic energy of the system :

$$KE_f = \frac{1}{2}(I_1+I_2)\omega^2$$

Loss in kinetic energy ;

$$\Delta KE = KE_i - KE_f = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}(I_1+I_2)\omega^2$$

From $(1)$ :

$$\Delta KE = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1+I_2)}$$

$$\Rightarrow \Delta KE = \frac{1}{2}\left(\frac{I_1 I_2}{I_1+I_2}\right)(\omega_1-\omega_2)^2$$

7. Mass per unit area of a circular disc of radius $'a'$ depends on the distance $r$ from its centre, as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre, is:

A. $2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5}\right)$

B. $2\pi a^4 \left(\frac{aA}{4} + \frac{B}{5}\right)$

C. $\pi a^4 \left(\frac{A}{4} + \frac{aB}{5}\right)$

D. $2\pi a^4 \left(\frac{A}{4} + \frac{B}{5}\right)$

Given,
mass per unit area of circular disc, $\sigma = A + Br$

Consider a small elemental ring of thickness $dr$ at a distance $r$ from the centre,

Area of the element $= 2\ \pi r dr$
Mass of the element, $dm = \sigma 2\pi r dr$

The moment of inertia of the ring about an axis, perpendicular to the plane and passing through its centre, is given by,

$$I = \int dmr^2 = \int \sigma 2\pi r dr . r^2$$

$$\Rightarrow I = 2\pi \int_0^a (A + Br) r^3 dr$$

$$\Rightarrow I = 2\pi \left[\frac{Aa^4}{4} + \frac{Ba^5}{5}\right]$$

$$\Rightarrow I = 2\pi a^4 \left[\frac{A}{4} + \frac{Ba}{5}\right]$$

Hence, option $(A)$ is correct.

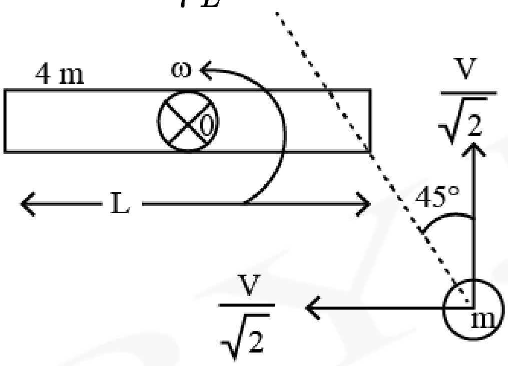
8. Consider a uniform rod of mass $M = 4m$ and length $L$ pivoted about its centre. A mass $m$ moving with a velocity $V$ making an angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod, and sticks to it. The angular speed of the rod-mass system just after the collision is:

A. $\frac{3}{7\sqrt{2}}\frac{V}{L}$

B. $\frac{3}{7}\frac{V}{L}$

C. $\frac{3\sqrt{2}}{7}\frac{V}{L}$

D. $\frac{4}{7}\frac{V}{L}$




Angular momentum of the rod-mass system about point $O$ is

$$\Rightarrow L = (mg \times 0) + \frac{mV}{\sqrt{2}} \times \frac{L}{2} = \frac{mVL}{2\sqrt{2}}$$

Let, $I$ be the moment of inertia about $O$ of the rod-mass system, and, $\omega$ be the angular speed just after collision.

$$I = \frac{4mL^2}{12} + \frac{mL^2}{4} = \frac{7}{12}mL^2$$

$$L = I\omega$$

$$\Rightarrow \frac{mVL}{2\sqrt{2}} = \frac{7}{12}mL^2 \times \omega$$

$$\therefore \omega = \frac{6V}{7\sqrt{2}L} = \frac{3\sqrt{2}V}{7L}$$

Hence, option $(C)$ is correct.

9. A uniform sphere of mass $500\ \mathrm{g}$ rolls without slipping on a plane horizontal surface with its centre moving at a speed of $5.00\ \mathrm{cm/s}$. Its kinetic energy is

A. $8.75 \times 10^{-4}\ \mathrm{J}$

B. $8.75 \times 10^{-3}\ \mathrm{J}$

C. $6.25 \times 10^{-4}\ \mathrm{J}$

D. $1.13 \times 10^{-3}\ \mathrm{J}$

$K.E$ of the sphere = translational $K.E + \text{rotational K.E}$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Where, I = moment of inertia,
$\omega$ = Angular, velocity of rotation
m = mass of the sphere
v = linear velocity of centre of mass of sphere

$$\because \text{ Moment of intertia of sphere } I = \frac{2}{5}mR^2$$

$$\therefore\ K.E = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \omega^2$$

$$\Rightarrow K.E = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v}{R}\right)^2 \left(\because \omega = \frac{v}{R}\right)$$

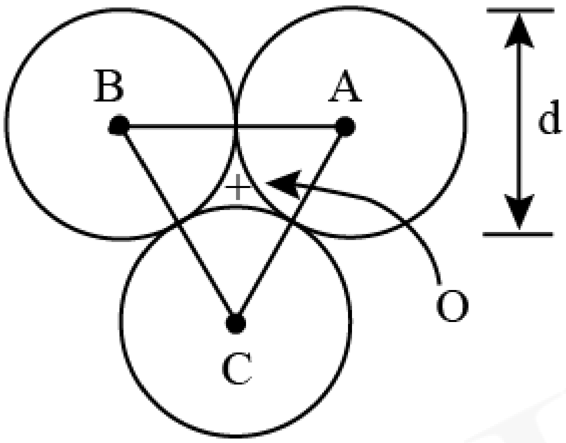
$$\Rightarrow KE = \frac{1}{2}\left(\frac{2}{5}mR^2 + mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow KE = \frac{1}{2}mR^2 \times \frac{7}{5} \times \frac{v^2}{R^2} = \frac{7}{10} \times \frac{1}{2} \times \frac{25}{10^4}$$

$$\Rightarrow KE = \frac{35}{4} \times 10^{-4}\mathrm{J}$$

$$\Rightarrow KE = 8.75 \times 10^{-4}\ \mathrm{J}$$

10. Three solid spheres each of mass $m$ and diameter $d$ are stuck together such that the lines connecting the centres form an equilateral triangle of side of length $d$. The ratio $\frac{I_0}{I_A}$ of moment of inertia $I_0$ of the system about an axis passing the centroid and about centre of any of the spheres $I_A$ and perpendicular to the plane of the triangle, is:

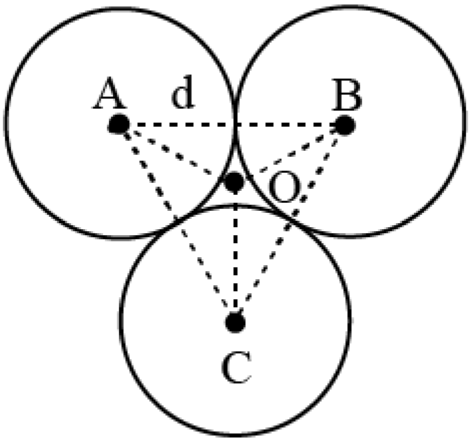



A. $\frac{13}{23}$

B. $\frac{15}{13}$

C. $\frac{23}{13}$

D. $\frac{13}{15}$




Moment of inertia of a sphere, about an axis passing through $O$, is,

$$I_1 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 + m(AO)^2$$

and $AO = \dfrac{d}{\sqrt{3}}$

Moment of inertia of the system about $O$, is

$$I_0 = 3I_1 = 3\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{\sqrt{3}}\right)^2\right]$$

$$\Rightarrow I_0 = \frac{13}{10}md^2$$

Similarly, Moment of inertia of the system about $A$, is

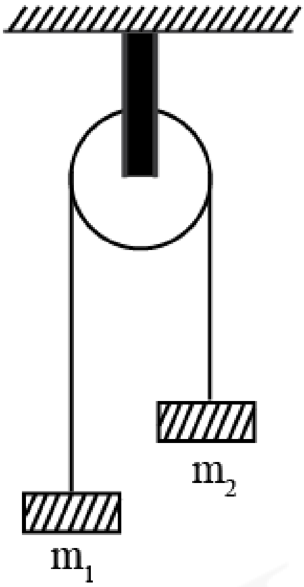
$$I_A = 2\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + md^2\right] + \frac{2}{5}m\left(\frac{d}{2}\right)^2$$

$$\Rightarrow I_A = \frac{23}{10}md^2$$

$$\therefore \frac{I_0}{I_A} = \frac{\frac{13}{10}md^2}{\frac{23}{10}md^2} = \frac{13}{23}$$

Hence, $(A)$ is the correct answer.

11. A uniformly thick wheel, with moment of inertia $I$ and radius $R$, is free to rotate about its centre of mass (see fig.). A massless string is wrapped over its rim and two blocks of masses $m_1$ and $m_2 > m_2$ are attached to the ends of the string. The system is released from rest. The angular speed of the wheel, when $m_1$ descents through a distance $h$, is




A. $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{1/2}$

B. $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{1/2}$

C. $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{1/2} gh$

D. $\left[\frac{(m_1 + m_2)}{(m_1 + m_2)R^2 + I}\right]^{1/2} gh$

Using the principal of conservation of energy,
$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$

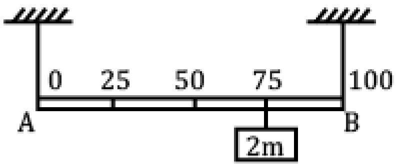
As, $v = \omega R$

$$\Rightarrow (m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega^2 R^2) + \frac{1}{2}I\omega^2$$

$$\Rightarrow (m_1 - m_2)gh = \frac{\omega^2}{2}[(m_1 + m_2)R^2 + I]$$

$$\Rightarrow \omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

Hence, $(A)$ is the correct answer.

12. 


Shown in the figure is a rigid and uniform one meter long rod $AB$ held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass $m$ and has another weight of mass $2m$ hung at a distance of $75\ \mathrm{cm}$ from $A$. The tension in the string at $A$ is :

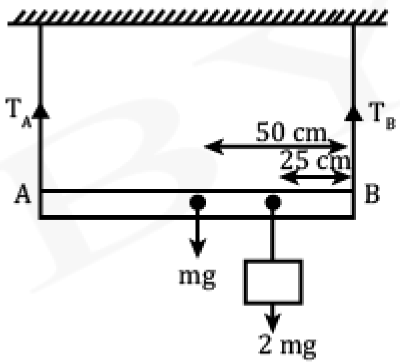
A. $0.5mg$

B. $2mg$

C. $0.75mg$

D. $1mg$

Net torque, $\tau_{net}$ about $B$ is zero at equilibrium,




$\Rightarrow (T_A \times 100) - (mg \times 50) - (2mg \times 25) = 0$

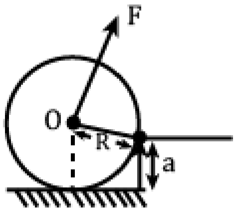
$\Rightarrow T_A \times 100 = 100mg$

Therefore, Tension in the string at $A$

$T_A = 1mg$

Hence, option $(D)$ is correct.

13. A uniform cylinder of mass $M$ and radius $R$ is to be pulled over a step of height $a$ $(a < R)$ by applying a force $F$ at its centre $O$ perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of $F$ required is :

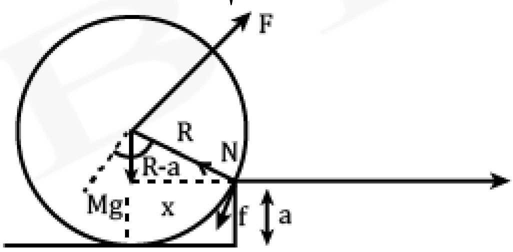



A. $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ ✓

B. $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

C. $Mg\frac{a}{R}$

D. $Mg\sqrt{1-\frac{a^2}{R^2}}$




With respect to the point of contact on the step,

For step up,

$\tau_F \geq \tau_{mg}$

$\Rightarrow F \times R \geq Mg \times x$

From the figure, $x = \sqrt{R^2-(R-a)^2}$

$\Rightarrow F_{min} = \frac{Mg}{R} \times \sqrt{R^2-(R-a)^2}$

$\Rightarrow F_{min} = Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

Hence, option $(A)$ is correct.

14. Moment of inertia of a cylinder of mass $M$, length $L$ and radius $R$ about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder to be made for a given mass of a material, the ratio $\frac{L}{R}$ for it to have minimum possible $I$ is-

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\sqrt{\frac{3}{2}}$

D. $\sqrt{\frac{2}{3}}$

Given, $I = \frac{MR^2}{4} + \frac{ML^2}{12}$

$$\because \quad V = \pi R^2 L \quad \Rightarrow \quad R^2 = \frac{V}{\pi L}$$

$$\Rightarrow I = \frac{M}{4} \times \frac{V}{\pi L} + \frac{ML^2}{12} = \frac{MV}{4\pi L} + \frac{ML^2}{12}$$

For $I$ to be minimum, $\frac{dI}{dL} = 0$

$$\frac{dI}{dL} = -\frac{MV}{4\pi L^2} + \frac{M \times 2L}{12} = 0$$

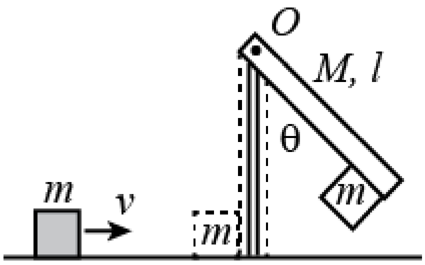
$$\Rightarrow \left(\frac{R}{L}\right)^2 = \frac{2}{3}$$

$$\therefore \frac{L}{R} = \sqrt{\frac{3}{2}}$$

Hence, $(C)$ is the correct answer.

15. A block of mass $m = 1\ \mathrm{kg}$ slides with velocity $v = 6\ \mathrm{ms}^{-1}$ on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about $\mathrm{O}$ and swings as a result of the collision, making angle $\theta$ before momentarily coming to rest. If the rod has mass $M = 2\ \mathrm{kg}$ and length $l = 1\ \mathrm{m}$, the value of $\theta$ is approximately-

[Take $g = 10\ \mathrm{ms}^{-2}$]

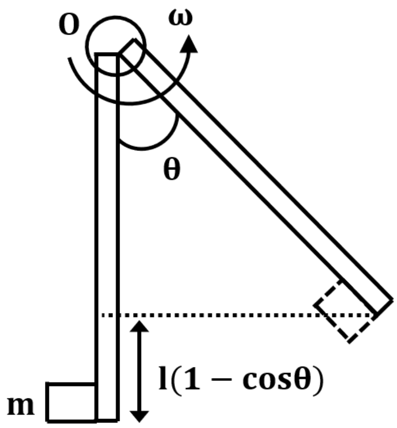



A. $63^\circ$

B. $55^\circ$

C. $69^\circ$

D. $49^\circ$




Using conservation of angular momentum, about point O is,

$$mvl = \left(ml^2 + \frac{Ml^2}{3}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{3mv}{3ml + Ml}$$

Now using energy conservation, after collision

$$\frac{1}{2}I_{rod+mass}\omega^2 = Mg\frac{l}{2}(1-\cos\theta) + mgl(1-\cos\theta)$$

$$\Rightarrow \frac{1}{2}\left(ml^2 + \frac{Ml^2}{3}\right)\left(\frac{3mv}{3ml+Ml}\right)^2 = l(1-\cos\theta)\left(\frac{Mg}{2} + m\right)$$

$$\Rightarrow \frac{l}{2}\times\frac{3mv^2}{3ml+Ml} = \frac{l}{2}(1-\cos\theta)\times(2mg+Mg)$$

On putting the values we get,

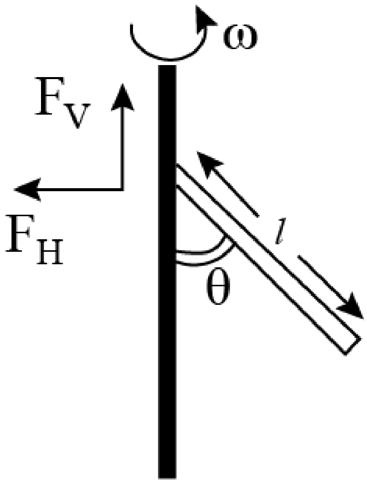
$$\frac{108}{200} = 1 - \cos\theta$$

$$\cos\theta = 0.46$$

$$\Rightarrow \quad \theta = 63^\circ$$

Hence, $(A)$ is the correct answer.

16.




A uniform rod of length $l$ is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed $\omega$ the rod makes an angle $\theta$ with it (see figure). To find $\theta$, equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass $(CM)$ to the torque provided by the horizontal and vertical forces $F_H$ and $F_V$ about the $CM$. The value of $\theta$ is then such that:

A. $\cos\theta = \frac{2g}{3l\omega^2}$

B. $\cos\theta = \frac{g}{2l\omega^2}$

C. $\cos\theta = \frac{g}{l\omega^2}$

D. $\cos\theta = \frac{3g}{2l\omega^2}$

Vertical force, $F_V = mg$

Horizontal force $=$ Centripetal force

$$\Rightarrow F_H = m\omega^2 \frac{l}{2}\sin\theta$$

Torque due to vertical force

$$\tau_V = mg\frac{l}{2}\sin\theta$$

Torque due to horizontal force

$$\tau_H = m\omega^2 \frac{l}{2}\sin\theta \frac{l}{2}\cos\theta$$

$$\tau_{net} = \frac{dL}{dt}$$

$$\Rightarrow mg\frac{l}{2}\sin\theta - m\omega^2\frac{l}{2}\sin\theta\frac{l}{2}\cos\theta = \frac{ml^2}{12}\omega^2\sin\theta\cos\theta$$

$$\Rightarrow \cos\theta = \frac{3}{2}\frac{g}{\omega^2 l}$$

Hence, option $(D)$ is correct.

17. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$, made of the same material. If the ratio of their moments of inertia $I_1$ and $I_2$, respectively, about their axes is $I_1 : I_2 = 1 : 16$, then the value of $\alpha$ is :

A. $2\sqrt{2}$

B. $\sqrt{2}$

C. $2$

D. $4$

Let $\rho$ be the density of the discs and $t$ is their thickness.

Moment of inertia of the disc is given by,

$$I = \frac{MR^2}{2} = \frac{[\rho(\pi R^2)t]R^2}{2} = \frac{\rho\pi t R^4}{2}$$

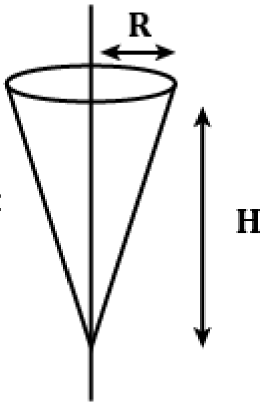
$$\Rightarrow\ I \propto R^4 \quad \because\ \rho \text{ and } t \text{ are same for the discs}$$

$$\therefore \frac{I_2}{I_1} = \left(\frac{R_2}{R_1}\right)^4 = \alpha^4 = \frac{16}{1}$$

$$\Rightarrow \alpha = 2$$

Hence, $(C)$ is the correct answer.

18. Shown in the figure, is a hollow ice-cream cone (it is open at the top). If its mass is $M$, radius of its top is $R$ and height $H$ then, its moment of inertia about its axis is:




A. $\frac{MR^2}{2}$

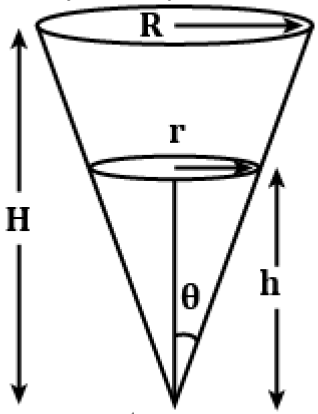
B. $\frac{M(R^2 + H^2)}{4}$

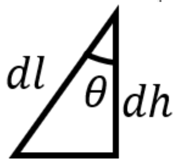
C. $\frac{MH^2}{3}$

D. $\frac{MR^2}{3}$

Hollow ice-cream cone can be assumed as stack of rings having different radius, so

$I = \int dI = \int dm(r^2) \ldots . (i)$




From diagram,

$\frac{r}{h} = \tan\theta = \frac{R}{H}$ or $r = \frac{R}{H}h \ldots . (ii)$

and, $\cos\theta = \frac{dh}{dl} \ldots \ldots (iii)$

Now,

$dm = \frac{M}{\pi R l} \times (2\pi r dl)$, where $l$ is slant height of cone given by $l = \sqrt{H^2 + R^2}$

so, from eq $(i)$ we have

$$I = \int \frac{M}{\pi R l} \times (2\pi r dl).r^2 = \int \frac{2Mr^3 dl}{Rl}$$

$$\Rightarrow I = \int_0^H \frac{2M(h\tan\theta)^3 dh}{R\sqrt{H^2 + R^2}\cos\theta}$$

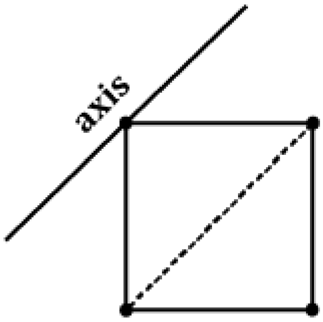
$$\Rightarrow I = \left(\frac{2M\tan^3\theta}{R\sqrt{H^2 + R^2}\cos\theta}\right)\int_0^H h^3 dh$$

$$\Rightarrow I = \left(\frac{2M\tan^3\theta}{R\sqrt{H^2 + R^2}\cos\theta}\right) \times \frac{H^4}{4}$$

$$\Rightarrow I = \left[\frac{2M\left(\frac{R}{H}\right)^3}{\left(R\sqrt{H^2 + R^2}\right) \times \left(\frac{H}{\sqrt{H^2 + R^2}}\right)}\right] \times \frac{H^4}{4}$$

$$\Rightarrow I = \frac{MR^2}{2}$$

19. Four point masses, each of mass $m$ are fixed at the corners of a square of side $l$. The square is rotating with angular frequency $\omega$, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is:



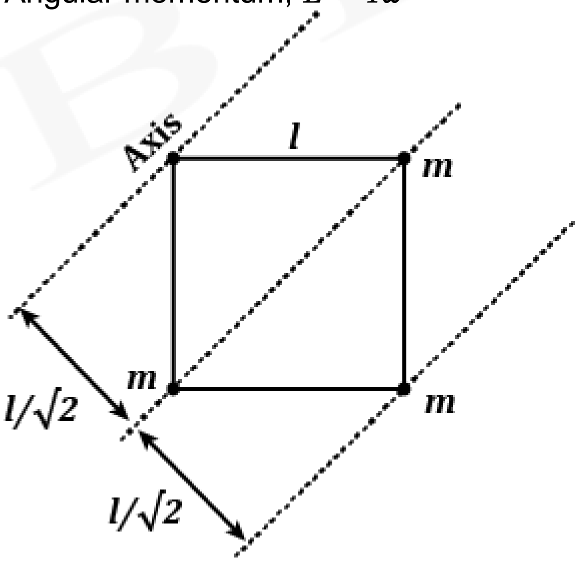

A. $ml^2\omega$

B. $4\,ml^2\omega$

C. $3\,ml^2\omega$

D. $2\,ml^2\omega$

Angular momentum, $L = I\omega$




Moment of inertia of the square

about the given axis will be given as,

$$I = m(0)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 \times 2 + m(\sqrt{2}l)^2$$

$$= \frac{2ml^2}{2} + 2ml^2 = 3ml^2$$

So, angular momentum will be
$L = I\omega = 3ml^2\,\omega$

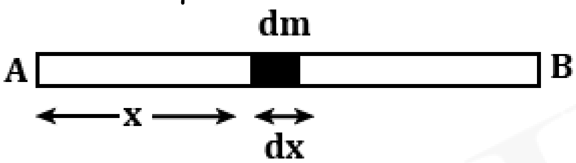
20. The linear mass density of a thin rod $AB$ of length $L$ varies from $A$ to $B$ as $\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where $x$ is the distance from $A$. If $M$ is the mass of the rod then its moment of inertia about an axis passing through $A$ and perpendicular to the rod is :

A. $\frac{5}{12}ML^2$

B. $\frac{7}{18}ML^2$

C. $\frac{2}{5}ML^2$

D. $\frac{3}{7}ML^2$




Mass of the small element of the rod
$dm = \lambda dx$

Moment of inertia of small element,
$dI = dm\, x^2 = \lambda_0\left(1+\frac{x}{L}\right).x^2 dx$

Moment of inertia of the complete rod can be obtained by integration

$$I = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right) dx$$

$$= \lambda_0 \left| \frac{x^3}{3} + \frac{x^4}{4L} \right|_0^L = \lambda_0\left[\frac{L^3}{3} + \frac{L^3}{4}\right]$$

$$\Rightarrow I = \frac{7\lambda_0 L^3}{12} \quad .......(i)$$

Mass of the thin rod,

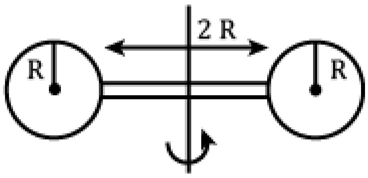
$$M = \int_0^L \lambda dx = \int_0^L \lambda_0\left(1+\frac{x}{L}\right) dx = \frac{3\lambda_0 L}{2}$$

$$\therefore \lambda_0 = \frac{2M}{3L}$$

$$\therefore I = \frac{7}{12}\left(\frac{2M}{3L}\right) L^3 \Rightarrow I = \frac{7}{18}ML^2$$

Hence, option $(B)$ is correct.

21. Two identical spherical balls of mass $M$ and radius $R$ each are stuck on two ends of a rod of length $2R$ and mass $M$ (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is




A. $\frac{137}{15}MR^2$

B. $\frac{17}{15}MR^2$

C. $\frac{209}{15}MR^2$

D. $\frac{152}{15}MR^2$

For ball, using parallel axes theorem,for ball moment of inertaia,

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2 = \frac{22}{5}MR^2$$

For two balls $2I_{ball} = 2 \times \frac{22}{5}MR^2$

For rod

$$I_{rod} = \frac{M(2R)^2}{12} = \frac{MR^2}{3}$$

Moment of inertia of the system is

$$I_{system} = 2I_{ball} + I_{rod}$$

$$\frac{44}{5}MR^2 + \frac{MR^2}{3} = \frac{137}{15}MR^2$$

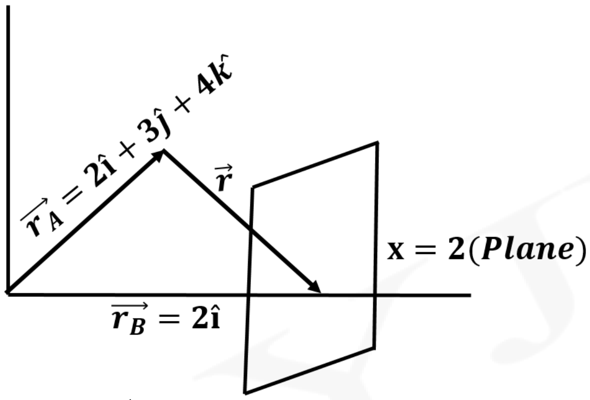
Topic : Rotational Motion

1. A force $\overrightarrow{F} = 4\hat{i} + 3\hat{j} + 4\hat{K}$ is applied on an intersection point of $x = 2$ plane and $x - \text{axis}$. The magnitude of torque of this force about a point $(2, 3, 4)$ is (Round off to the Nearest Integer)

Accepted Answers

20 20.0

Solution:




Given: $\overrightarrow{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

Position of intersection point where force is applied, $\overrightarrow{r}_B = 2\hat{i}$

Point about which torque to be calculated, $\overrightarrow{r}_A = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Now, from figure $\overrightarrow{r}_A + \overrightarrow{r} = \overrightarrow{r}_B$

$\overrightarrow{r} = \overrightarrow{r}_B - \overrightarrow{r}_A$

$\overrightarrow{r} = (-3\hat{j} - 4\hat{k})$

We know that,

$\overrightarrow{\tau} = \overrightarrow{r} \times \overrightarrow{F}$

$$\overrightarrow{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$= -16\hat{j} - 12\hat{k} = \sqrt{(-16)^2 + (-12)^2}$

$= \sqrt{400} = 20$

Correct Answer: $20$

2. The angular speed of truck wheel is increased from $900 \text{ rpm}$ to $2460 \text{ rpm}$ in $26$ seconds. The number of revolutions by the truck wheel during this time is ______.

(Assuming the acceleration to be uniform)

Accepted Answers

728 728.0 728.00

Solution:

Given :

$$\omega_f = 2460 \times \frac{2\pi}{60} = 82\pi \text{ rad/s}$$

$$\omega_i = 900 \times \frac{2\pi}{60} = 30\pi \text{ rad/s}$$

$$t = 26 \text{ s}$$

We know that,

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{82\pi - 30\pi}{26} = 2\pi \text{ rad/s}^2$$

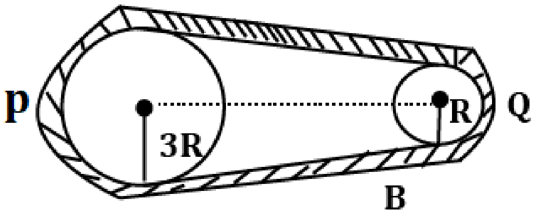
Angular displacement of the wheel is given by,

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \theta = 30\pi \times 26 + \frac{1}{2} \times 2\pi \times 26^2 = 1456\pi$$

Number of revolution $= \dfrac{\theta}{2\pi} = \dfrac{1456\pi}{2\pi} = 728$

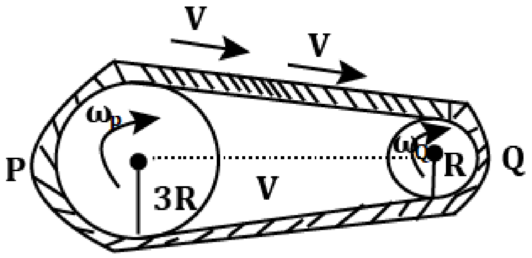
3. In the given figure , two wheels $P$ and $q$ are connected by a belt $B$. The radius of $P$ is three times as that of $Q$. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_P}{I_Q}\right)$ will be $x : 1$. The value of $x$ will be




Accepted Answers

9 9.0 9.00

Solution:




Let $\omega_P$ and $\omega_Q$ be the angular velocity of wheels $P$ and $Q$ respectively, and since, rotational kinetic energy is same for them, the tangential velocity will also be same.

$\Rightarrow \omega_P(3R) = \omega_Q(R)$

$\Rightarrow \frac{\omega_Q}{\omega_P} = \frac{3}{1} \ldots (1)$

Also, from the relation of rotational kinetic energy,

$\frac{1}{2} I_P (\omega_P)^2 = \frac{1}{2} I_Q (\omega_Q)^2$

$\frac{I_P}{I_Q} = \left(\frac{\omega_Q}{\omega_P}\right)^2 = \frac{9}{1}$[from eq $(1)$]

Hence, value of $x$ will be $9$.

4. A body rotating with an angular speed of $600 \mathrm{~rpm}$ is uniformly accelerated to $1800 \mathrm{~rpm}$ in $10 \mathrm{~sec}$. The number of rotations made in the process is ____

Accepted Answers

200

Solution:

$$\omega=\frac{2 \pi N}{60}$$

$$\omega_{0}=\frac{2 \pi \times 600}{60}=20 \pi \mathrm{~rad} / \mathrm{s}$$

$$\omega_{f}=\frac{2 \pi \times 1800}{60}=60 \pi \mathrm{~rad} / \mathrm{s}$$

Using equation,
$\omega_{f}=\omega_{0}+\alpha t$

$$60 \pi=20 \pi+(\alpha \times 10)$$

$$\Rightarrow \alpha=4 \pi \mathrm{~rad} / \mathrm{s}^{2}$$

Using, $\theta=\omega_{0} t+\frac{1}{2} \alpha t^{2}$

$$\theta=(20 \pi \times 10)+\frac{1}{2} \times 4 \pi \times 10^{2}=400 \pi \mathrm{~rad}$$

So, number of rotations will be,
$n=\frac{400 \pi}{2 \pi}=200$

5. A solid disc of radius $20 \mathrm{~cm}$ and mass $10 \mathrm{~kg}$ is rotating with an angular velocity of $600 \mathrm{rpm}$, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc to rest in $10 \mathrm{~s}$ is $\pi \times 10^{-1} \mathrm{~Nm}$.

Accepted Answers

4

Solution:

Torque is defined as,

$$\tau=\frac{\Delta L}{\Delta t}=\frac{I\left(\omega_f-\omega_i\right)}{\Delta t}$$

Here, for the given axis of rotation, $I=\frac{M R^2}{2}$

$$\tau=\frac{\left(\frac{M R^2}{2}\right) \times[0-\omega]}{\Delta t}$$

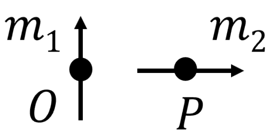
Converting, $1 \mathrm{rpm}=\frac{2 \pi}{60} \mathrm{rad} \mathrm{~s}^{-1}$

$$\Rightarrow \tau=\frac{10 \times\left(20 \times 10^{-2}\right)^2}{2} \times \frac{600 \times 2 \pi}{60 \times 10}$$

$$\Rightarrow \tau=0.4 \pi=4 \pi \times 10^{-1} \mathrm{~Nm}$$

Hence, $4$ is the correct answer.

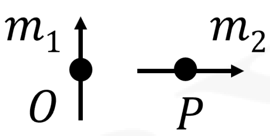
6. Two short magnetic dipoles, $m_1$ and $m_2$ each having magnetic moment of $1 \text{ A-m}^2$ are placed at points $O$ and $P$ respectively. The distance between $OP$ is $1 \text{ m}$. The torque experienced by the magnetic dipole $m_2$ due to the presence of $m_1$ is _______ $\times 10^{-7}$ N-m.




Accepted Answers

1 1.0 1.00

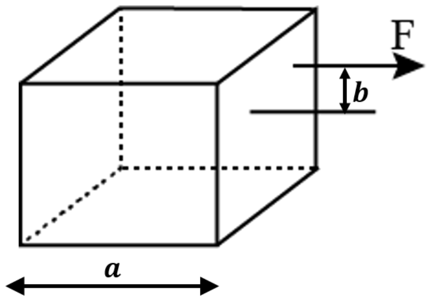
Solution:




$$\vec{\tau} = \vec{m}_2 \times \vec{B}_1$$

$$\Rightarrow \tau = m_2 B_1 \sin\theta = m_2 \times \frac{\mu_0}{4\pi}\frac{m_1}{r^3} \times \sin\theta$$

$$\Rightarrow \tau = 1 \times \frac{\mu_0}{4\pi}\frac{1}{1^3} \times \sin 90^\circ = 1 \times 10^{-7} \text{ N-m}$$

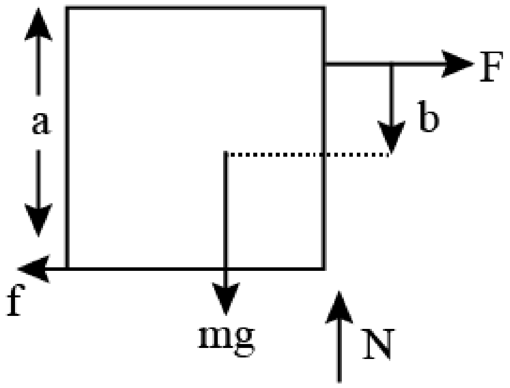
7. 


Consider a uniform cubical box of side $'a'$ on a rough floor that is to be moved by applying minimum possible force F at a point $'b'$ above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \dfrac{b}{a}$ for the box not to topple before moving is .

Accepted Answers

50

Solution:




For the box to slide
$F=\mu mg=0.4mg$

For the box not to topple, torque due to $F$ should be less than or equal to torque due to $mg$.

$$\Rightarrow F\left(\frac{a}{2}+b\right)\leq mg\frac{a}{2}$$

$$\Rightarrow 0.4mg\left(\frac{a}{2}+b\right)\leq mg\frac{a}{2}$$

$$\Rightarrow 0.2a+0.4b\leq 0.5a$$

$$\Rightarrow \frac{b}{a}\leq\frac{3}{4}$$

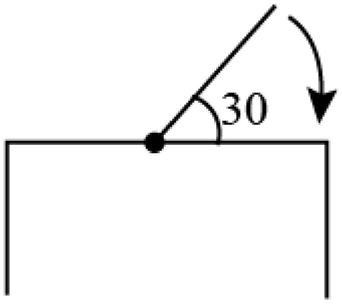
$$\Rightarrow b\leq 0.75\ a$$

But, here, for the box, the maximum value of $b$ can be equal to $0.5a$.

$$\Rightarrow \frac{100b}{a}=50$$

Hence, $50$ is the correct answer.

8. One end of a straight, uniform $1 \mathrm{~m}$ long bar is pivoted on a horizontal table. It is released from rest, when it makes an angle 30° from the horizontal (see figure). Its angular speed , when it hits the table, is given as $\sqrt{n} \mathrm{~s}^{-1}$, where $n$ is an integer. The value of $n$ is .




Accepted Answers

15 15.0 15.00

Solution:

Here, length of bar, $1=1 \mathrm{~m}$

and, angle, $\theta=30^{\circ}$

Now, $\Delta PE=\Delta KE$

$$mgh=\frac{1}{2} I \omega^{2}$$

$$\Rightarrow(mg) \frac{l}{2} \sin 30^{\circ}=\frac{1}{2}\left(\frac{m l^{2}}{3}\right) \omega^{2}$$

$$\Rightarrow g \times \frac{1}{4}=\frac{l}{6} \times \omega^{2}$$

$$\Rightarrow \omega=\sqrt{\frac{3 g}{2 l}}=\sqrt{\frac{3 \times 10}{2 \times 1}}=\sqrt{15} \mathrm{~rad} / \mathrm{s}$$

$\therefore n=15$

9. A person of $80\ \mathrm{kg}$ mass is standing on the rim of a circular platform of mass $200\ \mathrm{kg}$ rotating about its axis at $5$ revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _____.

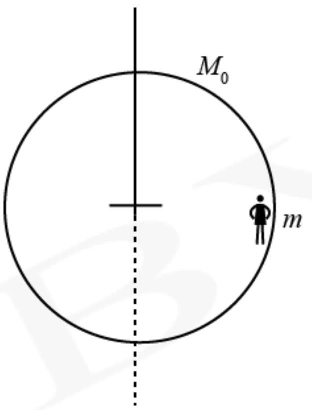
Accepted Answers

9 9.0

Solution:

Let, $M_0 = 200\ \mathrm{kg}\ ;\ m = 80\ \mathrm{kg}$

$\omega_1 = 5\ \mathrm{rpm}$




Using conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$

$$\left(\frac{M_0R^2}{2} + mR^2\right)\omega_1 = \left(\frac{M_0R^2}{2} + m(0)^2\right)\omega_2$$

$$\left(\frac{200R^2}{2} + 80R^2\right)\times 5 = \left(\frac{200R^2}{2} + 80(0)^2\right)\omega_2$$

$\Rightarrow\ \ \omega_2 = 9\ \mathrm{rpm}$