



UNITS, DIMENSION, MEASUREMENTS AND PRACTICAL PHYSICS

- **Fundamental or base quantities** : The quantities which do not depend upon other quantities for their complete definition are known as fundamental or base quantities. e.g. length, mass, time, etc.
- **Derived quantities** : The quantities which can be expressed in terms of the fundamental quantities are known as derived quantities . e.g. Speed, volume, acceleration, force, pressure, etc.
- **Units of physical quantities**: The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the unit of that quantity.
$$\text{Physical Quantity} = \text{Numerical Value} \times \text{Unit}$$

Systems of Units

Sr. No.	MKS	CGS	FPS	MKSQ	MKSA
(i)	Length (m)	Length (cm)	Length (ft)	Length (m)	Length (m)
(ii)	Mass (kg)	Mass (g)	Mass (pound)	Mass (kg)	Mass (kg)
(iii)	Time (s)	Time (s)	Time (s)	Time (s)	Time (s)

Fundamental Quantities in S.I. System and their units

Sr. No.	Physical Quantity	Name of Unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	electric Current	ampere	A
6.	Luminous Intensity	candela	cd
7.	Amount of Substance	mole	mol

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Base Quantity	SI Units		
	Name	Symbol	Definition
Length	Meter	m	The meter is the length of the path travelled by light in vacuum during a time interval of $1/(299,792,458)$ of a second (1983)
Mass	Kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures, at Sèvres, near Paris, France. (1889)
Time	Second	s	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom (1967)
Electric Current	ampere	A	the ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	Kelvin	K	The kelvin, is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water. (1967).
Amount of Substance	Mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	Candela	Cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of $1/683$ watt per steradian (1979).

• Supplementary Units:

Radian (rad) - for measurement of plane angle

Steradian (sr) - for measurement of solid angle

• Dimensional Formula :

Relation which express physical quantities in terms of appropriate powers of fundamental units.

• Use of dimensional analysis :

→ To check the dimensional correctness of a given physical relation

→ To derive relationship between different physical quantities

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→ To convert units of a physical quantity from one system to another

$$n_1 u_1 = n_2 u_2 \Rightarrow n_2 = n_1 \frac{u_1}{u_2} = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c \text{ where } u = M^a L^b T^c$$

• Limitations of this method :

- In mechanics the formula for a physical quantity depending on more than three other physical quantities cannot be derived. It can only be checked.
- This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + \frac{1}{2}at^2$ also can't be derived.
- The relation derived from this method gives no information about the dimensionless constants.
- If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.
- It gives no information whether a physical quantity is a scalar or a vector.

SI PREFIXES

The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain powers of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^9	giga	G	10^{-6}	micro	μ
10^6	mega	M	10^{-9}	nano	n
10^3	kilo	k	10^{-12}	pico	p
10^2	hecto	h	10^{-15}	femto	f
10^1	deca	da	10^{-18}	atto	a

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Units of important Physical Quantities

Physical Quantity	Unit	Physical Quantity	Unit
Angular Acceleration	Rad-s ⁻²	Frequency	Hertz (Hz)
Moment of inertia	Kg – m ²	Resistance	Kgm ² A ⁻² s ⁻³
Self-inductance	Henry (H)	Surface tension	Newton/m
Magnetic flux	Weber (Wb)	Universal gas Constant	Joule K ⁻¹ mol ⁻¹
Pole strength	A-m	Dipole moment	Coulomb-meter
Viscosity	Poise	Stefan constant	Watt m ⁻² K ⁻⁴
Reactance	Ohm	Permittivity of Free space (ϵ_0)	Colulomb ² /N-m ²
Specific heat	J/kg°C	Permeability of Free space (μ_0)	Weber/A-m
Strength of Magnetic field	Newton A ⁻¹ m ⁻¹	Planck's constant	Joule-sec
Astronomical Distance	Parsec	Entropy	J/K

Dimensions of important Physical Quantities

Physical Quantity	Dimensions	Physical Quantity	Dimensions
Momentum	M ¹ L ¹ T ⁻¹	Capacitance	M ⁻¹ L ⁻² T ⁴ A ²
Calorie	M ¹ L ² T ⁻²	Modulus of Rigidity	M ¹ L ⁻¹ T ⁻²
Latent heat Capacity	M ⁰ L ² T ⁻²	Magnetic Permeability	M ¹ L ¹ T ⁻² A ⁻²
Self inductance	M ¹ L ² T ⁻² A ⁻²	Pressure	M ¹ L ⁻¹ T ⁻²
Coefficient of Thermal Conductivity	M ¹ L ¹ T ⁻³ K ⁻¹	Planck's constant	M ¹ L ² T ⁻¹
Power	M ¹ L ² T ⁻³	Solar constant	M ¹ L ⁰ T ⁻³
Impulse	M ¹ L ¹ T ⁻¹	Magnetic flux	M ¹ L ² T ⁻² A ⁻¹
Hole mobility in a Semi conductor	M ⁻¹ L ⁰ A ¹ T ²	Current density	M ⁰ L ⁻² T ⁰ A ¹
Bulk modulus of elasticity	M ¹ L ⁻¹ T ⁻²	Young modulus	M ¹ L ⁻¹ T ⁻²
Potential energy	M ¹ L ² T ⁻²	Magnetic field intensity	M ⁰ L ⁻¹ T ⁰ A ¹
Gravitational constant	M ⁻¹ L ³ T ⁻²	Magnetic Induction	M ¹ T ⁻² A ⁻¹
Light year	M ⁰ L ¹ T ⁰	Permittivity	M ⁻¹ L ⁻³ T ⁴ A ²
Thermal resistance	M ⁻¹ L ⁻² T ³ K	Electric field	M ¹ L ¹ T ⁻³ A ⁻¹
Coefficient of viscosity	M ¹ L ⁻¹ T ⁻¹	Resistance	ML ² T ⁻³ A ⁻²

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Sets of Quantities having same dimensions

Sr. No.	Quantities	Dimensions
1.	Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility, etc.	$[M^0 L^0 T^0]$
2.	Mass and inertia	$[M^1 L^0 T^0]$
3.	Momentum and impulse.	$[M^1 L^1 T^{-1}]$
4.	Thrust, force, weight, tension, energy gradient.	$[M^1 L^1 T^{-2}]$
5.	Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density.	$[M^1 L^{-1} T^{-1}]$
6.	Angular momentum and Planck's constant (h).	$[M^1 L^2 T^{-1}]$
7.	Acceleration, g and gravitational field intensity.	$[M^0 L^1 T^{-2}]$
8.	Surface tension, free surface energy (energy per unit area), force gradient, spring constant.	$[M^1 L^0 T^{-2}]$
9.	Latent heat capacity and gravitational potential.	$[M^0 L^2 T^{-2}]$
10.	Thermal capacity, Boltzmann constant, entropy.	$[ML^2 T^{-2} K^{-1}]$
11.	Work, torque, internal energy, potential energy, kinetic energy, moment of force, (q^2/C) , (LI^2) , (qV) , (V^2C) , (I^2Rt) , $\frac{V^2}{R}t$, (VIt) , (PV) , (RT) , (mL) , $(mc \Delta T)$	$[M^1 L^2 T^{-2}]$
12.	Frequency, angular frequency, angular velocity, velocity gradient, radioactivity $\frac{R}{L}, \frac{1}{RC}, \frac{1}{\sqrt{LC}}$	$[M^0 L^0 T^{-1}]$
13.	$\left(\frac{1}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{L}{R}\right), (RC), (\sqrt{LC}), \text{time}$	$[M^0 L^0 T^1]$
14.	$(VI), (I^2R), (V^2/R), \text{Power}$	$[M L^2 T^{-3}]$



Some Fundamental Constants

Gravitational constant(G)	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Speed of light in vacuum (c)	$3 \times 10^8 \text{ ms}^{-1}$
Permeability of vacuum (μ_0)	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Permittivity of vacuum (ϵ_0)	$8.85 \times 10^{-12} \text{ F m}^{-1}$
Planck constant (h)	$6.63 \times 10^{-34} \text{ Js}$
Atomic mass unit (amu)	$1.66 \times 10^{-27} \text{ kg}$
Energy equivalent of 1 amu	931.5 MeV
Electron rest mass (m_e)	$9.1 \times 10^{-31} \text{ kg} \equiv 0.511 \text{ MeV}$
Avogadro constant (N_a)	$6.02 \times 10^{23} \text{ mol}^{-1}$
Faraday constant (F)	$9.648 \times 10^4 \text{ C mol}^{-1}$
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Wien constant (b)	$2.89 \times 10^{-3} \text{ mK}$
Rydberg constant (R_∞)	$1.097 \times 10^7 \text{ m}^{-1}$
Triple point for water	273.16 K (0.01°C)
Molar volume of ideal gas (NTP)	$22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3 \text{ mol}^{-1}$

Key Points

- Trigonometric functions $\sin\theta$, $\cos\theta$, $\tan\theta$ etc and their arrangements are dimensionless.
- Dimensions of differential coefficients $\left[\frac{d^n y}{dx^n} \right] = \left[\frac{y}{x^n} \right]$
- Dimensions of integrals $\left[\int y dx \right] = [yx]$
- We can't add or subtract two physical quantities of different dimensions.
- Independent quantities may be taken as fundamental quantities in a new system of units.



PRACTICAL PHYSICS

Rules for counting significant Figures

For a number greater than 1

- All non-zero digits are significant.
- All zeros between two non-zero digits are significant. Location of decimal does not matter.
- If the number is without decimal part, then the terminal or trailing zeros are not significant.
- Trailing zeros in the decimal part are significant.

For a number less than 1

Any zero to the right of a non-zero digit is significant. All zeros between decimal point and first non-zero digit are not significant.

Ex. 0.108 \rightarrow 3SF,
 $1.23 \times 10^{-19} \rightarrow$ 3SF,

40.000 \rightarrow 5SF,
0.0018 \rightarrow 2SF

Rounding off

Rounding Rules for Whole Numbers

Rounding rules for whole numbers is as follows:

- To get an accurate final result, always choose the smaller place value.
- Look for the next smaller place which is towards the right of the number that is being rounded off to. For example, if you are rounding off a digit from tens place, look for a digit in the ones place.
- If the digit in the smallest place is less than 5, then the digit is left untouched. Any number of digits after that number becomes zero and this is known as **rounding down**.
- If the digit in the smallest place is greater than or equal to 5, then the digit is added with +1. Any digits after that number become zero and this is known as **rounding up**.

Rounding Rules for Decimal Numbers

Rounding rules for decimal numbers are as follows:

- Determine the rounding digit and look at its righthand side.
- If the digits at the righthand side are less than 5, consider them as equal to zero.

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- If the digits at the righthand side are greater than or equal to 5, then add to that digit and consider all other digits as zero.

Example:

$$\begin{array}{lll} 6.87 \rightarrow 6.9, & 6.84 \rightarrow 6.8, & 6.85 \rightarrow 6.8, \\ 6.75 \rightarrow 6.8, & 6.65 \rightarrow 6.6, & 6.95 \rightarrow 7.0 \end{array}$$

Order of magnitude :

Power of 10 required to represent a quantity

$$49 = 4.9 \times 10^1 \approx 10^1 \Rightarrow \text{order of magnitude} = 1$$

$$51 = 0.51 \times 10^2 \approx 10^2 \Rightarrow \text{order of magnitude} = 2$$

$$0.051 = 5.1 \times 10^{-2} \approx 10^{-2} \Rightarrow \text{order of magnitude} = -2$$

Propagation of combination of errors

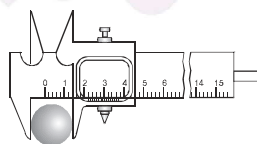
- **Error in Summation and Difference** : $x = a + b$ then $\Delta x = \pm (\Delta a + \Delta b)$
- **Error in Product and Division** : A physical quantity X depend upon Y & Z as $X = Y^a Z^b$ then maximum possible fractional error in X .

$$\frac{\Delta X}{X} = |a| \frac{\Delta Y}{Y} + |b| \frac{\Delta Z}{Z}$$

- **Error in Power of a Quantity** : $x = \frac{a^m}{b^n}$ then $\frac{\Delta x}{x} = \pm \left[m \left(\frac{\Delta a}{a} \right) + n \left(\frac{\Delta b}{b} \right) \right]$

- **Least count** : The smallest value of a physical quantity which can be measured accurately with an instrument is called the least count of the measuring instrument.

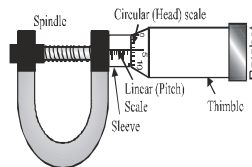
- **Vernier Callipers**: Least count = $1\text{MSD} - 1\text{VSD}$
(MSD \rightarrow main scale division, VSD \rightarrow Vernier scale division)



Ex. A vernier scale has 10 parts, which are equal to 9 parts of main scale having each parts equal to 1 mm then least count = $1 \text{ mm} - \frac{9}{10} \text{ mm} = 0.1 \text{ mm}$ [$\because 9 \text{ MSD} = 10 \text{ VSD}$]

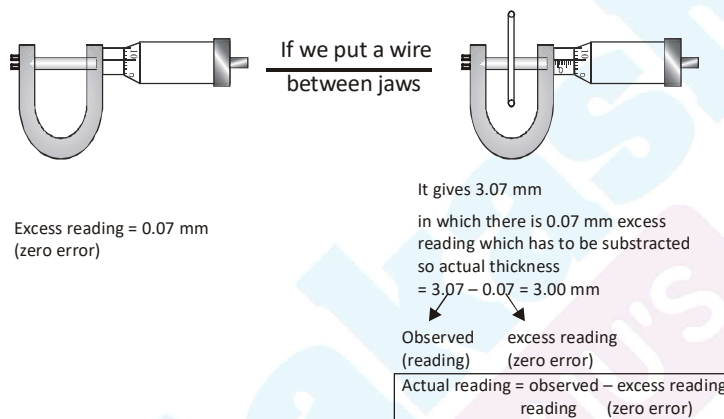
• Screw Gauge :

$$\text{Least Count} = \frac{\text{Pitch}}{\text{total no. of divisions on circular scale}}$$



Zero Error :

If there is no object between the jaws (i.e. jaws are in contact), the screwgauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called zero error.



Excess reading = 0.07 mm (zero error)

Ex. The distance moved by spindle of a screw gauge for each turn of head is 1mm. The edge of the thimble is provided with an angular scale carrying 100 equal divisions. The least

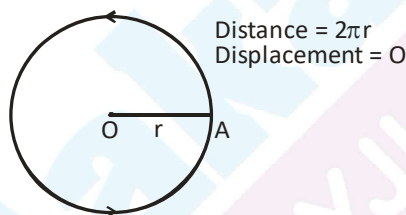
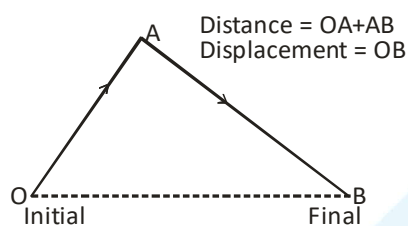
$$\text{count} = \frac{1\text{mm}}{100} = 0.01 \text{ mm}$$



MOTION IN 1-D

1. Distance and Displacement

Distance	Displacement
(i) Length of actual path covered between the initial and final positions/points	(i) Length of the shortest path between initial and final points
(ii) Scalar quantity	(ii) Vector quantity
(iii) Can have only +ve values	(iii) Can have -ve, 0, +ve values.



- Distance \geq |displacement| and average speed \geq |average velocity|
 - If distance $>$ |displacement| this implies
 - (a) atleast at one point in path, velocity is zero.
 - (b) the body must have retarded during the motion.

2. Displacement in terms of position vector : Let a body is displaced from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ then its displacement is given by vector \overrightarrow{AB}

$$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$$

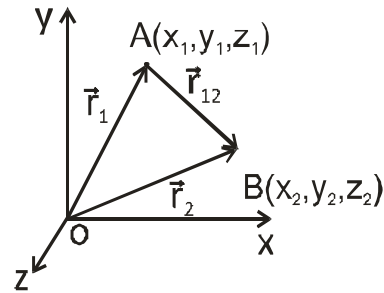
$$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$$

$$\text{From } \Delta OAB \quad \vec{r}_1 + \vec{r}_{12} = \vec{r}_2 \Rightarrow \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$



$$\vec{r}_{12} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\vec{r}_{12}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



3. Velocity & Acceleration :

- Velocity** $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$
- For uniform motion**

Average speed = |average velocity| = |instantaneous velocity|

- Average velocity** = $\frac{\text{Displacement}}{\text{time interval}} = \vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$
- Average speed** = $\frac{\text{Distance Travelled}}{\text{time interval}}$
- Instantaneous velocity** = $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$ if a body moving along a straight line travels distance S_1 in time t_1 , S_2 in time t_2 , S_n in time t_n then its average speed or average velocity is $\langle V \rangle = \frac{S_1 + S_2 + \dots + S_n}{t_1 + t_2 + \dots + t_n}$
- If a body moving along a straight line travels distance S_1 with velocity V_1 , distance S_2 with velocity V_2 , distance S_3 with V_3 ,then

$$\text{average velocity } \langle V \rangle = \frac{S_1 + S_2 + \dots}{\frac{S_1}{V_1} + \frac{S_2}{V_2} + \dots}$$

here if $S_1 = S_2 = S_3 = \dots = S_n$

$$\text{then } \langle V \rangle = \frac{1 + 1 + \dots}{\frac{1}{V_1} + \frac{1}{V_2} + \dots}$$



- A particle moving along a straight line covers the first half distance with velocity V_1 and the remaining distance with velocity V_2 then its average velocity $\langle V \rangle = \frac{2V_1V_2}{V_1 + V_2}$

If it covers the first half time with velocity V_1 and next half time with V_2 , then

$$\langle V \rangle = \frac{V_1 + V_2}{2}$$

- A particle moved from A to B along a straight line with velocity V_1 and then from B to A with velocity V_2 . In this case its average velocity is zero but average speed is $\frac{2V_1V_2}{V_1 + V_2}$

- Acceleration** $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

- Average acceleration** = $\frac{\text{Total change in velocity}}{\text{total time taken}} = \vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$

- A particle moves along a straight line with acceleration a_1 for time t_1 , a_2 for t_2 , then

$$\text{average acceleration} = \frac{a_1t_1 + a_2t_2 + \dots + a_nt_n}{t_1 + t_2 + t_3 + \dots + t_n}$$

- If a body starts from rest and moves with an acceleration which increases at a steady rate with time, then $s \propto t^3$, $v \propto t^2$, $a \propto t$.

- A particle moving along a straight line with uniform acceleration crosses points A and B with velocities V_1 and V_2 . If C is the mid point of AB, it crosses C with a velocity equal

$$\text{to } \sqrt{\frac{V_1^2 + V_2^2}{2}}.$$

- If a particle starts from rest and moves with uniform acceleration 'a' such that it travels

$$\text{distances } S_m \text{ and } S_n \text{ in the } m^{\text{th}} \text{ and } n^{\text{th}} \text{ seconds then } a = \frac{S_m - S_n}{(m - n)}.$$

- A particle starts from rest and moves along a straight line with uniform acceleration. If S is the distance travelled by it in n seconds and S_n is the distance travelled in the n^{th}

$$\text{second, then } \frac{S_n}{S} = \frac{(2n - 1)}{n^2}.$$

- If a body starting from rest moves with acceleration α for certain time and then



deceleration at the rate β . Until it stops and 't' is the total time of its motion, maxi-

imum velocity of the body $(v) = \frac{\alpha\beta t}{(\alpha + \beta)}$

average velocity = $v/2$

distance travelled by the body $(S) = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$

- If a bullet loses $(1/n)^{\text{th}}$ of its velocity while passing through a plank, then the minimum

number of planks required to stop the bullet is $\left(\frac{n^2}{2n-1} \right)$

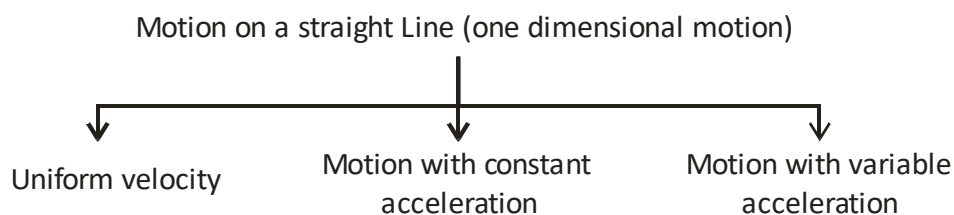
- Two particles move with constant velocities V_1 and V_2 along X and Y axes. If they are at distances x and y from the origin 'O', they will be closest to each other after time

$\left(\frac{V_1 x + V_2 y}{V_1^2 + V_2^2} \right)$



MOTION IN 1-D

4. Problem Solving Strategy :



(i) $s = vt$

(ii) $a = 0$

(i) $s = \left(\frac{u+v}{2} \right) t$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

(iv) $v = u + at$

(v) $s_{nth} = u + (2n-1)\frac{a}{2}$

(vi) For retardation, 'a' will be negative

(i) If $a = f(t), a = \frac{dv}{dt}$

(ii) If $a = f(s), a = v \frac{dv}{ds}$

(iii) If $a = f(v), a = \frac{dv}{dt}$

(iv) $v = \frac{ds}{dt}$

(v) $s = \int v dt$

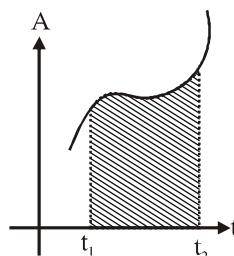
(vi) $v = \int a dt$

5. Graphical Representation :

- Area under the curve in 1-D motion

(i) $a = \frac{dv}{dt} = v \frac{dv}{dx}$

$$\Rightarrow a = \frac{dv}{dt} \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt \Rightarrow v_2 - v_1 = \int_{t_1}^{t_2} a dt$$

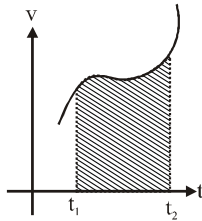


\Rightarrow Change in velocity = Area between acceleration curve and time axis, from t_1 to t_2 .



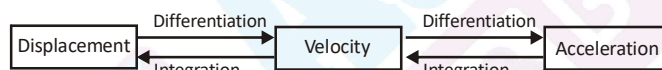
$$(ii) \quad v = \frac{dx}{dt} \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v dt \Rightarrow x_2 - x_1 = \int_{t_1}^{t_2} v dt$$

\Rightarrow Change in position = displacement = area between velocity curve and time axis, from t_1 to t_2 .



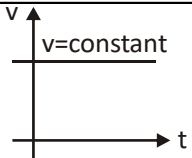
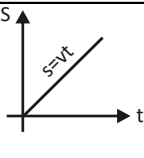
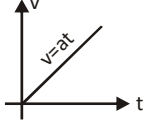
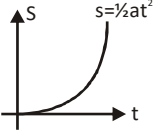
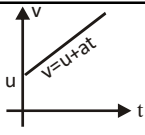
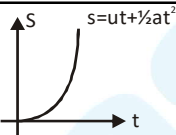
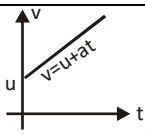
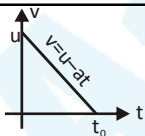
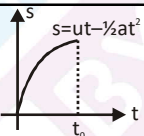
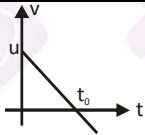
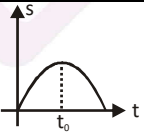
Important point about graphical analysis of motion

- Instantaneous velocity is the slope of position time curve. $\left(v = \frac{dx}{dt} \right)$
- Slope of velocity-time curve = instantaneous acceleration $\left(a = \frac{dv}{dt} \right)$
- v-t curve area gives displacement. $\left[\Delta x = \int v dt \right]$
- a-t curve area gives change in velocity. $\left[\Delta v = \int a dt \right]$





6. V-t and S-t graph for different equation of motion

S. No.	Different Cases	v-t Graph	s-t Graph	Important Points
1.	Uniform motion			(i) Slope of v-t graph = $v = \text{constant}$ (ii) In s-t graph, $s=0$ at $t=0$
2.	Uniformly accelerated motion with $u=0$ and $s=0$ at $t=0$			(i) $u=0$, i.e. $v=0$ at $t=0$ (ii) a or slope of v-t graph is constant (iii) $u=0$, i.e. slope of s-t graph at $t=0$, should be zero
3.	Uniformly accelerated motion with $u \neq 0$ but $s=0$ at $t=0$			(i) $u \neq 0$, i.e., v or slope of v-t graph at $t=0$ is not zero (ii) s or slope of s-t graph gradually goes on increasing
4.	Uniformly accelerated motion with $u \neq 0$ and $s=s_0$ at $t=0$			(i) $v=u$ at $t=0$ (ii) $s=s_0$ at $t=0$
5.	Uniformly retarded motion till velocity becomes zero			(i) slope of s-t graph at $t=0$ gives u (ii) Slope of s-t graph at $t=t_0$ becomes zero (iii) In this case u can't be zero
6.	Uniformly retarded then accelerated in opposite direction			(i) At time $t=t_0$, $v=0$ or slope of s-t graph is zero (ii) In s-t graph slope or velocity first decreases then increases with opposite sign.



7. Motion Under Gravity

• If a body is dropped from some height ($u=0$) -

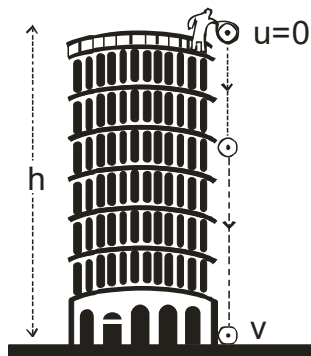
In this case, $u=0$, $a=+g$ then equations of motion are -

(i) $v = gt$

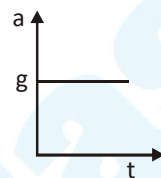
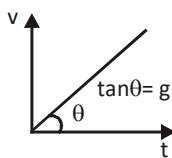
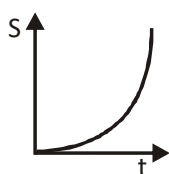
(ii) $h = \frac{1}{2}gt^2$

(iii) $v^2 = 2gh$

(iv) $h_n = \frac{g}{2}(2n-1)$



Graphs-



(v) A body is released from certain height h above the ground. The time after which it reached the ground is $\sqrt{\frac{2h}{g}}$.

Here if g vanishes exactly at the midway during its fall, the time of fall till then will be

$$t_1 = \sqrt{\frac{h}{g}}$$

The further time of fall is $t_2 = \frac{1}{2}\sqrt{\frac{h}{g}}$

Here for time ' t_1 ' it will move with uniform acceleration and for time ' t_2 ' it will move with uniform velocity \sqrt{gh} .

Here total time of its fall is $t_1 + t_2 = \frac{3}{2}\sqrt{\frac{h}{g}}$

$$\frac{t_1 + t_2}{t} = \frac{3}{2\sqrt{2}}$$



- (vi) When a body is dropped from a certain height above the ground it will reach the ground after time t_1 . But after time t_2 , the body is stopped and then released. The further time after which it reaches the ground from the instant is t . Then $t^2 = t_1^2 - t_2^2$ or

$$t = \sqrt{t_1^2 - t_2^2}$$

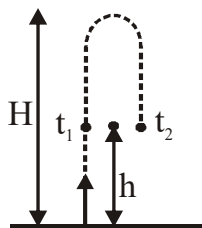
- (vii) If the distance travelled by a freely falling body in the first t seconds is equal to the distance travelled in the last second, the time of its fall is $(r^2 + 1)/2$

- (viii) A freely falling body acquires a velocity V after falling through a certain distance h .

The distance travelled by it in the next second is $V + \frac{g}{2}$ or $\left(\sqrt{2gh} + \frac{g}{2}\right)$.

- (ix) If air resistance is taken into consideration and two bodies of different masses are dropped simultaneously from the same height, heavier body reaches the ground earlier. If the two bodies have same mass but different sizes, smaller body reaches the ground first.

- (x) Gallileo's law of odd numbers : For a freely falling body ratio of successive distance covered in equal time interval ' t '

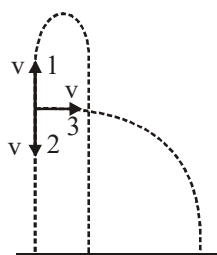


$$S_1 : S_2 : S_3 : \dots = 1 : 3 : 5 : \dots : 2n-1$$

- At any point on its path the body will have same speed for upward journey and downward journey.
- A body is thrown upward, downward & horizontally with same speed takes time t_1 , t_2 & t_3 respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ & height from where the particle was thrown is $H = \frac{1}{2} g t_1 t_2$



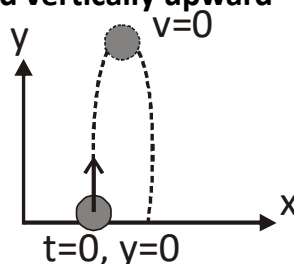
MOTION IN 1-D



If a body is projected vertically upward

(i) $u = gt$

(ii) $h = ut - gt^2$



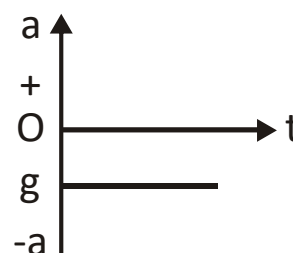
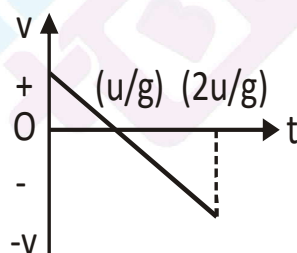
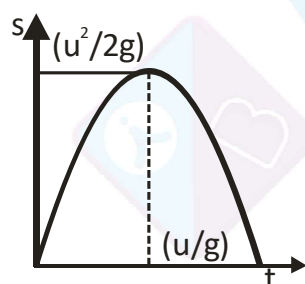
$$\text{or } h_{\max} = ut = -\frac{1}{2}gT^2 \Rightarrow h_{\max} = (gT)T - \frac{1}{2}gT^2$$

$$\text{or } h_{\max} = \frac{1}{2}gT^2$$

(iii) $v^2 = u^2 - 2gh$ or $0 = u^2 - 2gh_{\max}$

$$\text{or } u^2 = 2gh_{\max}$$

Graphs-



Note :-

(i) After attaining maximum height body turns and come back at ground.

(ii) Time taken during up flight and down flight are equal.

(iii) Time for one side $T = u/g$ and

Total time of flight $= 2T = 2u/g$

(iv) At each equal height from ground speed of body will be same either going up or coming down.

(v) A body projected vertically up crosses a point P at a height 'h' above the ground at time ' t_1 ' seconds and at time ' t_2 ' seconds while coming down. then total time of



its flight $T = t_1 + t_2$

Height of P is $h = \frac{1}{2}gt_1t_2$

Maximum height reached above the ground $H = \frac{1}{8}g(t_1 + t_2)^2$

Magnitude of velocity while crossing P is $\frac{g(t_2 - t_1)}{2}$

If the body crosses P after t_1 seconds and then reaches the ground after t_2 seconds from that point, the above conditions apply.

(vi) A body is dropped from the top of a tower of height 'h'. Simultaneously another body is projected vertically up with a velocity u from the foot of the tower.

(a) The separation between them after 't' seconds is $(h - ut)$

(b) The time after which they meet $t = \frac{h}{u}$

(c) The height at which they meet above the ground is $\left(h - \frac{gt^2}{2}\right)$

(d) The time after which their velocities are equal in magnitudes is $t = \frac{u}{2g}$

(e) If their speeds are equal after some time, the ratio of distances travelled by those two will be in the ratio 1:3.

(vii) A body is projected vertically up with velocity u_1 and after 't' seconds body is projected vertically up with a velocity u_2 . If $u_2 > u_1$, the time after which both the

bodies will meet with each other is $\frac{u_2t + \frac{1}{2}gt^2}{(u_2 - u_1) + gt}$ for the first body.

In this case if $u_1 = u_2 = u$ ie the two bodies are projected with same velocity, the time

after which they meet is $\left(\frac{u}{g} + \frac{t}{2}\right)$ for the first body and $\left(\frac{u}{g} - \frac{t}{2}\right)$ for the second body.

(viii) A rocket moves up with a resultant acceleration a . If its fuel exhausts completely after time 't' seconds the maximum height reached by the rocket above the ground is

$$h = \frac{1}{2}at^2 \left(1 + \frac{a}{g}\right)$$



MOTION IN 2-D

1. Projectile Motion

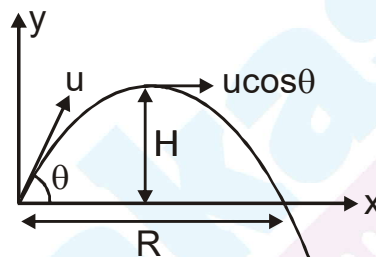
If a constant force (and hence constant acceleration) acts on a particle at an angle $\theta (\neq 0^\circ \text{ or } 180^\circ)$ with the direction of its initial velocity ($\neq \text{zero}$), the path followed by the particle is a parabola and the motion of the particle is called projectile motion. Projectile motion is a two dimensional motion, i.e., motion of the particle is constrained in a plane.

- Horizontal Motion**

$$u_x = u \cos \theta$$

$$a_x = 0$$

$$x = u_x t = (u \cos \theta) t$$



- Vertical Motion**

$$u_y = u \sin \theta \text{ and } a_y = -g ; y = u_y t + \frac{1}{2} a_y t^2 = u \sin \theta t - \frac{1}{2} g t^2$$

- Net acceleration** = $\vec{a} = a_x \hat{i} + a_y \hat{j} = -g \hat{j}$

- At any instant:** $v_x = u \cos \theta$, $v_y = u \sin \theta - g t$

2. Ground to Ground Projectile Motion

2.1 For projectile motion :

A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point so,

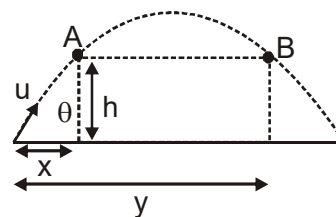
(a) $x + y = R$

(b) $t_1 + t_2 = T$

(c) $h = \frac{1}{2} g t_1 t_2$

(d) Average velocity from A to B is $u \cos \theta$

(e) If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be $(x/2)$.





2.2 Velocity of particle at time t :

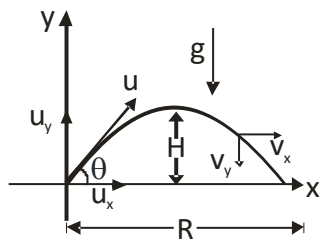
$$\vec{v} = v_x \hat{i} + v_y \hat{j} = u_x \hat{i} + (u_y - gt) \hat{j} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$$

If angle of velocity \vec{v} from ground is α , then

$$\tan \alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u \sin \theta - gt}{u \cos \theta} = \tan \theta - \frac{gt}{u \cos \theta}$$

- At highest point: $v_y = 0$, $v_x = u \cos \theta$

- Time of flight : $T = \frac{2u_y}{g} = \frac{2u \sin \theta}{g}$



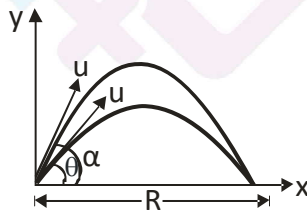
x-axis	y-axis
$a_x = 0$	$a_y = g$
$u_x = u \cos \theta$	$u_y = u \sin \theta$
$x = (u \cos \theta)t$	$y = (u \sin \theta)t - \frac{1}{2}gt^2$
$v_x = u \cos \theta$	$v_y = u \sin \theta - gt$

- Horizontal range : $R = (u \cos \theta)T = \frac{2(u \cos \theta)(u \sin \theta)}{g} = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$

It is same for θ and $(90^\circ - \theta)$ and maximum for $\theta = 45^\circ$

- For objects projected at complementary launch angle, range will be same

Here, $\alpha + \theta = 90^\circ$



- Maximum height : $H = \frac{u_y^2}{2g} = \frac{u^2 \sin^2 \theta}{2g} = \frac{1}{8}gT^2$

- $\frac{H}{R} = \frac{1}{4} \tan \theta$

- Equation of Trajectory : $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} = x \tan \theta \left(1 - \frac{x}{R}\right)$

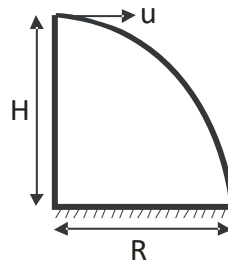


3. Projectile Motion From Some Height

3.1 Projectile thrown horizontally

- Time of flight $T = \sqrt{\frac{2h}{g}}$

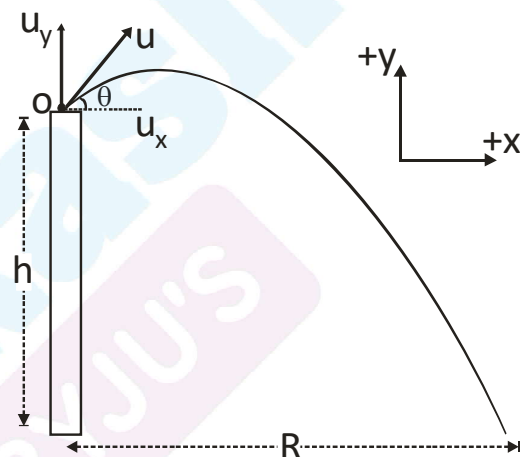
- Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$



- Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$

3.2 Projection at an angle θ above horizontal

x-axis	y-axis
$u_x = u \cos \theta$	$u_y = u \sin \theta$
$a_x = 0$	$a_y = -g$



- Time of flight (T) $S_y = u_y t + \frac{1}{2} a_y t^2$

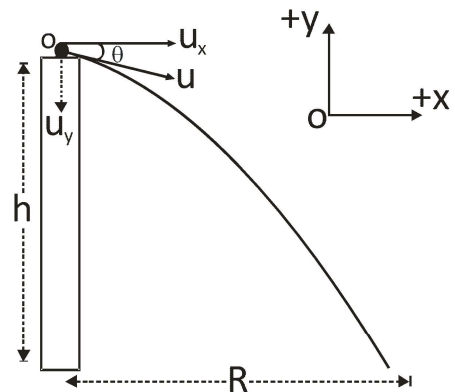
Putting, $s_y = -h$, $a_y = -g$ and $t = T$
we get time of flight T.

- Range, $R = u_x T = (u \cos \theta) T$



3.3 Projection at an angle θ below horizontal

x-axis	y-axis
$u_x = u \cos \theta$	$u_y = -u \sin \theta$
$a_x = 0$	$a_y = -g$



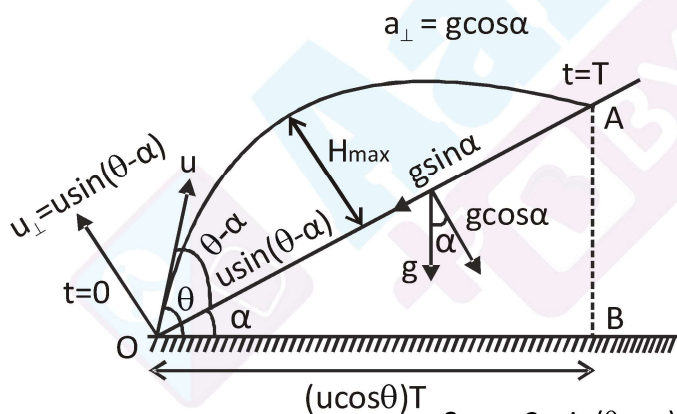
- Time of flight (T) $S_y = u_y t + \frac{1}{2} a_y t^2$

Putting $s_y = -h$, $a_y = -g$ and $t = T$, we get time of flight.

- Range, $R = u_x T = (u \cos \theta) T$

4. Projectile Motion on Inclined Plane

4.1 Projectile motion on inclined plane-up motion



- Time of flight

$$T = \frac{2u_{\perp}}{g_{\perp}} = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$$

- Maximum height

$$H_{\max} = \frac{u_{\perp}^2}{2g_{\perp}} = \frac{u^2 \sin^2(\theta - \alpha)}{2g \cos \alpha}$$

- Range of inclined plane

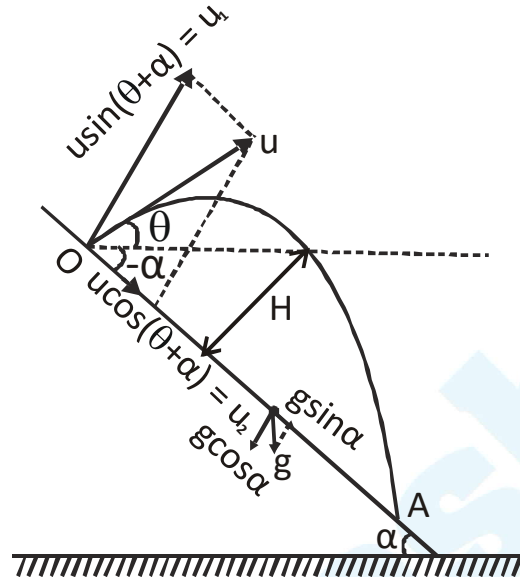
$$R = OA = \frac{2u^2 \sin(\theta - \alpha) \cos \theta}{g \cos^2 \alpha}$$

- Maximum range

$$R_{\max} = \frac{u^2}{g(1 + \sin \alpha)} \text{ at angle } \theta = \frac{\pi}{4} + \frac{\alpha}{2}$$



4.2 Projectile motion of inclined plane (2-D): Down motion (put $\alpha = -\alpha$ in above)



- Time of flight : $T = 2t_H = \frac{2u_{\perp}}{a_{\perp}} = \frac{2u \sin(\theta + \alpha)}{g \cos \alpha}$
- Maximum height $H = \frac{u_{\perp}^2}{2a_{\perp}} = \frac{u^2 \sin^2(\theta + \alpha)}{2g \cos \alpha}$
- Range of inclined plane $R = OA = \frac{2u^2 \cos \theta \sin(\theta + \alpha)}{g \cos^2 \alpha}$
- Maximum range $R_{\max} = \frac{u^2}{g(1 - \sin \alpha)}$ at angle $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$



5. Comparison of projection motion

5.1 If a body is projected at angles θ and $(90^\circ - \theta)$ with the same initial velocity

Parameter	for θ	for $(90^\circ - \theta)$	Relation
Range	$R_1 = \frac{u^2 \sin^2 \theta}{g}$	$R_2 = \frac{u^2 \sin 2\theta}{g}$	$R_1 = R_2 = R$
Maximum height	$H_1 = \frac{u^2 \sin^2 \theta}{2g}$	$H_2 = \frac{u^2 \cos^2 \theta}{2g}$	$H_1 + H_2 = \frac{u^2}{2g}$ $u = \sqrt{2g(H_1 + H_2)}$ $\frac{H_1}{H_2} = \tan^2 \theta$ $R^2 = 16H_1 H_2$
Time of flight	$T_1 = \frac{2u \sin \theta}{g}$	$T_2 = \frac{2u \cos \theta}{g}$	$\frac{T_1}{T_2} = \tan \theta$ $T_1 T_2 = \frac{2R}{g}$

6. Some important trajectory and their results

- If the equation of trajectory of a projectile is $y = Ax - Bx^2$
 - Its angle of projection with horizontal is $\theta = \tan^{-1}(A)$
 - Horizontal range $R = (A/B)$
 - Maximum height $H = (A^2 / 4B)$
 - Time of flight $T = \left(\sqrt{\frac{2}{Bg}} \right) A$
- The horizontal vertical displacements of a projectile are given by $x = at; y = bt - ct^2$ then
 - Velocity of projection is $\sqrt{a^2 + b^2}$
 - Angle of projection is $\tan^{-1}(a/b)$
 - Acceleration of the projectile = $2c$
 - Maximum height reached = $b^2 / 4c$
 - Horizontal range = ab/c
- A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant bullet leaves the gun, the monkey falls under gravity. The bullet hits the monkey (for any muzzle velocity of the bullet).



4. A ship is moving with a constant velocity V_1 in the horizontal direction. If a ball is projected vertically up with a velocity V_2 from its deck, its path is parabola for a person on the ground. If u is resultant velocity, here $u \cos \theta = V_1$ and $u \sin \theta = V_2$.

If A is an observer on the ship and B is an observer at rest on the ground.

$$\text{for A and B } T = \frac{2V_2}{g}; H = \frac{V_2^2}{2g}$$

$$\text{for A, horizontal range} = 0; \text{ for B, horizontal range} = \frac{2V_1V_2}{g}$$

5. If a body is projected horizontally from certain height above the ground and another is released simultaneously from the same height, both reach the ground after the same time but with different speeds.
6. A body at certain height exploded into two fragments of unequal masses and they move in opposite direction with velocities u_1 and u_2 .

The time after which their displacement vectors (ie then position vectors with respect

$$\text{to initial position) are perpendicular to each other is } \frac{2\sqrt{u_1u_2}}{g}$$

$$\text{The time after which their velocity vectors are perpendicular to each other } \sqrt{\frac{u_1u_2}{g}}$$

7. An object is projected vertically up from the top of a tower which reaches the ground after t_2 sec. then

$$\text{height of that tower} = \frac{1}{2}gt_1t_2$$

$$\text{velocity of projection} = \frac{g(t_1 - t_2)}{2}$$

If that object is projected horizontally with the same velocity or if it is released from the same point, it reaches the ground after $\sqrt{t_1t_2}$ sec.



RELATIVE MOTION

1. Relative Motion :

- Motion is a combined property of the object under study as well as the observer.
- Motion is always defined with respect to an **observer or reference frame**.
- It is always relative, there is no such thing as **absolute motion or absolute rest**.
- All parameters like position, velocity and acceleration are “**frame dependent**”.

2. Relative Motion in 1D :

Relative position : $\vec{x}_{BA} = \vec{x}_B - \vec{x}_A$

Differentiating both sides, we get

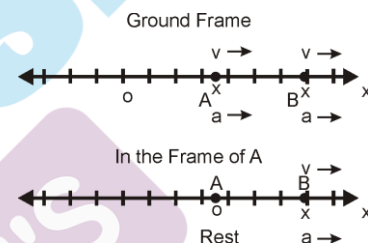
$$\frac{d}{dt} \vec{x}_{BA} = \frac{d}{dt} \vec{x}_B - \frac{d}{dt} \vec{x}_A$$

Relative velocity : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Differentiating again on both sides, we get

$$\frac{d}{dt} \vec{v}_{BA} = \frac{d}{dt} \vec{v}_B - \frac{d}{dt} \vec{v}_A$$

Relative acceleration : $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$



3. Relative Acceleration :

It is the rate at which relative velocity is changing.

$$a_{AB} = \frac{dv_{AB}}{dt} = \frac{dv_A}{dt} - \frac{dv_B}{dt} = a_A - a_B$$

Equations of motion when relative acceleration is constant.

$$v_{rel} = u_{rel} + a_{rel}t$$

$$s_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$$

$$v_{rel}^2 = u_{rel}^2 + 2a_{rel}s_{rel}$$



4. Relative Motion in 2D :

Relative position : $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$

Differentiating both sides, we get

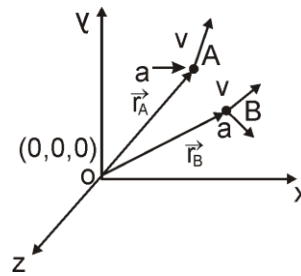
$$\frac{d}{dt} \vec{r}_{AB} = \frac{d}{dt} \vec{r}_A - \frac{d}{dt} \vec{r}_B$$

Relative velocity : $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Differentiating again on both sides, we get

$$\frac{d}{dt} \vec{v}_{AB} = \frac{d}{dt} \vec{v}_A - \frac{d}{dt} \vec{v}_B$$

Relative acceleration : $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$



5. Relative Motion in Lift :

Projectile motion in a lift moving with acceleration a upwards

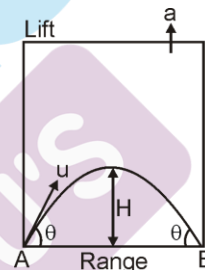
(1) In the reference frame of lift, acceleration of a freely falling object is $g+a$

(2) Velocity at maximum height = $u \cos \theta$

(3) $T = \frac{2u \sin \theta}{g+a}$

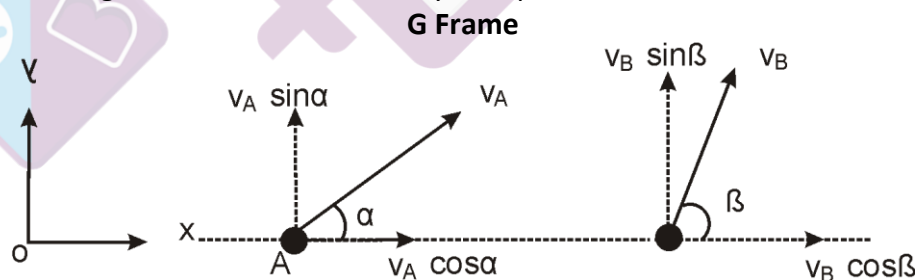
(4) Maximum height (H) = $\frac{u^2 \sin^2 \theta}{2(g+a)}$

(5) Range = $\frac{u^2 \sin 2\theta}{g+a}$



6. Velocity of Approach or Separation :

- It is the component of relative velocity of one particle w.r.t. to another, along the line joining them.
- If the separation is decreasing, it is called “velocity of approach” and if separation is increasing, then it is called “velocity of separation”.

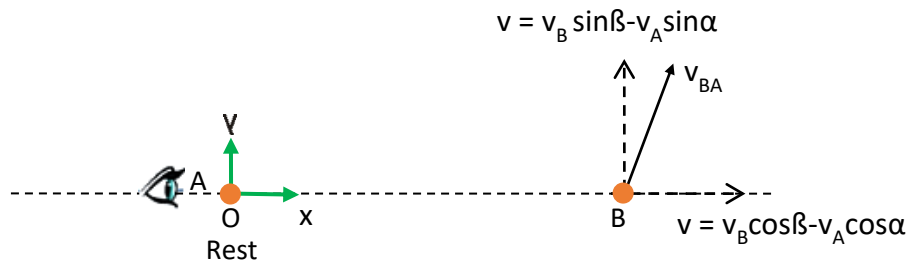


$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$\vec{v}_{BA} = (v_B \cos \beta \hat{i} + v_B \sin \beta \hat{j}) - (v_A \cos \alpha \hat{i} + v_A \sin \alpha \hat{j})$$



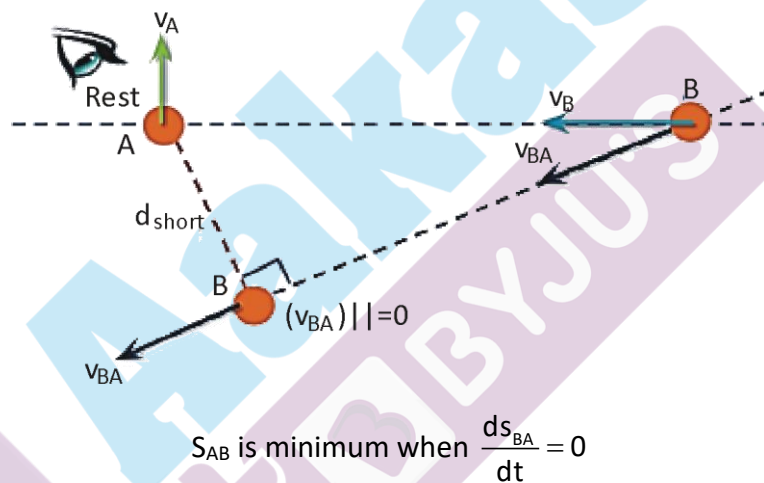
A Frame



v represents velocity of approach or separation

7. Minimum Distance :

- If the separation between two particles decreases and then increases, there will be a minimum distance at the transition.
- Velocity of approach changes to velocity of separation.



8. Relative Motion in River Flow :

If a man can swim relative to water with velocity \vec{v}_{mr} and water is flowing relative to ground with velocity \vec{v}_R , velocity of man relative to ground \vec{v}_m will be given by :

$$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$$

or
$$\vec{v}_m = \vec{v}_{mR} + \vec{v}_R$$

If $\vec{v}_R = 0$, then $\vec{v}_m = \vec{v}_{mR}$

in words, velocity of man in still water = velocity of man w.r.t. river



9. River Problem in One Dimension :

- Velocity of river is u & velocity of man in still water is v .

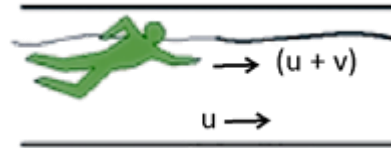
Case-1

Man swimming downstream (along the direction of river flow)

In this case velocity of river $v_R = +u$

velocity of man w.r.t. river $v_{mR} = +v$

now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = u + v$



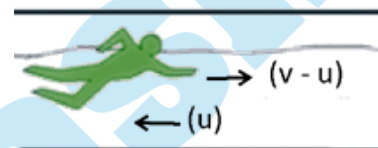
Case-2

Man swimming upstream (opposite to the direction of river flow)

In this case velocity of river $\vec{v}_R = -u$

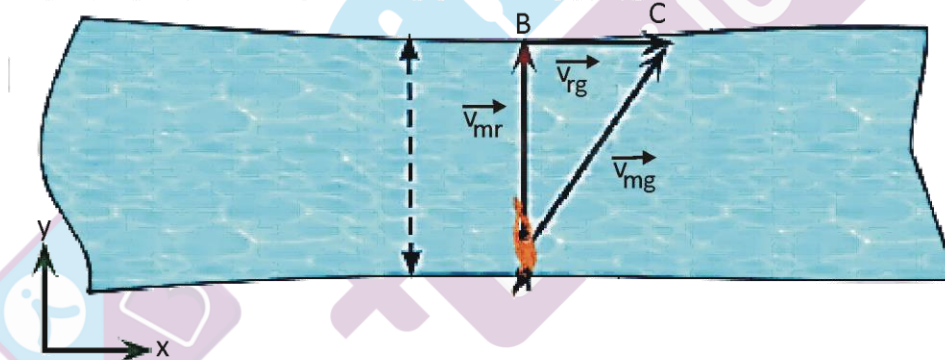
velocity of man w.r.t. river $\vec{v}_{mR} = +v$

now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = (v - u)$



10. Crossing of River (River problem in two dimension :)

Let the speed of the river be u , and speed of the man in still water is v .



\vec{v}_{rg} = velocity of the river w.r.t. ground

\vec{v}_{mr} = velocity of man relative to river

= velocity of man w.r.t. river

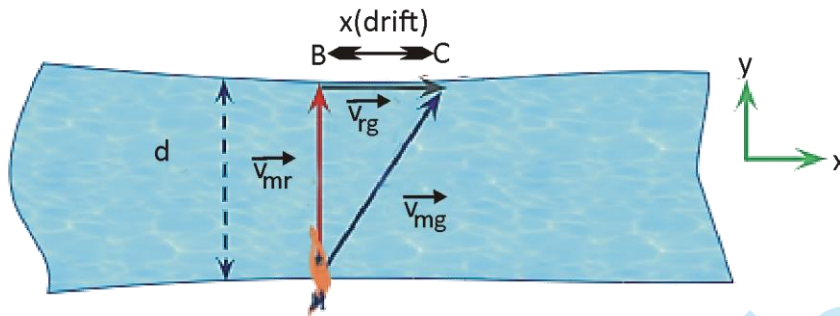
= velocity of man in still water

\vec{v}_{mg} = velocity of the man w.r.t. ground

$$\vec{v}_{mg} = \vec{v}_{mr} + \vec{v}_{rg}$$



- **Case-I** : When the man swims perpendicular to the river flow:



- The velocity of man w.r.t. ground, (\vec{v}_{mg})

$$\vec{v}_{mg} = \vec{v}_{mr} + \vec{v}_{rg} = v\hat{j} + u\hat{i}$$

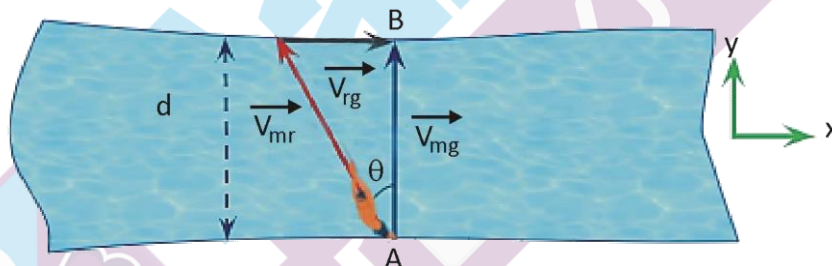
- The time required to cross the river,

$$t = \frac{d}{v_y} = \frac{d}{v}$$

The drift of swimmer,

$$x = v_x \times t = u \times \frac{d}{v}$$

- **Case-II** : When the man crosses river perpendicularly. ($v_{rg} < v_{mr}$)



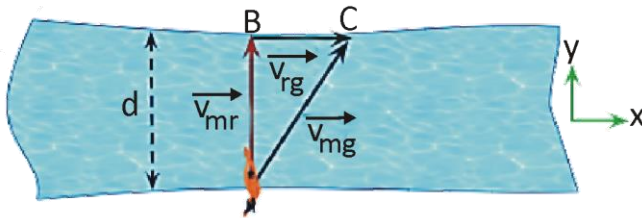
The velocity of man w.r.t ground, (\vec{v}_{mg})

$$\vec{v}_{mg} = \vec{v}_{mr} + \vec{v}_{rg} = -v\sin\theta\hat{i} + v\cos\theta\hat{j} + u\hat{i} = (u - v\sin\theta)\hat{i} + v\cos\theta\hat{j}$$

The time required to cross the river

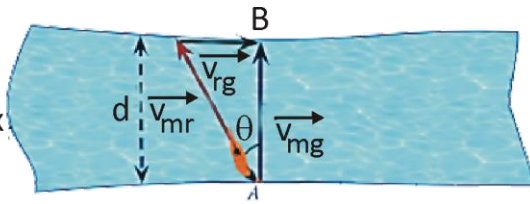
$$t = \frac{d}{v_y} = \frac{d}{v\cos\theta}$$

- Angle for zero drift $\theta = \sin^{-1}\left(\frac{u}{v}\right)$



Case I : Represents the case of minimum time

$$t_{\min} = \frac{d}{v}$$

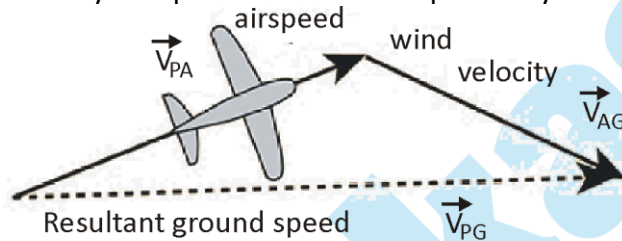


Case II : Represents the case of shortest path or zero drift.

$$\theta = \sin^{-1}\left(\frac{u}{v}\right)$$

11. Wind-Aeroplane problems :

This is very similar to boat river flow problems. The only difference is that boat is replaced by aeroplane and river is replaced by wind.



$$\text{So, } V_{PG} = V_{PA} + V_{AG}$$

Here V_{PA} = Aeroplane speed with respect to air.

V_{AG} = Wind velocity with respect to ground.

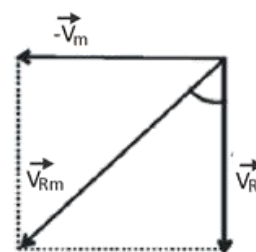
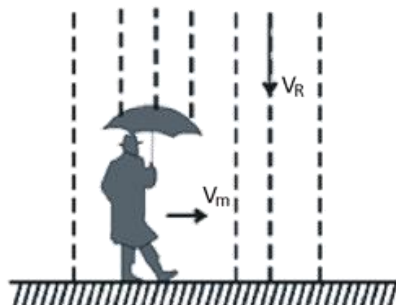
V_{PG} = Aeroplane speed with respect to ground.

12. Rain Problem :

If rain is falling vertically with a velocity \vec{v}_R and an observer is moving horizontally with velocity \vec{v}_m , the velocity of rain relative to observer will be :

$$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m \quad \text{or} \quad v_{Rm} = \sqrt{v_R^2 + v_m^2}$$

and direction = $\tan^{-1} \frac{v_m}{v_R}$ with the vertical as shown in figure.





LAWS OF MOTION

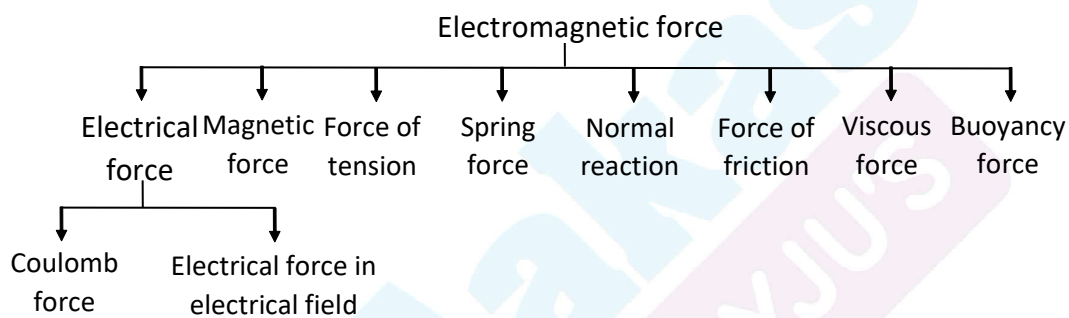
1. Force :

A push or pull that one object exerts on another.

1.1 Forces in nature

There are four fundamental forces in nature :

1. Gravitational force
2. Electromagnetic force
3. Strong nuclear force
4. Weak force



1.2 Types of forces on macroscopic objects :

(a) Field Forces or Range Forces:

These are the forces in which contact between two objects is not necessary.

- Ex.** (i) Gravitational force between two bodies.
 (ii) Electrostatic force between two charges.

(b) Contact Forces:

Contact forces exist only as long as the objects are touching each other.

- Ex.** (i) Normal force. (ii) Frictional force

(c) Attachment to Another Body:

Tension (T) in a string and spring force ($F = kx$) comes in this group.



2. Newton's first law of motion (Galileo's law of Inertia) :

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Inertia : Inertia is the property of the body due to which body oppose the change in its state. Inertia of a body is measured by mass of the body.

$$\text{inertia} \propto \text{mass}$$

3. Momentum :

It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$

SI Unit : kg m s^{-1} **Dimensions** : $[M L T^{-1}]$

4. Newton's second law :

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt} \quad (\text{Linear momentum } \vec{p} = m\vec{v})$$

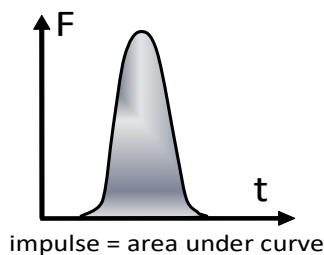
*For constant mass system $\vec{F} = m\vec{a}$

5. Impulse :

1. When a large force acts on a body for a short interval of time, the change in momentum of the body is called impulse.
2. Impulse of a force is the product of force and time interval during which it acts.
3. Impulse $\vec{I} = \vec{F}t$
4. If force \vec{F}_1 acts on a body for time t_1 , \vec{F}_2 for t_2 ,then total change in momentum is equal to $\vec{F}_1 t_1 + \vec{F}_2 t_2 + \vec{F}_3 t_3 + \dots$
5. Force \vec{F}_1 acts on a body at rest for time t_1 and then force \vec{F}_2 brings the body to rest in time t_2 . Then $\vec{F}_1 t_1 = \vec{F}_2 t_2$
6. Impulse, change in momentum and force will be along the direction.

5.1 Force time graph and impulse :

Impulse is the area under the curve of the force versus time graph.



Impulse = product of force with time.

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F} dt$

If constant force acts for an interval Δt then Impulse = $\vec{F} \Delta t$

5.2 Impulse - Momentum theorem

Impulse of a force is equal to the change of momentum

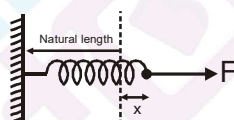
$$\vec{F} \Delta t = \Delta \vec{p}$$

6. Newton's third law of motion :

Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude and in opposite direction.

7. Spring Force (According to Hooke's law) :

In equilibrium $F = kx$ (k is spring constant)

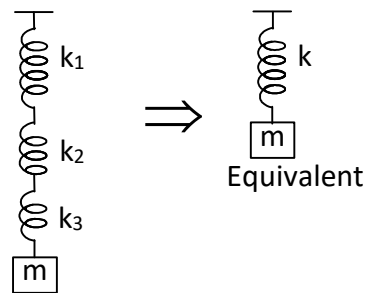


Note : Spring force is non impulsive in nature.

7.1 Combination of springs

(i) Spring in series :

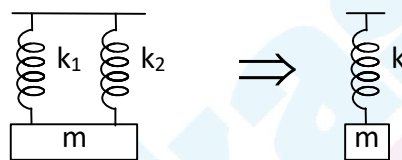
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$



where k is equivalent spring constant.

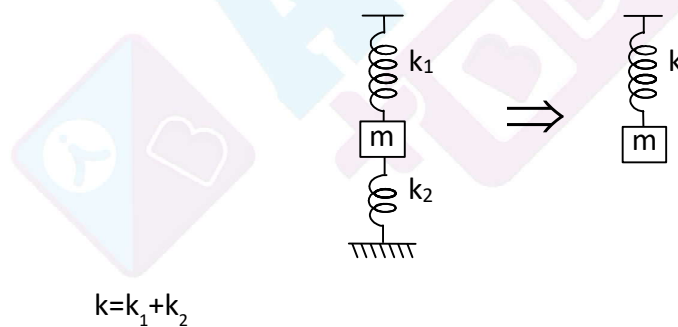
In general, $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$

(ii) Spring in parallel :



In general, $k = k_1 + k_2$
 $k = k_1 + k_2 + k_3 + \dots$

(iii)



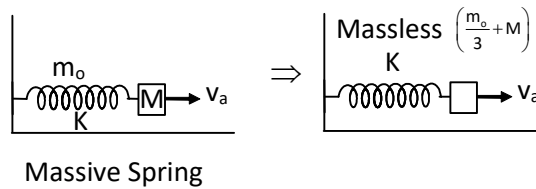
(iv) If spring of spring constant k and length l is cut into two pieces of length l_1 and l_2 , then

$$k \propto \frac{1}{l}; \quad k_1 \propto \frac{1}{l_1}; \quad k_2 \propto \frac{1}{l_2}$$

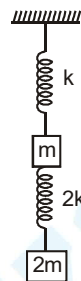
$kl = \text{constant}$



(v) If spring is massive then the effective mass of spring is $\frac{m_0}{3}$, where m_0 is mass of spring.

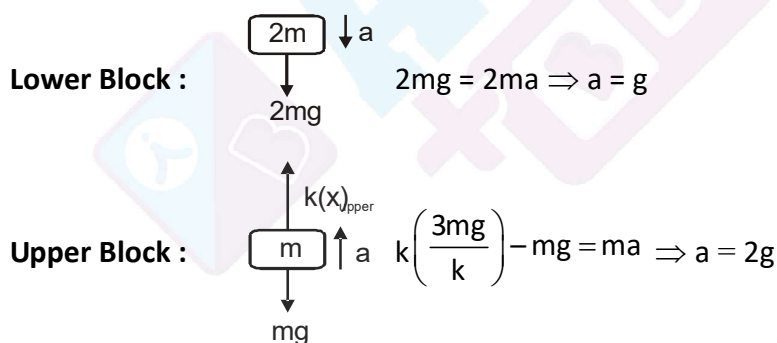


Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.



Sol.: Initial stretches $x_{\text{upper}} = 3mg/k$ and $x_{\text{lower}} = mg/k$

On cutting the lower spring, by virtue of non-impulsive nature of spring the stretch in upper spring remains same immediately after cutting the spring, Thus,

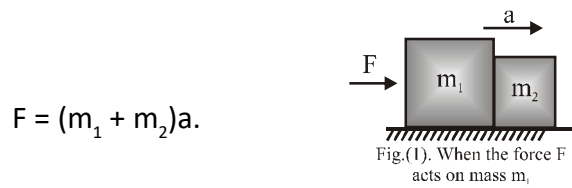


8. Motion of bodies in contact :

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of Contact. These two bodies will move with same acceleration a .



(i) When the force F acts on the body with mass m_1 as shown in fig. (1)



If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1 : $(F - f_1) = m_1 a$
 For body m_2 : $f_1 = m_2 a$

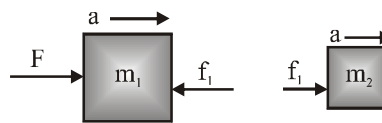


Fig.-1(a) : F.B.D. representation of action and reaction forces

For body m_2 : $f_1 = m_2 a \Rightarrow$ action of m_1 on m_2 : $f_1 = \frac{m_2 F}{m_1 + m_2}$

9. Dependent Motion of Connected Bodies :

Method I: Method of constraint equations

$$\sum x_i = \text{constant} \Rightarrow \sum \dot{x}_i = 0 \Rightarrow \sum \ddot{x}_i = 0$$

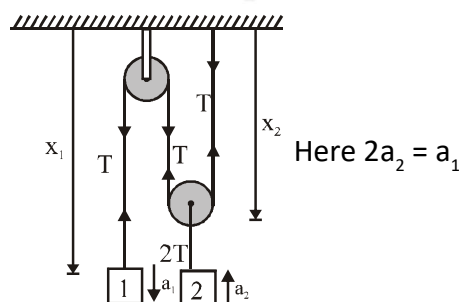
*For n moving bodies we have x_1, x_2, \dots, x_n

*No. of constraint equations = no. of strings

Method II: Method of virtual work

The sum of scalar products of forces applied by connecting links of constant length and displacement of corresponding contact points equal to zero.

$$\sum \vec{F}_i \cdot \delta \vec{r}_i = 0 \Rightarrow \sum \vec{F}_i \cdot \vec{v}_i = 0 \Rightarrow \sum \vec{F}_i \cdot \vec{a}_i = 0$$





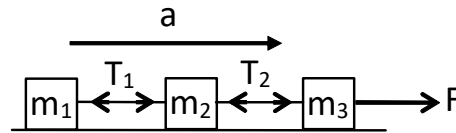
9.1 Connected bodies

1. Masses m_1, m_2, m_3 are connected by light string and are pulled as shown. Then

For m_1 : $T_1 = m_1 a$

For m_2 : $T_2 - T_1 = m_2 a$

For m_3 : $F - T_2 = m_3 a$



a) acceleration of the system $a = \frac{F}{(m_1 + m_2 + m_3)}$

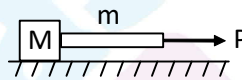
b) $T_2 = m_1 a$

c) $T_2 = (m_1 + m_2) a$

(T_1 and T_2 are tensions in the strings)

2. A block of mass M is pulled by a rope of mass m by a force P on a smooth horizontal plane.

Acceleration of the block $a = \frac{P}{(M + m)}$



Force exerted by the rope on the block is $\frac{MP}{(M + m)}$

3. A rope of length L (uniform rope) is pulled by a constant force F . The tension in the rope at a distance x from the end where the force applied is $F \left(1 - \frac{x}{L} \right)$.

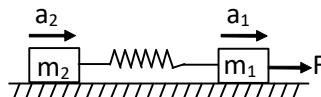


4. If two bodies are connected by a spring and the system is pulled as shown, then accelerations of both bodies are different.

for m_1 , $F - f = m_1 a_1$

for m_2 , $f = m_2 a_2$

(f is spring force)



9.2 Pulley system

A single fixed pulley changes the direction of force only and in general assumed to be mass-less and frictionless.

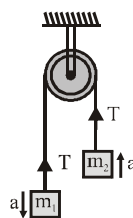


(a) Let $m_1 > m_2$

now for mass m_1 , $m_1 g - T = m_1 a$

for mass m_2 , $T - m_2 g = m_2 a$

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g \quad \& \quad T = \frac{2m_1 m_2}{(m_1 + m_2)} g$$



$$\text{Acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

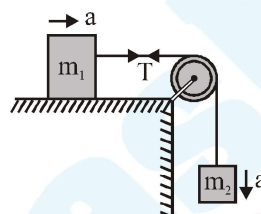
$$\text{Reaction at the suspension of pulley } R = 2T = \frac{4m_1 m_2 g}{(m_1 + m_2)}$$

(b) For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2 g - T = m_2 a$

$$\text{Acceleration } a = \frac{m_2 g}{(m_1 + m_2)}$$

$$\text{and } T = \frac{m_1 m_2}{(m_1 + m_2)} g$$



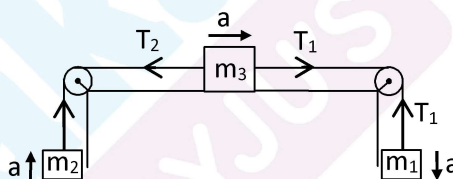
(c) For m_1 : $m_1 g - T_1 = m_1 a$

For m_2 : $T_2 - m_2 g = m_2 a$

For m_3 : $T_1 - T_2 = m_3 a$

\Rightarrow Acceleration of the system is

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + m_3}$$



9.3 Monkey climbing a rope

Case-I : When monkey moves up with constant speed :

In this case, $T = mg$

where m = mass of the monkey,

T = tension in the rope.

Case-II : When monkey is accelerated upwards

$$T - mg = ma$$

Case-III : When monkey accelerated downwards,

$$mg - T = ma$$





10. Frame of Reference :

Any co-ordinate system relative to which the motion, position of a body can be described is called a frame of reference.

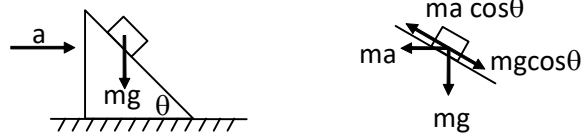
- **Inertial frames of reference** : A reference frame which is either at rest or in uniform motion along the straight line. A non-accelerating frame of reference is called an inertial frame of reference.
 - (i) All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
 - (ii) In inertial frame, acceleration of a body is caused by real forces.
 - (iii) We may consider the earth as inertial frame if we neglect the small acceleration due to rotation of the earth around itself and around the sun.
 - (iv) In inertial frame events are described by real forces only :
- **Non-inertial frame of reference** : An accelerating frame of reference is called a non-inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force
 - (i) All the accelerated and rotating frames are non-inertial frames of reference.
 - (ii) Even if no external force is acting on a particle, in the accelerated frame, it will appear that a force is acting on it which is known as pseudo force or fictitious force.
 - (iii) In non inertial frame events are described by both real and pseudo forces.

11. Pseudo force:

The force on a body due to acceleration of non-inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$ where \vec{a}_0 is acceleration of non-inertial frame with respect to an inertial frame and m is mass of the particle or body, The direction of pseudo force must be opposite to the direction of acceleration of the non inertial frame.

When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a noninertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

- (i) An object kept on a smooth inclined plane can be kept stationary to the incline by giving a horizontal acceleration of $g \tan \theta$ as shown.



Here $ma \cos \theta = mg \sin \theta$

$$a = g \tan \theta$$

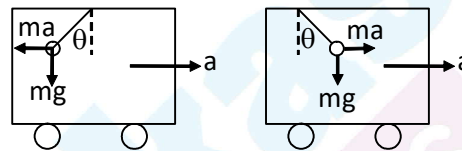
(Force ma on the object is inertial or pseudo force)

Here $F = (M+m)a = (M+m)g \tan \theta$ (M

(ii) A pendulum is suspended from the roof of a moving car.

a) If the car is moving with uniform velocity, the position of the bob does not change.

b) If the car moves with acceleration or retardation, the bob moves in opposite direction to that of acceleration.

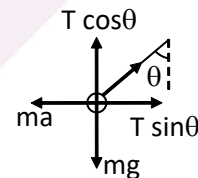


The forces acting on the bob are, weight mg , tension in the string T and pseudo force ma

For the equilibrium of bob.

$$T \sin \theta = ma \text{ and } T \cos \theta = mg$$

$$\Rightarrow \tan \theta = \frac{a}{g} \text{ or } a = g \tan \theta \text{ and } T = m \sqrt{a^2 + g^2}$$



(iii) In the above case if the car is moving along a horizontal circular track, pseudo force is $m\omega^2 r$

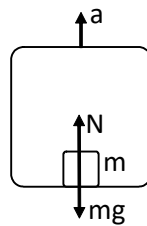
$$\text{Then } \tan \theta = \frac{ma}{mg} = \frac{m\omega^2 r}{mg} \text{ or } \omega^2 r = g \tan \theta$$

Here $m\omega^2 r$ centrifugal force.

12. Man in a Lift :

(a) If the lift is moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift. So apparent weight $W' = Mg = \text{Actual weight}$.

(b) If the lift is accelerated upward with constant acceleration a . Then forces acting on the man w.r.t. observed inside the lift are

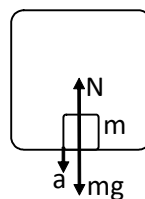


(i) Weight $W=Mg$ downward

(ii) Fictitious force $F_0=Ma$ downward.

So apparent weight $W'=W+F_0=Mg+Ma=M(g+a)$

(c) If the lift is accelerated downward with acceleration $a < g$.



Then w. r.t. observer inside the lift fictitious force $F_0=Ma$ acts upward while weight of man $W=Mg$ always acts downward.

So apparent weight $W' = W - F_0 = Mg - Ma = M(g-a)$

Special Case:

If $a = g$ then $W' = 0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration $a > g$. Then as in Case (c). Apparent weight $W' = M(g-a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

13. Variable Mass System :

1. For any variable mass system force is rate of change of momentum $\Rightarrow F = \left(\frac{dm}{dt} \right) v$

2. If the fuel burns in a rocket at the rate of $\left(\frac{dm}{dt} \right)$ and hot gases eject with a speed V

relative to rocket, thrust on the rocket is $F = \left(\frac{dm}{dt} \right) v$

here $\frac{dm}{dt}$ is mass rat of fuel burnt.



If the rocket moves up with an acceleration 'a', then $\left(\frac{dm}{dt}\right)V = M(g + a)$ here M is mass of the rocket.

If $a = 0$, here $\left(\frac{dm}{dt}\right)V = Mg$.

3. Gravel is dropped on a conveyor belt at the rate of $\frac{dm}{dt}$. Extra force required to keep

the belt moving with velocity V is $\left(\frac{dm}{dt}\right)V$.

4. A jet of water from a tube of area of cross section A strikes a wall normally with a velocity V and then flows down along the wall.

The force exerted on the wall is $F = AV^2\rho$

If water bounces back with velocity u, then $F = A\rho V(v + u)$

If water bounce back with same velocity V, then $F = 2A\rho V^2$

If water strick at an angle θ to the surface of wall and reflects, then

$F = 2A\rho V^2 \sin\theta$

KEY POINTS

- Aeroplanes always fly at low altitudes because according to Newton's III law of motion as aeroplane displaces air & at low altitude density of air is high.
- Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.
- Pulling a lawn roller is easier than pushing it because pushing increases the apparent weight and hence friction.



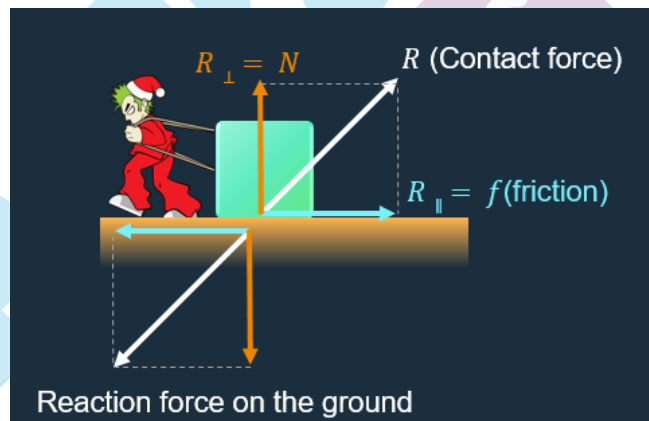
FRICION

1. Friction :

- Friction is a force related to two surfaces in contact, which opposes the relative motion between them.

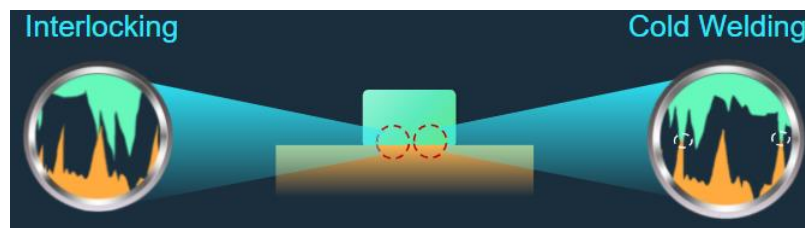
A. Macroscopic View:

- When two bodies are kept in contact, electromagnetic forces act between the microscopic particles between the surfaces of the bodies. As a result, each body exerts a contact force on the other.
- Contact force can be acting in any direction.
- Friction-Component of contact force:** The component of the contact force which is parallel to the surface in contact is called friction or frictional force.



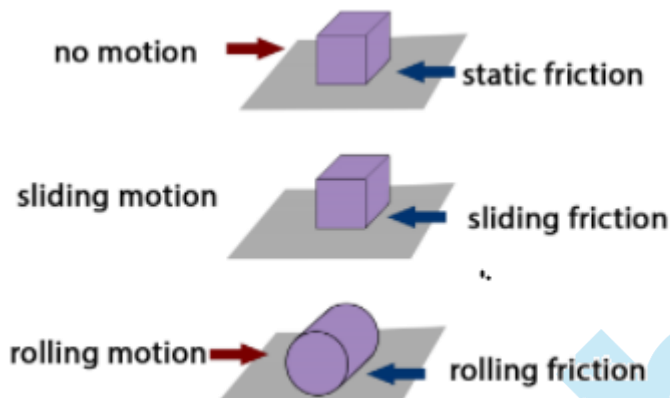
B. Microscopic View:

Interlocking	Cold Weld
The interlocking between irregularities gives rise to friction.	Bonds are formed at contact points due to high pressure, which opposes relative motion that leads to friction.





2. Types of Friction :



A. Static friction:

- The magnitude of the static force is equal in magnitude and opposite in direction to the force applied.
- When the force increases, at some point maximum static friction is reached.
- Coefficient of Static Friction:

$$F_s^{\max} = \mu_s N \text{ and } F_s \leq \mu_s N$$

Where

- F_s is force of static friction
- μ_s is the coefficient of static friction
- N is the normal force
- F_s^{\max} is the maximum force of static friction

B. Kinetic Friction : Kinetic friction is given by

$$f_k = \mu_k N$$

- For an object, the direction of kinetic is opposite to its relative velocity with respect to the other object in contact.
- Not necessarily opposite to the force applied.
- Coefficient of kinetic friction (μ_k) depends on nature of 'surfaces of materials in contact.'
- (μ_k) is independent of the area of the surface in contact.
- (μ_k) is independent of the relative speed of the surfaces in contact.

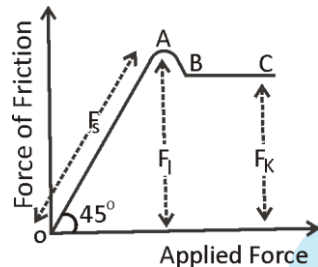


Sliding Friction :

- Sliding friction is defined as the resistance that is created between any two objects when they are sliding against each other.
- Coefficient of Sliding Friction :

$$F_s = \mu_s F_n$$

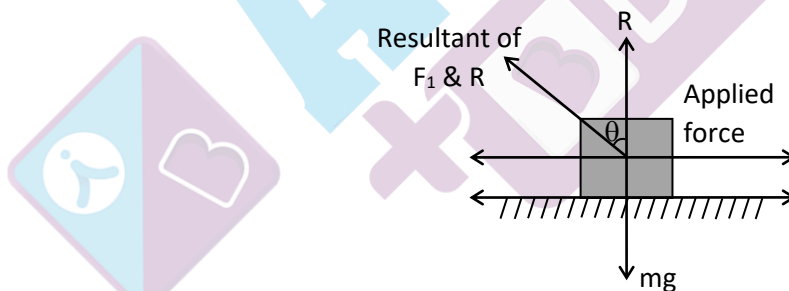
3. Graph between Applied force and Force of Friction :



- The part of the curve represents **static friction (F_s)**. Its value increases with the applied force.
- At point A, the static friction is maximum. This represents the **limiting friction (F_l)**.
- Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve shows the **kinetic friction (F_k)**.
- As the portion BC of the curve is parallel to x-axis therefore kinetic friction does not change with the applied force, it remains constant.

4. Angle of Friction :

The angle which the resultant of limiting friction and normal reaction makes, with the normal reaction is called '**angle of friction**'.



By diagram $\tan \theta = \frac{F_l}{R}$

$$\therefore \tan \theta = \mu_s$$

Angle of Friction $\theta = \tan^{-1}(\mu_s)$

Hence coefficient of static friction is equal to tangent of the angle of friction.

Angle of Repose / Sliding :

It is defined as the maximum angle of inclination of a plane with the horizontal at which a body placed on it just begins to sliding down.



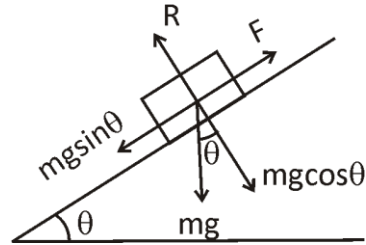
From diagram –

$$F = mg \sin \theta$$

$$R = mg \cos \theta$$

$$\text{So that } \mu_s = \frac{F}{R} = \frac{mg \sin \theta}{mg \cos \theta}$$

$$\text{or } \mu_s = \tan \theta$$



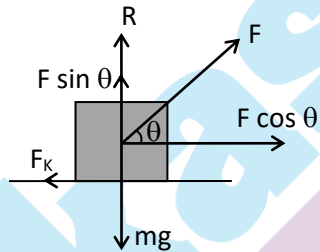
Angle of Repose $\theta = \tan^{-1}(\mu_s)$

Hence the coefficient of static friction is equal to the tangent of angle of repose.

Also **Angle of repose = Angle of Friction**

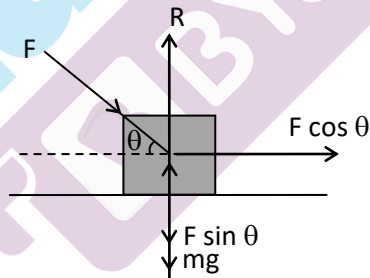
5. Pulling and Pushing of a body :

A. Pulling :



$$F_k = \mu_k (mg - F \sin \theta)$$

B. Pushing :



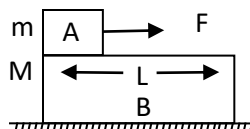
$$\text{or } F_k' = \mu_k (mg + F \sin \theta)$$

Note –

Friction force is more in the case of push. Hence it is easier to pull than to push a body.

6. Motion of two bodies one resting on the other :

(A) A force F is applied on the upper body –





(i) When there is no friction

(a) The body A will move on body B with acceleration

$$a_A = \frac{F}{m}$$

(b) The body B remains at rest $a_B = 0$

(c) If L is the length of body B then body A will fall from B after time t.

$$t = \sqrt{\frac{2L}{a_A}} \quad (\text{by } s = \frac{1}{2}at^2)$$

(ii) If friction is present between A and B only and applied force is less than limiting friction ($F < F_e$) between A and B.

(a) The body A will not slide on body B till $F < F_e$.

Where $F_e = \mu_s mg$

(b) Combined system $(m+M)$ will move together with common acceleration

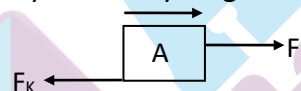
$$a_A = a_B = \frac{F}{m+M}$$

(iii) If friction is present between A and B only and applied force is greater than limiting friction ($F > F_e$) between A and B.

In this condition the two bodies will move in the same direction but with different acceleration.

Here force of kinetic friction $F_k = \mu_k mg$ will oppose the motion of A while will cause the motion of B.

by free body diagram of A

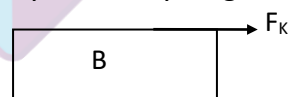


$$F = F_k = ma_A$$

$$a_A = a_A = \frac{F - F_k}{m}$$

where $F_k = \mu_k mg$

by free body diagram of B



$$F_k = Ma_B$$

where $a_B = \frac{F_k}{M}$

As both the bodies are moving in the same direction

Acceleration of body A w.r.t. B $a_{AB} = a_A - a_B$

So that body a will fall from B after time t

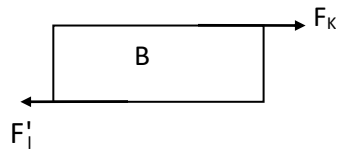
$$t = \sqrt{\frac{2L}{a_{AB}}}$$



(iv) If there is friction between body B and floor

Limiting friction between B and floor (F_l) = $\mu'(m + M)g$

Kinetic friction between A and B (F_k) = $\mu_k mg$



Body B will move only if $F_k > F_l'$

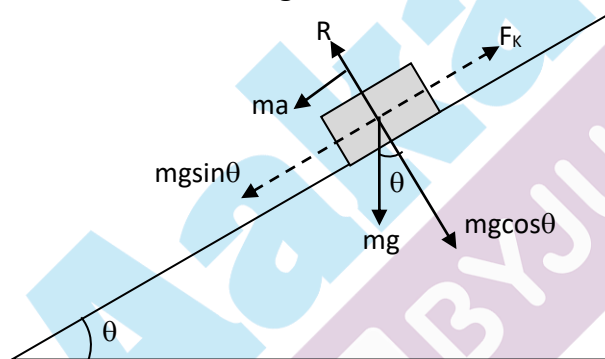
and then $F_k = F_l' = Ma_B$

However if B does not move then static friction will work (Not limiting friction) between body B and the floor.

friction force = applied force (F_k)

7. Acceleration of a block over an inclined plane :

A. Acceleration of a block sliding down :



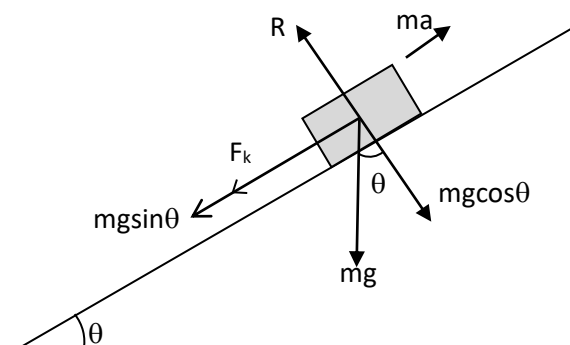
From figure $ma = mg \sin \theta - F_k$

or $ma = mg \sin \theta - \mu_k R$

$\Rightarrow ma = mg \sin \theta - \mu_k mg \cos \theta$

\therefore acceleration $a = g[\sin \theta - \mu_k \cos \theta]$

B. Retardation of a block sliding up :





From figure $ma = mg \sin \theta + F_k$

or $ma = mg \sin \theta + \mu_k mg \cos \theta$

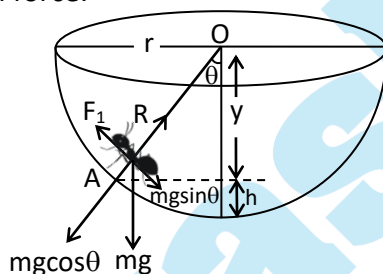
\therefore Retardation $a = g[\sin \theta - \mu_k \cos \theta]$

Note –

For frictionless inclined plane $\mu_k = 0$ and $a = g \sin \theta$

8. Motion of an insect in the rough bowl :

The insect will crawl up the bowl till the component of its weight along the bowl is balanced by limiting frictional force.



From diagram –

$$R = mg \cos \theta$$

$$F_1 = mg \sin \theta$$

$$\text{Hence } \tan \theta = \frac{F_1}{R}$$

$$\text{or } \tan \theta = \mu$$

$$[\text{as } F_1 = \mu R]$$

$$\text{or } \frac{\sqrt{r^2 - y^2}}{y} = \mu \Rightarrow y = \frac{r}{\sqrt{1 + \mu^2}}$$

So that the maximum height to which the insect can crawl up in the bowl is

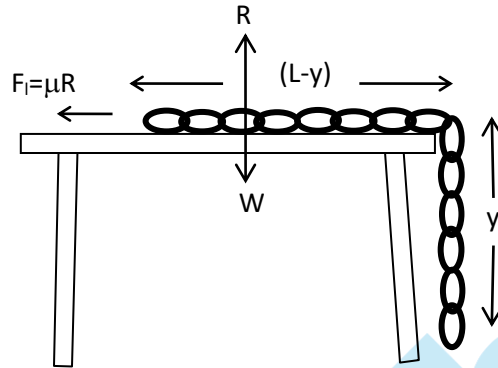
$$h = r - y$$

or

$$h = r \left[1 - \frac{1}{\sqrt{1 + \mu^2}} \right]$$



9. Maximum length of hung chain on table :

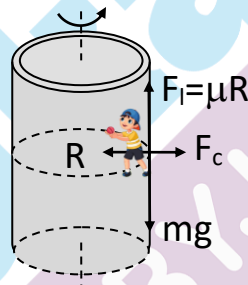


Limiting force of friction = Weight of hanging part of chain

$$\mu R = w \quad \Rightarrow \quad \mu W' = w$$

$$\Rightarrow \mu \frac{M}{L} (L - y) g = \frac{M}{L} y g \quad \text{or} \quad y = \frac{\mu L}{1 + \mu}$$

10. Sticking of a person with the wall of rotor :



The person remains stuck to the wall when

Friction Force = Weight of person

$$\mu R = mg$$

or $\mu F_c = mg$ [$F_c \rightarrow$ centrifugal force]

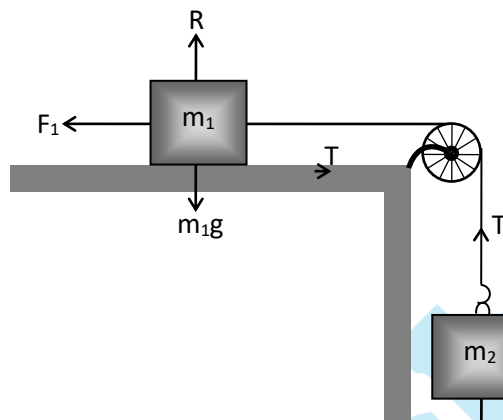
$$\mu m r w_{\min}^2 = mg$$

$$\therefore w_{\min} = \sqrt{\frac{g}{\mu r}}$$



11. Minimum mass hung from the string to just start the motion :

A. When mass m_1 is placed on a rough horizontal surface :



At limiting condition

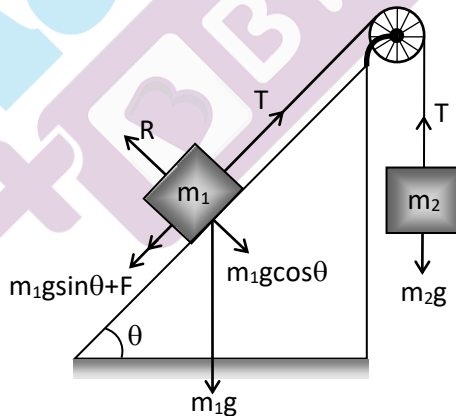
$$T = F_f$$

$$m_2g = \mu R \Rightarrow m_2g = \mu m_1g$$

$$\therefore \boxed{m_2 = \mu m_1}$$

$$\text{Also } \therefore \boxed{\mu = \frac{m_2}{m_1}}$$

B. When mass m_1 is placed on a rough inclined surface :



At limiting condition

$$T = m_1g \sin \theta + \mu R$$

$$\text{or } T = m_1g \sin \theta + \mu m_1g \cos \theta$$

$$\text{Also } T = m_2g$$

Hence on solving we get

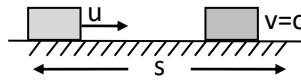


$$m_2 = m_1[\sin\theta + \mu\cos\theta]$$

and $\mu = \left[\frac{m_2}{m_1\cos\theta} - \tan\theta \right]$

12. Stopping of Block due to friction on horizontal road :

A. Distance travelled before coming to rest :



Retarding Force $F = ma = \mu R$

or $ma = \mu mg$

$$\therefore a = \mu g$$

From $v^2 = u^2 + 2as$

$$0 = u^2 - 2\mu gs$$

$$\text{or } s = \frac{u^2}{2\mu g}$$

B. Time taken to come to rest :

From $v = u + at$

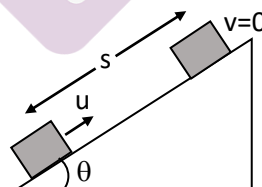
$$0 = u - \mu gt$$

$$[As v = 0, a = -\mu g]$$

$$\therefore t = \frac{u}{\mu g}$$

13. Stopping of Block due to friction on inclined road :

A. Distance travelled before coming to rest :



$$\text{Retardation } a = g[\sin\theta + \mu\cos\theta]$$

From $v^2 = u^2 + 2as$

$$0 = u^2 - 2g[\sin\theta + \mu\cos\theta]s$$

$$\therefore s = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$$