



Fundamentals of Mathematics

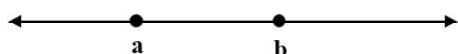
INEQUALITIES

Intervals

The set of numbers between any two real numbers is called interval. The following are the types of interval.

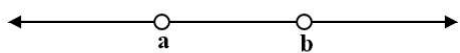
Closed interval

$$x \in [a, b] \equiv \{x : a \leq x \leq b\}$$



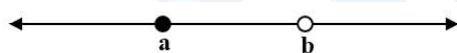
Open Interval

$$x \in (a, b) \equiv \{x : a < x < b\}$$

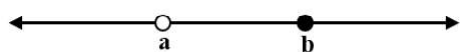


Semi-Open or Semi Closed interval

$$x \in [a, b) = \{x : a \leq x < b\}$$



$$x \in]a, b] \text{ or } (a, b] = \{x : a < x \leq b\}$$



Note:

A set of all real numbers can be expressed as $(-\infty, \infty)$

$$x \in (-\infty, a) \cup (b, \infty) \Rightarrow x \in \mathbb{R} - [a, b]$$

$$x \in (-\infty, a] \cup [b, \infty) \Rightarrow x \in \mathbb{R} - (a, b)$$



Some Important Facts about Inequalities

Some very useful points to remember:

(i) $a \leq b$ either $a < b$ or $a = b$

(ii) $a < b$ and $b < c \Rightarrow a < c$ (transition property)

(iii) $a < b \Rightarrow -a > -b$, i.e., inequality sign reverses if both sides are multiplied by a negative number.

(iv) $a < b$ and $c < d \Rightarrow a + c < b + d$

(v) If both sides of an inequality are multiplied (or divided) by a positive number, inequality does not change. When both sides are multiplied (or divided) by a negative number, inequality gets reversed.

i.e., $a < b \Rightarrow ka < kb$ if $k > 0$ and $ka > kb$ if $k < 0$

(vi) $0 < a < b \Rightarrow a^r < b^r$ if $r > 0$ and $a^r > b^r$ if $r < 0$

(vii) $a + \frac{1}{a} \geq 2$ for $a > 0$ and equality holds for $a = 1$

(viii) $a + \frac{1}{a} \leq -2$ for $a < 0$ and equality holds for $a = -1$

(ix) **Squaring an inequality:**

If $a < b$, then $a^2 < b^2$ does not follow always:

Consider the following illustrations:

$2 < 3 \Rightarrow 4 < 9$, but $-4 < 3 \Rightarrow 16 > 9$

If $-5 < x < 4 \Rightarrow 0 \leq x^2 < 25$

In fact $a < b \Rightarrow a^2 < b^2$ follows only when absolute value of a is less than the absolute value of b or distance of a from zero is less than the distance of b from zero on real number line.

(x) **Law of reciprocal:**

If both sides of inequality have same sign then

i.e., $a > b > 0 \Rightarrow \frac{1}{a} < \frac{1}{b}$ and $a < b < 0 \Rightarrow \frac{1}{a} > \frac{1}{b}$



But if both sides of inequality have opposite signs, then after reciprocal, sign of inequality does not change,

$$\text{i.e. } a < 0 < b \Rightarrow \frac{1}{a} < \frac{1}{b}$$

$$\text{If } x \in [a, b] \Rightarrow \begin{cases} \frac{1}{x} \in \left[\frac{1}{b}, \frac{1}{a} \right], & \text{if } a \text{ and } b \text{ have same sign} \\ \frac{1}{x} \in \left(-\infty, \frac{1}{a} \right] \cup \left[\frac{1}{b}, \infty \right), & \text{if } a \text{ and } b \text{ have opposite signs} \end{cases}$$

Wavy Curve Method

1. The wavy Curve method or method of intervals is a strategy used to solve inequality of form $f(x)/g(x) > 0, (>, <, \geq, \leq)$

2. This method uses the fact that $f(x)/g(x)$ may change signs at critical points.

3. STEPS:

a) **Step 1:** Bring everything to L.H.S and make R.H.S = 0

b) **Step 2:** Factorize the given expression into as many linear factors as possible.

c) **Step 3:** Identify the critical points. (The values of x at which each individual factor is equal to zero)

d) **Step 4:** Plot the critical points on the real number line and determine the regions.

e) **Step 5:** Determine the sign of the given expression in the regions.

f) **Step 6:** Now according to the sign make the required interval. (as per the requirement of the Question).

Shortcut for Sign Convention :

- (i) Reduce the given inequality to as many linear factors as possible.
- (ii) Observe the powers, even or odd for the respective linear factors.
- (iii) For even power, sign of inequality remains unchanged.
- (iv) For odd power, sign of inequality will be changed.

Frequently used Inequalities

(i) $(x-a)(x-b) < 0 \Rightarrow x \in (a, b)$, where $a < b$

(ii) $(x-a)(x-b) > 0 \Rightarrow x \in (-\infty, a) \cup (b, \infty)$, where $a < b$.

(iii) $x^2 \leq a^2 \Rightarrow x \in [-a, a]$



(iv) $x^2 \geq a^2 \Rightarrow x \in (-\infty, -a] \cup [a, \infty)$

(v) If $ax^2+bx+c<0, (a>0) \Rightarrow x \in (\alpha, \beta)$, where $\alpha, \beta (\alpha < \beta)$ are roots of the equation $ax^2+bx+c=0$

(vi) If $ax^2+bx+c>0, (a>0) \Rightarrow x \in (-\infty, \alpha) \cup (\beta, \infty)$ where $\alpha, \beta (\alpha < \beta)$ are roots of the equation $ax^2+bx+c=0$





MODULUS

1. Magnitude of x = Absolute value of $x = |x| = \text{Mod}(x)$

2. $|x|$ = Distance of x from zero along the number line = Length of $AB = AB = |AB| \therefore |x| \geq 0$.

$$3. |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$4. |px + q| = \begin{cases} px + q, & x \geq -\frac{q}{p} \\ -(px + q), & x < -\frac{q}{p} \end{cases}$$

5. Properties involving inequalities: $|x+y| \leq |x| + |y|$; $x, y \in \mathbb{R}$.

Case 1 : $|x+y| = |x| + |y| \Leftrightarrow x \text{ \& } y \text{ are both of the same signs or either of them can be equal to zero i.e. } xy \geq 0$

Case 2: $|x+y| < |x| + |y| \Leftrightarrow x \text{ \& } y \text{ are both of opposite Signs i.e. } xy \leq 0$.

6. Properties involving inequalities:

Case-1: $|x - y| = |x| - |y| \Leftrightarrow x \text{ \& } y \text{ are both of same signs \& } |x| > |y|$.

Case-2: $|x - y| > |x| - |y|$

(a) $x \text{ \& } y$ are both of same signs but $|x| < |y|$ or

(b) $x \text{ \& } y$ are both of opposite signs.

7. Reverse Triangle Inequality:

$$||a| - |b|| \leq |a| + |b|$$

$$8. ||a| - |b|| = |a + b|, \text{ iff } ab < 0$$

9. Properties involving inequalities:

$$1. |x| \leq a; a > 0. \Leftrightarrow -a \leq x \leq a$$

$$2. |x| < a; a > 0. \Leftrightarrow -a < x < a$$

$$3. |x| \geq a; a > 0. \Leftrightarrow x \leq -a \text{ or } x \geq a$$



$$4. |x| > a; a > 0. \Leftrightarrow x < -a \text{ or } x > a$$

$$5. a \leq |x| \leq b; a, b > 0. \Leftrightarrow x \in [-b, -a] \cup [a, b]$$

$$6. a < |x| < b; a, b > 0. \Leftrightarrow x \in (-b, -a) \cup (a, b)$$

$$7. |x + y| < |x| + |y| \text{ if } x \text{ and } y \text{ have opposite signs.}$$

$$|x - y| < |x| + |y| \text{ if } x \text{ and } y \text{ have same signs.}$$

$$|x + y| = |x| + |y| \text{ if } x \text{ and } y \text{ have same sign or at least one of them is zero.}$$

$$|x - y| = |x| + |y| \text{ if } x \text{ and } y \text{ have opposite signs or at least one of them is zero.}$$





SET

Set : A set is a collection of well defined objects which are distinct from each other
 Sets are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by a, b, c ... etc.
 If a is an element of a set A, then we write $a \in A$ and say a belongs to A.
 If a does not belong to A then we write $a \notin A$.

Some important number sets :

N = Set of all natural numbers

$$= \{1, 2, 3, 4, \dots\}$$

W = Set of all whole numbers

$$= \{0, 1, 2, 3, \dots\}$$

Z or I set of all integers

$$= \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Z^+ = Set of all +ve integers

$$= \{1, 2, 3, \dots\} = N.$$

Z^- = Set of all -ve integers

$$= \{-1, -2, -3, \dots\}$$

Z_0 = The set of all non-zero integers.

$$= \{\pm 1, \pm 2, \pm 3, \dots\}$$

Q = The set of all rational numbers.

$$= \left\{ \frac{p}{q} : p, q \in I, q \neq 0 \right\}$$

R = the set of all real numbers.

R-Q = The set of all irrational numbers

Methods to Write a Set :

(i) Roster Method : In this method a set is described by listing elements, separated by commas and enclose them by curly brackets

(ii) Set Builder Form : In this case we write down a property or rule which gives us all the elements of the set

$$A = \{x : P(x)\}$$

Types of Sets :

Null set or Empty set : A set having no element in it is called an Empty set or a null set or void set it is denoted by ϕ or $\{ \}$

A set consisting of at least one element is called a non-empty set or a non-void set.



Singleton : A set consisting of a single element is called a singleton set.

Finite Set : A set which has only finite number of elements is called a finite set.

Order of a finite set : The number of elements in a finite set is called the order of the set A and is denoted $O(A)$ or $n(A)$. It is also called cardinal number of the set.

Infinite set : A set which has an infinite number of elements is called an infinite set.

Equal sets : Two sets A and B are said to be equal if every element of A is a member of B , and every element of B is a member of A .

If sets A and B are equal. We write $A = B$ and A and B are not equal then $A \neq B$

Equivalent sets : Two finite sets A and B are equivalent if their number of elements are same

ie. $n(A) = n(B)$

Note : Equal set always equivalent but equivalent sets may not be equal

Subsets : Let A and B be two sets if every element of A is an element of B , then A is called a subset of B we write $A \subseteq B$

Proper subset : If A is a subset of B and $A \neq B$ then A is a proper subset of B and we write $A \subset B$

Note-1 : Every set is a subset of itself i.e. $A \subseteq A$ for all A

Note-2 : Empty set ϕ is a subset of every set

Note-3 : Clearly $N \subset W \subset Z \subset Q \subset R \subset C$

Note-4 : The total number of subsets of a finite set containing n elements is 2^n

Universal set : A set consisting of all possible elements which occur in the discussion is called a Universal set and is denoted by U

Note : All sets are contained in the universal set

Power set : Let A be any set. The set of all subsets of A is called power set of A and is denoted by $P(A)$

Some Operation on Sets :

(i) **Union of two sets** : $A \cup B = \{x : x \in A \text{ or } x \in B\}$

(ii) **Intersection of two sets** : $A \cap B = \{x : x \in A \text{ and } x \in B\}$

(iii) **Difference of two sets** : $A - B = \{x : x \in A \text{ and } x \notin B\}$

(iv) **Complement of a set** : $A' = \{x : x \notin A \text{ but } x \in U\} = U - A$

(v) **De-Morgan Laws** : $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

(vi) $A - (B \cup C) = (A - B) \cap (A - C)$; $A - (B \cap C) = (A - B) \cup (A - C)$

(vii) **Distributive Laws** : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(viii) **Commutative Laws** : $A \cup B = B \cup A$; $A \cap B = B \cap A$

(ix) **Associative Laws** : $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

(x) $A \cap \phi = \phi$; $A \cap U = A$

$A \cup \phi = A$; $A \cup U = U$

(xi) $A \cap B \subseteq A$; $A \cap B \subseteq B$



(xii) $A \subseteq A \cup B$; $B \subseteq A \cup B$

(xiii) $A \subseteq B \Rightarrow A \cap B = A$

(xiv) $A \subseteq B \Rightarrow A \cup B = B$

Disjoint Sets :

If $A \cap B = \phi$, then A, B are disjoint.

Note : $A \cap A' = \phi$ \therefore A, A' are disjoint.

Symmetric Difference of Sets :

$$A \Delta B = (A - B) \cup (B - A)$$

• $(A')' = A$

• $A \subseteq B \Leftrightarrow B' \subseteq A'$

If A and B are any two sets, then

(i) $A - B = A \cap B'$

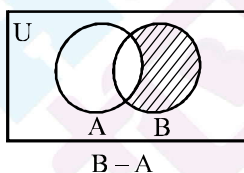
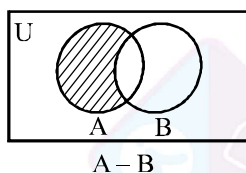
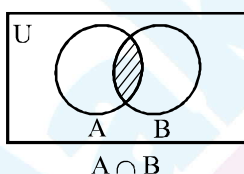
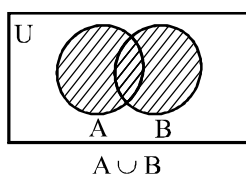
(ii) $B - A = B \cap A'$

(iii) $A - B = A \Leftrightarrow A \cap B = \phi$

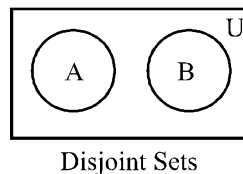
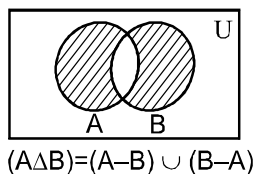
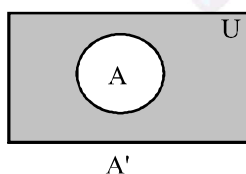
(iv) $(A - B) \cup B = A \cup B$

(v) $(A - B) \cap B = \phi$

(vi) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$



Clearly $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$



Note : $A \cap A' = \phi$, $A \cup A' = U$



Some important Results on Number of Elements in sets :

If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint non-void sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$ i.e. $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \Delta B) = \text{No. of elements which belong to exactly one of A or B}$

$$= n((A - B) \cup (B - A))$$

$$= n(A - B) + n(B - A) \quad [\because (A - B) \text{ and } (B - A) \text{ are disjoint}]$$

$$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$$

$$= n(A) + n(B) - 2n(A \cap B)$$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B, C

$$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$$

(vii) number of elements in exactly one of the sets A, B, C

$$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$$

(viii) $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

(x) If $A_1, A_2 \dots A_n$ are finite, then

$$n\left(\bigcap_{i=1}^n A_i\right) = \sum_{i=1}^n n(A_i) - \sum_{1 \leq i < j \leq n} n(A_i \cap A_j) + \sum_{1 \leq i < j < k \leq n} n(A_i \cap A_j \cap A_k) - \dots + (-1)^{n-1} n(A_1 \cap A_2 \cap \dots \cap A_n)$$



LOGARITHM

- Logarithm of a number**

The logarithm of a number N to the base ' a ' is the exponent indicating the power to which the base ' a ' must be raised to obtain the number N . This number is designated as $\log_a N$.

(a) $\log_a N = x$, read as log of N to the base $a \Leftrightarrow a^x = N$, $N > 0$, $a > 0$, $a \neq 1$, $x \in \mathbb{R}$.

If $a = 10$ then we write $\log N$ or $\log_{10} N$ and if $a = e$ we write $\ln N$ or $\log_e N$ (Natural log)

(b) $\log_a 1 = 0$

(c) $\log_a a = 1$

(d) $\log_{1/a} a = -1$

(e) $\log_a (x \cdot y) = \log_a x + \log_a y$; $x, y > 0$

(f) $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$; $x, y > 0$

(g) $\log_a x^p = p \log_a x$; $x > 0$

(h) $\log_{a^q} x = \frac{1}{q} \log_a x$; $x > 0$

(i) $\log_a x = \frac{1}{\log_x a}$; $x > 0$, $x \neq 1$

(j) $\log_a x = \frac{\log_b x}{\log_b a}$; $x > 0$, $a, b > 0$, $b \neq 1$, $a \neq 1$

(k) $\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$; $a, b, c > 0, \neq 1$ ($d > 0$)

(l) $a^{\log_a x} = x$; $a > 0$, $a \neq 1$

(m) $a^{\log_b c} = c^{\log_b a}$; $a, b, c > 0$; $b \neq 1$

(n) $\log_a x < \log_a y \Leftrightarrow \begin{cases} x < y & \text{if } a > 1 \\ x > y & \text{if } 0 < a < 1 \end{cases}$

(o) $\log_a x = \log_a y \Rightarrow x = y$; $x, y > 0$; $a > 0$, $a \neq 1$

(p) $e^{\ln_a x} = a^x$

(q) $\log_{10} 2 = 0.3010$; $\log_{10} 3 = 0.4771$; $\ln 2 = 0.693$, $\ln 10 = 2.303$

(r) If $a > 1$ then $\log_a x < P \Rightarrow 0 < x < a^P, x > 1$

(s) If $a > 1$ then $\log_a x > P \Rightarrow x > a^P, x > 1$

(t) If $0 < a < 1$ then $\log_a x < P \Rightarrow x > a^P, x > 1$

(u) If $0 < a < 1$ then $\log_a x > P \Rightarrow 0 < x < a^P, x > 1$



QUADRATIC EQUATION

1. Solution of quadratic equation & relation between roots & Co-efficients :

(a) The solutions of the quadratic equation, $ax^2+bx+c=0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(b) The expression $b^2 - 4ac = D$ is called the discriminant of the quadratic equation.

(c) If α & β are the roots of the quadratic equation $ax^2+bx+c=0$ then ;

(i) $\alpha+\beta = -b/a$ (ii) $\alpha\beta = c/a$ (iii) $|\alpha-\beta| = \sqrt{D}/|a|$

(d) Quadratic equation whose roots are α & β is $(x-\alpha)(x-\beta) = 0$ i.e. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e. $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$.

2. Nature of roots :

(a) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{R}$ & $a \neq 0$ then :

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal).

(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal).

(iii) $D < 0 \Leftrightarrow$ roots are imaginary.

(iv) If $p + iq$ is one root of a quadratic equation, then the other root must be the conjugate $p - iq$ & vice versa.

($p, q \in \mathbb{R}$) & $i = \sqrt{-1}$).

(b) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{Q}$ & $a \neq 0$ then :

(i) If D is a perfect square, then roots are rational.

(ii) if $\alpha = p + \sqrt{q}$ is one root in the irrational (where p is rational and \sqrt{q} is a surd) then other root will be $p - \sqrt{q}$

3. Common roots of two quadratic equations :

(a) Only one common root.

Let α be the common root of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$ then

$$a\alpha^2 + b\alpha + c = 0$$

$$\& a'\alpha^2 + b'\alpha + c' = 0. \text{ By Cramer's Rule}$$

$$\frac{\alpha^2}{bc' - b'c} = \frac{\alpha}{a'c - ac'} = \frac{1}{ab' - a'b}$$



Therefore, $\alpha = \frac{ca' - c'a}{ab' - a'b} = \frac{bc' - b'c}{a'c - ac'}$

So the condition for a common root is $(ca' - c'a)^2 = (ab' - a'b)(bc' - b'c)$

(b) If both roots are same then $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

4. Roots under particular cases :

Let the quadratic equation $ax^2 + bx + c = 0$ has real roots and

(a) If $b = 0 \Rightarrow$ roots are of equal magnitude but of opposite sign

(b) If $c = 0 \Rightarrow$ one root is zero other is $-b/a$

(c) If $a = c \Rightarrow$ roots are reciprocal to each other, $b \neq 0$.

(d) If $\left. \begin{matrix} a > 0 & c < 0 \\ a < 0 & c > 0 \end{matrix} \right\} \Rightarrow$ roots are of opposite signs

(e) If $\left. \begin{matrix} a > 0, & b > 0 & c > 0 \\ a < 0, & b < 0 & c < 0 \end{matrix} \right\} \Rightarrow$ both roots are negative

(f) If $\left. \begin{matrix} a > 0, & b < 0 & c > 0 \\ a < 0, & b > 0 & c < 0 \end{matrix} \right\} \Rightarrow$ both roots are positive

(g) If sign of $a =$ sign of $b \neq$ sign of $c \Rightarrow$ Greater root in magnitude is negative.

(h) If sign of $b =$ sign of $c \neq$ sign of $a \Rightarrow$ Greater root in magnitude is positive.

(i) If $a + b + c = 0 \Rightarrow$ one root is 1 and second root is c/a .

5. Maximum & Minimum values of quadratic expression :

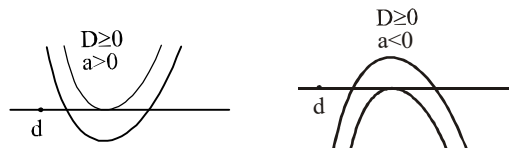
Maximum & Minimum values of expression $y = ax^2 + bx + c$ is $\frac{-D}{4a}$ which occurs at $x = (-b/2a)$ according as $a < 0$ or $a > 0$.

$y \in \left[\frac{-D}{4a}, \infty \right)$ if $a > 0$ & $y \in \left(-\infty, \frac{-D}{4a} \right]$ if $a < 0$.

6. Location of roots :

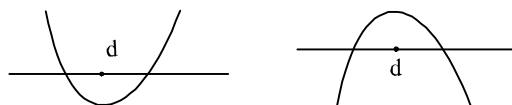
Let $f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$, $a \neq 0$

(a) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'd' are $D \geq 0$; $a \cdot f(d) > 0$ & $(-b/2a) > d$.

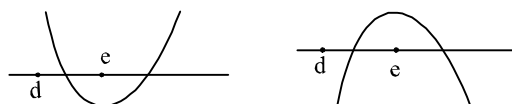




(b) Conditions for the both roots of $f(x) = 0$ to lie either side of the number 'd' i.e. number 'd' lies between the roots of $f(x) = 0$ is $D > 0$, $a \cdot f(d) < 0$.

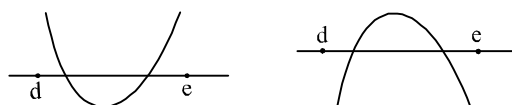


(c) Conditions for exactly one root of $f(x) = 0$ to lie in the interval (d,e) i.e.. $d < x < e$ is $D > 0$, $f(d) \cdot f(e) < 0$



(d) Conditions that both roots of $f(x) = 0$ are in between d & e are (here $d < e$).

$D \geq 0$; $a \cdot f(d) > 0$ & $a \cdot f(e) > 0$; $d < (-b/2a) < e$



7. General quadratic expression in two variables ::

$f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factors if $\Delta = abc +$

$$2fgh - af^2 - bg^2 - ch^2 = 0 \text{ OR } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0, h^2 - ab > 0$$

8. Theory of equations :

If $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ are the roots of the equation ;

$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n = 0$ where a_0, a_1, \dots, a_n are constants $a_0 \neq 0$ then,

$$\sum \alpha_1 = -\frac{a_1}{a_0}, \sum \alpha_1 \alpha_2 = \frac{a_2}{a_0},$$

$$\sum \alpha_1 \alpha_2 \alpha_3 = -\frac{a_3}{a_0}, \dots, \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n = (-1)^n \frac{a_n}{a_0}$$

Note :

(i) Every odd degree equation has at least one real root whose sign is opposite to that of its last term, given coefficient of highest degree term is (+) ve {If not then make it (+)ve}.

Ex. $x^3 - x^2 + x - 1 = 0$

(ii) Even degree polynomial whose last term is (-)ve & coefficient of highest degree term is (+) ve has at least two real roots, one (+)ve & one (-)ve.

(iii) If equation contains only even power of x & all coefficients are (+)ve, then all roots are imaginary.



RELATIONS AND FUNCTIONS

RELATIONS

Introduction :

Cartesian product : Let A and B be any two non empty sets then the Cartesian product of sets A and B, denoted by $A \times B$, is defined as the set consisting of all ordered pairs (a, b) for which $a \in A$ and $b \in B$.

Let A and B be two non empty sets. Then a relation R from A to B is a subset of $A \times B$ thus, R is a relation from A to B $\Leftrightarrow R \subseteq A \times B$.

Total Number of Relations : Let A and B be two non-empty finite sets consisting of m and n elements respectively then $A \times B$ consists of mn ordered pairs. So, total number of subsets of $A \times B$ is 2^{mn} .

Domain and Range of a relation : Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called to domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Domain (R) = $\{a : (a, b) \in R\}$

and, Range (R) = $\{b : (a, b) \in R\}$

It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B.

Inverse Relation : Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1} , is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

Also, Dom(R) = Range(R^{-1}) and Range (R) = Dom (R^{-1})

TYPES OF RELATIONS :

In this section we intend to define various types of relations on a given set A.



Void Relation : Let A be a set. Then $\phi \subseteq A \times A$ and so it is a relation on A . This relation is called the void or empty relation on A .

Universal Relation : Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A . This relation is called the universal relation on A .

Identity Relation : Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A .

In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only.

Reflexive Relation : A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R on a set A is not reflexive if there exists an element $A \in A$ such that $(a, a) \notin R$.

Every Identity relation is reflexive relation but every reflexive relation is not an identity relation.

Symmetric Relation : A relation R on a set A is said to be a symmetric relation iff

$$(a, b) \in R \Rightarrow (b, a) \in R \text{ for all } a, b \in A$$

$$\text{i.e. } a R b \Rightarrow b R a \text{ for all } a, b \in A.$$

Transitive Relation : Let A be any set. Relation R on A is said to be a transitive relation iff

$$(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R \text{ for all } a, b, c \in A$$

Antisymmetric Relation : Let A be any set. A relation R on set A is said to be an antisymmetric relation iff

$$(a, b) \in R \text{ and } (b, a) \in R \Rightarrow a = b \text{ for all } a, b \in A$$

Equivalence Relation : A relation R on a set A is said to be an equivalence relation on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Note : It is not necessary that every relation which is symmetric and transitive is also reflexive.



FUNCTIONS

1. Definition :

Relation $f : A \rightarrow B$ is called a function if each and every element of A is uniquely mapped to the elements of set B . Set A is called the domain and set B is called the codomain $f : A \rightarrow B, y = f(x)$, x is called argument or independent variable and y is called dependent variable.

Pictorially : $\xrightarrow[\text{input}]{x} \quad f \quad \xrightarrow[\text{output}]{f(x)=y}$

y is called the image of x & x is the pre-image of y , under f . Every function $f : A \rightarrow B$ satisfy the following conditions.

- (i) $f \subset A \times B$ (ii) $\forall a \in A \exists b \in B$ such that $(a, b) \in f$ and
- (iii) If $(a, b) \in f$ & $(a, c) \in f \Rightarrow b = c$

Note: Number of functions for $f : A \rightarrow B$ is $(n(B))^{n(A)}$

2. Domain, Co-domain & Range Of A Function :

Let $f : A \rightarrow B$ be a function, then the set A is known as the domain of ' f ' & the set B is known as co-domain of ' f '. The set of f images of all the elements of A is known as the range of ' f '. Thus:

Domain of $f = \{x \mid x \in A, (x, f(x)) \in f\}$

Range of $f = \{f(x) \mid x \in A, f(x) \in B\}$

Range is a subset of co-domain.

3. Important types of function :

(a) Polynomial function :

If a function ' f ' given by $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n$ where n is a non negative integer and $a_0, a_1, a_2, \dots, a_n$ are real numbers and $a_0 \neq 0$, then f is called a polynomial function of degree n .

Note :

(i) A polynomial of degree one with no constant term is called an odd linear function.

i.e. $f(x) = ax, a \neq 0$

(ii) There are two polynomial functions, satisfying the relation;

$f(x) \cdot f(1/x) = f(x) + f(1/x)$. They are

(a) $f(x) = 1 + x^n$ &

(b) $f(x) = 1 - x^n$, where n is a positive integer.

(iii) Domain of a polynomial function is \mathbb{R}

(iv) Range of odd degree polynomial is \mathbb{R} whereas range of an even degree polynomial is never \mathbb{R} .



(b) Algebraic function :

A function 'f' is called an algebraic function if it can be constructed using algebraic operations (such as addition, subtraction, multiplication, division and taking radicals) starting with polynomials.

(c) Rational function :

A rational function is a function of the form $y = f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ & $h(x)$ are polynomials & $h(x) \neq 0$,

Domain : $\mathbb{R} - \{x \mid h(x)=0\}$

Any rational function is automatically an algebraic function.

(d) Exponential and Logarithmic Function :

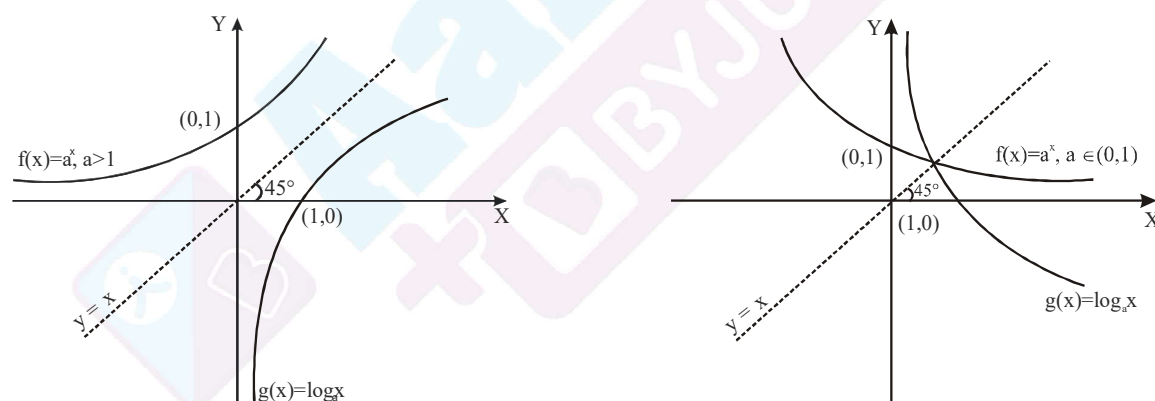
A function $f(x) = a^x$ ($a > 0$), $a \neq 1$, $x \in \mathbb{R}$ is called an exponential function. The inverse of the exponential function is called the logarithmic function, i.e. $g(x) = \log_a x$.

Note that $f(x)$ & $g(x)$ are inverse of each other & their graphs are as shown. (functions are mirror image of each other about the line $y = x$)

Domain of a^x is \mathbb{R} , $a > 0$

Range \mathbb{R}^+

Domain of $\log_a x$ is \mathbb{R}^+ , $a > 0$, $a \neq 1$ **Range** \mathbb{R}



(e) Absolute value function :

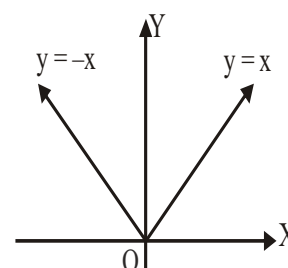
It is defined as

$$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Also defined as $\max\{x, -x\}$

Domain : \mathbb{R}

Range : $[0, \infty)$





Note : $f(x) = \frac{1}{|x|}$, **Domain :** $\mathbb{R} - \{0\}$, **Range :** \mathbb{R}^+

Properties of modulus function :

For any $x, y, a \in \mathbb{R}$.

(i) $|x| \geq 0$

(ii) $|x| = |-x|$

(iii) $|xy| = |x| |y|$

(iv) $\left| \frac{x}{y} \right| = \frac{|x|}{|y|} : y \neq 0$

(v) $|x| = a \Rightarrow x = \pm a$ (vi) $\sqrt{x^2} = |x|$

(vii) $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$. where a is positive.

(viii) $|x| \leq a \Rightarrow x \in [-a, a]$ where a is positive.

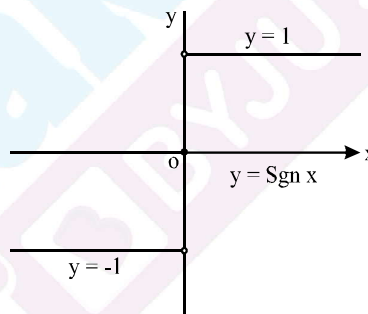
(ix) $|x| > |y| \Rightarrow x^2 > y^2$.

(x) $||x| - |y|| \leq |x| + |y| = \begin{cases} (a) |x| + |y| = |x + y| \Rightarrow xy \geq 0 \\ (b) |x| + |y| = |x - y| \Rightarrow xy \leq 0 \end{cases}$

(f) Signum function :

Signum function $y = \text{sgn}(x)$ is defined as follows

$$y = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases} = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$



Domain : \mathbb{R}

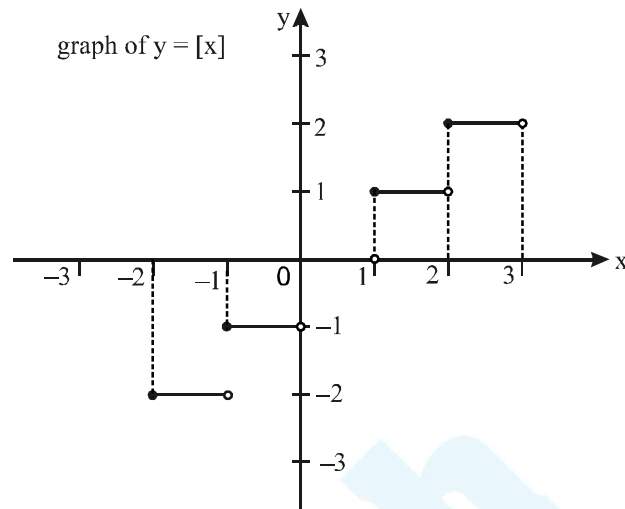
Range : $\{-1, 0, 1\}$

(g) Greatest integer or step function :

The function $y = f(x) = [x]$ is called the greatest integer function where $[x]$ denotes the greatest integer less than or equal to x . Note that for :



x	$[x]$
$[-2, -1)$	-2
$[-1, 0)$	-1
$[0, 1)$	0
$[1, 2)$	1



Domain : \mathbb{R}

Range : \mathbb{I} (Set of integers)

Properties of greatest integer function :

(i) $[x] \leq x < [x] + 1$ and $x - 1 < [x] \leq x$, $0 \leq x - [x] < 1$

(ii) $[x] + [-x] = \begin{cases} 0, & x \in \mathbb{I} \\ -1, & x \notin \mathbb{I} \end{cases}$

(iii) $[x + I] = [x] + I$, if I is an integer

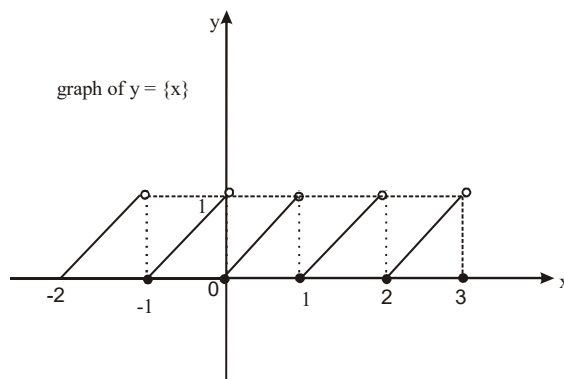
(iv) $[x + y] \geq [x] + [y]$

Note : $f(x) = \frac{1}{[x]}$ **Domain :** $\mathbb{R} - [0, 1)$ **Range :** $\{x \mid x = \frac{1}{n}, n \in \mathbb{I}_0\}$

(h) Fractional part function :

It is defined as : $g(x) = \{x\} = x - [x]$ e.g.

x	$\{x\}$
$[-2, -1)$	$x+2$
$[-1, 0)$	$x+1$
$[0, 1)$	x
$[1, 2)$	$x-1$



Domain : \mathbb{R}

Range : $[0, 1)$

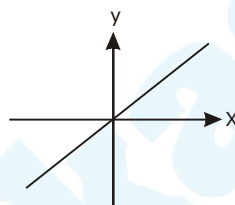
Period : 1

Note : $f(x) = \frac{1}{\{x\}}$

Domain : $\mathbb{R} - \mathbb{I}$

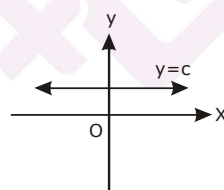
Range : $(1, \infty)$

(i) Identity function :



The function $f : A \rightarrow A$ defined by $f(x) = x \forall x \in A$ is called the identity function on A and is denoted by I_A .

(j) Constant function :



$f: A \rightarrow B$ is said to be constant function if every element of A has the same f image in B . Thus $f : A \rightarrow B ; f(x) = c \forall x \in A$, $c \in B$ is constant function

Domain : \mathbb{R} **Range :** $\{c\}$

(k) Trigonometric functions :

(i) Sine function $f(x) = \sin x$

Domain : \mathbb{R} **Range :** $[-1, 1]$, **Period :** 2π

(ii) Cosine function $f(x) = \cos x$



Domain : \mathbb{R} **Range:** $[-1, 1]$, **Period :** 2π

(iii) Tangent function $f(x) = \tan x$

Domain : $\mathbb{R} - \left\{ x \mid x = \frac{(2n+1)\pi}{2}, n \in \mathbb{I} \right\}$ **Range :** \mathbb{R} , **Period :** π

(iv) Cosecant function $f(x) = \operatorname{cosec} x$

Domain : $\mathbb{R} - \{x \mid x = n\pi, n \in \mathbb{I}\}$ **Range :** $\mathbb{R} - (-1, 1)$, **Period :** 2π

(v) Secant function $f(x) = \sec x$

Domain : $\mathbb{R} - \{x \mid x = (2n+1)\pi/2 : n \in \mathbb{I}\}$

Range : $\mathbb{R} - (-1, 1)$, **Period :** 2π

(vi) Cotangent function

$f(x) = \cot x$

Domain : $\mathbb{R} - \{x \mid x = n\pi, n \in \mathbb{I}\}$, **Range :** \mathbb{R} , **Period :** π

(I) Inverse Trigonometric Functions :

(i) $f(x) = \sin^{-1} x$ **Domain :** $[-1, 1]$ **Range :** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $f(x) = \cos^{-1} x$ **Domain :** $[-1, 1]$ **Range :** $[0, \pi]$

(iii) $f(x) = \tan^{-1} x$ **Domain :** \mathbb{R} **Range :** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $f(x) = \cot^{-1} x$ **Domain :** \mathbb{R} **Range :** $(0, \pi)$

(v) $f(x) = \operatorname{cosec}^{-1} x$ **Domain :** $\mathbb{R} - (-1, 1)$ **Range :** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(vi) $f(x) = \sec^{-1} x$ **Domain :** $\mathbb{R} - (-1, 1)$ **Range :** $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

4. Equal or identical function :

Two functions f & g are said to be equal if :

(a) The domain of f = the domain of g

(b) The range of f = range of g and

(c) $f(x) = g(x)$, for every x belonging to their common domain (i.e. should have the same graph)

5. Algebraic operations on functions :

If f & g are real valued functions of x with domain set A , B respectively, $f + g$, $f - g$, $(f \cdot g)$ & (f/g) as follows :

(a) $(f \pm g)(x) = f(x) \pm g(x)$ domain in each case is $A \cap B$

(b) $(f \cdot g)(x) = f(x) \cdot g(x)$ domain is $A \cap B$

(c) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ domain $A \cap B - \{x \mid g(x) = 0\}$



6. Classification of functions :

(a) One-One function (Injective mapping) :

A function $f : A \rightarrow B$ is said to be a one-one function or injective mapping if different elements of A have different f images in B . Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$.

Note:

(i) Any continuous function which is entirely increasing or decreasing in whole domain is one-one.

(ii) Any line parallel to x-axis cuts the graph of the function at one point. Then the function is one-one.

(iii) If $f : A \rightarrow B$ is one-one function, then number of one-one functions are ${}^{n(B)}P_{n(A)}$.

(b) Many-one function :

A function $f : A \rightarrow B$ is said to be a many one function if two or more distinct elements of A have the same f image in B .

Thus $f : A \rightarrow B$ is many one if $\exists x_1, x_2 \in A$, $f(x_1) = f(x_2)$ but $x_1 \neq x_2$

Note : If a continuous function has local maximum or local minimum, then $f(x)$ is many-one because atleast one line parallel to x-axis will intersect the graph of function atleast twice.

Total number of function = number of one-one functions + number of many-one function

(c) Onto function (Surjection) :

A function $f : A \rightarrow B$ is said to be an onto function is every element of B has atleast one pre image in set A .

Range of f = codomain of f .

Note: If A and B are two sets having m and n elements respectively such

that $1 \leq n \leq m$, then number of onto functions from A to B is $\sum_{r=1}^n (-1)^{n-r} \times {}^nC_r r^m$.

(d) Into function :

If $f : A \rightarrow B$ is such that there exists atleast one element in co-domain which is not the image of any element in domain, then $f(x)$ is into.

Number of into functions = total functions – onto functions.

Note :

(i) If ' f ' is both injective & surjective, then it is called a **Bijjective** mapping. The bijective functions are also named as invertible functions.

(ii) If a set A contains n distinct elements then the number of different functions defined from $A \rightarrow A$ is n^n & out of it $n!$ are one one and rest are many one.

(iii) $f : \mathbb{R} \rightarrow \mathbb{R}$ is a polynomial

(a) Of even degree, then it will neither be injective nor surjective.

(b) Of odd degree, then it will always be surjective, no general comment can be given on its injectivity.

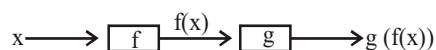


7. Composite of uniformly & Non-Uniformly defined function:

Let $f:A \rightarrow B$ & $g:B \rightarrow C$ be two functions. Then the function $g \circ f:A \rightarrow C$ defined by $(g \circ f)(x) = g(f(x))$

$\forall x \in A$ is called the composite of the two functions f & g .

Hence in $g \circ f(x)$ the range of ' f ' must be a subset of the domain of ' g '



Properties of composite functions:

(a) In general composite functions is not commutative i.e. $g \circ f \neq f \circ g$.

(b) The composite of functions is associative i.e. if f, g, h are three functions such that $f \circ (g \circ h)$ & $(f \circ g) \circ h$ are defined, then $f \circ (g \circ h) = (f \circ g) \circ h$.

(c) The composite of two bijections is a bijection i.e. if f & g are two bijections such that $g \circ f$ is defined, then $g \circ f$ is also a bijection.

(d) If $g \circ f$ is one-one function then f is one-one but g may not be one-one.

(e) If $f:A \rightarrow B, g:B \rightarrow C$ are two functions and $g \circ f:A \rightarrow C$ is onto then $g:B \rightarrow C$ is onto.

8. Homogeneous functions :

A function is said to be homogeneous with respect to any set of variables when each of its terms is of the same degree with respect to those variables.

For examples $5x^2 + 3y^2 - xy$ is homogenous in x & y . Symbolically if, $f(tx, ty) = t^n f(x, y)$ then $f(x, y)$ is homogeneous function of degree n .

9. Bounded function :

A function is said to be bounded if $|f(x)| \leq M$, where M is a finite quantity.

10. Implicit & explicit function :

A function defined by an equation not solved for the dependent variable is called an **implicit function**. e.g. the equations $x^3 + y^3 = 1$ & $x^y = y^x$, defines y as an implicit function. If y has been expressed in terms of x alone then it is called an **Explicit function**.

11. Inverse of a function :

Let $f:A \rightarrow B$ be a one-one & onto function, then there exists a unique function $g:B \rightarrow A$ such that $f(x) = y \Leftrightarrow g(y) = x, \forall x \in A \text{ & } y \in B$. Then g is said to be inverse of f .

Thus $g = f^{-1}: B \rightarrow A = \{(f(x), x) | (x, f(x)) \in f\}$.



Properties of inverse function :

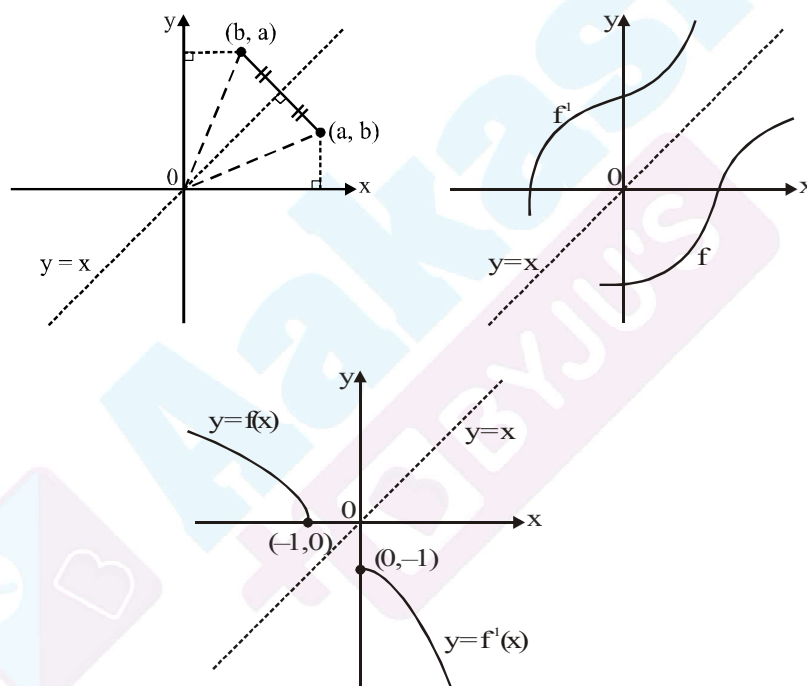
(a) The inverse of a bijection is unique.

(b) If $f : A \rightarrow B$ is a bijection & $g : B \rightarrow A$ is the inverse of f , then $f \circ g = I_B$ and $g \circ f = I_A$, where I_A & I_B are identity functions on

the sets A & B respectively. If $f \circ f = I$, then f is inverse of itself. **(c)** The inverse of a bijection is also a bijection.

(d) If f & g are two bijections $f : A \rightarrow B$, $g : B \rightarrow C$ & $g \circ f$ exist, then the inverse of $g \circ f$ exists and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(e) If $f : A \rightarrow B$ is bijective, $a \in A$, $b \in B$, $f(a) = b$ if and only if $f^{-1}(b) = a$, the point (a, b) is on the graph of ' f ' if and only if the point (b, a) is on the graph of f^{-1} and we get the point (b, a) from (a, b) by reflecting about the line $y = x$.



The graph of f^{-1} is obtained by reflecting the graph of f about the line $y = x$.

12. Odd & even functions :

If a function is such that whenever ' x ' is in it's domain ' $-x$ ' is also in it's domain & it satisfies $f(-x) = f(x)$ it is an even function

$f(-x) = -f(x)$ it is an odd function



Note :

- (i) A function may neither be an odd nor an even.
- (ii) Inverse of an even function in its entire domain is not defined, as it is many -one function.
- (iii) Every even function is symmetric about the y-axis & every odd function is symmetric about the origin.
- (iv) Every function which has '-x' in it's domain whenever 'x' is in it's domain, can be expressed as the sum of an even & an odd function .

$$\text{i.e. } f(x) = \underbrace{\frac{f(x) + f(-x)}{2}}_{\text{EVEN}} + \underbrace{\frac{f(x) - f(-x)}{2}}_{\text{ODD}}$$

- (v) The only function which is defined on the entire number line & even and odd at the same time is $f(x) = 0$
- (vi) If $f(x)$ and $g(x)$ both are even or both are odd then the function $f(x).g(x)$. will be even but if any one of them is odd & other even, then $f.g$. will be odd.

13. Periodic function :

A function $f(x)$ is called periodic if there exists a least positive number $T(T > 0)$ called the period of the function such that $f(x + T) = f(x)$, for all values of x within the domain of $f(x)$.

Note : :

- (i) Inverse of a periodic function does not exist.
- (ii) Every constant function is periodic, but its fundamental period is not defined.
- (iii) If $f(x)$ has a period T & $g(x)$ also has a period T then it does not mean that $f(x) + g(x)$ must have a period T . e.g. $f(x) = |\sin x| + |\cos x|$.
- (iv) If $f(x)$ has period p and $g(x)$ has period q . then period of $f(x) + g(x)$ will be LCM of p & q provided $f(x)$ & $g(x)$ are non interchangeable. If $f(x)$ & $g(x)$ can be interchanged by adding a least positive number r , then smaller of LCM & r will be the period.
- (v) If $f(x)$ has period p , then $\frac{1}{f(x)}$ and $\sqrt{f(x)}$ also has a period p .
- (vi) If $f(x)$ has period T then $f(ax + b)$ has a period T/a ($a > 0$)
- (vii) $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|$ & $|\csc x|$ are periodic function with period π .
- (viii) $\sin^n x, \cos^n x, \sec^n x, \csc^n x$, are periodic functions with period 2π when 'n' is odd or π when n is even.
- (ix) $\tan^n x, \cos^n x$, are periodic function with period π .

14. General :

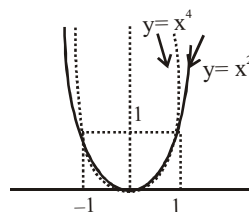
If x, y are independent variables, and if :

- (a) $f(xy) = f(x) + f(y) \Rightarrow f(x) = k \ln x$
- (b) $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n, n \in \mathbb{R}$ or $f(x) = 0$
- (c) $f(x + y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx}$ or $f(x) = 0$
- (d) $f(x + y) = f(x) + f(y) \Rightarrow f(x) = kx$, where k is a constant.

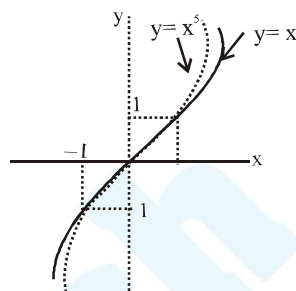


15. Some Basic Functions & Their Graphs:

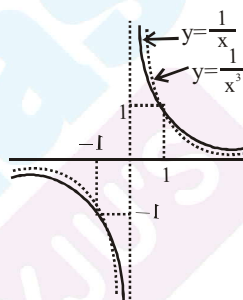
(a) $y = x^{2n}$, where $n \in \mathbb{N}$



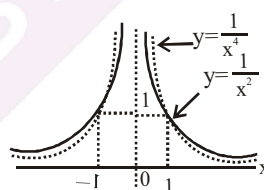
(b) $y = x^{2n+1}$, where $n \in \mathbb{N}$



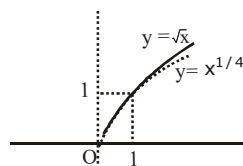
(c) $y = \frac{1}{x^{2n-1}}$ where $n \in \mathbb{N}$



(d) $y = \frac{1}{x^{2n}}$ where $n \in \mathbb{N}$

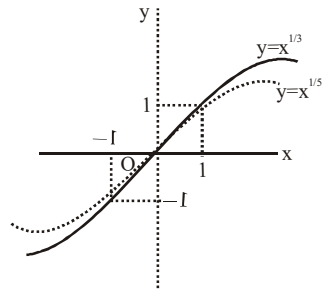


(e) $y = x^{1/2n}$ where $n \in \mathbb{N}$

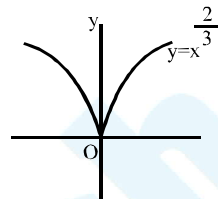




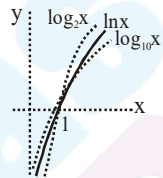
(f) $y = x^{\frac{1}{2n+1}}$ where $n \in \mathbb{N}$



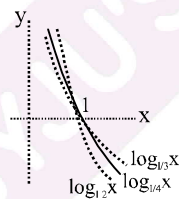
$y = x^{2/3}$



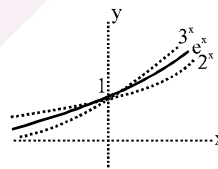
(g) $y = \log_a x$
when $a > 1$



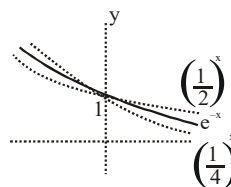
when $0 < a < 1$
 $y = \log_a x$



(h) $y = a^x$
 $a > 1$



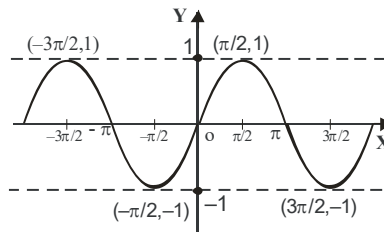
$0 < a < 1$
 $y = a^x$



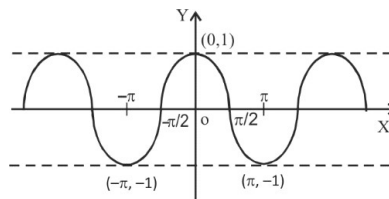


(f) Trigonometric functions :

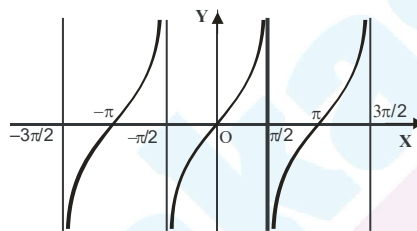
(i) $f(x) = \sin x$



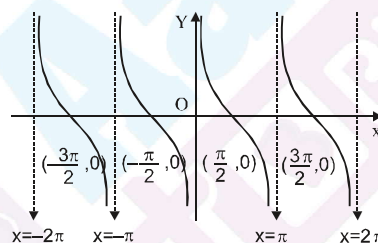
(ii) $f(x) = \cos x$



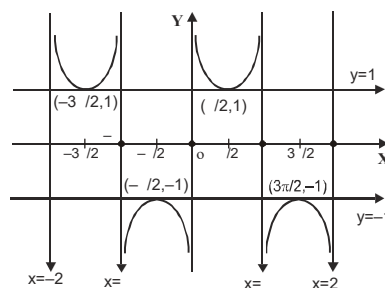
(iii) $y = \tan x$



(iv) $f(x) = \cot x$

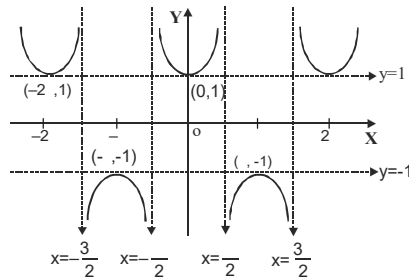


(v) $f(x) = \operatorname{cosec} x$

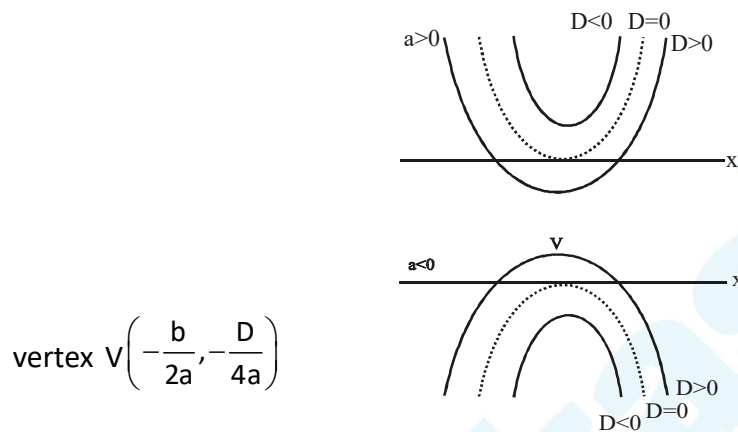




(vi) $f(x) = \sec x$



(j) $y = ax^2 + bx + c$



vertex $V\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

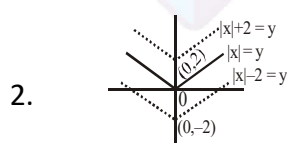
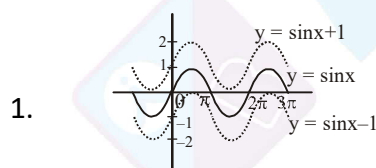
where $D = b^2 - 4ac$

16. Transformation of graph:

(a) when $y = f(x)$ transforms to $y = f(x) + k$ if $k > 0$ then shift graph of $f(x)$ upwards through k units.

if $k < 0$ then shift graph $f(x)$ downwards through k units.

Examples:



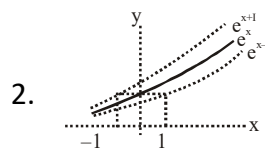
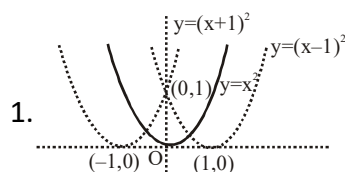
(b) $y = f(x)$ transforms to $y = f(x + k)$:

if $k > 0$ then shift graph of $f(x)$ through k units towards left.

if $k < 0$ then shift graph of $f(x)$ through k units towards right.



Examples:

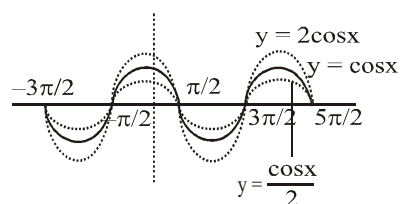


(c) $y = f(x)$ transformations to $y = kf(x)$:

if $k > 1$ then stretch graph of $f(x)$ k times along y -axis

if $0 < k < 1$ then shrink graph of $f(x)$, k times along y -axis

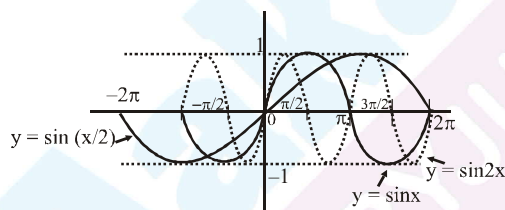
Examples :



(d) $y = f(x)$ transforms to $y = f(kx)$:

if $k > 1$ then shrink graph of $f(x)$. ' k ' times along x -axis if $0 < k < 1$ then stretch graph of (x) , ' k ' times along x -axis

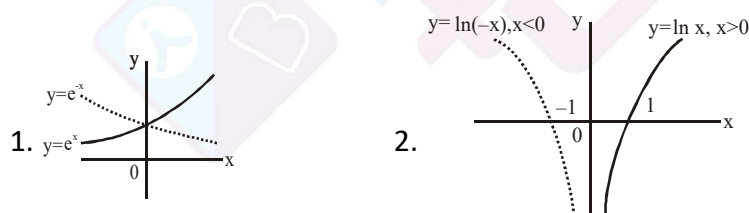
Example:



(e) $y = f(x)$ transforms to $y = f(-x)$:

Take mirror image of the curve $y = f(x)$ in y -axis as plane mirror

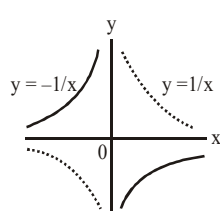
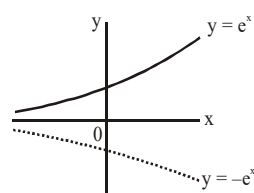
Example:



(f) $y = f(x)$ transforms to $y = -f(x)$:

Take image of $y = f(x)$ in the x axis as plane mirror

Examples :

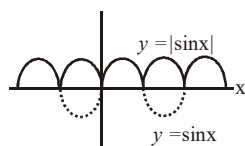
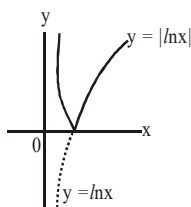




(g) $y = f(x)$ transforms to $y = |f(x)|$:

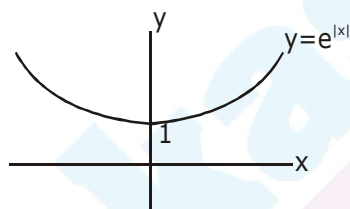
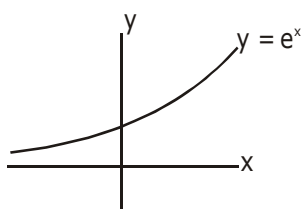
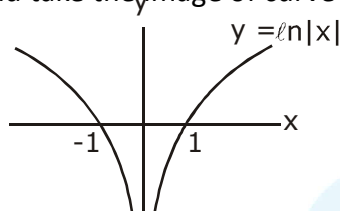
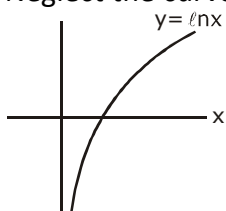
Take mirror image of the portion of the graph of $f(x)$ which lies below x-axis w.r.t. x-axis.

Examples :



(h) $y = f(x)$ transforms to $y = f(|x|)$

Neglect the curve for $x < 0$ and take the image of curve for $x \geq 0$ about y-axis



(i) $y = f(x)$ transforms to $|y| = f(x)$:

Remove the portion of graph which lies below x-axis & take mirror image {in x axis} of remaining portion of graph

Examples:

