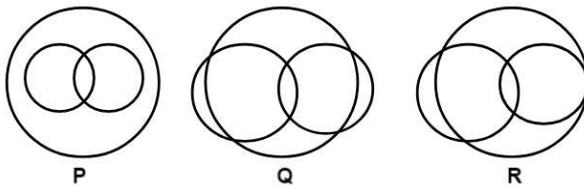Subject: Mathematics

1. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is

   A. 4

   B. 2

   C. 1

   D. 3

2. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statements.

   A. P and R

   B. P and Q

   C. Q and R

   D. None of these

3. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

   A. 63

   B. 54

   C. 38

   D. 36

4. The set of all values of $k > -1$, for which the equation
$(3x^2 + 4x + 3)^2 - (k+1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$
has real roots, is

A. $\left(\dfrac{1}{2}, \dfrac{3}{1}\right] - \{1\}$

B. $\left[-\dfrac{1}{2}, 1\right)$

C. $[2, 3)$

D. $\left(1, \dfrac{5}{2}\right]$

5. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is

A. 1

B. 2

C. 4

D. 0

6. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is

A. 8

B. 4

C. 6

D. 2

7. Let S be the set of all real roots of the equation,
   $3^x (3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

   A. is a singleton.

   B. is an empty set.

   C. contains at least four elements.

   D. contains exactly two elements.

8. Let $a, b \in R, a \neq 0$, such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to:

   A. 24

   B. 25

   C. 26

   D. 28

9. The set of all real values of λ for which the quadratic equation,
   $\left(\lambda^2 + 1\right) x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0, 1)$ is:

   A. $(-3, -1)$

   B. $(2, 4]$

   C. $(1, 3]$

   D. $(0, 2)$

10. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is:

   A. $\dfrac{4}{9}$

   B. 2

   C. 1

   D. $\dfrac{15}{8}$

11. The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in \mathbb{R}$, is always positive, is

   A. 8

   B. 7

   C. 6

   D. 3

12. If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0, k \in R$, then the real root of this equation

   A. exists and is equal to 1

   B. exists and is equal to $-\dfrac{1}{2}$

   C. exists and is equal to $\dfrac{1}{2}$

   D. does not exist

13. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = 2x-1$ and $g : \mathbb{R}-\{1\} \to \mathbb{R}$ be defined as $g(x) = \dfrac{x - \dfrac{1}{2}}{x-1}$. Then the composition function $f(g(x))$ is:

   A. both one-one and onto

   B. onto but not one-one

   C. neither one-one nor onto

   D. one-one but not onto

14. Let $R = \{(P,Q) \mid P \text{ and } Q \text{ are at the same distance from the origin}\}$ be a relation, then the equivalence class of $(1, -1)$ is the set :

   A. $S = \{(x,y) \mid x^2 + y^2 = 1\}$

   B. $S = \{(x,y) \mid x^2 + y^2 = 4\}$

   C. $S = \{(x,y) \mid x^2 + y^2 = \sqrt{2}\}$

   D. $S = \{(x,y) \mid x^2 + y^2 = 2\}$

15. The inverse of $y = 5^{\log x}$ is:

   A. $x = 5^{\log y}$

   B. $x = y^{\log 5}$

   C. $x = y^{\frac{1}{\log 5}}$

   D. $x = 5^{\frac{1}{\log y}}$

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16. If the function are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions: $f + g, f - g, \dfrac{f}{g}, \dfrac{g}{f}, g - f$ where $(f \pm g)(x) = f(x) \pm g(x), \left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

   A. $0 < x \leq 1$

   B. $0 \leq x < 1$

   C. $0 \leq x \leq 1$

   D. $0 < x < 1$

17. Let $\mathbb{N}$ be the set of natural numbers and a relation R on $\mathbb{N}$ be defined by
    $R = \left\{(x, y) \in \mathbb{N} \times \mathbb{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\right\}$.
    Then the relation R is

   A. an equivalence relation

   B. reflexive and symmetric, but not transitive

   C. reflexive but neither symmetric nor transitive

   D. symmetric but neither reflexive nor transitive

18. If the function $f : \mathbb{R} - \{1, -1\} \to A$ defined by $f(x) = \dfrac{x^2}{1 - x^2}$, is surjective, then A is

   A. $\mathbb{R} - [-1, 0)$

   B. $\mathbb{R} - \{-1\}$

   C. $\mathbb{R} - (-1, 0)$

   D. $[0, \infty)$

19. If g is the inverse of a function f and $f'(x) = \dfrac{1}{1 + x^5}$, then $g'(x)$ is equal to:

    A. $1 + x^5$

    B. $5x^4$

    C. $\dfrac{1}{1 + \{g(x)\}^5}$

    D. $1 + \{g(x)\}^5$

20. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals :

    A. $2f_1(x)f_1(y)$

    B. $2f_1(x + y)f_1(x - y)$

    C. $2f_1(x)f_2(y)$

    D. $2f_1(x + y)f_2(x - y)$

Subject: Mathematics

1. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is

2. If $A = \{x \in \mathbb{R} : |x-2| > 1\}, B = \{x \in \mathbb{R} : \sqrt{x^2-3} > 1\}$,
    $C = \{x \in \mathbb{R} : |x-4| \geq 2\}$ and $\mathbb{Z}$ is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap \mathbb{Z}$ is

3. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0, \alpha > \beta$ and
    $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of
    $\left(\dfrac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right)$ is equal to

4. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to

5. Let $f(x)$ be a polynomial of degree 3 such that $f(k) = -\dfrac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10f(10)$ is equal to

6. Let $\lambda \neq 0$ be in $\mathbb{R}$. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\dfrac{\beta\gamma}{\lambda}$ is equal to

7. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\dfrac{1}{x}\right) = bx + \dfrac{\beta}{x}, x \neq 0$, then the value of the expression $\dfrac{f(x) + f\left(\dfrac{1}{x}\right)}{x + \dfrac{1}{x}}$ is

8. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

9. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \rightarrow S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to

10. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one–one}\}$ is