

Subject: Mathematics

1. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is

☐ A. 4

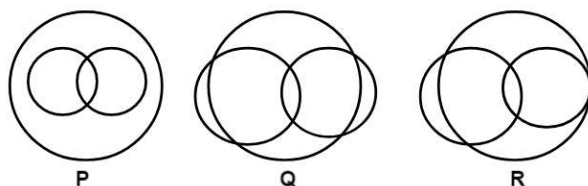
☒ B. 2

☐ C. 1

☐ D. 3

Given: $x^2 - |x| - 12 = 0$
 $\Rightarrow x^2 - 4|x| + 3|x| - 12 = 0$
 $\Rightarrow (|x| - 4)(|x| + 3) = 0$
 $\Rightarrow |x| = 4$ or $|x| = -3$ (rejected)
 $\Rightarrow x = \pm 4$
 \therefore Number of solutions = 2

2. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statements.



☐ A. P and R

☐ B. P and Q

☐ C. Q and R

☒ D. None of these

In all three (P), (Q), (R) there are some students who play all the three games
Hence none of the venn diagram is correct.

3. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

☐ A. 63

☐ B. 54

☐ C. 38

☒ D. 36

$C \rightarrow$ Coffee and $T \rightarrow$ Tea

$n(C) = 73\%$, $n(T) = 65\%$

$$n(\text{coffee}) = \frac{73}{100}$$

$$n(\text{tea}) = \frac{65}{100}$$

$$n(T \cap C) = \frac{x}{100}$$

$$n(C \cup T) = n(C) + n(T) - x \leq 100$$

$$= 73 + 65 - x \leq 100$$

$$\Rightarrow x \geq 38$$

$$\Rightarrow 73 - x \geq 0 \Rightarrow x \leq 73$$

$$\Rightarrow 65 - x \geq 0 \Rightarrow x \leq 65$$

$$\therefore 38 \leq x \leq 65$$

Therefore, $x \neq 36$

4. The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is

☒ A. $\left(\frac{1}{2}, \frac{3}{1}\right] - \{1\}$

☒ B. $\left[-\frac{1}{2}, 1\right)$

☒ C. $[2, 3)$

☒ D. $\left(1, \frac{5}{2}\right]$

$$3x^2 + 4x + 2 > 0 \quad \forall x \in \mathbb{R} \quad (\because D < 0)$$

$$(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$$

$$\Rightarrow \left(\frac{3x^2 + 4x + 3}{3x^2 + 4x + 2}\right)^2 - (k + 1)\left(\frac{3x^2 + 4x + 3}{3x^2 + 4x + 2}\right) + k = 0 \dots (i)$$

$$\text{Let } \frac{3x^2 + 4x + 3}{3x^2 + 4x + 2} = t$$

$$\Rightarrow t = \frac{3x^2 + 4x + 2 + 1}{3x^2 + 4x + 2} = 1 + \frac{1}{3x^2 + 4x + 2}$$

$$3x^2 + 4x + 2 \in \left[\frac{2}{3}, \infty\right)$$

$$\Rightarrow \frac{1}{3x^2 + 4x + 2} \in \left(0, \frac{3}{2}\right]$$

$$\Rightarrow t = 1 + \frac{1}{3x^2 + 4x + 2} \in \left(1, \frac{5}{2}\right]$$

$$\Rightarrow t^2 - (k + 1)t + k = 0 \text{ where } t \in \left(1, \frac{5}{2}\right] \dots (ii)$$

$$(ii) \text{ should have at least one root in } \left(1, \frac{5}{2}\right]$$

$$\Rightarrow (t - 1)(t - k) = 0$$

$$\Rightarrow t = 1, t = k$$

$$\therefore k \in \left(1, \frac{5}{2}\right]$$

5. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is

☒ A. 1

☐ B. 2

☐ C. 4

☐ D. 0

Given: $e^{4x} + 2e^{3x} - e^x - 6 = 0$

Let $e^x = t$

$$\Rightarrow t^4 + 2t^3 - t - 6 = 0$$

$$\Rightarrow t^4 + 2t^3 + t^2 - t^2 - t - 6 = 0$$

$$\Rightarrow (t^2 + t)^2 - (t^2 + t) - 6 = 0$$

Let $t^2 + t = z$

$$\Rightarrow z^2 - z - 6 = 0$$

$$\Rightarrow (z - 3)(z + 2) = 0$$

$$\Rightarrow z = 3 \text{ or } z = -2$$

$$\Rightarrow t^2 + t - 3 = 0 \text{ or } t^2 + t + 2 = 0 \text{ (rejected)}$$

$$\Rightarrow t = \frac{-1 \pm \sqrt{13}}{2}$$

Since, $e^x = t > 0$

$$\therefore e^x = \frac{-1 + \sqrt{13}}{2}$$

Hence, the given equation has only one real root.

Alternate Solution:

Let $e^x = t$

Now, assuming

$$f(t) = t^4 + 2t^3 - t - 6, \quad t > 0$$

$$\Rightarrow f'(t) = 4t^3 + 6t^2 - 1$$

$$\Rightarrow f''(t) = 12t^2 + 12t > 0 \quad [\because t > 0]$$

So, $f'(t)$ is always increasing.

$$\text{Also, } f'(0) = -1 < 0, \quad f'(1) = 9 > 0$$

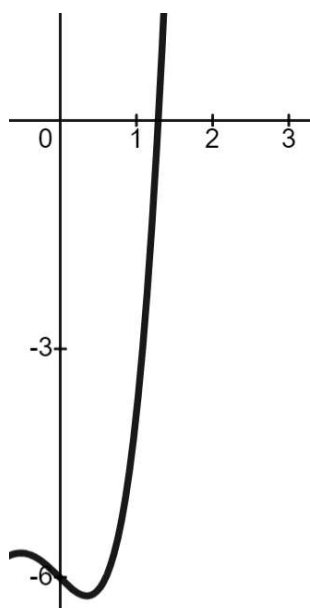
So, $f'(t) = 0$, for only one value of $t \in (0, 1)$.

Now, the nature of the graph is

$$f(0) = -6 < 0$$

$$f(1) = -4 < 0$$

$$f(2) > 0$$



Hence, $f(t) = 0$ has only one real solution.

6. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is

- ☒ A. 8
☒ B. 4
☒ C. 6
☒ D. 2

Let α, β be the roots of the quadratic equation.

Then, $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

$$\Rightarrow (\alpha^2 - 2)^2 - 2 = \alpha$$

$$\Rightarrow \alpha^4 - 4\alpha^2 - \alpha + 2 = 0$$

$$\Rightarrow (\alpha + 1)(\alpha - 2)(\alpha^2 + \alpha - 1) = 0$$

$$\Rightarrow (\alpha, \beta) = (-1, -1), (-1, 1), (2, 2), (2, -2), (-1, 2) \text{ and } \left(\frac{\sqrt{5}-1}{2}, -\frac{\sqrt{5}+1}{2} \right)$$

Hence there will be 6 possible values of (a, b) .

7. Let S be the set of all real roots of the equation,
 $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

- ☒ A. is a singleton.
☐ B. is an empty set.
☐ C. contains at least four elements.
☐ D. contains exactly two elements.

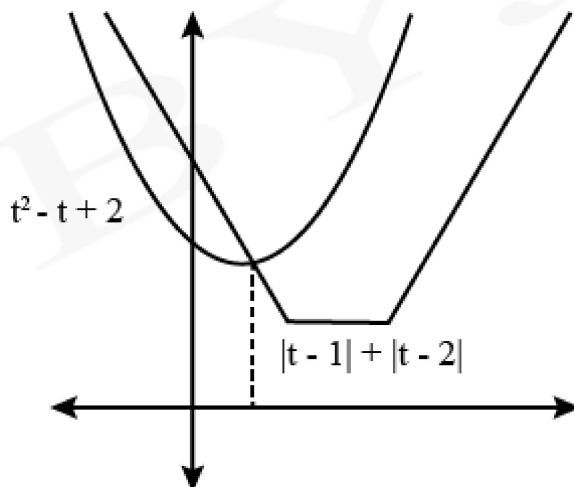
$$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$$

Let $3^x = t$

$$\begin{aligned} \text{Then } t(t - 1) + 2 &= |t - 1| + |t - 2| \\ \Rightarrow t^2 - t + 2 &= |t - 1| + |t - 2| \end{aligned}$$

We plot $t^2 - t + 2$ and $|t - 1| + |t - 2|$

As $t = 3^x$ is always positive, therefore only positive values of t will be the solution.



The graphs intersect for only positive value of t .

So, there is single value of x given by $x = \log_3 t$

Therefore, we have only one solution.

8. Let $a, b \in R, a \neq 0$, such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α , which is also a root of the equation $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to:

- ☒ A. 24
- ☒ B. 25
- ☐ C. 26
- ☐ D. 28

$ax^2 - 2bx + 5 = 0$ has both roots as α

$$\Rightarrow \text{Sum of roots, } \alpha + \alpha = 2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

And product of roots, $\alpha \cdot \alpha = \alpha^2 = \frac{5}{a}$

$$\Rightarrow b^2 = 5a(a \neq 0) \cdots (1)$$

$$\Rightarrow \alpha + \beta = 2b \text{ and } \alpha\beta = -10$$

β, α are the roots of $x^2 - 2bx - 10 = 0$.

$$\alpha = \frac{b}{a}$$

$$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$$

$$\Rightarrow 5a - 10a^2 - 10a^2 = 0 \quad (\text{Using (1)})$$

$$\Rightarrow a = \frac{1}{4} \Rightarrow b^2 = \frac{5}{4}$$

$$\Rightarrow \alpha^2 = 20, \beta^2 = 5 \Rightarrow \alpha^2 + \beta^2 = 25$$

9. The set of all real values of λ for which the quadratic equation, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0, 1)$ is:

- ☐ A. $(-3, -1)$
- ☐ B. $(2, 4]$
- ☒ C. $(1, 3]$
- ☐ D. $(0, 2)$

$$f(0)f(1) \leq 0$$

$$\Rightarrow (2)(\lambda^2 - 4\lambda + 3) \leq 0$$

$$\Rightarrow (\lambda - 1)(\lambda - 3) \leq 0$$

$$\Rightarrow \lambda \in [1, 3]$$

But at $\lambda = 1$, both roots are 1 so $\lambda \neq 1$

$$\therefore \lambda \in (1, 3]$$

10. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is:

- ☒ A. $\frac{4}{9}$
- ☒ B. 2
- ☒ C. 1
- ☒ D. $\frac{15}{8}$

The given equation is,
 $x^2 + (3 - \lambda)x + 2 = \lambda$

Rearranging the equation

$$x^2 + (3 - \lambda)x + 2 - \lambda = 0 \quad \dots (1)$$

Roots of the equation (1) are

$$\begin{aligned} x &= \frac{-(3 - \lambda) \pm \sqrt{(3 - \lambda)^2 - 4(2 - \lambda)}}{2} \\ &= \frac{-(3 - \lambda) \pm (\lambda - 1)}{2} \\ \Rightarrow x &= (-1) \text{ and } (\lambda - 2) \end{aligned}$$

According to the question,

Let sum of the squares of roots be $f(\lambda)$

$$f(\lambda) = (-1)^2 + (\lambda - 2)^2$$

$$f(\lambda) = \lambda^2 - 4\lambda + 5$$

$$f(\lambda) = (\lambda - 2)^2 + 1$$

$f(\lambda)$ is minimum when $(\lambda - 2)^2$ is minimum.

Therefore, $\lambda = 2$.

11. The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in \mathbb{R}$, is always positive, is

☐ A. 8

☒ B. 7

☐ C. 6

☐ D. 3

$$(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m) > 0$$

$$\therefore D < 0$$

$$\begin{aligned} 4(1 + 3m)^2 - 16(1 + 2m)(1 + m) &< 0 \\ \Rightarrow 1 + 9m^2 + 6m - 4 - 12m - 8m^2 &< 0 \\ \Rightarrow m^2 - 6m - 3 &< 0 \\ \Rightarrow (m - 3)^2 &< 12 \\ \Rightarrow -2\sqrt{3} < m - 3 < 2\sqrt{3} \\ \Rightarrow 3 - 2\sqrt{3} < m < 3 + 2\sqrt{3} \end{aligned}$$

Also

$$\begin{aligned} 2m + 1 &> 0 \\ \Rightarrow m &> -\frac{1}{2} \end{aligned}$$

Possible integral values of m are 0, 1, 2, 3, 4, 5, 6.

Hence, number of integral values of m is 7.

12. If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0, k \in R$, then the real root of this equation

- ☒ A. exists and is equal to 1
- ☒ B. exists and is equal to $-\frac{1}{2}$
- ☒ C. exists and is equal to $\frac{1}{2}$
- ☒ D. does not exist

Whenever the coefficients of a polynomial are real, complex roots exist in conjugate pair.

Here, the coefficients are real and one of the root is $2 + 3i$, So the other root must be $2 - 3i$. Let the other root be α .

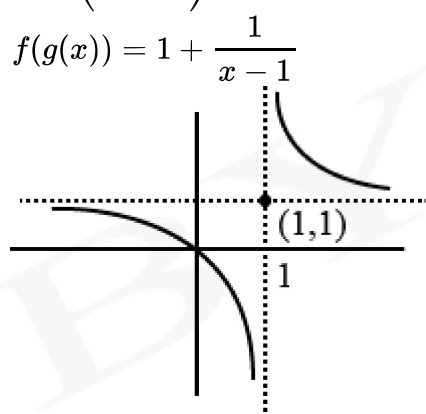
$$\begin{aligned} \text{Sum of the roots} &= 2 + 3i + 2 - 3i + \alpha = \frac{9}{2} \\ \Rightarrow \alpha &= \frac{1}{2} \end{aligned}$$

13. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 2x - 1$ and $g : \mathbb{R} - \{1\} \rightarrow \mathbb{R}$ be defined as

$$g(x) = \frac{x - \frac{1}{2}}{x - 1}. \text{ Then the composition function } f(g(x)) \text{ is:}$$

- ☒ A. both one-one and onto
- ☒ B. onto but not one-one
- ☒ C. neither one-one nor onto
- ☒ D. one-one but not onto

$$\begin{aligned} f(g(x)) &= 2g(x) - 1 \\ &= 2 \left(\frac{x - \frac{1}{2}}{x - 1} \right) - 1 = \frac{x}{x - 1} \end{aligned}$$



$\therefore f(g(x))$ is one-one, into.

14. Let $R = \{(P, Q) \mid P \text{ and } Q \text{ are at the same distance from the origin}\}$ be a relation, then the equivalence class of $(1, -1)$ is the set :

- ☒ A. $S = \{(x, y) \mid x^2 + y^2 = 1\}$
- ☒ B. $S = \{(x, y) \mid x^2 + y^2 = 4\}$
- ☒ C. $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$
- ☒ D. $S = \{(x, y) \mid x^2 + y^2 = 2\}$

Let $P(a, b)$ and $Q(c, d)$ be two points.

$$OP = OQ$$

$$\Rightarrow a^2 + b^2 = c^2 + d^2$$

$$R(x, y), S(1, -1)$$

$$\Rightarrow OR = OS \quad (\because \text{equivalence class})$$

$$\Rightarrow x^2 + y^2 = 2$$

15. The inverse of $y = 5^{\log x}$ is:

☐ A. $x = 5^{\log y}$

☐ B. $x = y^{\log 5}$

☒ C. $x = y^{\frac{1}{\log 5}}$

☐ D. $x = 5^{\frac{1}{\log y}}$

$$y = 5^{\log x}$$

Taking \log_5 on both sides, we get

$$\Rightarrow \log_5 y = \log x$$

$$\Rightarrow x = e^{\log_5 y}$$

$$\Rightarrow x = y^{\log_5 e} = y^{\frac{1}{\log_e 5}}$$

$$\therefore x = y^{\frac{1}{\log 5}}$$

16. If the function are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions: $f + g, f - g, \frac{f}{g}, \frac{g}{f}, g - f$ where

$$(f \pm g)(x) = f(x) \pm g(x), \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

☒ A. $0 < x \leq 1$

☒ B. $0 \leq x < 1$

☒ C. $0 \leq x \leq 1$

☒ D. $0 < x < 1$

$$f + g = \sqrt{x} + \sqrt{1-x}$$

$$\Rightarrow x \geq 0 \text{ \& } 1-x \geq 0 \Rightarrow x \in [0, 1]$$

$$f - g = \sqrt{x} - \sqrt{1-x}$$

$$\Rightarrow x \geq 0 \text{ \& } 1-x \geq 0 \Rightarrow x \in [0, 1]$$

$$\frac{f}{g} = \frac{\sqrt{x}}{\sqrt{1-x}}$$

$$\Rightarrow x \geq 0 \text{ \& } 1-x > 0 \Rightarrow x \in [0, 1)$$

$$\frac{g}{f} = \frac{\sqrt{1-x}}{\sqrt{x}}$$

$$\Rightarrow 1-x \geq 0 \text{ \& } x > 0 \Rightarrow x \in (0, 1]$$

$$g - f = \sqrt{1-x} - \sqrt{x}$$

$$\Rightarrow 1-x \geq 0 \text{ \& } x \geq 0 \Rightarrow x \in [0, 1]$$

$$\Rightarrow x \in (0, 1)$$

17. Let \mathbb{N} be the set of natural numbers and a relation R on \mathbb{N} be defined by
 $R = \{(x, y) \in \mathbb{N} \times \mathbb{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$.
 Then the relation R is

- ☒ A. an equivalence relation
- ☒ B. reflexive and symmetric, but not transitive
- ☒ C. reflexive but neither symmetric nor transitive
- ☒ D. symmetric but neither reflexive nor transitive

$$\begin{aligned}
 x^3 - 3x^2y - xy^2 + 3y^3 &= 0 \\
 \Rightarrow x^2(x - 3y) - y^2(x - 3y) &= 0 \\
 \Rightarrow (x - y)(x + y)(x - 3y) &= 0 \quad \dots (1)
 \end{aligned}$$

\therefore (1) holds for all (x, x)

$\therefore R$ is reflexive

If (x, y) holds, then (y, x) may or may not hold for factor $(x - 3y)$

For example $(3, 1)$

$\therefore R$ is not symmetric

Similarly factor $(x - 3y)$ doesn't hold for transitive

For example $(9, 3) \in R$ and $(3, 1) \in R$ but $(9, 1) \notin R$

Hence, relation R is reflexive but neither symmetric nor transitive

18. If the function $f : \mathbb{R} - \{1, -1\} \rightarrow A$ defined by $f(x) = \frac{x^2}{1 - x^2}$, is surjective, then A is

☒ A. $\mathbb{R} - [-1, 0)$

☐ B. $\mathbb{R} - \{-1\}$

☐ C. $\mathbb{R} - (-1, 0)$

☐ D. $[0, \infty)$

$f : \mathbb{R} - \{1, -1\} \rightarrow A$

$f(x) = \frac{x^2}{1 - x^2}$ is surjective.

As f is surjective, so every element in co-domain must have a pre-image in domain.

Let $y = \frac{x^2}{1 - x^2}$

$\Rightarrow y - yx^2 = x^2$

$\Rightarrow x^2(1 + y) = y$

$\Rightarrow x = \pm \sqrt{\frac{y}{1 + y}}$

For x to be defined,

$1 + y \neq 0 \Rightarrow y \neq -1 \dots (i)$

and $\frac{y}{1 + y} \geq 0$

$\Rightarrow y \in (-\infty, -1) \cup [0, \infty) \dots (ii)$

From (i) and (ii)

$y \in \mathbb{R} - [-1, 0)$

19. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to:

- ☐ A. $1+x^5$
- ☐ B. $5x^4$
- ☐ C. $\frac{1}{1+\{g(x)\}^5}$
- ☒ D. $1+\{g(x)\}^5$

$$g(x) = f^{-1}x$$

$$\Rightarrow f(g(x)) = x$$

$$\Rightarrow f'(g(x)) \cdot g'(x) = 1$$

$$\Rightarrow g'(x) = \frac{1}{f'(g(x))} \dots (1)$$

$$\text{As given, } f'(x) = \frac{1}{1+x^5}$$

$$\Rightarrow f'(g(x)) = \frac{1}{1+\{g(x)\}^5}$$

from equation (1)

$$g'(x) = 1 + \{g(x)\}^5$$

20. Let $f(x) = a^x$ ($a > 0$) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals :

- ☒ A. $2f_1(x)f_1(y)$
- ☐ B. $2f_1(x+y)f_1(x-y)$
- ☐ C. $2f_1(x)f_2(y)$
- ☐ D. $2f_1(x+y)f_2(x-y)$

Every function $f(x)$ can be represented as

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2},$$

where $\frac{f(x) + f(-x)}{2}$ is even function and $\frac{f(x) - f(-x)}{2}$ is odd function.

$$f(x) = a^x$$

$$\text{So, } f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$\begin{aligned} \therefore f_1(x+y) + f_1(x-y) &= \frac{a^{x+y} + a^{-(x+y)}}{2} + \frac{a^{x-y} + a^{-(x-y)}}{2} \\ &= \frac{a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})}{2} \\ &= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2} \\ &= 2f_1(x)f_1(y) \end{aligned}$$

Subject: Mathematics

1. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is

Accepted Answers

1 01 1.00 1.0

Solution:

$$\log_4(x - 1) = \log_2(x - 3)$$

$$\Rightarrow \frac{1}{2} \log_2(x - 1) = \log_2(x - 3)$$

$$\Rightarrow x - 1 = (x - 3)^2$$

$$\Rightarrow x^2 - 6x + 9 = x - 1$$

$$\Rightarrow x^2 - 7x + 10 = 0$$

$$\Rightarrow x = 2, 5$$

$x = 2$ is not possible as $\log_2(x - 3)$ is not defined at $x = 2$.

\therefore Number of solution = 1

2. If $A = \{x \in \mathbb{R} : |x - 2| > 1\}$, $B = \{x \in \mathbb{R} : \sqrt{x^2 - 3} > 1\}$, $C = \{x \in \mathbb{R} : |x - 4| \geq 2\}$ and \mathbb{Z} is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap \mathbb{Z}$ is

Accepted Answers

256 256.0 256.00

Solution:

$$A = \{x \in \mathbb{R} : |x - 2| > 1\}$$

$$|x - 2| > 1$$

$$\Rightarrow x - 2 > 1 \text{ or } x - 2 < -1$$

$$\Rightarrow x > 3 \text{ or } x < 1$$

$$\therefore A = (-\infty, 1) \cup (3, \infty)$$

$$B = \{x \in \mathbb{R} : \sqrt{x^2 - 3} > 1\}$$

$$x^2 - 3 \geq 0 \text{ and } x^2 - 3 > 1$$

$$\Rightarrow x^2 - 4 > 0$$

$$\Rightarrow x > 2 \text{ or } x < -2$$

$$\therefore B = (-\infty, -2) \cup (2, \infty)$$

$$C = \{x \in \mathbb{R} : |x - 4| \geq 2\}$$

$$x - 4 \geq 2 \text{ or } x - 4 \leq -2$$

$$x \geq 6 \text{ or } x \leq 2$$

$$\therefore C = (-\infty, 2] \cup [6, \infty)$$

$$\text{So, } (A \cap B \cap C)^c = (-\infty, -2) \cup [6, \infty)$$

$$\text{Now, } (A \cap B \cap C)^c \cap \mathbb{Z} = \{-2, -1, 0, 1, 2, 3, 4, 5\}$$

$$\Rightarrow n((A \cap B \cap C)^c \cap \mathbb{Z}) = 8$$

$$\therefore \text{Number of subset} = 2^8 = 256$$

3. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n , then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right)$ is equal to

Accepted Answers

1 1.0 1.00 01

Solution:

$$\alpha^{n-2}(\alpha^2 + 5\sqrt{2}\alpha + 10) = 0 \quad \dots (1)$$

$$\beta^{n-2}(\beta^2 + 5\sqrt{2}\beta + 10) = 0 \quad \dots (2)$$

From (2) - (1)

$$P_n + 5\sqrt{2}P_{n-1} = -10P_{n-2}$$

Now,

$$\frac{P_{17}(P_{20} + 5\sqrt{2}P_{19})}{P_{18}(P_{19} + 5\sqrt{2}P_{18})} = \frac{P_{17} \cdot (-10P_{18})}{P_{18} \cdot (-10P_{17})} = 1$$

4. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to

Accepted Answers

2 2.0 2.00 02

Solution:

$$\text{Let } e^x = t, (t > 0)$$

$$t^4 - t^3 - 4t^2 - t + 1 = 0$$

$$\Rightarrow \left(t^2 + \frac{1}{t^2} \right) - \left(t + \frac{1}{t} \right) - 4 = 0$$

$$\Rightarrow \left(t + \frac{1}{t} \right)^2 - \left(t + \frac{1}{t} \right) - 6 = 0$$

$$\text{Let } t + \frac{1}{t} = u \quad (u > 2)$$

$$(u - 3)(u + 2) = 0$$

$$\Rightarrow u = 3, -2 \text{ (rejected)}$$

For $u = 3$

$$\Rightarrow t + \frac{1}{t} = 3$$

$$\Rightarrow t^2 - 3t + 1 = 0$$

$$t = \frac{3 \pm \sqrt{5}}{2} = e^x$$

$$x = \ln \frac{3 + \sqrt{5}}{2}, \ln \frac{3 - \sqrt{5}}{2}$$

5. Let $f(x)$ be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10f(10)$ is equal to

Accepted Answers

26 26.0 26.00

Solution:

$$\text{Let } P(k) = kf(k) + 2$$

$$\Rightarrow kf(k) + 2 = a(x-2)(x-3)(x-4)(x-5)$$

$$\text{If } k = 0$$

$$\Rightarrow 2 = a(-2)(-3)(-4)(-5)$$

$$\therefore a = \frac{1}{60}$$

$$\Rightarrow kf(k) + 2 = \frac{1}{60}(x-2)(x-3)(x-4)(x-5)$$

$$\text{Putting } k = 10$$

$$\Rightarrow 10f(10) + 2 = \frac{1}{60} \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 28$$

$$\therefore 10f(10) = 26$$

$$\Rightarrow 52 - 10f(10) = 26$$

6. Let $\lambda \neq 0$ be in \mathbb{R} . If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to

Accepted Answers

18 18.0 18.00

Solution:

$$x^2 - x + 2\lambda = 0 \quad \begin{cases} \alpha \\ \beta \end{cases} \Rightarrow \alpha \cdot \beta = 2\lambda$$

$$3x^2 - 10x + 27\lambda = 0 \quad \begin{cases} \alpha \\ \gamma \end{cases} \Rightarrow \alpha \cdot \gamma = \frac{27\lambda}{3} = 9\lambda$$

Both equations have a common root α .

$$\therefore \frac{\alpha^2}{-27\lambda + 20\lambda} = \frac{\alpha}{6\lambda - 27\lambda} = \frac{1}{-10 + 3}$$

$$\Rightarrow \frac{\alpha^2}{-7\lambda} = \frac{\alpha}{-19\lambda} = \frac{1}{-7}$$

$$\Rightarrow \alpha^2 = \lambda$$

$$\text{Now, } (\alpha\beta) \cdot (\alpha\gamma) = (2\lambda)(9\lambda)$$

$$\frac{\beta \cdot \gamma}{\lambda} = 2 \times 9 \cdot \frac{\lambda}{\alpha^2} = 18$$

7. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \neq 0$, then the value of the

expression $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is

Accepted Answers

2 2.0 2.00

Solution:

$$af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \quad \dots (1)$$

Replace x by $\frac{1}{x}$

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x \quad \dots (2)$$

(1) + (2), we get

$$(a + \alpha) \left[f(x) + f\left(\frac{1}{x}\right) \right] = \left(x + \frac{1}{x} \right) (b + \beta)$$

$$\Rightarrow \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

8. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then the number of bijective functions $f : A \rightarrow A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

Accepted Answers

720 720.0 720.00

Solution:

Clearly $f(1), f(2)$ and $f(3)$ are the permutations of 0, 1, 2; and $f(0), f(4), f(5), f(6)$ and $f(7)$ are the permutations of 3, 4, 5, 6 and 7.

Total number of bijective functions $= 5!3! = 720$

9. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \rightarrow S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to

Accepted Answers

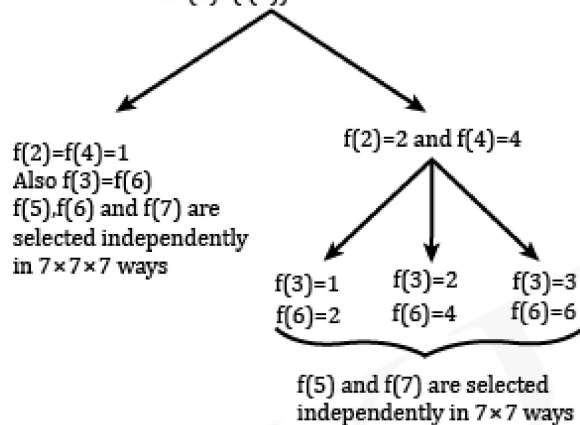
490 490.0 490.00

Solution:

Given $f(m \cdot n) = f(m) \cdot f(n)$

Clearly, $f(1) = 1$

and $f(4) = [f(2)]^2$



Total number of ways $= 7^3 + 3 \cdot 7^2 = 490$

10. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is

Accepted Answers

19 19.00 19.0

Solution:

$C = \{f : A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$

Case-I: If $f(x) = 2 \forall x \in A$, then number of function $= 1$.

Case-II : If $f(x) = 2$ for exactly two elements then total number of many-one function.
 $= {}^3C_2 \times {}^3C_1 = 9$

Case-III : If $f(x) = 2$ for exactly one elements then total number of many-one function
 $= {}^3C_1 \times {}^3C_1 = 9$

Total $= 1 + 9 + 9 = 19$