1. A body of mass $m = 10^{-2}\ kg$ is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10\ ms^{-1}$. If, after $10\ s$, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

   ☒ **A.** $10^{-3}\ kg\ m^{-1}$

   ☒ **B.** $10^{-3}\ kg\ s^{-1}$

   ☑ **C.** $10^{-4}\ kg\ m^{-1}$

   ☒ **D.** $10^{-1}\ kg\ m^{-1}s^{-1}$

   $m = 10^{-2}kg$

   $F = -kv^2$

   $v_0 = 10ms^{-1}$

   Also

   $u = v_0, v = \dfrac{v_0}{2}$

   $a = \dfrac{f}{m} = -\dfrac{kv^2}{m}$

   $a = \dfrac{dv}{dt}$

   $\dfrac{-kv^2}{m} = \dfrac{dv}{dt}$

   $\dfrac{-k}{m}\int_0^{10} dt = \int_{v_0}^{v_0/2} \dfrac{dv}{v^2}$

   $\dfrac{-k}{m} \times 10 = -[\dfrac{1}{v}]_{10}^{5}$

   $\dfrac{k}{m} \times 10 = \dfrac{1}{5} - \dfrac{1}{10}$

   $k \times 10^3 = \dfrac{1}{10}$

   $R = 10^{-4}\ kgm^{-1}$

2. A person trying to lose weight by burning fat lifts a mass of $10\ kg$ upto a height of $1\ m$, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work is done only when the weight is lifted up? Fat supplies $3.8 \times 10^7\ J$ of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$.

    ❌ **A.** $2.45 \times 10^{-3}\ kg$

    ❌ **B.** $6.45 \times 10^{-3}\ kg$

    ❌ **C.** $9.89 \times 10^{-3}\ kg$

    ✔ **D.** $12.89 \times 10^{-3}\ kg$

The net work done by the man will be 1000 times the work done in lifting $10\ kg$ to a height of $1\ m$.
Net work done $= 1000 \times 10 \times 9.8 \times 1\ J$
$= 9.8 \times 10^4\ J$
Let's assume $x\ kg$ of fat is burnt in doing this work. Energy balance will give the following equation.
$x \times \dfrac{20}{100} \times 3.8 \times 10^7 = $ Net work done $= 9.8 \times 10^4\ J$
$\Rightarrow x = 12.8947 \times 10^{-3}\ kg$

3. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when the particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x\,(= QR)$, are respectively close to
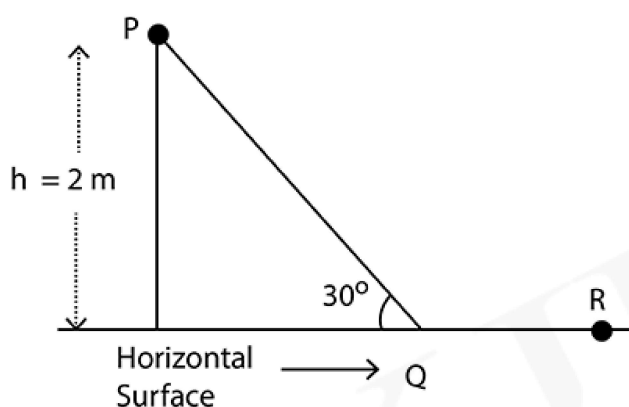
- ⊗ A. 0.2 and 6.5 m

- ⊗ B. 0.2 and 3.5 m

- ✔ C. 0.29 and 3.5 m

- ⊗ D. 0.29 and 6.5 m

All the potential energy is lost by dissipation due to work done by frictional force.
$P.E\ lost =$ Work done by friction

Work done by friction from P to Q $= \mu mg(\cos\theta)PQ = 2\sqrt{3}\mu mg$
Work done by friction from Q to R $= \mu mg \times QR$
$2mg = 2\sqrt{3}\mu mg + \mu mg \times QR$
$2 = 2\sqrt{3}\mu + \mu QR$ --- (1)
Since equal energies are lost along PQ and QR,
Work done by friction is the same on both path lengths.
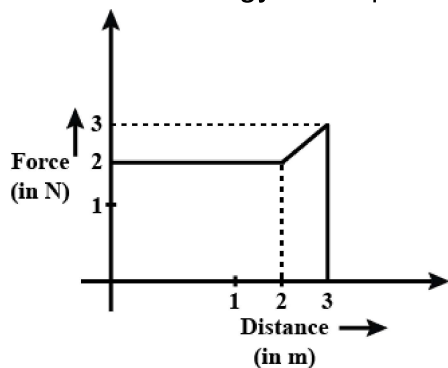$\mu mg \cos\theta \times PQ = \mu mg \times QR$
$PQ\cos\theta = QR$ --- (2)
From (1) and (2) we get,
$\mu = 0.29, QR = 3.5$

4. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled $3\ m$ is

(✓) **A.** $6.5\ J$

(✗) **B.** $2.5\ J$

(✗) **C.** $4\ J$

(✗) **D.** $5\ J$

According to Work Energy Theorem,
Work done by force on the particle $=$ Change in KE
Work done $=$ Area under $F - x$ graph
$= \int F.\,dx$
$= 2 \times 2 + \dfrac{(2+3) \times 1}{2} = 6.5\ J$
$W = KE_{final} - KE_{initial} = 6.5\ J$
$KE_{initial} = 0$
$\therefore KE_{final} = 6.5\ J$

5. A particle of mass 1 kg slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P(height 1 m) the kinetic energy of the particle (in J) is:(Figure drawn is schematic and not to scale; take $g = 10 \ m/s^2$)

Accepted Answers

10   10.0   10.00

Solution:
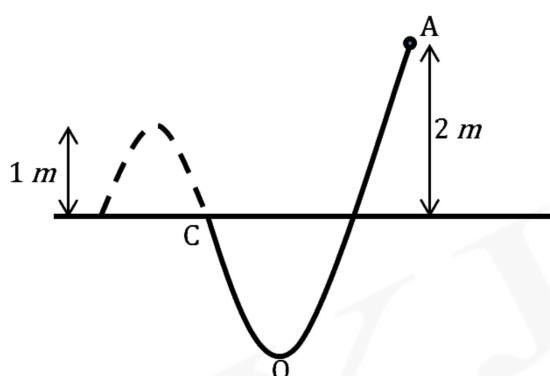
As the particle starts from rest the

total energy at point $A = mgh = T.E_A$(where $h = 2 \ m$)
After reaching point P

$$T.E_c = K.E. + mgh$$
By conservation of energy
$$T.E_A = T.E_p$$
$$\implies K.E. = mgh = 10 \ J$$

6. A $60\ HP$ electric motor lifts an elevator with a maximum total load capacity of $2000\ kg$. If the frictional force on the elevator is $4000\ N$, the speed of the elevator at full load is close to (Given $1\ HP = 746\ W$, $g = 10\ m/s^2$)

    ✗   **A.** $1.5\ m/s$

    ✗   **B.** $2.0\ m/s$

    ✗   **C.** $1.7\ m/s$

    ✔   **D.** $1.9\ m/s$

Friction will oppose the motion

Net force $= 2000 \times g + 4000 = 24000\ N$
Power of lift $= 60\ \text{HP}$
Power $=$ Force $\times$ Velocity
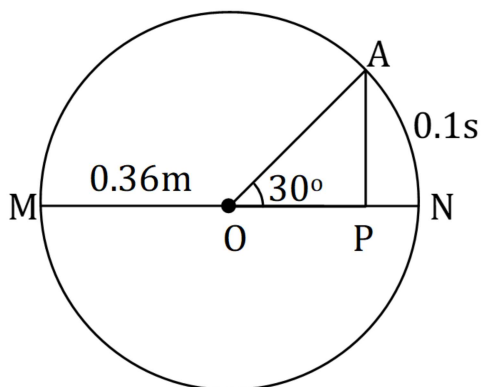$v = \dfrac{P}{F} = \dfrac{60 \times 746}{24000}$
$v = 1.86 \approx 1.9\ m/s$

7. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and cover $30°$ in 0.1 s. The perpendicular projection $'P'$ from $'A'$ on the diameter MN represents the simple harmonic motion of $'P'$. The restoring force per unit mass when P touches M will be:

- (✗) **A.** 100 N
- (✗) **B.** 50 N
- (✓) **C.** 9.87 N
- (✗) **D.** 0.49 N
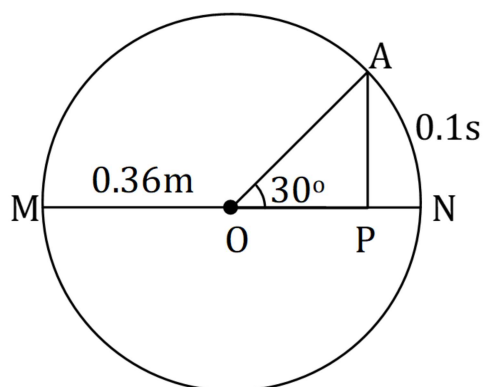
The point A covers $30°$ in 0.1 s.
From unitary method it means,

$\dfrac{\pi}{6} \to 0.1$ s

$1 \to \dfrac{0.1}{\dfrac{\pi}{6}}$ s

$2\pi \to \dfrac{0.1}{\dfrac{\pi}{6}} \times 2\pi$ s

$T = 1.2$ s

We know that $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{1.2}$

Restoring force $(F) = m\omega^2 A$

Then, restoring force per unit mass $\left(\dfrac{F}{m}\right) = \omega^2 A = \left(\dfrac{2\pi}{1.2}\right)^2 \times 0.36 \approx 9.87$ N

8. A modern Grand Prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_1 acting downwards on the car is -
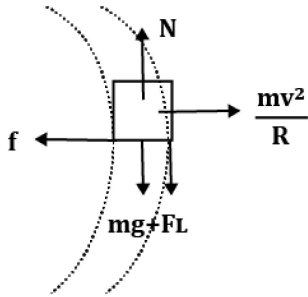
(Assume forces on the four tyres are identical and $g =$ acceleration due to gravity)
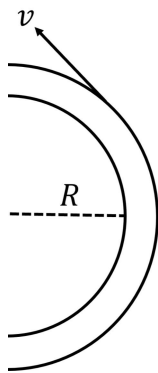
- ✔ A. $m\left(\dfrac{v^2}{\mu_s R} - g\right)$
- ✘ B. $m\left(\dfrac{v^2}{\mu_s R} + g\right)$
- ✘ C. $m\left(g - \dfrac{v^2}{\mu_s R}\right)$
- ✘ D. $-m\left(g + \dfrac{v^2}{\mu_s R}\right)$

On drawing the FBD of the car,

Applying the condition of equilibrium along the radial direction,

$$f = \frac{mv^2}{R}$$

$$\Rightarrow \mu_s N = \frac{mv^2}{R}$$

$$\Rightarrow \mu_s (mg + F_L) = \frac{mv^2}{R}$$

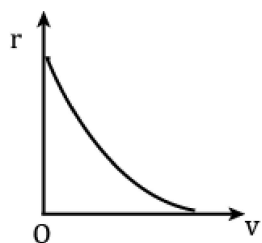$$\Rightarrow F_L = m\left(\frac{v^2}{\mu_s R} - g\right)$$

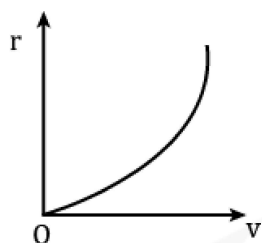Hence, option (A) is the correct answer.

9. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \dfrac{-c}{r}$, where c is positive constant. The correct radius (r)-velocity (v) graph of the particle's motion is :

A.

B.

C.

D.
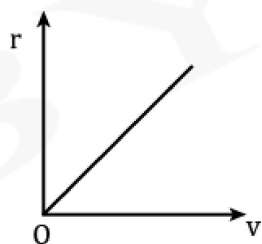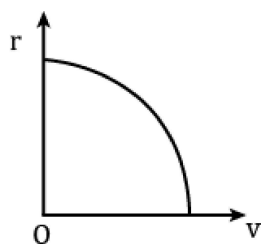
Circular motion and WPE

Given, potential field,

$$U(r) = \frac{-c}{r}$$

$$\Rightarrow F = -\frac{dU}{dr} = \frac{c}{r^2}$$

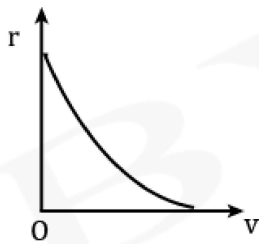We know that the centripetal force is given by,

$$F_c = \frac{mv^2}{r}$$

$$\therefore \frac{mv^2}{r} = \frac{c}{r^2}$$

$$\Rightarrow r = \frac{c}{mv^2}$$

$$\Rightarrow r \propto \frac{1}{v^2}$$

This situation is best represented by the graph of option (A).

10. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being $68 \,\text{kg}$. The mass of the elevator itself is $920 \,\text{kg}$ and it moves with a constant speed of $3 \,\text{ms}^{-1}$. The frictional force opposing the motion is $6000 \,\text{N}$. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator $(g = 10 \,\text{ms}^{-2})$ must be at least

   ✗ **A.**   56300 W

   ✗ **B.**   62360 W

   ✗ **C.**   48000 W

   ✔ **D.**   66000 W

As the lift is moving with a constant speed,

$$F = (10m + M)g + f$$

where,
$F =$ Force exerted by motor,
$m =$ mass of the person,
$M =$ mass of the elevator,
$f =$ frictional force

$$F = [((10 \times 68) + 920) \times 10] + 6000$$
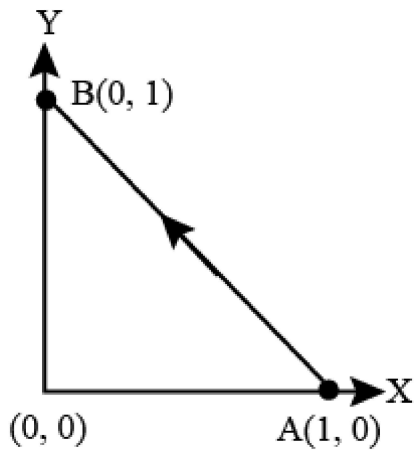
$$\Rightarrow F = 22000 \,\text{N}$$

$$\Rightarrow \text{Power}, P = Fv = 22000 \times 3$$

$$\therefore P = 66000 \,\text{W}$$

Hence, option (D) is correct.

11. Consider a force $\overrightarrow{F} = -x\hat{i} + y\hat{j}$. The work done by this force, in moving a particle, from point $A(1, 0)$ to $B(0, 1)$, along the line segment is: (all quantities are in SI units)

- ✕ **A.** 2
- ✕ **B.** $\dfrac{1}{2}$
- ✔ **C.** 1
- ✕ **D.** $\dfrac{3}{2}$

Work done, $W = \displaystyle\int \overrightarrow{F} \cdot \overrightarrow{ds}$

Here, $\overrightarrow{F} \cdot \overrightarrow{ds} = (-x\hat{i} + y\hat{j}).(dx\hat{i} + dy\hat{j}) = -xdx + ydy$

$\Rightarrow W = -\displaystyle\int_{1}^{0} xdx + \int_{0}^{1} ydy$

$= \left(0 + \dfrac{1}{2}\right) + \dfrac{1}{2} = 1$ J

Hence, (C) is the correct answer,

12. A spring mass system (mass m, spring constant k and natural length of spring l) rests in equilibrium, on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc, together with spring mass system, rotates about it's axis with an angular velocity $\omega, (k >> m\omega^2)$ the relative change in the length of the spring is best given by the option:

  (×)   A. $\sqrt{\dfrac{2}{3}\left(\dfrac{m\omega}{k}\right)}$

  (×)   B. $\dfrac{2m\omega^2}{k}$

  (✓)   C. $\dfrac{m\omega^2}{k}$

  (×)   D. $\dfrac{m\omega^2}{3k}$
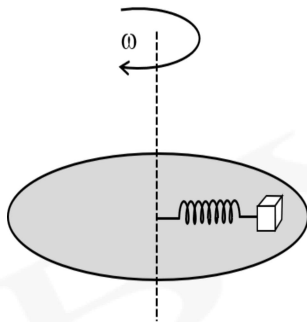
The free body diagram in the frame of disc is as shown below.

$$\therefore m\omega^2(l+x) = kx$$

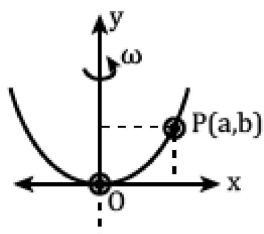$$\Rightarrow x = \frac{ml\omega^2}{k - m\omega^2}$$

For $k >> m\omega^2$

$$\Rightarrow \frac{x}{l} = \frac{m\omega^2}{k}$$

Hence, (C) is the correct answer.

BYJU'S
The Learning App

13. A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction)

✓ A. $2\sqrt{2gC}$

✗ B. $2\sqrt{gC}$

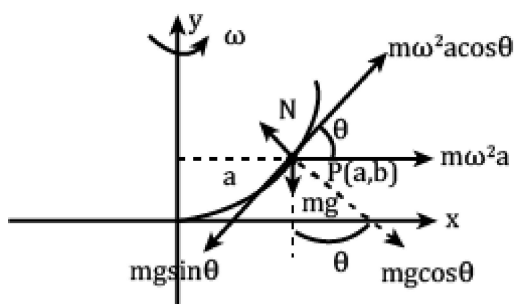✗ C. $\sqrt{\dfrac{2gC}{ab}}$

✗ D. $\sqrt{\dfrac{2g}{C}}$

Given that, $y = 4Cx^2$

$$\Rightarrow \frac{dy}{dx} = \tan\theta = 8Cx$$

At P, $\tan\theta = 8Ca$

For the bead to stay at point P,

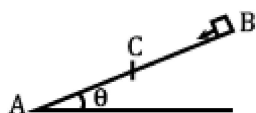$$m\omega^2 a \cos\theta = mg\sin\theta$$

$$\Rightarrow \omega = \sqrt{\frac{g\tan\theta}{a}}$$

$$\therefore \omega = \sqrt{\frac{g \times 8aC}{a}} = 2\sqrt{2gC}.$$

Hence, option (A) is correct.

14.

A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal, section BC is smooth, and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom $(\text{point } A)$ of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k \tan \theta$. The value of k is

[Mains-2020, Sep 3rd, Shift 1]

Accepted Answers

3

Solution:

If $AC = l$ then, according to question, $BC = 2l$ and $AB = 3l$.
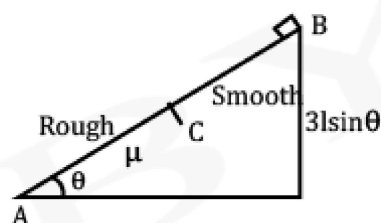
According to the Work - Energy Theorem,

$$W.D_{net} = \Delta K.E$$

Here, the block starts from rest and comes to rest at the end.

Therefore, work done by all forces is zero.

$$W_{mg} + W_{friction} = 0$$

$$\Rightarrow mg(3l) \sin \theta - \mu mg \cos \theta (l) = 0$$

$$\Rightarrow \mu mgl \cos \theta = 3mgl \sin \theta$$

$$\Rightarrow \mu = 3 \tan \theta = k \tan \theta$$

$$\therefore k = 3$$

Hence, 3 is the correct answer.

15. If the potential energy between two molecules is given by $U = -\dfrac{A}{r^6} + \dfrac{B}{r^{12}}$, then at equilibrium, separation between molecules and the potential energy respectively are:

   ⊗ **A.** $\left(\dfrac{B}{2A}\right)^{\frac{1}{6}}, -\left(\dfrac{A^2}{2B}\right)$

   ⊗ **B.** $\left(\dfrac{B}{A}\right)^{\frac{1}{6}}, 0$

   ✔ **C.** $\left(\dfrac{2B}{A}\right)^{\frac{1}{6}}, -\left(\dfrac{A^2}{4B}\right)$

   ⊗ **D.** $\left(\dfrac{2B}{A}\right)^{\frac{1}{6}}, \left(\dfrac{A^2}{2B}\right)$

Given : $U = -\dfrac{A}{r^6} + \dfrac{B}{r^{12}}$

For equilibrium,

$F = \dfrac{dU}{dr} = 0$

$\Rightarrow -(A(-6r^{-7})) + B(-12r^{-13}) = 0$

$\Rightarrow 0 = \dfrac{6A}{r^7} - \dfrac{12B}{r^{13}} \Rightarrow \dfrac{6A}{12B} = \dfrac{1}{r^6}$

$\therefore$ Separation between molecules, $r = \left(\dfrac{2B}{A}\right)^{1/6}$

Also, Potential energy,

U at $\left(r = \left(\dfrac{2B}{A}\right)^{1/6}\right) = -\dfrac{A}{2B/A} + \dfrac{B}{4B^2/A^2}$

$= \dfrac{-A^2}{2B} + \dfrac{A^2}{4B} = -\left(\dfrac{A^2}{4B}\right)$

16. A body of mass $2\ \text{kg}$ is driven by an engine delivering a constant power of $1\ \text{J/s}$. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m)

Accepted Answers

18   18.0   18.00

Solution:

Given, mass of the body, $m = 2\ \text{kg}$

Power delivered by engine, $P = 1\ \text{J/s}$

Time, $t = 9$ seconds

Power, $P = FV$

$\Rightarrow P = mav \quad [\because F = ma]$

$\Rightarrow m\dfrac{dv}{dt}v = P \quad \left(\because a = \dfrac{dv}{dt}\right)$

$\Rightarrow v\,dv = \dfrac{P}{m}dt$

Integrating both sides we get

$\Rightarrow \displaystyle\int_0^v vdv = \dfrac{P}{m}\int_0^t dt$

$\Rightarrow \dfrac{v^2}{2} = \dfrac{Pt}{m}$

$\Rightarrow v = \left(\dfrac{2Pt}{m}\right)^{1/2}$

$\Rightarrow \dfrac{dx}{dt} = \sqrt{\dfrac{2P}{m}}t^{1/2} \quad \left(\because v = \dfrac{dx}{dt}\right)$

$\Rightarrow \displaystyle\int_0^x dx = \sqrt{\dfrac{2P}{m}}\int_0^t t^{1/2}dt$

$\therefore \text{Distance, } x = \sqrt{\dfrac{2P}{m}}\dfrac{t^{3/2}}{3/2} = \sqrt{\dfrac{2P}{m}} \times \dfrac{2^{3/2}}{3}$

$\Rightarrow x = \sqrt{\dfrac{2 \times 1}{2}} \times \dfrac{2}{3} \times 9^{3/2} = \dfrac{2}{3} \times 27 = 18$

17. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired, and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved, is 30 m. What is the work done by the person, during the total movement of the box ?

(✗) **A.** 3280 J
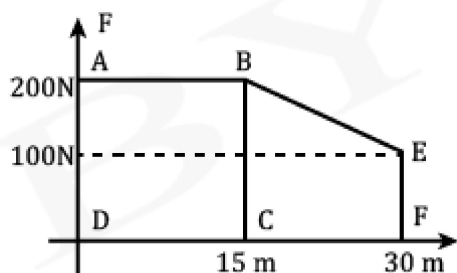
(✗) **B.** 2780 J

(✗) **C.** 5690 J

(✔) **D.** 5250 J

The given situation can be drawn graphically as shown in figure.

Work done $=$ Area under F-x graph

$=$ Area of rectangle ABCD $+$ Area of trapezium BCFE

$W = (200 \times 15) + \dfrac{1}{2}(100 + 200) \times 15$

$= 3000 + 2250 = 5250$ J

Hence, (D) is the correct answer.

18. A cricket ball of mass $0.15\ \mathrm{kg}$ is thrown vertically up by a bowling machine so that it rises to a maximum height of $20\ \mathrm{m}$ after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of $0.2\ \mathrm{m}$ while launching the ball, the value of F in N is (integer only)
$[g = 10\ \mathrm{ms}^{-2}]$
[Mains-2020, Sep 3rd, Shift 1]

Accepted Answers

150

Solution:

From work energy theorem,

$W = \Delta KE$

$F.x = \dfrac{1}{2}mv^2$

Here, $v^2 = 2gh$

$F \times \dfrac{2}{10} = \dfrac{1}{2} \times \dfrac{15}{100} \times 2 \times 10 \times 20$

$\therefore F = 150\ \mathrm{N}$

19. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?
[Mains-2019, Jan 9th, Shift 1]

(✗) **A.** 9 J

(✗) **B.** 12 J

(✗) **C.** 10 J

(✓) **D.** 15 J

Work done by the force is $W = \vec{F} \cdot \vec{d} = (3\vec{i} - 12\vec{j}) \cdot (4\vec{i}) = 12$ J
From work energy theorem.
$W_{net} = \Delta K.E. = K_f - K_i$
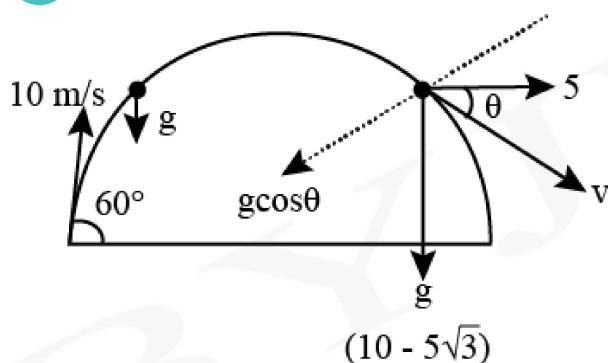$\Rightarrow 12 = K_f - 3$
$\therefore K_f = 15$ J.

20. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of $60°$ with the horizontal. The radius of curvature of its trajectory at $t = 1$ s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10 \text{ ms}^{-2}$, the value of R is:
(Take: $\tan 15° = 0.268; \cos 15° = 0.966$)

(×) A. 10.3 m

(✓) B. 2.8 m

(×) C. 2.5 m

(×) D. 5.1 m

At $t = 0$,
Horizontal component of velocity
$u_x = 10 \cos 60° = 5 \text{ ms}^{-1}$

Vertical component of velocity
$u_y = 10 \cos 30° = 5\sqrt{3} \text{ ms}^{-1}$

At $t = 1$ s,
Horizontal component of velocity
$v_x = 5 \text{ ms}^{-1}$

Vertical component of velocity
$v_y = u_y - gt = |(5\sqrt{3} - 10)| \text{ ms}^{-1}$

$\Rightarrow v_y = 10 - 5\sqrt{3} \text{ ms}^{-1}$

Centripetal acceleration, $a_c = \dfrac{v^2}{R}$

$\Rightarrow R = \dfrac{v_x^2 + v_y^2}{a_c}$

Here, $g \cos \theta$ points towards the centre and acts as the centripetal acceleration,

$\Rightarrow R = \dfrac{5^2 + (10 - 5\sqrt{3})^2}{g \cos \theta}$

$$\Rightarrow R = \frac{25 + (100 + 75 - 100\sqrt{3})}{10\cos\theta}$$

$$\Rightarrow R = \frac{100(2 - \sqrt{3})}{10\cos\theta}$$

At $t = 1 \rightarrow \dfrac{v_y}{v_x} = \tan\theta$,

$$\tan\theta = \frac{10 - 5\sqrt{3}}{5} = 2 - \sqrt{3} = 0.268$$

$$\Rightarrow \theta = 15°$$

$$\Rightarrow R = \frac{100(2 - \sqrt{3})}{10\cos 15} = \frac{100 \times 0.268}{10 \times 0.966} = 2.8 \text{ m}$$

Hence, option (B) is correct.

21. A body of mass $1\,\text{kg}$ falls freely from a height of $100\,\text{m}$, on a platform of mass $3\,\text{kg}$ which is mounted on a spring having spring constant $k = 1.25 \times 10^6\,\text{Nm}^{-1}$. The body sticks to the platform, and the spring's maximum compression is found to be x. Given that $g = 10\,\text{ms}^{-2}$, the value of x will be close to:

     (✕) **A.**   40 cm

     (✔) **B.**   4 cm

     (✕) **C.**   80 cm

     (✕) **D.**   8 cm

By the principle of conservation of energy, the initial gravitational potential energy will be equal to the spring potential energy at maximum compression.

$$\Rightarrow \frac{1}{2}kx^2 = mgh$$

$$\Rightarrow x^2 = \frac{2 \times 1 \times 10 \times 100}{1.25 \times 10^6}$$

$$\Rightarrow x^2 = 16 \times 10^{-4}$$

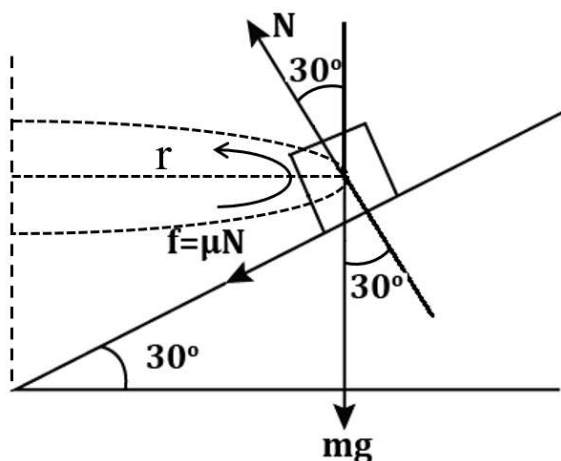$$\Rightarrow x = 4 \times 10^{-2}\,\text{m} = 4\,\text{cm}$$

Hence, option (B) is correct.

22. The normal reaction N for a vehicle of $800\ \mathrm{kg}$ mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _____ $\times 10^3\ \mathrm{kg\text{-}m/s^2}$.

Take $\cos 30^\circ = 0.87$ and $\mu = 0.2$

- ✔ **A.** 10.2
- ✘ **B.** 7.2
- ✘ **C.** 12.4
- ✘ **D.** 6.96

At maximum possible speed, friction will be limiting in nature.

∴ Balancing the forces in vertical direction,

$$N\cos 30^\circ - mg - \mu N \cos 60^\circ = 0$$

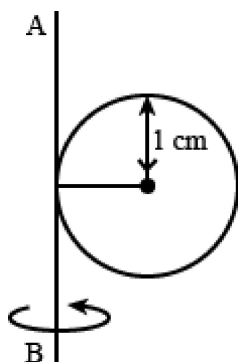$$\Rightarrow N(\cos 30^\circ - \mu \cos 60^\circ) = mg$$

$$\Rightarrow N = \frac{mg}{\cos 30^\circ - \mu \cos 60^\circ}$$

$$\Rightarrow N = \frac{800 \times 9.8}{(0.87 - 0.2 \times 0.5)} \approx 10.2 \times 10^3\ \mathrm{kg\text{-}m/s^2}$$

Hence, option (A) is the correct answer.

23. A metal coin of mass $5\ \mathrm{g}$ and radius $1\ \mathrm{cm}$ is fixed to a thin stick AB of negligible mass as shown in the figure The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in $5\ \mathrm{s}$, is close to :

$\times$ **A.** $4.0 \times 10^{-6}\ \mathrm{Nm}$

$\times$ **B.** $1.6 \times 10^{-5}\ \mathrm{Nm}$

$\times$ **C.** $7.9 \times 10^{-6}\ \mathrm{Nm}$

$\checkmark$ **D.** $2.0 \times 10^{-5}\ \mathrm{Nm}$

Angular accceletration,

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$= \frac{25 \times 2\pi - 0}{5}$$

$$= 10\,\pi\ \mathrm{rad/s}^2$$

$$\tau = I\alpha$$

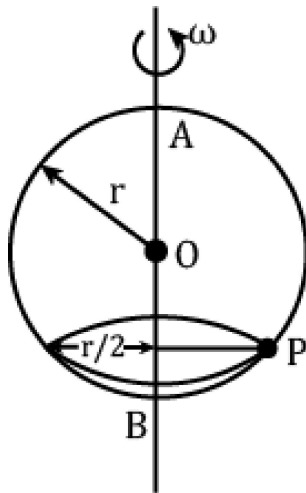$$\Rightarrow \tau = \left(\frac{5}{4}mR^2\right)\alpha$$

$$\approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$$

$$\approx 2.0 \times 10^{-5}\ \mathrm{Nm}$$
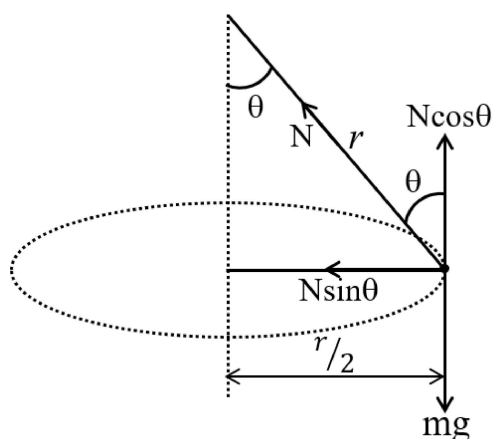
Hence, option (D) is correct.

24. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring, at position P, as shown. Then the value of ω^2 is equal to :

   ✗ **A.** $\dfrac{\sqrt{3g}}{2r}$

   ✓ **B.** $\dfrac{2g}{(r\sqrt{3})}$

   ✗ **C.** $\dfrac{(g\sqrt{3})}{r}$

   ✗ **D.** $\dfrac{2g}{r}$

Now, $\sin\theta = \dfrac{r/2}{r} = \dfrac{1}{2} \Rightarrow \theta = 30°$

From the free body diagram, we can write,

$N\sin\theta = m\omega^2\left(\dfrac{r}{2}\right)$

And, $N\cos\theta = mg$

$\Rightarrow \tan\theta = \dfrac{\omega^2 r}{2g}$

$\tan 30° = \dfrac{\omega^2 r}{2g}$

$\dfrac{1}{\sqrt{3}} = \dfrac{\omega^2 r}{2g}$

$\therefore \omega^2 = \dfrac{2g}{r\sqrt{3}}$

Hence, (B) is the correct answer.

25. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then

   ✔ **A.** $T \propto R^{(n+1)/2}$

   ✘ **B.** $T \propto R^{n/2}$

   ✘ **C.** $T \propto R^{3/2}$ for any n

   ✘ **D.** $T \propto R^{\frac{n}{2}+1}$

Given that,
$$F \propto \frac{1}{R^n} \Rightarrow F = \frac{k}{R^n}$$

And we know that,
$$F = m\omega^2 R$$

$$\therefore \frac{k}{R^n} = m\omega^2 R$$

$$\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}}$$

So, time period will be,

$$\Rightarrow T = \frac{2\pi}{\omega}$$

So, $T \propto R^{(n+1)/2}$

Hence, option (A) is correct.

26. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is:
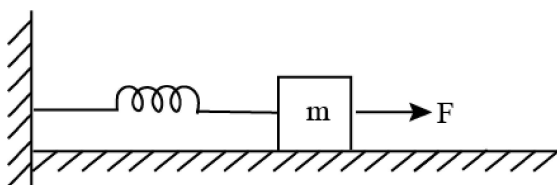
[Mains-2019, Jan 9th, Shift 1]

⊗ A. $\dfrac{2F}{\sqrt{mk}}$

⊗ B. $\dfrac{F}{\pi\sqrt{mk}}$

⊗ C. $\dfrac{\pi F}{\sqrt{mk}}$

✔ D. $\dfrac{F}{\sqrt{mk}}$

Maximum speed is at mean position or equilibrium .

At extreme position
$$F = kx \Rightarrow x = \frac{F}{k}$$

From work-energy theorem,
$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{k} = \frac{1}{2}mv^2$$

$$\therefore v_{\max} = \frac{F}{\sqrt{mk}}$$

Hence, option (D) is correct.

27. A force acts on a $2 \, \text{kg}$ object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?
[Mains-2019, Jan 9th, Shift 2]

 ⓧ **A.** 850 J

 ⓧ **B.** 950 J

 ⓧ **C.** 875 J

 ✓ **D.** 900 J

Position, $x = 3t^2 + 5$

$\therefore$ Velocity, $v = \dfrac{dx}{dt}$

$\Rightarrow v = \dfrac{d(3t^2 + 5)}{dt}$

$\Rightarrow v = 6t + 0$

At $t = 0$ sec $v_i = 0 \, \text{m/s}$
At $t = 5$ sec $v_f = 30 \, \text{m/s}$

According to work-energy theorem, $W = \Delta KE$.

$W = \dfrac{1}{2}mv_f^2 - \dfrac{1}{2}mv_i^2$

$W = \dfrac{1}{2}(2)(30)^2 - 0 = 900 \, \text{J}$

28. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by-

    ☒ **A.** $\dfrac{v^2}{g}$

    ☒ **B.** $\dfrac{2v^2}{7g}$

    ☑ **C.** $\dfrac{2v^2}{5g}$

    ☒ **D.** $\dfrac{v^2}{2g}$

    Using conservation of linear momentum,

    $mv = (m + M)v'$

    $v' = \dfrac{mv}{m + M} = \dfrac{mv}{m + 4m} = \dfrac{v}{5}$

    Using conservation of mechanical energy, we have

    $\dfrac{1}{2}mv^2 = \dfrac{1}{2}(m + 4m)\left(\dfrac{v}{5}\right)^2 + mgh$

    $\Rightarrow \quad h = \dfrac{2\,v^2}{5\,g}$

    Hence, (C) is the correct answer.

29. A uniform cable of mass $'M'$ and length $'L'$ is placed on a horizontal surface such that its $\left(\dfrac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be
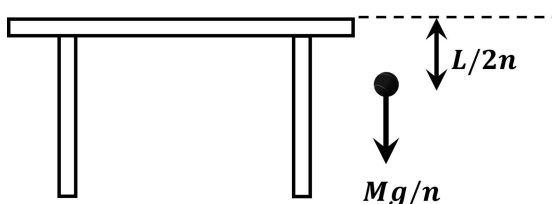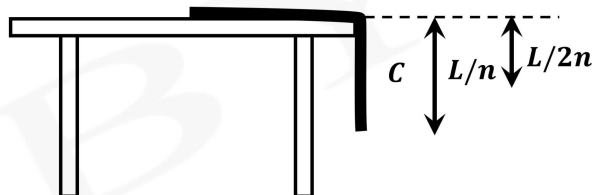
   ✔   **A.** $\dfrac{MgL}{2n^2}$

   ✘   **B.** $\dfrac{MgL}{n^2}$

   ✘   **C.** $\dfrac{2MgL}{n^2}$

   ✘   **D.** $nMgL$

Length of hanging part $= \dfrac{L}{n}$

Mass of hanging part $= \dfrac{M}{n}$

Weight of hanging part $= \dfrac{Mg}{n}$

Let $'C'$ be the centre of mass of the hanging part.

The hanging part can be assued to be a particle of weight $\dfrac{Mg}{n}$ at a distance $\dfrac{L}{2n}$ below the table top.

The work done in lifting it to the table top is equal to increase in its potential energy.

$\therefore \ W = \left(\dfrac{Mg}{n}\right)\left(\dfrac{L}{2n}\right)$
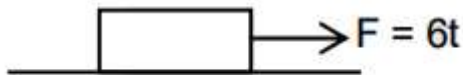
$\therefore \ W = \dfrac{MgL}{2n^2}$

Hence option (A) is correct.

30. A time dependent force $F = 6t$ acts on a particle of mass $1\ kg$. If the particle starts from rest, the work done by the force during the first 1 sec will be

   ✗ **A.** $18\ J$

   ✗ **B.** $22\ J$

   ✓ **C.** $4.5\ J$

   ✗ **D.** $9\ J$

$a = \dfrac{F}{m} = \dfrac{6t}{1} = 6t$

$\dfrac{dv}{dt} = 6t$

$\int_0^v dv = 6 \int_0^1 t\,dt$

$\Rightarrow v = 6\left[\dfrac{t^2}{2}\right]_0^1 = 3$

$W = \Delta K.E = K_f - K_i$

$\quad = \dfrac{1}{2}mv^2$

$\quad = \dfrac{1}{2} \times 1 \times 3^2$

$\quad = 4.5\ J$